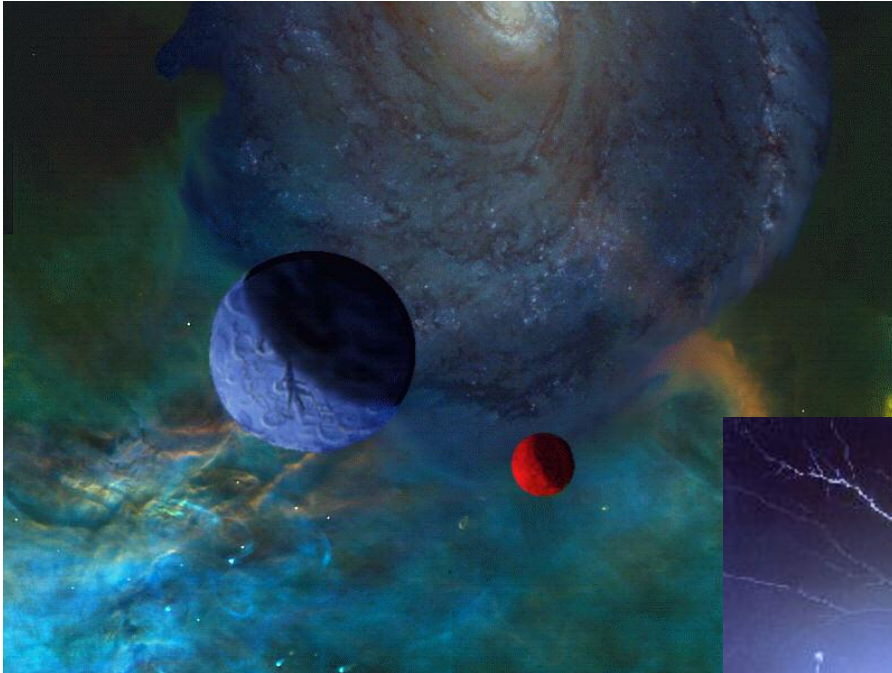
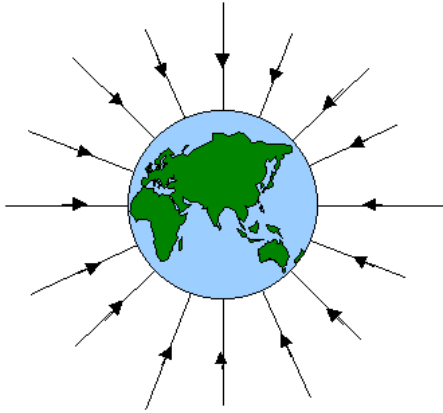


# 10.1 Describing Fields



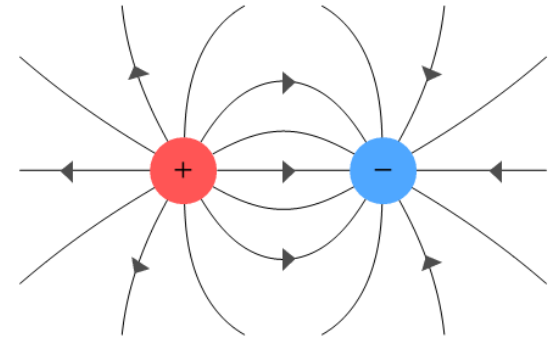
# Electric Fields & Gravitational Fields

The concept of field lines can be used to visually represent:



The gravitational field,  $g$ , around a mass (or collection of masses).

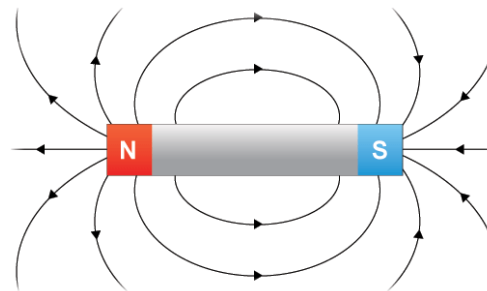
Units: **N kg<sup>-1</sup>**



The electric field,  $E$ , around a charge (or collection of charges).

Units: **N C<sup>-1</sup>**

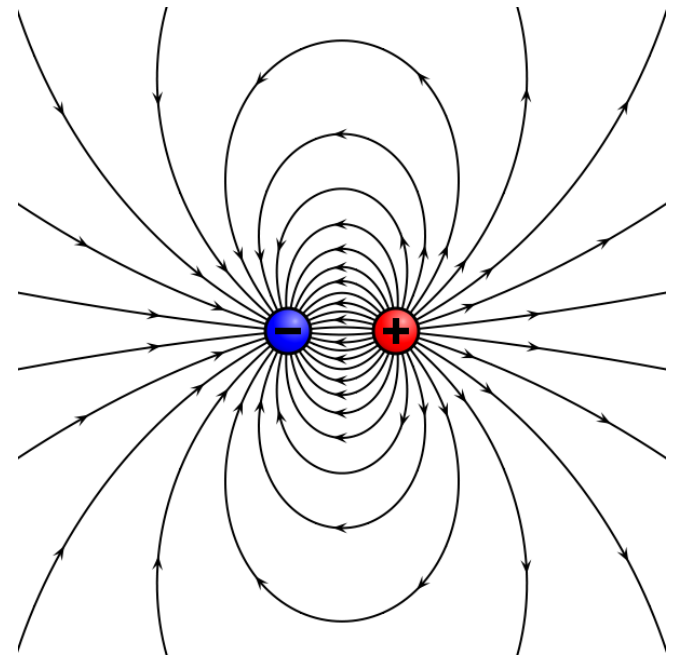
Magnetic fields can also be represented using field lines.



# Electric Fields & Gravitational Fields

Field lines represent both the **magnitude** and **direction** of the force that would be felt by the test object that is placed inside a field.

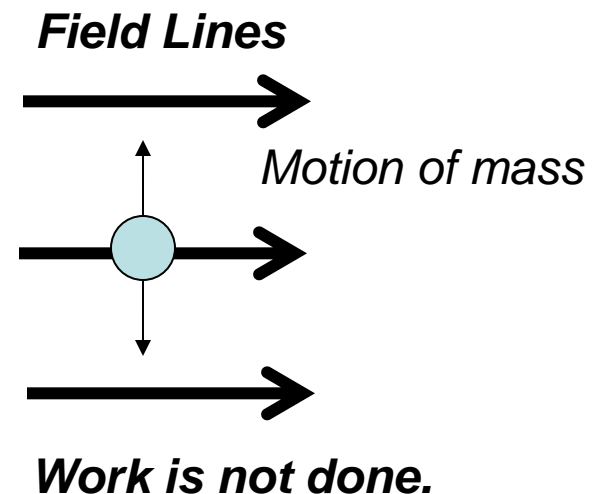
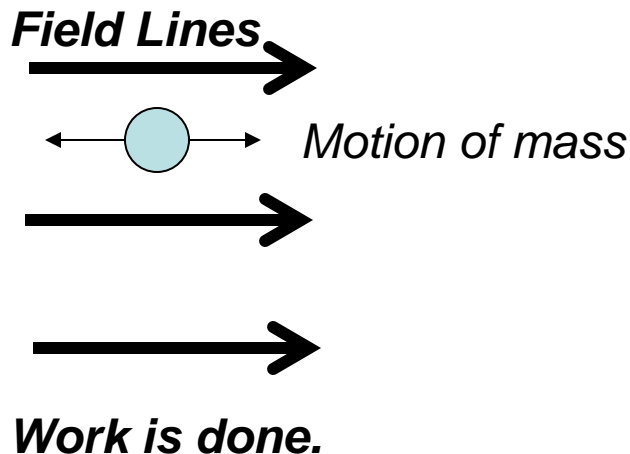
- The **magnitude** of the force of the force is represented by how close the field lines are to each other.
- The **direction** of the force is represented by the direction of the field lines.



# Electric Fields & Gravitational Fields

Hence for both electric and gravitational fields, as a test object is moved:

- **along a field line**, work will be done (force and displacement are in the same directions)
- **at right angles** to a field line, no work is done (force and displacement are perpendicular)



# Potential, $V$ , (Gravitational or Electric)

The potential (gravitational or electric) is defined as the energy per unit test point object that the object has as a result of the field it is placed in.

$$\text{Gravitational potential, } V = \frac{\text{Energy}}{\text{Mass}}$$

Units:  $\text{J kg}^{-1}$

$$\text{Electric potential, } V = \frac{\text{Energy}}{\text{Charge}}$$

Units:  $\text{J C}^{-1}$

# Potential Difference $\Delta V$ (Electric & Gravitational)

Potential is the energy per unit test object. In general, moving a mass between two points in a gravitational field and moving a charge between two points in an electric field means that work is done. When work is done, the potentials at the two points will be different, therefore resulting in a **potential difference**.

- If positive work is done **on** the test object as it moves between two points, then the potential between the two points must increase.
- If work is done **by** the test object as it moves between two points, then the potential between the two points must decrease.

# Potential Difference $\Delta V$ (Electric & Gravitational)

Gravitational Potential Difference between two points,

$$\Delta V_g = \frac{\textit{Work done in moving test mass}}{\textit{test mass}} \quad \text{Units: J kg}^{-1}$$

Electric Potential Difference between two points,

$$\Delta V_e = \frac{\textit{Work done in moving test charge}}{\textit{test charge}} \quad \text{Units: J C}^{-1}$$

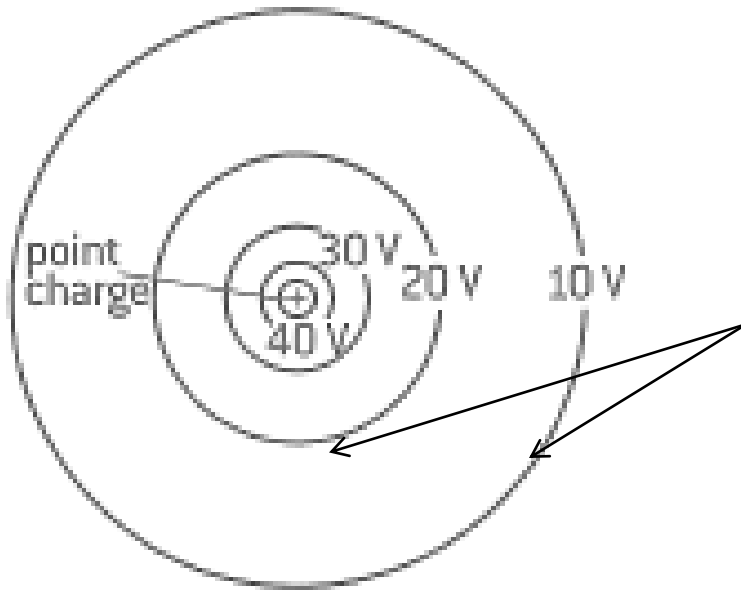
Thus to calculate the work done,  $W$ , in moving a charge  $q$  or mass  $m$  between two points in a field we have:

$$W = q\Delta V_e$$

$$W = m\Delta V_g$$

# Equipotential Surfaces

Using a charge as an example, we can represent how the electric potential varies around a charge by **identifying the regions** where the **potential is the same**. These are called **equipotential surfaces**.



