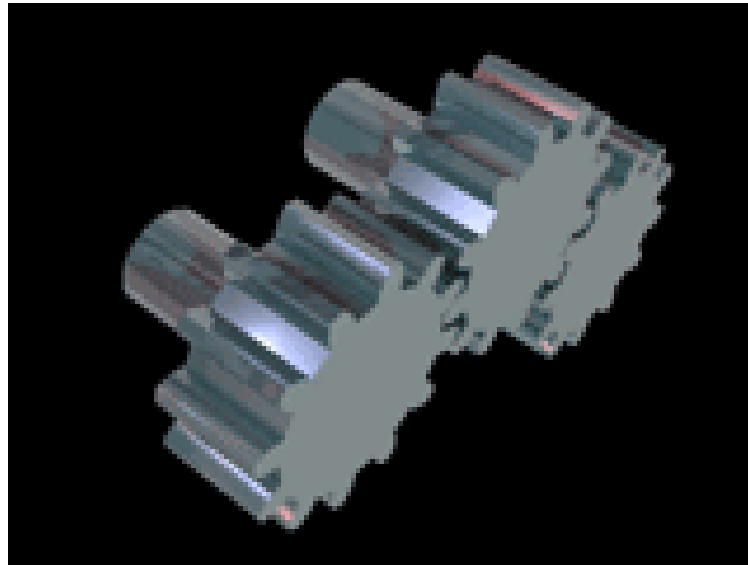


GEAR DESIGN

Reference Books: (i) *Machine Design* by R. S. Khurmi
(ii) Maitra G.M., Prasad L.V., “Hand book of Mechanical Design”, II Edition, Tata McGraw-Hill



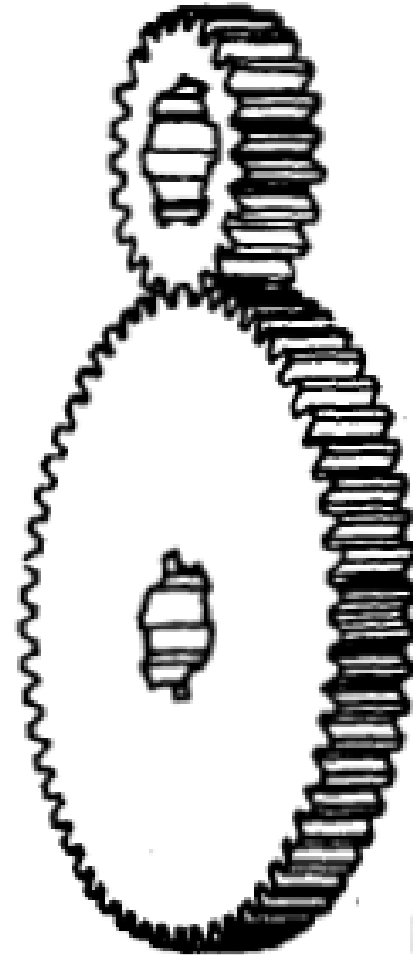
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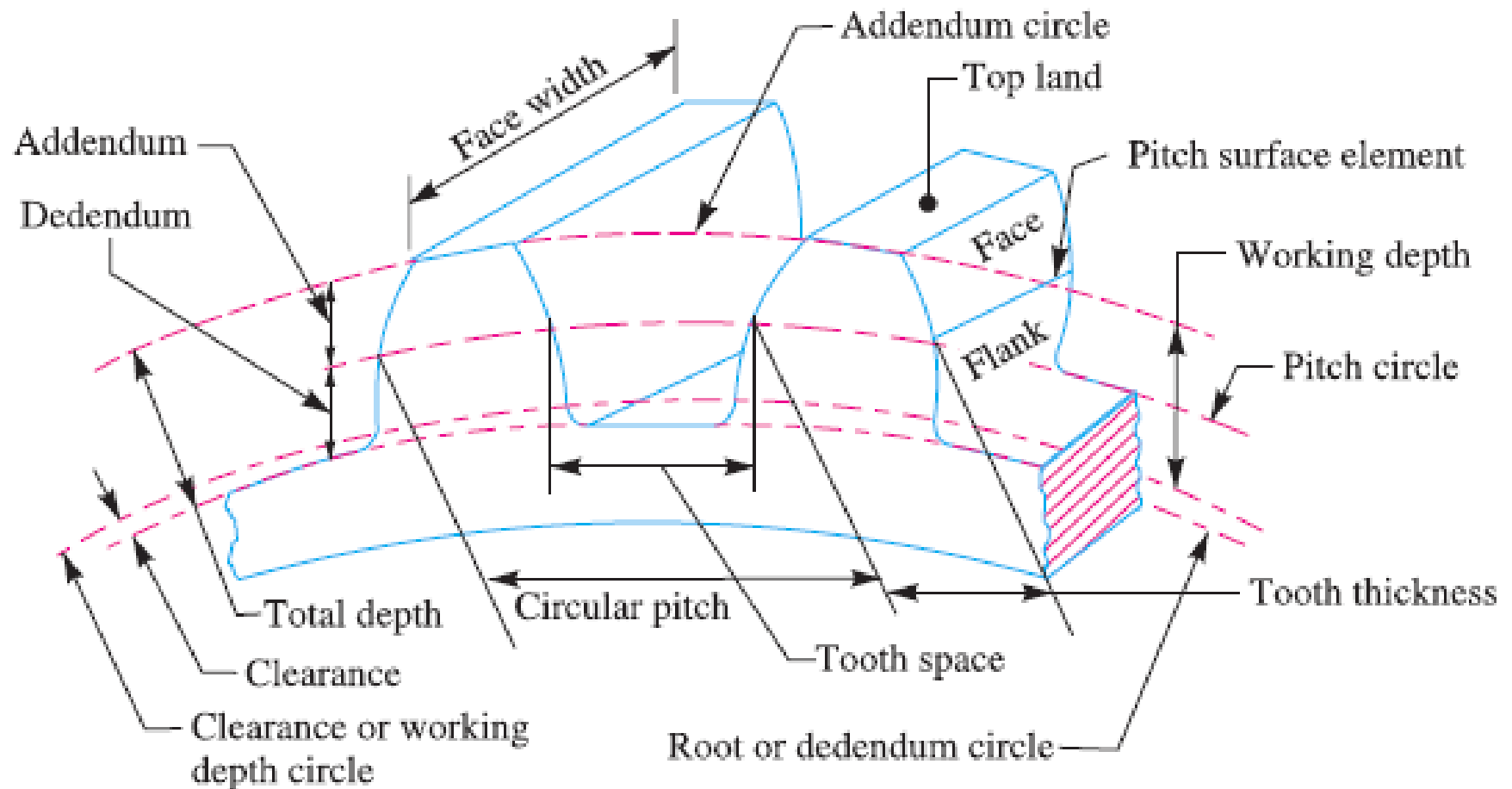
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SPUR GEAR DESIGN



Spur Gear Terminology



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Pressure angle or angle of obliquity: *It is the angle between the common normal to two gear teeth at the point of contact and the common tangent at the pitch point. It is usually denoted by ϕ . The standard pressure angles are $14\frac{1}{2}^\circ$ and 20° .*

Circular pitch. *It is the distance measured on the circumference of the pitch circle from a point of one tooth to the corresponding point on the next tooth. It is usually denoted by p_c .*

Mathematically,

$$\text{Circular pitch, } p_c = \pi D/T$$

where

D = Diameter of the pitch circle, and

T = Number of teeth on the wheel.

