Memories of Bill Tutte

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Introduction

Bill Tutte's codebreaking work in World War II is now well-known, and acknowledged for its national significance. But for most of his life he enjoyed the obscurity usually bestowed upon mathematicians, even those who are the undisputed leaders in their field. I was fortunate to know him well from about 1970 onwards, during which time he gradually emerged from the cloisters of the academy to become a national treasure. For some reason this shy and retiring man seemed to feel comfortable with me, and I was privileged to interact with him as a human being, rather than a character in a Hollywood movie. Alongside my personal recollections of Tutte the man, I shall describe how his wartime achievements were gradually revealed, and his reaction to that.

First contacts

I am aware that a common feature of articles like this is that they say more about the author than the subject, but some background is inevitable. I first heard about Tutte in the late 1960s, when he was a Distinguished Professor at the University of Waterloo in Canada and I was a Lecturer at the University of Southampton. I had no research to my name, but I had studied Algebraic Topology for Part 3 of the Cambridge Tripos, and was supposed to be writing a PhD thesis in that area. It was not going well, partly because my supervisor in Southampton had fled. By some chance that I cannot recall, I came across the work of Brooks, Smith, Stone, and Tutte on Squaring the Square, done when the four authors were undergraduates at Trinity College, Cambridge.[1] This fascinated me, and led to my first mathematical publication. Other excursions into Graph Theory followed, and in August 1968 I acquired Tutte's little book on Connectivity in Graphs.[2] As well as a careful exposition of the material referred to in its title, the book covered topics that seemed ripe for further research, such as Chapter 7 (automorphisms) and Chapter 8 (girth). It was clear that the author had obtained some remarkable results, which would surely form the basis of future research in Graph Theory, hopefully including my own. And so it was. With my first

research student, Derek Smith, I had obtained some results on a family of highly symmetrical graphs, and one of Tutte's ground-breaking discoveries[3] enabled us to prove a theorem that showed there are exactly 12 graphs in our family.

In April 1969 I was flattered to receive a letter from Tutte asking for an offprint of my paper on Squaring.[4] By this time the mathematical discipline now known as 'Combinatorics' was starting to flourish, and in July 1969 a meeting in Oxford devoted to the subject was attended by some famous mathematicians, including Paul Erdős and Roger Penrose. Tutte himself was not there, but many of the talks referred to his work.[5] Cedric Smith, one of the Trinity Four, talked about colouring problems and their relationship with an object which he called the 'dichromate', which is now always referred to as the 'Tutte polynomial'. Crispin Nash-Williams gave a wide-ranging talk which included a report on the work of Tutte and his colleagues in Waterloo on a related object known as the 'chromatic polynomial'. This is a function $P_G(x)$ such that, when x is a positive integer n, its value is the number of ways of colouring the vertices of the graph G with n colours, in such a way that no two adjacent vertices are coloured alike. It follows that if n is a root of the polynomial, then G cannot be coloured with n colours. So the famous Four Colour Conjecture (as it then was) states that, if G can be drawn in the plane without crossings, then the number 4 is not a root of $P_G(x)$. Nash-Williams reported that there had been extensive calculations of chromatic polynomials of planar graphs (by hand), followed by calculation of their roots (by electronic computer), and these had revealed that many of the roots were remarkably close to the number $2.618\ldots$. This number is τ^2, where τ is the golden ratio. $\frac{1}{2}\left(1+\sqrt{5}\right)$. Tutte had recently made progress towards explaining this observation, by showing that $P_G(\tau^2)$ must be small, (although it cannot actually be equal to 0).[6]

There were many talks at the Oxford meeting, and I must admit that I have no clear recollection of most of them. The details given above are based on the published versions, which did not appear until 1971. By that time I had heard Tutte himself talk about the 'golden root', in a seminar at University College London in June 1970. That was my first sighting of the great man. Fortunately I have retained the notes of that lecture, and some correspondence that followed it,[7] which led to significant developments. In the autumn of 1970 I had moved from Southampton to Royal Holloway, part of the University of London. My research student David Sands needed a topic for his thesis, and we settled on a study of the real and complex roots of chromatic polynomials. A colleague at Royal Holloway, Mark Damerell, also became interested in this topic, so some progress was possible.