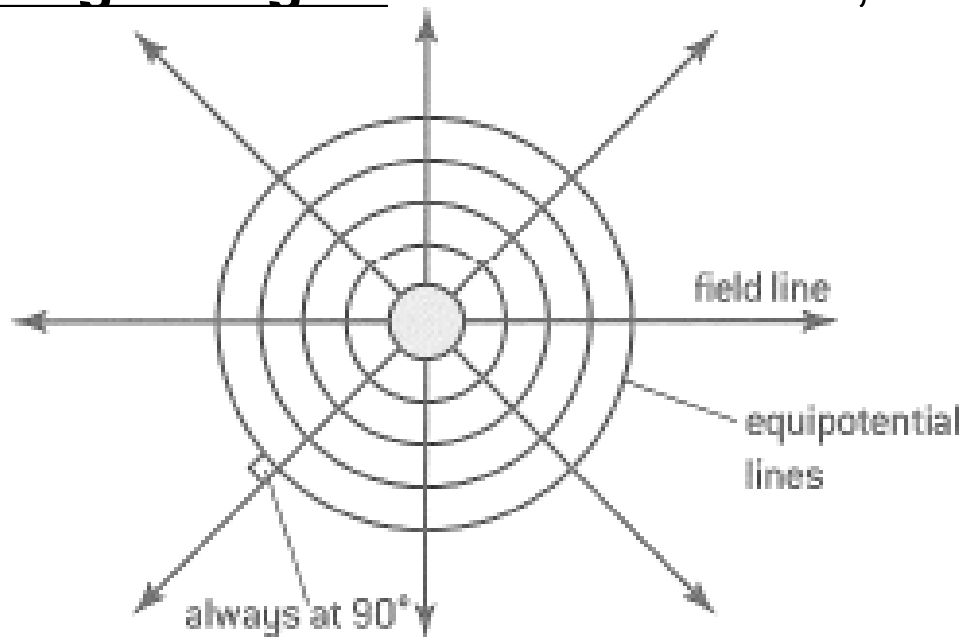
The lines around the point charge shown in the diagram connect all points around the charge with the same potential. These lines are **equipotentials**.

The same logic can be applied to masses with regards to gravitational potential (varies with height)



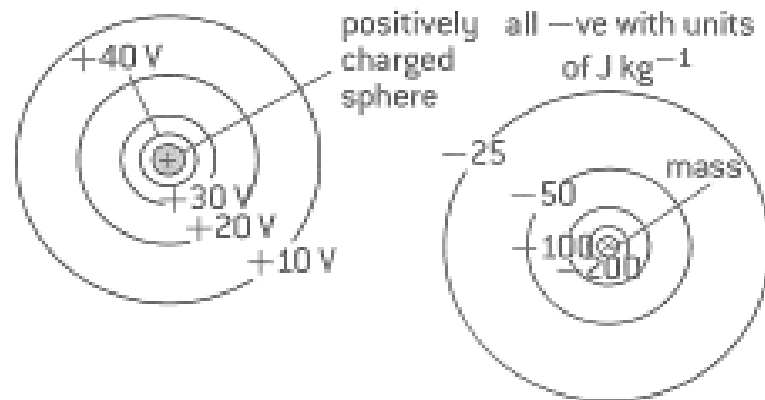
# Equipotential Surfaces & Field Lines

There is a simple relationship between field lines and lines of equipotential – they are always at right angles to one another. If we move along an equipotential, all the points are at the same potential thus there is no potential difference and no work done. When we move along an equipotential, we are moving at right angles to the field lines, hence no work is done.

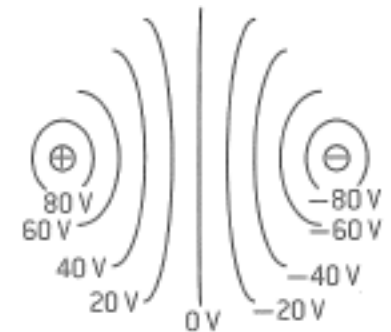


Field lines and equipotentials are at right angles

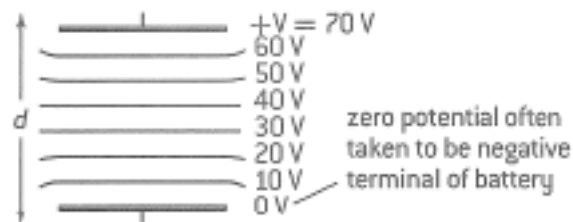
# Examples of Equipotentials



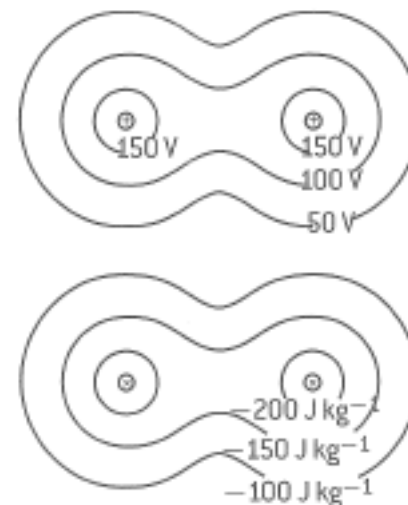
Equipotentials outside a charge-conducting sphere and a point mass.



Equipotentials for two point charges (equal and opposite charges).

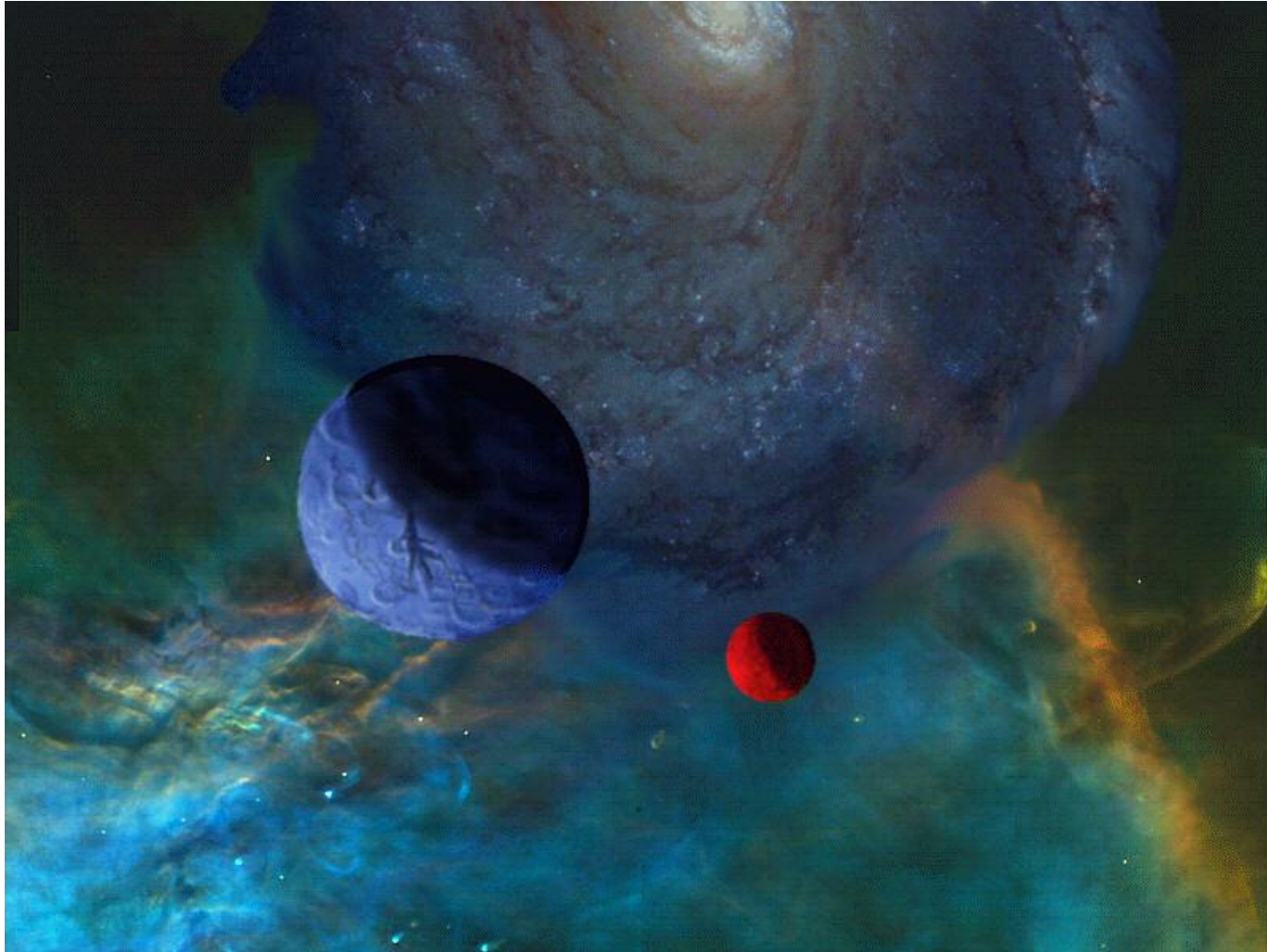


Equipotential lines between charged parallel plates.



Equipotentials for two point charges (same charge) and two point masses.

## 10.2 Gravitational Field, potential and energy



Parts adapted from Giancoli Lecture Powerpoint, EW

# 10.2 Gravitational field, potential and energy

- **Gravitational Potential and Gravitational Potential Energy**
- **Gravitational Potential due to point mass**
- **Relationship between Gravitational Field Strength and gravitational potential gradient**
- **Gravitational potential due to one or more point masses**
- **Equipotential surfaces due to one and two point point masses**
- **Relationship between equipotential surfaces and field lines**
- **Explain and derive Escape Speed**
- **Orbital Motion**
- **Weightlessness**

# Gravitational Potential Energy

## Recap:

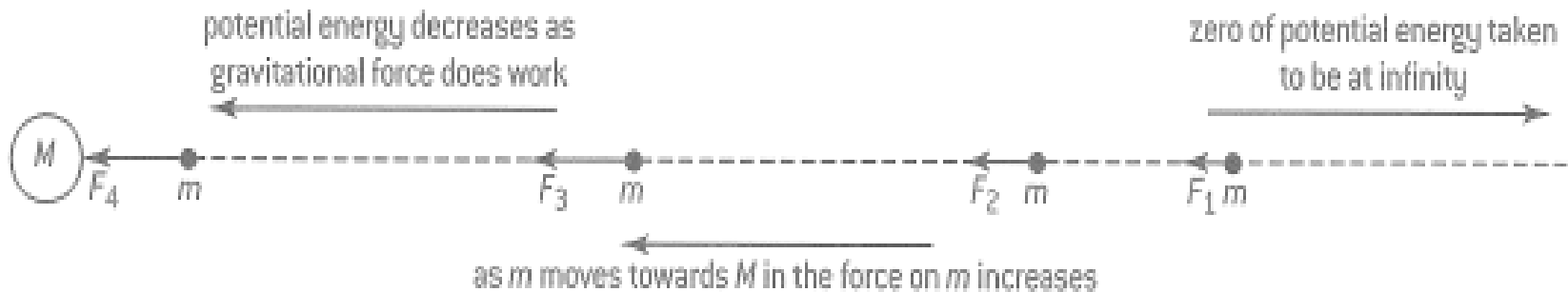
The difference in gravitational potential energy (GPE) when a mass moves between two different heights near the Earth's surface is given by  **$mg(h_2-h_1)$** .

## Points to note:

- This derivation/calculation assumes that the gravitational field strength  $g$  is a constant. However, Newton's theory of Universal Gravitation states that the field strength **MUST** change with distance. **Hence this equation can only be used if the distance moved is not very large.**
- The equation assumes that GPE has a magnitude of zero at the Earth's surface. **(Fundamentally 'incorrect')**

# Gravitational Potential Energy

The true zero of GPE is taken as infinity.



*E.g.*

If the potential energy of the mass,  $m$ , was zero at infinity and it lost potential energy moving towards mass  $M$ , the potential energy must be **negative** at a given point  $P$ .