A little consideration will show that the two gears will mesh together correctly, if the two wheels have the same circular pitch.

Note : If D_1 and D_2 are the diameters of the two meshing gears having the teeth T_1 and T_2 respectively; then for them to mesh correctly,

$$p_c = \frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2} \quad \text{or} \quad \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Continue...

Diametral pitch. It is the ratio of number of teeth to the pitch circle diameter in millimetres.

It denoted by *pd*. Mathematically,

$$\text{Diametral pitch, } p_d = \frac{T}{D} = \frac{\pi}{p_c} \quad \dots \left(\because p_c = \frac{\pi D}{T} \right)$$

where T = Number of teeth, and
 D = Pitch circle diameter.

Module. It is the ratio of the pitch circle diameter in millimetres to the number of teeth. It is usually denoted by *m*.

Mathematically, Module, $m = D / T$

The recommended series of modules in Indian Standard are 1, 1.25, 1.5, 2, 2.5, 3, 4, 5, 6, 8, 10, 12, 16, 20, 25, 32, 40 and 50.

The modules 1.125, 1.375, 1.75, 2.25, 2.75, 3.5, 4.5, 5.5, 7, 9, 11, 14, 18, 22, 28, 36 and 45 are of second choice.

Form of Gear Teeth

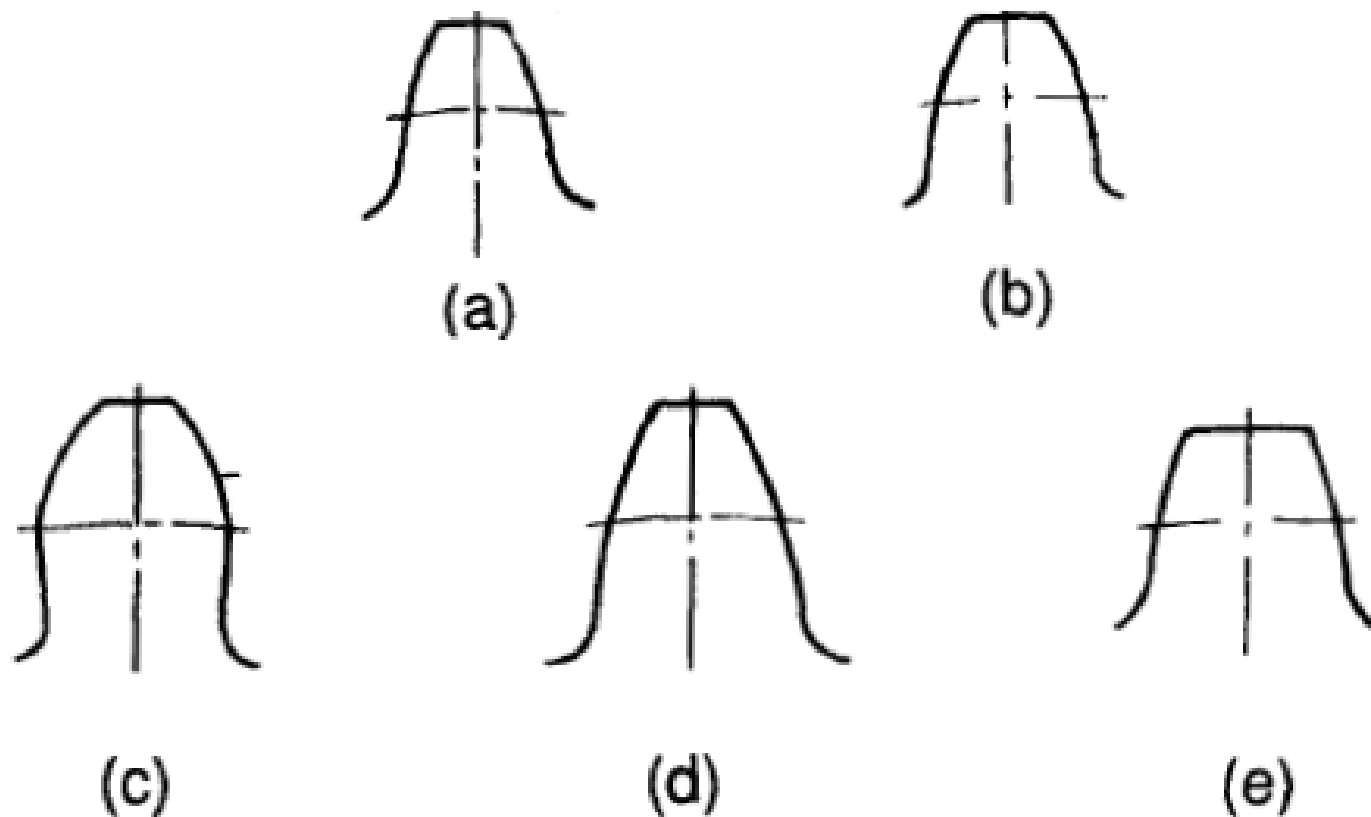
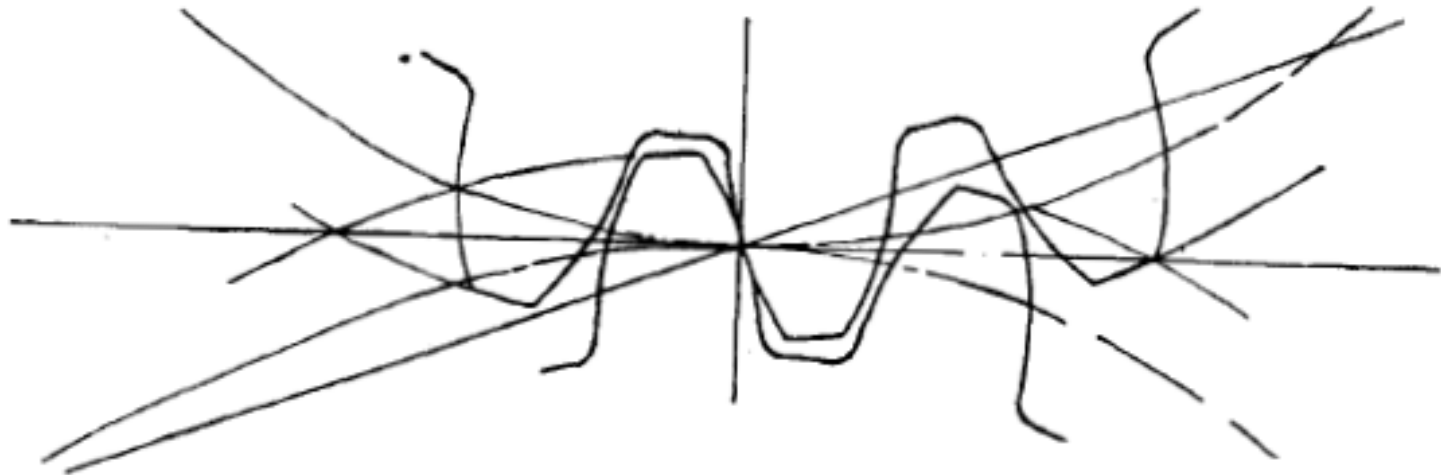