Soon after my arrival at Royal Holloway, the Head of Department told me that he wished to make use of some money that had accumulated in the Departmental Research Fund. As I was in regular contact with Bill Tutte, I suggested that we invite him for a short visit, together with about a dozen UK mathematicians whom we knew were working in his field. Rather to my surprise, Bill accepted, and the meeting was held during the week of 4 July 1971.[8]

At the meeting Bill was accompanied by his wife Dorothea, and it may have been from her that a rather surprising fact emerged. The main reason for their presence in the UK that summer was that Bill was spending some time in Cheltenham, working at the Government Communications Headquarters (GCHQ). I knew of this establishment, but I had not previously heard about Bill's connection with it. I think the fact that Bill had worked for 'the government' in the Second World War also emerged at this time, but I had no inkling of what he had done then, or what he was doing in 1971. In due course it became clear (by his silence) that Bill regarded signing the Official Secrets Act as a serious matter: 'he that hath a secret to keep must keep it secret that he hath a secret'. In 1971 the Cold War was still a fact of life, and it was to be many years before Bill would talk about the work for which he is now famous.

I think Bill enjoyed his visit to Royal Holloway.[9] Several of the participants were old friends, including Cedric Smith, Richard Rado, and Richard Guy. The younger generation was represented by John Conway, Dominic Welsh, Douglas Woodall, among others. Early in the meeting Conway mentioned in conversation that he had himself discovered a remarkable result on symmetric graphs, only to find that Tutte had published it many years earlier. He was persuaded to give a talk on his method, and in an aside to Bill he said something like 'I think you did this a long time ago', to which Bill replied 'Yes, but my notation was different'. Conway told us how his method could be implemented on a computer, a project in which he had been helped by Mike Guy (son of Richard Guy). Bill himself gave two talks, on 'The enumerative theory of chromatic polynomials' and `2-factors of bicubic graphs'. I think the first of these was concerned with his ongoing project on what he called 'chromatic sums'. He was hoping to approach the Four Colour Conjecture indirectly, by taking 'averages' of chromatic polynomials. Sadly, I cannot find any notes of either of the two talks he gave at Royal Holloway.[10]

Waterloo, 1973

The possibility of my paying a reciprocal visit to Waterloo had arisen, and in February 1972 there was an exchange of letters in which it was confirmed that I could go there for several months in 1973, financed by one of Bill's research grants. In these letters Bill continued to refer to his ongoing work on chromatic sums, and his hope that it might 'throw some light on the Four Colour Problem'.[11]

In due course, I was able to spend the months from January to April 1973 in Waterloo. Bill and Dorothea were splendid hosts. My wife and I were greeted by them at Toronto Airport, and they took us to our accommodation in the Married Students Residences, where we were very comfortable. Although the winter was severe (by British standards) they often drove us to places of interest, including Niagara Falls. I particularly remember being introduced to several species of exotic birds, including the magnificent red 'cardinals' that visited Bill and Dorothea's garden in West Montrose.

Bill was still making progress with his chromatic sums. This work involved a bewildering array of generating functions, some with three or four variables, and ingenious attempts to derive algebraic identities involving them. In retrospect, I can see why Bill was a successful codebreaker. The fact that he had studied Chemistry rather than Mathematics as an undergraduate meant that he had an individualistic approach to algebraic methods, which served him well.