# Gravitational Potential Energy

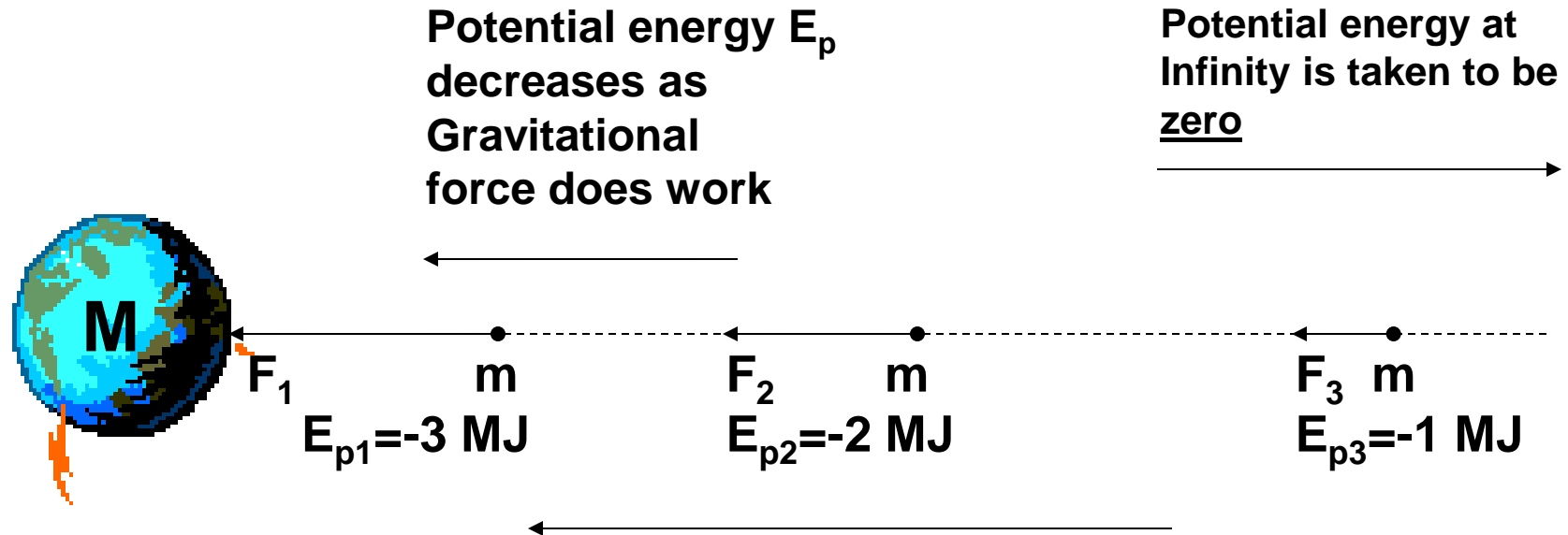
**Gravitational Potential Energy of a mass at any point in space is defined as the work done (by an external force) in moving it from infinity to that point.**

**The mathematics needed to work this is out of syllabus as it requires the use of calculus. It can be shown that:**

**Gravitational Potential Energy of a mass  $m$   
(due to mass  $M$ )**

$$E_p = -\frac{GMm}{r}$$

# Gravitational Potential Energy



$$F = \frac{GMm}{r^2}$$

As  $m$  moves towards  $M$ :

Force  $F$  on  $m$  increases:

$$F_1 > F_2 > F_3$$

$$E_p = -\frac{GMm}{r}$$

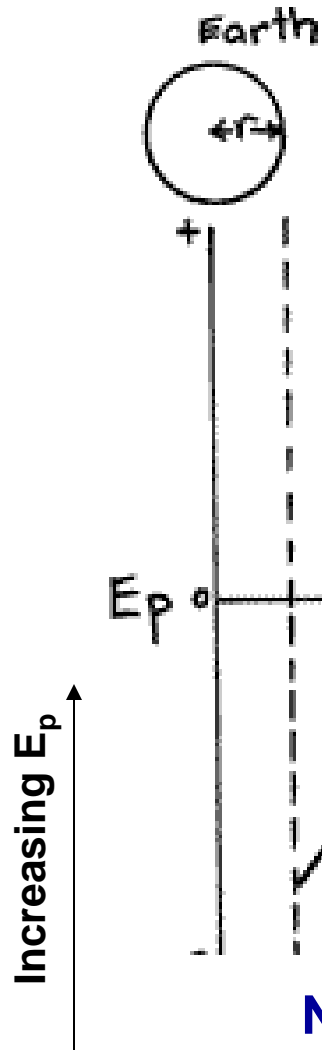
Potential Energy  $E_p$  decreases:

$$E_{p1} < E_{p2} < E_{p3}$$

E.g:  $-3 \text{ MJ} < -2 \text{ MJ} < -1 \text{ MJ}$



# Gravitational Potential Energy



When lifting an object against a gravitational field, e.g. launching a rocket, work is done on the object, that is, energy is transferred to the object. The object's gravitational potential energy,  $E_p$ , that is, the energy it has due to its position within the gravitational field, increases as a result.

An object only has zero  $E_p$  when it is no longer within the gravitational field, that is, at infinity.

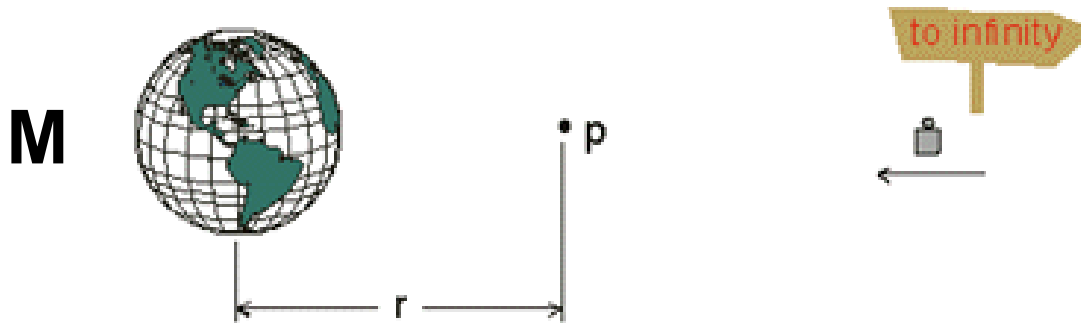
Notice that  $E_p$  increases by getting less negative.  
It increases to a maximum value of zero!

# Points to note about Gravitational Potential Energy

1.  $E_p$  is a scalar quantity measured in Joules (J).
2.  $E_p$  is independent of the path taken from infinity.
3.  $E_p$  at infinity is by definition taken to be ZERO.
4.  $E_p$  is always a negative value.
5. At the earth's surface, the difference in gravitational potential energies is given by  $\Delta E = mgh$ .
6. The equation  $\Delta E = mgh$  is an approximation because  $g$  is NOT constant with height. It can only be used if the vertical distance we move,  $h$ , is small.
7. Gradient of  $E_p$  against  $r$  graph = Gravitational Force.

# Gravitational Potential $V_g$

Gravitational Potential  $V_g$  due to a mass  $M$  at a point is defined as the work done per unit mass (by an external force) to bring a test mass from infinity to that point.

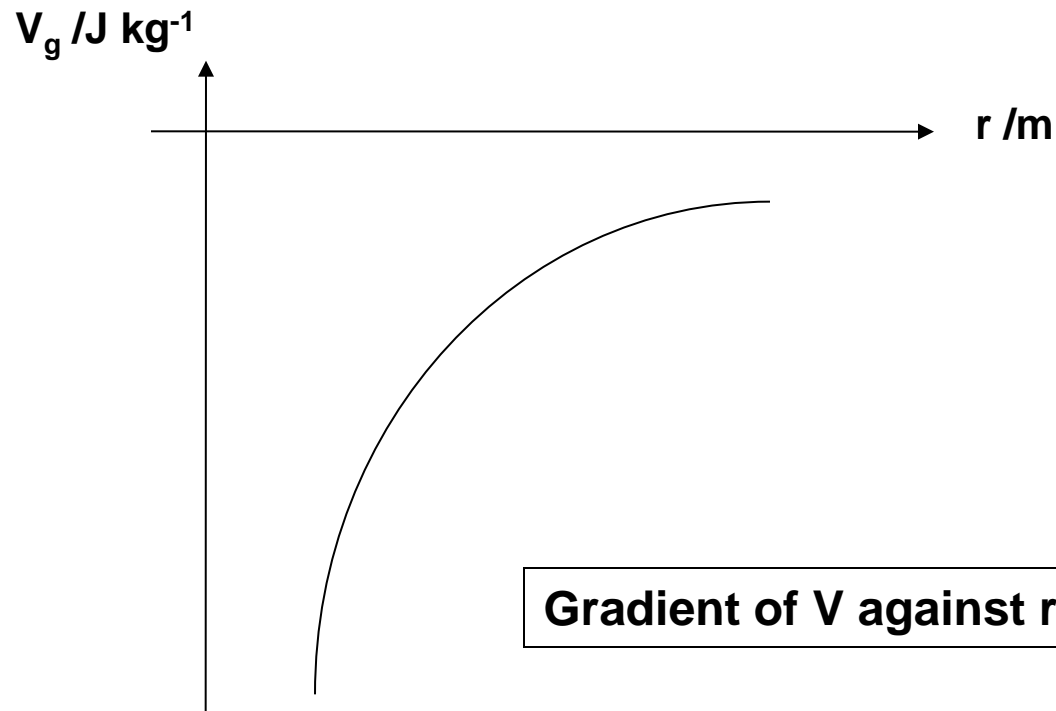


$$V_g = -\frac{GM}{r}$$

$$E_p = mV_g$$

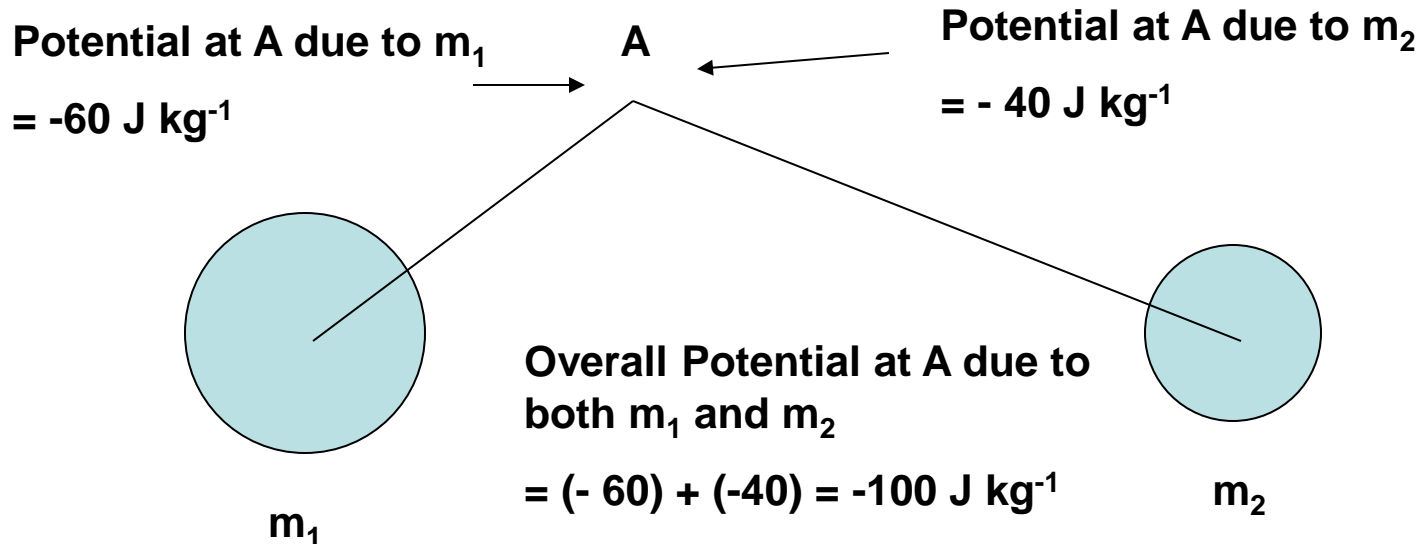
# Gravitational Potential $V_g$

$$V_g = -\frac{GM}{r}$$



# Gravitational Potential $V_g$

1.  $V_g$  is a scalar quantity measured in  $\text{J kg}^{-1}$ .
2. The gravitational potential as a result of lots of masses is just the addition of the individual potentials. This is an easy sum since  $V_g$  is a scalar quantity.



# Escape Speed $v_{\text{escape}}$



The Escape Speed a rocket is the speed needed to be able escape the gravitational attraction of the planet.

In other words, it has to get to an infinite distance away.

In reality it doesn't actually go to infinity. It just means that the rocket is effectively free of the gravitational attraction of the planet. We say that it has "escaped" the planet's gravitational pull.

