Topic 2.1
Motion

Contents

Distance, Displacement

Distance

Distance refers to the total length covered by a moving object irrespective of the direction of motion.

- has a unit of metres (m) or kilometres (km)
- A scalar quantity, only magnitude

Displacement

Displacement refers to the linear distance of the position of the moving object from a given reference point.

- has a unit of metres (m) or kilometres (km)
- A vector quantity with magnitude and direction

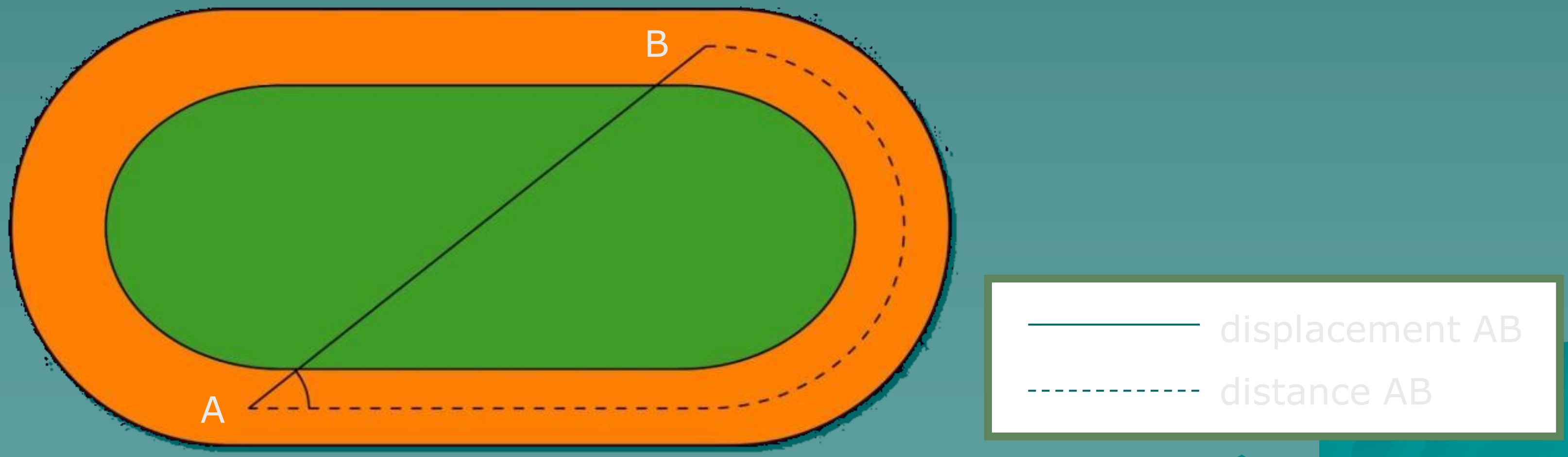
Graphical analysis of motion

distance

Distance refers to the total length covered by a moving object irrespective of the direction of motion.

displacement

Displacement refers to the linear distance of the position of the moving object from a given reference point.



Graphical analysis of motion

distance vs displacement

Distance	Displacement
scalar quantity	vector quantity
regardless of its direction	dependent on direction of motion

Speed, Velocity and Acceleration

speed

Speed of an object is defined as the distance travelled by the body per unit time.

- has a unit of metres per second (m s^{-1}) or kilometres per hour (km h^{-1})

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}$$

Speed, Velocity and Acceleration

average speed

Average speed of an object is defined as the total distance travelled by the object divided by the total time taken.

- has a unit of metres per second (m s^{-1}) or kilometres per hour (km h^{-1})

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

- for most journeys, speed is not constant

Speed, Velocity and Acceleration

instantaneous speed

Instantaneous speed is the speed of an object at any instant (any point in time).

- magnitude of the average speed lies between the highest (instantaneous) and the lowest (instantaneous) speeds in a journey

speedometer measures the instantaneous speed of the car

Speed, Velocity and Acceleration

speed measurement using a ticker-tape timer

- object moving with constant speed

- object moving with increasing velocity

- object moving with decreasing velocity

Speed, Velocity and Acceleration

velocity

Velocity is the distance travelled per unit time in a stated direction; or speed in a specified direction.

- has a unit of metres per second ($m\ s^{-1}$) or kilometres per hour ($km\ h^{-1}$)

$$\text{velocity} = \frac{\text{distance travelled in a stated direction}}{\text{time taken}}$$

Speed, Velocity and Acceleration

speed vs velocity

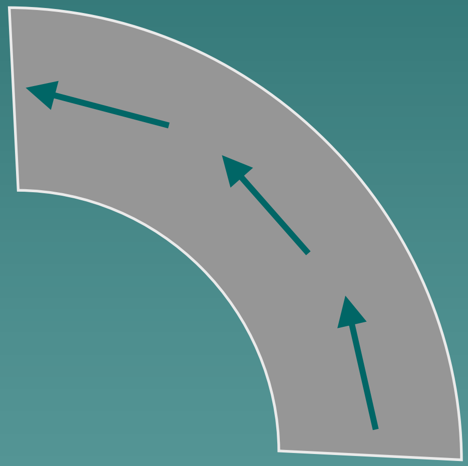
Speed	Velocity
scalar quantity	vector quantity
regardless of its direction	dependent on direction of motion

A negative velocity indicates that a body is moving in the opposite direction to the direction stated.

Speed, Velocity and Acceleration

velocity

Consider a vehicle travelling around a bend.



At a road bend, although the vehicle's speed is constant, its velocity is continuously changing.

Speed, Velocity and Acceleration

acceleration

Acceleration is defined as the rate of change of velocity.

- has a unit of metres per second square ($m\ s^{-2}$)

$$\text{acceleration} = \frac{\text{change of velocity}}{\text{time taken for the change}}$$

Definitions

Physical Quantities	Definitions
Distance	refers to the total length covered by a moving object irrespective of the direction of motion
Displacement	refers to the linear distance of the position of the moving object from a given reference point
Speed	is defined as the distance travelled by the body per unit time
Average Speed	is defined as the total distance travelled by the object divided by the total time taken
Instantaneous Speed	is the speed of an object at any instant (any point in time)
Velocity	is the distance travelled per unit time in a stated direction; or speed in a specified direction
Acceleration	is defined as the rate of change of velocity

speed, velocity and acceleration

acceleration

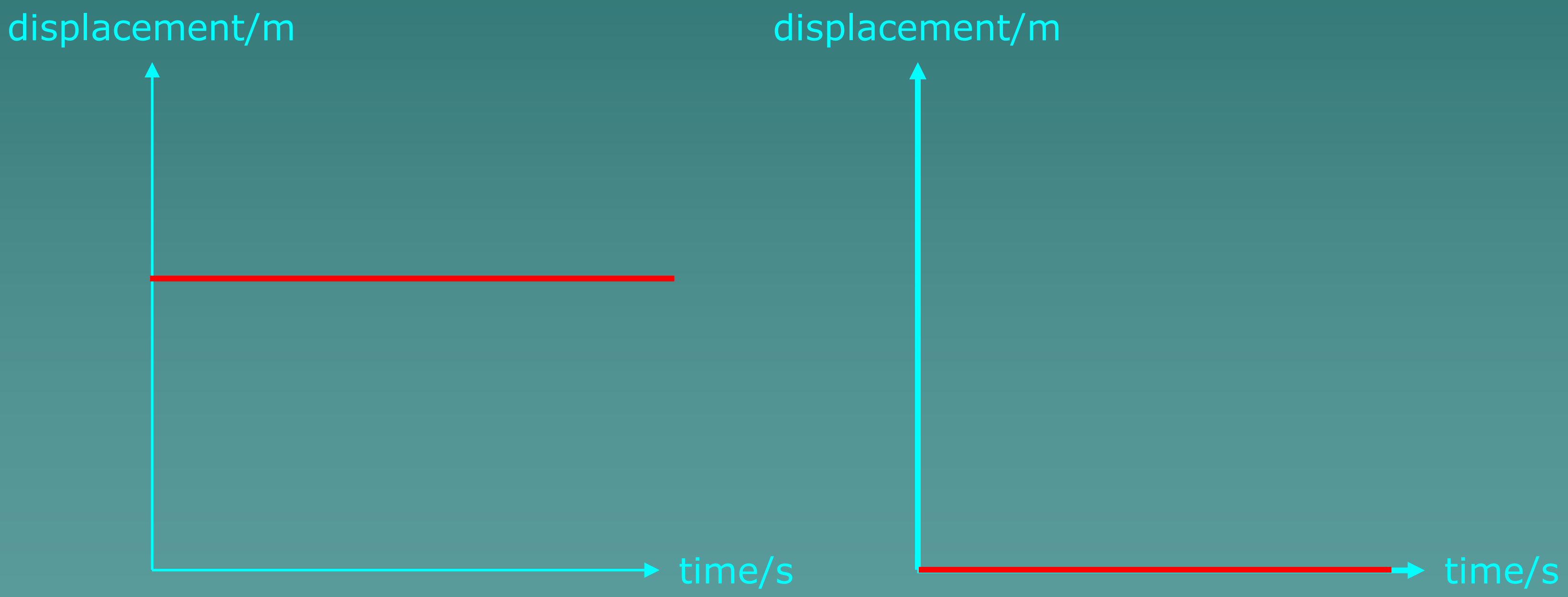
Uniform acceleration occurs when the velocity increases (or decreases) by the same amount per unit time.
[Or rate of change of velocity is constant/uniform]

No acceleration (acceleration = 0)	**Accelerating (positive acceleration)**	**Decelerating (negative acceleration)**
velocity of the moving object is constant throughout	velocity is increasing or velocity is decreasing in the opposite direction	velocity is decreasing or velocity is increasing in the opposite direction

graphical analysis of motion

displacement-time graphs

Gradient of the displacement-time graph gives the velocity of the object.

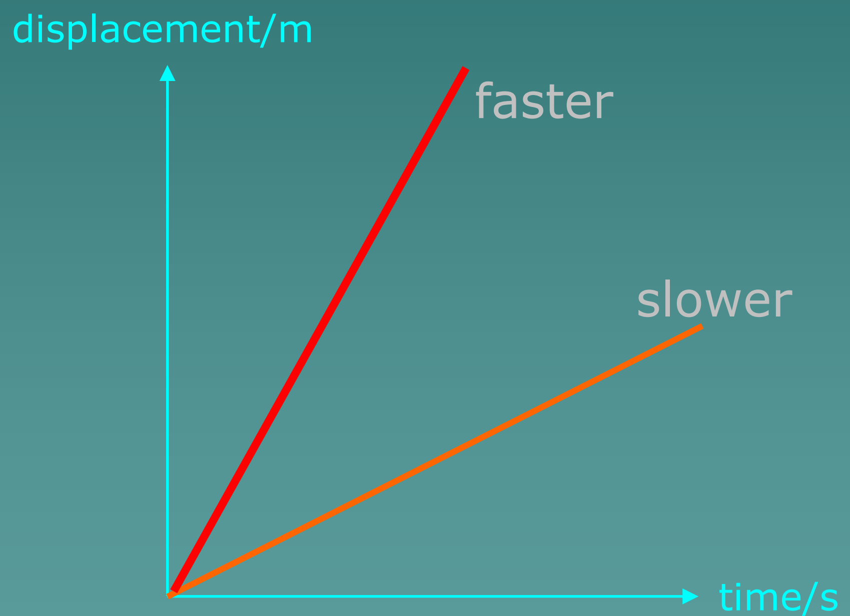


object is not moving

graphical analysis of motion

displacement-time graphs

The displacement-time graph of an object travelling with constant velocity is always a straight line.



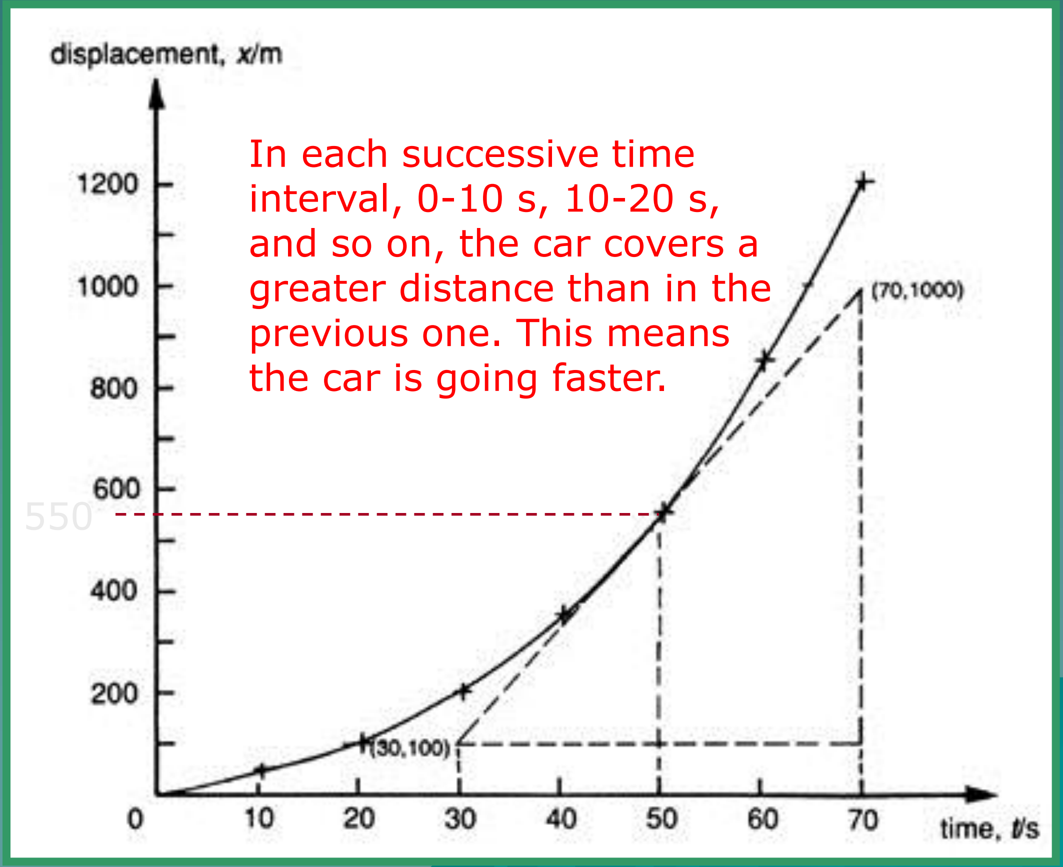
object is moving with constant velocity

graphical analysis of motion

displacement-time graphs

Time taken, *t*/s	0	10	20	30	40	50	60	70
Displacement, *x*/m	0	50	100	200	350	550	850	1200

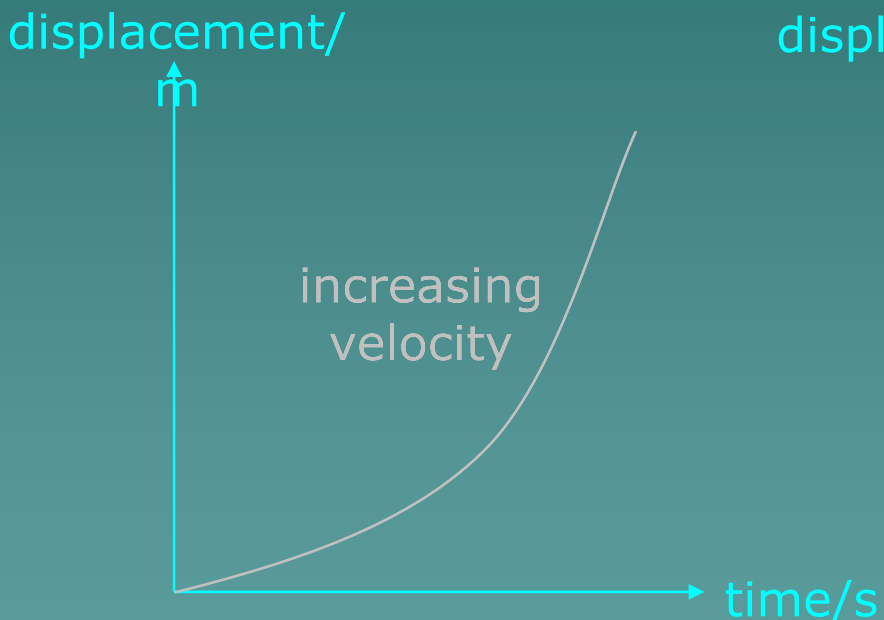
The instantaneous velocity of the car at a particular time can be obtained by finding the slope of the tangent to the graph (gradient) at that point in time.