My own project was a book on Algebraic Graph Theory, and I took the opportunity to trawl through Bill's publications, many of which were based on algebraic methods. Bill and I were particularly intrigued by a mysterious result on chromatic polynomials, which we referred to as the 'The Theorem of the Vanishing Coefficients'. It was the basis of a method of calculating chromatic polynomials by a multiplicative formula involving only a relatively small class of subgraphs. Bill had previously told me that he originally obtained this result in the late 1940s and had tried to get it published then. But the referee had said that it was 'not practical' and that 'it is easy to get complicated theorems in mathematics, but rather pointless'.[12] He had eventually published it in 1967 in the new *Journal of Combinatorial Theory*. Like the grumpy old referee, I found Bill's proof impenetrable, but in my opinion the result was very practical, and certainly by no means pointless. My view was reinforced by the link between chromatic polynomials and the 'partition functions' that were of great interest in theoretical physics. I managed to get a good enough grip on the material to put an outline of it in my book, but a neat proof of the key technical device still eluded me.

My time in Waterloo passed all too quickly. I returned to England with a fairly good appreciation of the work of the great man, and a fairly complete draft of my book. But I still had no idea whatsoever about Bill's work in the secret world of the codebreakers.

The blinkers start to slip, 1973-77

By 1973 a regular sequence of British Combinatorial Conferences had been established. Bill and Dorothea came to one held in Aberystwyth that year, and Bill gave a talk.[13] He was due to come to the next one in Aberdeen in 1975, but he was unable to attend because Dorothea was recovering from a recent operation. Fortunately he had prepared a talk, which was published in the proceedings.[14]

On 14 May 1977 Bill turned 60, an event that was noted in the mathematical community. In the spring he spent several weeks in Cambridge, where there was a conference and celebration dinner in Trinity College. He then returned to Waterloo, for another conference and celebration in July. Among the many tributes at Waterloo was one by Cedric Smith, which was enlivened by several anecdotes. He revealed that, in the Second World War, 'Tutte went off to do some very secret work', which was enough to get him elected to a Trinity fellowship, although the electors did not know what he had done.[15] The wider world was also starting to discover that great deeds had been done at Bletchley Park,

where the forerunner of GCHQ had been located in the 1940s. A book entitled *The Ultra Secret* had been published in 1974, and in 1977 there was a BBC television series called *The Secret War.* The headline revelation was that an electronic computer had been used at Bletchley, pre-dating the ones previously thought to have been the first of their kind. Presumably the computer had been used to help the codebreakers. The name of Alan Turing was mentioned, and the fact that in 1936 he had written a mathematical paper on the 'Turing Machine' led to speculation about what he had done. But we had no inkling of what Tutte might have achieved at Bletchley Park, and Smith did not enlighten us. Almost certainly that was because, like the fellows of Trinity, he did not know, even though he was Bill's closest friend and had corresponded with him in the war years.

It was fortunate that we did know about the mathematics that Bill had done after his time at Bletchley Park, and at Waterloo we heard many accounts of his work. Bill dedicated his own paper to his elder brother Joseph, who had died earlier that year. It was called 'All the King's Horses', and it was about a long-standing problem concerning the reconstruction of a graph from its subgraphs. He described a new version of the theory underlying the Theorem of the Vanishing Coefficients, so I returned to England for yet another conference (at Royal Holloway) with my head full of 'musters and clusters'.

Writing and travelling, 1978-85

In 1978 I met Bill twice, at conferences in the USA and Hungary.[16] We both gave invited talks in New York, but they were unrelated, and I cannot recall any conversations with him. The conference in Hungary took place in the city of Szeged. At that time it was difficult for westerners to meet Russian mathematicians, and although several Russians were listed as participants at the conference, some of them did not turn up. I was not aware that any significance should be attached to Bill's presence in Eastern Europe, although Hungary was changing rapidly, and the political situation, both internally and externally, was by no means stable. Bill's connections with GCHQ, historically and more recently, may well have led to him being advised to keep a low profile. I talked with him on several occasions, but my clearest memory is of a small gathering at László Lovász's home, where we had our first introduction to Rubik's cube.