## Derivation of expression for $V_{\text{escape}}$

$$E_p \text{ at surface of planet} = E_{\text{initial}} = -\frac{GMm}{R}$$

$R$  is the radius of the planet,  $m$  is the mass of the rocket

$$E_p \text{ at infinity} = E_{\text{final}} = 0 \text{ J}$$

The difference in  $E_p$  between the surface of the planet

$$\text{and infinity} = \Delta E_p = E_{\text{final}} - E_{\text{initial}} = \frac{GMm}{R}$$

$$\text{So Minimum Kinetic Energy needed} = \frac{GMm}{R}$$

$$\frac{1}{2}mv_{\text{escape}}^2 = \frac{GMm}{R}$$

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

**This derivation assumes  
the planet is isolated.**

## Derivation of expression for $V_{\text{escape}}$

$$v_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

We remember that  $g = \frac{GM}{R^2}$

Therefore we can get an alternative expression for  $v_{\text{escape}}$

$$v_{\text{escape}} = \sqrt{2gR}$$

**Substituting values for  $g$  and  $R$  gives a value of  $v_{\text{escape}}$  of about  $11 \text{ km s}^{-1}$  for the Earth.**

**Note that  $v_{\text{escape}}$  does not depend on the mass of the rocket.**



## Example

Mars has a mass of  $6.39 \times 10^{23}$  kg and a radius of  $3.39 \times 10^6$  m. Determine the speed at which a mass of 20 kg needs to be travelling in order to escape the gravitational attraction of Mars.

$$\begin{aligned} v_{esc} &= \sqrt{\frac{2GM}{R}} \\ &= \sqrt{\frac{2(6.67 \times 10^{-11})(6.39 \times 10^{23})}{(3.39 \times 10^6)}} \\ &= 5.0 \times 10^3 \text{ m s}^{-1} \end{aligned}$$

# Relationship between Gravitational Field Strength and Gravitational Potential Gradient

$$g = - \frac{\Delta V}{\Delta r}$$

This equation says that the gravitational field strength is equal to the negative of the gradient of the gravitational potential. Thus in a graph of gravitational potential versus distance, the negative of the slope is the gravitational field strength.

**Gravitational field strength = - Potential gradient**

**The gravitational field strength is equal to the negative potential gradient of the gravitational field.**

# Derivation of Relationship between Gravitational Field Strength and Gravitational Potential Gradient

Out of syllabus/For interest only

Change in gravitational potential is equal to the work done by an external force on a unit mass.

In other words, it is equal to the negative of the work done by gravity on a unit mass.

$$\Delta V = -\frac{W}{m}$$

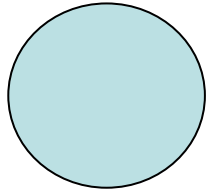
$$\Delta V = -\frac{F \times \Delta r}{m}$$

$$\Delta V = -\left(\frac{F}{m}\right)\Delta r$$

$$\Delta V = -g\Delta r$$

$$g = -\frac{\Delta V}{\Delta r}$$

# Energy of Orbiting Satellite



Satellite of mass,  $m$

Planet of mass,  $M$

$$\text{Gravitational Potential Energy } E_p = -\frac{GMm}{r}$$

Gravitational Force = Centripetal Force

$$\therefore \frac{GMm}{r^2} = \frac{mv^2}{r} \Rightarrow v = \sqrt{\frac{GM}{r}}$$

$$\text{Kinetic Energy } E_k = \frac{1}{2}mv^2 = \frac{1}{2} \frac{GMm}{r}$$

Total Energy = KE + PE

$$E = E_k + E_p$$

$$E = \frac{1}{2} \frac{GMm}{r} + \left(-\frac{GMm}{r}\right)$$

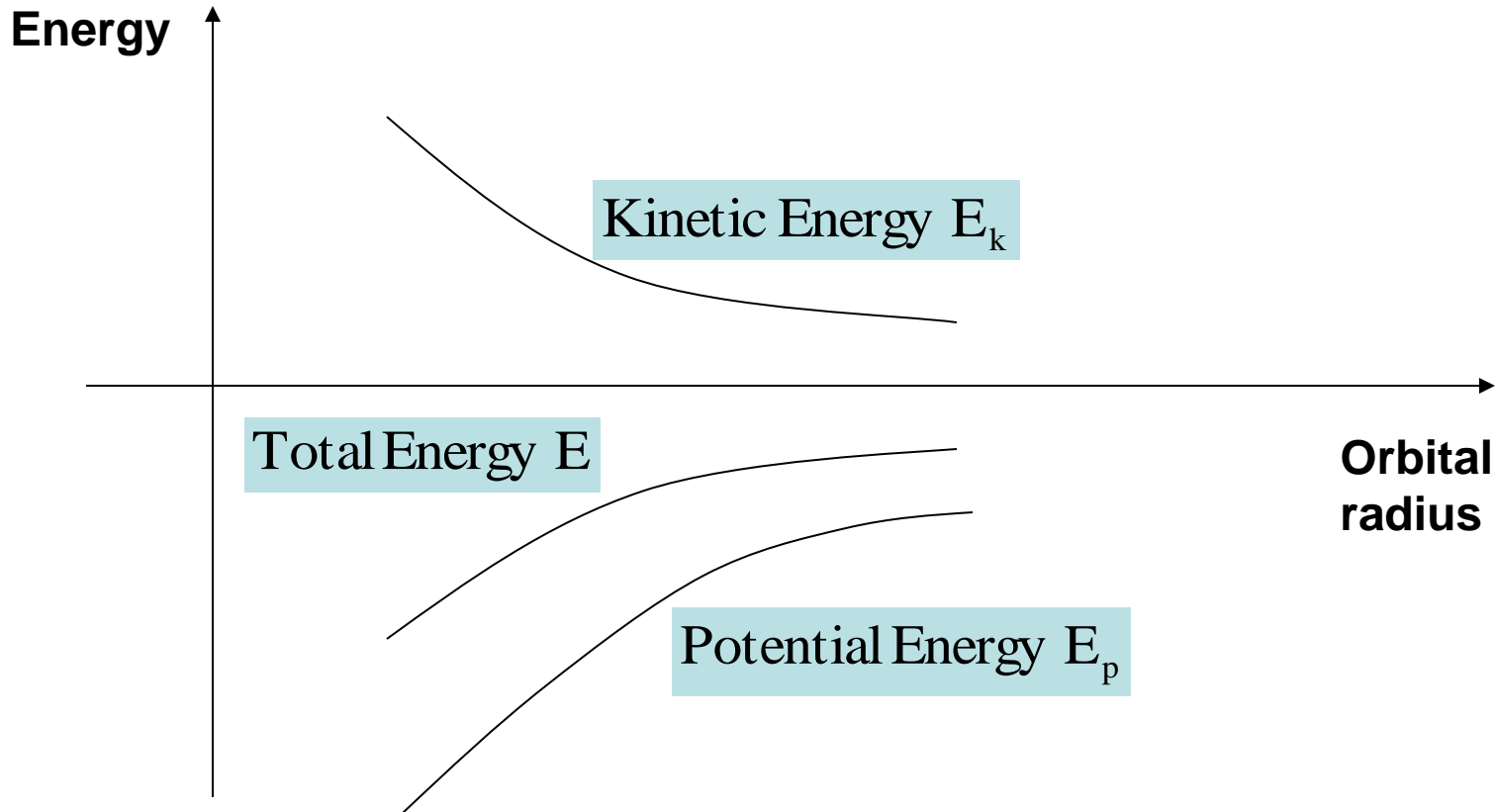
$$E = -\frac{1}{2} \frac{GMm}{r}$$

# Graph of PE, KE and Total Energy of an Orbiting Satellite

$$\text{PE} : E_p = -\frac{GMm}{r}$$

$$\text{KE} : E_k = \frac{1}{2} \frac{GMm}{r}$$

$$\text{Total Energy } E = -\frac{1}{2} \frac{GMm}{r}$$



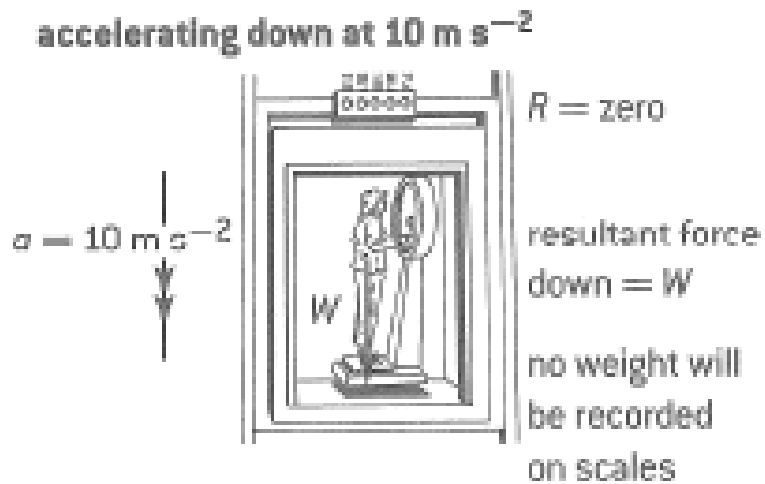
# Orbital Motion

**Geosynchronous satellites** orbit at much greater distances from the Earth and have orbital times equal to 24 hours. These satellites can be made to stay in the same area of sky and follows a figure of eight orbit, but the overhead position as viewed from the surface wanders.

**A geostationary orbit** is a special case of geosynchronous orbit where the satellite does not appear to move if viewed from the surface.

# Weightlessness

In the situation pictured in the figure below, It is possible for the scale to show a reading of zero. For example, if the lift cable breaks and the lift is accelerating downwards at a rate equal to the acceleration of gravity,  $g$ .

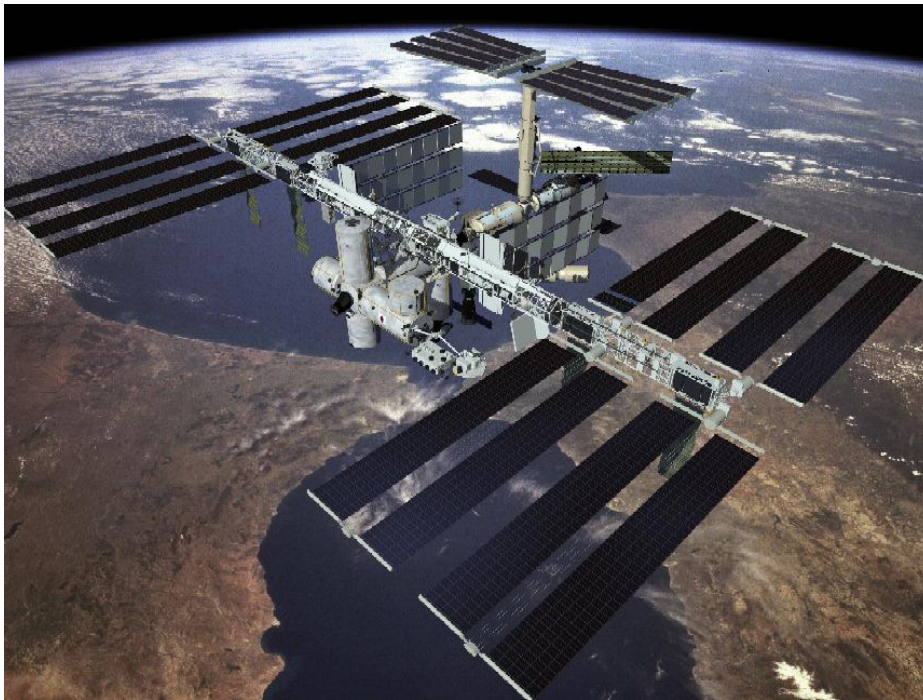


The person would appear to be weightless for the duration of the fall because the scale and person are all falling at the same acceleration hence there is no contact force between them. They are in free-fall together.

As the person is not truly 'weightless', it makes more sense to term this as apparent weightlessness.

# “Weightlessness” in orbital motion

An astronaut in an orbiting space station would also appear to be weightless. This is because the space station and the astronaut are both in free fall.



