

Creation Mathematics: The Harmonic Series and the Exponential

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Abstract

In the last 6 decades of modern Young-Earth Creationism, researchers have held fast to the literal interpretation of Genesis 1 as an historical account of a seven literal day work of a Creator God. In addition, many of these same researchers have done excellent work bringing awareness to the myriad examples of common mathematical patterns found throughout nature, such as Harmonic Series and the Fibonacci Series, attributing their existence to God. However, no connection has been made between these observed mathematical patterns and the 7-day creation pattern. To form a better view of the Genesis 1 Creation Account as a rotating, increasing pattern of a periodic nature, I assessed the Harmonic Series for suitability as a so-called “Creation Function”. I found that the step-wise output of seven rotations of the Harmonic Series generates a numerical pattern that is fundamental in character, logarithmic and correlates strongly to the Fibonacci series. The sequence obtained from the Harmonic Series also derives Euler’s natural logarithmic base, e . With these new findings taken together with the current body of research, an exponential model which exhibits a strong correlation to the growth and decay patterns that exist from the present-day though a literal 7-Day Creation account found in Genesis was obtained. I show that these numerical patterns are seen and exist from the astronomical scale to the that of the atomic. When we can associate a known numerical pattern to the Biblical Creation Account, it may help us focus a more thorough investigation of the world and the scriptures to achieve a Creation Model of Origins.

Introduction

With the advent of modern creationism owing much to the communications and activities of the Creation Research Society original “Team of Ten”, in 1963, these men agreed upon a statement of belief “All basic types of living things, including man, were made by direct creative acts of God during the Creation Week described in Genesis” (Rush, 1982). Over the last 57 years, the Creation Research Society has held true to its statement of beliefs by fearlessly conducting scientific research providing evidence of a recent special Creation. In particular, two such scientists, Dr. Don DeYoung and the late Dr. Glen Wolfrom published a book “Mathematics: The Language of Creation” in which many examples of mathematical patterns are given to illustrate a Creation imbued with inherent order giving credence to the existence of a Designer. DeYoung says that the mathematical patterns discussed in the book and existence of the Fibonacci numbers “throughout the botanical world is a mystery.” He goes even further; “One might suggest that we are observing the Creator’s fingerprint on his world; mathematics is surely the language of creation” (DeYoung and Wolfrom, 2017) The mission of the Creation Research Society is that it “pursues and supports original research verifying the creation model of origins as a means to reveal the Creator.” The purpose of this article is to offer a mathematical basis for the Genesis seven literal day Creation Account. I believe that this work is important because it recognizes that the Genesis Creation account as a historical record as well as a verifiable creation model of origins.

Modern young-Earth creationism depends upon a literal interpretation of the word “Day” as what can be referred to in modern times as a “24-hour” day. As will be discussed,

much scholarly work has been done establishing this fact. Coupled with very detailed genealogical chronologies and an equally detailed Flood Account given in calendar days, we must accept an Earth which is thousands of years old. In addition to this, we observe an orderly creation with periodicity and systems which obey laws, all of which we can express mathematically. We have observed mathematical patterns repeated throughout creation, and examples have been well documented in creation science with their existence attributed to a creative God. However, no mathematical connection has been made between these two vitally important concepts of a literal 7-day Creation and mathematical patterns in Young-Earth creationism. This work seeks to point out that the mathematical patterns we see in Nature are also seen in the Bible. And although most of the examples of these relationships exist in Genesis, these patterns have not yet been extended to the Genesis Creation Account. To have a seven-day creation pattern with the same mathematical basis as seen in Nature itself would be a valuable step forward in Young-Earth Creationism. It would create a new map overlay allowing a deeper understanding of recent origins.

This work brings together of some the current secular and creationist research on the topic of logarithmic and exponential patterns seen in nature. A new example of exponential growth and decay in the Flood Account is reported. Brought together, these very specific and repeating mathematical patterns seen in both nature and the Bible strengthens the case for Creation. The main task of this paper is to offer a well-known mathematical function known as the Harmonic Series, which is also logarithmic, which correlates to the Fibonacci Series and most importantly, generates a numerical pattern in the same way that the 7-day creation pattern is expressed in Genesis. This would represent

a so-called “Creation Function” which can be employed to further relate those same mathematical patterns seen in the Bible to those which we see in Nature.

First, consider the existence of fundamental constants in Nature. If we work backwards in time, beginning with the present day, it reveals the known exponential character of the Creation. This shows how the entire cosmos progresses exponentially according to the natural logarithmic constant, e . This research shows how the human population has increased exponentially since the Flood. Then additional research studies how the patriarchal ages decayed immediately after the Flood. Next, our own research suggests that this exponential progression also existed during the Flood.

Therefore, with a known exponential pattern from the present-day to the pre-Flood era, the Harmonic Series shows how it correlates with the seven-day creation account by producing a numerical logarithmic spiral pattern that makes 7 full rotations, just like 7 days is seven full rotations of the Earth. Our findings report that the step-wise numerical output of the Harmonic Series approximates the fundamental Euler Constant, e , with greater and greater precision with each turn, as well as exhibiting a high correlation to the Fibonacci sequence. These findings complete a survey of Genesis as a historical account by showing that the same exponential pattern exists from the present-day through the 7 literal days of Creation. The argument states that for any mathematical expression to be regarded as a “Creation Function” it must be fundamental in character, fit in with the known exponential pattern of the Universe, correlate to the Fibonacci Series and Golden Ratio, have copious examples in nature, and most importantly, exhibit predictive power.

The Natural Exponential and Logarithm: Fundamental Constants

Leonhard Euler was an 18th-Century mathematician and preacher's son. He is considered one of the greatest and most prolific mathematicians of all time. Euler discovered the base of natural logarithms, most commonly referred to as “ e ” (DeYoung and Wolfrom, 2017). This fundamental constant is an irrational number, and carries the approximate value $e \approx 2.7183$. Among the many applications of the natural logarithmic base, the main one that we will focus upon in this article is the exponential growth and decay model (Blitzer, 2003). This model is important in Nature to help characterize the growth rate of certain populations of organisms, as well as the decay of radioactive materials (Swokowski, 1987).

