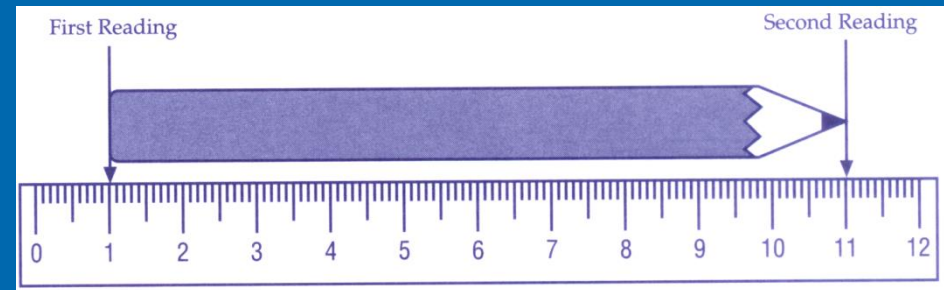
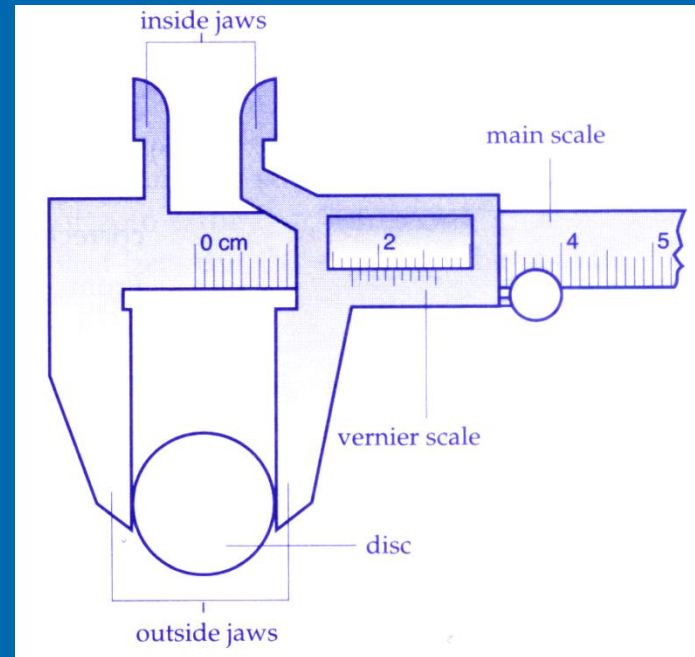
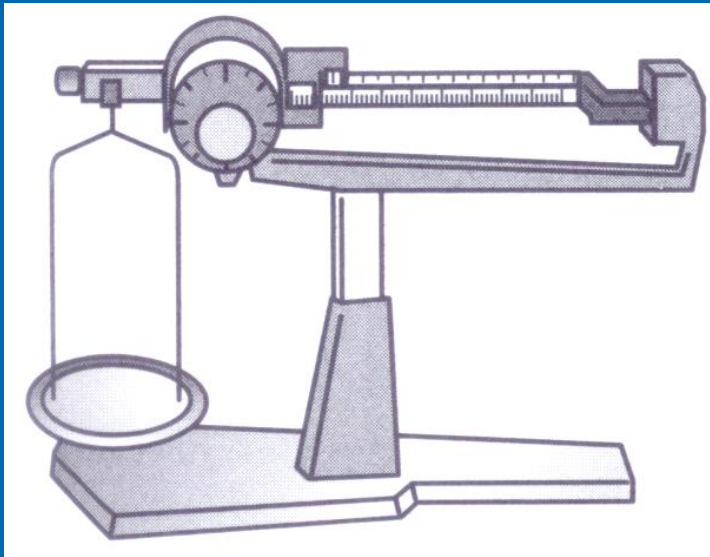


Topic 1.1-1.2



Measurement & Uncertainties

Nature of Science

- *All measurements have their limitations or uncertainties and it is important that we understand what the limitations are.*

Theory of Knowledge

- *“One aim of Physics is to give an exact picture of the material world. One achievement of Physics in the 20th Century is to show that this is not possible.”*
- *Can scientists ever be certain of their discoveries?*

Assessment statement

- State and compare quantities to the nearest order of magnitudes.
- State the ranges of magnitudes of distances, masses and times that occur in the universe, from smallest to greatest.
- State ratios of quantities as differences of orders of magnitude.
- Estimate approximate values of everyday quantities to one or two significant figures and/or to the nearest order of magnitude
- State the fundamental units in the SI system.
- Distinguish between fundamental and derived units and give examples of derived units.

Assessment statement

- **Convert between different units of quantities.**
- **State units in the accepted SI format.**
- **State values in scientific notation and in multiples of units with appropriate prefixes.**
- **Describe and give examples of random and systematic errors.**
- **Distinguish between precision and accuracy.**
- **Explain how the effects of random errors may be reduced.**
- **Calculate quantities and results of calculations to the appropriate number of significant figures.**

Assessment statement

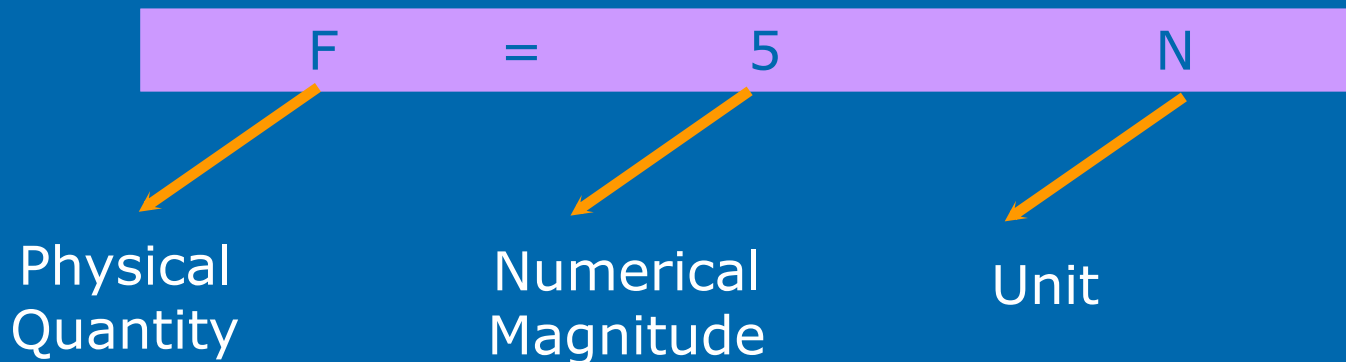
- State uncertainties as absolute, fractional and percentage uncertainties
- Determine the uncertainties in results
- Identify uncertainties as error bars in graphs
- State random uncertainty as an uncertainty range (\pm) and represent it graphically as an “error bar”
- Determine the uncertainties in the gradient and intercepts of a straight-line graph.

Measurement

Physical Quantities

Are measurable features of concepts
e.g. length of a table, mass of a bag of rice

consists of a numerical magnitude and a unit



Measurement

Physical Quantities

SI format of units: m s^{-1} , not m/s
 m s^{-2} , not m/s^2

$$\text{kg/m}^3 \equiv \text{kg m}^{-3}$$

$$\text{J/s} \equiv \text{J s}^{-1}$$

Measurement

Base Quantities & Units

International System of Units (S.I.) distinguishes SEVEN physical quantities as base or fundamental quantities

They were chosen arbitrarily and form the building blocks of all derived physical quantities

BASE QUANTITY	BASE UNIT	NAME
length	m	metre
mass	kg	kilogram
time	s	second
electric current	A	ampere
temperature	K	kelvin
amount of substance	mol	mole
* luminous intensity	cd	candela

Measurement

Derived Quantities & Units

Physical quantities with combination of various basic quantities through a defining equation

Derived Quantity	Formula	Derived unit
Area	$A = l^2$	m^2
Volume	$V = l^3$	m^3
Density	$\rho = m / V$	$kg\ m^{-3}$
Velocity	$v = \Delta s / \Delta t$	$m\ s^{-1}$
Acceleration	$a = \Delta v / \Delta t$	$m\ s^{-2}$
Force	$F = m\ a$	$kg\ m\ s^{-2}$
Pressure	$P = F / A$	$kg\ m^{-1}\ s^{-2}$
Work	$W = F\ s$	$kg\ m^2\ s^{-2}$
Power	$P = W / t$	$kg\ m^2\ s^{-3}$
Electric charge	$Q = I\ t$	$A\ s$

Example

The viscous drag force F of a sphere of radius r moving through a fluid with speed v is given by $F = 6\pi\eta rv$. What are the base units of the viscosity of the fluid η ?

- A $\text{kg m}^3 \text{s}^{-1}$
- B $\text{kg m}^{-1} \text{s}^{-1}$
- C $\text{kg}^{-1} \text{m s}$
- D $\text{kg m}^{-1} \text{s}^{-3}$

Solution

Unit of $F = \text{kg m s}^{-2}$

Unit of $6\pi\eta rv = [\eta] \text{ m} \cdot \text{m s}^{-1}$

Therefore, unit of η :

$$\text{kg m s}^{-2} = [\eta] \text{ m} \cdot \text{m s}^{-1}$$

$$[\eta] = \text{kg m}^{-1} \text{s}^{-1}$$

Example

Which one of the following are the base units for the volt V?

- A N m
- B $\text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$
- C $\text{kg m}^2 \text{s}^{-2} \text{C}^{-1}$
- D kg m s^{-3}

Solution

$$V = W / Q = F \times d / I \times t$$

$$\text{Unit of } V = \text{kg m s}^{-2} \cdot \text{m} / \text{A} \cdot \text{s} = \text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$$

B

Example

The unit of resistance, Ω , expressed in terms of base units is

A $\text{kg m}^3 \text{s}^{-2} \text{A}^{-2}$

B $\text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$

C $\text{kg m}^2 \text{A}^{-3}$

D $\text{kg m}^2 \text{A}^{-1} \text{s}^{-3}$

Solution

$$R = V / I$$

$$\text{Unit of } V = \text{kg m s}^{-2} \cdot \text{m} / \text{A} \cdot \text{s} = \text{kg m}^2 \text{s}^{-3} \text{A}^{-1}$$

$$\text{Unit of } R = \text{kg m}^2 \text{s}^{-3} \text{A}^{-1} / \text{A} = \text{kg m}^2 \text{s}^{-3} \text{A}^{-2}$$

B

Measurement

Approx. of order of magnitude of lengths/ distances

Lengths or distance	Approx. meters
Neutron or proton	10^{-15} m
Atom	10^{-10} m
wavelength of light	10^{-7} m
Sheet of paper	10^{-4} m
length of Finger nail	10^{-2} m
Tallest building	10^2 m
Mt Everest's height	10^4 m
Earth's diameter	10^7 m
Earth to sun	10^{11} m
Earth to alpha centuri	10^{16} m
Radius of local galaxy (Milky way)	10^{21} m
Radius of observable universe	10^{27} m

Measurement

Approx. of order of magnitude of time intervals

Time interval	Approx. seconds
Passage of light across a nucleus	10^{-24} s
Passage of light across an atom	10^{-20} s
Period of visible light	10^{-15} s
Passage of light across a room	10^{-8} s
Period of high frequency sound	10^{-4} s
Time between human heartbeat	10^0 s
One day	10^5 s
One year	3×10^7 s
Human life span	2×10^9 s
Length of recorded history	10^{11} s
Humans on earth	10^{14} s
Life on earth	10^{17} s
Age of universe	10^{19} s

Measurement

Approx. of order of magnitude of masses

Object	Approx. kg
Electron	10^{-30} kg
Proton, nucleus	10^{-27} kg
DNA molecule	10^{-17} kg
Bacterium	10^{-15} kg
Mosquito	10^{-5} kg
Plum	10^{-1} kg
Human	10^2 kg
laden oil super tanker	10^8 kg
total mass of atmosphere	10^{18} kg
Earth	6×10^{24} kg
Sun	2×10^{30} kg
Galaxy	10^{41} kg
Total mass of observable universe	10^{52} kg

Measurement

Approx. of order of magnitude of energies

Range of energies	Approx./ J
Energy needed to remove electron from the surface of metal	10^{-18} J
Energy in the beat of fly's wing	10^{-4} J
Kinetic energy of Tennis ball during game	10^0 J
Energy needed to charge a car battery	10^6 J
Energy in a lightning strike	10^{10} J
Energy released by annihilation of 1 kg of matter	10^{14} J
Energy released in an earthquake	10^{20} J
Energy radiated by Sun in 1s	10^{26} J
Energy released in Supernova	10^{44} J

Measurement

Estimating ratio of order of magnitude

What is the ratio of the diameter of an atom to its nucleus?

Answer

$$10^{-10} / 10^{-15}$$

$$= 10^5 \text{ or 5 orders of magnitude}$$

Estimating dimensions of brick, mass of an apple, duration of a heartbeat, room temperature

Measurement

Prefixes

Another feature of the S.I. Units makes use of prefixes to indicate decimal multiples or submultiples of all units

Prefix	Abbre	Value
yotta	Y	10^{24}
zetta	Z	10^{21}
exa	E	10^{18}
peta	P	10^{15}
tera	T	10^{12}
giga	G	10^9
mega	M	10^6
kilo	k	10^3
hecto	h	10^2
deka	da	10^1

Prefix	Abbre	Value
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}
femto	f	10^{-15}
atto	a	10^{-18}
zepto	z	10^{-21}
yocto	y	10^{-24}

Measurement

Conversion between units

Useful Conversion factors:

Joules and kilowatt-hour

Joules and electron-volt

Years and seconds

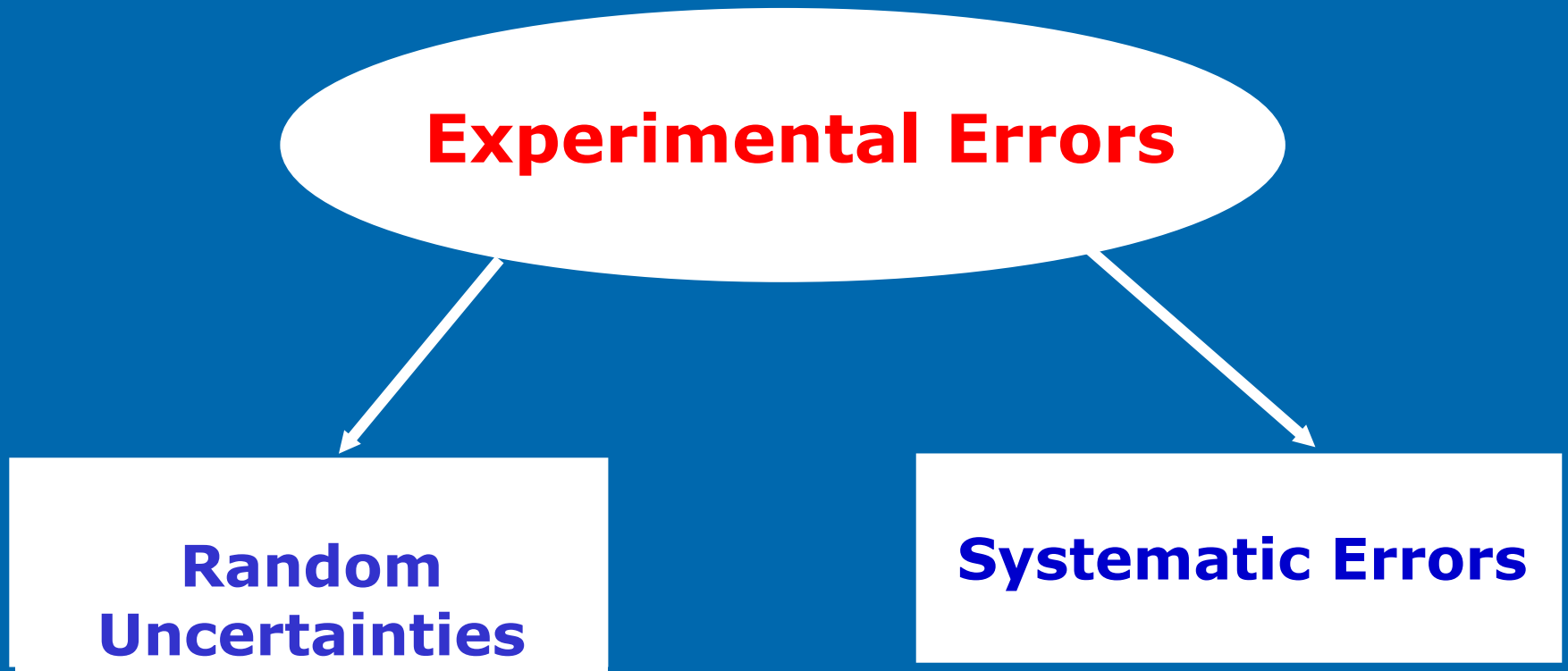
Error & Uncertainty

Measurement error

Instrument	Instrumental Uncertainties (\pm)	Example
Metre-rule	0.001 m 0.1 cm 1 mm	0.543 m 54.3 cm 543 mm
Vernier caliper	0.01 cm (0.1 mm)	2.53 cm
Vernier caliper (more accurate version)	0.002 cm (0.02 mm)	1.276 cm
Vernier microscope	0.01 cm	6.48 cm
Micrometer screw gauge	0.01 mm	1.57 mm
Digital stopwatch	0.1 s	9.85 s $\cong 9.9 \text{ s}^*$
Thermometer	0.2 °C 0.5 °C	27.8 °C 67.5 °C
Electronic balance	0.01 g 0.001 g	4.03 g 1.789 g
Protractor	1°	39°

* Average human reaction is about 0.2 s, it is reasonable to round off the time obtained from a digital stopwatch to 1 decimal place

Measurement error



Measurement error

Random Errors

Unpredictable deviations of a measured value (reading) from the actual value. Each reading has an equal chance to fall above or below the actual value.

Different magnitudes and signs in repeated measurements.

Inability to obtain true value due to:

- * limitations in the accuracy of a particular measuring technique (period of a pendulum; timing only one oscillation instead of 20)
- * limited sensitivities of instruments as given by the instrumental uncertainties

Measurement error

Random Errors

Examples are:

- * variation in conditions in the measuring instruments
- * variation arising from the inability of an observer to measure small intervals.
- * variation due to fluctuating external conditions (e.g. change in temperature during an experiment)

Can be reduced by taking the average of all measurements

Measurement error

Systematic Errors

- Errors in measurements which occur according to some fixed rule or pattern such that they yield a consistent over-estimation or under-estimation of the true value.
- Same error in magnitude and sign for repeated measurements under the same conditions
- Cannot be reduced by taking the average of a few measurement
- Sources of these errors can be identified and accounted for

Measurement error

Systematic Errors

Examples are:

- * Zero errors of instruments
- * Human reaction time
- * Extra counts in a Geiger-Muller (G.M.) counter due to background radiation
- * Wrong assumption made, such as use of $g=9.61 \text{ ms}^{-2}$

Distinction between Precision and Accuracy

Precision

It refers to the repeatability of the measurement.

High precision means 'small scatter' and 'low uncertainty'.

Set A

Diameter, D/mm	Deviation, d ($d = D_{ave} - D$)
0.38	0.02
0.36	0.04
0.40	0.00
0.44	0.04
0.42	0.02

Average Dia, $D_{ave} = 0.40$ mm

Mean devia = $0.12/5 = 0.024$ mm

Set B

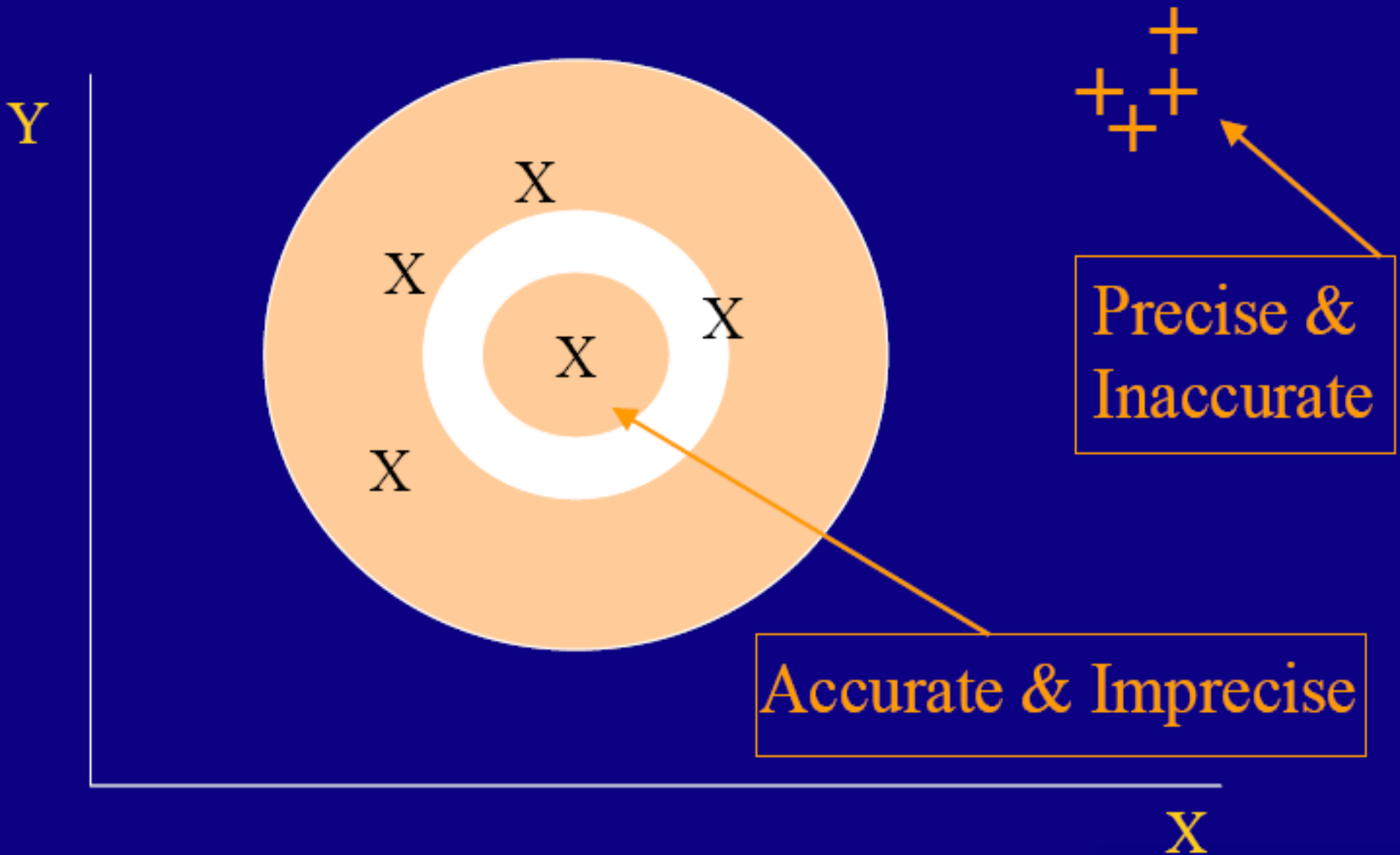
Diameter, D/mm	Deviation, d ($d = D_{ave} - D$)
0.40	0.00
0.41	0.01
0.39	0.01
0.42	0.02
0.38	0.02

Average Dia, $D_{ave} = 0.40$ mm

Mean devia = $0.06/5 = 0.012$ mm

**Mean Deviation is small, so set B is
MORE PRECISE**

Accuracy & Precision



Systematic Errors Vs Random Uncertainties



Large Systematic Error
(Inaccurate)

Large Random uncertainties
(Imprecise)

Large Systematic Error
(Inaccurate),

Small Random uncertainties
(Precise)

Systematic Errors Vs Random Uncertainties



Small Systematic Error
(Accurate)

Large Random uncertainties
(Imprecise)



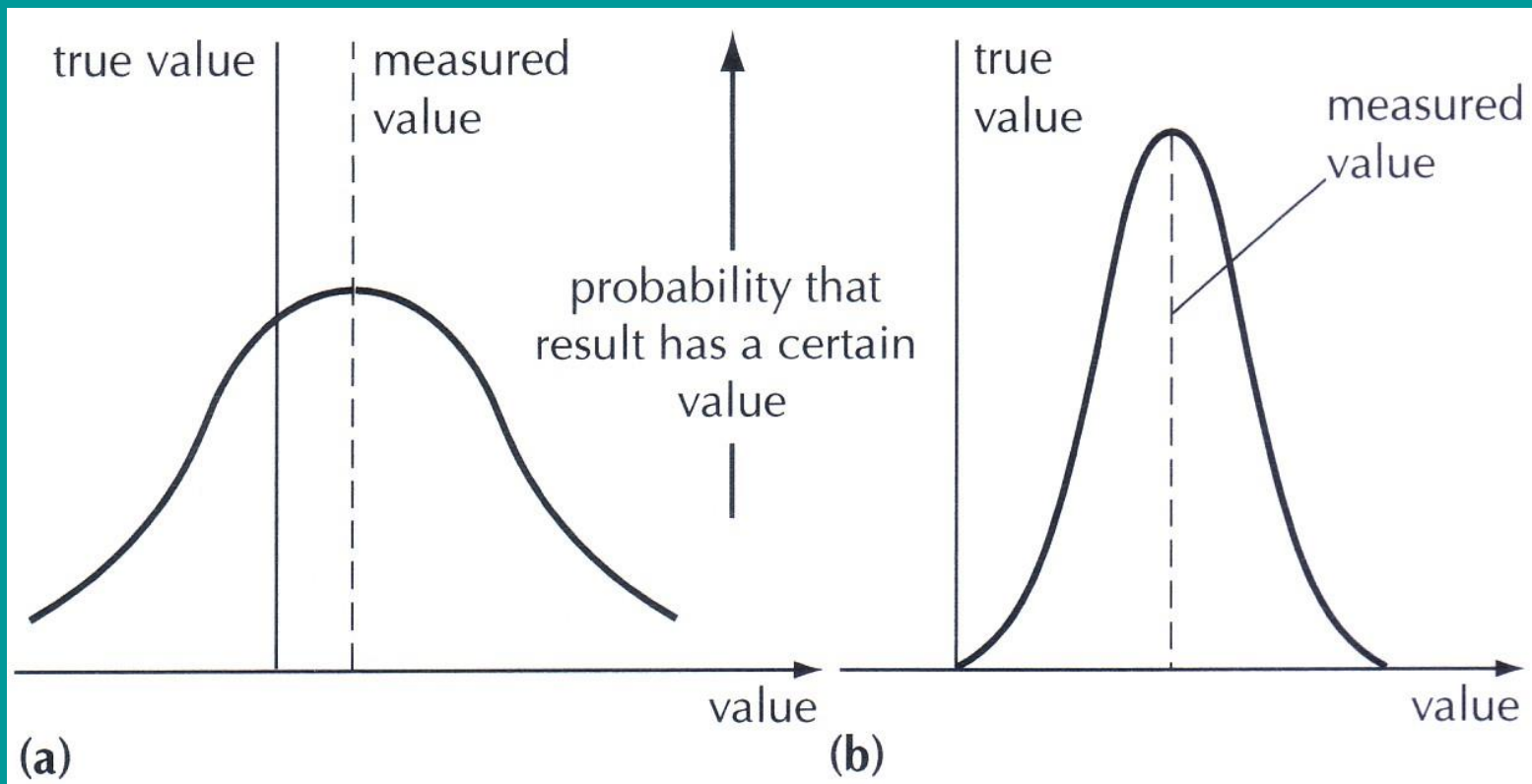
Very Small Systematic Error
(Very Accurate),

Very Small Random
uncertainties
(Very Precise)

Distinction between Precision and Accuracy

Accuracy

- A measurement is accurate if it is close to the true value



Summary

Readings with small random error are said to be precise.

Readings with small systematic error are said to be accurate.

Example

A steel rule can be read to the nearest millimeter. It is used to measure the length of a bar whose true length is 895 mm. Repeated measurements give the following readings

Length / mm 892, 891, 892, 891, 891, 892

Are the readings accurate and precise to within 1 mm?

	Results are accurate to within 1 mm	Results are precise to within 1 mm	
A	No	No	
B	No	Yes	
C	Yes	No	
D	Yes	Yes	

Solution

B

Example

Which of the following experimental techniques *does not* reduce the random error of the quantity being investigated?

- A calibrating the Y-sensitivity of the oscilloscope before measuring a voltage
- B measuring several internodal distances on a standing wave to find the mean internodal distance
- C timing a large number of oscillations to find a period
- D plotting a graph of voltage and current readings for an ohmic device and using its gradient to find resistance

Solution

A

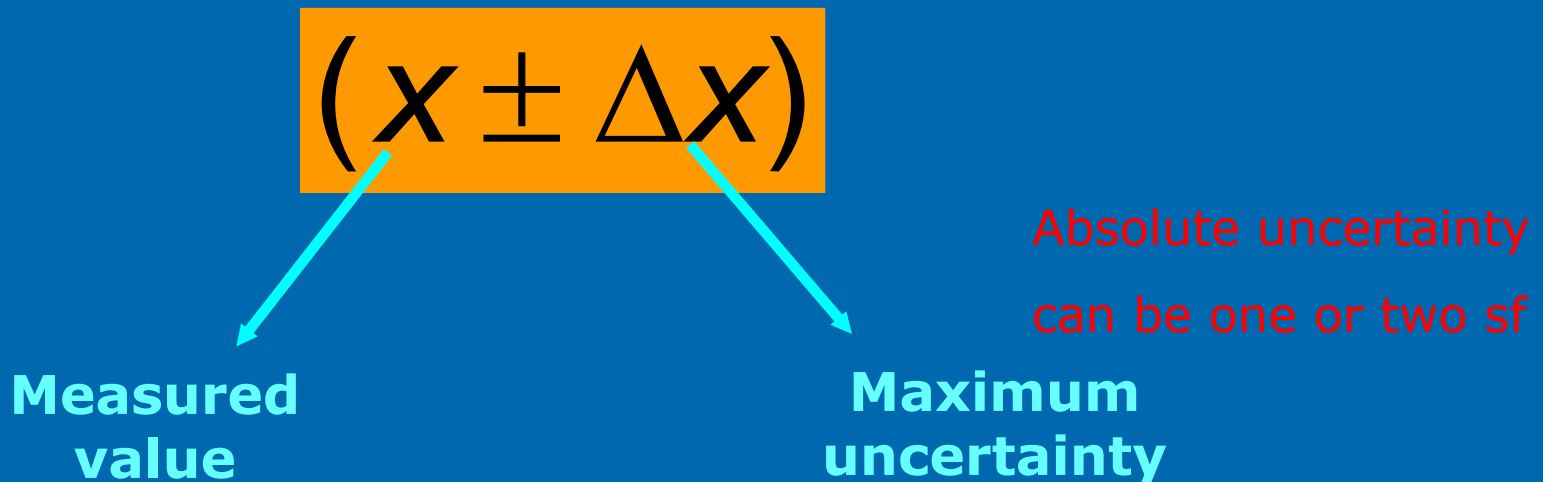
Example

A student measures the diameter of a cylindrical wooden pencil with a ruler. How could he increase the precision of the measurement?

