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John Reynolds of the Mint: A mathematician in the service of king and commonwealth

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Abstract

In his youth, John Reynolds showed a talent for arithmetic and was destined for a career as a mathematician at the Tower Mint in London. He became skilled in the algorithms needed to determine the correct relationship between the weight and purity of coins and their values. This was a matter of national importance, and his work came to the attention of King James I, who reigned from 1603 to 1625, and his chief ministers, including Robert Cecil and Francis Bacon. It seemed that John might attain high office himself, but the murky administration of the early Stuart period cast its shadow over his career. Nevertheless, for the next forty years he continued to play a major part in the nation's affairs. He produced books of tables for the valuation of coins in the commercial world, and for the highly technical work of the assayers. Also, he was actively involved in the production of standard measures and instruments used by the excise officers. His life and works illustrate how mathematical ideas were employed by the English government in the period of the early Stuart kings and the Commonwealth.

Zusammenfassung

In seiner Jugend zeigte John Reynolds Talent für Arithmetik und war bestimmt für eine Karriere als Mathematiker an der Münze im Tower of London. Er wurde ein Experte für diejenigen Algorithmen, die nötig sind, um das richtige Verhältnis zwischen Gewicht und Reinheit der Münzen und ihrer Werte zu bestimmen. Das war eine Sache von nationaler Bedeutung, und King James I., der von 1603 bis 1625 regierte, sowie seine führenden Minister, darunter Robert Cecil und Francis Bacon, wurden auf Reynolds' Wirken aufmerksam. Es bestand die Chance, dass John selbst in eine höhere Position berufen werden könnte, aber die undurchsichtige Administration der frühen Stuartperiode warf Schatten auf seine Karriere. Dennoch spielte er eine führende Rolle in den Geschicken der Nation für die nächsten vierzig Jahre. Er veröffentlichte Bücher mit Tafeln für die Berechnung von Münzwerten in der Handelswelt und für die stark technische Arbeit der Metallprüfer. Er war auch aktiv in die Produktion von Standardmaßen und Instrumenten für die Steuerbehörden involviert. Sein Leben und Werk illustrieren, wie mathematische Ideen von der englischen Regierung in der Zeit der ersten Stuarts und des frühen Commonwealth verwendet wurden.

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1. John Reynolds

On 12 October 1599 a young man named John Reynolds signed the Apprentice Book of the Goldsmiths Company of London.[1] He was probably about 14 years old at the time, the usual age for starting an apprenticeship, but we cannot be sure of his exact age because no record of his birth has been found. The entry in the Apprentice Book tells us that he was the 'sonne of William Reynolds of London Citizen and Fletcher', and that his master was to be Richard Rogers. The latter was an important man, a member of a family that had long been associated with the Tower Mint in London, and recently appointed Comptroller there. He would also become Prime Warden of the Goldsmiths Company in 1601 (Forbes, 1999, 67–70). Rogers would have known that his new apprentice was a bright young man, but he could not have foreseen the extent to which John Reynolds would figure in the affairs of the Goldsmiths, the Mint, and the nation, for the next 60 years. At that time the Mint played a more central part in financial and economic matters than is now the case, because it produced coins made of precious metal, which had an intrinsic value.

Why did this young man receive such an opportunity? Almost certainly the main reason was that he had displayed a talent for arithmetic, a skill much needed at the Mint, as we shall see. But at the end of the sixteenth century apprenticeships were not filled by open competition, and John Reynolds' success must have been partly due to some family connections with the Mint in the Tower.

In fact, John Reynolds' father William, the Citizen and Fletcher named in the Apprentice Book, was not simply a maker of arrows: he was Her Majesty's Fletcher, responsible for ensuring that the nation was supplied with the equipment needed for its defence. On 1 June 1587, when England was busily preparing for the onslaught of the Spanish armada, it was recorded in the State Papers that he had made an official visit to Portsmouth, and had checked the store of arrows kept there.[2] He had been accompanied on his visit by William Hopkins, the Queen's Master Smith in the Tower, who had checked the calivers and harquebuses (early firearms). Hopkins had several official duties in the Tower (Challis, 1978, 35; Challis, 1992a, 256), and both Reynolds and Hopkins were powerful agents of the government. As was the custom, they were also in business on their own account. Furthermore, they were personal friends, and they had many other friends and acquaintances among the officials and entrepreneurs who were based in the Tower of London. That was the background to the selection of John Reynolds for his apprenticeship with Richard Rogers.

More details can be gleaned from the will[3] of William Reynolds, written in September 1597, shortly before his death. Indeed, this document contains most of what little we know about the Reynolds family. We read that William lived in one of the houses on old London Bridge, and that his wife had died before him and had been buried in the church of St Magnus the Martyr which stood at the northern end of the bridge. William wished to be buried there too. He had three children, Mary, Jeremy, and John, all of whom were minors at the time of his death. The person named in the will as their guardian was Thomas Hopkins, and it turns out that he was a son of William Hopkins, the aforementioned Master Smith.[4]

William Reynolds' will confirms that he was a prosperous man. He owned 'tenements and houses' in Blackman Street in the parish of Newington, Surrey, about a kilometre due south of London Bridge. This property was bequeathed to Jeremy, who was presumably the elder son. John got all his father's goods and chattels, and a house in Bourne, Lincolnshire, although it was stipulated that the current occupier could continue to live there during his lifetime. William also made small bequests to his brothers Harry and Richard, and his sister Ann, but the will gives no details about where they lived or worked.

The will does throw more light on William's business activities. He had had 'greate dealings' with John Hayborne, 'her Majesties pike maker', and there is much detail about how their accounts should be settled.

[1] Goldsmiths Hall (London): Apprentice Book 1, f.127v; Court Minutes O.1, 61.

[2] Calendar of State Papers Domestic 1581–1590, 414.

[3] The National Archives (Kew): PROB 11/90/288.

[4] Will of Mary Hopkins, William's widow and mother of Thomas. The National Archives (Kew): PROB 11/91/202.

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Further information about his association with the Tower is provided by an annex to the will, known as a 'sentence'.[5] Here we discover that William had been acquainted with William Painter, who was 'Clerke of the great Ordnance in the Tower', as well as being a notable playwright. Reynolds had been present when Painter uttered his dying wishes in 1594,[6] and the sentence deals with some issues about a house formerly owned by Reynolds and now occupied by Painter's widow.

Now we have a rough sketch of the young man looking forward to his apprenticeship at the Tower in 1599. Although an orphan, John Reynolds was already the owner of property and well set up financially; furthermore, he could read, write, and (presumably) do arithmetic rather well. So why should he be of interest to historians of mathematics?

In this paper it will be argued that Reynolds was a practitioner of mathematics of a rather unusual kind. He does appear in Eva Taylor's classic work on the mathematical practitioners of Tudor and Stuart England, but only in a minor role—and with some misleading information (Taylor, 1967, 89, 194). As we shall see, he spent much of his time doing difficult arithmetical calculations, and this work was influential at the highest levels of national policy. He advised the leading ministers of the time. He compiled books of tables that helped the mercantile classes to manage their financial affairs. He carried out scientific work that led to more reliable methods of collecting the revenue. And, as might be expected, the twists and turns of his career throw light on the practice of mathematics in a turbulent period of English history.

2. An unusual apprenticeship

John Reynolds' apprenticeship lasted from 1599 to 1606. In a document[7] written in 1609 it is stated that he had been 'trained ten years in the service of the Mint', so we must conclude that his apprenticeship was spent working at the Mint in the Tower of London, rather than learning the trade of a practising goldsmith. Indeed, it is clear that his apprenticeship was regarded as training for a career in the service of the Mint, and that his talent for arithmetic was a major factor.

The Tower had originally been built in the eleventh century, to assert the military might of the Norman conquerors, but it had gradually become a large and complex set of buildings, fulfilling many functions. For example, it was often used to imprison famous people, such as Sir Walter Raleigh (or 'Ralegh'), who was kept there from 1603 until his execution in 1618. A Mint had been in operation in the Tower since the second half of the thirteenth century and, by the time of John Reynolds' apprenticeship, it was by far the most important mint in England (Challis, 1992a, 1992b).

The work at the Tower included the assaying and refining of metals, processes that were an important part of the making of gold and silver coins, and the new apprentice would have had to learn about the complicated calculations that were involved. At that time the value of a coin depended on the amount of precious metal that it contained, and great care had to be taken to ensure that this was correct. There were two factors, the weight of the coin and its fineness (purity). For silver coins, 37 units of pure silver were alloyed with 3 units of base metal, to make 40 units of alloy. This was the *sterling* standard, which was (and still is) the legal standard of all silver goods in England: we now express it decimally as .925. So, when hallmarked silver in the form of old plate was converted into coins, there was a good chance that it was of the correct standard. But much of the silver bullion sent to the mint came from abroad, in the form of all kinds of foreign coin, and its fineness varied. For this reason, Reynolds would soon have become familiar with a 'ready reckoner', listing the price that the Mint would pay for silver bullion. An extract from such a document, dating from 1577, is shown in Figure 1 (Challis, 1992a, 257; Williams, 1995, 253).

[5] The National Archives (Kew): PROB 11/92/360.

[6] The National Archives (Kew): PROB 11/87/51.

[7] Calendar of State Papers Domestic 1580–1625 Addenda, 520.

		The ounce is worth					The pound wt is worth					
oz	dwt	S	D	qrs	m		L	S	D	qrs	m	
10	0	4	4	2	5	0	2	12	8	2	0	0
10	1	4	4	3	5	¼	2	12	11	2	3	¾
10	2	4	5	0	5	½	2	13	11	2	3	¾
...												
...												
11	2	4	10	2	0	0	2	18	6	0	0	0
...												

Figure 1. Transcript of part of a ready reckoner for the purchase of silver, 1577.

A glance at the table is enough to confirm that it was intended to be used by people who not only had expertise in arithmetic, but also familiarity with systems of measurement that are no longer in common use today. Complicated arithmetical operations were to play a significant part in John Reynolds' working life for over sixty years, and they will be considered in detail in the rest of this paper. From the start, he would have needed instruction in arithmetic to back up the expertise available at the Mint, and his position merited the best tuition available. In 1597, a man who was later to become tutor to the king's son, Prince Henry, recommended a certain John Goodwyn as a teacher for those 'wishing instruction in mathematical practice' (Barlow, 1597). The fact that Goodwyn did indeed teach Reynolds was confirmed in 1650, when it was written that 'Mr Goodwyn was Master in the Mathematicks to Mr Reynolds, as himself hath told me' (Wybard, 1650, 265).