Another important Euler discovery is his special identity that relates the natural base e to pi:

$$(1) e^{i\pi} + 1 = 0,$$

where $\pi \approx 3.1416$,

$$i = \sqrt{-1}$$

Another way to write Euler's Equation:

$$(2) e^{ix} = \cos x + i \sin x,$$

where $i = \sqrt{-1}$

x is an angle in radians

$$360^\circ = 2 \pi \text{ radians}$$

Euler's equation displays “symmetry, wide physical application, and connections between diverse mathematical disciplines” (DeYoung, 2015). This equation is referred to as an identity, because if you have e , you get pi and vice versa. The natural base e and pi are

considered fundamental constants because, besides mathematics, they are used in just about every area of human inquiry including, but not limited to chemistry, physics, biology, economics, and engineering. And these two fundamental values are inextricably tied together by a beautiful identity. Refer to Figure 1 to see a graph of the Euler Equation. Notice how e is used to create a rotating spiral pattern as we increase the angle x from 0° to 360° . We will discuss Euler's equation more later in this article, and see why it is so important as an identity. Next, we will see how the natural base e is seen throughout the Creation.

The Entire Cosmos Proceeds Exponentially

From atoms to galaxies, the entire cosmos has been expressed exponentially when surveyed from a mass to size ratio standpoint. An examination of Figure 2 illustrates that each step from atom to DNA to bacteria to insect to Man to planet to Star to galaxy follows an exponential growth curve with a correlation coefficient of 0.93 (Batarseh, 2008). The big-picture, "God's Eye" view of Creation is one that has an unmistakable exponential character. We now observe the spiral patterns of galaxies with arms extending down to their resident stars, solar systems and planets. Not only does the straight line of Figure 2 run through each scalar demarcation of existence, but that thread also seems to wrap around each member. This is because each galaxy, as an individual element of the collective, also contains the smaller members, such as stars which contain planets, which contain people in the case of Earth, and so on. In God's Creation, it seems that everything is connected by a very visible mathematical thread, and exists harmoniously.

We have seen that when things grow, they grow exponentially. But why? In his article *Thoughts on Half-Life*, Dr. DeYoung says that change is "geometric or exponential

in nature” when a particular group of objects have an equal probability of undergoing spontaneous change (DeYoung, 2017). The more common term for this is “half-life”. The mathematical equation for calculating how long it takes for half of a group of objects to change also includes the fundamental constant “ e ”. However, as DeYoung points out, this pattern is not limited to decay, but can also model growth. This is possible through cell division. Instead of a particular quantity of objects being reduced in size by half, when an organism grows via cell division, the quantity of cells doubles according to the same mathematical formula using the natural base “ e ”.

Another important example of exponential growth is that of the human population since the Flood. Dr. Monty White of Answers in Genesis answers the question how can there be “Billions of People in Thousands of Years?” It only requires 32 of our “doublings” to achieve a population of 8.6 Billion people (White, 2006). White compares his model to the world population at the time of his writing, the Beginning with a population of 2, and ending in 2006 with a population of 6.5 Billion people (White, 2006). To double a population every 150 years over a total 4800 years is a 0.46% continuous exponential growth rate. For comparison, I cite a secular population study of Australia focusing on a similar time period of 5000 years ago. They compared their exponential model to their radiocarbon dating of rock shelters and were able to fit their exponential “5 Ka Acceleration Model” with a 97.3% correlation suggesting that they saw a noteworthy population explosion in Australia 5000 years ago. Population growth studies fit that same well-known exponential growth model with great confidence.

We see that God designed an orderly World with rules and common fingerprints we recognize mathematically. But to truly attribute this work to God, shouldn't we try to

match these fingerprints to those see in His Word? Do we see numerical examples of exponential growth and decay in the Word of God? And do we see a continuance of this pattern that we have established working backwards from the known present-day to the Patriarch's age decay reported in Genesis at end of the Flood?

Logarithms and the Exponential in the Bible: The Pattern Continues from Patriarchal Age Decay Period Through the Pre-Flood Ark Construction

Genesis states that pre-Flood humans lived to be nearly 1000 years old. The oldest man, Methuselah, lived to a great age of 969 years. Noah, the builder of the Ark, lived to be 950 years old. But then something curious happens to the life-span of post-Flood humans. We read in Genesis of an apparent decay in Patriarchal Ages immediately after the Flood. Charles A. Glatt Jr. studied this decay and found that it was indeed, exponential based upon the natural base e with a correlation coefficient of -0.850 . Glatt summarized his findings in this excerpt from his Creation Research Society Quarterly paper "*Patriarchal Life Span Exponential Decay*":

“This single correlation between Genesis life spans and the years following the Flood is of interpretable value to report again to review the work that has been done on this subject since 1948, to incorporate RATE project results, present the time-based analysis's correlation coefficient, express the natural (Napierian, base e) curve as part of the family of the universe's most common curve, the logarithmic curve...(and) explore anomalies in relation to space expansion...” (Glatt, 2016).

In this article, my findings strengthen his position that the base e logarithmic curve is indeed a universal mathematical relationship.

Bodie Hodge of Answers in Genesis published an article “Biblical overview of the Flood Timeline” in 2010. He tabulates the calendar dates referenced in in Genesis chapters

7 and 8 and also reports the duration of each Flood stage (Hodge, 2010). We have taken his data and plugged it into our own Table 1. Since these are integer values that are subject to inclusive, exclusive, start, and ending considerations, we abide +/- 1-day discrepancies between our tables, as they do not affect the conclusions of this work greatly. Also, the instances of durations given that are not explicitly associated with a calendar datum are not incorporated. Arguments for or against Hodge's description of how the Flood proceeded are not discussed, because the calendar dates present a clear beginning, mid-stages and ending to the Flood. The time periods not associated with a calendar date in the Bible are used directly in our analysis without needing to plug these values precisely into the timeline.

Referring to Table 1, the first date given is 2/27, the 47th day of the first year of the Flood. Our research has found it significant to include the first day of that year 1/1 in the table because it is implicit with the assertion of a date in a calendar-like fashion. The significance will be revealed as the data are presented. The second calendar date given in 7/17 of the same year, and a 150-day period is mentioned specifically in the account. Hodge says that the 150-day period is "including the initial 40 days" (Hodge, 2010). Since the 40 days itself is not explicitly associated with a calendar date, I place it in a separate column along with the 110-day period implied by Hodge.

These 150 days is a ramp-up period of the Flood. We know this from the context of the story, but also noticed that the ratio of 110 Days to 40 Days is a close 98.8% match to the natural base, e , implying the growth from 40 days to 110 days could be viewed as exponential growth of the Flood waters coming from both the rains (40 days) and fountains

(150 days). If you include the 47-day Ark assembly and loading period from 1/1 to 2/17, we obtain a 197-day period of total ramp-up.