- A Use a micrometer with zero error and take one value of the diameter.
- B Take the average value of several measurements of the diameter along different parts of the pencil using the ruler.
- C Take the average value of several measurements of the diameter along different parts of the pencil using vernier calipers without zero error.
- D Take the average value of several measurements of the diameter along different parts of the pencil using vernier calipers with zero error.

Uncertainty & Error

For any measurement, it is common to express any measured quantity as:



Measurement must have the same number of decimal places as the maximum uncertainty.

Eg. (12.0 ± 0.1) cm

Uncertainty & Error

Absolute, Fractional Error and Percentage Error

$$(x \pm \Delta x)$$

1) **Absolute error** of $X = \Delta x$

2) **Fractional error** of $X = \frac{\Delta x}{x}$

3) **Percentage error** of $X = \frac{\Delta x}{x} \times 100\%$

(Percentage error is useful to indicate suitability of the chosen instrument to measure a given quantity)

Uncertainty & Error

ADDING of two or more physical quantities

Add the **absolute** uncertainties

If

$$C = A + B$$

Maximum uncertainties

$$\Delta C = \Delta A + \Delta B$$

Percentage uncertainties

$$\frac{\Delta C}{C} \times 100\%$$

Uncertainty & Error

ADDING of two or more physical quantities

Example 1

Two strings with length A and length B are (1.0 ± 0.1) cm and (2.5 ± 0.1) cm respectively.

What is

- (a) the total length, **L**, when both strings are tied together
- (b) the percentage uncertainty of **L**

(a) 3.5 ± 0.2 cm

(b) $(0.2 / 3.5) 100 = 5.7 \%$

Uncertainty & Error

SUBTRACTION of two or more physical quantities

Add the **absolute** uncertainties

If

$$F = D - E$$

Maximum uncertainties

$$\Delta F = \Delta D + \Delta E$$

Percentage uncertainties

$$\frac{\Delta F}{F} \times 100\%$$

Uncertainty & Error

SUBTRACTION of two or more physical quantities

Example 2

Two strings with length D and length E are (1.0 ± 0.1) cm and (2.5 ± 0.1) cm respectively.

What is

- (a) the difference in length, **F**,
- (b) the percentage uncertainty of **F**

(a) 1.5 ± 0.2 cm

(b) $(0.2 / 1.5) 100 = 0.1 \%$

Uncertainty & Error

Multiplication of two or more physical quantities

Add the fractional & percentage uncertainties

If

$$M = B \times A$$

Fractional uncertainty of M

$$\frac{\Delta M}{M} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Uncertainty & Error

Division of two or more physical quantities

Add the fractional & percentage uncertainties

If

$$D = \frac{A}{B}$$

Fractional uncertainty of D

$$\frac{\Delta D}{D} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$$

Treatment of Error

Uncertainties involving **indices**

(These equations will be added in IB Data booklet)

If

Maximum fractional uncertainty

1)

$$A = B^n$$

$$\frac{\Delta A}{A} = n \frac{\Delta B}{B}$$

2)

$$A = B^n \times C^m$$

$$\frac{\Delta A}{A} = n \frac{\Delta B}{B} + m \frac{\Delta C}{C}$$

3)

$$A = \frac{B^n}{C^m}$$

$$\frac{\Delta A}{A} = n \frac{\Delta B}{B} + m \frac{\Delta C}{C}$$

Uncertainty & Error

Example 4

The density of the material of a rectangular block was determined by measuring the mass and linear dimensions of the block.

The results obtained, together with their uncertainties are shown below.

Mass = $(25.0 \pm 0.1)\text{g}$

Length = $(5.00 \pm 0.01)\text{ cm}$

Breath = $(2.00 \pm 0.01)\text{ cm}$

Height = $(1.00 \pm 0.01)\text{ cm}$

The density was calculated to be 2.50 g cm^{-3}

What was the uncertainty in this result?

Answer: $\pm 0.05\text{ g cm}^{-3}$

$$\frac{\Delta\rho}{\rho} = \frac{0.01}{5.00} + \frac{0.01}{2.00} + \frac{0.01}{1.00} + \frac{0.1}{25.0}$$

$$\frac{\Delta\rho}{2.50} = 0.021$$

$$\Delta\rho = (2.50)(0.021)$$

$$\Delta\rho = 0.05$$

Uncertainty & Error

Example 5

In an experiment to determine the acceleration of free fall g , the period of oscillation T and length l of a simple pendulum were measured. The uncertainty in the measurement of l was estimated to be 4%, and that of T , 1%.

The value of g was determined using the formula

$$g = \frac{4\pi^2 l}{T^2}$$

What is the uncertainty in the calculated value of g ?

$$\begin{aligned}\text{Percentage uncertainty of } g &= 4 + 2(1) \\ &= 6 \%\end{aligned}$$

Example

In determining the acceleration of free fall, g , using the formula for the period of a simple pendulum,

$$T = 2\pi\sqrt{\frac{l}{g}}$$

The length of the pendulum was found to be $l = (0.500 \pm 0.001) \text{ m}$, while the time was measured to be $T = (1.42 \pm 0.02) \text{ s}$. What should the student record as the value of g ?

- A $(9.8 \pm 0.2) \text{ m s}^{-2}$
- B $(9.8 \pm 0.3) \text{ m s}^{-2}$
- C $(9.79 \pm 0.03) \text{ m s}^{-2}$
- D $(9.789 \pm 0.295) \text{ m s}^{-2}$

Solution

$$g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 (0.500)}{1.42^2} = 9.789 \text{ ms}^{-2}$$

$$\frac{\Delta g}{9.789} = \frac{0.001}{0.500} + 2 \frac{0.02}{1.42} \Rightarrow \Delta g = 0.3 \text{ ms}^{-2}$$

$$g = (9.8 \pm 0.3) \text{ ms}^{-2}$$

Example

The volume of a cylinder was found by measuring its diameter and height to within 0.01 and 0.03 fractional uncertainty respectively.

The percentage uncertainty in the calculated volume of metal cylinder is at most

- A 2 %
- B 3 %
- C 4 %
- D 5 %

Solution

$$V = \pi r^2 h = \pi \left(\frac{d}{2} \right)^2 h$$

$$\frac{\Delta V}{V} = \frac{\Delta h}{h} + 2 \frac{\Delta d}{d} = 0.03 + 2(0.01) = 0.05$$

$$\frac{\Delta V}{V} \times 100\% = 5\%$$

D

Example

A student makes measurements from which he calculates the density of a liquid to be 951.5 kg m^{-3} . He estimates that his result is only accurate to $\pm 4\%$. The density for the liquid should be written as

- A $(951.50 \pm 38.06) \text{ kg m}^{-3}$
- B $(952.0 \pm 38.1) \text{ kg m}^{-3}$
- C $(952 \pm 38) \text{ kg m}^{-3}$
- D $(950 \pm 40) \text{ kg m}^{-3}$

Solution

$$\frac{\Delta\rho}{\rho} \times 100\% = 4\% \Rightarrow \Delta\rho = 38.06 \approx 40 \text{ kg m}^{-3}$$

Mathematical and Graphical Techniques

Mathematical and Graphical Techniques

Approximation

Estimated values of everyday quantities to one or two significant digits and/or to the nearest order of magnitude.

Reasonable estimate of common quantities (eg dimensions of a brick, mass of an apple, duration of a heartbeat or room temperature are expected).

State and explain simplifying assumptions in approaching and solving problems.

(eg reasonable assumptions that certain quantities may be neglected, others ignored (eg heat losses, internal resistance), or that behaviour is approximately linear.

Mathematical and Graphical Techniques

Approximation

Simple calculations:

$$\begin{aligned} 1) \quad 174 \div 118 &= 180 \div 120 \\ &= 3/2 \\ &= 1.5 \end{aligned}$$

$$\begin{aligned} 2) \quad 6.3 \times 7.6/4.9 &= 6 \times 8/5 \\ &= 48/5 \\ &= 50/5 \\ &= 10. \end{aligned}$$

Mathematical and Graphical Techniques

Recall: significant figures

Value	No. of significant figures
0.5	1
0.50	2
0.500	3
0.05	1
0.050	2
5	1
5.0	2
5.00	3
1.52	3
1.52×10^4	3
1.5×10^2	2
1.50×10^2	3
150	2 or 3 (ambiguous)

Mathematical and Graphical Techniques

Handling Numbers

Multiplication and division of numbers

Keep the same number of significant figures in the product or quotient as in the least accurate factor

$$\begin{array}{ccccccc} \text{(a)} & 16.42 & \times & 0.211 & = & \mathbf{3.46} \\ & (4 \text{ sig. fig.}) & & (3 \text{ sig. fig.}) & & (3 \text{ sig. fig.}) \end{array}$$

$$\begin{array}{ccccccc} \text{(b)} & 5.6 & \times & 0.530 & = & \mathbf{3.0} \\ & (2 \text{ sig. fig.}) & & (3 \text{ sig. fig.}) & & (2 \text{ sig. fig.}) \end{array}$$

Mathematical and Graphical Techniques

Handling Numbers

Multiplication and division of numbers

Keep the same number of significant figures in the product or quotient as in the least accurate factor

$$\begin{array}{ccccccc} \text{(c)} & 6.5 & \div & 14.50 & = & \mathbf{0.45} \\ & (2 \text{ sig. fig.}) & & (4 \text{ sig. fig.}) & & (2 \text{ sig. fig.}) \end{array}$$

$$\begin{array}{ccccccc} \text{(d)} & 100.2 & \div & 0.5 & = & \mathbf{2 \times 10^2} \\ & (4 \text{ sig. fig.}) & & (1 \text{ sig. fig.}) & & (1 \text{ sig. fig.}) \end{array}$$

Mathematical and Graphical Techniques

Handling Numbers

Adding and subtracting numbers

Final value has the same number of decimal places or is in the same place value as the least accurate factor

$$(a) \quad 60.5 \quad + \quad 1.53 \quad = \quad \mathbf{62.0}$$

$$(b) \quad 2.432 \quad + \quad 1.7 \quad = \quad \mathbf{4.1}$$

$$(c) \quad 2.921 \quad + \quad 0.7 \quad + \quad 3 \quad = \quad \mathbf{7}$$

$$(d) \quad 15.4 \quad - \quad 0.232 \quad = \quad \mathbf{15.2}$$

$$(e) \quad 120 \quad - \quad 18.3 \quad = \quad \mathbf{102}$$

Mathematical and Graphical Techniques

Displaying data on Graph

Decide on most suitable scale.

Include or suppress zero as necessary

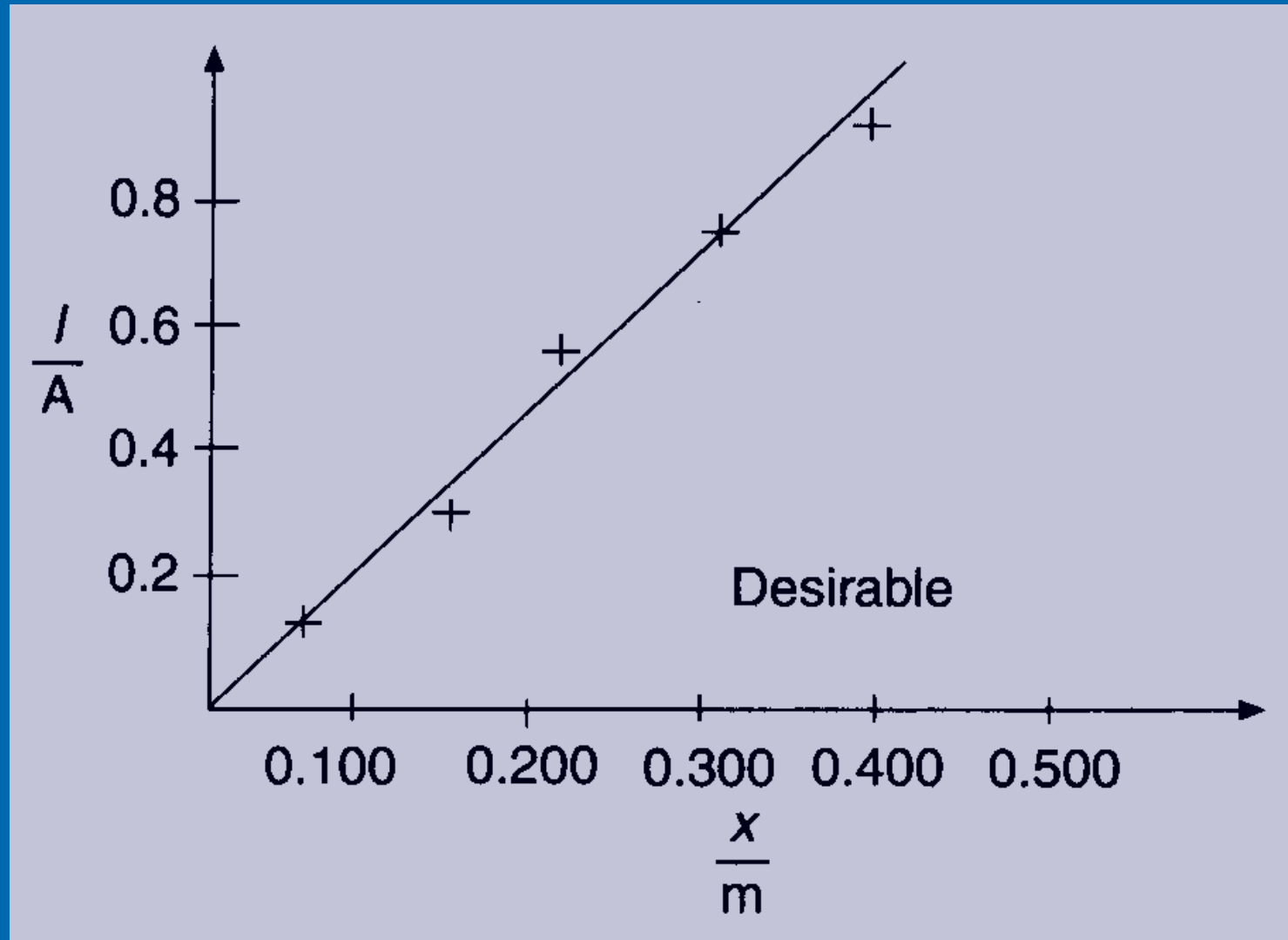
When determining the gradient of a straight line.

- * Draw a large triangle (broken lines)
- * Label on the graph the coordinates used to find the gradient

Evidence of how a reading is obtained from a graph is shown using dotted reference lines from co-ordinate to both axes.