My mind is similarly blank about Bill's visit to England in 1979, but I know it took place, because I have notes of his lecture at University College London, dated 13 December 1979.[17]

In 1982 the Department of Combinatorics and Optimisation in Waterloo organised a major colloquium to mark its 25th anniversary. Tutte spoke about 'Map Colourings and Differential Equations', a further twist in his study of chromatic sums.[18] I was present, but I cannot recall details of any conversations with Bill at that time, although there must have been several. I would almost certainly have tried to interest him in my own work, which harked back to one of his early discoveries on symmetric graphs. I had travelled to Waterloo

from Columbus, Ohio, where I had given a lecture about cubic graphs with the highest possible symmetry (technically known as '5-arc-regular'). Tutte had discovered the first such graph back in the 1940s, and only a few more had been discovered since then, so I was pleased to have found some new ones

At this time Bill was working on a new book, entitled simply *Graph Theory*. It was 'an attempt at the reference book I would have liked to have had in 1936-40', as he wrote in his Introduction.[19] I found the book quite hard going, but there were many gems waiting to be unearthed. In preparation for his official retirement in 1985 Bill gave a more friendly exposition of his life's work in a series of lectures entitled *Graph Theory As I Have Known It.* The notes of these lectures lay untouched for many years, but in 1997 I received a query from Oxford University Press asking for my opinion of a book with that title. I was delighted to support the publication – the fact that the book mentioned my name occasionally had no bearing, of course.[20]

Cedric Smith's discreet and unspecific comment in 1977 about Bill's 'very secret work' was my first indication that he might have been part of something of national importance. Over the next few years hints about the work done at Bletchley Park began to appear in the popular press and the media: most of them were speculations about the activities of Alan Turing and some kind of computer. In 1983 Andrew Hodges published his biography of Turing.[21] The book was well-received, but from my point of view it was disappointing, because the account of the work done at Bletchley Park was rather patchy. It is now clear that the gaps occurred because some very significant material was still covered by the Official Secrets Act at that time. However, it was good to find that Tutte was mentioned, albeit briefly. On page 230 he is credited with a 'decisive observation' concerning a new and troublesome kind of cipher known as 'Fish', and there is mention of a subsequent contribution by Turing. On pages 267-8 it is stated that an electronic machine called 'Colossus' had been developed to deal with the Fish messages, but Tutte is not mentioned. (A footnote on page 332 refers to Tutte in a different context.[22])

In retirement, 1986-97

Bill's 'retirement' was, course, only nominal. He continued to work on his long-term research projects, but I think he found travelling less attractive, especially when Dorothea's health began to decline. He had remained in contact with the family of his elder brother Joseph, who had died in 1977. Joseph's daughter, Jeanne Youlden, still lived in Newmarket, near Bill's childhood home: he visited her regularly, and she accompanied him on his installation as a Fellow of the Royal Society of London in May 1987.[23] Jeanne and Bill were actually quite close in age, since Jeanne's father was 16 years older than Bill. Her three children knew that their mild-mannered Uncle Bill was a distinguished mathematician, and that he had done some secret work in wartime, but they must have been surprised when the full significance of his codebreaking activities was gradually revealed.

The next revelations were partly the result of the publication in 1993 of a book called *Codebreakers*, which contained reminiscences of some of the people who had worked at Bletchley Park. Several mathematicians whose names were familiar to me from my student days were mentioned, including the Cambridge group of Dennis Babbage, Shaun Wylie and Derek Taunt. Indeed, it was clear that the entire mathematical community had been strongly represented at Bletchley Park, a fact that came as a surprise. It was disappointing to find that there was only one reference to Tutte, relating to work on the German ciphers that were collectively known as 'Fish', and in particular the one called 'Tunny'.[24] I think this book suggested to Bill that some of his work at Bletchley Park was no longer covered by the Official Secrets Act (and he may well have sought official advice about that) .

In 1994 Dorothea, Bill's wife and companion of more than forty years, died of cancer. Her presence had always helped him to overcome his natural shyness, and he must have been bereft. He was still in good health, and he had to make plans for his future. He might have stayed in Canada, with his academic family at the University of Waterloo, but in 1996 he decided to return to English roots, and his relatives in Newmarket.