The next calendar date given is 10/1 of that same year. We noticed that the ratio of 197 Days ramp-up to this next period reported of 73 Days given of is a close 99.3% match to the natural base, e , implying that the decay from 197 ramp-up days to 73 days of receding waters could be viewed as exponential decay of the Flood waters.

The next two calendar dates given is 1/1 and 2/27 of the following year, respectively. I noticed that the ratio of 90 Days to this next period reported of 57 Days given of is a close 97.7% match to the Golden Ratio, phi, ϕ , the decay from 90 days of receding waters to 57 days of drying. This poses the question of why this single phi appears among all the examples of the natural base towards the end of the article.

That final calendar date given was 2/27 when God called Noah, family and the animals that remained with him out of the Ark. Hodge shows that the Noah/Bird Period lasted 21 days, and may have occurred in the final month of that first year, yet concurrently with our 90-day period (Hodge, 2010). We noticed that the ratio of 57 Days to this previous period reported of 21 Days, is a very close 99.7% match to the natural base, e . This growth period from 21 days of Noah interacting with his Birds to 57 days of God drying the Earth for inhabitation could be viewed as an exponential growth of the Earth's contents, for example floral growth.

It is noted here that these Flood dates are taken from the KJV version of the Bible, which is based upon the Masoretic texts. The Septuagint account reports different calendar dates. We use the Masoretic dates because it has been determined that "Septuagint corruptions disqualify it for creationist research (Tompkins et al, 2019). Although the

focus of this research is not the Septuagint, our research does show that every Flood datum given from the Masoretic text has an exponential/Golden ratio character and this evidence is lost in the Septuagint. The significance of the three exponential ratios found in the Flood Account is a topic for further research, but it is supposed that they link qualitatively and quantitatively with the 7-Day literal Creation Account.

Taken together, multiplying the correlations of each step of the Flood Account to its respective fundamental constant figures to a correlation of 95.6%. With these, we have placed the exponential natural base, e at the doorstep of the Creation Account. Is there a continuation of Glatt's "universally common curve" in the 7-day Creation? Next, we examine a well-known mathematical relationship suggested by DeYoung called the Harmonic Series.

The Harmonic Series Derives the Logarithmic Natural Base "e"

The Harmonic Series is defined as the sum of an infinite series of constants $1/n$, where $n = \{1, 2, 3, \dots\}$ and as such may be expressed in summation notation, as in equation (3):

$$(3) \sum_{n=1}^n \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} \right\} = S_n$$

If the sequence of partial sums, S_n , tends to a limit as n goes to infinity, it would be considered convergent (Edwards and Penney, 1994). This concept is surprising because it is counter-intuitive that adding an infinite series of numbers together could result in a single, finite number. An example of such a case is equation (4), in which an infinite sum of constants $1/(2n)$ does add up to exactly 1.

$$(4) \sum_{n=1}^{n=\infty} \frac{1}{2n} = \left\{ \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots + \frac{1}{n} \right\} = 1$$

This may be visualized by dividing a 1 x 1 square in half, then dividing the 1/2 by half, giving 1/4, and dividing that 1/4 in half, giving 1/8, and so on forever. This all can be accomplished inside that very finite 1 x 1 square. However, in the previous case of the Harmonic Series (3), the sum diverges towards infinity as n becomes large. We will further discuss how in the next section. In regard to these two examples of infinite series, DeYoung points out that “these and other paradoxes show surprise answers in math, the fascinating language of creation” (DeYoung and Wolfrom, 2016). We will significantly unpack the Harmonic Series to suggest that it is indeed, the language of Creation. We will begin with some examples in Nature.

A very musical member of our world is the bird. What compels a bird to sing? Is it biological or “cultural”? A biological cause could be that a given bird makes a sound like a whistle when it is breathing because its windpipe has a shape conducive to that particular pitch. A “cultural” cause would imply that the bird learns a song and chooses how to sing it for its own perceived benefit. A recent study suggest that the hermit thrush “prefers” to sing in harmonic series, a fundamental component of human music (Doolittle, et al., 2014). This idea that a bird can decide whether or not to sing a particular song, which would drastically affect how effectively that bird mates or not seriously complicates evolutionary thought. Doolittle’s study says that not all birds sing songs comprised of harmonic pitches, but there are an increasing number of studies highlighting more examples of harmonic preference (Doolittle, et al., 2014). The existence of the harmonic series in Creation, but not as a consequence of a physical reality but more of a spiritual one gives weight to the idea that God created these birds and put these songs in their hearts, just as He imbued our creation with a musical quality.

When one plays a note on a piano, what is actually happening is that a hammer inside the piano case is striking a metal string that is under a precise amount of tension. This impulse of energy from our hand is transmitted to the piano string and causes it to become energetic and vibrate. It is the vibration that we hear and the fundamental frequency of that vibration is determined by the length of the string and the tension thereof (Serway et al, 2000). But what many do not realize is that what is heard is actually many tones mixed together in what is commonly referred to as harmony. This special mixture of tones derived from one string mathematically follows its namesake series, the Harmonic Series. All of these frequencies exist together at the same time on the same piano string, without cancelling each other out. Many systems of music are held in the math behind musical instruments (Harkelroad, 2006).

This is significant to our argument because it is important when creating a system that one does not upset or destroy components of that system previously constructed when adding new components. As we will see later in this article, God created the Earth system over the course of 7 days. He was careful not to destroy on Day 2 what He created on Day 1, or Day 3 what He created on Day 2. In other words, the 7 Days of Creation were harmonious. And this harmonious process can be modelled mathematically with the Harmonic Series equation (3), and it is shown next how this calculation creates the natural logarithmic base “*e*”. The proof that the Harmonic Series diverges to infinity demonstrates its logarithmic character by showing that its partial sum S_n will always be larger than an integration of $1/n$ over the same interval (Hughes-Hallet, et al., 1998).

$$(5) S_n = 1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} > \int_1^{n+1} \frac{1}{x} dx = \ln(x + 1)$$

When one uses the heights of rectangles of area $1/n$ each to approximate the area under the curve of $\ln(x+1)$, there will always be a portion of the rectangle that will appear above the same curve, implying that the combined area of S_n will always be greater. And since $\ln(x+1)$ gets arbitrarily large as x approaches infinity, then so must S_n , with “ln” being the natural logarithmic function with natural base “ e ”. The exponential character of equation (3) is shown in Table 2. Under a step-wise analysis, very time S_n increases by 1, n will have increased by a factor approaching “ e ”. We see for small n the error is large, but in 7 iterations the ratio n_{step}/n_{step-1} is within 0.9997 of natural base e . With all of the exponential evidence from the present-day through the 7-day Creation Fibonacci correlation, we obtain a complete exponential model of the Creation. We now do a qualitative comparison of the seven-day creation pattern to the Harmonic Series pattern.