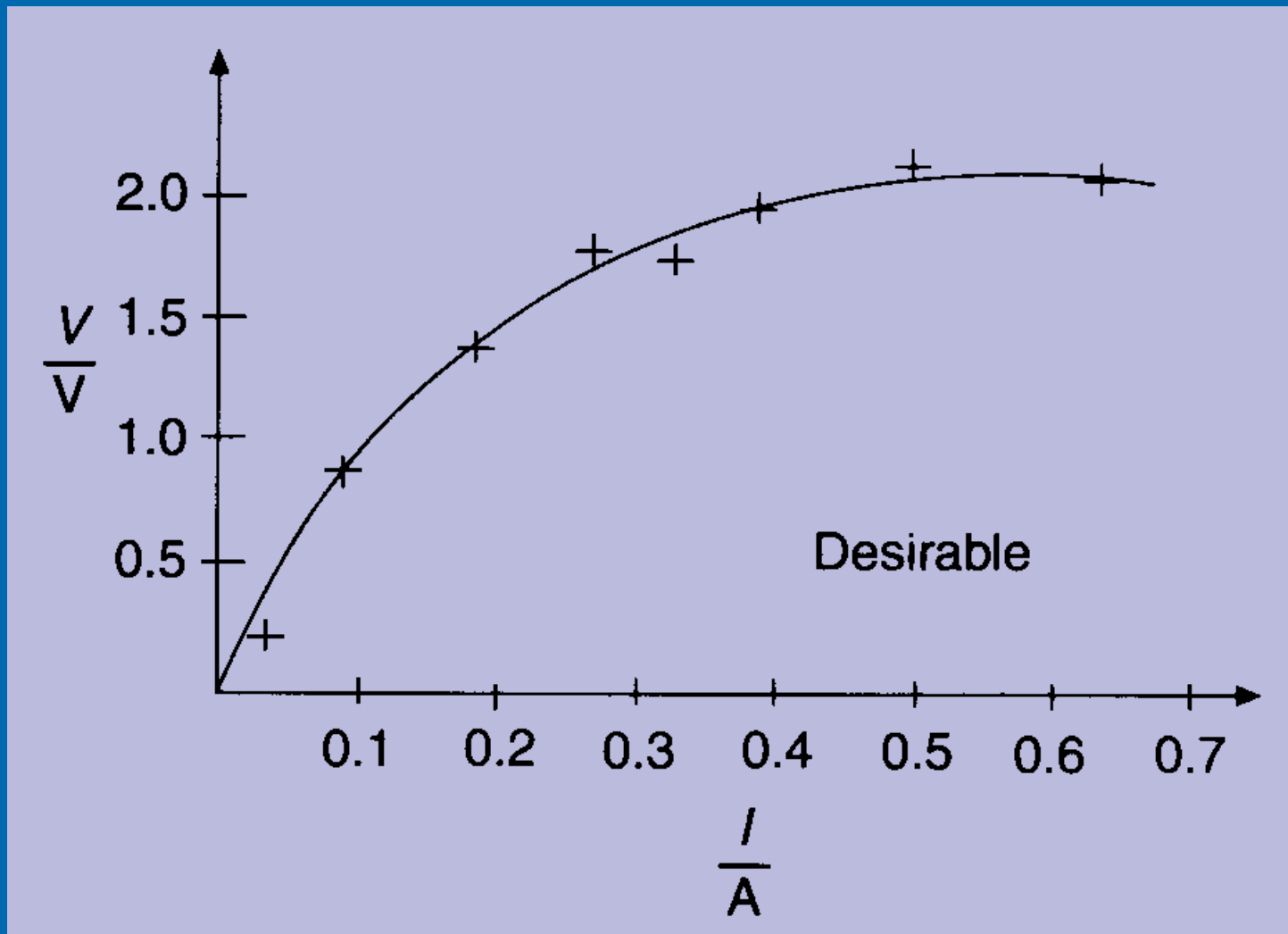
Mathematical and Graphical Techniques

Displaying data on Graph



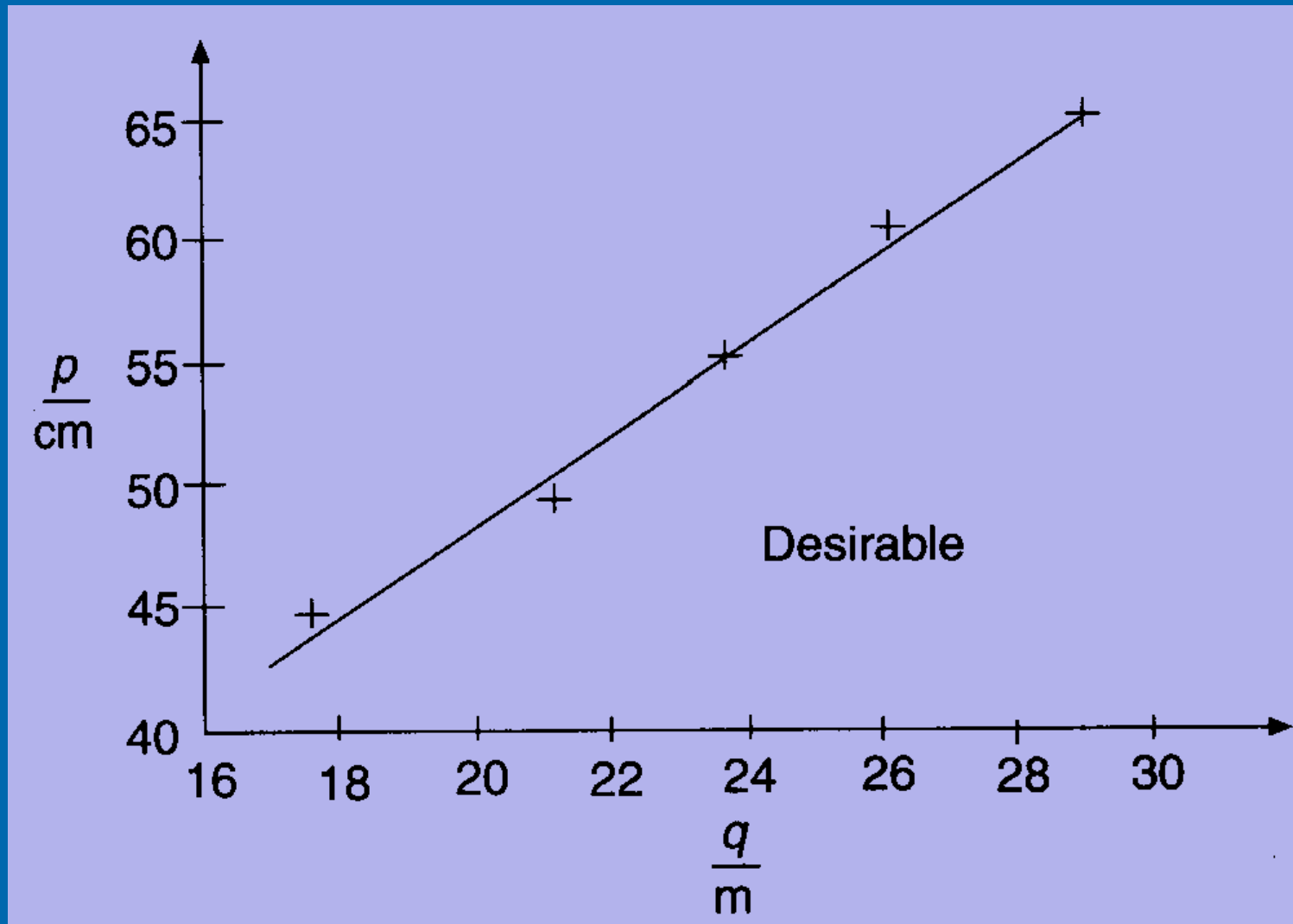
Mathematical and Graphical Techniques

Displaying data on Graph



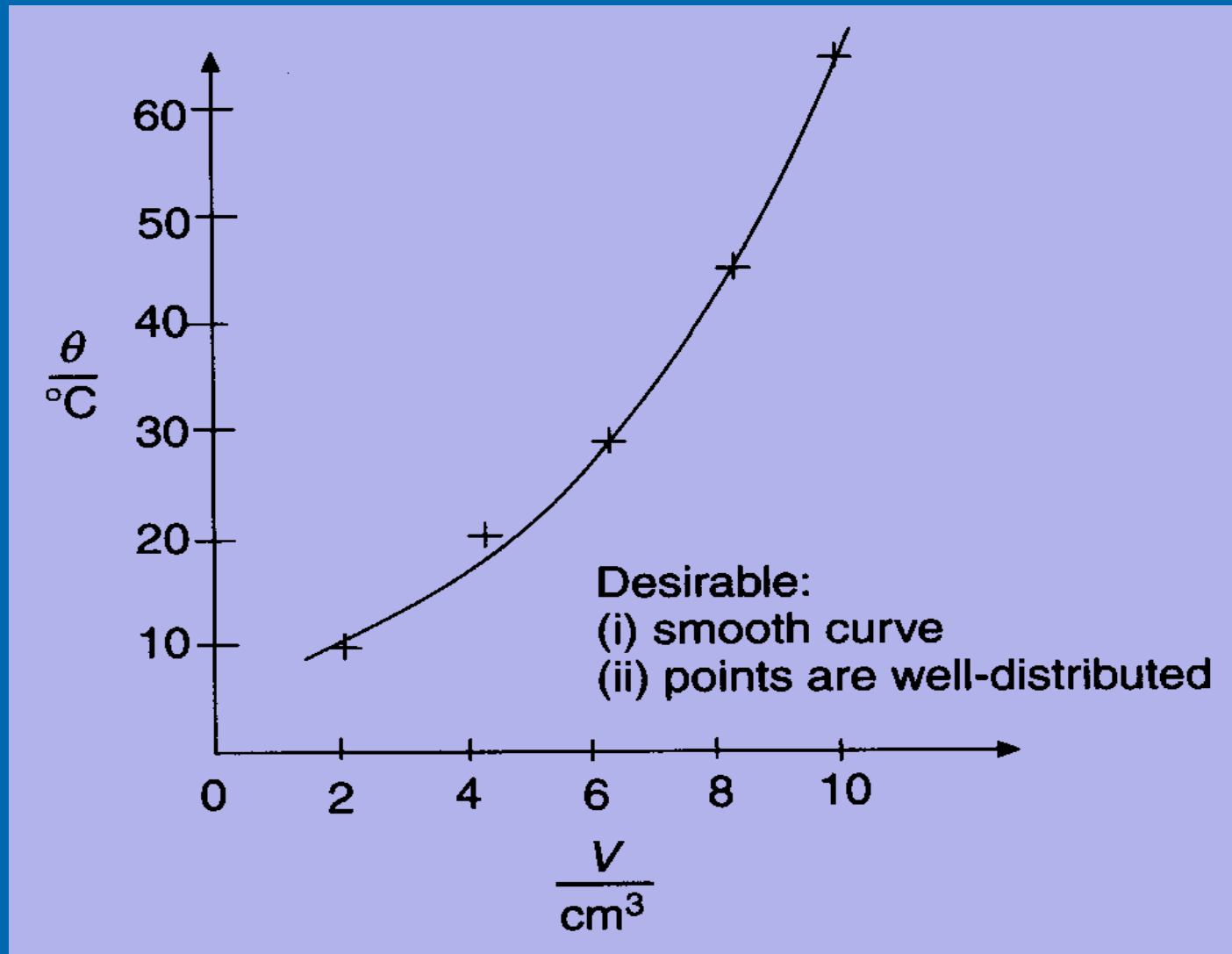
Mathematical and Graphical Techniques

Displaying data on Graph



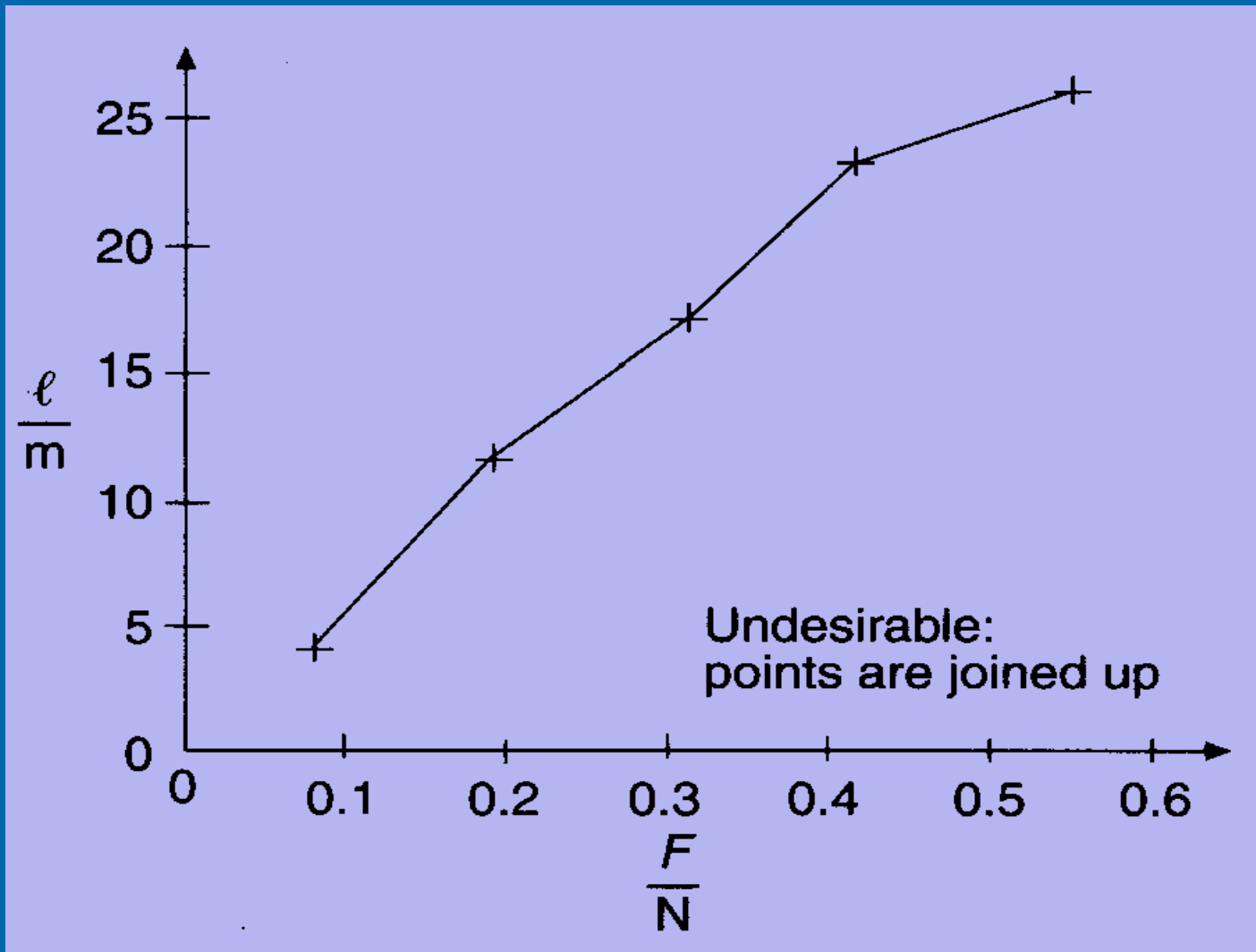
Mathematical and Graphical Techniques

Displaying data on Graph



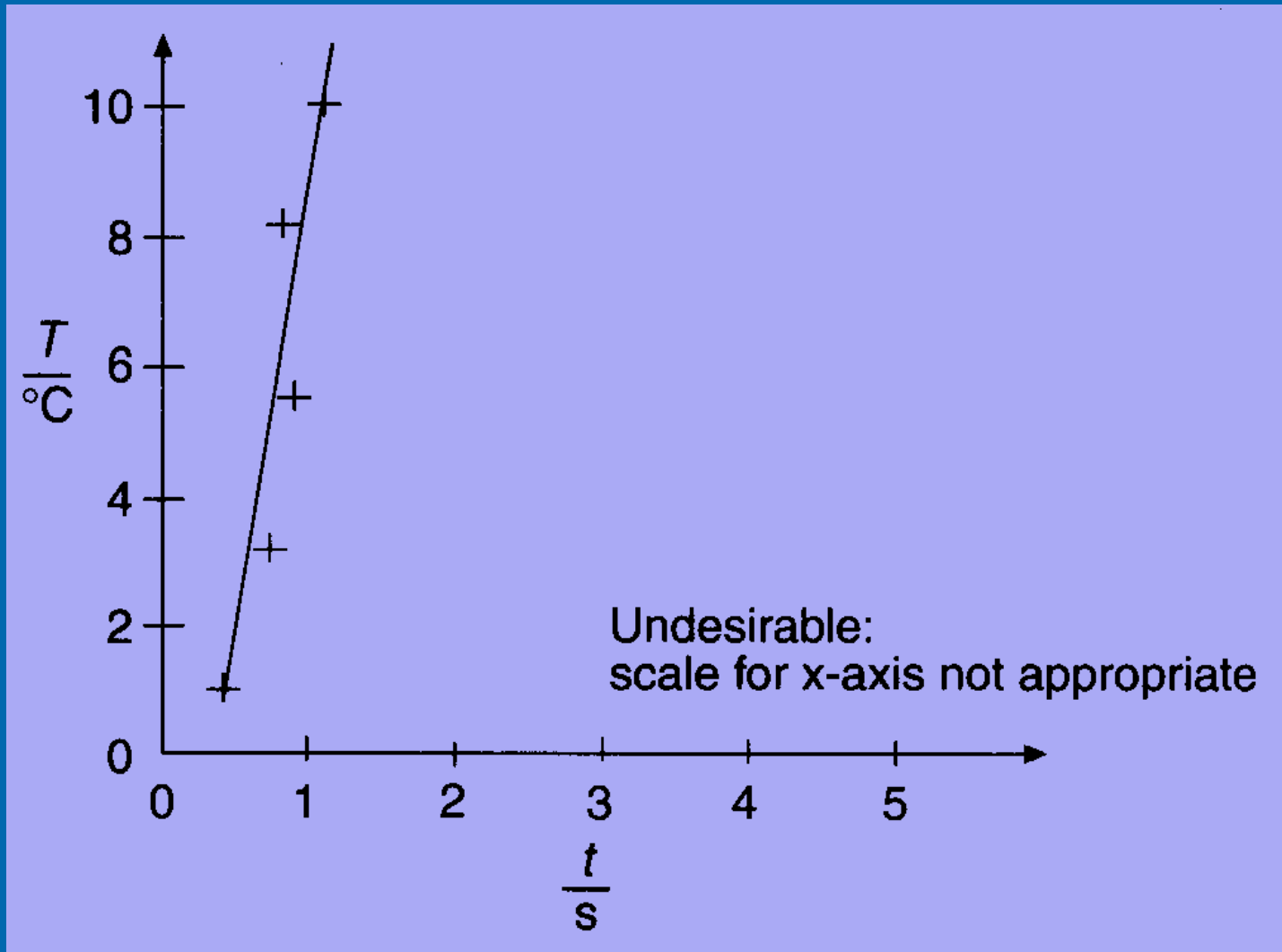
Mathematical and Graphical Techniques