Fig. 2.7 Comparison of gear tooth profiles
(a) Cycloidal, (b) Involute, (c) $14\frac{1}{2}^\circ$ full-depth involute,
(d) 20° full-depth involute, (e) 20° stub involute

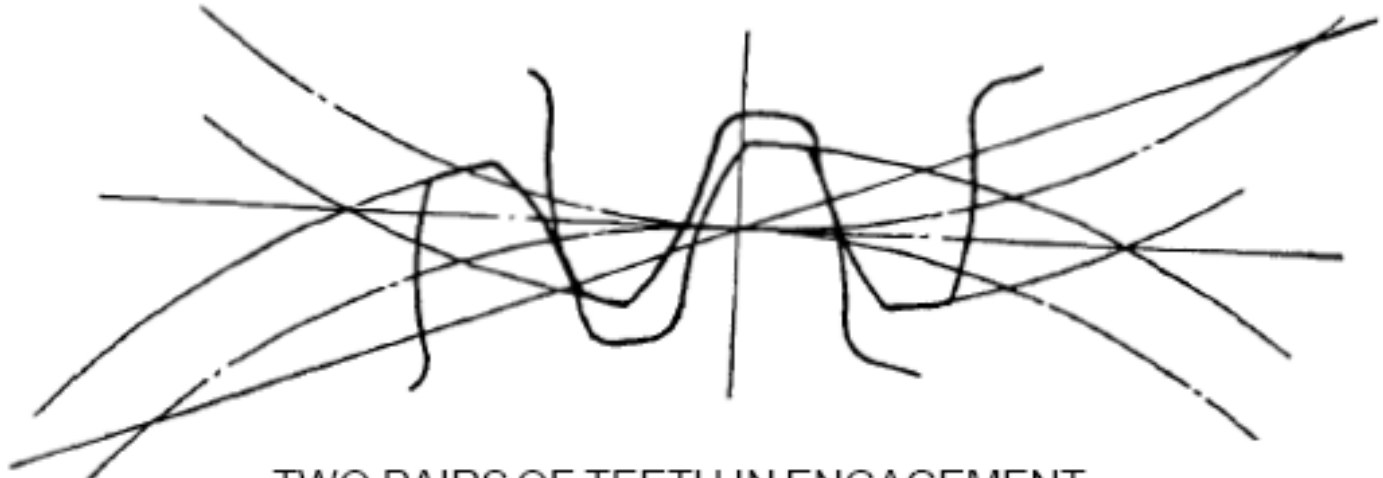
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- The $14\frac{1}{2}^\circ$ ***composite system is used for general purpose gears. It is stronger but has no interchangeability.***
- The tooth profile of this system has cycloidal curves at the top and bottom and involute curve at the middle portion. The teeth are produced by formed milling cutters or hobs. The tooth profile of the $14\frac{1}{2}^\circ$ ***full depth involute system was developed for use with gear hobs for spur and*** helical gears.
- The tooth profile of the 20° ***full depth involute system may be cut by hobs. The increase of the*** pressure angle from $14\frac{1}{2}^\circ$ to 20° results in a stronger tooth, because the tooth acting as a beam is wider at the base. The 20° ***stub involute system has a strong tooth to take heavy loads.***