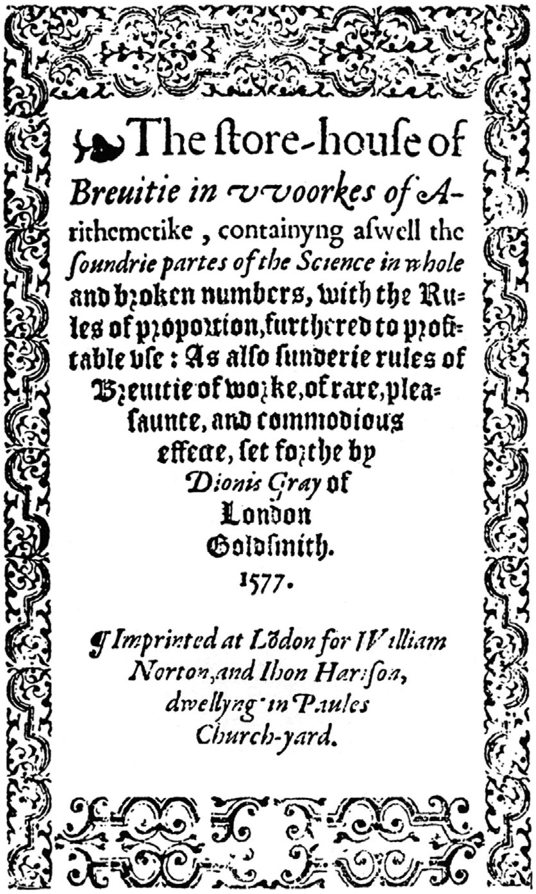
With Goodwyn, Reynolds would have studied some of the printed textbooks of arithmetic that were available at the end of the sixteenth century. In English, the classic text was Robert Recorde's *Ground of Artes*, first published in 1543. Recorde wrote several popular books on mathematics: he was also a surveyor of the mines and, for a short time, the comptroller of a small mint at Durham House in London. The *Grounde of Artes* described two methods of calculation: the first, using counters laid out on a cloth or board, went back to ancient times, and was mainly used for simple calculations in trade. But in the sixteenth century it was gradually being supplanted by another, more efficient, method, known as 'pen-reckoning'. This is the method still in use today, although it has been transformed and partly replaced by electronic devices. In pen-reckoning, numbers are represented by strings of digits in the Hindu-Arabic notation 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, and the working is done by manipulating these symbols on paper. Although this notation had been introduced into Europe as far back as the late tenth century, the old Roman numerals, I, V, X, L, C, D, M were still in use at this time, especially in official records. The great advantage of the new system was that the basic arithmetical operations, particularly multiplication and division, could be done efficiently by algorithms—that is, step-by-step procedures guaranteed to produce the right answer, provided the symbols were manipulated correctly.

By the end of the sixteenth century a few other textbooks of arithmetic were available in English. One of them (Gray, 1577) was written by a London goldsmith, and it had several features that would have made it particularly appropriate for study by an apprentice working at the Mint (Figure 2).

3. Algorithms: pure and applied

John Reynolds had to master two aspects of arithmetic, which can be roughly categorized as 'pure' and 'applied'. The pure aspect—the algorithms for the basic arithmetical operations—had to be applied to the complicated systems of units used for money, weights and measures.

By far the most difficult operation was division. In modern terms, a fundamental theorem about natural numbers states that, given a *divisor* x and a *dividend* y, then there is a *quotient* q and a *remainder* r such that $y = qx + r$, where r is a number between 0 and $q - 1$. In the 16th century, several methods for finding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Breuitie of worke, of rare, plea-
ſaunte, and commodious
effecte, ſet forthe by
Dionis Gray of
London
Goldſmith.
1577.

¶ *Imprinted at Lōdon for William*
Norton, and Ihon Hariſon,
dwellyng in Paules
Church-yard.

Figure 2. Title page of Dionis Gray's textbook of arithmetic.

q and r, given x and y, were known. The most popular was the 'scratch' method. However, the steps required mental agility, and only people who had an innate familiarity with numbers could manage them successfully. In his 1577 book Dionis Gray had set out the calculation for the case $x = 24$, $y = 56847$ as shown on the left in Figure 3. As can be seen, the method involved scratching out the numbers as they were dealt with, and this gave the method its name. Eventually, all that remains are the answers: $q = 2368$ and $r = 15$.

The contorted nature of the scratch method can be appreciated by comparison with the method of 'long division', which gradually displaced it, and which is still used today. In long division (Figure 3, right) the first step is the calculation $56 = 2 \times 24 + 8$. In the scratch method, as described by Cajori (1896, 148), this would have been done in two stages. First $5 = 2 \times 2 + 1$, so 5 is scratched and replaced by 1, the quotient 2 is inserted on the right, and the first digit (2) of the divisor is scratched. The first two digits of the dividend are now 16; the digit 4 in the divisor remains, and since $16 = 2 \times 4 + 8$, 16 is scratched and replaced by 8. The process continues, digit by digit, requiring great care and concentration. Although it now seems clear that long division is the better method, it was slow to take hold in England. In Nicholas Hunt's *Hand-maid to Arithmetic* (Hunt, 1633), the scratch method was explained at length, and was followed by a section on 'Italian Division'. That name covered a variety of alternative methods, some of which resemble long division, but the modern method did not take over completely in England until the eighteenth century.

The division algorithm was an essential tool for solving problems about proportion. For centuries, this part of arithmetic was based on the 'Rule of Three', also known as the 'Golden Rule'. In modern terms, if

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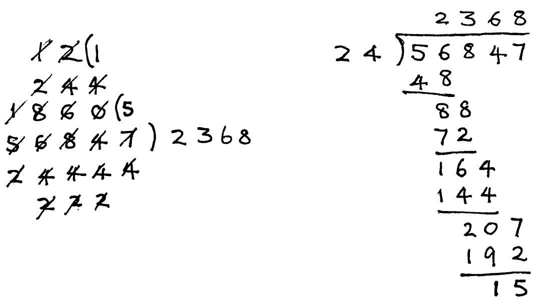

Figure 3. Dividing 56847 by 24, by the scratch method, and by long division.

four numbers, a, b, c, d, are such that (a, b) and (c, d) are both in the same direct relationship, so that the ratios a/b and c/d are equal, then any one of them can be calculated from the other three by multiplication and division. In particular, $d = bc/a$. There were many variations of this rule: Dionis Gray (1577, 63–77) described no less than seven of them, each with its own name, such as 'the Backer Rule'. That was the version in which the numbers (a, b) and (c, d) are in inverse relationship, so that ab and cd are equal, and $d = ab/c$. When these rules were taught by rote, it is easy to see why they were unpopular. If there was any attempt at justification, it would be done by means of verbal description and examples—the modern form of algebraic symbolism, which helps us understand what is going on, was not used in 1600. However, John Reynolds was clearly able to deal with complicated numerical problems, and the Golden Rule and the Backer Rule were to appear frequently in his later writings (Figure 9).

We cannot be sure that Reynolds used Gray's book, but there were few alternatives in English (De Morgan, 1847). Furthermore, Gray also dealt with the systems of money, weights, and measures that were then in use, and these were particularly important in calculations where extreme precision was needed, such as the valuation of precious metals. The traditional English system of accounting for money, as in Figure 1, was based on pounds (denoted by a form of the letter L), shillings (S), and pence (D), where

$$\text{one pound} = 20 \text{ shillings}, \qquad \text{one shilling} = 12 \text{ pence}.$$

There was also a smaller unit, a farthing, equal to a quarter of penny, which was used for buying and selling everyday items, such as bread. But the goldsmiths, who dealt in precious metals, required an even smaller unit, known as a 'mite'. According to Gray (1577, 5) this was equal to one sixth of a farthing, so 24 mites made one penny. There was no coin corresponding in value to a mite, it was purely a money-of-account. This system of money-units was awkward to use in practice, because the ratios between the units were irregular: 6 mites made a farthing, 4 farthings made a penny, 12 pence made a shilling, 20 shillings made a pound. Of course, there were historical reasons for the ratios, but the irregularity of the system added considerably to the complexity of arithmetic in practice.

John Reynolds would have had to learn how to apply the techniques described by Gray and others to the range of practical problems that faced the officers of the mint in their daily grind. He would have been helped by manuscripts that described these problems specifically, such as the one written by the master-worker Sir Richard Martin and presented to the new king James soon after his accession (Challis, 1992a, 258–259 and Figure 20). Another manuscript written around the same time was entitled *The Gouldesmithes Storehouse*. Several versions of this have survived, at least one of them containing a copy of the ready reckoner referred to in Figure 1 (Jenstad, 1998).

That document listed the values in pounds, shillings, pence, quarters (farthings), mites, and even fractions of a mite. But, in order to use it, John had also to understand the system for stating the fineness of silver alloy. This was based on the proportion of pure silver in the alloy, by weight, and the weight-system was as complicated as the money-system. The system used for coins and precious metals was known as 'troy'

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weight. The troy pound was about 373 g in modern terms, and the troy ounce and the pennyweight, were defined by the rules

$$\text{one pound} = 12 \text{ ounces}, \qquad 1 \text{ ounce} = 20 \text{ pennyweights}.$$

This 'troy pound of weight' and the 'pound of money' seem to have had a common origin, and the 'pennyweight' had originally signified the weight of coin worth a penny, but that was no longer true in the sixteenth century. The larger weight-units, like the money-units, were further subdivided when extreme accuracy was essential, as we shall see.

The fineness of alloyed silver was expressed in terms of the number of ounces and pennyweights of pure silver in a pound of mixed metal. For example, the second line of the document shown in Figure 1 relates to silver of fineness 10 ounces and 1 pennyweight, which is 201 parts of silver in 240. The basis for the table was the fixed price that the Mint paid for a troy pound of silver of the *sterling* standard (222 parts in 240): at that time this was 2 pounds 18 shillings and 6 pence (= 702 pence). The second and third columns of the table give the corresponding price for silver at all levels of fineness, per ounce and per pound, calculated using the Golden Rule. For silver of fineness 10 ounces and 1 pennyweight the rule says that if 222 ($= a$) parts are worth 702 ($= b$) pence, then 201 ($= c$) parts are worth

$$d = (bc)/a = (702 \times 201)/222 \text{ pence}.$$

After removing common factors, multiplying, and dividing, the answer is 635 pence and 22/37 parts of penny. For practical purposes this had to be expressed in the usual money-units, so the table gives the price of one troy pound of silver with fineness 10 ounces and 1 pennyweight as

2 pounds 12 shillings 11 pence 2 farthings 3 3/4 mites.

The arithmetic for the last part of the calculation is quite tedious, and we shall return to it later.

This extreme precision was needed because, when bullion was purchased in very large amounts, a small variation in the price per ounce could make a big difference. The table would have been used by the people who received the coin and bullion brought for coining. They would weigh a consignment of silver, and have it assayed to establish its fineness. Then they could work out the total value of the consignment, by multiplication, using the figures given in the table. John Reynolds would have known how to do these calculations, but he would also have had to learn how to compile such a table, because the basic fixed price for a pound of standard silver might change. To put this achievement in context, we should recall that, at this time, most people had no knowledge of arithmetic at all. A small minority could calculate with counters and express the results in roman numerals. The receivers of bullion were among the very few who were familiar with Arabic numerals, and could perform the basic operations with them. Compiling the bullion tables (as opposed to using them) required the highest levels of expertise, specifically a mastery of the division algorithm and the application of the Golden Rule in various forms.

In the first quarter of the seventeenth century the practice of arithmetic would change significantly. The widespread adoption of decimal fractions meant that, in principle, calculations with precise quantities, like the 2 pounds 12 shillings 11 pence 2 farthings 3 3/4 mites mentioned above, could be done with the number of pence in decimal form, 635.594594. In that way, the same arithmetical procedures as for whole numbers could be used. Decimal fractions do indeed appear in John Reynolds' later work (Section 11), and there is also evidence that he was familiar with the other great innovation of the period, logarithms (Section 12).

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4. Trouble with the coins—and the king

The primary function of the Mint was to make coins of gold and silver. In the sixteenth century Henry VIII had experimented with debasement of the currency, but Elizabeth had successfully restored its integrity. The result was a bewildering array of coins, particularly in gold. There were two standards of fineness for gold, and, just to add to confusion, the terminology was different from that used for silver.