The Harmonious, Creative Progression of Seven Literal Days and Nights in Succession: A Qualitative Analysis

There has been spirited theological debate for generations on whether or not the “Days” of Creation were literal 24-hour days, and those who study the Word of God have never gained a consensus as to the meaning of the days of Genesis (Lewis, 1989). In recent times, the Young Earth Creation movement has solidified around the literal meaning of “Day” and for good reason. An in-depth literary analysis was done by the late Dr. Gerhard Hasel, of Andrews University, and he concluded that these were indeed literal days.

“The author of Genesis 1 could not have produced more comprehensive and all-inclusive ways to express the idea of a literal "day" than the ones that were chosen. There is a complete lack of indicators from prepositions, qualifying expressions, construct phrases, semantic-syntactical connections, and so on, on the basis of which the designation "day" in the creation week could be taken to be anything different than a regular 24-hour day.” (Hasel, 1994).

One ideology that gained traction many years ago is the “Day-Age” theory, that attributes arbitrarily large periods of time for each “day” of creation. This is popular because of the belief in theistic evolution and the confusion over the age of the Earth. Theistic Evolution is easily refuted by the Bible itself when it says in Romans 5:12 that “through one man sin entered the world, and death through sin...”, while evolution requires death to further its process of selection well before humans could evolve. James Stambaugh of the Institute of Creation Research makes an interesting point about the semantics of the Creation Account passage. It seems as though God himself foreknew that this confusion would exist and He phrased the Creation account in a repetitive, almost laborious manner on purpose:

“God, through the ‘pen’ of Moses, is being redundant for redundancy’s sake. God is going out of His way to tell us that the ‘days’ of creation were literal solar days. He has used the word *yôm*, and combined this with a number and the words ‘morning’ and ‘evening’. God has communicated the words of Genesis 1 in a specific manner, so that the interpreter could not miss His point. God could not have communicated the timing of creation more clearly than He did in Genesis 1.” (Stambaugh, 1991).

These solar days are important because they imply physical rotation relative to a light source. There is no better definition of a “day” than one complete physical rotation of the Earth. And since the focus of the entire Genesis 1 creative act is the Earth, we say that the seven days were seven rotations of Creation. This view allows there to be seven discrete literal days, but also takes into account that each step of Creation must take into account all of the other past steps and make room for the next steps. In Henry M. Morris’ work *Scientific Creationism*, he states that “each stage was an appropriate preparation for the succeeding stage and for all of them the ultimate purpose of providing a suitable home for man” (Morris, 1985). Creation proceeded without disturbing what had been done

already and summed together to form and inhabit the ultimate creation, human beings. The latter being perfectly suitable to exist alongside and have dominion over the former. This is truly harmony in nature, characterized at each step by the statement “it was good”. Consider the Asian Carp’s presence in the Great Lakes watershed. Area environmental organizations has been working for years to keep this invasive species out of the Great Lakes, for fear it “may damage an ecosystem” (Zhang, et al., 2015). There are approximately 32,500 species of fish worldwide (Nelson, 2006). If we believe that two carp species can destroy the largest freshwater habitat on the planet, it would seem that more people would be open to the prospect of a Creator who was able to distribute all of these 32,500 fish varieties harmoniously among the diverse water habitats of our planet.

At the end of the Creation Account, we are commanded “to be fruitful and multiply” and we observe that this process is accomplished via reproduction and cell division. And as discussed earlier, processes involving division create an exponential growth curve. With this commandment, God put an exponential growth process into motion from the Beginning. The Gospel of Luke chapters 3:23-38 contains a genealogy of the lineage of Christ. We count 77 people in the genealogy from God to Christ inclusive. The command for Adam to multiply, a purely mathematical concept, coupled with the precise number of steps to the birth of Christ implies that God uses the power of the exponential. We also see that there were actually 12 steps to Creation, each “day” actually being comprised of a night and a day, or half rotations of the Creation. Next, we will apply the Harmonic Series equation to this 12-step process to see if there is any more correlation to other mathematical patterns or fundamental constants observed in Nature.

The Harmonic Series Function Applies to the 6-Day Active Creation Pattern: A Quantitative Analysis

The usual reading of the Genesis Creation Account places emphasis on what was created on a particular Creation Day. These details are of utmost importance, but do not represent all of the information that may be gleaned from the story. We focus here on the mathematical pattern of twelve half-days of active Creation. We have established that a day is a rotation of Earth, which implies that days can be divided in half and the whole active process may be analyzed as a series of twelve half-rotations. And with rotational periodicity, we can graph the Harmonic Series and show that with each rotation, the step count n increases by a factor of e the natural exponential base, creating a logarithmic spiral pattern. In addition, we know that each step of the Creation was good at the time it occurred and continued to be good up until, and after the Creation process was completed. Again, this means that the process must build upon, protect and make room for the next steps, not an easy task, considering two fish in the wrong lake can destroy everything done that Creation Day.

We have already seen in Table 3 that it can represent 7 steps for 7 days and within that time period, it derives the natural logarithmic base “ e ”. In Table 3, we apply the Harmonic Series to 12 half-days. The table features several columns of information as the 12 steps proceed, beginning with an initial uncounted step we refer to as “Day Zero”. The Day Zero concept is not the focus of this paper, but it is included to illustrate how purely the Harmonic Series may be applied to the Creation Pattern, and also how it provides potential new clues as to the nature of the Creation Act. And a desirable feature of a

Creation Model of Origins is not only one that matches up with what we can see now, but inspires new ideas which promotes new research.