At the beginning of 1997 he wrote telling me about the work he had been doing 'while hiding away in Newmarket these last few months'.[25] He had turned again to the roots of chromatic polynomials of planar graphs, and was trying to understand why these roots seemed to cluster around numbers of the form $2 + 2\cos(2\pi/n)$, the famous 'golden root' being the case $n = 5$. I invited him to talk to our mathematics seminar at the LSE, and he agreed, but warned me that he would be spending the month of May in Canada. Of course, that would be his eightieth birthday. I was on Sabbatical Leave, and was delighted when Chris Godsil offered to finance a research visit to Waterloo, so I was able to attend the celebrations.

Bill's birthday was on 14 May. A few days earlier an article had appeared in the *New Scientist* under the title 'Colossal Adventures'.[26] It was about efforts to build a replica of Colossus, the Bletchley Park computer, and there was some information, quoted from the *Codebreakers* book, about how Tutte's work had led to its construction. Tony Sale, the main architect of the re-built machine, was quoted as saying that Tutte's achievement was 'the greatest intellectual feat of the whole war.' I think Bill was slightly amused by this, and I recall him telling me about the pivotal role played by large sheets of squared paper, a resource with which Bletchley Park was well-supplied. A few years later I was taught to do elementary arithmetic using squared paper, although post-war austerity meant that the sheets were very small. But I digress.

On his return to England, Bill fulfilled his promise to visit the LSE. His talk was arranged for 19 June 1997, and we had probably the largest audience yet for a mathematical talk at the LSE. However, Bletchley Park was not mentioned. My clearest recollection is of an incident with a coffee-machine, illustrating Bill's fascination with (and ignorance of) machines of any kind.[27]

His own words, 1998-2000

In 1998 the University of Waterloo set up a Centre for Applied Cryptographic Research. Bill was duly installed as the Honorary Director and, at the opening ceremony on 19 June 1998, he gave a talk on 'Fish and I'.[28] He began by referring to the *Codebreakers* book, and proceeded to set out in detail what he remembered about his own contributions. In fact (with the aid of squared paper) he had discovered a period of 41 in a long string of about 4000 bits, which was known to have been part of a key for the Tunny cipher. With the help of colleagues, his method was then applied to the entire string, and the result was a virtual reconstruction of the Tunny machine. This was an amazing breakthrough, but it did not immediately lead to a practical method of decrypting current messages – which was, of course, the object of the exercise. He went on to describe how he had discovered a way of doing this, which he called the 'statistical method'. It was clear that his method would work, but it required more squared paper than even Bletchley could provide. He also invented a method (known as 'rectangling') that facilitated the process of breaking the Tunny messages when the keys were changed frequently . The calculations involved in these methods had to be mechanized, and this led to Bletchley Park's most famous invention, the electronic computer Colossus. Tutte himself played no part in the making of Colossus but it was his discoveries that demanded it.

My next meeting with Bill was at the Seventeenth British Combinatorial Conference, held in Canterbury in July 1999. His talk on 'The Coming of the Matroids' explained how some of his work at Bletchley Park had influenced his mathematical career.[29] (A 'matroid' is a mathematical structure in which some, but not all, of the properties of matrices are taken as axioms.) He had explored this idea in his unpublished PhD thesis, and later developed it into a theory that found many applications.

Bill appeared to be in good shape at this time. He had returned to live in Canada, possibly because he appreciated the recognition that he received there. Even so, some loose ends remained, as I was to discover in my capacity as Honorary Librarian of the London Mathematical Society (LMS). In August 2000 I received a letter from Judith Field (on behalf of the British Society for the History of Mathematics) to the President of the LMS.[30] She told us that In 1945 a 500-page 'General Report on Tunny with Emphasis on Statistical Methods' had been prepared. It had recently been declassified, and Judith was seeking financial backing for a project that would make it available online. This was duly approved, and the document eventually appeared, although I do not know whether the LMS was ever asked for its contribution.[31]

The General Report on Tunny made few references to individual contributions, but its publication must have convinced Bill that the time was ripe for further personal

reminiscences. He had been worried by the fact that he had not had the opportunity to make amendments to the published version of 'Fish and I' and he now took the opportunity to expand and correct that article. The new article was probably written in August 2000, soon after the declassification of the Tunny Report. It contains more details of his own contributions, and a few anecdotes.[32]