For gold, the fineness was expressed as so many parts in 24, where a part was called a carat. The carat was not a unit of weight, merely a number; so, for example, gold of 12 carats was 50 per cent pure gold, and gold of 18 carats was 75 per cent pure gold. The carat was subdivided into sub-units, and here too confusion reigned. The sub-unit was called the grain, the same name as the weight-unit equal to one twenty-fourth of a pennyweight, but this carat-grain was simply a number, an abbreviation for one-quarter of a carat. The carat-grain could itself be divided into fractions, but no special name was given to them.

Since the fourteenth century the traditional standard for gold coins in England had been effectively pure gold, but it was difficult to refine gold so that all impurities were removed, and so the nominal standard was 23 carats $3^1/_2$ grains, equal to a fineness of 191/192. A lower standard of 22 carats, or 11/12, had been introduced by Henry VIII. It was known as crown gold. Both these standards had continued under Elizabeth, because the old standard was considered to be important for the prestige of the nation, while coins of the crown gold standard were found to be more durable than the traditional ones.

The accession of James as king of England in 1603 created more problems, because he was also king of Scotland, where everything was done differently. The Scottish coins had different names, different weights and different values from the English ones. Commendably, James set out to reconcile the coinages of Scotland and England, and to do so he proposed to introduce a new system, based on a new gold coin, to be known as a *unite*. It was thought that the integrity of the coinage played an important part in the health of the national economy, and so there were many difficulties: the divergence of the English and Scottish systems was only one of them. Eventually it was agreed that the unite should be made of 22 carat gold, and valued at one English pound, or twenty shillings. On 11 November 1604 the Mint received the indenture instructing it to make gold coins at the rate of £37 4s from a troy pound of crown gold, and silver coins at the rate of £3 2s from a troy pound of sterling silver. These numbers had been chosen carefully, after much deliberation, and arguments about them would continue to play a large part in the affairs of the Mint for many years. (See Section 7.)

The most noteworthy feature of the new coinage, from the viewpoint of the ordinary citizen, was that the new gold and silver coins were to be lighter than the ones previously in circulation. Weighing gold coins was already well-established in England, as a first line of defence against the work of the forgers and clippers. It was to become much more common, as shown by the fact that large numbers of coin-weights specially intended for this purpose have survived. The standard work on this subject (Withers, 1993) lists many types, particularly from the period 1610–1640. The frequent alterations of coins and values was, as we shall see, just one more reason for checking coins in the daily business of buying and selling.

It was possible to infer the correct weight of a new gold unite from the figures given in the Mint indenture, but the exact weight of each coin was not specified: only the average weight was fixed, and that indirectly. Furthermore, even if the indenture had been freely available (which it was not), few people would have been able to cope with the arithmetic involved. For this reason, the publication of a royal proclamation on 16 November 1604 was welcome, because it listed the correct weights of all the new coins (Figure 4).[8]

We do not know who was responsible for drawing up the table, but it must have been someone highly skilled in arithmetic. There is a draft copy[9] of the proclamation with corrections in the hand of Robert Cecil, the king's chief minister, and it is possible that he did the calculations himself, or at least checked

[8] Larkin and Hughes (1973, 97–103, item 47).

[9] The National Archives (Kew): SP 14/10/20.

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		Peny weights 20.	Graines 24.	Mites 20.	Droits 24.	Periots 20.	Blanks 24.	
Pieces of Gold of	xx.s.	06	10	16	18	10		Of these 37.li.4.s. make a pound weight Troy.
	x.s.	03	05	08	09	05		
	v.s.	01	14	14	04	12	12	
	iiii.s.	01	06	19	08	10		
	ii.s.vi.d.	00	19	07	02	06	06	
Pieces of Silver of	v.s.	19	08	10	08			Of these 3.li.2.s. make a pound weight Troy.
	ii.s.vi.d.	09	16	05	04			
	xii.d.	03	20	18	01	10		
	vi.d.	01	22	09	00	15		
	ii.d.	00	15	09	16	05		
	i.d.	00	07	14	20	02	12	
	ob.	00	03	17	10	01	06	

Figure 4. Table of weights from the 1604 proclamation.

some of them. The draft must have been approved by the officers of the Mint, and it would have been an obvious exercise for their mathematical apprentice, John Reynolds, as well as a valuable introduction to the workings of high government.

It is worth looking more closely at how the calculation could have been done in 1604, when the result was expressed with extreme precision, but without the use of decimal fractions. At that time, the decimal notation would have been understood by few readers of the royal proclamation. Consequently, the results were stated using an elaborate system of subdivisions of the troy weight-system. The calculations began in a manner similar to that used to produce a Ready Reckoner, as described in Section 3.

The first step is to apply the Golden Rule. Given that 37 pounds and 4 shillings ($a = 744$ shillings) worth of gold coin is to be made from one troy pound ($b = 240$ pennyweights) of standard gold, the unite worth $c = 20$ shillings must require

$$d = (bc)/a = (240 \times 20)/744$$

pennyweights of gold. After simplification and multiplication, the number reduces to 200/31. The division algorithm gives the quotient 6 and the remainder 14, so the answer is 6 and $14/31$ pennyweights. But that is only the first part of the calculation.

The next step is to convert the fraction $14/31$ of a pennyweight into grains. Since 24 grains make a pennyweight, this is $336/31$ grains, and division yields the quotient 10, with remainder 26. Hence the weight of a gold unite should be 6 pennyweights 10 and $26/31$ grains. A figure of 6 pennyweights and 10 (or 11) grains was good enough for most commercial purposes, because the accuracy of the scales and weights used in the routine business of trade was limited. But, for greater accuracy, the calculation can be continued in the same way, using an artificial system of sub-units. The system used in the proclamation involved mites, droits, periots and blanks, defined by the rules

$$1 \text{ grain} = 20 \text{ mites}, \qquad 1 \text{ mite} = 24 \text{ droits}, \qquad 1 \text{ droit} = 20 \text{ periots}, \qquad 1 \text{ periot} = 24 \text{ blanks}.$$

Even the best balances available at that time could not detect the weight of a droit, so the smaller units were indeed artificial. The final result stated in the proclamation,

$$6 \text{ pennyweights } 10 \text{ grains } 16 \text{ mites } 18 \text{ droits } 10 \text{ periots},$$

seems to be wrong by one periot; but a periot is only about 7 micrograms, so that is hardly serious.

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5. An officer of the Mint

John Reynolds was made free of his apprenticeship on 28 November 1606.[10] It is likely that he continued to work at the Mint, and in October 1607 he was officially appointed to the position for which his apprenticeship had prepared him. Mint appointments had to be ratified at the highest level, and so it was recorded in the State Papers that he would serve as Assistant to the Assay Master, and receive a salary of £40 per annum.[11] The Assay Master's position was held jointly by two men, Walter Williams and Andrew Palmer, both of whom came from families with long histories at the Mint. Palmer had been appointed only recently (1605), and he would have been John's immediate superior. John would have known him well, and of course he had another friend and mentor in his apprentice-master Richard Rogers, who was to remain as comptroller until 1636.

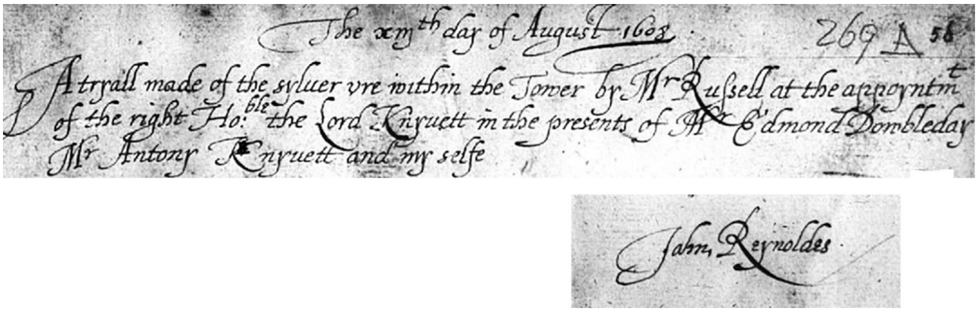
Despite his training, John would surely have found his official position more demanding than his role as an apprentice. His apprenticeship would have prepared him for the fact that the Mint's organisational structure was byzantine in its complexity, but now he had to be part of that structure. It is unhelpful to think of the Mint at that time in modern managerial terms, and any attempt to unravel the details of who actually did what is fraught with difficulty. Richard Rogers and Andrew Palmer, John's superiors, were in the second tier of mint officers, subordinate to the warden and the master-worker. The warden and the master supposedly had complementary roles, but back in Elizabeth's reign the situation had become confused when Sir Richard Martin had held both positions for several years. In 1599 Martin had been replaced as warden by Thomas Knyvet, a courtier with little knowledge of coinage and minting, and, almost inevitably, Knyvet and Martin were soon involved in a series of legal battles. This situation must have cast its shadow over almost all the operations of the Mint (Challis, 1992a, 259). Knyvet's position was much enhanced in 1605 when, with the aid of his friend Edmund Doubleday, he was instrumental in preventing Guy Fawkes from blowing up the king and the parliament. In 1608 he was rewarded with the title of Lord Knyvet of Escrick.

It is likely that John's daily tasks would have been of the kind for which he had received careful training during his apprenticeship. But many and varied were the projects that came the way of the Tower Mint, and, within a year of his arrival, John became involved in one of them. King James, who relied on the Mint for a significant portion of his income, was always looking for alternative sources of bullion, and from his point of view the best source was silver-bearing ore dug out of the ground, since all mining rights belonged ultimately to him. He was therefore very interested when a new vein of silver ore was discovered at Hilderston in central Scotland. The Hilderston ore was said to produce particularly fine silver, but that had to be confirmed. After much ado, involving several hopeful entrepreneurs as well as the king's chief minister Robert Cecil, and his cousin Sir Francis Bacon, a large quantity of Hilderston ore was shipped to London, and the Tower Mint was instructed to make an assay of it.

The assay took place in August 1608, under the supervision of Thomas Russell, an expert on mining and assaying. The potential importance of the outcome was signalled by the presence of the warden, Lord Knyvet, together with his friend Edmund Doubleday, now established in the post of teller at the Mint, and later to become joint warden. Also present was Anthony Knyvet, the warden's nephew, who later also became an officer at the Mint. The assay masters were represented by John Reynolds, who presented a detailed report on the proceedings (Figure 5).

The report[12] is particularly interesting because it describes the practical details of the assay, as well as the factors affecting the extraction of the precious metal on an industrial scale, in a precise and logical fashion. Although the title and signature are in John's hand, the main part of the report is in a different hand,

[10] Goldsmiths Hall (London): Court Minutes O.3, 487.
[11] Calendar of State Papers Domestic 1603–1610, 373.
[12] Calendar of State Papers Domestic 1603–1610, 452; The National Archives (Kew): SP14/35. See also Pastorino (2009).


The xiijth day of August 1608 269 A 58
A tryall made of the syluer vre within the Tower by Mr Russell at the appointmt of the right Ho:ble the Lord Knyuett in the presents of Mr Edmond Dowbleday Mr Antony Knyuett and my selfe

John Reynoldes

Figure 5. Title and signature for an assay of silver, 1608. The signature is very similar to a later example (Figure 11), but with an extra ‘e’.

possibly transcribed by a clerk who was familiar with the style used in official communications. Most of the numerical information is written in roman numerals, although arabic numerals occur in a few places.