We will go through one iteration of the Creation Process. Referring to Table 3, the “Step” column of the first row begins with the basis of Day Zero. From there, the second row of the table represents the first step, Night 1.0, during which the “ n_{step} ” column variable “2” is plugged into the “Harmonic Series” equation. The equation output, known as the partial sum, S_n , is tabulated in the “ S_n Step Finish” column, which in this case equals the partial sum “1.5”. With this, the sum was increased from 1.0 to 1.5, or a half-rotation, by the Harmonic Series equation. The next two columns “Step Range” and “Fibonacci Compare” will be explained in the next section. The new partial sum “1.5”, sets the process up for the next half-rotation represented in the third row of the table, Step “Day 1.5”, plugging the next “ n_{step} ” of 4 into the next Harmonic Series equation. This repeating process continues in the same fashion through Step “Day 6.5” the last half-day of the active portion of Creation. The set of “ n_{step} ” numbers tabulated in (6) is important, because these are the numbers that create the complete half-rotations of Creation.

$$(6) \{1, 2, 4, 7, 11, 19, 31, 51, 83, 137, 227, 373, 616\}.$$

This repeating, expanding process creates the logarithmic spiral pattern seen in Figure 2. We already have shown that the Harmonic Series can be applied to the study of sound, and that sound is periodic in nature (Serway et al, 2000). With this in mind, we graph the Harmonic Series using trigonometry and parametric functions. This is very similar to Figure 1 how the Euler formula is plotted with its complex plane on the y-axis and real planes on the x-axis. Using the Harmonic Series equation (3), we graph parametrically for $n = 1$ through 31:

$$(7) (x, y) = (n \cos[2\pi \text{ mod } 1[S_n]], n \sin[2\pi \text{ mod } 1[S_n]])$$

We take the modulus of S_n to isolate the remainder which increases periodically from 0 to 1 and we multiply it by 2π to obtain an angle in radians. Now we can view the Harmonic Series as a series of divisions of a polar plot with the magnitude of (x,y) increasing with n , creating the spiral pattern in Figure 2. We arbitrarily limited the plot to $n = 31$ for the sake of clarity and simplicity. The set of integers we tabulated in (6) originates from every time the spiral plot touches or crosses the x-axis or “DAY/NIGHT” line. With this plot, we obtain a rotating, increasing pattern, which as an equiangular, logarithmic spiral pattern, keeps its proportion as it grows (Thompson, 1961). This is vitally important for growing organic systems as such could be suitable for a continuous 6-day Creation process.

With this logarithmic spiral derived from the Harmonic Series and applied directly to the Creation Account, we have completed the connection between the exponential procession of our cosmos today to the exponential progression of the Genesis Creation. We have both qualitatively and quantitatively established this connection. That single sequence of 13 integers (6) whose ratios approximate the natural logarithmic base “ e ” is a constructive proposition. But (6) also derives another interesting sequence of numbers that correlates strongly to another popular mathematical pattern that has been widely observed in nature. This forms an independent second numerical connection between a Harmonic Series based 6-day active Creation and our observations in Nature today.

Harmonic Series Parallels Fibonacci Sequence

To begin to answer the possibility that sequence (6) is merely an arbitrary set of numbers with limited potential to add new information to the Creation Account, we refer

again to Table 3. For each “Step” there is a “Step Range” defined by subtracting the current n_{step} from the preceding n_{step-1} . An example would be Night 3.0 with its Step Range of 8. To visualize this, we refer to Figure 2, and we count the number of data points along the spiral that are on the DAY side of the x-axis, beginning with n_{step-1} 11. We count the following points {11, 12, 13, 14, 15, 16, 17, 18} a total of 8. Another example would be Day 3.5 with its Step Range of 12. We count the number of data points along the spiral that are on the NIGHT side of the x-axis, beginning with n_{step-1} 19. We count the following points {19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30} a total of 12.

From all of the “Step Range” entries in Table 3, we obtain a new series of integers:

$$(8) \{1, 1, 2, 3, 4, 8, 12, 20, 32, 54, 90, 146, 243\}$$

If we compare this set of numbers with that of the “Fibonacci Compare” column

$$(9) \{1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233\}$$

we see a 99.9% correlation. The numerical series (9) is commonly referred to as the Fibonacci Series or “Fibonacci Numbers”, described by Italian mathematician Leonardo Fibonacci in his 1202 book *Liber Abaci* (DeYoung, 2018). The Fibonacci Series is an infinite sequence which is generated by adding two consecutive numbers in the series to get the next number. Even though this is a self-defined mathematical construct, we do see these individual numbers or pairs and triplets of these integers in nature, as in the populations of breeding rabbits, phyllotaxis of plants, sunflower seed heads, and in finance (DeYoung, 2018).

The phyllotaxis examples in DeYoung and Wolfrom’s *Mathematics: The Language of Creation* are interesting when compared to how the Harmonic Series is brought forth in Figure 2. In phyllotaxis, angular leaf placement around an axis is determined by taking the

ratio of two Fibonacci numbers, such that after a certain Fibonacci total of leaves, a complete 360° rotation around the axis is completed. To create the Harmonic Series spiral, each 360° rotation around an axis is accomplished by summing two Fibonacci-like summations of fractions of a 360° circle.

There are many more examples of Fibonacci numbers in Nature. We will highlight here that these numbers are seen from the atomic scale through the astronomical scale.

Fred Willson, of the *Good Science* program, said of DNA:

“When we realize that the information to produce these spirals and numbers in living things is stored in DNA, should we then be surprised to find that the DNA molecule is 21 angstroms in width and the length of one full turn in its spiral is 34 angstroms, both Fibonacci numbers?” (Willson, 2002).

Fundamental electric charges are Fibonacci based, with values of 1/3, 2/3, 1 (Thornton, 2002). Also in Table 4, with the phyllotaxis data, we tabulated the ratios of planetary orbital periods. Each planet’s orbital period around the Sun from Pluto to Mars is a Fibonacci fraction of its neighbor (Willson, 2002). With Fibonacci numbers, just as we did the exponential function, we have blazed an independent trail from the astronomical, to the scale of every-day things such as plants and tree all the way down to the sub-atomic scale. With the Harmonic Series strong correlation to the Fibonacci Series, we submit that the Harmonic Series’ numerical output is by no means arbitrary. But some could say that “not everything is Fibonacci, not even Fibonacci”.

Dr. Harry Wiant, in his CRSQ article *Relation of southern pine cone spirals to the Fibonacci series*, said that “almost without fail, counts may be observed which are adjacent numbers in the Fibonacci series (e.g., 5 right, 8 left)” (Wiant, 1973). What about those instances where Fibonacci fails? The plant does not die, it manages to survive based upon

something even more fundamental. A study was done by the Royal Society to determine how common the Fibonacci Spiral counts were in the well-known Sunflower seed head. It was determined that majority of the seed heads were indeed Fibonacci, but there were significant examples of Fibonacci ± 1 (Swinton, et. al, 2016). The existence of Fibonacci ± 1 in a viable plant implies that the set of Harmonic Series (8), with its Fibonacci ± 1 character could be a better view of how nature continues to progress, however imperfect. One would never expect an ear of corn to always have the exact same number of kernels.