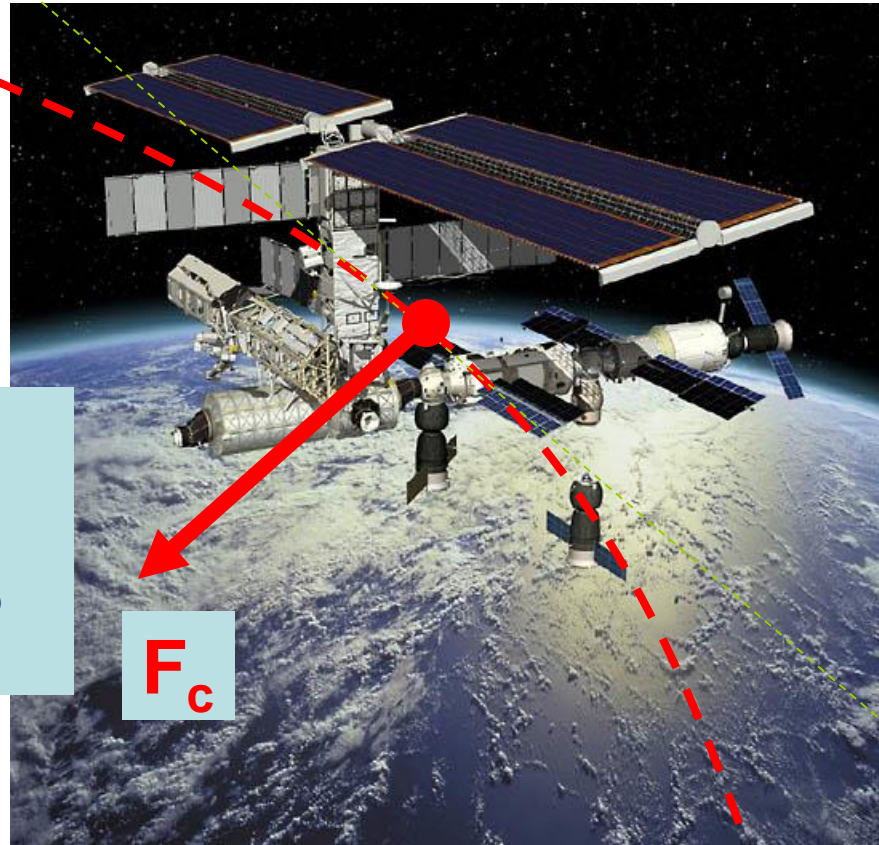
The gravitational pull on the astronaut provides the centripetal force needed for the astronaut to stay in orbit. The same is true for the space station. Therefore there is no contact force between the satellite and the astronaut, so, we have apparent weightlessness.



# “Weightlessness” in orbital motion

Orbital path  
is a circle

Gravitation attraction on  
space station (and the  
astronaut inside) provides  
centripetal force needed to  
stay in orbit



## 10.2 Electric field, potential and energy



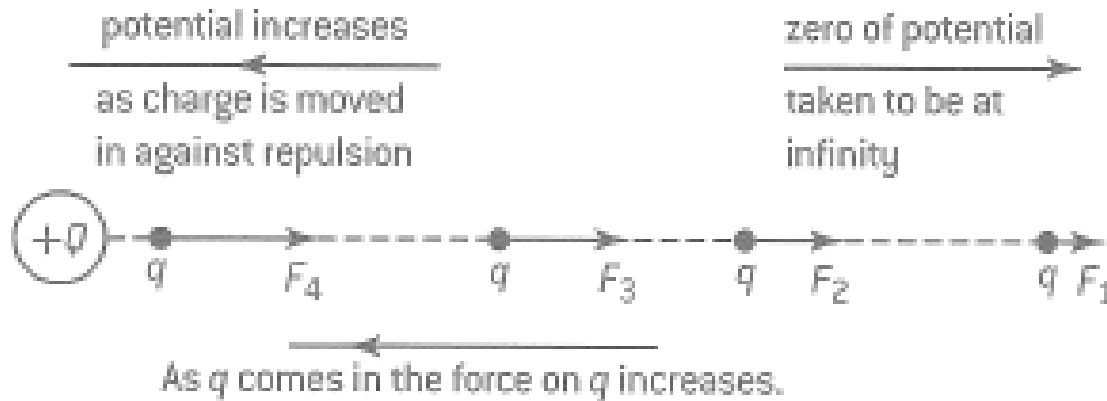
Parts adapted from Giancoli Lecture Powerpoint, Eric Wee

## **10.2 Electric field, potential and energy**

- Electric potential and electric potential energy**
- Expression for electric potential**
- Formula for electric field strength and potential gradient**
- Potential due to one or more point charges**
- Equipotential surfaces and electric field lines**

# Electric Potential

The definition of electric potential is very much like that of gravitational potential in that the potential is zero at infinity.



If the positive charge  $q$  is moved closer to  $Q$ , work must be done on  $q$ . Hence potential increases. This is because the two charges repel and so a force must be applied to  $q$  to make it move closer to  $Q$ .

# Electric Potential

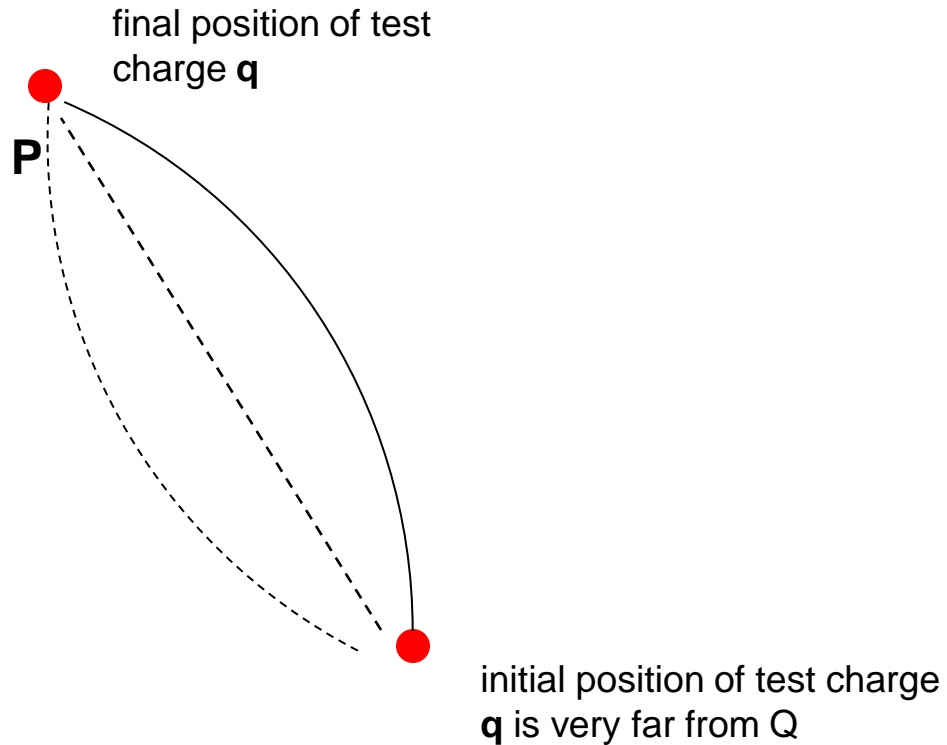
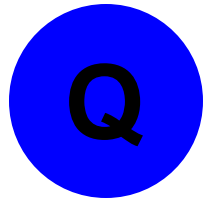
1. An electric charge creates an electric field in the space around it. It also creates a related quantity, an *electric potential*.
2. Consider a positive charge  $Q$  and a positive test charge  $q$ . If the charge  $q$  is moved closer to  $Q$ , work must be done on  $q$ . Hence potential increases. This is because the two charges repel and so a force must be applied to  $q$  to make it move closer to  $Q$ .
3. If the work done in moving the positive test charge  $q$  from very far away (at infinity) to some position  $P$  near  $Q$  is  $W$ , then the quantity

$$V = \frac{W}{q}$$

**Note:** Infinity is a convenient way of saying a point far removed from all other electrical influences

defines the potential at  $P$ .

**4. Definition:** The electric potential at a point in an electric field is defined as the work done per unit charge in moving a small positive test charge from infinity to that point.



5. The work done to bring the small positive test charge  $q$  from far away to a point  $P$  near the charge  $Q$  goes into electric potential energy. The route taken by the charge  $q$  to get to  $P$  does not affect the amount of work done.
6. The electric potential at infinity is taken as zero.
7. The unit of *electric potential* is the *volt*, and  $1 \text{ V} = 1 \text{ J C}^{-1}$ .

8. The *electric potential energy*  $E_p$  of the test charge  $q$  is given by the following:

$$E_p = Vq$$

where  $V$  is the electric potential at some point .

9. The *change in electric potential energy*  $\Delta E_p$  of the test charge  $q$  moving between two points is given by the following:

$$\Delta E_p = \Delta Vq$$

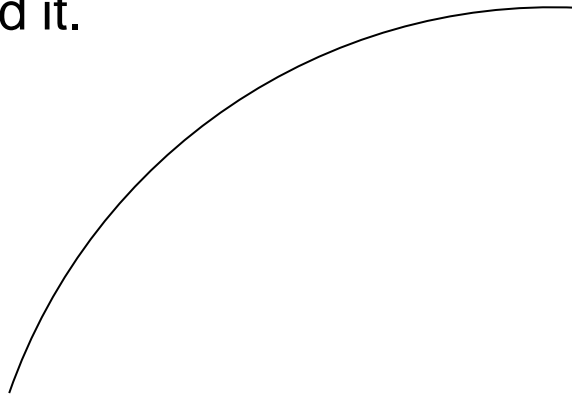
where  $\Delta V$  is the electric potential difference between two points.

## 10. Concept of potential difference $\Delta V$

Consider now an arrangement of charges that creates an electric potential in the space around it.

point A at 15 V

point B at 28 V



A charge of 2 C is initially at A is moved to B.

$$\begin{aligned}\text{The change in electric potential energy } \Delta E_p &= E_B - E_A = q (V_B - V_A) \\ &= 2 (28 - 15) = 26 \text{ J}\end{aligned}$$

This is the amount of work that must be done.

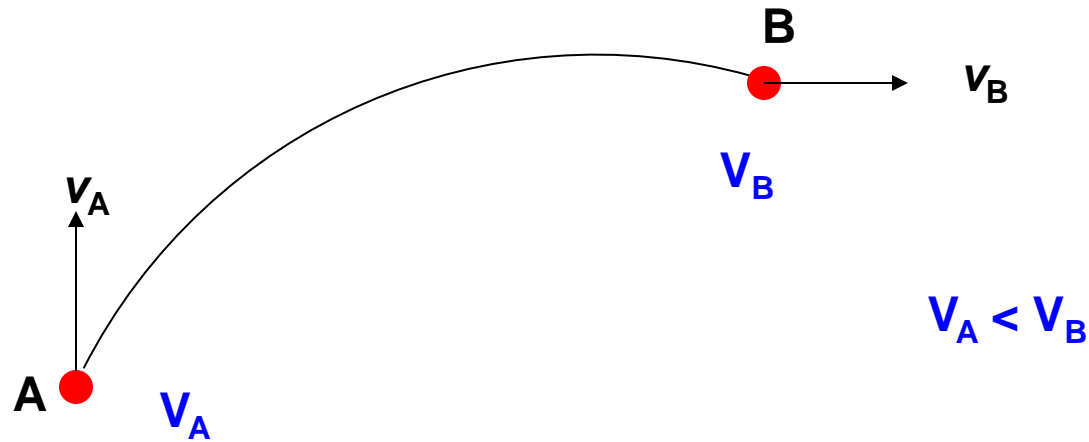
In general, the work that must be done on a charge  $q$  to move it from point A, where the potential is  $V_A$ , to a point B where the potential is  $V_B$  is given by

$$W = \Delta E_p = q (V_B - V_A) = q\Delta V$$

$$W = \Delta V q$$



## 11. Conservation of mechanical energy



Consider a charge  $q$  of mass  $m$  moving from point A to point B,

work done by the electric force acting on the charge is opposite to the work done on the charge by an external force and so

$$W_{\text{electric}} = -q (V_B - V_A)$$

we also know that the work done by the net force on a body equals to the change in kinetic energy. Thus

$$W_{\text{electric}} = -q (V_B - V_A) = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

which simplifies to

$$\frac{1}{2} m v_A^2 + q V_A = \frac{1}{2} m v_B^2 + q V_B$$

## 12. Electric field strength $E$ between parallel plates

Since electric field strength  $E$  is uniform in between the plates, a small positive test charge  $q$ , if released, will accelerate uniformly by an electric force given by  $F = E q = ma$