In summary, the assay found that a ton of Hilderston ore yielded 440 troy ounces of silver, with an unusually high level of fineness, 11 ounces 16 pennyweights in a pound. This is 236 parts in 240, much better than the sterling standard of 11 ounces 2 pennyweights, or 222 parts in 240. The value of the silver to the Mint, per ton of ore, was declared to be £117 2s 1d. To arrive at this figure, John had to take account of several factors, and the calculations required him to display his arithmetical expertise. The silver coins, of sterling fineness, had to be struck at the face value of 62 shillings per pound. The cost of coining also had to be considered, as well as the fact that extracting the silver would require 500 pounds of lead per ton of ore. The final figure of £117 2s 1d corresponds to a valuation of 63 shillings and 10½ pence per pound for the Hilderston silver, reflecting its superiority to sterling silver.

Later in 1608 another batch of Hilderston ore was assayed by Russell and Doubleday at the Maresfield ironworks in Sussex. The potential importance of the result meant that both Russell and Doubleday reported directly to Robert Cecil on their findings.[13] In fact, the outcome was less favourable than the Tower assay, indicating that the ore was of variable quality. This was confirmed after an extensive mining operation, when only a small amount of high-grade ore was found, and the project was soon abandoned. But the report presented by John Reynolds was exemplary, and it would have brought him to the notice of important men such as Robert Cecil, as well as his cousin Francis Bacon, who was solicitor-general at that time. It may even have been seen by the king himself, for he liked to dabble in these matters.

6. A startling recommendation

John Reynolds’ first year as an officer of the Mint had gone well. He had doubtless been involved in the routine work of assaying and coining, which, as we have seen, was based on extremely complicated calculations. He had also made his mark with his report on the assay of the Hilderston silver ore, a matter that had aroused the interest of the most powerful men in the land. But the background to his work at the Mint was far from ideal.

In 1609 Sir Richard Martin was still in post as master-worker, although he was now very old. He had been master since 1582, and also warden for a time, until that post was taken over by Thomas (now Lord) Knyvet. The series of disputes between Martin and Knyvet had come to a head in 1607, when a judgement was imposed by the Lord Chancellor. This had involved various financial settlements, as well as the removal of Sir Richard’s son, who had been joint master since 1599, and must have hoped to succeed his father in

[13] Calendar of State Papers Domestic 1603–1610, 474 (items 23, 24, 26).

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due course.[14] Obviously, the officers of the Mint were concerned about the succession, and there must have been many discussions of the possibilities.

In November 1609 the matter was deemed sufficiently serious to warrant a letter to the Lord Treasurer, a post now held by the Earl of Salisbury (Robert Cecil), in addition to his role as Secretary of State. The signatories were Richard Rogers, Francis Goston, Alexander King, Paul Swallow and Charles Anthony, who held (respectively) the offices of Comptroller, joint Auditors, Surveyor of the Meltings and Engraver. Rogers was, of course, John Reynolds' apprentice-master, and well aware of his capabilities. Nevertheless, the recommendation of the officers was startling.[15]

> John Reynolds has been trained ten years in the service of the Mint, and is become expert in making assays of gold and silver, trial of mines and ores, melting, rating and co-mixing gold and silver to any kind of standard, so that he is sufficiently informed in any service that concerns the master-worker's place. His endeavours are still better known to Lord Knyvet, Warden of the Mint.

The letter was accompanied by a note from Knyvet, supporting the recommendation.

Salisbury was surely aware of the disturbed state of the Mint, and he was probably receptive to any suggestion for a peaceful transition. He may even have suggested it himself. But the mastership required a range of skills far beyond those that John had displayed thus far in his official position as Assistant to the Assay Master. His father's career as the Queen's Fletcher may have helped to reinforce his suitability. Whatever the reason, the recommendation was accepted, and John Reynolds was officially granted a reversion on the post of master-worker.[16] In theory, that meant that he would succeed Sir Richard Martin when the post became vacant. In practice, it meant that John's influence on the day-to-day operations of the Mint, in all its aspects, was greatly enhanced.

From John Reynold's personal point of view, his position and prospects at the Mint would have enabled him to marry and start a family. However, very little is known about this aspect of his life. Much later, in 1648, the will[17] of his brother Jeremy mentions Anne, 'wife of Richard Nicholls and daughter to my said brother John', so we may conclude that John did indeed marry at some point.[18]

7. The importance of arithmetic in the nation's affairs

The subtlety of the arithmetic required to keep the nation's coins in good order had become familiar to John Reynolds in the days of his apprenticeship. For example, he would have understood the significance of some of the numbers in James's proclamation of 1604 (Figure 4). Was there a reason why the price of a troy pound of crown gold (£37 4s) was exactly 12 times the price of a troy pound of sterling silver (£3 2s)? Indeed, this was no accident. In Western Europe at that time the standard of value was silver, and the ratio between gold and silver, 12 to 1 in this case, played an important part in the financial health of the nation.

[14] The removal of Richard Martin junior is overlooked in some accounts of the Mint's history, but it is confirmed in the sources cited by Challis (1992a, 261).

[15] Calendar of State Papers Domestic 1580–1625 Addenda, 520.

[16] Calendar of State Papers Domestic 1603–1610, 560; The National Archives (Kew): C66/1829 no. 10.

[17] The National Archives (Kew): PROB 11/205/384.

[18] This conclusion is confirmed by some unspecific references to his 'family' in other documents. There are also some genealogical records which may be relevant, but they cannot, as yet, be regarded as conclusive. Perhaps the most promising link is that on 10 January 1612 a child named Anne Reynolds, whose father was named John, was christened at St Brides, Fleet Street (FHL microfilm 380154). Two boys named Thomas and Richard Nicholls, sons of a Richard Nicholls and his wife Anne, and so possibly John's grandsons, were christened at St Clement Danes on 4 August 1637 and 10 August 1638 respectively (FHL microfilm 1042327). Given the lack of firm evidence about descendants of John, it may be pertinent to note that neither his brother Jeremy nor his sister Mary were married, according to Jeremy's will.

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If other countries set different ratios, the coins would tend to be exported or imported. We shall have more to say about this in due course, but first we must look more closely at the arithmetic.

W.A. Shaw's classic *History of Currency 1252–1894* analyzed the monetary history of Europe over many centuries, using the fluctuations in the ratio as a guide. He began by stressing that the arithmetic is actually more complex than the neat 12 to 1 ratio might suggest (Shaw, 1895, xiv–xvi). That is because it did not take account of all relevant factors, such as the fineness of the coins. The gold and silver coins of other countries were not of the same fineness as those in England, and the true ratio had to be calculated in terms of pure gold and pure silver. Since crown gold at 22 carats was $11/12$ pure, a troy pound of pure gold was actually worth $12/11$ times £37 4s; similarly, sterling silver at 11oz 2 dwt was $37/40$ pure, and a troy pound of pure silver was worth $40/37$ times £3 2s. The resulting ratio for the pure metals works out at $666/55$, or about 12.1 to 1 in modern decimal notation. In the last years of Elizabeth's reign the ratio in England had been effectively about 11 to 1, which was low, in comparison with other countries.

The fineness factor had also to be accounted for when setting the value of gold minted at the old standard of 23 carats $3^1/_2$ grains, or 191 parts in 192. This had been the traditional standard for several centuries, before Henry had begun to tamper with the currency, and it was known as 'angel gold', from the coin known as an angel, which bore a dramatic representation of Archangel Michael slaying a dragon. Elizabeth had long issued angels with this fineness, and for reasons of prestige, James wished to continue doing so. Consequently, he needed to determine the value of a troy pound of angel gold, so that the ratio was the same as that for crown gold, $666/55$. In fact, the Mint indenture for the coinage of angel gold did not appear until 16 July 1605, eight months after the one for crown gold. In modern terms, the problem is to find the value of x such that x pounds of angel gold, $191/192$ fine, makes the ratio between pure gold and pure silver $666/55$. The solution is $x = 17763/440$, which means that angels should be coined at the rate of £40 7s 5d per troy pound. The indenture actually set the rate at £40 10s, probably because that gives an integer (81) for the number of ten-shilling angels per troy pound. Inevitably, the average weight of an angel was still an awkward number, seventy-one and one-ninth grains (and there was no accompanying royal proclamation to help the public check their coins individually).

Although the value of gold had been raised in England, it was still undervalued in comparison with other countries, such as Holland. In 1607 the king issued a proclamation[19] reminding his subjects that the exportation of coin and bullion was unlawful, quoting statutes going back to the time of Edward I. But the trade in precious metals was well-established, and the proclamation was ineffective. By 1610 the loss of gold was causing great anxiety, and this led to lengthy discussions in the Privy Council.[20] The king himself was involved in these discussions, and both Salisbury and Francis Bacon were active in the matter.

It appears that Salisbury was originally against the idea of another increase in the ratio, but eventually he relented, and preparations were put in hand (Spedding, 1868, 243–244). In October 1611 Bacon sent Salisbury an amended draft of the proclamation, with some suggestions of his own.[21] The proclamation was eventually published on 23 November.[22] The preamble claims that extensive consultations had taken place, involving 'divers Gentlemen of quality and discretion' as well as 'sundry Merchants of every Trade', 'Officers of Our Mint', and 'Goldsmiths of the best sort'. The proposals had probably created some consternation at the Mint, and John Reynolds, as the presumptive master, would surely have been concerned. The main effect was to raise the values of the gold coins by one-tenth. A gold unite, previously worth 20 shillings, was now worth 22 shillings, and there were some very awkward denominations, such as a *thistle crown* at 4s $4^1/_2d$. This proclamation did not match its 1604 predecessor in transparency, since it specified only the official values of the various coins (in Roman numerals), not their weights. Its obscurity

19 Larkin and Hughes (1973, 158–161, item 73).

20 The background is described in detail by Supple (1959, Chapter 8, especially 181 and the references given there).

21 Calendar of State Papers Domestic 1611–1618, 83, item 103; The National Archives (Kew): SP 14/66/103.

22 Larkin and Hughes (1973, 272–276, item 122).

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was compounded by a clumsy attempt to revive an old idea about allowing for the fact that coins became worn or clipped in use, a suggestion apparently due to Francis Bacon. The intention was that light coins should circulate at their full face-value—provided they were not too light. Fixing a small allowance meant that there was a *least current weight* for each coin. The practical implications of this idea had never been properly worked out, and the wording of the proclamation was unclear. It was stated that anyone might lawfully refuse to accept a coin that fell below its least current weight, but it did not say what that weight was, and it did not say explicitly that coins above the least current weight should be accepted at their full face-value.

8. The new master

Robert Cecil (Lord Salisbury) died on 24 May 1612. He had been Secretary of State since 1596 and Lord Treasurer since 1608. Opinions differ as to his achievements as the king's chief minister, but there can be no doubt that his death heralded a decline in the management of the nation's affairs. That decline was to affect all areas of government, including the Mint, and its presumptive master, John Reynolds.

In Cecil's life-time the king had been restrained from exercising the royal prerogative, especially in the more technical aspects of government. Now he was free to act arbitrarily, and often foolishly. The position of Secretary of State was left vacant, with the king himself purporting to act as his own Minister, assisted by his favourite courtier, Robert Carr. Carr had been given the title of Viscount Rochester in 1611, and he became earl of Somerset in 1613. However, by 1615 he was being replaced in the king's affections by George Villiers, who was soon to play a significant part in the fate of the Mint. In 1614 the post of Secretary of State was taken over by Sir Ralph Winwood, and he was joined in 1616 by Sir Thomas Lake. These two men were rivals, as well as colleagues, and they too would have a part to play in our story.