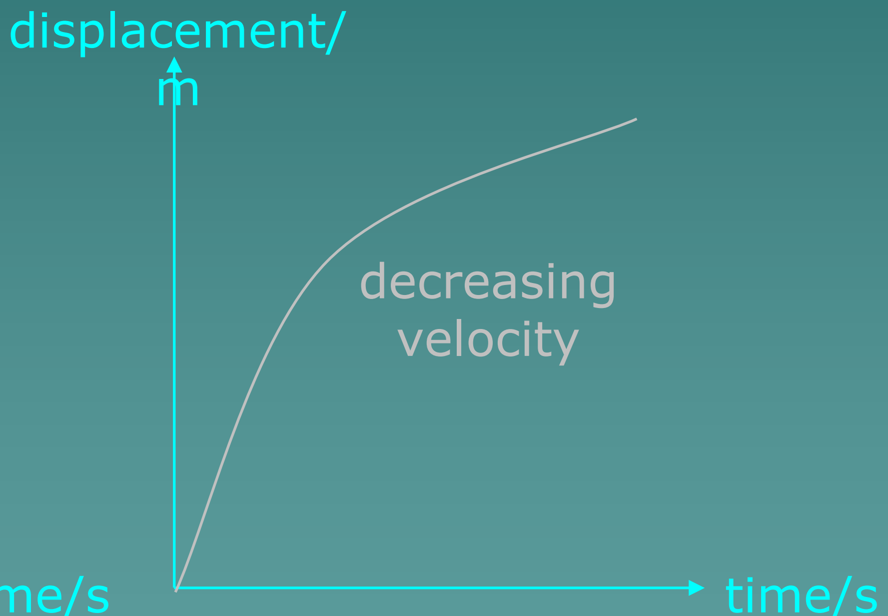


graphical analysis of motion

displacement-time graphs

Gradient of a tangent to the displacement-time graph of an object travelling with non-uniform velocity gives its instantaneous velocity at a given time.



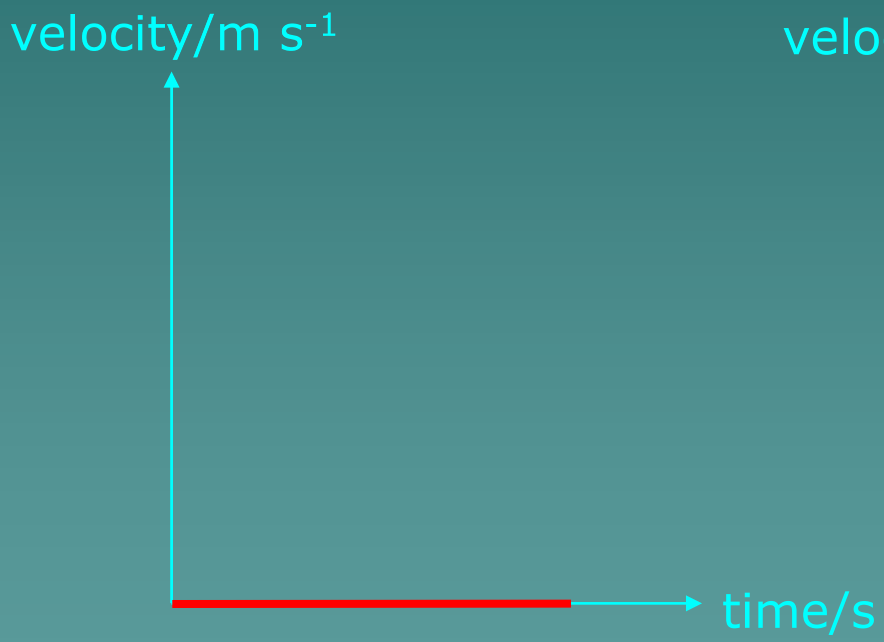


object has variable velocity

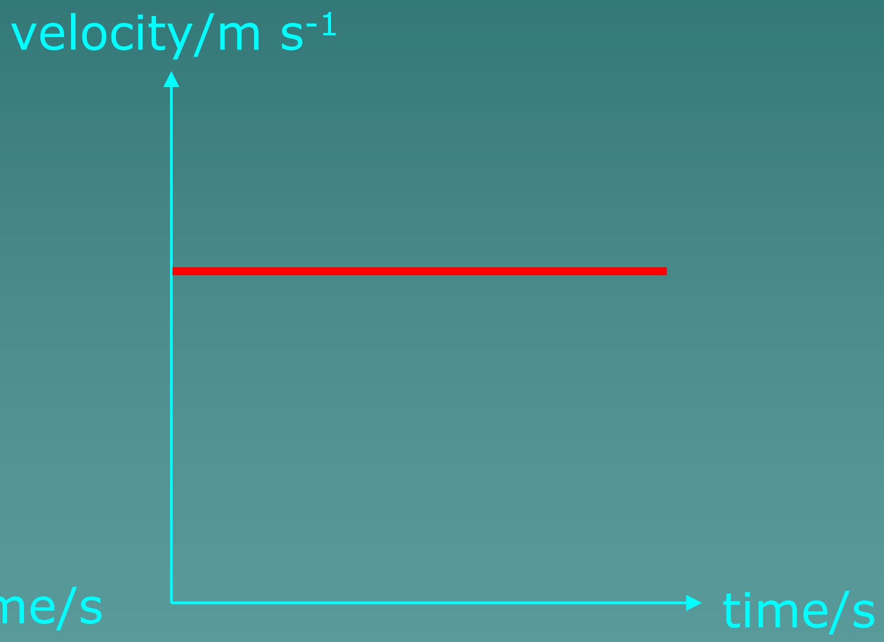
graphical analysis of motion

velocity-time graphs

Gradient of the velocity-time graph gives the acceleration of a moving body.



object is not moving

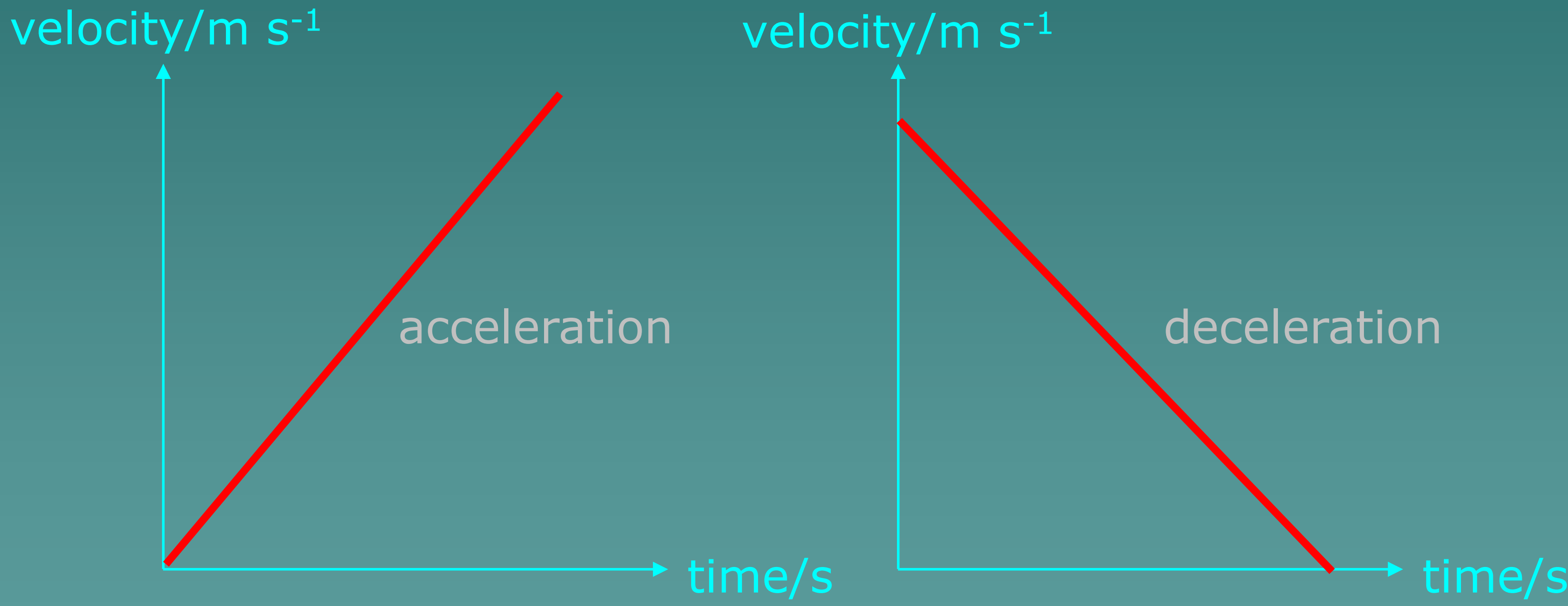


object moves with constant velocity

graphical analysis of motion

velocity-time graphs

Gradient of the velocity-time graph gives the acceleration of a moving body.

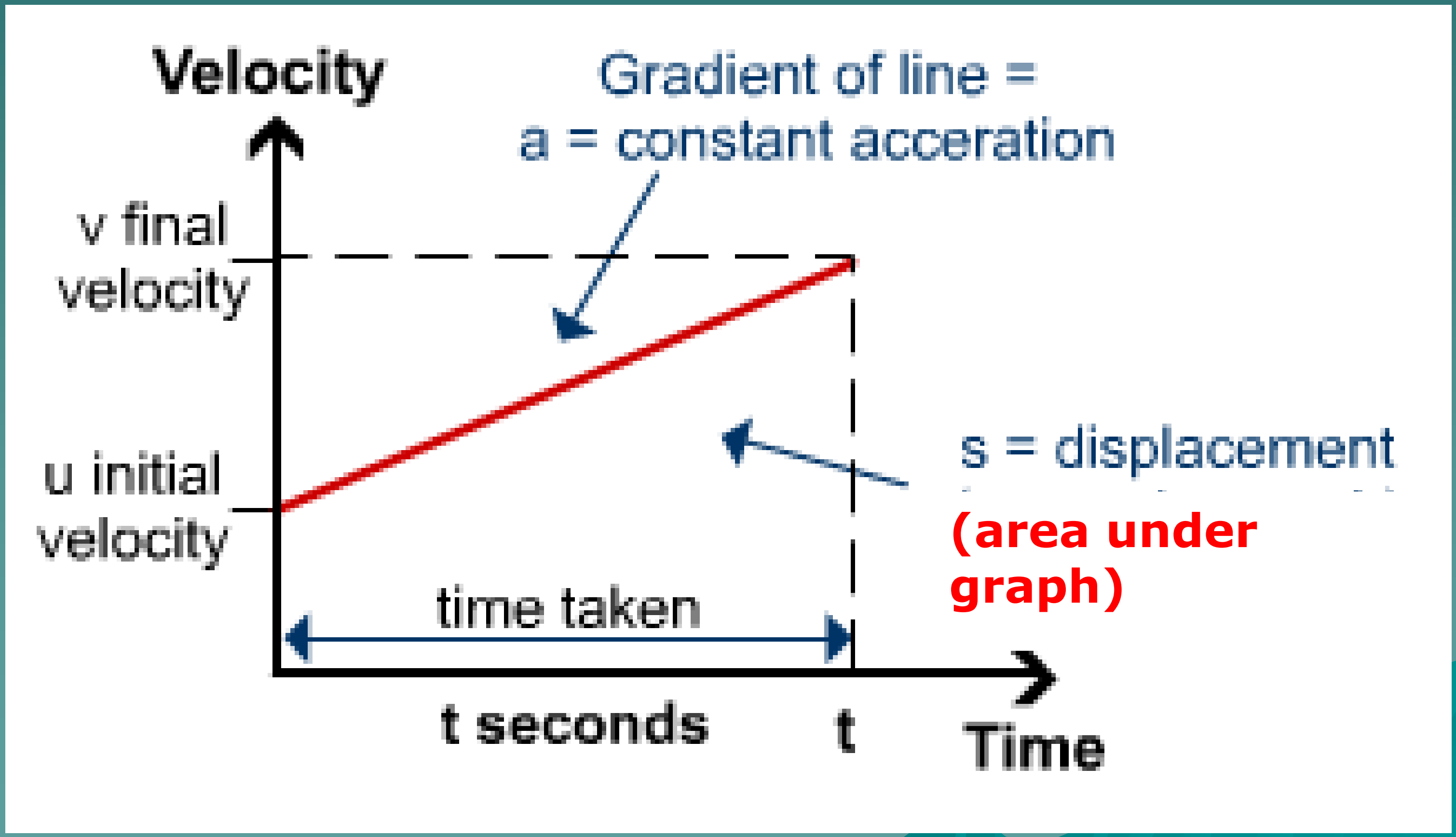


object has constant acceleration/deceleration

graphical analysis of motion

velocity-time graphs

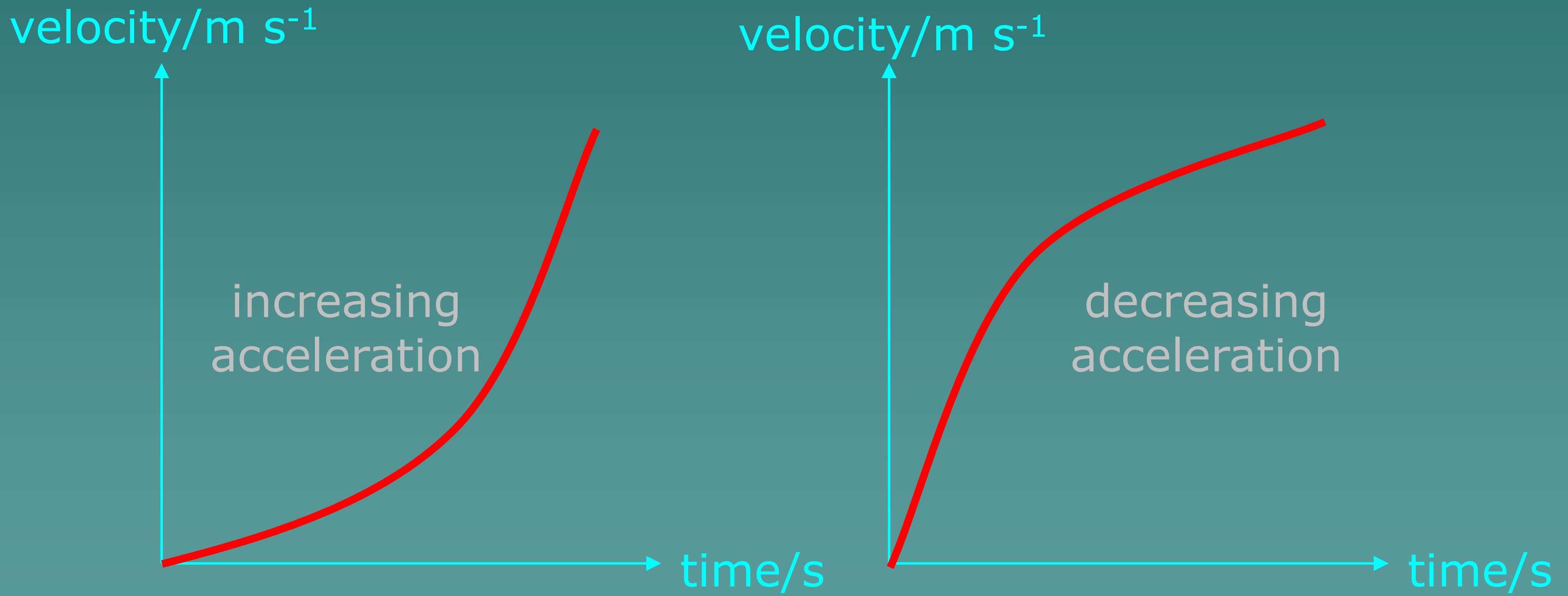
Gradient of the velocity-time graph gives the acceleration of a moving body.



graphical analysis of motion

variable acceleration

Gradient of the velocity-time graph gives the acceleration of a moving body.



object has variable acceleration/deceleration

graphical analysis of motion

variable acceleration

Not all objects move with constant acceleration. Most vehicles move with accelerations that keep changing.