Gear Motion Mechanism

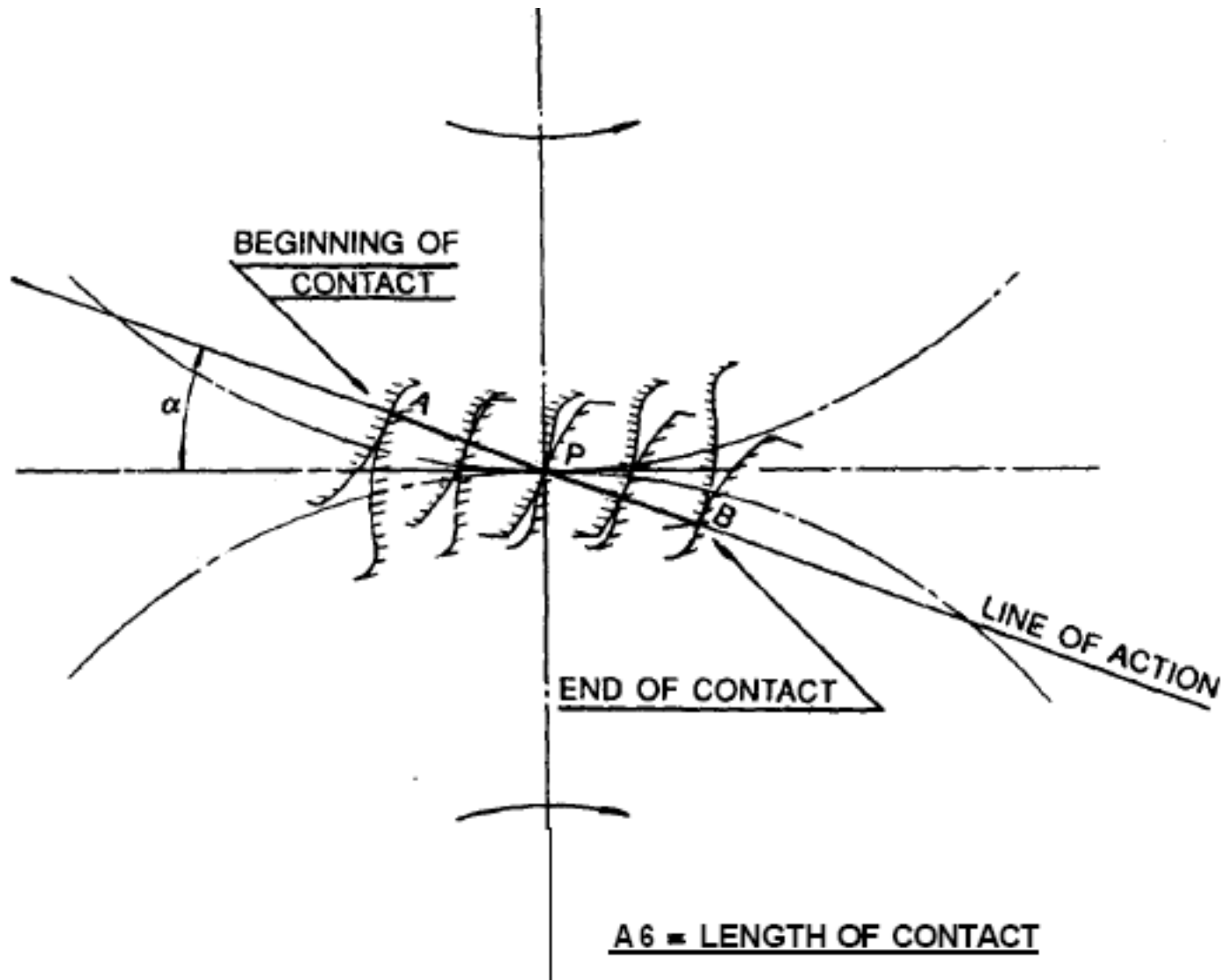


ONE PAIR OF TEETH IN ENGAGEMENT

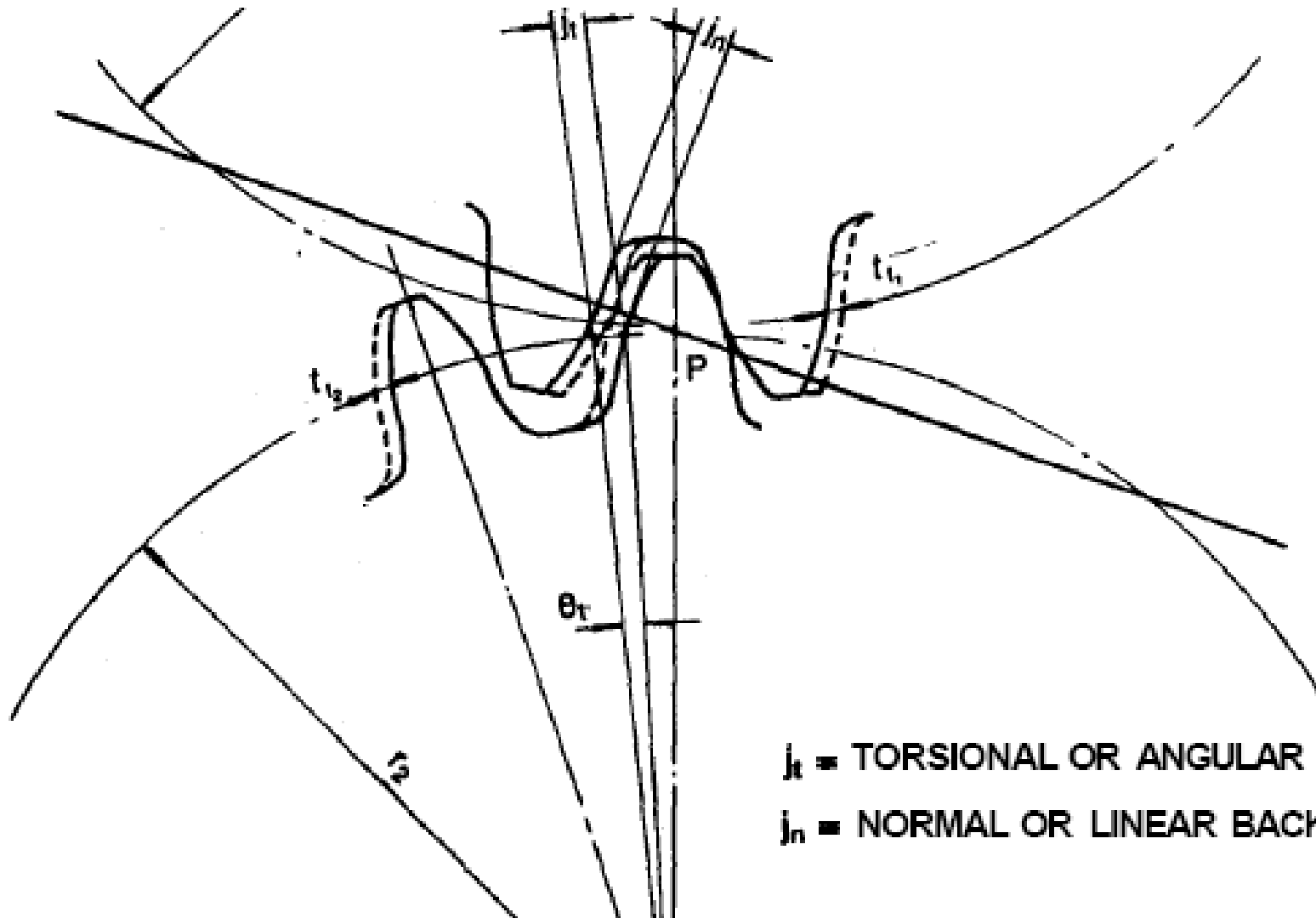


TWO PAIRS OF TEETH IN ENGAGEMENT

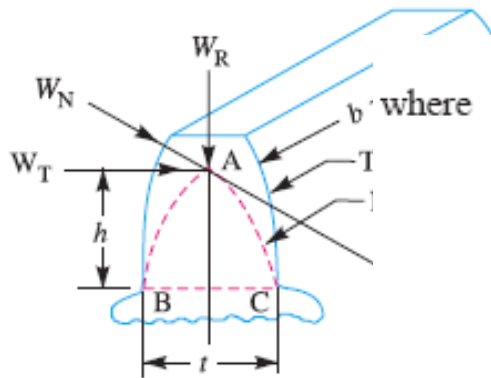
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Beam Strength of Spur Gear Teeth – Lewis Equation



$$\sigma_w = M \cdot y / I \quad \dots (i)$$

M = Maximum bending moment at the critical section $BC = W_T \times h$,

W_T = Tangential load acting at the tooth,

h = Length of the tooth,

y = Half the thickness of the tooth (t) at critical section $BC = t/2$,

I = Moment of inertia about the centre line of the tooth $= b \cdot t^3 / 12$,

b = Width of gear face.

Substituting the values for M , y and I in equation (i), we get

$$\sigma_w = \frac{(W_T \times h) \cdot t / 2}{b \cdot t^3 / 12} = \frac{(W_T \times h) \times 6}{b \cdot t^2}$$

or

$$W_T = \sigma_w \times b \times t^2 / 6 h$$

In this expression, t and h are variables depending upon the size of the tooth (*i.e.* the circular pitch) and its profile.

Let

$t = x \times p_c$, and $h = k \times p_c$; where x and k are constants.

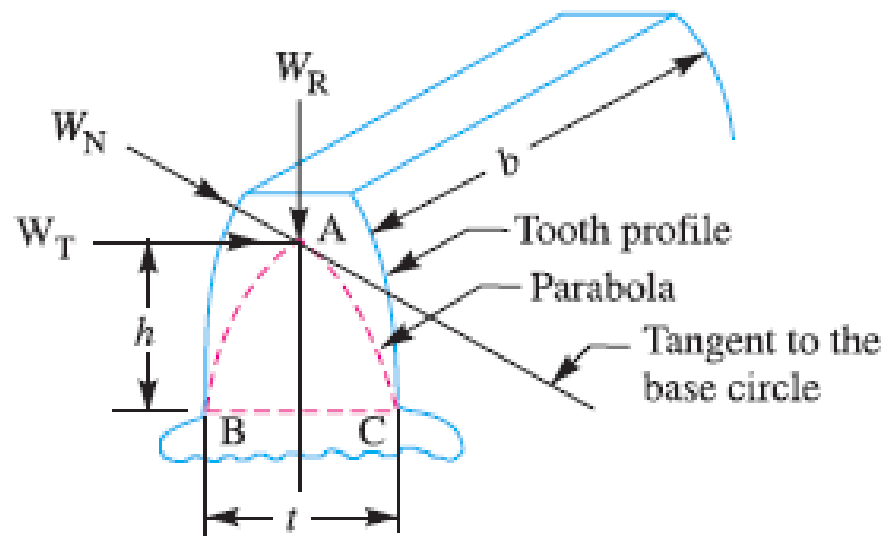
$$\therefore W_T = \sigma_w \times b \times \frac{x^2 \cdot p_c^2}{6k \cdot p_c} = \sigma_w \times b \times p_c \times \frac{x^2}{6k}$$

Substituting $x^2 / 6k = y$, another constant, we have

$$W_T = \sigma_w \cdot b \cdot p_c \cdot y = \sigma_w \cdot b \cdot \pi m \cdot y \quad \dots (\because p_c = \pi m)$$

The quantity y is known as **Lewis form factor** or **tooth form factor** and W_T (which is the tangential load acting at the tooth) is called the **beam strength of the tooth**.