Work done by the electric field  $W = F d = E q d$

Since work done is given by  $W = q\Delta V$

so we get  $\Delta V = E d$

or more commonly, it is expressed in the form

$$E = \frac{\Delta V}{d}$$

where  $\Delta V$  is the potential difference between the plates, and  $d$  is their separation

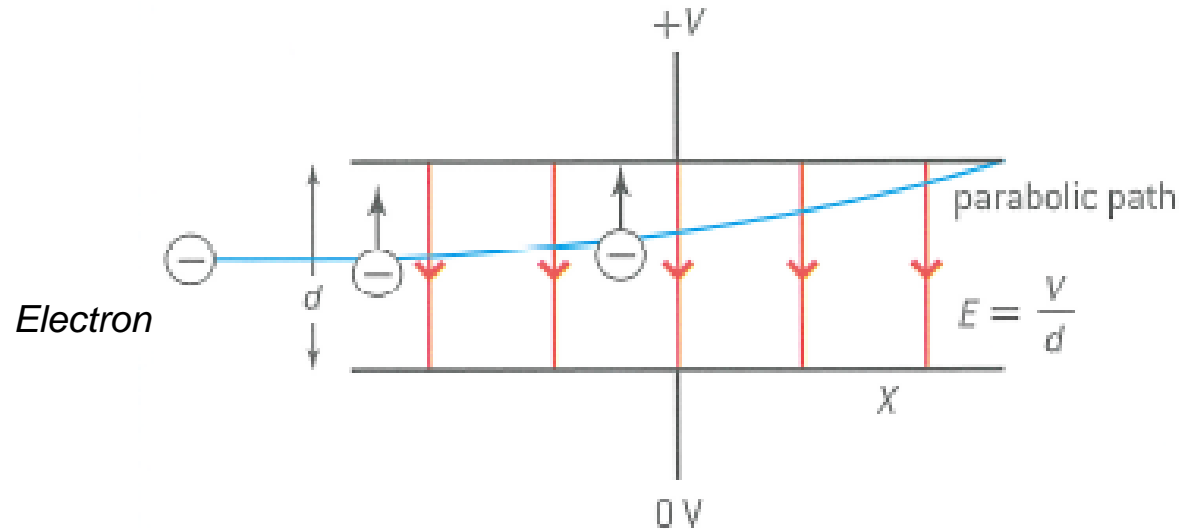
# Electronvolt

1. The energy scale that characterizes the atomic world is one of about  $10^{-18}$  J. This is a tremendously small amount of energy by macroscopic standards; the joule is not an appropriate energy unit.
2. A more convenient unit is the electronvolt, eV.
3. One electronvolt is defined as the work done when a charge equals to one electron charge is taken across a potential difference of one volt.
4. Thus  $1 \text{ eV} = 1.6 \times 10^{-19} \text{ C} \times 1 \text{ V} = 1.6 \times 10^{-19} \text{ J}$
5. If a charge of two electron charges is taken across a potential difference of 2 V, the work done will then be 4 eV.

## Question

1. What is the speed of a mass  $1.6 \times 10^{-27}$  kg whose kinetic energy is 5000 eV?  
(  $10^6 \text{ ms}^{-1}$  )

# Charges Moving In Electric Field



*Motion of electron entering uniform electric field between parallel plates*

For a uniform field, Field strength  $E = \frac{\Delta V}{\Delta d}$

Electron is accelerated upwards, hence it experiences a force  $F = ma$

$$\therefore a = \frac{F}{m} = \frac{qE}{m} = \frac{qV}{md} \quad \text{where } m \text{ is the mass of the electron.}$$

To determine the final speed of the electron, we find the the resultant of  $v_x$  (remains constant) and  $v_y$  (need to determine via kinematics).

## Questions

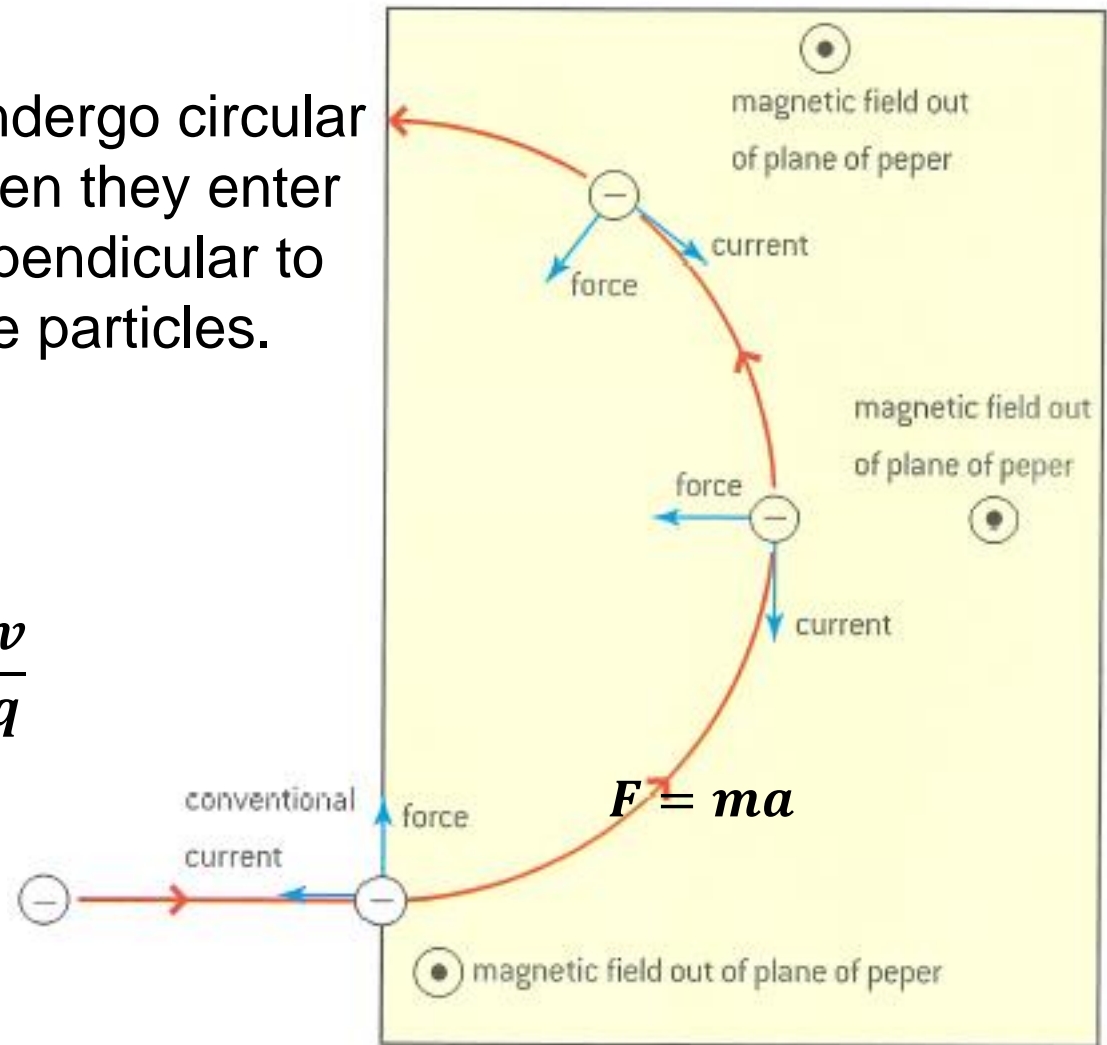
1. A charge of  $5.0 \mu\text{C}$  and mass  $2.0 \times 10^{-6} \text{ kg}$  is shot with a speed of  $300 \text{ ms}^{-1}$  between two parallel plates kept at a potential of  $200 \text{ V}$  and  $500 \text{ V}$ . What will be the speed be when the charge travels from the  $200 \text{ V}$  plate to the  $500 \text{ V}$  plate? ( $297 \text{ ms}^{-1}$ )
2. What must the initial velocity of an electron be if it is to reach from one plate at  $2.0 \text{ V}$  to the other plate at  $0 \text{ V}$  and momentarily stops there? (Charge of one electron =  $1.6 \times 10^{-19} \text{ C}$  and mass of an electron =  $9.1 \times 10^{-31} \text{ kg}$ )  
  
( $8.4 \times 10^5 \text{ ms}^{-1}$ )

# Charges Moving In Magnetic Fields

Recap: charged particles undergo circular motion and hence orbits when they enter magnetic fields that are perpendicular to the direction of motion of the particles.

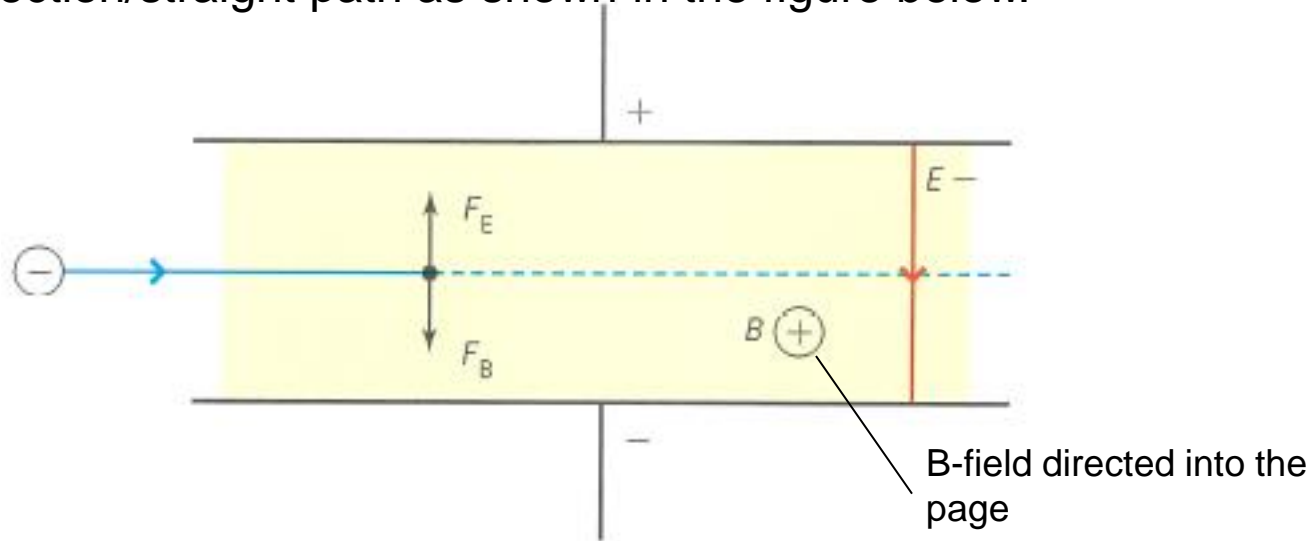
$$Bqv = \frac{mv^2}{r}$$

$$\text{Radius of circle, } r = \frac{mv}{Bq}$$



# Charges Moving In Magnetic & Electric Field

Despite the fact that magnetic fields lead to circular orbits and electric fields lead to parabolic trajectories, we can use them to accelerate charged particles in a single direction/straight path as shown in the figure below.



We achieve this by orientating the magnetic and electric fields at right angles to each other. In the figure above, the magnetic force on the electron is downwards while the electric force is upwards – they balance each other out.

$$F = qE \quad \& \quad F = Bqv$$

$$\Rightarrow v = \frac{E}{B}$$

This means that there is one particular speed - for a given ratio of  $E$  and  $B$  at which the forces acting on the electron are balanced.



# Electric potential & electric potential energy

1. If the potential at some point P in an electric field is  $V_P$ , and we place a charge  $q$  at P, the quantity  $E_p = qV_P$  is the electric potential energy of the charge  $q$ .