Displaying data on Graph



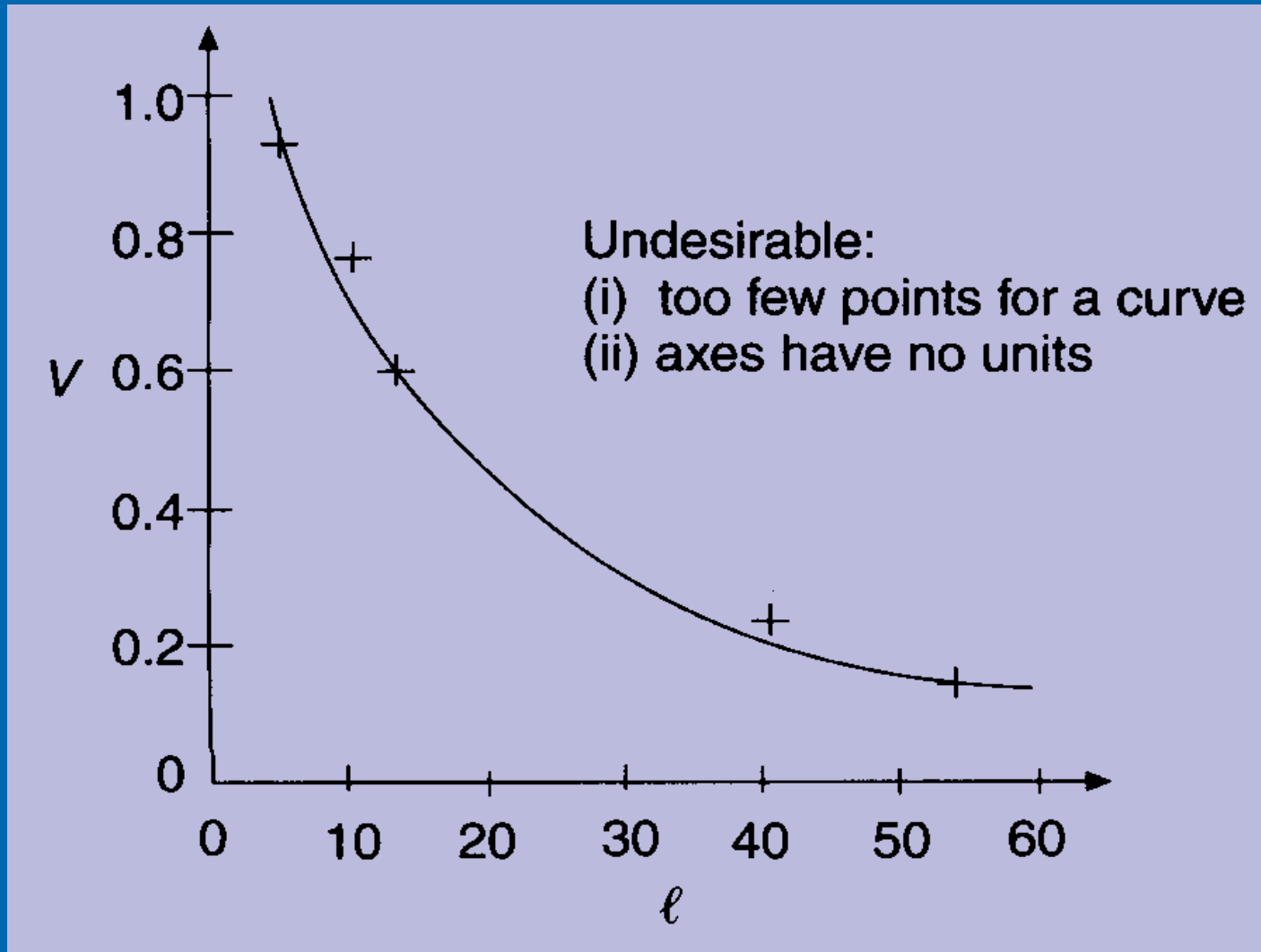
Mathematical and Graphical Techniques

Displaying data on Graph



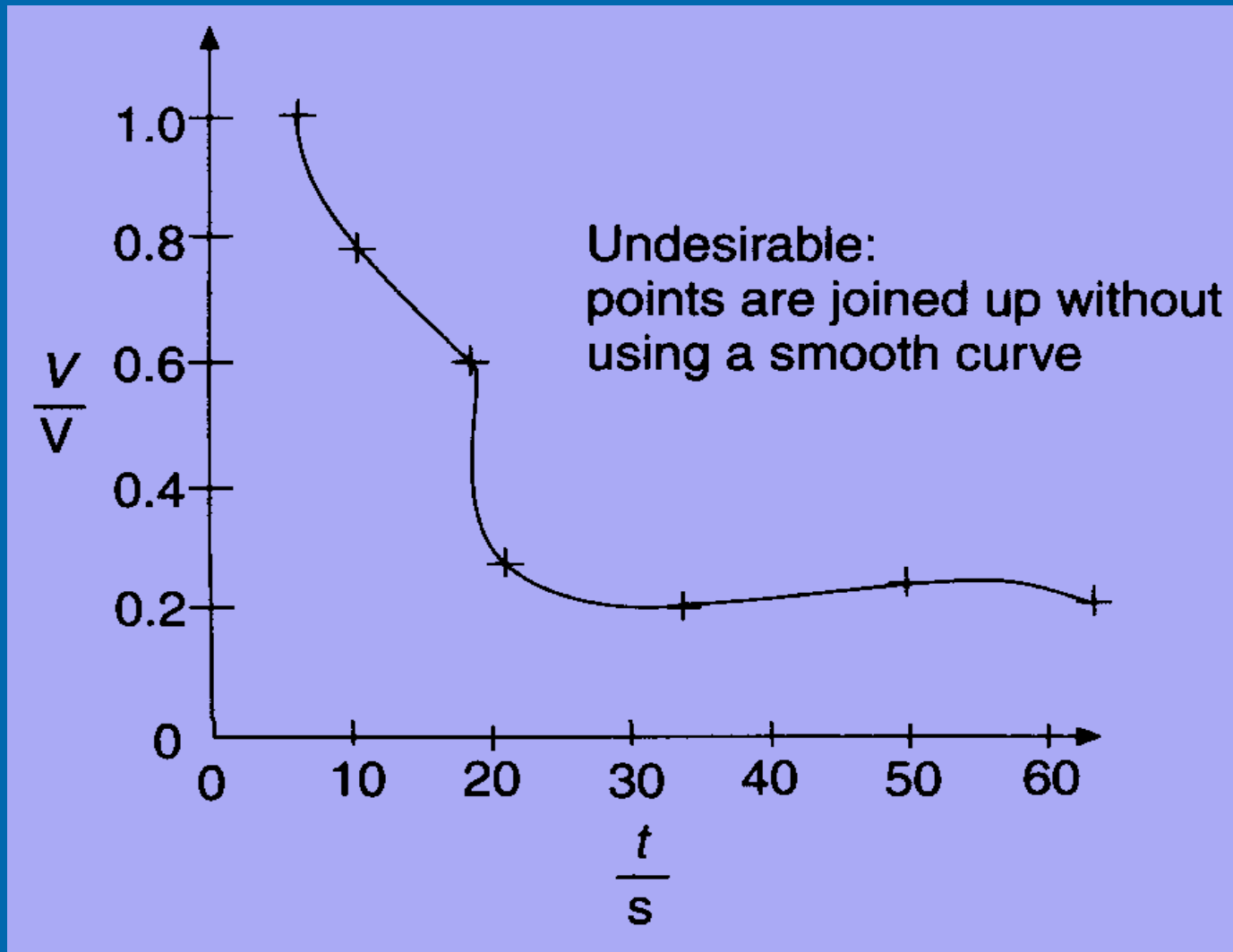
Mathematical and Graphical Techniques

Displaying data on Graph



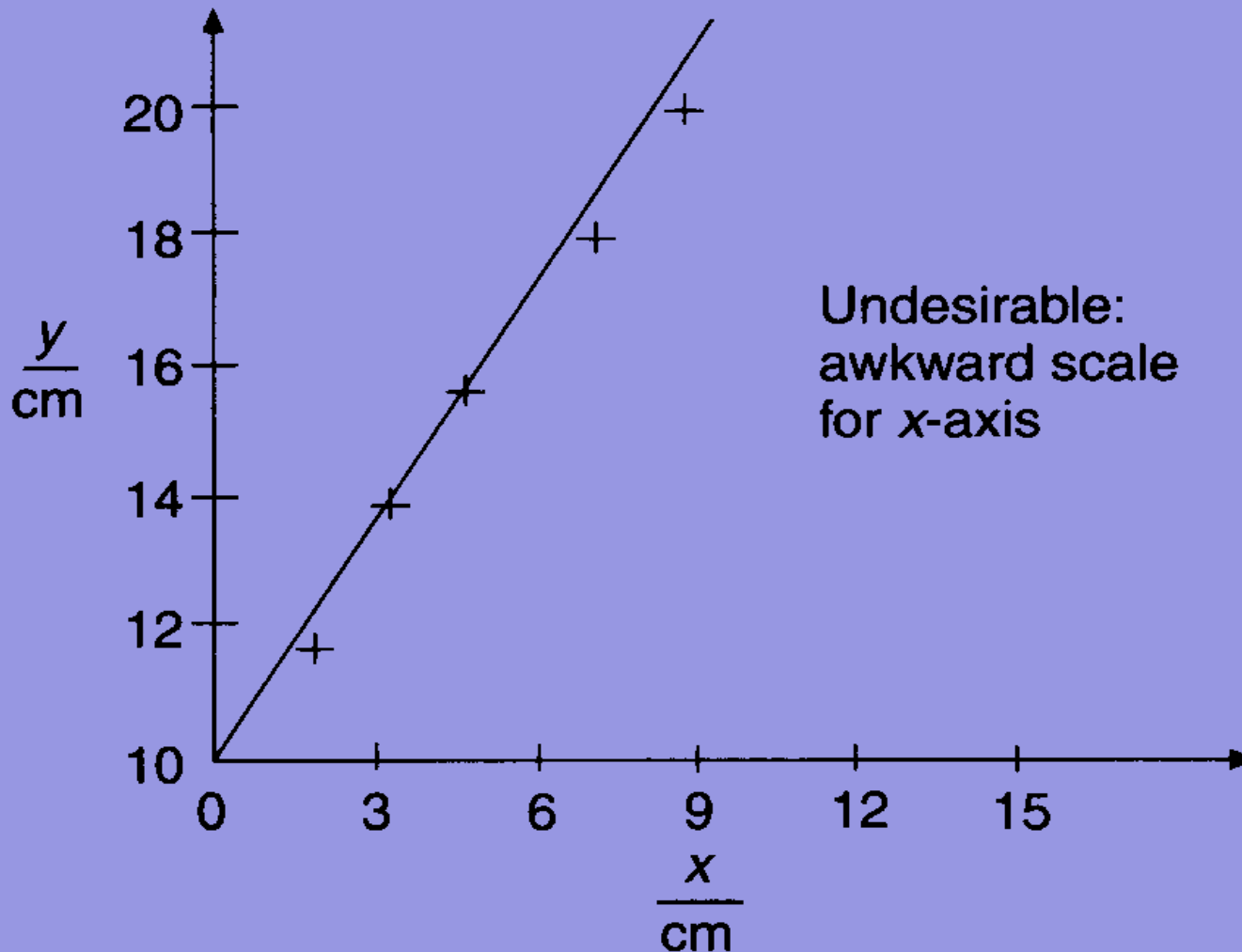
Mathematical and Graphical Techniques

Displaying data on Graph



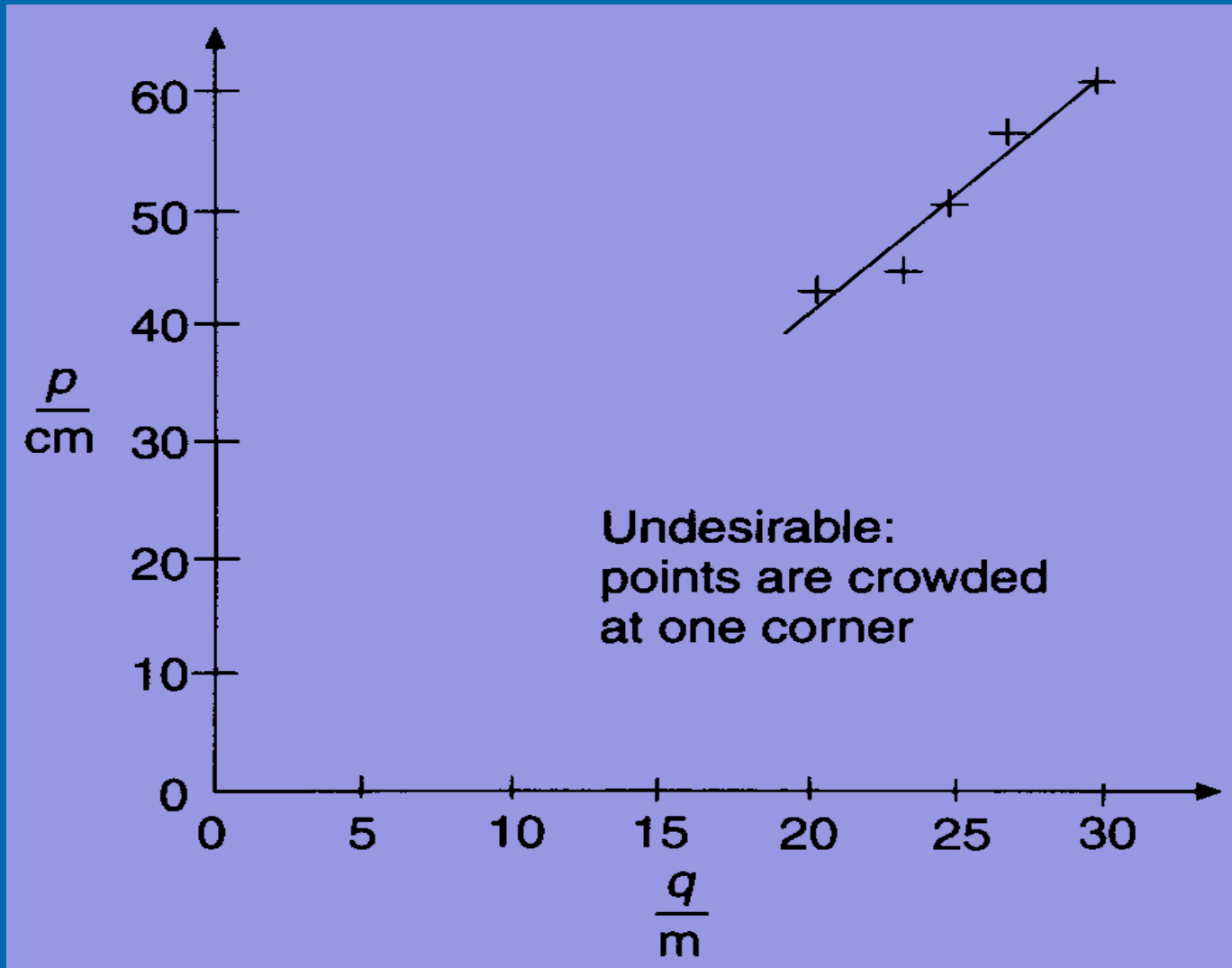
Mathematical and Graphical Techniques

Displaying data on Graph



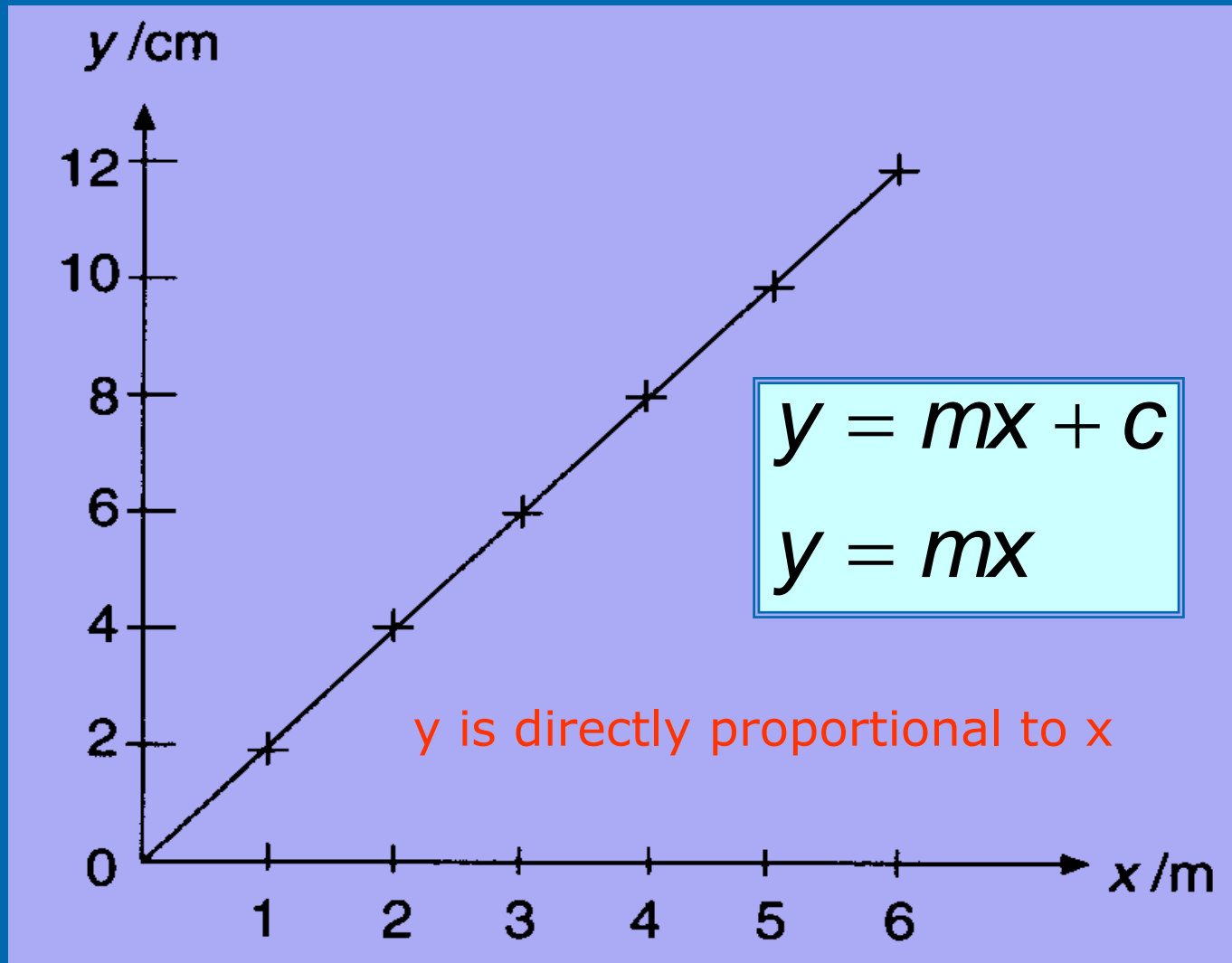