Design of Spur Gear



$$W_T = \sigma_w \cdot b \cdot p_c \cdot y = \sigma_w \cdot b \cdot \pi m \cdot y \quad \dots (\because p_c = \pi m)$$

$$\begin{aligned} y &= 0.124 - \frac{0.684}{T}, \text{ for } 14\frac{1}{2}^\circ \text{ composite and full depth involute system.} \\ &= 0.154 - \frac{0.912}{T}, \text{ for } 20^\circ \text{ full depth involute system.} \\ &= 0.175 - \frac{0.841}{T}, \text{ for } 20^\circ \text{ stub system.} \end{aligned}$$

$$\sigma_w = \sigma_o \times C_v$$

σ_o = Allowable static stress, and

C_v = Velocity factor.

$$C_v = \frac{3}{3 + v}, \text{ for ordinary cut gears operating at velocities upto } 12.5 \text{ m / s.}$$

$$= \frac{4.5}{4.5 + v}, \text{ for carefully cut gears operating at velocities upto } 12.5 \text{ m/s.}$$

$$= \frac{6}{6 + v}, \text{ for very accurately cut and ground metallic gears operating at velocities upto } 20 \text{ m / s.}$$

$$= \frac{0.75}{0.75 + \sqrt{v}}, \text{ for precision gears cut with high accuracy and operating at velocities upto } 20 \text{ m / s.}$$

$$= \left(\frac{0.75}{1 + v} \right) + 0.25, \text{ for non-metallic gears.}$$

Dynamic Tooth Load Checking

$$W_D = W_T + W_I = W_T + \frac{21 v (b.C + W_T)}{21 v + \sqrt{b.C + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

- K = A factor depending upon the form of the teeth.
- = 0.107, for $14\frac{1}{2}^\circ$ full depth involute system.
 - = 0.111, for 20° full depth involute system.
 - = 0.115 for 20° stub system.

Static Tooth Load Checking

Static tooth load or beam strength of the tooth,

$$W_S = \sigma_e \cdot b \cdot p_c \cdot y = \sigma_e \cdot b \cdot \pi \cdot m \cdot y$$

For safety, against tooth breakage, the static tooth load (WS) should be greater than the dynamic load (WD)

Wear Tooth Load Checking

$$W_w = D_p \cdot b \cdot Q \cdot K$$

W_w = Maximum or limiting load for wear in newtons,

D_p = Pitch circle diameter of the pinion in mm,

b = Face width of the pinion in mm,

Q = Ratio factor

$$= \frac{2 \times V.R.}{V.R. + 1} = \frac{2 T_G}{T_G + T_p}, \text{ for external gears}$$

$$= \frac{2 \times V.R.}{V.R. - 1} = \frac{2 T_G}{T_G - T_p}, \text{ for internal gears.}$$

$V.R.$ = Velocity ratio = T_G / T_p ,

K = Load-stress factor (also known as material combination factor) in N/mm^2 .

The wear load (W_w) should not be less than the dynamic load (W_D).

$$K = \frac{(\sigma_{es})^2 \sin \phi}{1.4} \left(\frac{1}{E_p} + \frac{1}{E_G} \right)$$

σ_{es} = Surface endurance limit in MPa or N/mm^2 ,

ϕ = Pressure angle,

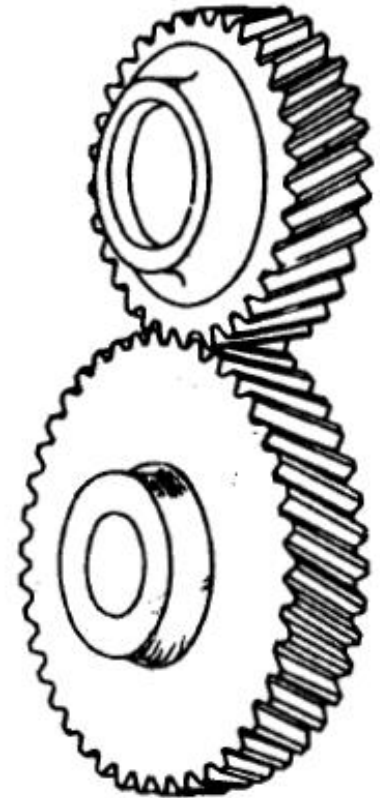
E_p = Young's modulus for the material of the pinion in N/mm^2 , and

E_G = Young's modulus for the material of the gear in N/mm^2 .

Parallel Axis Helical Gear Design

Introduction

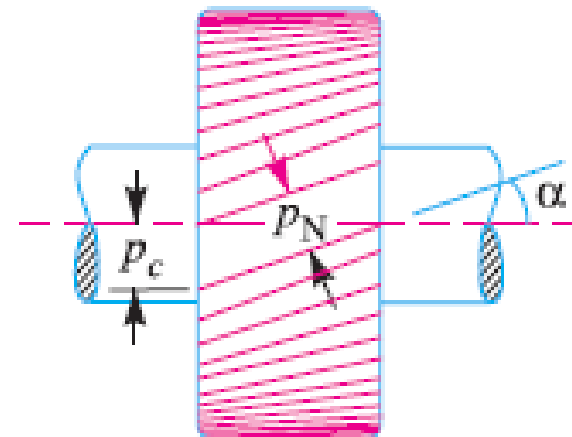
- A helical gear has teeth in form of helix around the gear. Two such gears may be used to connect two parallel shafts in place of spur gears.
- The helixes may be right handed on one gear and left handed on the other. The pitch surfaces are cylindrical as in spur gearing, but the teeth instead of being parallel to the axis, wind around the cylinders helically like screw threads. The teeth of helical gears with parallel axis have line contact, as in spur gearing.
- This provides gradual engagement and continuous contact of the engaging teeth. Hence helical gears give smooth drive with a high efficiency of transmission



Helical Gear Terminology

- **1. Helix angle.** It is a constant angle made by the helices with the axis of rotation.
- **2. Axial pitch.** It is the distance, parallel to the axis, between similar faces of adjacent teeth. It is the same as circular pitch and is therefore denoted by p_c . The axial pitch may also be defined as the circular pitch in the plane of rotation or the diametral plane.
- **3. Normal pitch.** It is the distance between similar faces of adjacent teeth along a helix on the pitch cylinders normal to the teeth. It is denoted by p_N . The normal pitch may also be defined as the circular pitch in the normal plane which is a plane perpendicular to the teeth. Mathematically, normal pitch,

$$p_N = p_c \cos \alpha$$



Continue...

- **4. Face Width:**

$$b = \frac{1.15 p_c}{\tan \alpha} = \frac{1.15 \times \pi m}{\tan \alpha} \dots (\because p_c = \pi m)$$

b = Minimum face width, and
 m = Module.