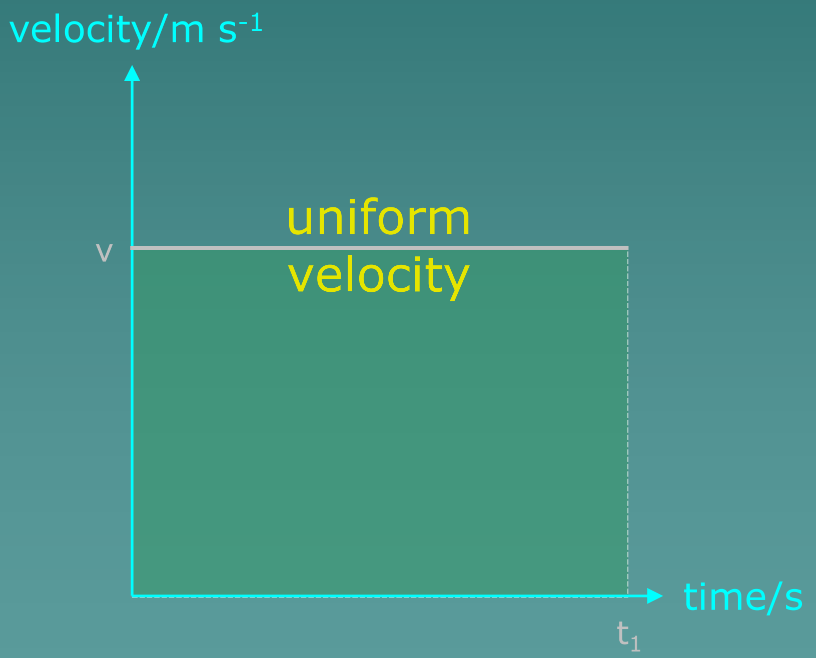
The acceleration or deceleration of the object at any point in time is still given by the gradient of the graph at that point.

velocity-time graph of a car on a straight road where it has to stop twice because of traffic lights

graphical analysis of motion

area under a velocity-time graph

The area under the velocity-time graph gives the distance travelled by the moving object.

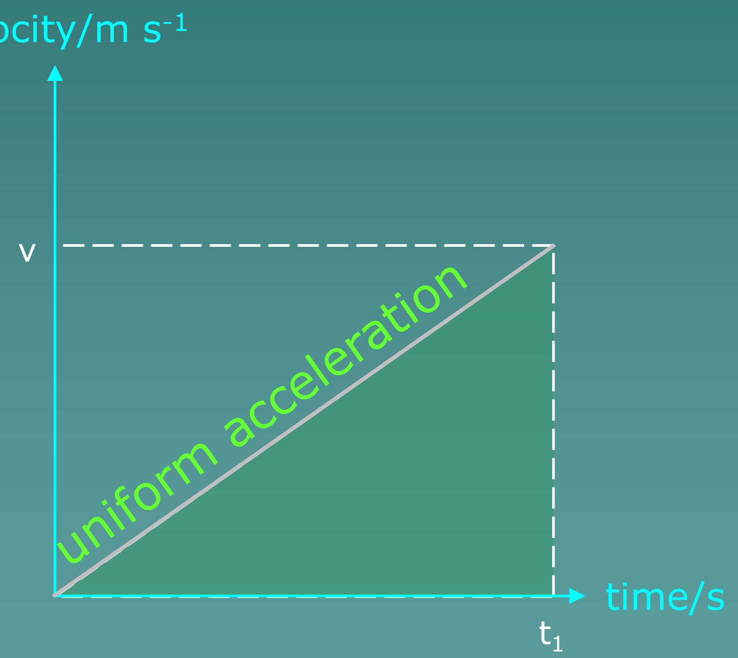


distance travelled = vt_1

graphical analysis of motion

area under a velocity-time graph

The area under the velocity-time graph gives the distance travelled by the moving object.

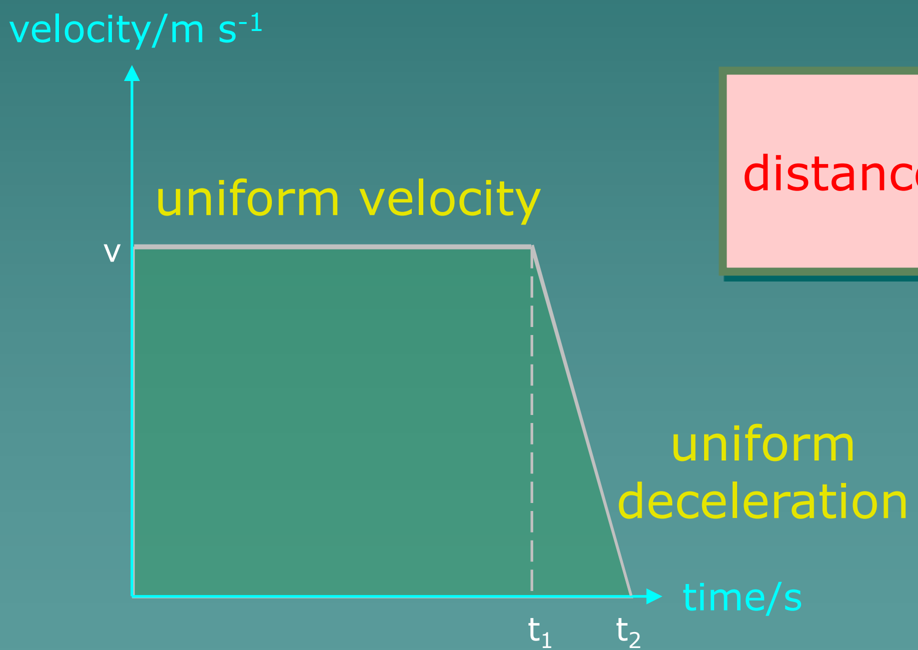


$$\text{distance travelled} = \frac{1}{2} v t_1$$

graphical analysis of motion

area under a velocity-time graph

The area under the velocity-time graph gives the distance travelled by the moving object.

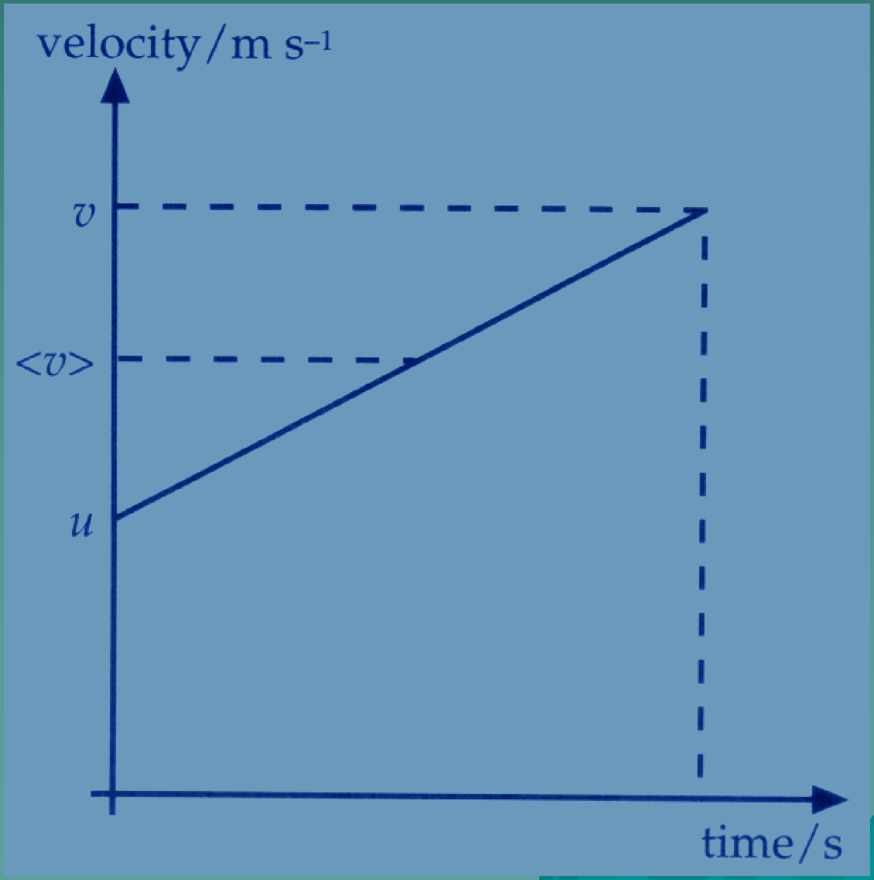


$$\text{distance travelled} = \frac{1}{2} v(t_1 + t_2)$$

area of trapezium = ½ x (a + b) x height

Equation of motion for uniformly accelerated linear motion

Consider the motion of a particle that accelerates from an initial velocity *u* to a final velocity *v* with constant acceleration *a*.



Equation of motion for uniformly accelerated linear motion

Equation 1

- For constant acceleration, velocity increases uniformly with time

$$<v> = \frac{(u+v)}{2}$$

- From definition, displacement (S) during time t,

$$s = <v> t$$

$$\boxed{s = \frac{1}{2}(u+v)t}$$

Equation 1

Equation of motion for uniformly accelerated linear motion

Equation 2

- From definition, acceleration is

$$a = \frac{\Delta v}{\Delta t} = \frac{(v - u)}{t}$$

$$\boxed{v = u + at}$$

Equation 2

Equation of motion for uniformly accelerated linear motion

Equation 3

- From Equation 1 and Equation 2,

$$s = \frac{1}{2}(u + v)t$$

$$s = \frac{1}{2}ut + \frac{1}{2}(u + at)t$$

$$s = ut + \frac{1}{2}at^2$$

Equation 3

Equation of motion for uniformly accelerated linear motion

Equation 4

- From Equation 1 and Equation 2,

$$s = \frac{1}{2}(u+v)t$$

$$s = \frac{1}{2}(u+v)(\frac{v-u}{a})$$

$$s = \frac{1}{2a}(v^2 - u^2)$$

$$v^2 = u^2 + 2as$$

Equation 4

Linear Motion Under The Action of Gravity

Two Situations

- Object is released and allowed to fall under gravity
- Object is projected vertically upwards such that it reaches a certain maximum height and then falls back

Linear Motion Under The Action of Gravity

Object is released and allowed to fall under gravity

- Displacement, velocity and acceleration are all pointing in the same direction
- If no air resistance,

Acceleration = 9.81 m s^{-2}, downward

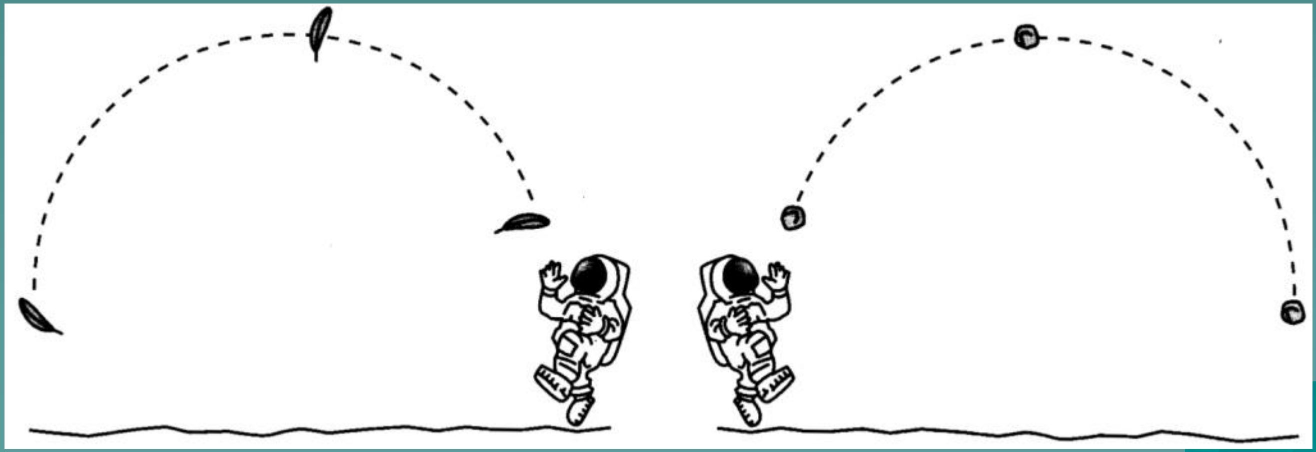
$$v = u + at$$
$$v = 0 + gt$$
$$v = gt$$

Linear Motion Under The Action of Gravity

Object is released and allowed to fall under gravity

- Displacement, velocity and acceleration are all pointing in the same direction
- If <u>no air resistance</u>,