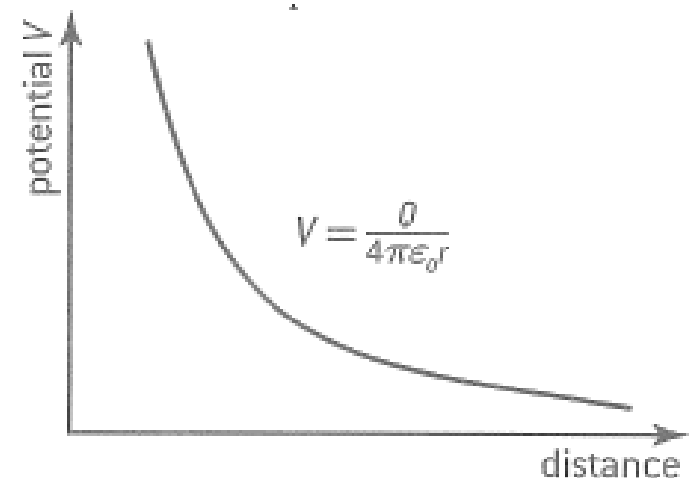
2. For a point charge  $Q$ , the electric potential at a point P, a distance  $r$  is given by

$$V = \frac{Q}{4\pi\epsilon_0 r} = \frac{kQ}{r}$$

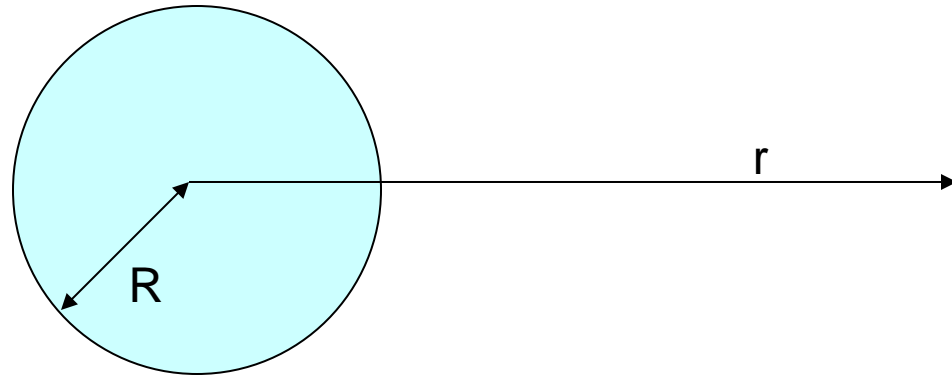
The proof of this equation is by using calculus (not required)

3. Thus the electric potential energy of a test charge  $q$  placed at P would then be

$$E_p = \frac{Qq}{4\pi\epsilon_0 r}$$



4. Consider a sphere of radius  $R$  with a charge  $Q$ .



On the surface of the sphere, the potential is  $V = \frac{Q}{4\pi\epsilon_0 R}$

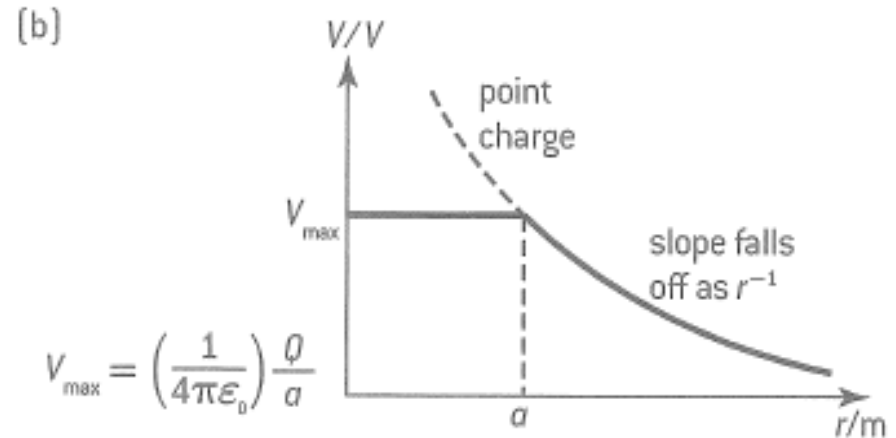
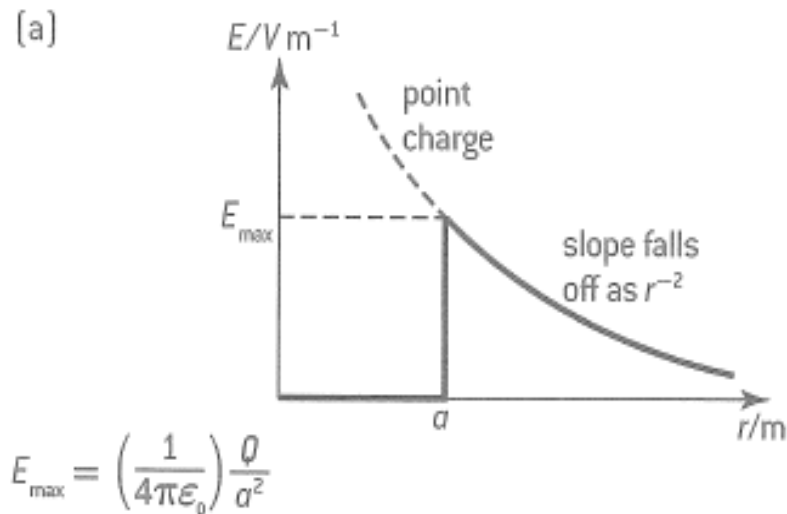
At a distance  $r$  from the centre of the sphere,  $V = \frac{Q}{4\pi\epsilon_0 r}$

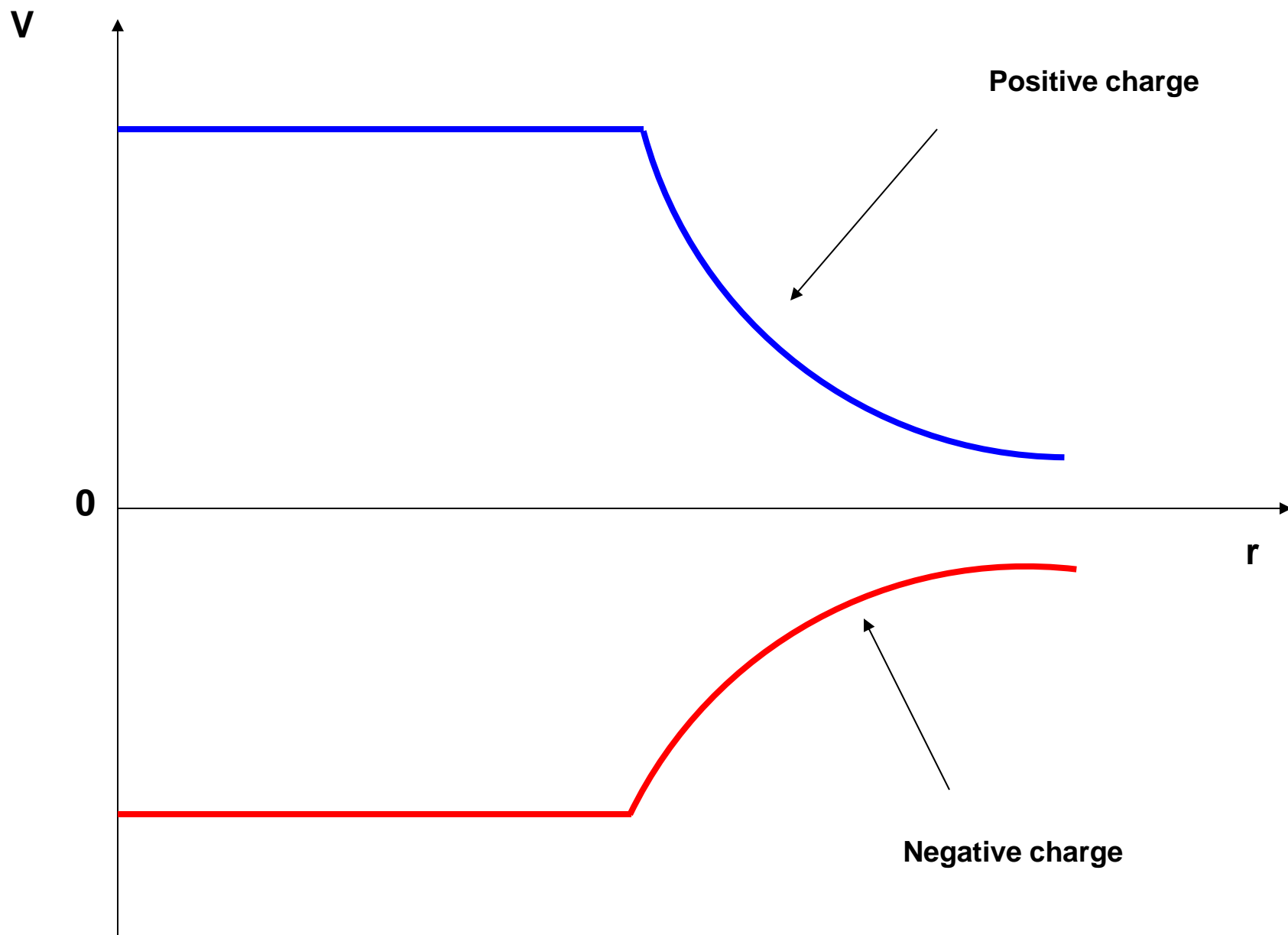
But at any point inside the sphere the electric potential is constant and has the same value as the potential at the surface.

5. Charges will distribute themselves uniformly on the outside of a conducting sphere.

- **Outside the sphere:** the field lines and equipotential surfaces are the same as if all the charge was concentrated at a point in the centre of the sphere.
- **Inside the sphere:** there is no net contribution from the charges outside the sphere and the electric field is zero. The potential gradient is thus zero meaning that every point inside the sphere is at the same potential – the potential at the sphere's surface.

The graphs below show how the field and potential vary for a sphere of radius  $a$ .





## Examples

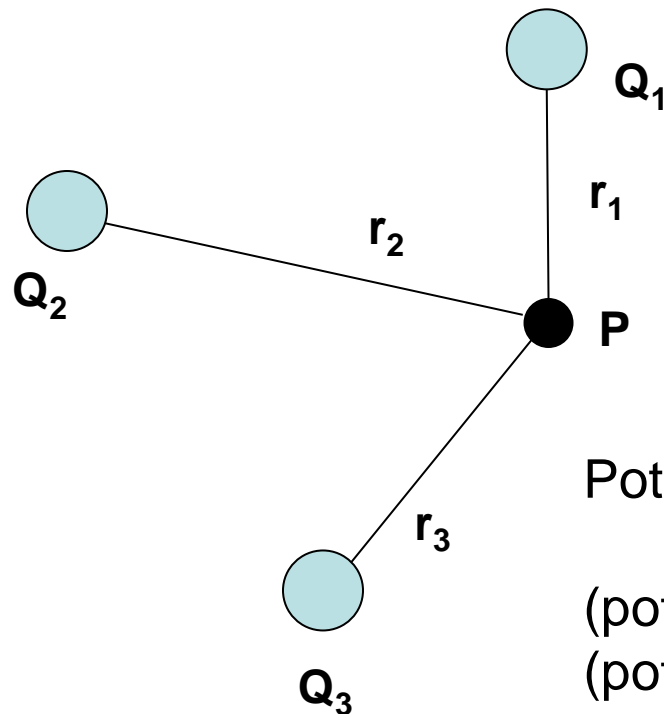
1 Find the electric potential energy between the proton in a hydrogen atom and an electron orbiting the proton at a radius  $0.5 \times 10^{-10}$  m. The proton has a charge  $1.6 \times 10^{-19}$  C, equal and opposite to that of the electron.

From the formula

$$\begin{aligned} E_p &= \frac{Qq}{4\pi\epsilon_0 r} \\ &= 9 \times 10^9 \times \frac{1.6 \times 10^{-19} \times (-1.6 \times 10^{-19})}{0.5 \times 10^{-10}} \\ &= -4.6 \times 10^{-18} \text{ J} \end{aligned}$$

## Potential Due to Multiple Charges

The potential due to multiple charges at one point can be found by adding up the individual potentials due to the individual charges at that point.