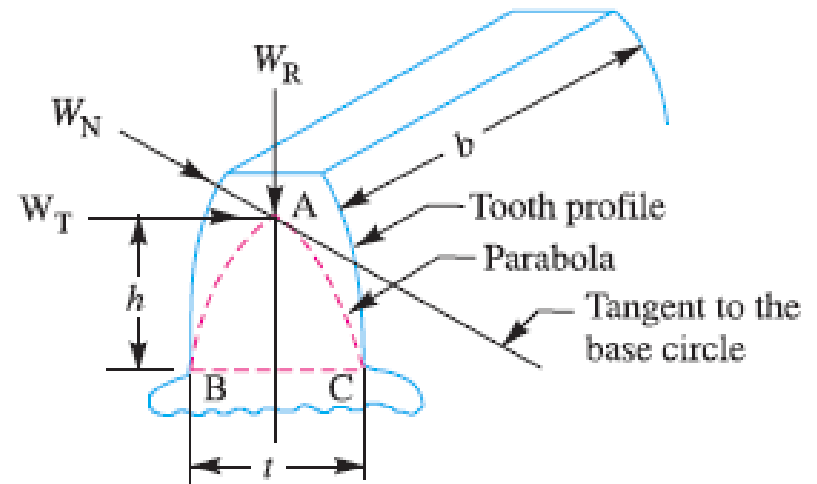
5. Formative or Equivalent Number of Teeth for Helical Gears

$$T_E = T / \cos^3 \alpha$$

T = Actual number of teeth on a helical gear, and

α = Helix angle.

Beam Strength of Helical Gear Teeth – Lewis Equation



$$W_T = (\sigma_o \times C_v) b \cdot \pi m y'$$

W_T = Tangential tooth load,

σ_o = Allowable static stress,

C_v = Velocity factor,

b = Face width,

m = Module, and

y' = Tooth form factor or Lewis factor corresponding to the formative or virtual or equivalent number of teeth.

$$C_v = \frac{6}{6 + v}, \text{ for peripheral velocities from 5 m / s to 10 m / s.}$$

$$= \frac{15}{15 + v}, \text{ for peripheral velocities from 10 m / s to 20 m / s.}$$

$$= \frac{0.75}{0.75 + \sqrt{v}}, \text{ for peripheral velocities greater than 20 m / s.}$$

$$= \frac{0.75}{1 + v} + 0.25, \text{ for non-metallic gears.}$$

Dynamic Tooth Load Checking

$$W_D = W_T + \frac{21 v (b.C \cos^2 \alpha + W_T) \cos \alpha}{21 v + \sqrt{b.C \cos^2 \alpha + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_P} + \frac{1}{E_G}}$$

- K = A factor depending upon the form of the teeth.
- = 0.107, for $14\frac{1}{2}^\circ$ full depth involute system.
 - = 0.111, for 20° full depth involute system.
 - = 0.115 for 20° stub system.

Static Tooth Load Checking

$$W_s = \sigma_e . b . \pi m . y'$$

For safety, against tooth breakage, the static tooth load (WS) should be greater than the dynamic load (WD)

Wear Tooth Load Checking

$$W_w = \frac{D_p \cdot b \cdot Q \cdot K}{\cos^2 \alpha}$$

The wear load (Ww) should not be less than the dynamic load (WD).

$$K = \frac{(\sigma_{es})^2 \sin \phi_N}{1.4} \left[\frac{1}{E_p} + \frac{1}{E_G} \right]$$

ϕ_N = Normal pressure angle.

Design of Bevel Gear

Strength of Bevel Gears

$$W_T = (\sigma_o \times C_v) b \pi m y' \left(\frac{L - b}{L} \right)$$

σ_o = Allowable static stress,

C_v = Velocity factor,

$$= \frac{3}{3 + v}, \text{ for teeth cut by form cutters,}$$

$$= \frac{6}{6 + v}, \text{ for teeth generated with precision machines,}$$

v = Peripheral speed in m / s,

b = Face width,

m = Module,

y' = Tooth form factor (or Lewis factor) for the equivalent number of teeth,

L = Slant height of pitch cone (or cone distance),

$$= \sqrt{\left(\frac{D_G}{2} \right)^2 + \left(\frac{D_P}{2} \right)^2}$$

D_G = Pitch diameter of the gear, and

D_P = Pitch diameter of the pinion.

$$y'_P = 0.124 - \frac{0.686}{\tau}$$

$$y'_G = 0.124 - \frac{0.686}{T_{EG}}$$

$$T_{EP} = T_P \cdot \sec \theta_{P1}$$

$$T_{EG} = T_G \cdot \sec \theta_{P2}$$

$$\theta_{P1} = \tan^{-1} \left(\frac{\sin \theta_S}{V.R + \cos \theta_S} \right)$$

$$\theta_{P2} = \tan^{-1} \left(\frac{\sin \theta_S}{\frac{1}{V.R} + \cos \theta_S} \right)$$

θ_{P1} = Pitch angle for the pinion,

θ_{P2} = Pitch angle for the gear,

θ_S = Angle between the two shaft axes,

Standard Proportions

1. Addendum, $a = 1\ m$
2. Dedendum, $d' = 1.2\ m$
3. Clearance $= 0.2\ m$
4. Working depth $= 2\ m$
5. Thickness of tooth $= 1.5708\ m$

where m is the module.