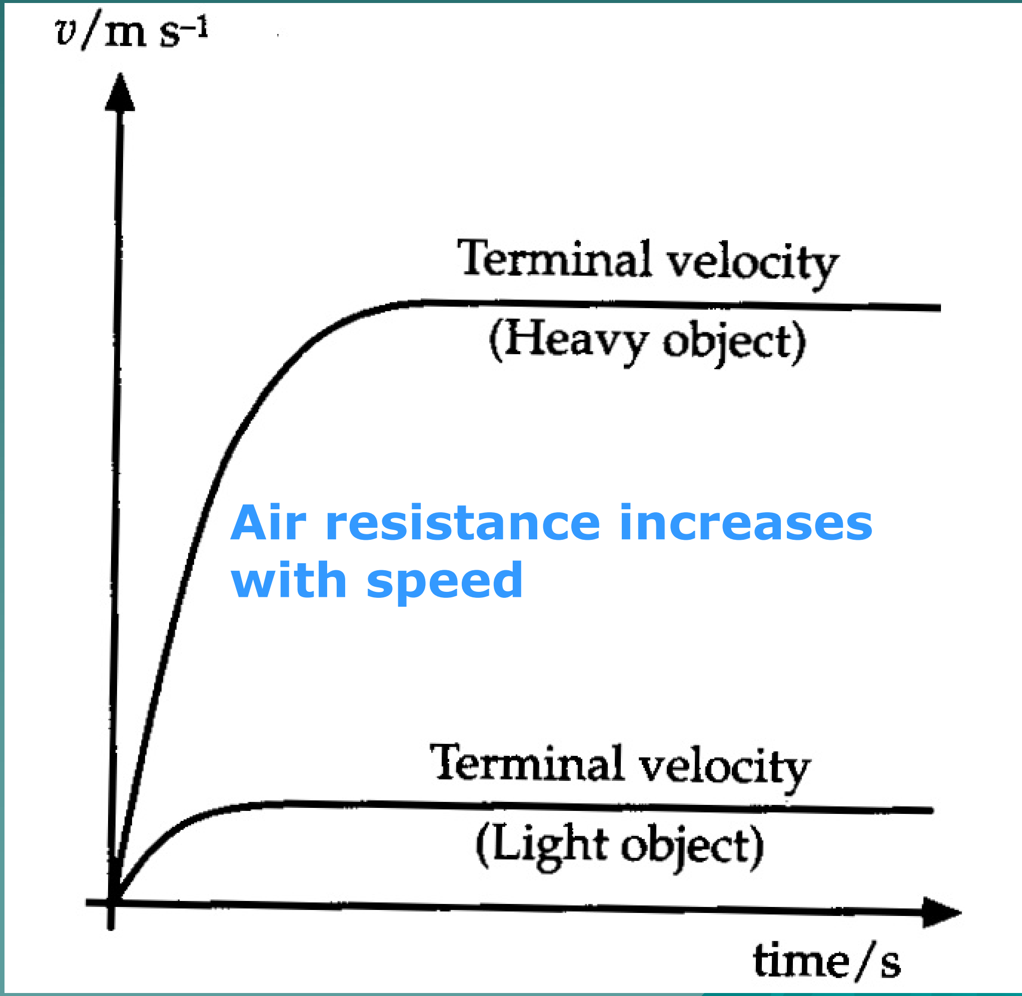
Acceleration = 9.81 m s^{-2}, downward



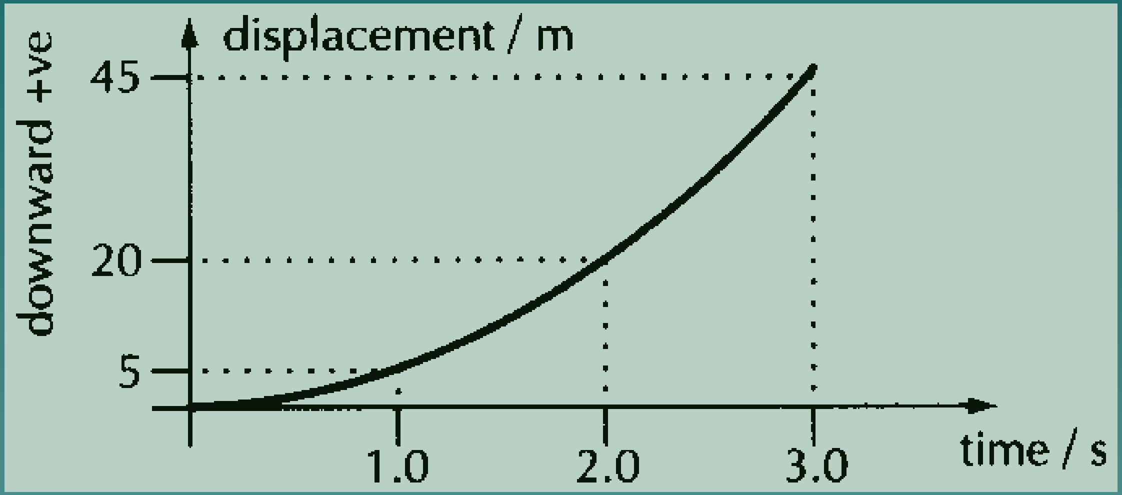
Absence of air resistance means all objects fall with the same acceleration

Linear Motion Under The Action of Gravity

In the presence of air resistance,

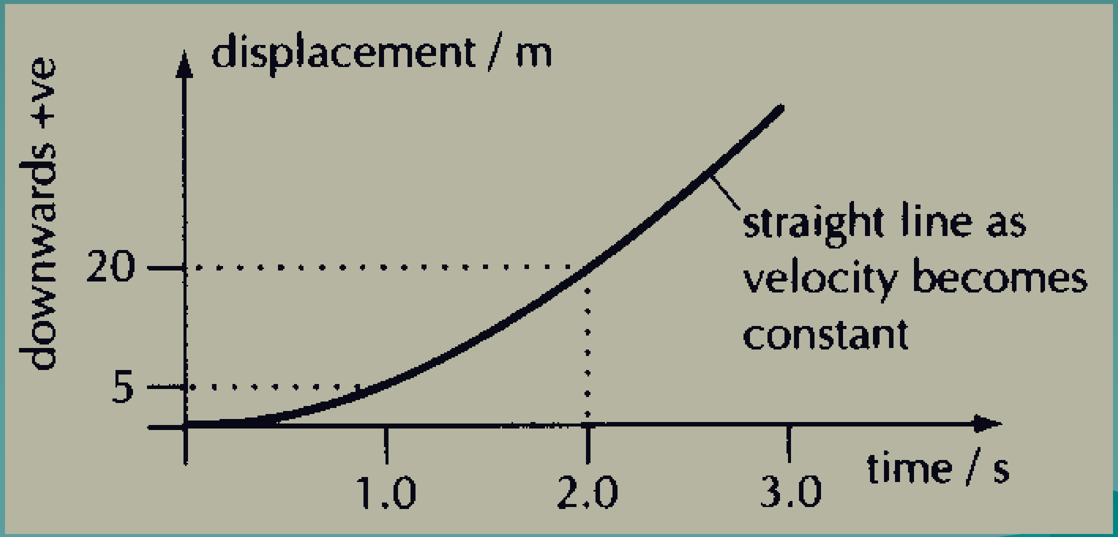


Linear Motion Under The Action of Gravity



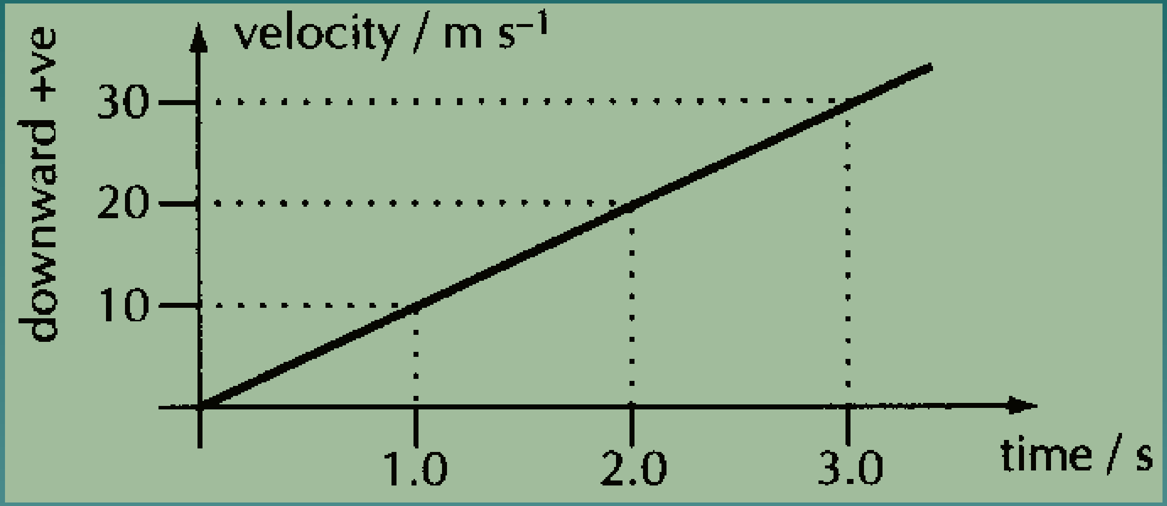
No air resistance

Assume $g = 10\ m\ s^{-2}$



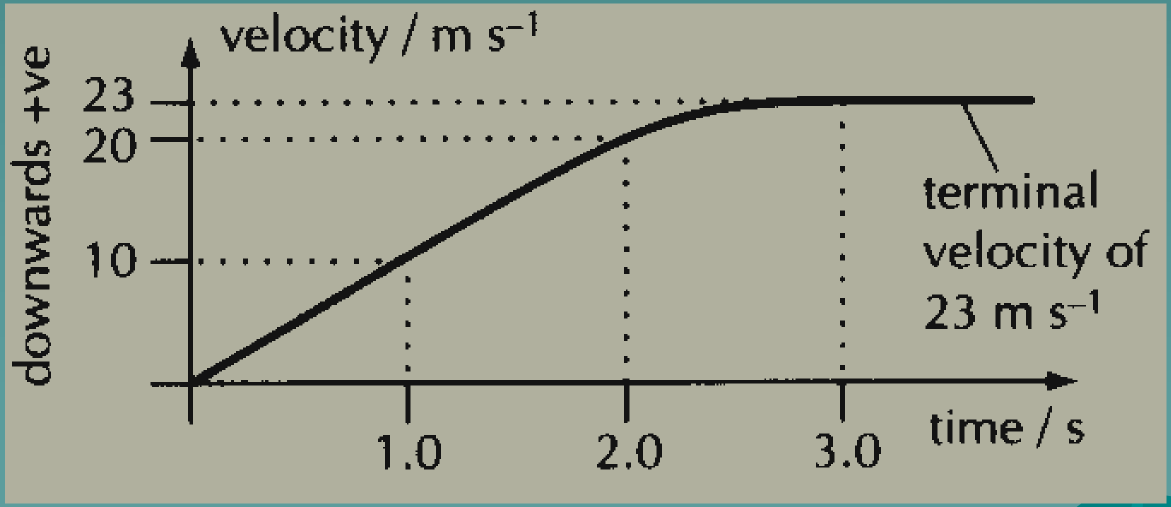
With air resistance

Linear Motion Under The Action of Gravity



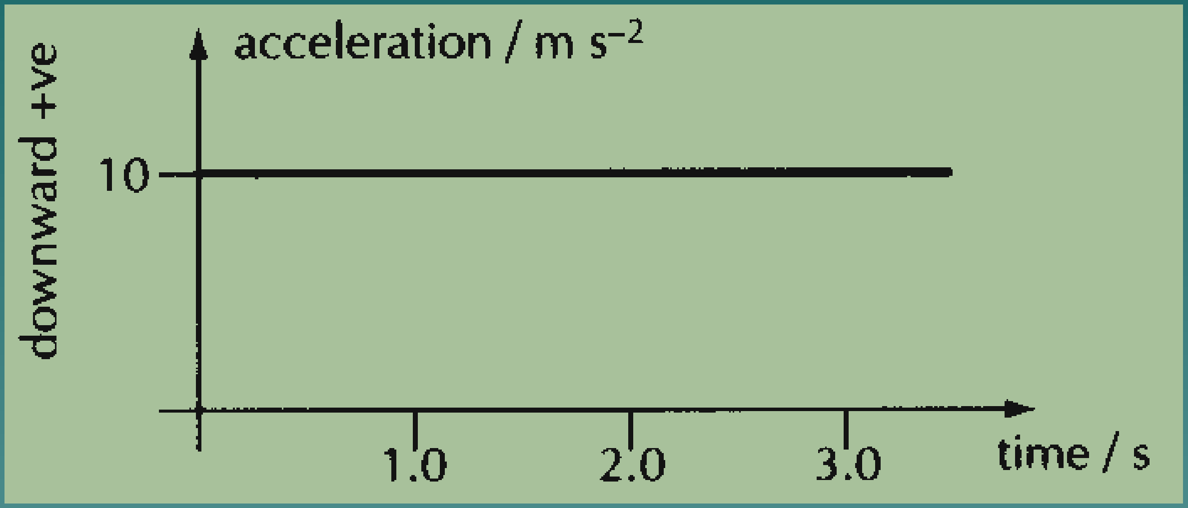
No air resistance

Assume $g = 10\ m\ s^{-2}$



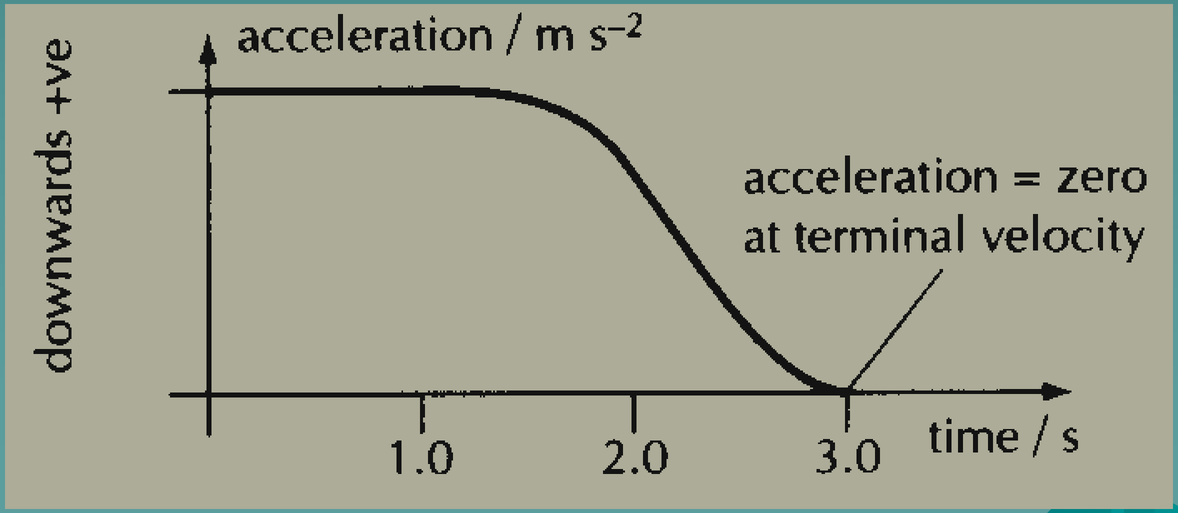
With air resistance

Linear Motion Under The Action of Gravity



No air resistance

Assume $g = 10\ m\ s^{-2}$

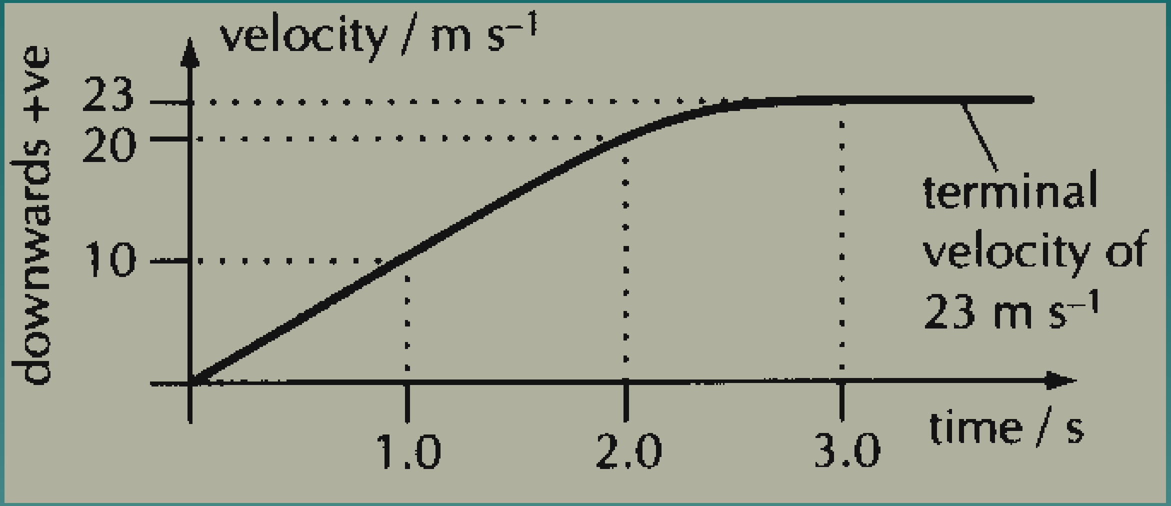


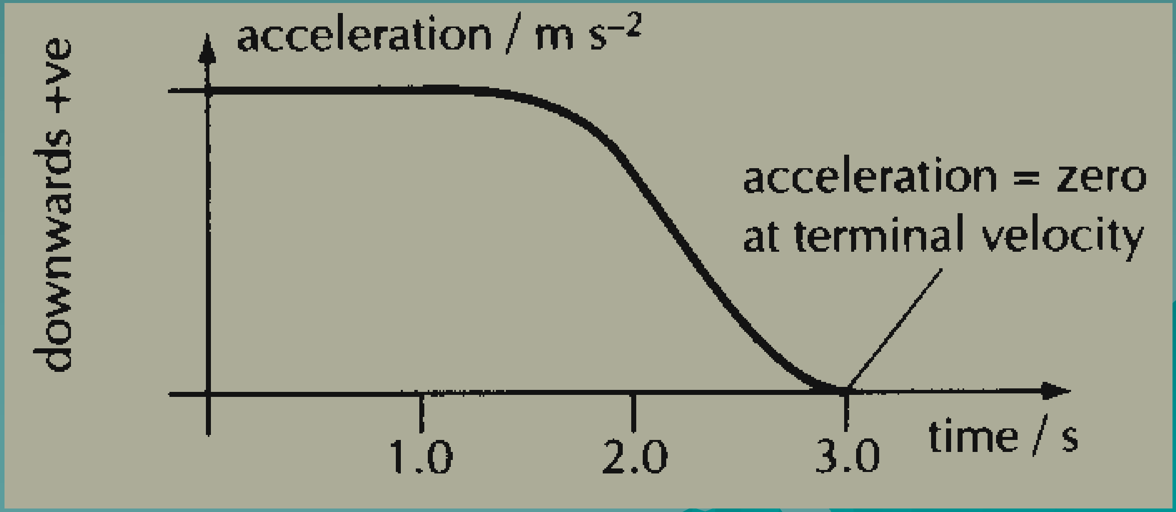
With air resistance

Terminal Velocity

Terminal velocity is the constant velocity that is reached by a falling object when the drag force equals in magnitude to the force that is accelerating the object.