The 1611 proclamation had simply raised the value of gold coins by one-tenth (in terms of the silver standard) without altering their weight or fineness. So the ratio had been increased in the same proportion, from about 12.1 to about 13.3. The round figure of ten per cent was in fact too round, because the ratio was now higher than in other countries, and silver was soon flowing out. The Privy Council was again concerned, and Sir Francis Bacon seems to have taken the lead (Spedding, 1868, 255–259). He drew up a proclamation[23] for the valuation of foreign coins, which he hoped would bring them to the Mint for coining. A new indenture was issued on 18 May 1612, but since the composition of the coins had not been altered, the content was only superficial: a troy pound of gold, which had formerly produced £37 4s of coin, was now required to produce £40 18s worth. Here Bacon's liking for round numbers surfaced again, because the correct figure for an increase of 10 per cent has a few extra pence. (Eventually, the figure was rounded up to £41.)

In these troubled times the Council continued to be bombarded by advice from the apparently everlasting Master of the Mint, Sir Richard Martin.[24] Some of his utterances imply that he had no faith in his colleagues in the Tower. We can only speculate as to the part played by Martin's heir-presumptive, John Reynolds, but it seems likely that he was one of the group of officers who offered their own advice, such as a proposal for making the silver coins lighter.[25]

Sir Richard Martin died in July 1617. Although his exact age is not known, he was surely very old, and his death must have been expected for some time. Some state papers of uncertain date appear to anticipate a difficult transition to a new Master, and so it was. In the event, Secretary Winwood wrote to Secretary Lake on 8 August 1617, recalling that a reversion on Martin's office had been granted to 'one Reynolds', and

[23] Larkin and Hughes (1973, 279–281, item 124).

[24] BL Add MS 10113 contains many papers submitted by Martin.

[25] Calendar of State Papers Domestic (1611–1618, 396).

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enclosing a commission for the king to sign.[26] The implication is that Lake was the Secretary who had regular access to the king, and he was aware of the uncertainties of the king's pleasure. In these circumstances it is no surprise that the commission was returned by Lake a few days later, presumably unsigned.[27] Hints of what was afoot are contained in Lake's reply, which mentions several other matters, with instructions about them issued in the name of George Villiers, now the earl of Buckingham. A few weeks later, all was made clear. John Chamberlain was an acute observer of the London scene, and he wrote many letters which throw light on the murky dealings of James and his entourage (McClure, 1939). On 18 October he reported on the latest gossip surrounding the court and, in particular, the earl of Buckingham,

> whose brother[28] Sir Edward Villiers they say shalbe Master of the Mint, an office lately voyde by the death of Sir Richard Martin who was held to be very neere an hundred years old: there was one that had a patent for the place in reversion, but how he is put of[f] I know not.

No documentary evidence of the way in which John Reynolds was 'put off' is known, but subsequent events imply that he retained an explicit commission from the king to continue his mathematical work at the Mint.

9. More proclamations and some advice

The appointment of Edward Villiers was a severe setback for John Reynolds. Although he retained his position as Assistant to the Assay Master, his influence at the Mint must have been greatly diminished. The situation could only get worse, and it did so when Villiers began to act in a way that must have been expected, given the manner of his appointment. He saw the Mint primarily as an opportunity to make money for himself, rather than the public, and the records show that he amassed a large fortune (Challis, 1992a, 271). However, Reynolds' expertise in arithmetic remained a valuable asset and, unknowingly, the stage was being set for him to play another important part in the nation's affairs.

The complexities of James's coinage, and the implications for the economic health of the nation have already been mentioned (Section 7 and footnote 21). In November 1618 the attention of the Privy Council turned once again to what was described as the 'decay of his Majesty's Mint', evidenced by the scarcity of gold and silver coins.[29] Francis Bacon was now Lord Chancellor, and his influence on the proceedings is clear. The officers of the Mint were ordered to provide statistical tables of the bullion coined, and several possible causes of the problem were examined in detail. After much deliberation, the outcome of these activities was a new proclamation,[30] issued in July 1619, 'for reforming sundry inconveniences touching the Coynes of this Realme'. New gold coins with convenient values were to be issued, such as a twenty-shilling piece, to be known as a *laurel*. The old coins, such as the unite valued at 22 shillings, would continue to circulate with their awkward values, so that there were more than ten denominations of gold coins in circulation. The proclamation repeated the rules about least current weight; for example, a laurel could be refused if it was more than two grains lighter than its proper weight. The figures were now given in arabic numerals but, as in 1611, the proper weight was not stated in the proclamation, and could only be deduced from the instructions given to the Mint. These were expressed in the traditional format, requiring that 41 laurels should be struck from each pound of gold. Since the pound contained 5760 grains, it was possible to calculate the correct weight of a laurel in grains, mites, and so on, although few people had the ability to do this. The king was aware of the problem, and his proclamation proposed a solution.

26 Calendar of State Paoers Domestic (1611–1618, 479); The National Archives (Kew): SP 14/93 no. 6.

27 Calendar of State Papers Domestic (1611–1618, 480, item 16).

28 Actually, half-brother.

29 Acts of the Privy Council (1617–1618, 302–320).

30 Larkin and Hughes (1973, 436–439, no. 189).

Figure 6. A brass weight for checking a laurel of 20s after 1619, illustrated ×2.

> And to the intent all Our loving Subjects in all parts of this Our Realme may have good and just Weights for the true weighing of all the said Coynes of Gold, and true and upright graines & half-graines for the remedies and abatements aforesaid, We have commanded the Master of Our Mint within Our Tower of London now being (the appointment thereof being proper & peculiar to Us oneley) to prepare and make ready before the first day of September next a sufficient number of upright Balances and true Weights, aswell of every severall peece of Gold lawfully current in this Our Realme... And We strictly forbid all Our subjects and others whatsoever, to have or use any other weights than as aforesaid for the said Coyness of Gold...

John Chamberlain's comment on the provision of balances and weights was probably typical: 'all that have any store by them shalbe great loosers, without gain to any but to the master of the mint' (McClure, 1939, vol. 2, 258). Indeed, the reluctance of the king's 'loving subjects' to comply with his wishes resulted in the proclamation being re-issued in February 1620, regretting that 'not any, or very few' had taken up the offer.[31]

The fact that significant numbers of contemporary coin-weights have survived suggests that there were other, cheaper, sources. The weights are square pieces of brass, usually bearing an image resembling the obverse of the intended coin, and the value (Figure 6). Some of them have been countermarked with a crowned I (for James), as ordered in the 1620 proclamation. Several varieties, both with and without the countermark, are listed in the *Corpus of British Coin-Weights* (Withers, 1993), which reinforces the conclusion that the intended monopoly was not effective.

Coin-weights would play a major part in the career of John Reynolds, but for the time being he had other concerns. He was now in his early 30s, working in a job with an obscure title and little prospect of advancement. His expertise in arithmetic, and his extensive knowledge of the business of assaying and coinage, deserved better. When the post of Chief Assayer at the Goldsmiths Hall fell vacant, he applied, and in October 1619 he was appointed (Forbes, 1999, 104–106). He was required to take up residence at the Hall with his family, but allowed to retain his official position at the Mint. Although his tenure began well, by 1625 there were clear signs of difficulty between John and his employers. At the heart of the problem was a disagreement about the practical implementation of the sterling standard. The matter rumbled on for several years, with many twists and turns, until he finally left the Company in 1630 (Forbes, 1999, 108–119). From the mathematical point of view his work for the Goldsmiths was routine, because (unlike his work at the Mint) there were no external factors to be considered. The complexity of the currency, and the repeated attempts to maintain its position internationally, offered greater scope for his skills.

In 1625 King James died and was succeeded by his son Charles. A proclamation[32] issued in May 1627 was a typically Stuart attempt to stem the outflow of silver, by establishing a monopoly of the Exchanges.

[31] Larkin and Hughes (1973, 460–464, no. 196).

[32] Larkin (1983, 144–153, no. 69).

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This long-winded proclamation also re-iterated some rules, originally set out in 1620, concerning the 'odde' pieces of gold. For example, the circulation of the old unite, worth 22 shillings, was subject to the following rules:

(1) a coin lacking not more than 3 grains should be current at its full value;
(2) a coin lacking more than 3 grains but not more than 6 grains should be current, but at a reduction of 2d per grain for each grain above 3;
(3) a coin lacking more than 6 grains should not be current, but should be taken to the king's Exchanges, where it would be redeemed for its value as bullion.

The effect of the proclamation was not entirely clear, because the correct weights of the coins were not stated explicitly. Thus the commercial world was ready for a little book published later in 1627, entitled *An Advice Touching the Currancie in payment of our English Golde. As also, A Table of the Severall Worths of all Pieces, uncurrant through want of weight, at his Ma*[ties] *Exchanges in London* (Reynolds, 1627). The book began with a careful account of the various proclamations and their provisions, which was followed by tables stating the correct weights of the coins and, in particular, the exact values at the Exchanges for the coins that were not current.

The basis for the values listed in the *Advice* was the price paid for a troy pound of crown gold at Exchanges. It was not stated, but (with some difficulty) it can be deduced from the table, and appears to be about £39 12s per troy pound. (By comparison, the Mint was required to produce £41 of coin per troy pound.) The £39 12s figure has some advantages for calculations based on the Golden Rule, because it is an exact multiple of 12 shillings. Thus, if a troy pound (5760 grains) was worth £39 12s (= 792 shillings), then one grain was worth 11/80 of a shilling, or 33/5 farthings, a number that one might hope to remember. Indeed, if the money-system of the 1620s had been decimal, it would have been very convenient. Decimalisation of money in the UK was to be delayed by a few centuries, although decimal arithmetic became commonplace later in the 17th century. The awkwardness of the $33/5$ figure was reflected in the tables, because rounding the values into farthings produced variations in the steps for a difference of one grain. Even the rule that a difference of 5 grains in weight should correspond to a difference in value of 33 farthings ($= 8^1/_4 d$) was not always respected.

There is no name on the title page of the *Advice*, but it has always been assumed that its author was John Reynolds. This conclusion is based on the resemblance between the book and a similar one, published a few years later, which we shall describe in Section 10 of this paper (Reynolds, 1651). For the later book the case for his authorship is compelling, albeit circumstantial. But Reynolds was apparently not the only person who took the opportunity to assist the money-men with their sums. One version of the *Advice* (BL 1139 c.22) refers to another anonymous booklet, *The Free Exchanger*, which it rejects as being 'full of errors'.

10. The perils of precision

The problems created by the complexity of the gold coinage were particularly acute for the mercantile classes in London.[33] The proclamations of 1619 and 1620 had been ineffective in restricting the production of coin-weights to the Mint, and there were many types in use, some of them made in the Low Countries. The merchants complained that there was 'great deceit' in these weights, even though they all bore the image of the king's head. Some of the variation was surely due to poor workmanship, but it was also pointed out that some weights were specifically intended to check the full 'mint weight' of the coin, while

[33] Calendar of State Papers Domestic 1631, 4.