Check for Dynamic Load

$$W_D = W_T + \frac{21 \nu (b.C + W_T)}{21 \nu + \sqrt{b.C + W_T}}$$

$$C = \frac{K.e}{\frac{1}{E_p} + \frac{1}{E_G}}$$

Checking for Wear Load

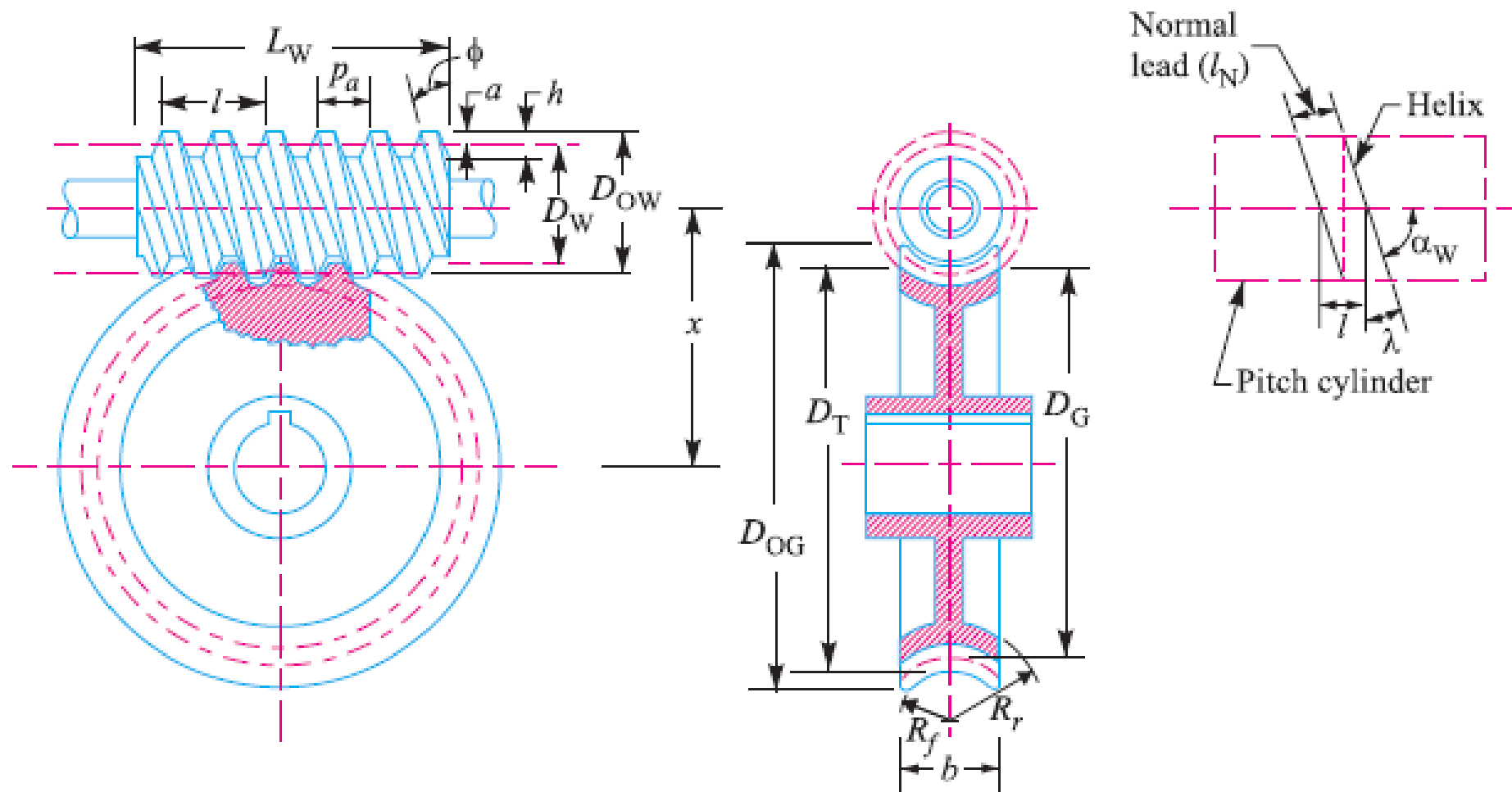
$$W_w = \frac{D_p . b . Q . K}{\cos \theta_{p1}}$$

$$Q = \frac{2 T_{EG}}{T_{EG} + T_{EP}}$$

$$W_s = \sigma_e . b . \pi m . y' \left(\frac{L - b}{L} \right)$$

$$K = \frac{(\sigma_{es})^2 \sin \phi}{1.4} \left[\frac{1}{E_P} + \frac{1}{E_G} \right]$$

Worm Gear Design



Design a worm gear set to deliver 12 kW from a shaft rotating at 1500 rpm to another rotating at 75 rpm.

Solution:

20° normal pressure angle worm gear is assumed for which the lead angle should not exceed 25° (Table 1) and Z_2 minimum is 21 (Table 2). Allowing 6° lead per thread of the worm, the worm could have 4 or less teeth. $Z_1 = 4$ or quadruple threaded worm is assumed

WORM GEARS – GEOMETRY AND NOMENCLATURE

Table 1. Maximum Worm Lead Angle and Worm Gear Lewis Form Factor for Various Pressure angles.

Pressure Angle Φ_n (Degrees)	Maximum Lead Angle λ (degrees)	Lewis form factor y	Modified Lewis form factor Y
14.5	15	0.100	0.314
20	25	0.125	0.393
25	35	0.150	0.473
30	45	0.175	0.550

WORM GEARS–DESIGN GUIDELINES

Table 2. Minimum number of teeth in the worm gear

Pressure angle ϕ_n	14.5°	17.5°	20°	22.5°	25°	27.5°	30°
Z_2 minimum	40	27	21	17	14	12	10

$$i = n_1 / n_2 = 1500 / 75 = 20 = Z_2 / Z_1$$

$$\omega_1 = 2\pi n_1 / 60 = 2 \times 3.14 \times 1500 / 60 = 157 \text{ rad/s}$$

$$Z_2 = i \times Z_1 = 20 \times 4 = 80$$

A centre distance of 250 mm (as per R10 series) is assumed.

$$\frac{C^{0.875}}{3.0} \leq d_1 \leq \frac{C^{0.875}}{1.7}$$

$$d_1 \geq C^{0.875} / 3 = 250^{0.875} / 3 \geq 42 \text{ mm and}$$

$$d_1 \leq C^{0.875} / 1.7 = 75 \text{ mm.}$$

$d_1 = 72 \text{ mm}$ is taken.

Since $d_1 \approx 4p_2$ or circular pitch,

$$p_2 = d_1/4 = 72 / 4 = 18 \text{ mm}$$

$m = p / \pi = 18/3.14 = 5.73 \text{ mm}$ take standard module of 6mm.

Hence, $d_2 = m Z_2 = 6 \times 80 = 480 \text{ mm}$.

Actual centre distance: $C = 0.5 (d_1 + d_2)$

$$= 0.5(72+480) = 276 \text{ mm}.$$

Check for $d_1 \leq C^{0.875}/1.7 \leq 80.4 \text{ mm}$, $d_1 = 80 \text{ mm}$ is taken.

$$C = 0.5(d_1 + d_2) = 0.5(80+480) = 280 \text{ mm}$$

$$\text{Lead} = N_{tw} \times p_a = 4 \times 18.84 = 75.36 \text{ mm}$$

$$\tan \lambda = L / \pi d_1 = 75.36 / 3.14 \times 72 = 0.3333$$

$$\lambda = 18.43^\circ = \psi$$

$$\omega_2 = (2\pi n_2 / 60) = (2 \times 3.14 \times 75 / 60) = 7.85 \text{ rad/s}$$

$$V_2 = \omega_2 r_2 = 7.85 \times (0.5 \times 480) \times 10^{-3} = 1.884 \text{ m/s}$$

$$F_t = 1000W / V = 1000 \times 12 / 1.884 = 6370 \text{ N}$$

$$b \leq 0.5 d_{a1}, \quad b \leq 0.5(d_1 + 2m) \leq 0.5 \times (80+2 \times 6) \leq 46$$

$b = 45 \text{ mm}$ is assumed.

$$Y = 0.393 \text{ from Table 1 for } \phi_n = 20^\circ$$

Table 1. Maximum Worm Lead Angle and Worm Gear Lewis Form Factor for Various Pressure angles.

Pressure Angle Φ_n (Degrees)	Maximum Lead Angle λ (degrees)	Lewis form factor y	Modified Lewis form factor Y
14.5	15	0.100	0.314
20	25	0.125	0.393
25	35	0.150	0.473
30	45	0.175	0.550

$$F_d = F_{2t} \left(\frac{6.1 + V_2}{6.1} \right) = 6370 \times \left(\frac{6.1 + 1.884}{6.1} \right) = 8133 \text{ N}$$

Choosing phosphor bronze for the gear and heat treated C45 steel for the ground worm,

$[\sigma_b] = 80 \text{ MPa}$ from Table 3

Beam strength of the worm gear

$$F_b = [\sigma_b] b m Y = 80 \times 45 \times 6 \times 0.393$$

$$= 8489 \text{ N}$$

WORM GEARS – BENDING AND SURFACE FATIGUE STRENGTHS

Table 3. Permissible stress in bending fatigue

Material of the gear	$[\sigma_b]$ MPa
Centrifugally cast Cu-Sn bronze	23.5
Phosphor bronze	80
Aluminium alloys Al-Si	11.3
Zn Alloy	7.5
Cast iron	11.8

$F_b (8489) > F_d (8133)$ Hence the design is safe from bending fatigue consideration.
Check for the wear strength.

$$F_w = d_2 b K_w = 480 \times 45 \times 0.518 = 11189 \text{ N}$$

$K_w = 0.518$ for steel worm vs bronze worm gear with $\lambda < 25^\circ$ from Table 4.
 $F_w (11189) > F_d (8133)$, the design is safe from wear strength consideration.