Drag force is the retarding force that is due to air resistance. Provided the density of air stays constant, the drag force is directly proportional to the speed of the object.





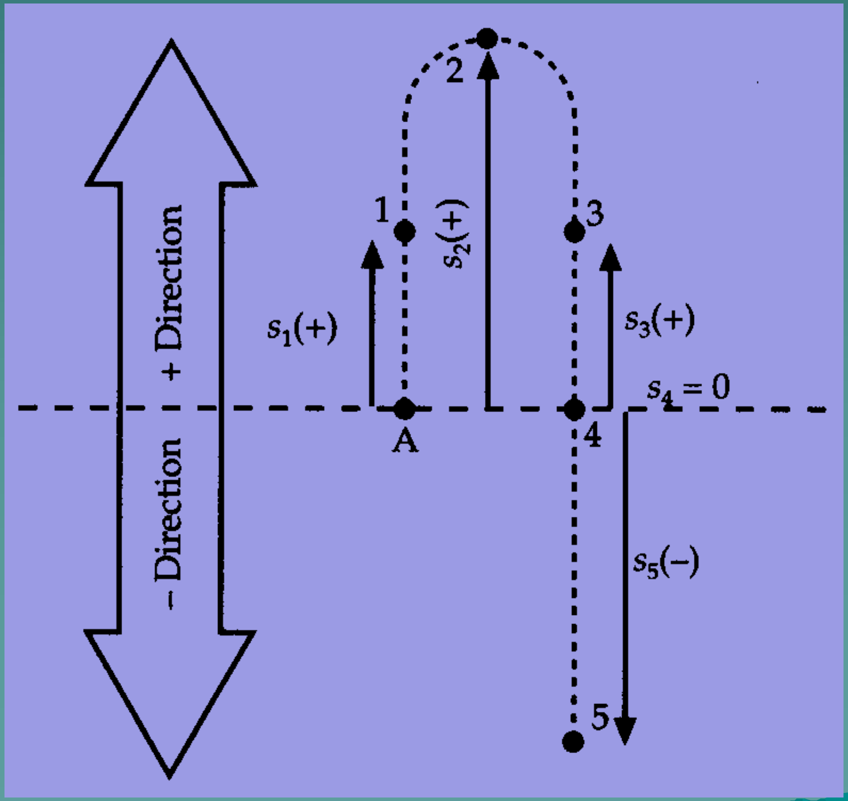
Methods of solving Kinematics Problems

- Draw diagrams to gain a better understanding of the problem
- List
  - the unknown quantities that are given
  - the unknown quantities required
- Define the positive direction and use the correct sign convention for all the known and unknown quantities.
- Often, the direction of the initial velocity u is taken as the positive direction. A vector (v, s or a) which has an opposite direction is given a negative sign
- The direction of the force acting on the body is the same as the direction of the acceleration

Methods of solving Kinematics Problems

- Example: A ball is projected upwards from point A

  The positive direction is defined as the ball's initial direction of travel



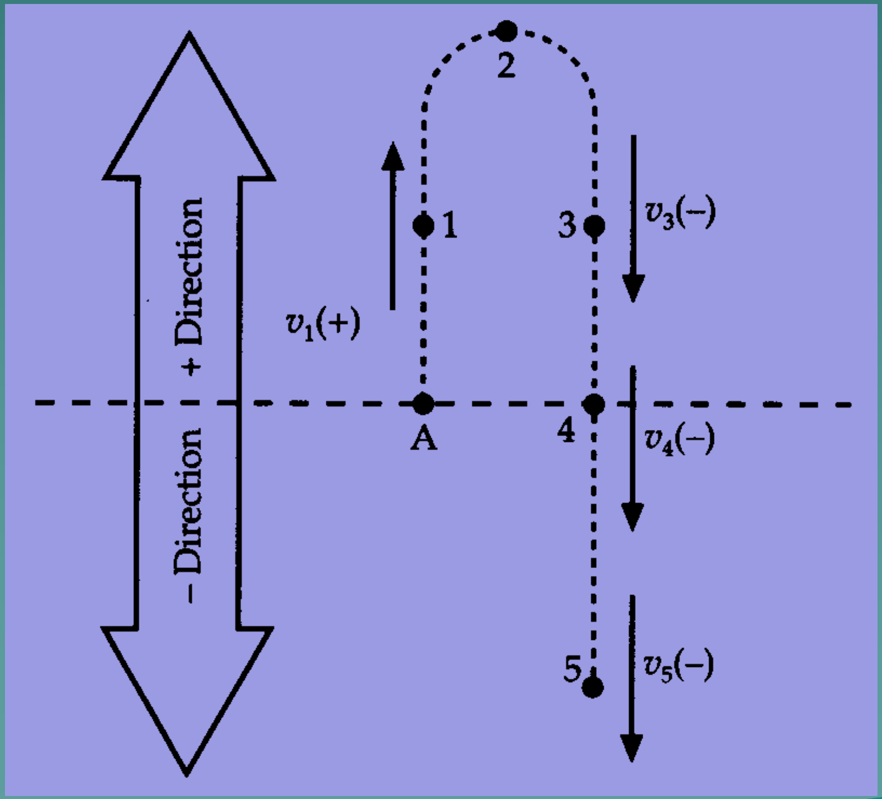
Point	s / m	a / m s^{-2}
1	+	-
2	+	-
3	+	-
4	0	-
5	-	-

Displacements of projected ball

Methods of solving Kinematics Problems

- Example: A ball is projected upwards from point A

  The positive direction is defined as the ball's initial direction of travel



Point	v / m	a / m s^{-2}
1	+	-
2	0	-
3	-	-
4	-	-
5	-	-

Velocities of projected ball

Solving Kinematics Problems

Example 1

A motorist traveling at 13 ms^{-1} approaches traffic lights, which turn red when he is 25 m away from the stop line. His reaction time is 0.70 s and the condition of the road and his tyres is such that the car cannot slow down at a rate of more than 4.5 ms^{-2}. If he brakes fully, how far from the stop line will he stop, and on which side of it?

2.9 m after the stop line

Solving Kinematics Problems

Example 2

A jogger with a constant velocity of 4.0 m s^{-1} runs by a stationary dog. After 1.0 s, the dog decides to chase the jogger. The dog accelerates at 1.5 m s^{-2}.

(a) How long does it take the dog to catch the jogger (if 0 time was when the dog starts running)?

(b) How far away from the spot where the dog was sitting has the jogger gone when she is caught by the dog?

(Assume the jogger is listening to her iPod and does not realise the dog chasing her until it is too late. In other words, assume constant velocity for the jogger)

(a) 6.2 s (b) 29 m

Solving Kinematics Problems

Example 3

A lunar landing module is descending to the Moon's surface at a steady velocity of 10 m s^{-1}. At a height of 120 m, a small object falls from its landing gear.

Taking the Moon's gravitational acceleration as 1.6 m s^{-2}, at what speed does the object strike the Moon?

22 m s^{-1}

Solving Kinematics Problems

Example 4

A balloon is going up at a steady velocity of 12 m s^{-1}. At a height of 32 m above the ground, an object is dropped from it. How long does it take for the object to reach the ground?

$t = 1.2$ s

Solving Kinematics Problems

Example 5

A ball is thrown vertically upwards from the ground with an initial velocity *u*. The time taken by the ball to reach a height *h* is t_1. The ball then takes a further time of t_2 to return to the ground. Find in terms of t_1, t_2 and *g*,