Fundamental Constants and Numerical Triad Seen in Geometry, Scripture, Fibonacci and Harmonic Series.

When I first began studying the Harmonic Series sequence (8) back in 2005, I was very skeptical of it until I noticed a numerical triad in the Flood Account that virtually matched three numbers of sequence (8). These triads are shown in Table 5. It was years later that I discovered that the Fibonacci Series had the triad and that this triad also was very close to being a “Magic Triangle”, a right triangle with sides of exactly 3, 4, 5 (or multiples thereof). As an anecdote, there is a fifth example of the triad in Noah’s Ark which is beyond the scope of this article. It has been suggested that the Holy Trinity may be represented in mathematics as a triangle (DeYoung and Wolfrom, 2017). Although theirs was an equilateral triangle, which makes sense, our magic triangle seen in scripture may be a sign of the Trinity expressed in Nature as an efficient pattern for growth and stability. Either way, the existence of this triad from 4 independent sources is more evidence of the non-arbitrary nature of the Harmonic series sequence (8).

Also, in *Mathematics, The Language of Creation*, Dr. DeYoung cites several examples of “Fundamental Numbers in Nature”, and says “one might suggest that these numbers were selected by God to shape the fabric of creation.” (DeYoung and Wolfrom, 2017)”. The first set of numbers he suggests is the Fibonacci Series (9). We have already shown that the Harmonic Series Sequence (8) correlates 99.9% with that of Fibonacci. The next example he gives is the Golden Ratio. This special ratio is derived from the Fibonacci Sequence (9), and therefore will correlate strongly with the Harmonic Sequence (8). The next numbers he suggests is e , and pi. We have shown that the Harmonic Series derives the natural logarithmic base “ e ” and pi is derived from e using the Euler Identity (1). The next example given is the fine structure constant “ α ” or 137. At this juncture, we can provide only an anecdotal example from the Harmonic Series. An examination of row 10 “Night 5.0” of Table 3 shows that $n = 137$ is the beginning of Night 5. The significance of this an advanced topic of my research, and will be left to a future article. The last example he gives is the Paul Dirac’s “Large Numbers Hypothesis” stating that many of the ratios of size scales in the Universe are on the order of 10^{40} , or 40 orders of magnitude (DeYoung and Wolfrom, 2017). We can cite another example that the ratio of the strong force to the gravitational force is approximately 10^{39} (Thornton and Rex, 2002). This translates to 90 orders of natural magnitude or e^{90} . How this relates to the Harmonic Series is anecdotal for our purpose now, but one may imagine that using the Harmonic Series as we have in Creation, with 6 full days relating to e^6 , how 90 full days of the Flood could relate to e^{90} , suggesting a rational basis for the existence of such large numbers in Creation after the Flood. The Harmonic Series (8) is seen in scripture, nature, geometry and mathematics,

and shows its fundamental character suggesting that it is not merely a set of “random numbers.”

Summary

With the Harmonic Series equation, we have provided that missing mathematical link from the 7-literal day Genesis Creation Account to the mathematical patterns seen in nature today, such as the Fibonacci series. Two independent paths show that the 6 active, literal days of Creation were exponential in character; thereby advancing a Universe that is also exponential at all scales from the galactic down to the sub-atomic. With these we propose that the Harmonic Series equation, as a so-called “Creation Function”, is a Creation Model of Origins that may be employed to answer further questions such as: Where did the actual Fibonacci Series come from? What was is the nature of a “Day Zero”? What is the significance of the 3 exponential growth and decays of the Flood based upon the natural logarithmic base e ? If the material basis of the days of the Creation advanced exponentially, did the days of the Flood also?

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Tables

Table 1: Time Spans Between Calendar Flood Events as Reported

Reference	Date Noah	Days	Total Days	Comment
Gen 7:11	1/1/600	0	0	Ark Assembly
Gen 7:11	2/17	47	47	Flood Begins/Rain/Fountains
Gen 7:24, 8:3,4	7/17	150	197	Ark Comes to Rest
Gen 8:5	10/1	73	270	Drying, First See Mtn. Tops
Gen 8:13	1/1/601	90	360	Remove Hull, See Dryness
Gen 8:14	2/27	57	417	Earth is Dry. Noah Disembark
Gen 7:12,17,8:6		40		Active Flood Time, (Noah sees)
Gen 8:6		21		Bird Wait/Drying Time
Gen 7:24		110	150-40	150 Days includes the 40 Days
Activity	Ratio	=	Actual	Comment
Fountains/Rain	110/40	2.75	2.718	Natural Log Base e (growth)
Flooding/Drying 1	197/73	2.70	2.718	Natural Log Base e (decay)
Drying 2/Drying 3	90/57	1.58	1.618	Golden Ratio ϕ (decay)
Drying 3/Bird Dry	57/21	2.71	2.718	Natural Log Base e (growth)

Table 1 shows the Flood Account Dates. There are three periods of the Flood that show exponential growth or decay, based upon the ratios of the numbers of days of the respective periods. There is also an instance of the Golden Ratio during the second drying phase. All of data participate in some sort of growth or decay based upon known mathematical constants.

Table 2: The Harmonic Series Function Derives the Natural Base “e”

Step	Harmonic Series	n	S_n	Ratio n_{step} / n_{step-1}	=	Compare to e
0	$\sum_{n=1}^{n=1} \frac{1}{n} = \left\{ \frac{1}{1} \right\}$	1	1	-	-	-
1	$\sum_{n=1}^{n=4} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right\}$	4	2.08	4/1	4	2.718
2	$\sum_{n=1}^{n=11} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{11} \right\}$	11	3.02	11/4	2.750	2.718
3	$\sum_{n=1}^{n=31} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{31} \right\}$	31	4.03	31/11	2.818	2.718
4	$\sum_{n=1}^{n=83} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{83} \right\}$	83	5.00	83/31	2.677	2.718
5	$\sum_{n=1}^{n=227} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{227} \right\}$	227	6.00	227/83	2.735	2.718
6	$\sum_{n=1}^{n=616} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{616} \right\}$	616	7.00	616/227	2.714	2.718
7	$\sum_{n=1}^{n=1674} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{1674} \right\}$	1674	8.00	1674/616	2.718	2.718
8	$\sum_{n=1}^{n=4550} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{4550} \right\}$	4550	9.00	4550/1674	2.718	2.718
9	$\sum_{n=1}^{n=12368} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{12368} \right\}$	12368	10.00	12368/4550	2.718	2.718

Table 2 Illustrates the stepwise output of the Harmonic Series. As the steps proceed, the ratio between n_{step} values approximate the natural logarithmic base e , and by the 7th step the approximation is within 99.9%.