Mathematical and Graphical Techniques

Displaying data on Graph



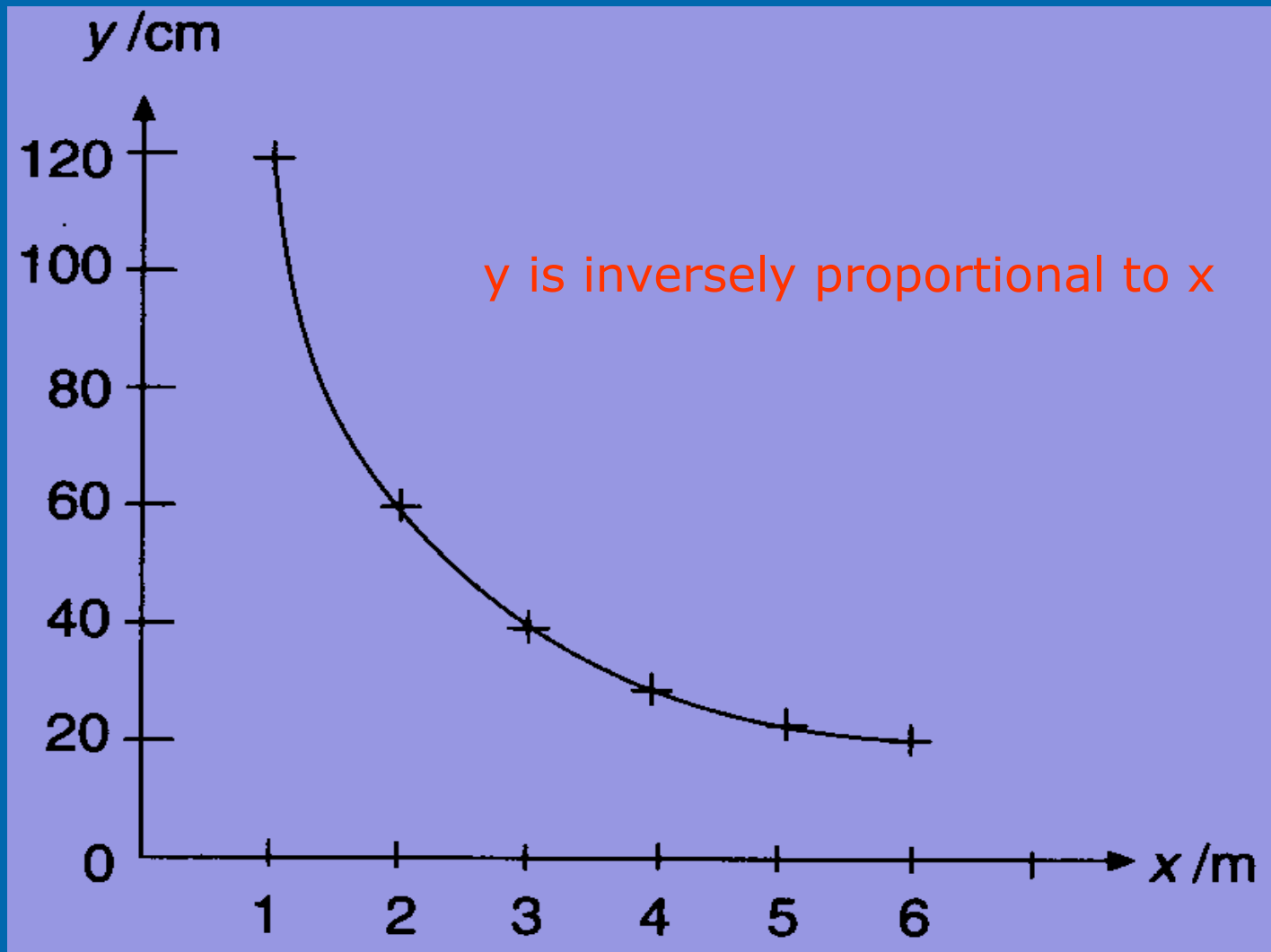
Mathematical and Graphical Techniques

Interpreting Graph



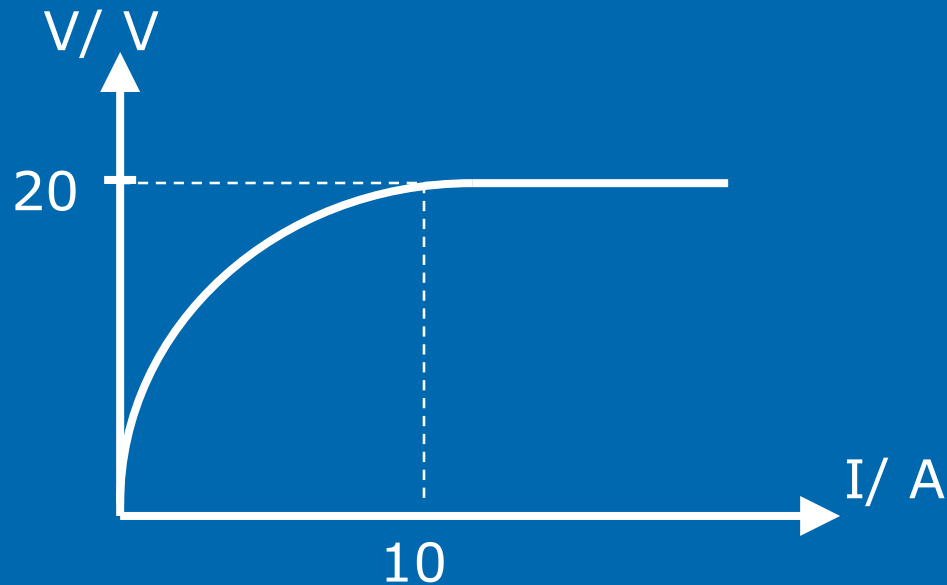
Mathematical and Graphical Techniques

Interpreting Graph



Mathematical and Graphical Techniques

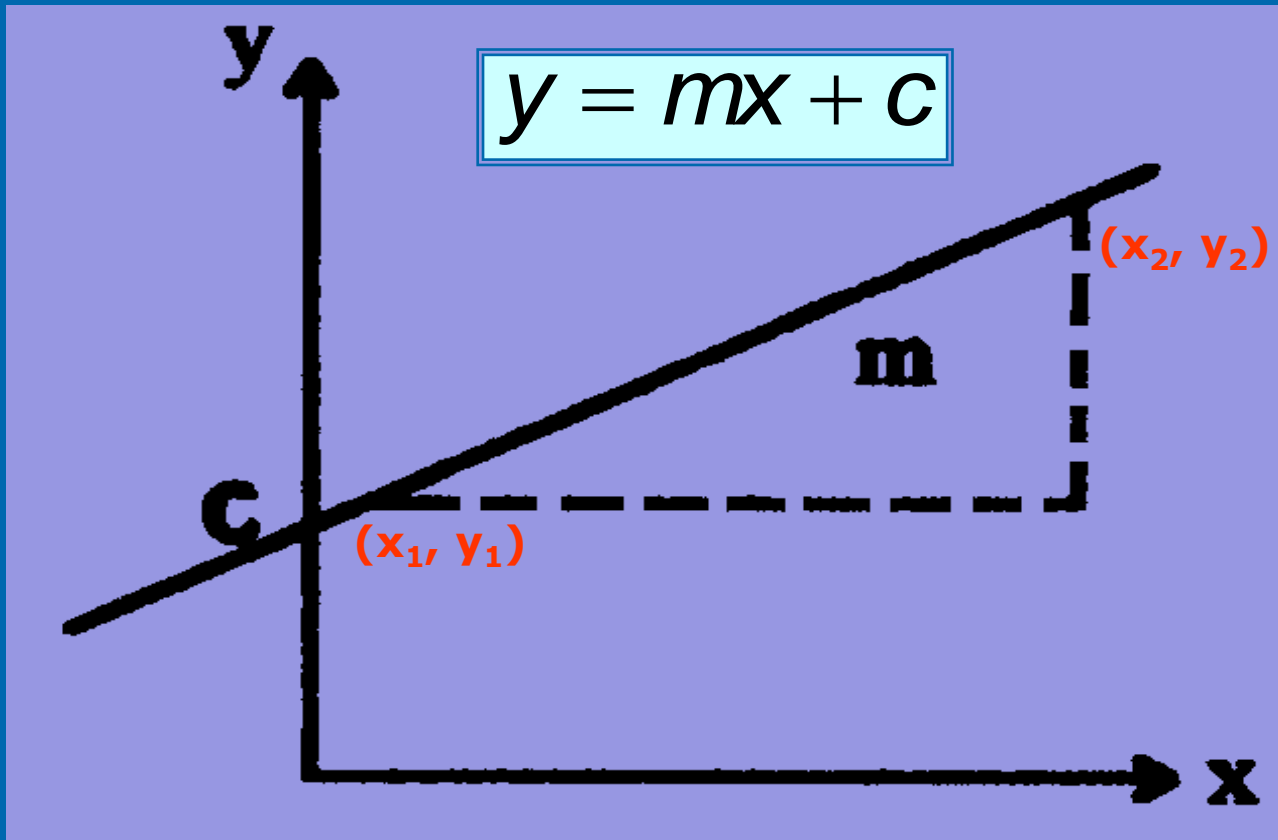
Interpreting Graph



As current approaches 10 A, the voltage approaches 20 V.
Beyond a current of 10 A, the voltage remains constant at 20 V.