An accurate historical account of what had been achieved at Bletchley Park was now being pieced together. Alan Turing's tragic death in 1954 meant that he had had no chance to speak for himself. It was now clear that his wartime work was mainly concerned with the Enigma cipher, and the design of electro-mechanical machines (known as 'bombes') that facilitated its decryption. Before the war he had written his seminal paper on the 'Turing Machine' (an abstract concept) and after the war he had made significant contributions to the construction of machines that were among the first electronic computers. But Turing had no hand in the Colossus project. That was due to the conceptual ideas of Max Newman and the practical wizardry of Tommy Flowers.

Tutte had made several important contributions to the Tunny project. His discovery of the period of 41 was his 'Jane Austen moment' – the insight that characterizes genius. His statistical method was more of a 'Sherlock Holmes moment' – putting together the pieces of a jigsaw that led to a clear conclusion. It is ironic that the mathematical details of the statistical method (as extended by Jack Good and others) seem to have been the justification for the long delay in revealing some of Bletchley Park's secrets. Are there more documents still locked away?

Conclusion

After Bill returned to Canada, it was good to hear that his achievements were being recognized officially there. In 2001 he was inducted as an Officer of the Order of Canada, the highest award he ever received. The award was well-timed, because in March 2002 he was diagnosed with lymphoma of the spleen, and six weeks later, a few days before his 85th birthday, he died.

The sad news spread quickly in the mathematical community. I was concerned that Bill's achievements should be properly covered in the national newspapers, and I hurriedly composed an obituary. This was published in the *Independent* on 9 May 2002. The *Times*, *Guardian*, *Telegraph* and *Scotsman* also published obituaries, but I was glad that I had decided to write one myself, because Tutte's work as the master builder of Graph Theory was mentioned only briefly in the others.[33] In 2004 the *Journal of Combinatorial Theory (Series B)* devoted a special issue to Bill's memory, and I was pleased to be able to contribute an article about some work on his pet topic, chromatic polynomials.

A coherent account of all that had been done at Bletchley Park was still hard to find at that time. Although the official reasons for obfuscation had been largely removed, a mist of

fact and fiction had formed, and it took some time to clear. Alan Turing had become an icon, and fictional works based loosely on his life proliferated.[34] The Colossus too had become a celebrity. Slowly, more sober accounts began to appear, and the historical record gradually acquired some semblance of reality.[35] In 2011 Tutte's work at Bletchley Park reached a wider audience than he would ever have thought possible when the BBC broadcast a television programme about 'Bletchley Park's Lost Heroes'.[36]

The 'Lost Heroes' programme convinced the people of Newmarket that Tutte was a figure of national importance. The local newspaper campaigned for a Memorial Fund to be established, and they pointed out that his work had received no recognition whatsoever from the government. As a result, the Prime Minister, David Cameron, wrote to Jeanne Youlden to express his appreciation of Bill's wartime achievements.[37] The Memorial Fund was duly established, and in 2014 an impressive sculptural memorial was opened. In 2017 the centenary of Bill's birth was marked by a blue plaque.[38] Commemorations also took place elsewhere in the UK and in Canada and Australia.

I think it is appropriate to end with a personal assessment of how the long delay in acknowledging his wartime achievements affected Bill. He was not a man to seek the limelight, and his post-war work as a mathematician brought him the kind of respect that he valued, because it came from people who could fully appreciate what he did. In my opinion he would have been unhappy if he had had to deal with the kind of attention that was given to Turing (whose early death saved him from that problem). When the stories of Bletchley Park began to leak out, Bill was not moved to emphasize the part that he had played. Latterly, when a clearer picture began to emerge, he wanted to put on record his own memories of the details and the chronology, but he displayed no rancour about the delay. Now he is gone, and will forever remain (to use a quotation he used himself in a different context) 'silent upon a peak in Darien'.[39]