(a) the initial velocity *u*,

(b) the height *h*, and

(c) the maximum height *H* reached by the ball

$$(a)\ u = \frac{1}{2}g(t_1 + t_2)$$

$$(b)\ h = \frac{1}{2}g t_1 t_2$$

$$(c)\ H = \frac{1}{8}g(t_1 + t_2)^2$$

KINEMATICS

Average Speed

$$\text{Average Speed} = \frac{\text{Total Distance}}{\text{Total Time Taken}}$$

SI unit : $\mathrm{m\ s^{-1}}$

Acceleration

Rate of Change of Velocity

$$a = \frac{v-u}{t} \qquad a = \frac{dv}{dt}$$

SI unit : $\mathrm{m\ s^{-2}}$

Velocity

Rate of Change of Displacement

SI unit : $\mathrm{m\ s^{-1}}$

to define

to define

to define

Distance

SI unit : m

Time

SI unit : s

Displacement

$\Delta x = x_f - x_i$

SI unit : m

uses

uses

uses

KINEMATICS

describes motion (a = constant) with

Equations of Motion

$$s = \tfrac{1}{2}(u+v)t$$

$$v = u + at$$

$$s = ut + \tfrac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

describes projectile motion with

Equations of Motion

y-axis

$$y = \tfrac{1}{2}(u_y + v_y)t$$

$$v = u_y + a_y t$$

$$y = u_y t + \tfrac{1}{2}a_y t^2$$

$$v_y^2 = u_y^2 + 2a_y y$$

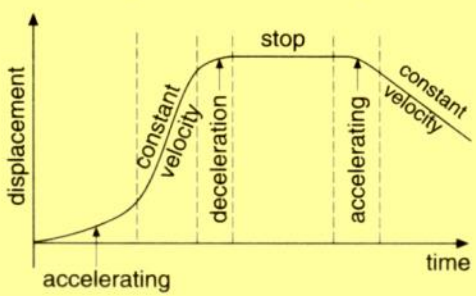
x-axis

$$x = u_x t$$

describes motion with

Graphs

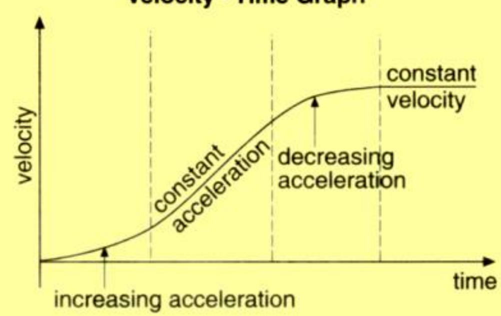
Displacement - Time Graph



Gradient of $(s-t)$ graph gives

Velocity

Velocity - Time Graph



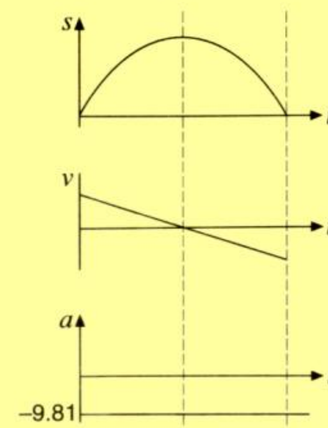
Gradient of $(v-t)$ graph gives

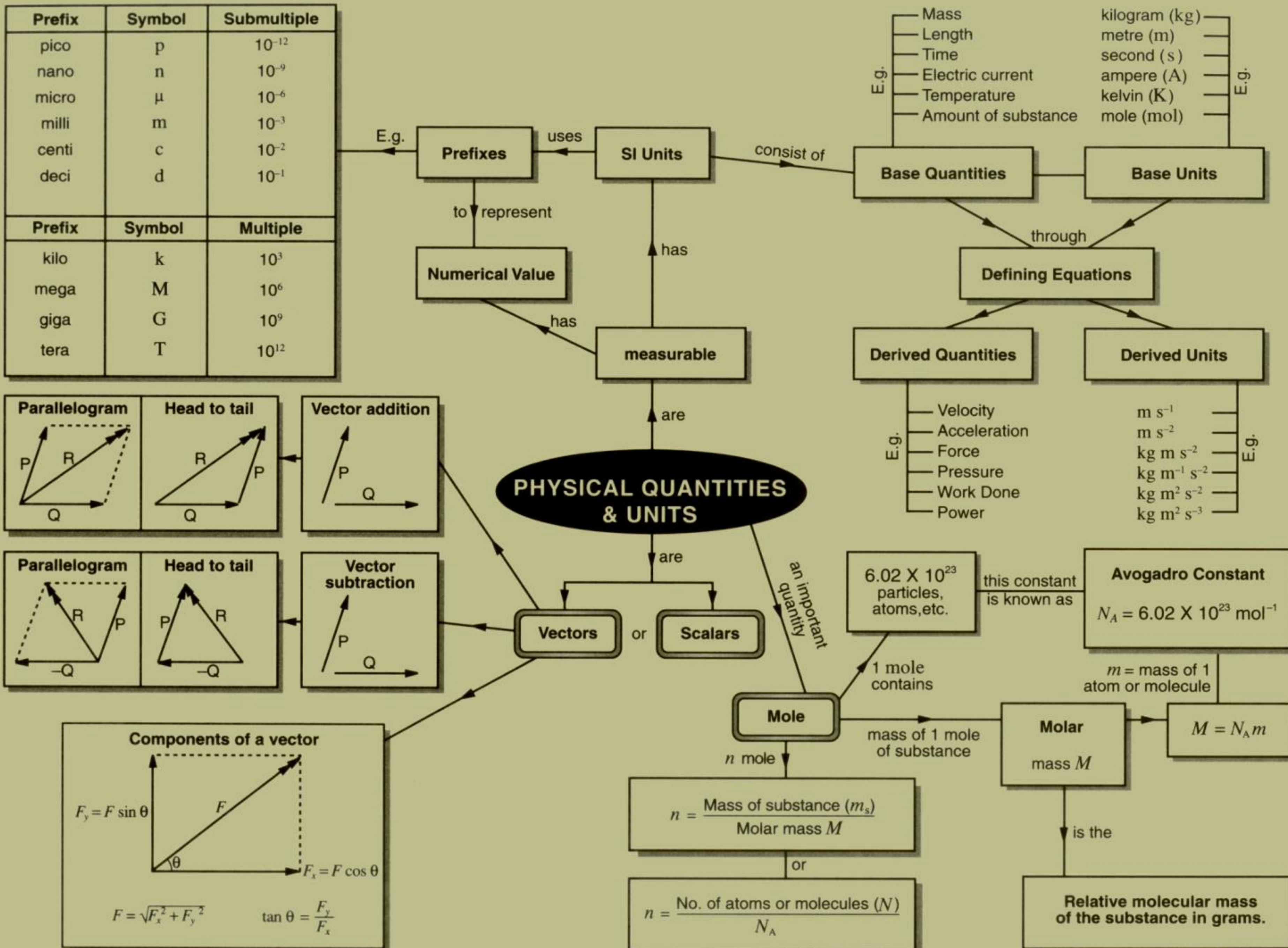
Acceleration

Area under $(v-t)$ graph gives

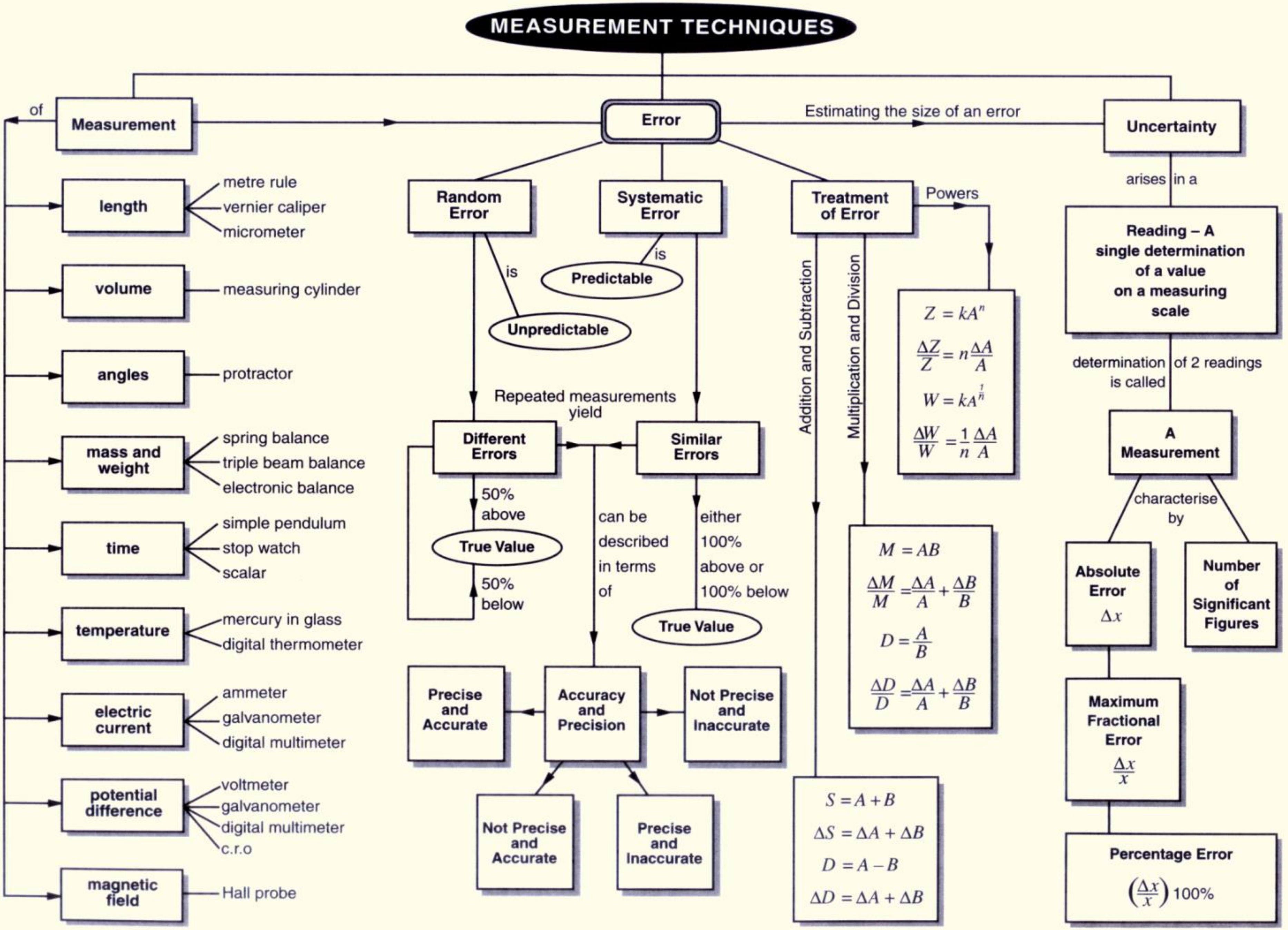
Displacement

Graphs for an object thrown up into the air





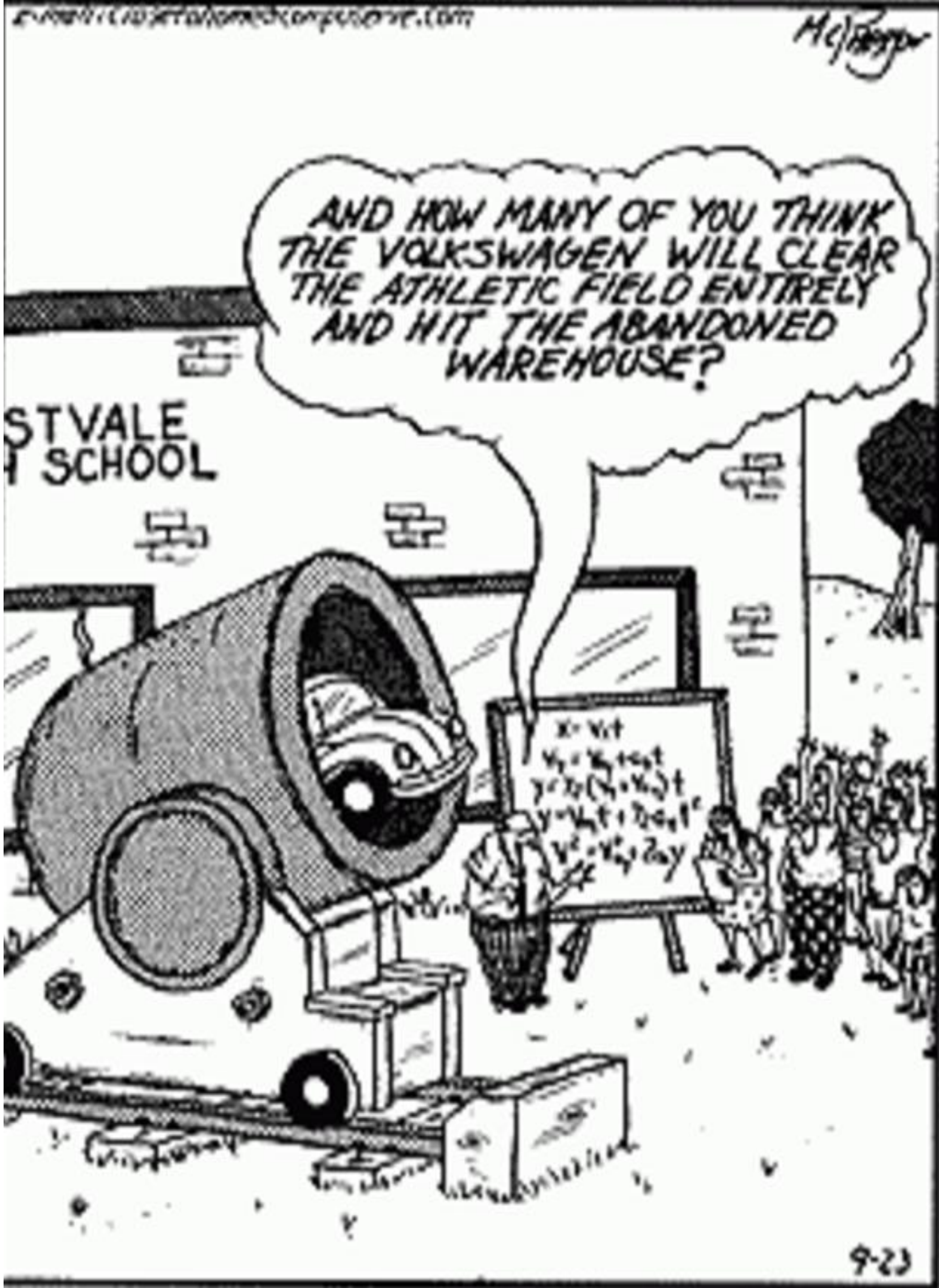
Prefix	Symbol	Submultiple
pico	p	10^{-12}
nano	n	10^{-9}
micro	μ	10^{-6}
milli	m	10^{-3}
centi	c	10^{-2}
deci	d	10^{-1}
Prefix	**Symbol**	**Multiple**
kilo	k	10^{3}
mega	M	10^{6}
giga	G	10^{9}
tera	T	10^{12}



MEASUREMENT TECHNIQUES
of
Measurement
Error
Estimating the size of an error
Uncertainty
length
metre rule
vernier caliper
micrometer
volume
measuring cylinder
angles
protractor
mass and weight
spring balance
triple beam balance
electronic balance
time
simple pendulum
stop watch
scalar
temperature
mercury in glass
digital thermometer
electric current
ammeter
galvanometer
digital multimeter
potential difference
voltmeter
galvanometer
digital multimeter
c.r.o
magnetic field
Hall probe
Random Error
is
Unpredictable
Systematic Error
is
Predictable
Treatment of Error
Powers
Addition and Subtraction
Multiplication and Division
$Z = kA^n$
$\frac{\Delta Z}{Z} = n\frac{\Delta A}{A}$
$W = kA^{\frac{1}{n}}$
$\frac{\Delta W}{W} = \frac{1}{n}\frac{\Delta A}{A}$
Repeated measurements yield
Different Errors
Similar Errors
50% above
True Value
50% below
can be described in terms of
either 100% above or 100% below
True Value
Precise and Accurate
Accuracy and Precision
Not Precise and Inaccurate
Not Precise and Accurate
Precise and Inaccurate
$M = AB$
$\frac{\Delta M}{M} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
$D = \frac{A}{B}$
$\frac{\Delta D}{D} = \frac{\Delta A}{A} + \frac{\Delta B}{B}$
$S = A + B$
$\Delta S = \Delta A + \Delta B$
$D = A - B$
$\Delta D = \Delta A + \Delta B$
arises in a
Reading – A single determination of a value on a measuring scale
determination of 2 readings is called
A Measurement
characterise by
Absolute Error Δx
Number of Significant Figures
Maximum Fractional Error $\frac{\Delta x}{x}$
Percentage Error
$\left(\frac{\Delta x}{x}\right)$ 100%

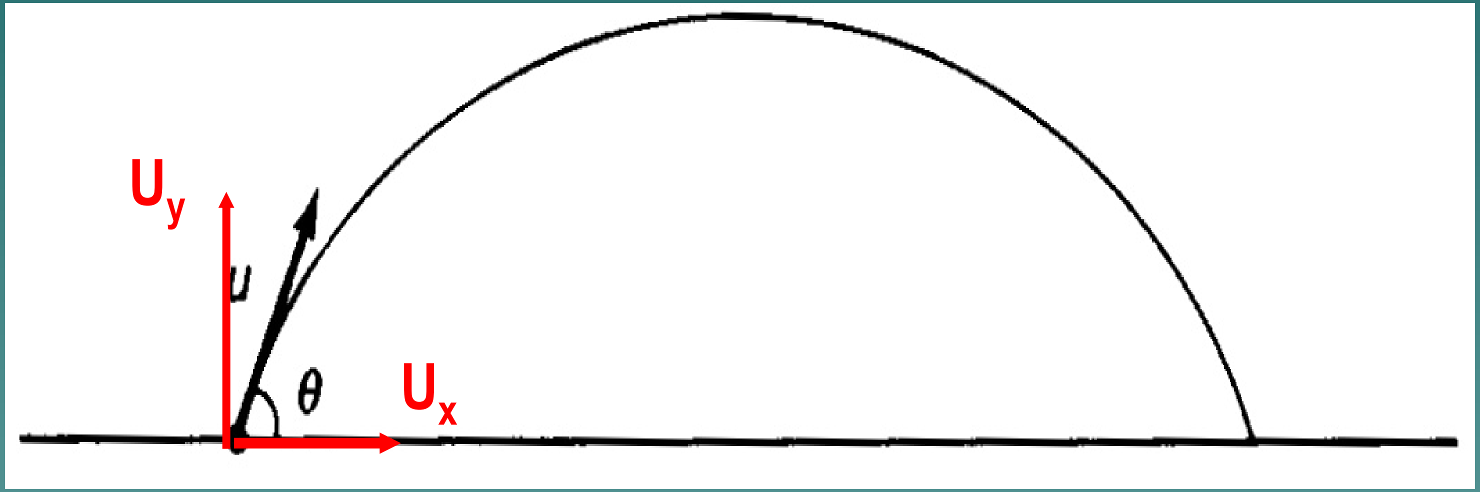
Projectile Motion

Kinematics in 2-Dimensions



Projectile Motion

- If a body is projected into the air at an angle θ to the horizontal, the body will move in a **parabolic path** (in the absence of air resistance)



Taking upward direction as positive,

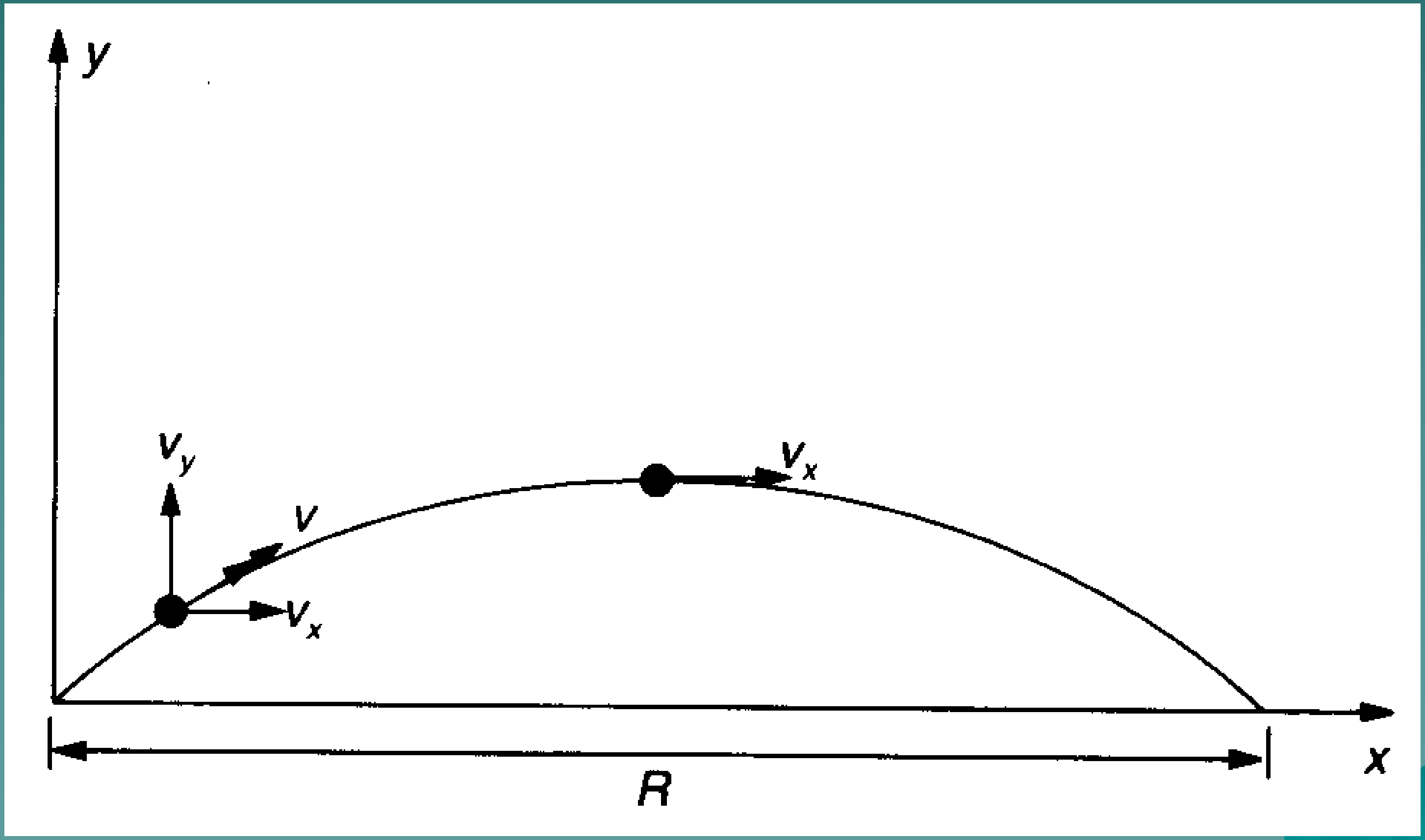
$$U_x = U\cos\theta$$

$$U_y = U\sin\theta$$

The vertical component Uy and horizontal component Ux; of velocity are <u>independent</u> of each other.