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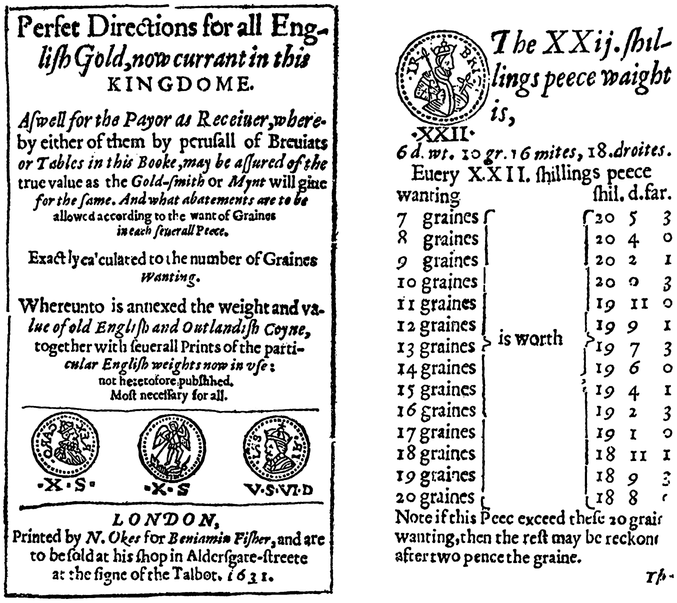

Perfet Directions for all English Gold, now currant in this KINGDOME.

*Aswell for the Payor as Receiuer, where*by either of them by perusall of Breuiats *or Tables in this Booke, may be assured of the* true value as the *Gold-smith* or *Mynt* will giue *for the same. And what abatements are to be* allowed according to the want of Graines *in each seuerall Peece.*

Exactly ca'culated to the number of Graines *Wanting.*

Whereunto is annexed the weight and va*lue of old English and Outlandish Coyne,* together with seuerall Prints of the parti*cular English weights now in vse:* not heretofore publshed. Most necessary for all.

•X•S• •X•S V•S•VI•D

LONDON,
Printed by *N. Okes* for *Beniamin Fisher*, and are to be sold at his shop in Aldersgate-streete at the signe of the Talbot. *1631.*

The XXij. shillings peece waight is,

•XXII•

6 d. wt. 10 *gr.* 16 *mites*, 18. *droites.*

Euery X.X I I. shillings peece

wanting		shil.	d.	far.
7 graines	is worth	20	5	3
8 graines		20	4	0
9 graines		20	2	1
10 graines		20	0	3
11 graines		19	11	0
12 graines		19	9	1
13 graines		19	7	3
14 graines		19	6	0
15 graines		19	4	1
16 graines		19	2	3
17 graines		19	1	0
18 graines		18	11	1
19 graines		18	9	3
20 graines		18	8	[illegible]

Note if this Peec exceed these 20 grai[ns] wanting, then the rest may be reckon[ed] after two pence the graine.

Th-

Figure 7. The title page and a table from the *Perfet Directions*.

others were intended to check the 'least current weight'. After discussion in the Lord Mayor's Court, the matter was referred to the Privy Council in April 1631, and it was decided to revive the old policy of monopoly, by ensuring that all gold-weights were produced by a single, reliable, maker. However, the Council no longer had confidence in the Mint's ability to do the job, and on 17 May 1631 the king instructed the Attorney General to offer the position of maker of 'weights and balances for the King's moneys of gold' to Sir Thomas Aylesbury, for life.[34] Aylesbury had not previously figured in the annals of coinage or weight-making, but that was clearly not an important consideration, in the King's opinion. He had attended Westminster School and Christ Church Oxford, before rising steadily through the ranks of the civil service, under the patronage of Buckingham. He had also acquired a reputation for competence in mathematics, mainly as a result of his association with the group of scholars surrounding Thomas Harriot, and his links with the mathematical community continued for the rest his life (Biggs, 2017b, 2018). Although the officers of the Mint were not given charge of the new gold-weights, Aylesbury clearly needed the help of some of them, and John Reynolds was an obvious choice.

In fact, it seems that Reynolds took charge of the operation in an executive capacity, not merely as a consultant on arithmetic. The number of weights required was large, and there was a great deal of preparatory work. Brass of the right quality was essential, and the dies had to be designed and cut. One important decision was made quite early in the process: the new weights should be round, in order that they could be easily distinguished from the old square ones. The new weights were not ready until December 1632, when they were announced by royal proclamation.[35] In the meantime Reynolds had the opportunity to produce an updated version of his *Advice*. It was entitled *Perfet*[36] *Directions for all English Gold* (Reynolds, 1631), and it was noteworthy because it contained illustrations of the new round coin-weights (Figure 7).

In the *Perfet Directions* the table for each coin was headed by a drawing of the relevant coin-weight, and a statement of the 'mint weight' of the coin. This was given with a higher level of precision than in the *Advice*; for example, the figure for the 22-shilling piece is 6 pennyweights 10 grains 16 mites 18

[34] Calendar of State Papers Domestic 1631, 46.

[35] Larkin (1983, 366–369, no. 164).

[36] An old spelling derived from the Middle English word.

droits, whereas in the *Advice* there were no droits. The valuations for the light coins were also new, because the price paid for bullion at the exchanges had increased. The book was apparently successful, and new editions were published in 1632 and 1633. In these editions. *Perfect* was spelled in the modern way, and the supplementary material at the end of the book was amplified. This material is worth studying in detail, because it illustrates many of the currency problems of the time—arithmetical, metrological, and practical.

First, there is a section giving valuations, as bullion, for the great variety of gold coins circulating in England. These included old English gold, such as the 'Harry Soveraine', actually a half-sovereign of debased (20 carat) gold, issued by Henry VIII in the 1540s. Also listed were over twenty foreign gold coins, such as the 'Flemish spur royall' and 'Duckets of Aragon'. The list is followed by several other useful tables. The first is a Ready Reckoner for exchanging gold and silver of various finenesses, similar to the 1577 example described in Figure 1 of this paper, but with revised prices. For example, an ounce of silver, with fineness 10 oz, was now worth 4 shillings 6 pence 1 mite and 11 'parts of a mite'. The last figure indicates that Reynolds had adopted a new way of converting the results of dividing by 37 into fractions of a mite (see Section 3). A footnote to the table informs the reader that 'here 37 parts make a mite'; in other words, the number of parts is just the remainder after division by 37. In this form the answer is exact, and there is no need for the futile procedure of approximating by halves and quarters of a mite. This section is followed by a table giving the value of small amounts, in pennyweights and grains, of crown gold. It reveals that the price was now exactly 40 pence per pennyweight, which translates neatly into £40 per pound troy, a small increase on the £39 12s which prevailed at the time of the *Advice*. The new figure is also equivalent to 5 pence for 3 grains, and consequently the tables for light coin, such as the one shown in Figure 7, display a period of 3. This feature would have made the calculations relatively easy.

Next, the reader is informed that the new weights are sold by John East, goldsmith, at the sign of the crown in Maiden Lane. Almost certainly East was not the only seller of the weights, but the mention of him here is significant, because the East family and the Reynolds family were linked, both by marriage and by their work at the Tower. It will be recalled that John Reynolds had an elder brother, Jeremy, and the Mint records show that they shared some of the duties there from 1615 until 1630. Jeremy Reynolds died in 1648, and in his will[37] he left twenty shillings to Mary East, described as 'daughter to my kinswoman Mary the wife of John East'. The kinswoman may well be the Mary Braye who had married John East at Caddington in Bedfordshire in 1617, but her relationship with Jeremy is unknown. We do know that John East, goldsmith, was employed at the Mint as under-engraver from 1634 until his death in 1652, and that his brother James East was involved in making farthing tokens from 1643 onwards (Peck, 1970, 47). These snippets of information suggest that much activity in the Tower, including the making of coin-weights, was undertaken by people employed at the Mint, but not as part of their official duties.

The book ends with a collection of pictures of the eight weights for the gold coins most in use, yet another reminder of the complexity of the currency. These eight coins were by no means the only ones described in the book: there were 14 current coins for which individual tables were provided, as well as the extensive list of 'Old English and Outlandish Gold Coynes'.

The production and distribution of Aylesbury's new weights did not go well; indeed, Aylesbury and Reynolds had a spectacular falling-out. In June 1633 John Reynolds wrote to the Privy Council, saying that he had been confined to the Fleet prison by Aylesbury, who alleged that he had produced false weights.[38] After some delay he was eventually released, but it was declared that he had been at fault.[39] The magnitude of the enterprise is revealed by the numbers of weights involved—24,000 sets are mentioned in the documents—but the precise reason for the dispute is not clear. One likely cause of disagreement was in-

37 The National Archives (Kew): PROB 11/208/384.

38 Calendar of State Papers Domestic 1633–1634, 359–361. See also Biggs (2017a).

39 Larkin (1983, 420–423, no. 183).

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Figure 8. Grain weights, countermarked I (James) and C (Charles), actual size. The number of grains is indicated by the dots.

herent in the provisions for checking the deficiency of light coin. The proclamation of December 1632 had called for the new coin-weights to be

> of the full and just size according to the true Weights of the Gold Moneyes, and Graines and halfe Graines shall be apart by themselves, to show the remedies and abatements as they ought to bee, and none of them shall be made with the remedies and abatements purposely taken off...

The Graines and halfe Graines were small pieces of thin brass sheet (Figure 8), and their use was a fiddling business. In everyday commerce, it was necessary only to know that a coin was above the least current weight, not that it was of full mint weight, and it is therefore possible that Reynolds had produced weights of the former level, for practical convenience, but contrary to the proclamation.

11. A brief and easie way

In 1631 Sir Thomas Aylesbury's standing in the mathematical community had been confirmed by the publication of Harriot's *Artis Analyticae Praxis*, which Aylesbury had supported financially. Following his victory over Reynolds in the matter of the coin-weights, Aylesbury's standing in the royalist administration was also enhanced. In 1635 the position of Master of the Mint fell vacant, and Aylesbury was appointed, jointly with Sir Ralph Martin. But his authority began to crumble with that of his royalist masters and, after Parliament took control of the Mint in 1642, he was removed. He went first to the royal stronghold at Oxford, and after the execution of Charles in 1649 he moved to the continent, where he died in 1657. On the other hand, John Reynolds was a great survivor. Despite his dispute with Aylesbury he continued to work at the Tower Mint during the latter's tenure as Master, and some records of his routine work in 1639–1640 have been found (Challis, 1992a, 307 and note 210). He was also active in the scientific community at that time, an aspect of his work that will be described in the following section.

The execution of the king was followed by sweeping changes of personnel at the Mint, but Reynolds survived (Challis, 1992a, 325). Indeed, it was not long before he was once again involved in matters of national importance. The Council of State was greatly concerned about the coinage, and in December 1649 they considered evidence from Reynolds about the 'fluctuation of the coin'.[40] In June 1650 the Council resolved to investigate the business of making weights and scales for checking gold coins, a process that must also have involved Reynolds. No documentary evidence of the outcome of this investigation has been found, and the artefactual evidence suggests that nothing was done, because coin-weights do not re-appear on the English scene until the latter part of the reign of Charles II (Withers, 1993). The lack of progress may have some bearing on the fact that, in December 1650, Reynolds was summoned to appear before the Council, to answer for some 'dangerous words spoken by him'.[41]

After this rebuke, Reynolds seems to have focused on his arithmetical activities. In November 1651 he was one of the signatories to a set of tables prepared for the Council, listing the weight, fineness, and

[40] Calendar of State Papers Domestic 1649–50, 462.
[41] Calendar of State Papers Domestic 1650, 469.