Table 3: The Harmonic Series Function Applied to 6-Day Active Creation

Step	n_{step}	Harmonic Series	S_n Step Finish	$n_{step} - n_{step-1}$ Step Range	Fibonacci Compare
Day Zero	1	$\sum_{n=1}^{n=1} \frac{1}{n} = \left\{ \frac{1}{1} \right\} = S_n$	1	1	1
Night 1.0	2	$\sum_{n=1}^{n=2} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} \right\} = S_n$	1.5	1	1
Day 1.5	4	$\sum_{n=1}^{n=4} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} \right\} = S_n$	2.08	2	2
Night 2.0	7	$\sum_{n=1}^{n=7} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{7} \right\} = S_n$	2.59	3	3
Day 2.5	11	$\sum_{n=1}^{n=11} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{11} \right\} = S_n$	3.02	4	5
Night 3.0	19	$\sum_{n=1}^{n=19} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{19} \right\} = S_n$	3.55	8	8
Day 3.5	31	$\sum_{n=1}^{n=31} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{31} \right\} = S_n$	4.03	12	13
Night 4.0	51	$\sum_{n=1}^{n=51} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{51} \right\} = S_n$	4.52	20	21
Day 4.5	83	$\sum_{n=1}^{n=83} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{83} \right\} = S_n$	5.00	32	34
Night 5.0	137	$\sum_{n=1}^{n=137} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{137} \right\} = S_n$	5.50	54	55
Day 5.5	227	$\sum_{n=1}^{n=227} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{227} \right\} = S_n$	6.00	90	89
Night 6.0	373	$\sum_{n=1}^{n=373} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{373} \right\} = S_n$	6.50	146	144
Day 6.5	616	$\sum_{n=1}^{n=616} \frac{1}{n} = \left\{ \frac{1}{1} + \frac{1}{2} + \dots + \frac{1}{616} \right\} = S_n$	7.00	243	233

Table 3 illustrates the stepwise output of the Harmonic Series along with the total and comparison to Fibonacci Series. As the Day 6.5 Step Range diverges from Fibonacci 233, this is the step where the natural base “e” becomes within 99.8%. I am unaware of any Fibonacci Spiral Count of 233 that occurs in Nature.

Table 4: From Planets to Plants: Fibonacci Ratios are Observed

<i>Fraction of a circle between adjacent leaves on a stem</i>	<i>Angle between adjacent leaves</i>	<i>Periodicity of leaves</i>	<i>Plant Examples</i>	<i>Planetary Orbit Around Sun Time Ratio Example</i>
1/2	180°	2	Elm, lime, linden, mulberry	Uranus: Neptune (1:2)
2/5	144°	5	Apple, apricot, cherry, holly, oak, plum	Jupiter: Saturn (2:5)
5/13	138.5°	13	Almond, white pine, willow	Mars: Asteroids (5:13)
3/8	135°	8	Pear, poplar, rose, sunflower, sycamore	Asteroids: Jupiter (3:8)
1/3	120°	3	Alder, beech, birch, blackberry, hazel	Saturn: Uranus (1:3)

Table 4. The influence of the Fibonacci numbers is seen on an astronomical scale as well as that of the plant world. Ratios of Fibonacci numbers determine the angle in which leaves are arranged around the axis of a branch. This is very similar to how the Harmonic Series spiral proceeds around its axis, with every 180° representing Fibonacci-like numbers. Phyllotaxis from DeYoung and Wolfrom. Planetary ratios from Willson.

Table 5: A Reoccurring Triad Found in Math and Scripture

<i>Harmonic Series (8)</i>	<i>Fibonacci Series (9)</i>	<i>Flood Account Days, (Table 1)</i>	<i>Two Magic "3, 4, 5" Triangles</i>
146	144	73 (146)	144
90	89	90	90
54	55	57	54

"3, 4, 5" Triangles x 18

Table 5. Creation Triads found from Harmonic Series, Fibonacci Series, Flood Account, Magic Triangle. The Flood Account first drying phase was 73 days, and its doublet (146) foreshadows upcoming research featuring a 5th Triad example.

Figure 1: A graph of the Euler Equation

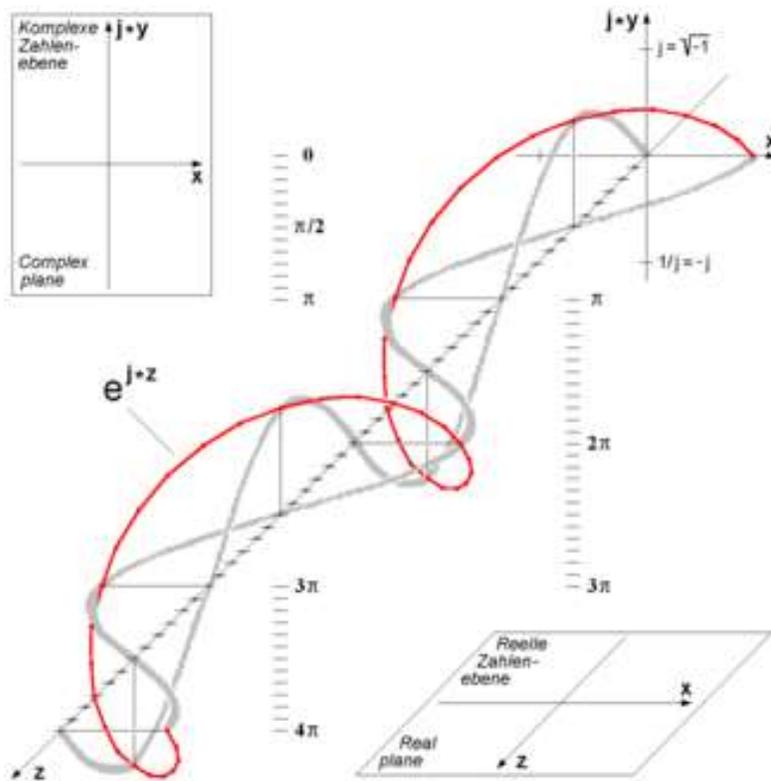


Figure 1 graphically illustrates the Euler Identity. Notice how the Euler Formula plots a spiral as the angle increases from 0 to 2π radians (0 to 360 degrees). Looking from the side or top, you will see a trigonometric sine wave developed by the natural logarithmic base, “e” taken to the power of angles 0 to 2π radians. Looking down through the middle (+z axis), you will see a circle, from which we derive pi. Study of this graph reveals the visual relationship between e and pi.