Projectile Motion

- The projectile motion is two-dimensional in the x-y plane.



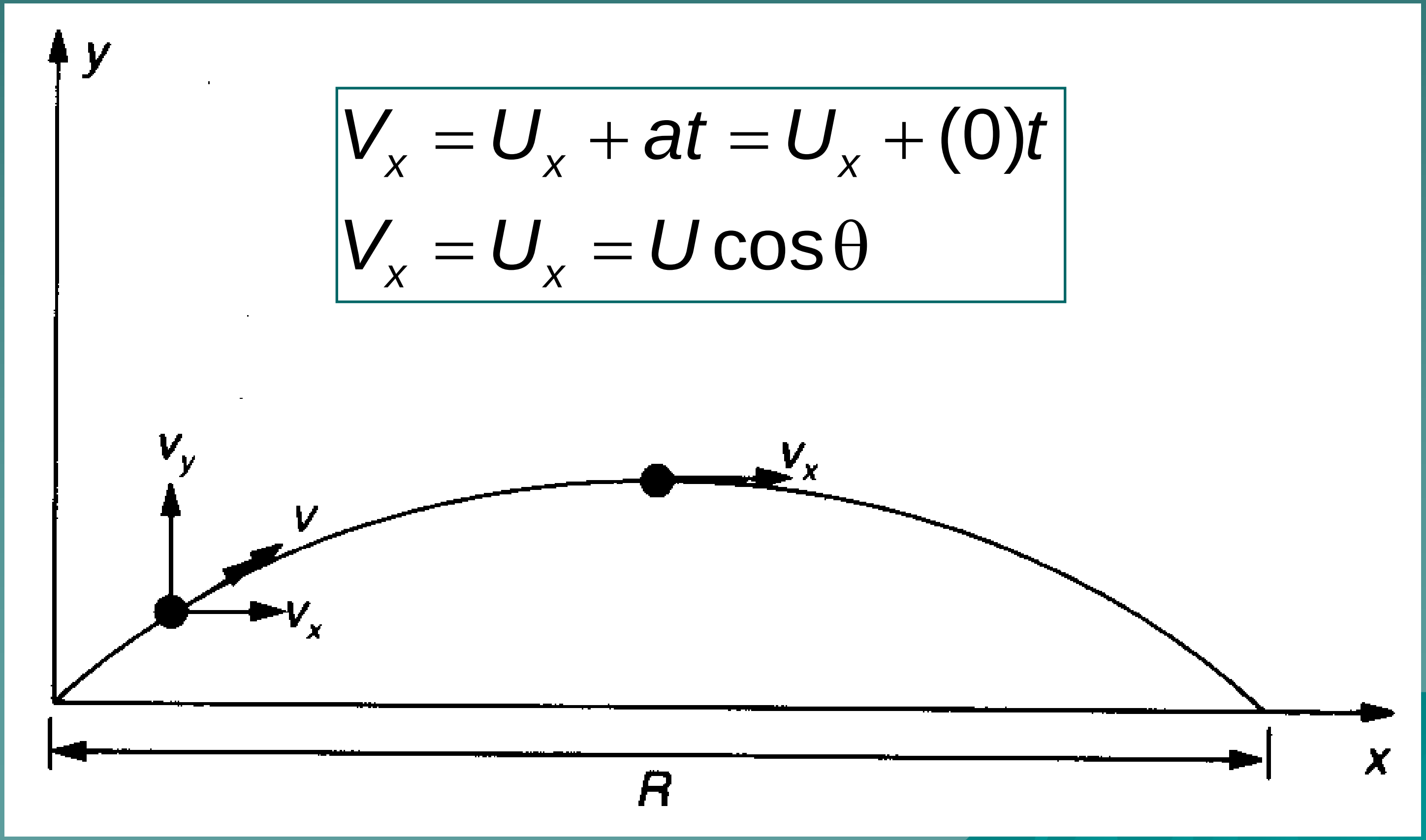
Projectile Motion

Horizontal direction

- $a_x = 0$, if the air resistance is negligible
- Motion will be uniform, $V_x = U_x$; i.e horizontal component of the velocity is the same throughout the motion

$$V_x = U_x + at = U_x + (0)t$$

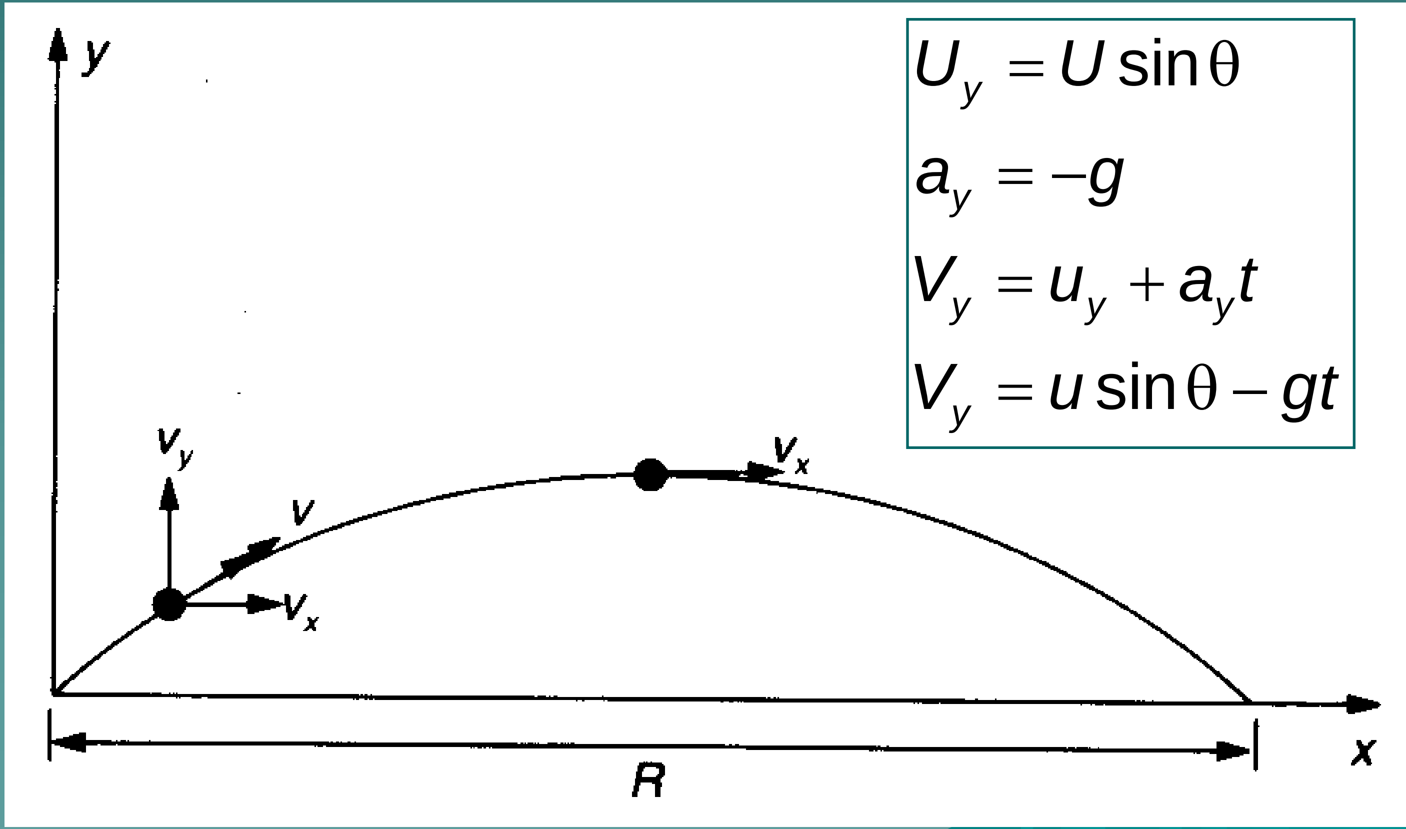
$$V_x = U_x = U\cos\theta$$



Projectile Motion

Vertical direction

- $a_y = -g$, if the air resistance is negligible
- Vertical motion will be non-uniform



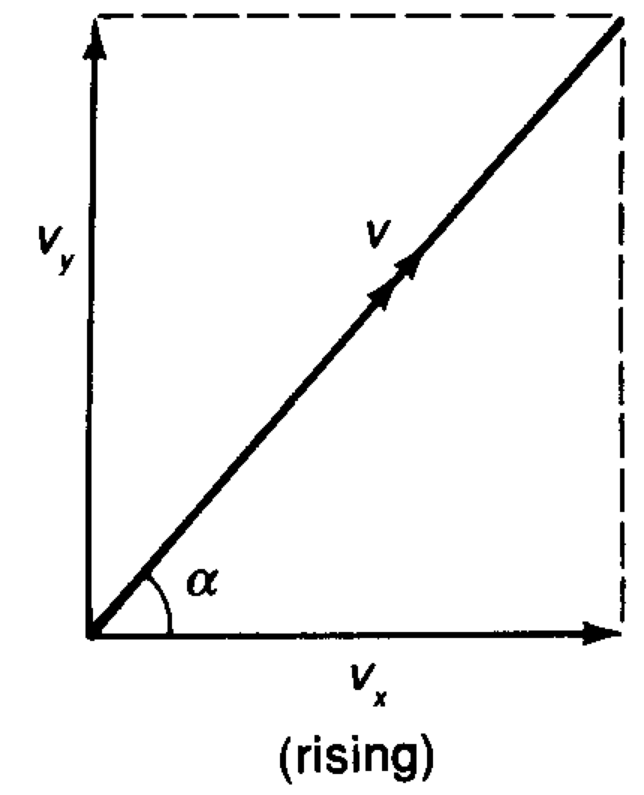
$$U_y = U\sin\theta$$

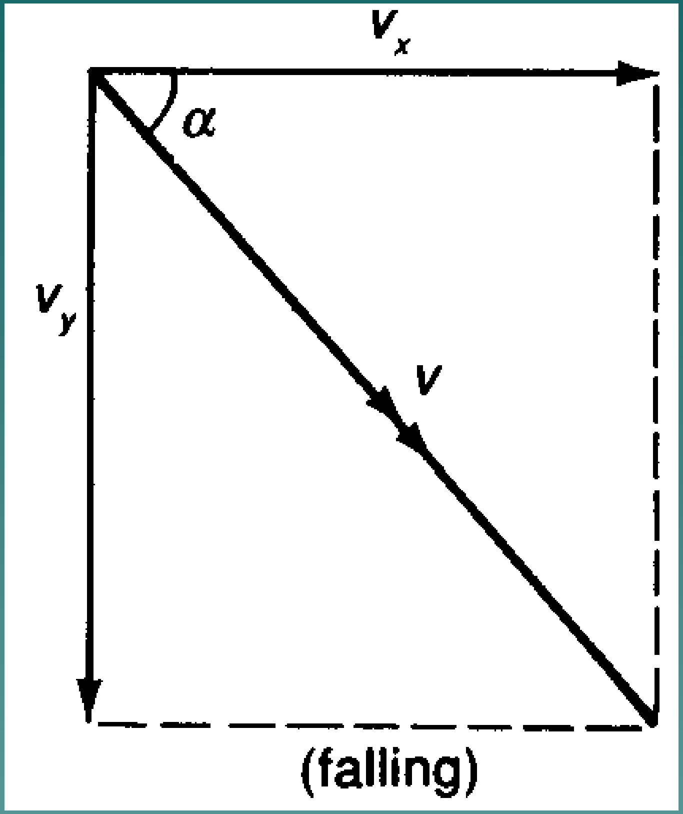
$$a_y = -g$$

$$V_y = u_y + a_y t$$

$$V_y = u\sin\theta - gt$$

Projectile Motion





Resultant Velocity

$$V = \sqrt{V_x^{\,2} + V_y^{\,2}}$$

Direction

$$\tan\alpha = \frac{V_y}{V_x}$$

Projectile Motion

*Try to prove:

1. Horizontal Displacement, x

$$x = (u\cos\theta)t$$

2. Vertical Displacement, y

$$y = (u\sin\theta)t - \frac{1}{2}gt^2$$

3. Horizontal Range, R

$$R = \frac{u^2 \sin 2\theta}{g}$$

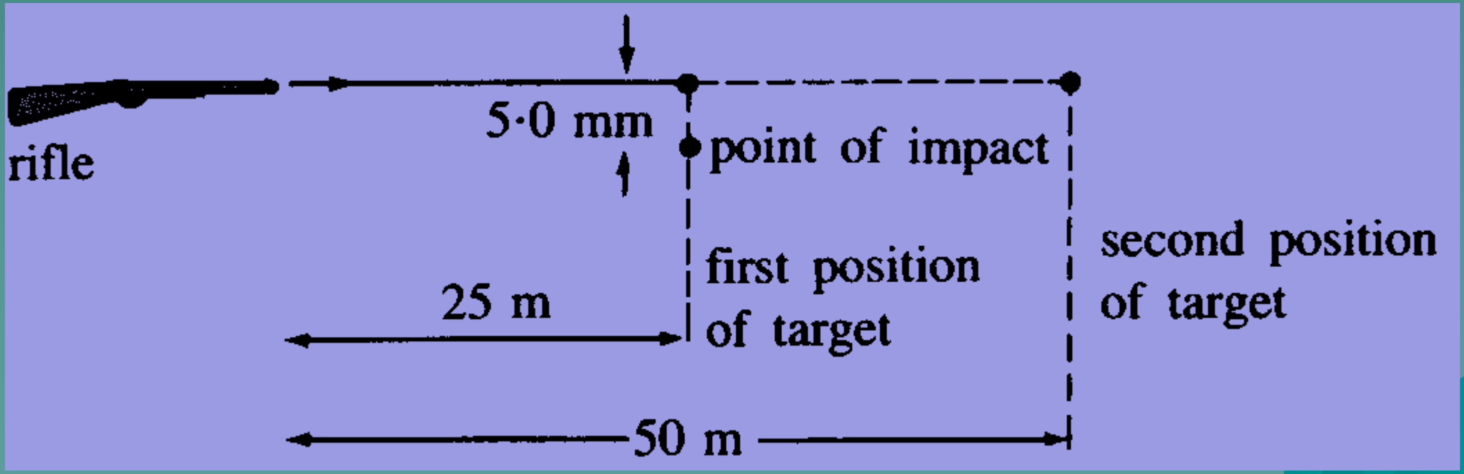
What is the condition for maximum range?