Mathematical and Graphical Techniques

Types of Graph



Where m is the gradient of the graph

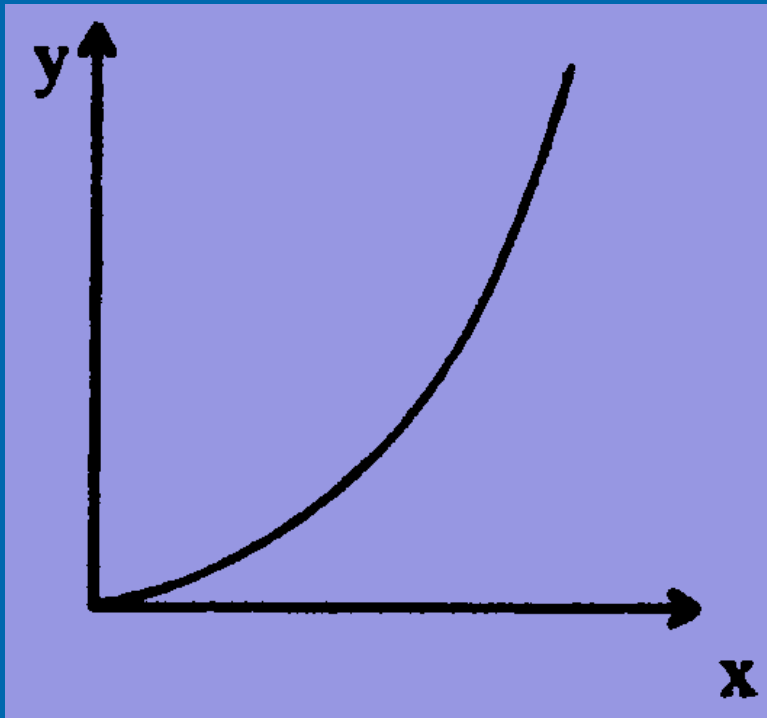
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

y varies linearly with x

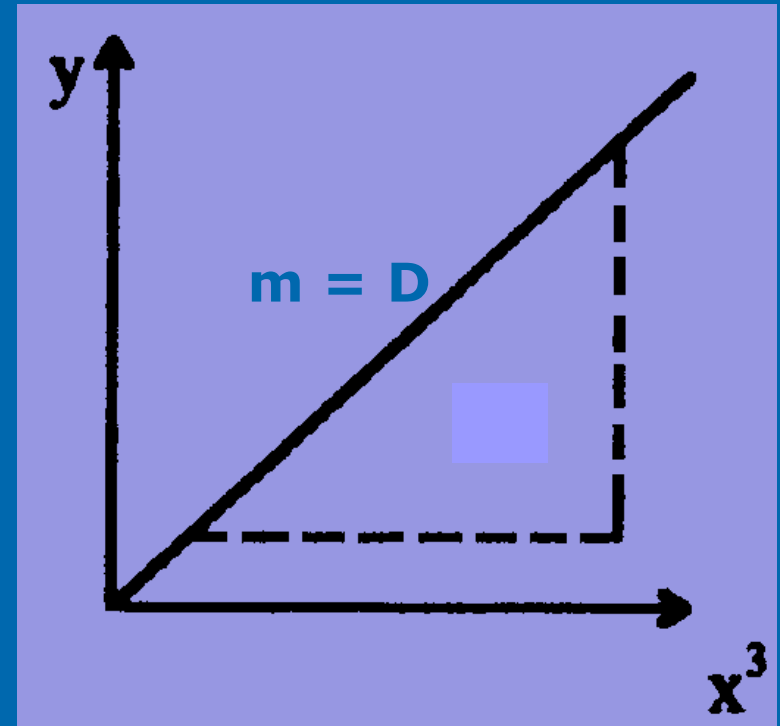
Mathematical and Graphical Techniques

Plot a suitable graph to determine the value of D

$$y = Dx^3$$



Graph of y against x

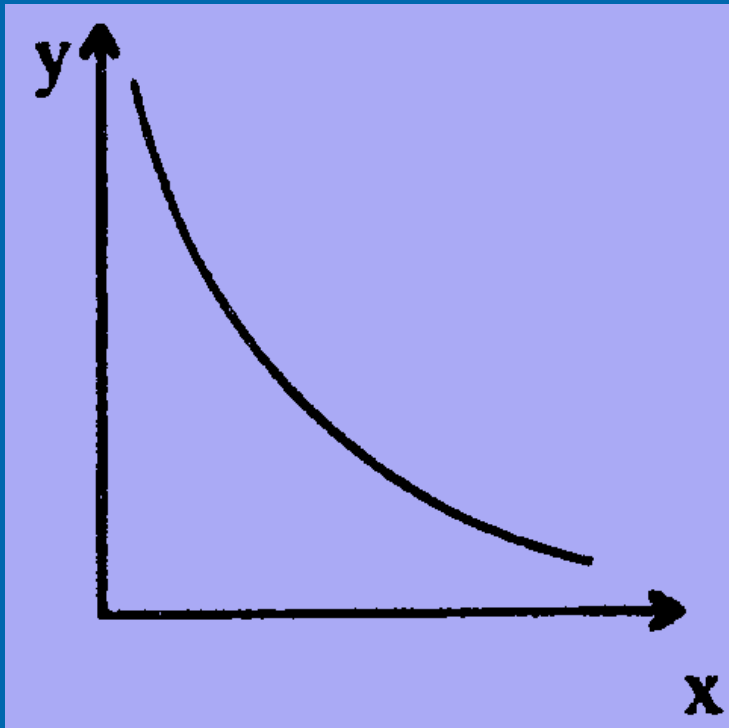


Graph of y against x^3

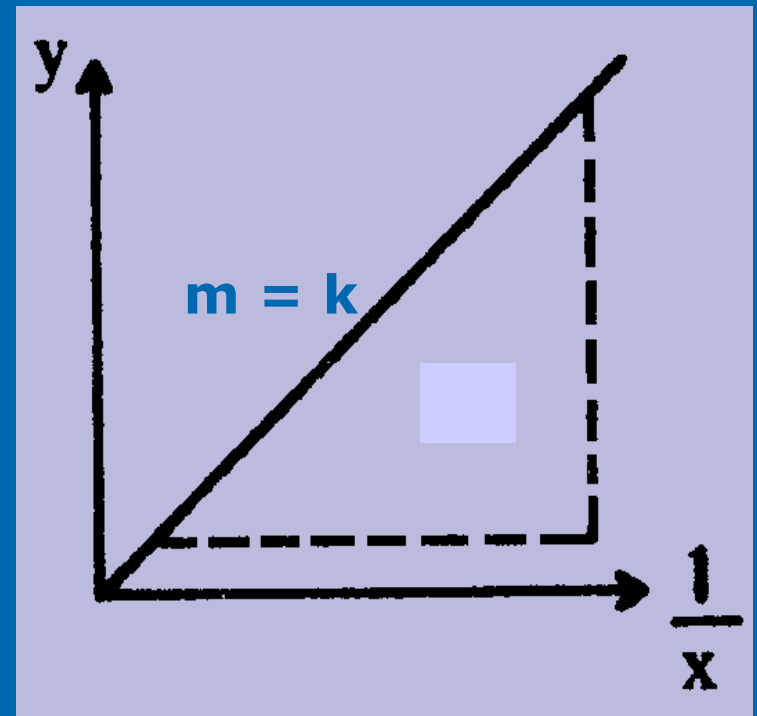
Mathematical and Graphical Techniques

Types of Graph

$$y = \frac{k}{x}$$



Graph of y against x

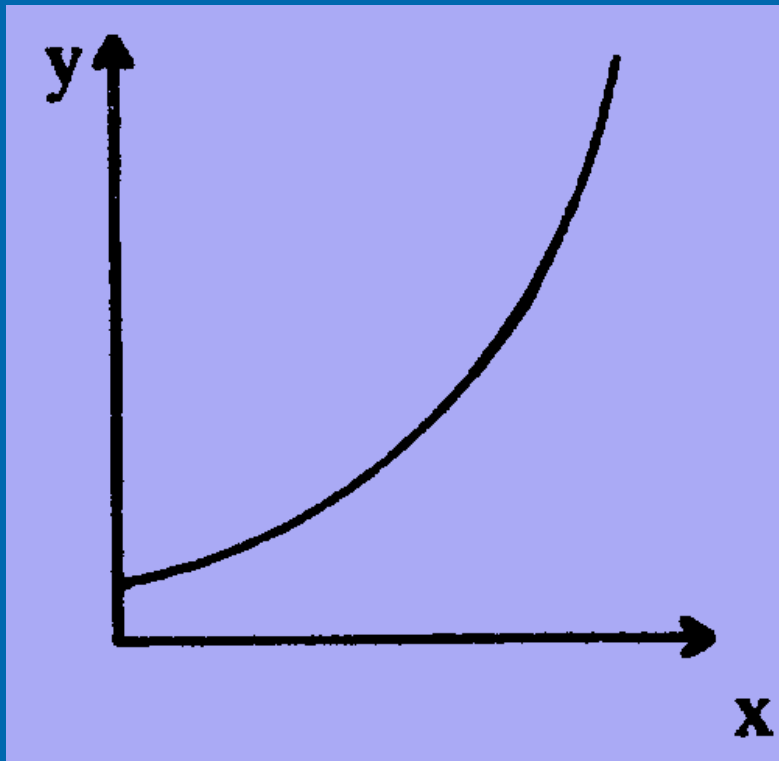


Graph of y against $1/x$

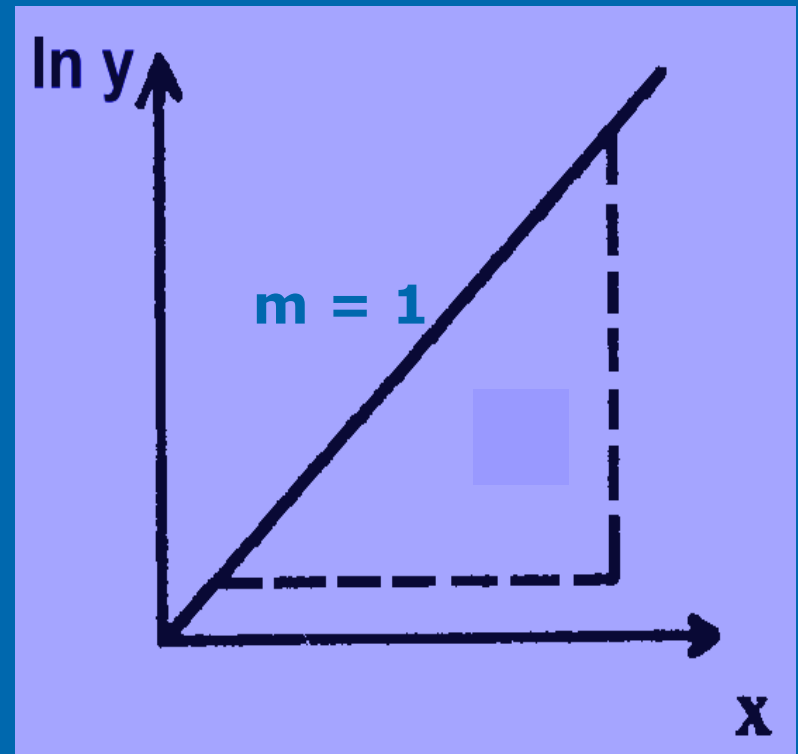
Mathematical and Graphical Techniques

Types of Graph

$$y = e^x$$



Graph of y against x



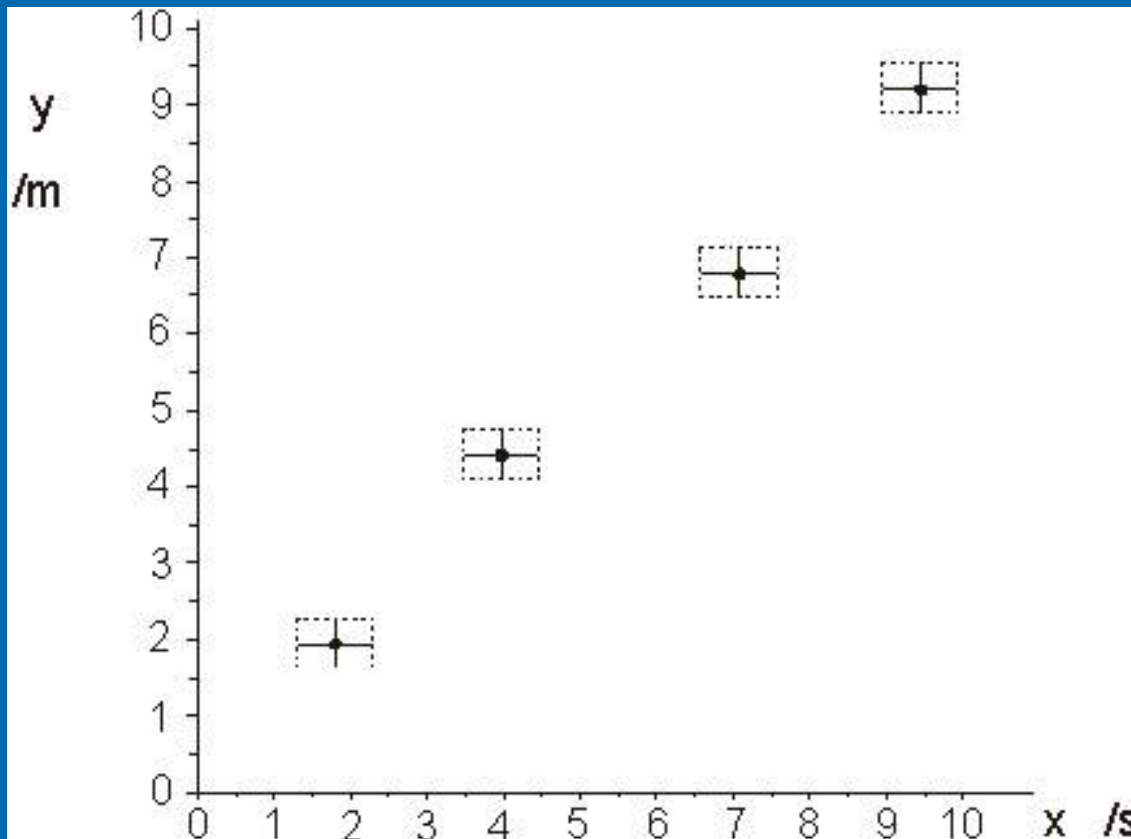
Graph of $\ln y$ against x

Mathematical and Graphical Techniques

Error bars

When uncertainty is taken into account in the plotting of a graph, error bars are drawn.