Figure 2: Logarithmic Scale of Variations

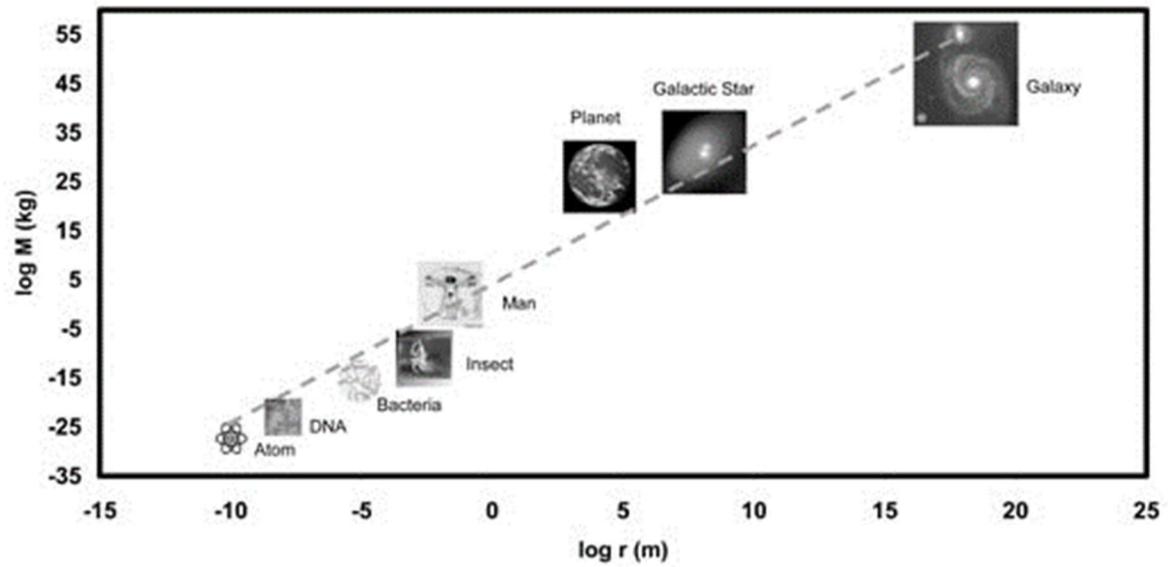


Figure 2 shows the Logarithmic scale of variations of some of the structures found in our universe in terms of mass (M) and size (r). Despite the vast differences of mass and size between these structures, this figure demonstrates the logarithmic correlation that exists between mass and size in the universe, implying coherence and order. From (Batarseh, 2008).

Figure 3: Harmonic Series Polar Graph

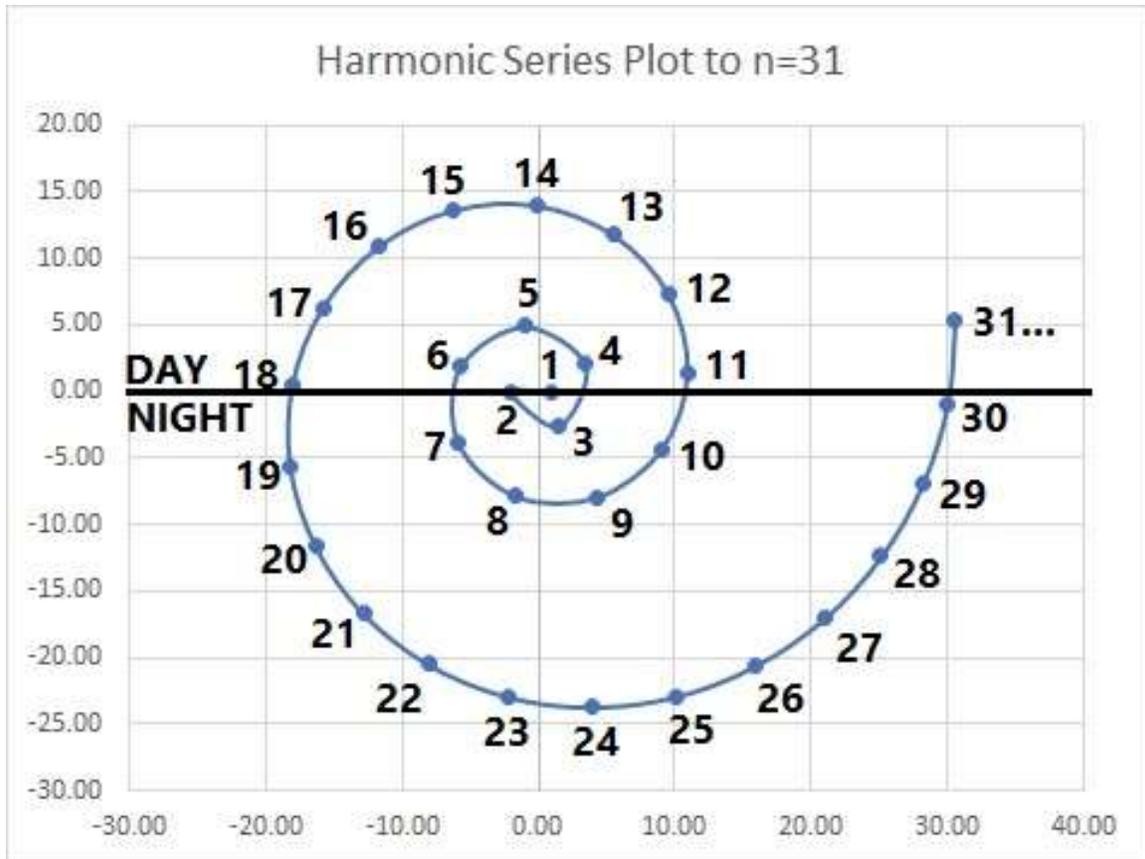


Figure 3 shows the Harmonic Series data to $n=31$. Similarly, to the Euler Formula, the Harmonic Series data is plotted parametrically on the x (DAY/NIGHT) and y (up and down) axes. The amplitude of the spiral increases with n as the spiral turns around z-axis (pointing into the page). Every time the spiral makes 1 complete rotation, n increases by a factor of 2.718, otherwise known as the natural logarithmic base, e .