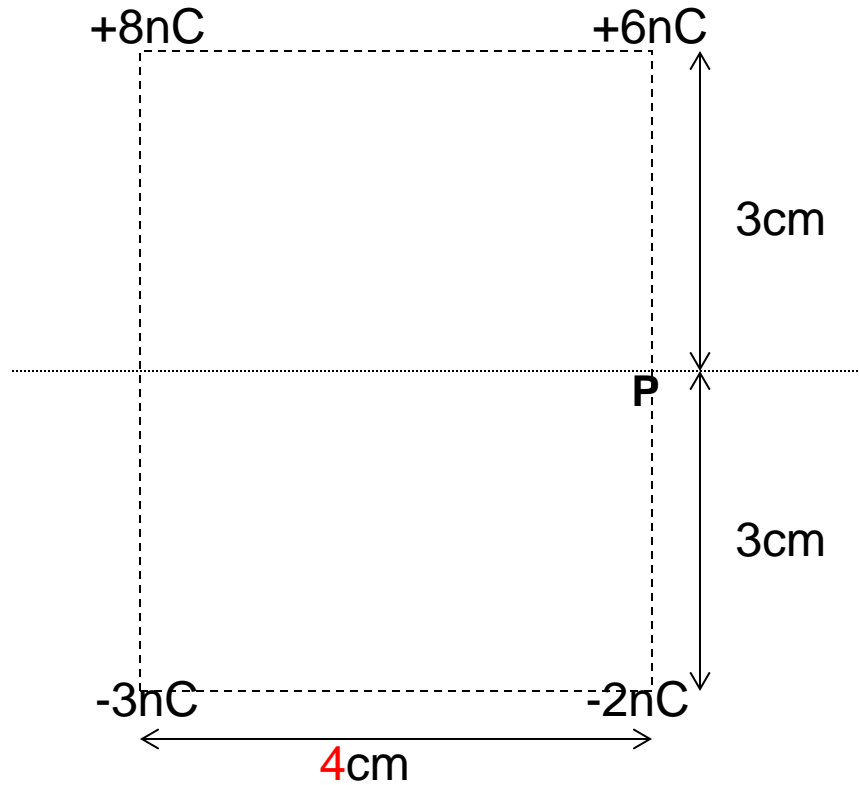


Potential at P =

(potential due to  $Q_1$ ) + (potential due to  $Q_2$ ) +  
(potential due to  $Q_3$ )

***Remember: The electric potential at any point outside a charged sphere is exactly the same as if all the charge had been concentrated at its centre.***

2. The diagram shows four charges placed at the corners of a rectangle. Find the potential of the point P. (2100 V)



3. Find the electric potential a distance of  $0.50 \times 10^{-10}$  m from the proton of the hydrogen atom.

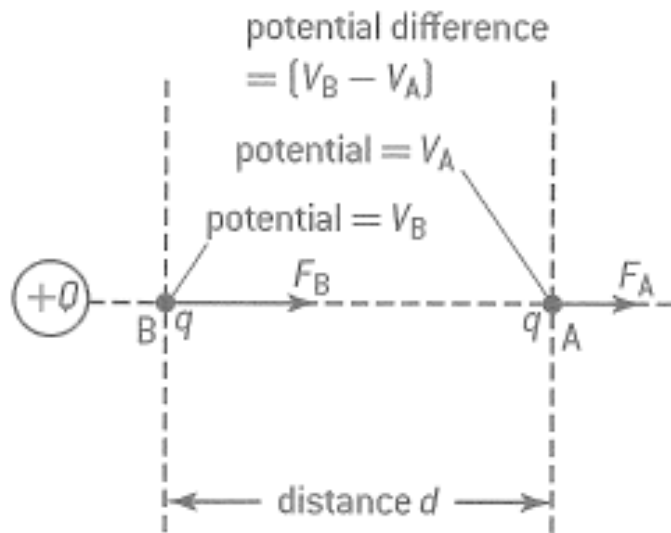
$$V = \frac{Q}{4\pi\epsilon_0 r}$$

$$= 9 \times 10^9 \times \frac{1.60 \times 10^{-19}}{0.50 \times 10^{-10}}$$

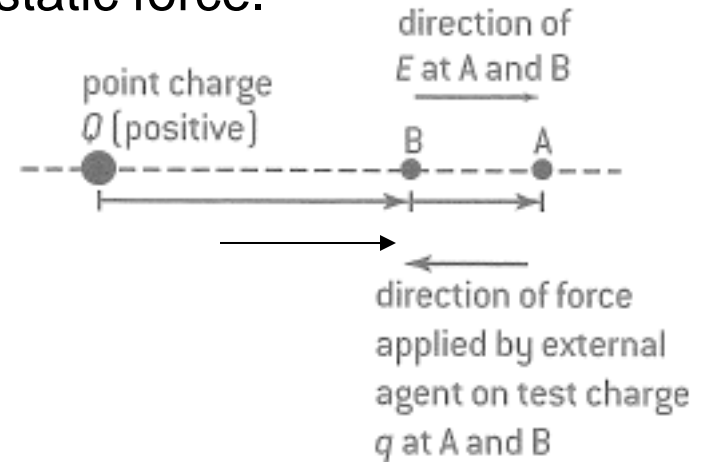
$$= 29 \text{ V}$$



# Relation between electric field strength and electric potential gradient



Bringing a positive charge from A to B means work needs to be done against the electrostatic force.



The work done  $W = -qEx$

[negative sign arises because the direction of the force needed to do the work is opposite to the direction of  $E$ ]

$$\therefore E = -\frac{1}{q} \frac{W}{x} = -\frac{\Delta V}{\Delta x} \left[ \text{since } \Delta V = \frac{W}{q} \right] \quad \text{Units: } \text{V m}^{-1}, \text{N C}^{-1}$$

**Electric field strength = - Potential gradient**

Note: The area under a electric field versus distance graph gives us the potential difference.

# COMPARISON BETWEEN ELECTRIC & GRAVITATIONAL FIELD

## Electrostatics

Force can be attractive or repulsive

Coulomb's law – for point charges

$$F_E = \frac{q_1 q_2}{4\pi\epsilon_0 r^2} = k \frac{q_1 q_2}{r^2}$$

Electric field

electric field

charge producing field

$$E = \frac{F}{q_2} = \frac{q_1}{4\pi\epsilon_0 r^2} = k \frac{q_1}{r^2}$$

test charge

Electric potential due to a point charge

$$V_e = \frac{q_1}{4\pi\epsilon_0 r} = k \frac{q_1}{r}$$

Electric potential gradient

$$E = - \frac{\Delta V_e}{\Delta r}$$

Electric potential energy

$$E_p = qV_e = \frac{q_1 q_2}{4\pi\epsilon_0 r} = k \frac{q_1 q_2}{r}$$

## Gravitational

Force always attractive

Newton's law – for point masses

$$F = G \frac{m_1 m_2}{r^2}$$

Gravitational field

gravitational field

mass producing field

$$g = \frac{F}{m_2} = \frac{Gm_1}{r^2}$$

test mass

Gravitational potential due to a point mass,  $m_1$

$$V_g = - \frac{Gm_1}{r}$$

Gravitational potential gradient

$$g = - \frac{\Delta V_g}{\Delta r}$$

Gravitational potential energy

$$E_p = mV_g = - \frac{GMm}{r}$$

# Uniform Fields

Field strength is equal to the negative of potential gradient.

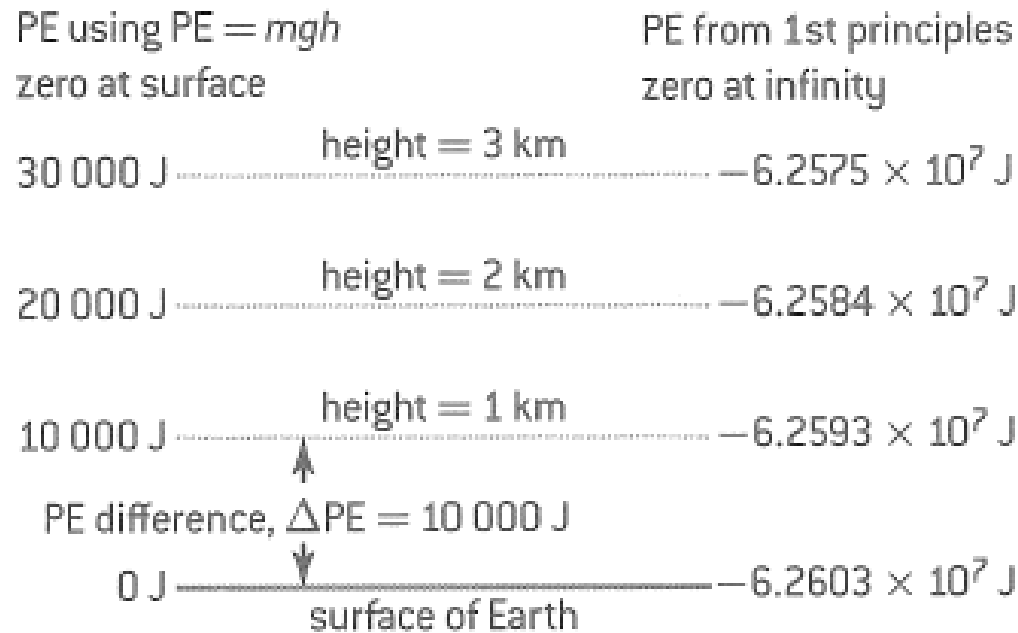
A constant field thus means:

- A constant potential gradient i.e. a given increase in distance will equate to a fixed change in potential.
- In 3D this means that the equipotential surfaces will be flat planes that are equally space apart. In 2D equipotential surfaces will be equally spaced.
- Field lines (perpendicular to equipotential surfaces) will be equally spaced parallel line.

# Uniform Fields

## Constant Gravitational Field

The gravitational field near the surface of a planet is effectively constant. At the surface of the Earth, the field lines will be perpendicular to the Earth's surface. Since  $g = 9.81 \text{ m s}^{-2}$ , the potential gradient must also be  $9.81 \text{ J kg}^{-1} \text{ m}^{-1}$ .



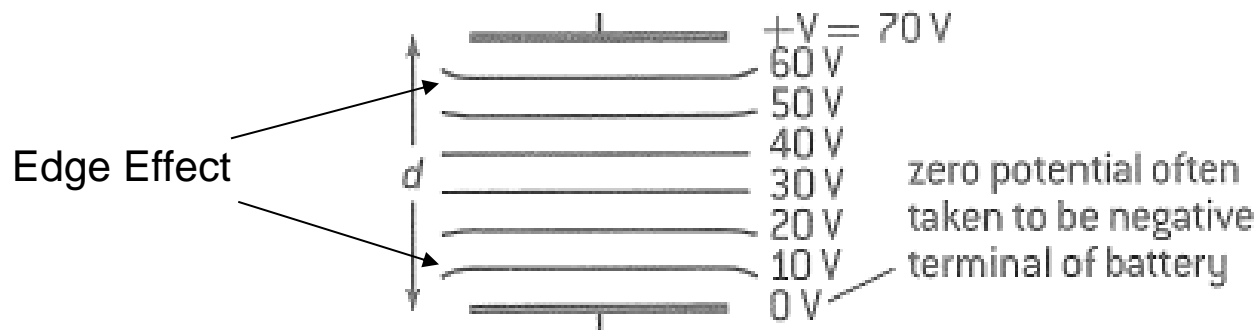
# Uniform Fields

## Constant Electric Field

- The electric field in between charged parallel plates is effectively constant in the middle section.
- In the figure below, the potential difference between the plates is  $V$  and the separation is  $d$ . Thus the electric potential gradient is  $\frac{V}{d}$  and the constant field in the centre of the plates,  $E = -\frac{V}{d}$ .

*Units:  $V\ m^{-1}$  or  $J\ C^{-1}$  (Both can be used)*

- Strictly speaking, the electric field between two charged parallel plates cannot remain uniform throughout the plates and there will be an edge effect.



Equipotential lines between charged plates