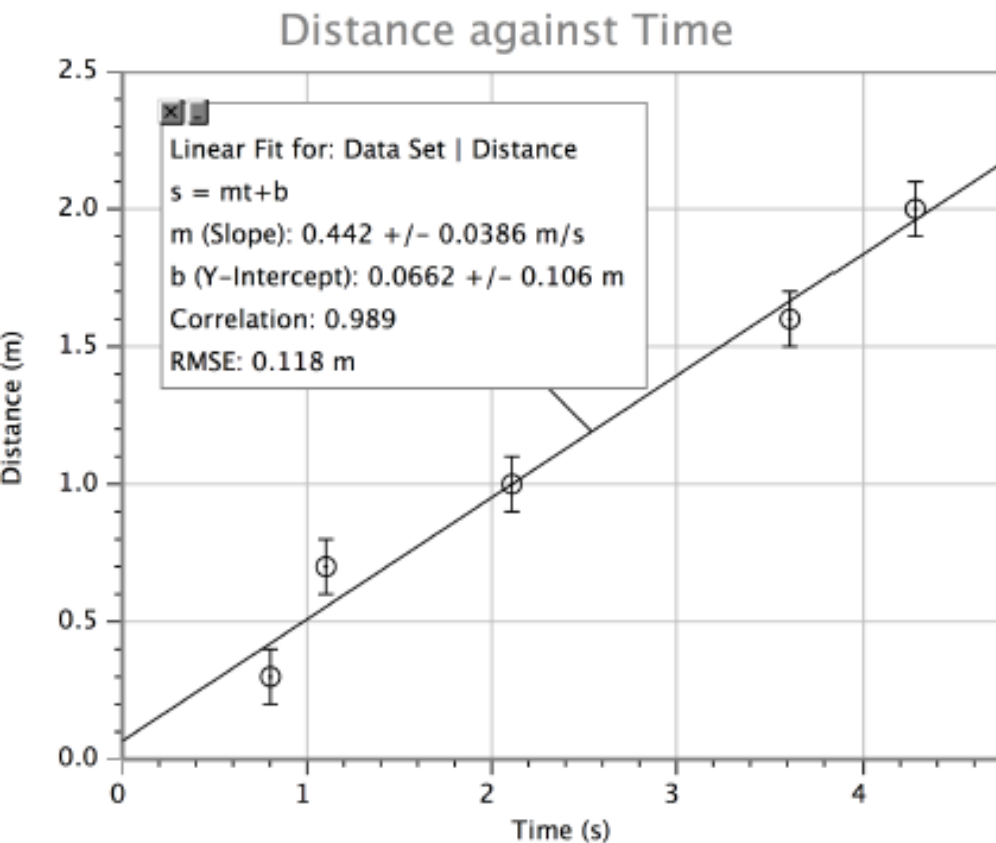
Vertical and horizontal errors result in an error rectangle.



x was measured
to ± 0.5 s

y was measured
to ± 0.3 m

Line of best fit- by using linear fit function on computer

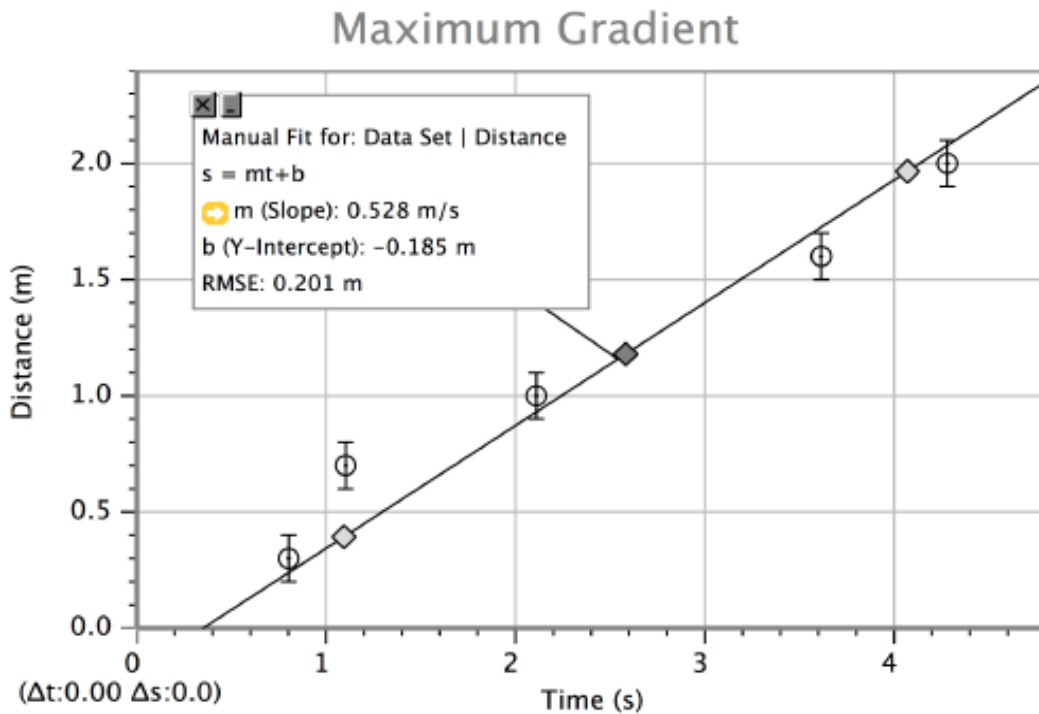


The gradient of the best-fit linear line and the y-intercept using standard deviation are:

$$m_{\text{best}} = 0.442 \pm 0.0386$$

$$y_{\text{best}} = 0.0662 \pm 0.106$$

Maximum gradient

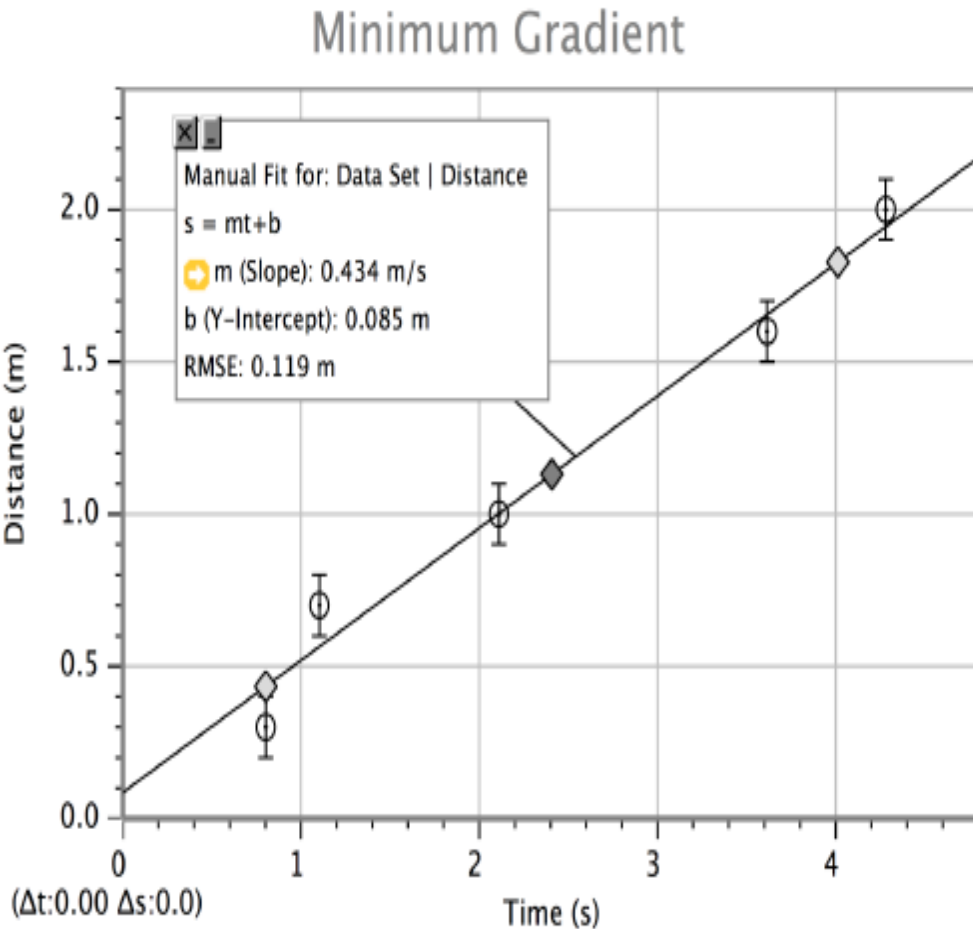


**Method by Eye: Maximum Gradient,
Minimum Intercept**

$$m_{\max} = 0.528$$

$$y_{\min} = -0.185$$

Minimum gradient



**Method by Eye: Minimum Gradient,
Maximum Intercept**

$$m_{\min} = 0.434$$

$$y_{\max} = 0.085$$

Finding $m \pm \Delta m$

$$\Delta m = \frac{\text{Range}}{2} = \frac{m_{\text{max}} - m_{\text{min}}}{2} = \frac{0.528 - 0.434}{2} = 0.047$$

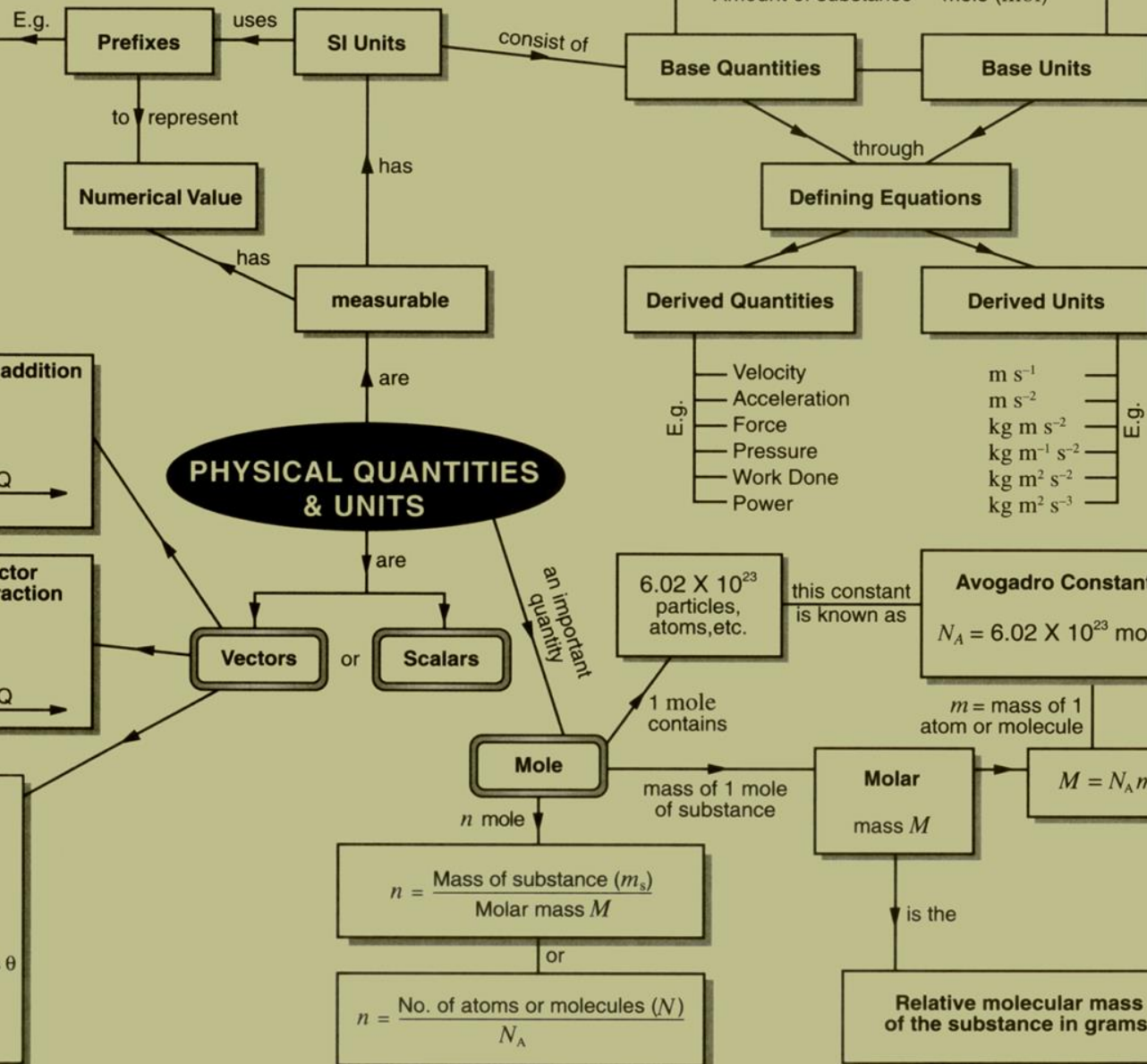
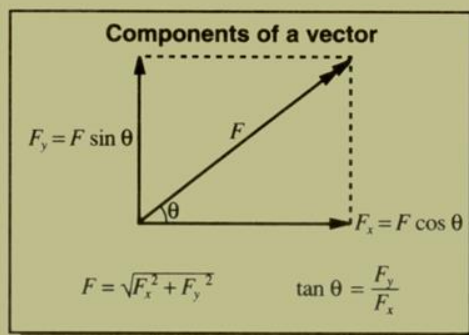
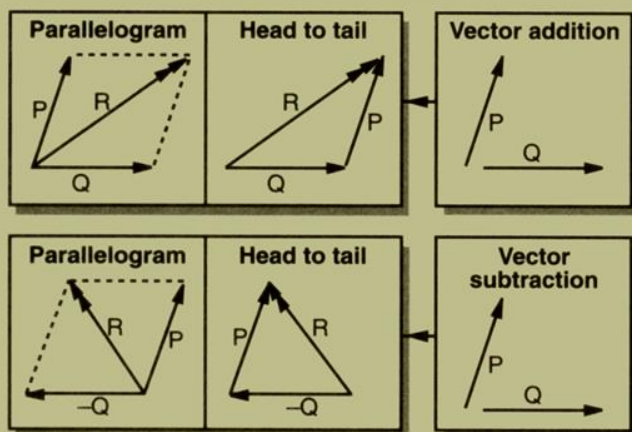
$$m_{\text{best}} \pm \Delta m = 0.442 \pm 0.047 \approx 0.44 \pm 0.05$$

Manual method on ICT

- The manual fit method accounts for all or nearly all the uncertainty ranges of all the data points while **standard deviation ignores uncertainties and relates the regression line to the data points.**

Prefix	Symbol	Submultiple
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}

Prefix	Symbol	Multiple
kilo	k	10^3
mega	M	10^6
giga	G	10^9
tera	T	10^{12}



MEASUREMENT TECHNIQUES