Table 4. Worm Gear Wear Factors K_w

Material		K_w (MPa)		
Worm	Gear	$\lambda < 10^\circ$	$\lambda < 25^\circ$	$\lambda > 25^\circ$
Steel, 250 BHN	Bronze	0.414	0.518	0.621
Hardened steel (Surface 500 BHN)	Bronze	0.552	0.690	0.828
	Chill-cast Bronze	0.828	1.036	1.243
Cast iron	Bronze	1.036	1.277	1.553

AGMA recommendation for the axial length of the Worm is, L_w

$$L_w = p_a \left(4.5 + \frac{Z_2}{50} \right) = 18.84 \times \left(4.5 + \frac{80}{50} \right) = 115 \text{ mm}$$

Worm Velocity $V_1 = \omega_1 r_1 = 157 \times 0.04 = 6.28 \text{ m/s}$

$$V_s = \frac{V_1}{\cos \lambda} = \frac{6.28}{\cos 18.43^\circ} = 6.62 \text{ m/s}$$

From the graph 1, for $V_s = 6.62 \text{ m/s}$ $f = 0.025$

WORM GEARS –FRICTION

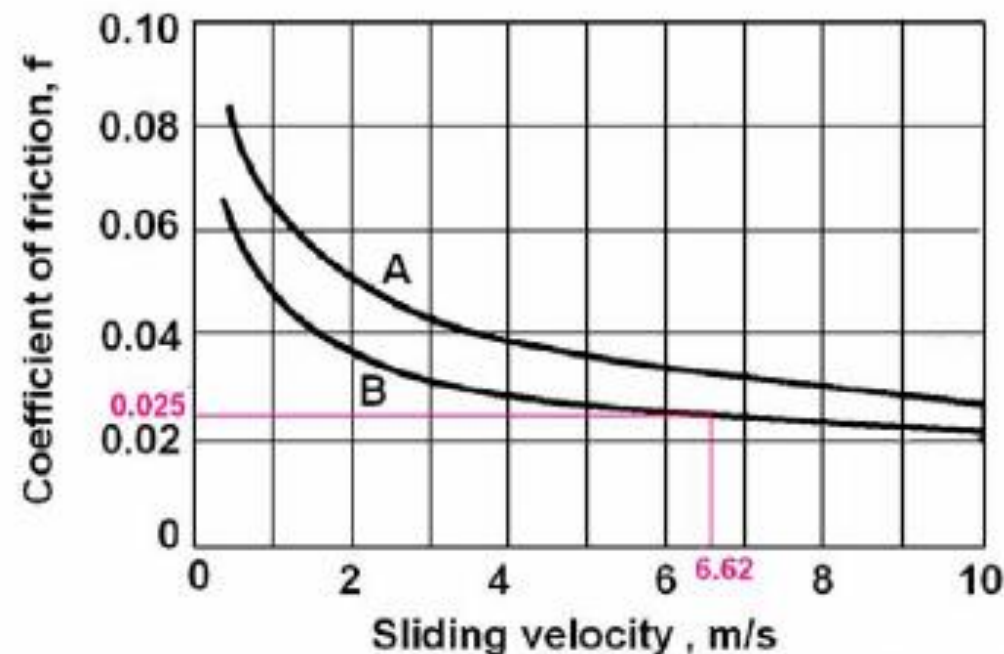


Fig.1. Friction of well lubricated worm gears, A for cast iron worm and worm gear and B for case hardened steel worm and phosphor bronze worm gear.

$$F_n = \frac{F_{t2}}{\cos \phi_n \cos \lambda} = \frac{6370}{\cos 20^\circ \cos 18.43^\circ} = 7145 \text{ N}$$

$$\begin{aligned} \eta &= \frac{\cos \phi_n - f \tan \lambda}{\cos \phi_n + f \cot \lambda} \\ &= \frac{\cos 20^\circ - 0.025 \tan 18.43^\circ}{\cos 20^\circ + 0.025 \cot 18.43^\circ} \\ &= 0.918 \end{aligned}$$

$$\begin{aligned}\text{Heat generated during operation: } H_g &= (1-\eta)W \\ &= (1-0.918)12000 = 984 \text{ Nm}\end{aligned}$$

Surface area A for conventional housing designs may be roughly estimated from the equation:

$$A = 14.75 C^{1.7}$$

Where A is in m^2 and C (the distance between the shafts) is in m.

$$A = 14.75 \times 0.28^{1.7} = 1.694 \text{ m}^2$$

Heat generated during operation: H_g

From Fig. 2 the $C_H = 32 \text{ Nm/s/m}^2/^{\circ}\text{C}$ for $n_1 = 1500 \text{ rpm}$

$$\begin{aligned}H_d &= C_H A (T_o - T_a) \quad \text{assuming } T_a = 35^{\circ}\text{C} \\ &= 32 \times 1.694 \times (T_o - 35^{\circ}) = 54.21 (T_o - 35^{\circ}) \\ &= H_g = 984 \text{ Nm}\end{aligned}$$

$T_o = 53.2^{\circ}\text{C} < 93^{\circ}\text{C}$ permissible for oil.

Hence the design is OK

WORM GEARS – THERMAL CAPACITY

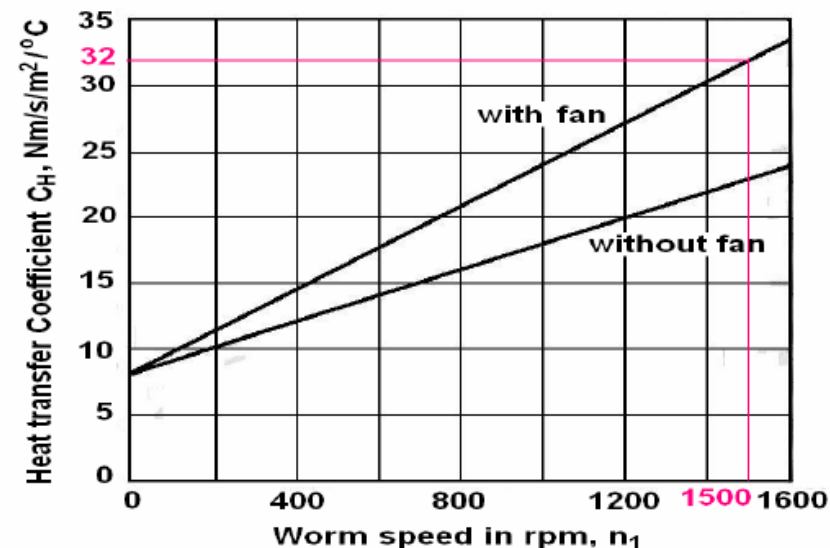


Fig. 2 . Influence of worm speed on heat transfer.