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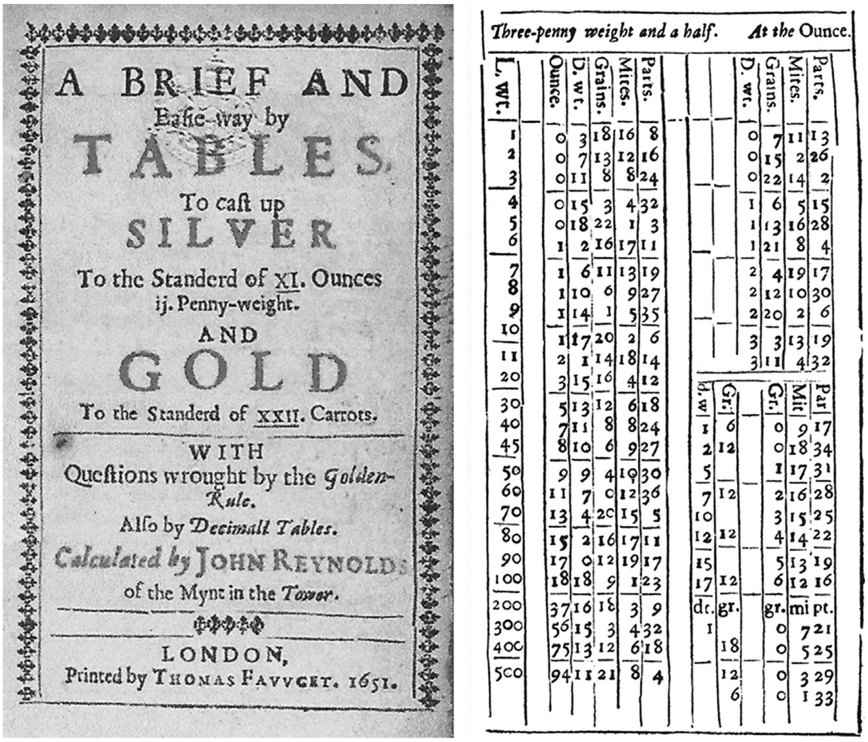

A BRIEF AND
Easie way by
TABLES
To cast up
SILVER
To the Standerd of XI. Ounces
ij. Penny-weight.
AND
GOLD
To the Standerd of XXII. Carrots.
WITH
Questions wrought by the *Golden-Rule.*
Also by *Decimall Tables.*
Calculated by JOHN REYNOLDS
of the Mynt in the *Tower.*
LONDON,
Printed by THOMAS FAVVCET. 1651.

Three-penny weight and a half. *At the* Ounce.

L. wt.	Ounce.	D. wt.	Grains.	Mites.	Parts.	D. wt.	Grains.	Mites.	Parts.
1	0	3	18	16	8	0	7	11	13
2	0	7	13	12	16	0	15	2	26
3	0	11	8	8	24	0	22	14	2
4	0	15	3	4	32	1	6	5	15
5	0	18	22	1	3	1	13	16	28
6	1	2	16	17	11	1	21	8	4
7	1	6	11	13	19	2	4	19	17
8	1	10	6	9	27	2	12	10	30
9	1	14	1	5	35	2	20	2	6
10	1	17	20	2	6	3	3	13	19
11	2	1	14	18	14	3	11	4	32
20	3	15	16	4	12				
30	5	13	12	6	18				
40	7	11	8	8	24				
45	8	10	6	9	27				
50	9	9	4	19	30				
60	11	7	0	12	36				
70	13	4	20	15	5				
80	15	2	16	17	11				
90	17	0	12	19	17				
100	18	18	9	1	23				
200	37	16	18	3	9				
300	56	15	3	4	32				
400	75	13	12	6	18				
500	94	11	21	8	4				

d.w	Gr.	Gr.	Mit	Par
1	6	0	9	17
2	12	0	18	34
5		1	17	31
7	12	2	16	28
10		3	15	25
12	12	4	14	22
15		5	13	19
17	12	6	12	16
dr.	gr.	gr.	mi	pt.
1		0	7	21
	18	0	5	25
	12	0	3	29
	6	0	1	33

Figure 9. The title page of Reynolds' tables, and a typical page. © 1651 British Library Board 1651/173.

valuation of the numerous foreign coins that were circulating in England at that time (Shaw, 1896, 85–90). The preparation of the tables required practical expertise in assaying, as well as arithmetical skill in making the calculations. But, from a mathematical point of view, a more interesting example of his work is the book which appeared in 1651 (Figure 9). This contains an extensive set of tables and, unlike the *Advice* and the *Perfect Directions*, some examples of their use. The Golden Rule and the Backer Rule are mentioned frequently, and there are some intriguing signs of the use of decimals.

The *Brief and Easie Tables* were intended for use by an assay master whose task was to produce silver (or gold) of standard fineness. The material available would be of varying levels of fineness; for example, the Spanish silver coins brought to the London mint were not of sterling standard. The strategy employed by the assayers was simple—in outline. Metal that was better than standard would be assayed and weighed, and a measure of its 'Betterness' calculated; call it B. The same would be done for metal that was worse, leading to a measure of its 'Worseness', W. The aim was to arrange that $B = W$, in which case all the metal could be melted down together and standard silver would result. However, that was very unlikely, and so the amounts had to be adjusted, which could be done in several ways. For example, if $B > W$ then W could be increased by adding a certain amount of base metal; while if $B < W$ then W could be reduced, by using only some of the worse material. The aim (usually) would be to use as much of the available silver as possible. The major difficulty was to calculate the measures B and W, and to determine exactly the changes required to make them equal.

In order to explain how the calculations were done, it is necessary to recall some of the details presented in Sections 3 and 4 of the present paper. When silver was received at the mint, it would be cast into ingots, and their weight and fineness determined. The traditional way of reporting the fineness was in terms of its relation to the standard. So if an ingot was reported to be (say) *three and a half pennyweights better*, that meant that its fineness was 11 oz $5^1/_2$ dwt in the pound, since the standard was 11 oz 2 dwt. This data was the basis for calculating the measures B and W, but there were complications. The fact that the standard for silver, when written as a fraction, is 37/40 is particularly significant, because the assayers used measurements expressed in terms of standard silver, rather than pure silver, and so they were sure to involve division by 37. Indeed, as we shall see, Reynolds' tables can be regarded as a list of the results of dividing certain quantities by 37.

The first half of the *Brief and Easie* tables is devoted to the assay of silver. Most of the tables are headed by words like 'three penny-weight and a half' (Figure 9). This means that the table is intended to be used for calculations involving an ingot which is three and a half pennyweights better (or worse) than standard. Each table refers to an integral number of half-pennyweights, call this number z. Since there are 480 half-pennyweights in a pound, the equivalent fineness as a fraction is $z/480$, and the entries in the table for z give the weight of *standard* silver that corresponds to various weights of an ingot that is better than standard by this amount. To compile the tables, Reynolds worked as follows. An ingot weighing m ounces which is z half-pennyweights better will contain $mz/480$ ounces of pure silver, over the standard, and this is equivalent to $mz/444$ ounces of standard silver. For example, taking $m = 12$ and $z = 7$, the figure for 12 ounces (= one pound) in the 'three-penny weight and a half' table should be 7/37 ounces. As noted in Section 3 of this paper, the old system of sub-units of the ounce did not allow such results to be written down exactly. To overcome this difficulty Reynolds resorted to the device which he had used in the tables at the end of the *Perfect Directions*. If the '37-th part' of a mite is regarded as an artificial unit, then the exact answer can be written as 3 pennyweights 18 grains 16 mites and 8 parts, and that is how it appears in the table.

The tables were calculated by this complicated procedure because the art of assaying was rooted in the distant past. For example, the betterness and worseness had to be calculated separately because negative numbers were regarded as meaningless, and the measurements were expressed in terms of standard silver because the aim of the assayer was to achieve that level of fineness. The practical usefulness of the tables depended on two fundamental properties. Nowadays we are fortunate to be able to use elementary algebra to explain what is going on, but Reynolds would have relied on his deep intuitive understanding of the procedures. (An extended version of the following account can be found in the Appendix.)

The basis of the method is the assignment of a number to an ingot. Suppose the ingot weighs m units and has fineness f, where f is a number between 0 and 1, and denote the standard fineness by s. As described above, Reynolds' method is to multiply m by $(f - s)/s$, assuming f is greater than s. So the tables give the number

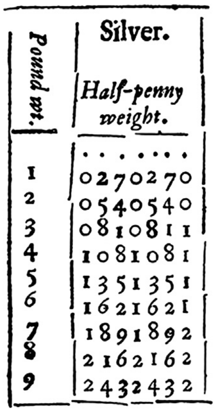
$$r = m(f - s)/s,$$

which represents the *betterness* of the ingot. When f is smaller than s, the tables give the *worseness* of the ingot, $r = m(s - f)/s$. The key properties of r determine its behaviour when ingots are combined—that is, when they are melted together to form a new ingot.

The addition property: *if we combine two ingots with betterness r_1 and r_2, the result is an ingot with betterness $r_1 + r_2$.*

The standarding property: *if we have an ingot with betterness r_B and an ingot with worseness $r_W < r_B$, then combining them and adding an amount $r_B - r_W$ of base metal produces an ingot of standard fineness.*

The standarding property is the key to the mystery, since it tells us how to make an ingot of standard fineness by adding the right amount of base metal, which has fineness 0. Reynolds gives an example with four ingots, two better than standard and two worse. The first step is to use the tables to read off the betterness or worseness of each ingot. For example, the first ingot weighs 40 lb 6 oz 10 dwt and its fineness is $16\frac{1}{2}$ dwt better, so it turns out that its betterness is 36 oz 3 dwt 4 gr 4 mites (plus some parts, which Reynolds ignores). Similarly, he finds the betterness or worseness of the other ingots. Then, combining the two better ingots into one, and the two worse ingots into another one, and using the addition property, he obtains the following results:



Pound wt.	Silver. Half-penny weight.
	. . . .
1	0270270
2	0540540
3	0810811
4	1081081
5	1351351
6	1621621
7	1891892
8	2162162
9	2432432

Figure 10. A ‘Decimall Table’. © 1651 British Library Board 1651/173.

	ounces	dwts	grains	mites
r_B	57	3	21	10
r_W	53	4	3	3
$r_B - r_W$	3	19	18	7

Since r_B exceeds r_W by the amount shown, it follows from the standarding property that, when that amount of base metal is melted together with the four given ingots, the result will be an ingot of standard silver. The advantage of the procedure is clear: by adding a relatively small amount of cheap metal the required standard is achieved.

Reynolds must have realized that it is not completely obvious that this method works, and that is probably why he does the same example again, by a slightly different method. Now the betterness and the worseness are calculated directly (without the tables) in terms of *pure* silver, and the difference is adjusted rather mysteriously at the final stage, by an ‘Addition’ which converts it into ‘standard’ terms.

The title page of the book contains the words ‘Also by Decimall Tables’. Decimal notation for fractions had been promoted by Simon Stevin[42] towards the end of the sixteenth century, and it was soon adopted by astronomers and mathematicians. Reynolds must have been aware of the trend, because he had used decimal fractions in the 1640s, in his metrological work. He had determined the specific gravity of many substances, and stated the results in decimal form; for example, he had found that a cubic inch of rain water weighed 0.579036 averdepois ounces (Davies, 1748). But in financial and commercial circles the old systems of fractions and sub-units were slow to disappear. Although Reynolds’ book does indeed contain some brief ‘decimall tables’, like the one shown in Figure 10, there are no examples of how they might have been used.