Solving Kinematics Problems

Example 6

When a rifle is fired horizontally at a target P on a screen at a range of 25 m, the bullet strikes the screen at a point 5.0 mm below P. The screen is now moved to a distance of 50 m and the rifle again fired horizontally at P in its new position.

Assuming that air resistance may be neglected, what is the new distance below P at which the screen would now be struck?

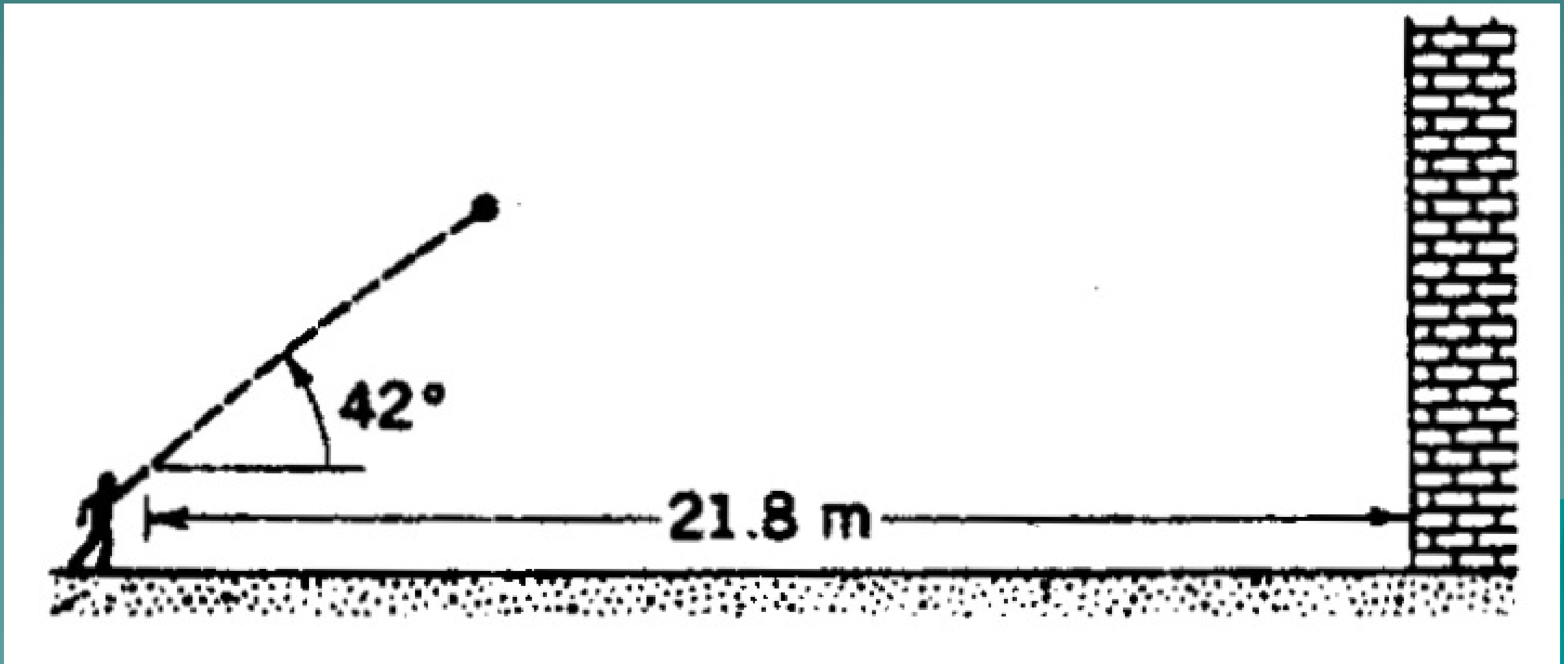


20 mm

Solving Kinematics Problems

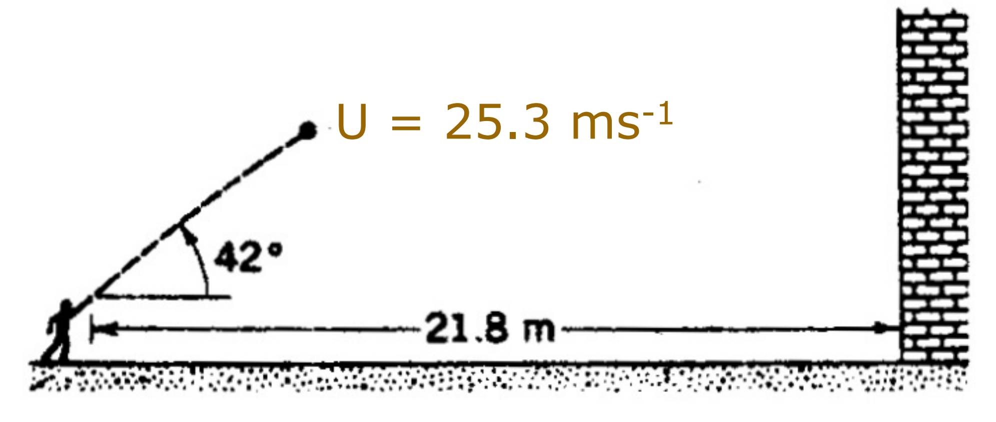
Example 7

You throw a ball with a speed of 25.3 m s^{-1} at an angle of 42.0^{o} above the horizontal directly toward a wall as shown below. The wall is 21.8 m from the release point of the ball.



Solving Kinematics Problems

Example 7 (continue)



(a) How long is the ball in the air before it hits the wall?

(b) How far above the release point does the ball hit the wall?

(c) What are the horizontal and vertical components of its velocity as it hits the wall?

(d) Has it passed the highest point on its trajectory when it hits?

(a) $t = 1.16$ s, (b) 13.0 m, (c) 18.8 m s^{-1}, 5.55 m s^{-1}, (d) No

Solving Kinematics Problems

Example 8

You throw a ball from a cliff with an initial velocity of 15.0 ms^{-1} at an angle of 20^o below the horizontal. The height of the cliff is 1000 m above the ground. Find

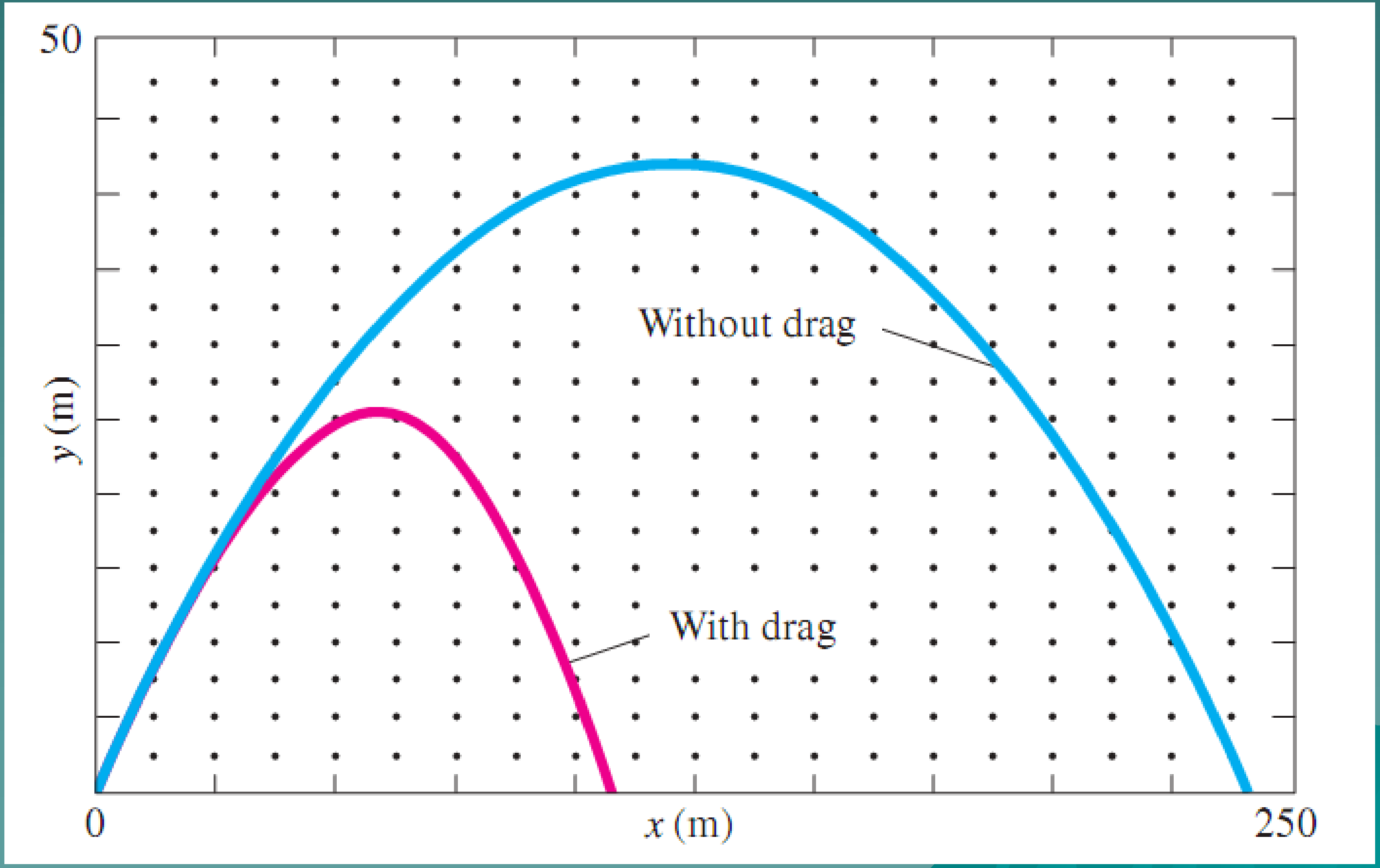
(a) its horizontal displacement as it hits the ground

(b) its vertical displacement 2.3 s later

(a) 194 m, (b) 37.7 m

Projectile Motion in the real world

- In the presence of air resistance, the path of the projectile is no longer symmetric about the maximum. We say that the projectile motion is no longer parabolic.



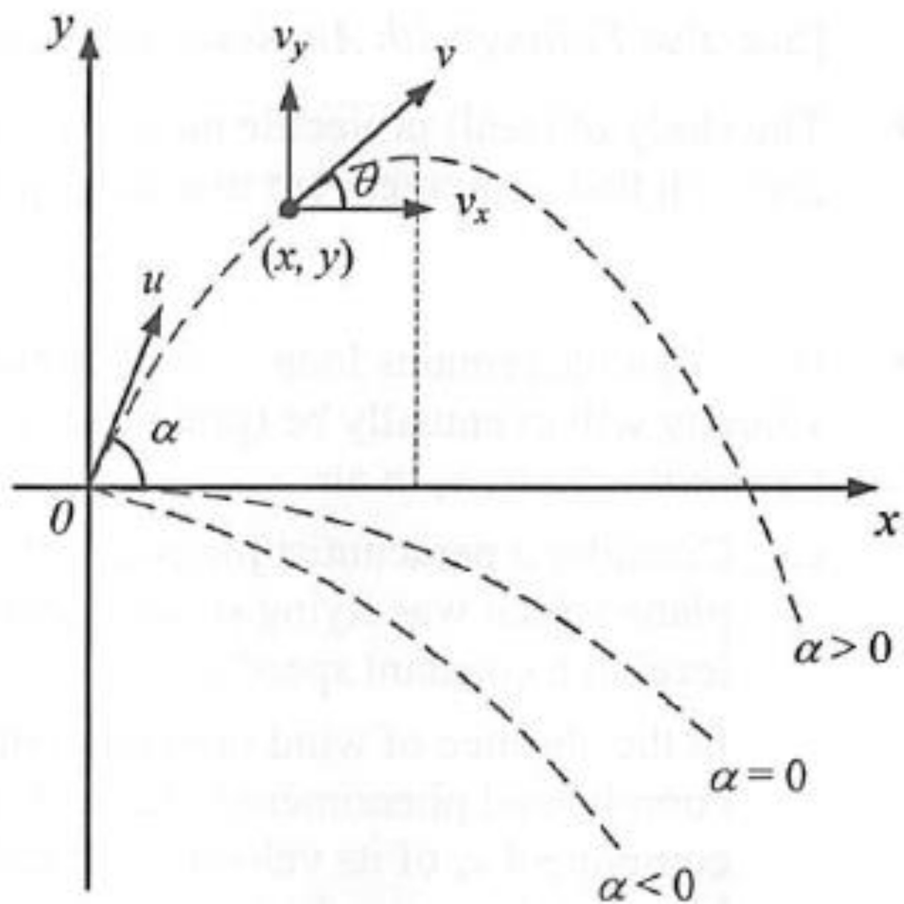
- Without air resistance, a *projectile* undergoes *inertial* motion with *uniform velocity* in the *horizontal* direction and, simultaneously but independently, *free-falling* motion with *uniform acceleration* in the *vertical* direction.

  i.e. The components of its acceleration (a_x, a_y) in the x-y plane are given by:

  $$a_x = 0 \quad \text{and} \quad a_y = -g$$

- With initial speed u and projection angle α, the trajectory of a projectile could be as shown.

  The angle α may also be zero or negative and the corresponding graphs could be as shown:



- The equations of motion (in the usual notations) include:

$$\boxed{\begin{aligned} u_x &= u\cos\alpha \\ u_y &= u\sin\alpha \\ v_x &= u_x = u\cos\alpha \quad \text{(constant)} \\ v_y &= u_y - gt = u\sin\alpha - gt \\ x &= u_x t = (u\cos\alpha)t \\ y &= u_y t - \tfrac{1}{2}gt^2 = (u\sin\alpha)t - \tfrac{1}{2}gt^2 \end{aligned}}$$

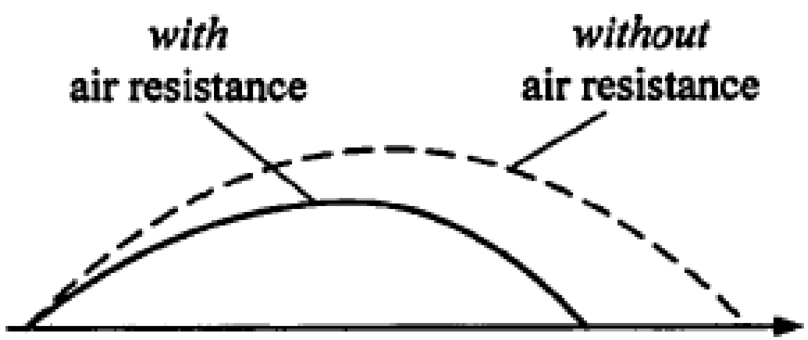
- The *magnitude* v and *direction* θ of the velocity at any instant can be calculated from the components v_x and v_y as follows:

$$\boxed{v = \sqrt{v_x^{\ 2} + v_y^{\ 2}} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right)}$$

- The vertical (y-) range of motion should be small compared to the Earth's radius so that the value of g may be taken as (approximately) constant.

- **The effects of air resistance on projectile motion are as follows:**
  - **The horizontal component v_x of the velocity will *decrease continuously*, tending towards zero.**
  - **The vertical component v_y of the velocity will *decelerate more when rising* and *accelerate less when falling*.**

    The effects are identical to those of a body falling with air resistance.



A trajectory with air resistance (——), compared with one without air resistance (- - -), could be as shown (above).