Even so, the table has some interesting features. The entries are the decimal representations of fractions of the form $n/37$, such as $1/37 = 0.027027027\ldots$, and it is apparent that the digits repeat with period 3, as do all the entries in the table. The book contains no comment on this feature, although it must have been noticed by the author and some of his readers. As far as we know, repeating decimals were not studied systematically until 1685 when John Wallis discussed them in his *Treatise of Algebra*. He gave some rules for finding the period of a repeating decimal, and wrote that ‘I do not remember that I have found it so considered by any other’ (Wallis, 1685, 327). He explained that $1/37$ repeats with period 3 because 37 is a divisor of 999. This means that, when we use long division to divide 37 into $1.00000\ldots$, we get a remainder of 1 after three steps, and so the working repeats. The phenomenon is not so obvious when the

42 Stevin (1585), also published in French in the same year.

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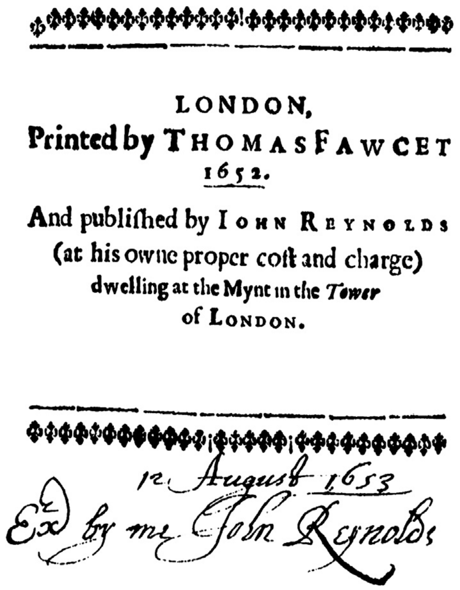
LONDON,
Printed by THOMAS FAWCET
1652.
And publiſhed by IOHN REYNOLDS
(at his owne proper coſt and charge)
dwelling at the Mynt in the Tower
of LONDON.

12 August 1653
Ex'd by me John Reynolds

Figure 11. John Reynolds' endorsement of his book, dated 12 August 1653. © 1651 British Library Board 1651/173.

old 'scratch' method of division is used. More generally, if the prime number p is a divisor of $10^m - 1$, then the decimal form of $1/p$ will repeat with period m.

The *Brief and Easie* tables were reprinted in 1679 in a book entitled *A New Touchstone for Gold and Silver Wares* (Badcock, 1679). This was essentially a polemic against the malpractices of the goldsmiths, and reached a wider audience than the original 1651 edition. The original is very rare; one of the few extant copies is in the British Library, and it has been endorsed by John Reynolds himself (Figure 11).

12. The final years

When John Reynolds endorsed a copy of his book in August 1653 he must have been close to 70 years old. It is not clear that his remark about 'dwelling at the Mynt in the Tower' should be taken to mean that he actually resided there, but certainly he occupied some rooms, where he continued with his official activities—and others, as we shall see. As an example of his ongoing Mint activities, it is known that in 1654 he was among those paid for assaying a large quantity of silver.[43] In 1658 he took part in Cromwell's funeral procession; and in 1659 he made an assay of copper from Nova Scotia.[44]

This was a time when great advances were being made in mathematics and science. Back in the 1640s Reynolds had been involved in the construction and verification of standard measures of capacity, such as the wine pottle 'tried by John Renalds at the Tower', dated 1641 (Connor, 1987, 155). A few years later he made precise measurements of the specific gravity of many substances, in a series of experiments described in detail by John Wybard (1650). After his death the results were communicated to the Royal Society, and much later they were rescued and published (Davies, 1748, 424–425). This work had brought Reynolds into contact with prominent mathematical practitioners. Among them were the aforementioned Wybard, Edmund Wingate (author of a very popular 'Arithmetic'), Baptist Sutton, Henry Sutton, and Henry

[43] Calendar of State Papers Domestic 1654, 456.
[44] Calendar of State Papers Colonial (America and West Indies 1), 478.

Phillipppes, all of whom are noticed in the classic work of Eva Taylor (1967). The introduction of an excise tax on alcohol in 1643 presented another set of problems, related to his metrological work, but of a more practical kind. His standard measures were perfect cylinders, but the barrels used for beer and wine were irregular in shape, and it was difficult to measure their contents accurately. This was a long-standing problem, and the existence of the excise tax greatly increased its importance in the nation's economy. The task of the excise officers was known as *gauging*, and accounts of it soon began to appear in commercial manuals. In 1656 the third edition of Henry Phillippes' *Purchasers Pattern* contained a chapter on gauging, which referred to both Reynolds and his mentor John Goodwyn as 'ancient artists' in this field (Phillippes, 1656, 178).

Evidence that John Reynolds continued his interest in gauging is provided by an instrument designed by him and made by Henry Sutton in 1656 (Jardine, 2016). The instrument is a wooden folding rule, inscribed 'Mr John Reynolds Diagonall Line for Gauging of Casks &c'. It was intended for use with one of the simplest methods of estimating the capacity of a barrel, the diagonal rod. The rod is inserted through the bung-hole in the middle of the barrel, and poked into the furthest corner, to give a single measurement, say x inches. Based on the principle that the volume of a barrel is proportional to the cube of its linear dimensions, the capacity in cubic inches is thus given by a formula we would write as kx^3, where the value of k can be estimated from a typical barrel. Of course, barrels differ in shape as well as size, but the method seems to have worked remarkably well, and the diagonal rod was still in use in the 19th century. Reynolds' rule was not itself a measuring rod, but it was presumably designed to convert the reading from a rod into gallons, the measure required by the excise officers. There was the further complication that the official gallon differed in size according to whether ale or wine was being measured, so two separate scales were needed (Biggs, 2017b). However, the most intriguing aspect of the rule is the presence on one of its faces of a scale that has been described as 'logarithmic'. By the 1650s logarithms were being used routinely in scientific work, but they were slow to take hold in the commercial world. This instrument suggests that Reynolds was familiar with the principle of the logarithmic scale, which had been incorporated into a practical calculating device, the slide rule, by Oughtred and others in the 1630s. But Reynolds' rule cannot be called a slide rule, because there is no sliding part.

The mention of Reynolds in Henry Phillippes' book, and the existence of artefacts bearing his name, would have made him familiar to the officers of the excise. Some of these men were themselves practitioners of mathematics in a minor way. For example, there is mention of Reynolds in a letter to John Collins, written in 1663 (Rigaud, 1841, 99–101). Collins was the accountant at the Excise Office, and the writer was Michael Dary, a self-taught mathematician, who was an excise officer in Bristol. He asked Collins to present his respectful duty to 'grandsire Reynolds'. The significance of the term grandsire is not clear: it may have been simply a courtesy title for a man who was then nearing the age of 80. Whatever the meaning, the letter implies that Reynolds was one of Collins' many acquaintances in the new world of mathematics that accompanied the restoration of the monarchy.

By the time the great plague reached London in 1665 Reynolds' life was drawing to a close. Sir Robert Moray, a close friend of Charles II, wrote about Mint affairs to the then Master, Sir Henry Slingsby, who had decamped to Yorkshire. He reported that Reynolds' death was imminent, and that he (Moray) had obtained an assurance from the king himself that the post of Assistant Assay Master should not be filled until Slingsby returned to London (Challis, 1991, 176). Perhaps that was a sign that an almost unknown man, with a modest title, had played an important part in the nation's affairs.

No record of John Reynolds' death has been found, and no will, possibly because of the disturbed state of London at the time of the plague. As far as we know, he did not make any original contributions to mathematics; but his legacy is still with us, in the form of books and artefacts that illuminate a remarkable period in the story of mathematics and the story of his country.

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Appendix A. Proofs of the rules mentioned in Section 11

The notion of 'proof' was formalised by the Greeks in their study of geometry, but in arithmetic the word was used in a different way. It usually meant checking the result of a calculation by doing it another way—for example, by 'casting out nines.' In the 16th century the development of algebraic symbolism made it possible to justify complicated arithmetical procedures by elementary algebra, and that was the sense in which algebra was used in Section 3 of the present paper to 'prove' the Golden Rule and the Backer Rule. The first part of this Appendix contains a similar discussion of the rules set out in Section 11. The second part contains a more abstract approach to the same rules, exhibiting their similarity to the most famous innovation of 17th century arithmetic, logarithms.

The purpose of Reynolds' *Brief and Easie Tables* was to assign to an ingot of mass m units and fineness f a *betterness* or *worseness* (measured in the same units as m), defined by

$$r = \pm m \frac{(f - s)}{s},$$

where s is the standard fineness. If the ingot is better ($f > s$) we take the plus sign, if the ingot is worse ($f < s$) we take the minus sign.

The proofs by elementary algebra are simplified if we rewrite the definition in the form

$$mf = s(m \pm r).$$

The addition rule Suppose we have two ingots whose mass, fineness, and betterness satisfy the definition: that is, $m_i f_i = s(m_i + r_i)$ for $i = 1, 2$. The result of combining them is an ingot with mass $m = m_1 + m_2$ and fineness f given by

$$mf = m_1 f_1 + m_2 f_2 = s(m_1 + r_1) + s(m_2 + r_2) = s\big(m + (r_1 + r_2)\big).$$

By definition, the betterness r is given by $mf = s(m + r)$, and so it follows that $r = r_1 + r_2$. A similar argument works if the ingots are worse.

The standarding rule Suppose we have an ingot with betterness r_B, and another with worseness r_W, where $r_B > r_W$. We want to show that if we combine them and add $r_B - r_W$ units of alloy (base metal), the result is an ingot of fineness s.

Consider the alloy as an ingot with mass $m_A = r_B - r_W$ and fineness $f_A = 0$. Its worseness is

$$r_A = -(r_B - r_W) \times (-s/s) = r_B - r_W.$$

By the addition rule, when the alloy-ingot is combined with the ingot of worseness r_W the result is an ingot with worseness r_B. Combining this with the ingot of betterness r_B, we obtain an ingot of standard fineness.

The proof of the standarding rule given above is probably a good reflection of the thought-processes of the medieval assayers who developed the method. A direct proof by elementary algebra is also possible.

To adopt the viewpoint of modern algebra, we must first define the terms 'ingot' and 'combining.' (We shall also abandon the distinction between better and worse, since negative numbers are now well-understood, mostly.) Thus, an *ingot* is a pair of real numbers (m, f), where $m > 0$ and $0 \leq f \leq 1$. If I denotes the set of all ingots, *combining* is a binary operation $\circ : I \times I \to I$ defined by

$$(m_1, f_1) \circ (m_2, f_2) = \big(m_1 + m_2, (m_1 f_1 + m_2 f_2)/(m_1 + m_2)\big).$$

The 'betterness-worseness' is a function $r : I \to \mathbb{R}$, and the addition rule says that, for any ingots a and b,

$$r(a \circ b) = r(a) + r(b).$$

At this point, we observe the analogy between r and the logarithm. The latter is a function $\log : \mathbb{R}^+ \to \mathbb{R}$ such that $\log(a \times b) = \log(a) + \log(b)$, where $\mathbb{R}^+$ denotes the positive real numbers and the binary operation is multiplication.

The analogy can be carried a bit further. When logarithms were first introduced there were several versions that satisfied the basic addition rule. The rise of decimal notation led quickly to the decision to impose the conditions $\log(1) = 0$ and $\log(10) = 1$, thus defining uniquely what we now call the *common logarithm*. In the case of the r-function, a modern algebraist might well begin by checking that any function of the form $r(m, f) = m(cf + d)$ will satisfy the addition rule. However, in this case the normalisation was not the result of clear-thinking by experts, it was the inevitable outcome of centuries of practical assaying. This imposed two conditions. First, standard silver is neither better nor worse, and so $r(m, s) = 0$, for all values of m. Second, an alloy-ingot is totally worse, so $r(m, 0) = -m$, for all values of m. Applying these conditions to the general form $m(cf + d)$ we obtain $c = 1/s$ and $d = -1$, which is precisely the function tabulated by Reynolds.

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