Topic 2.4

The motion of objects can be described precisely in terms of their distance travelled, displacement, speed, velocity and acceleration. However what actually makes a body move? Can we predict the motion of objects?

Momentum & Impulse

Contents

- Newton's Second Law expressed in terms of Rate of Change of Momentum
- Impulse & Force-Time Graphs
- Conservation of Linear Momentum
- Elastic Collisions, Inelastic Collisions & Explosions

Definition of Linear Momentum

The linear momentum of an object is defined as the product of its mass and velocity.

$$p = mv$$

The SI units for momentum is kg m s^{-1}.

Impulse of a force

From Newton's Second law of motion

$$F = \frac{\Delta p}{\Delta t}$$

p = linear momentum

m = mass of body

V = velocity of body

$$\Delta p = F \Delta t$$

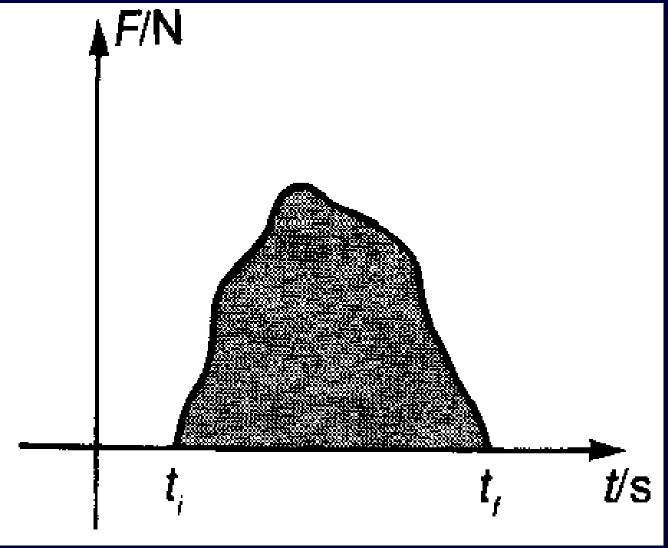
Impulse

The change in momentum, Δp, is called Impulse.

The units for momentum may also be N s as can be seen from the impulse formula.

Definition of Impulse

Impulse is defined as the change in momentum caused by a force.



Impulse = Area under Force-time graph

Note: Area under Force-displacement graph is equal to Work Done.

Use of Impulse in Newton's Second Law

Newton's Second law of motion

$$F = \frac{dp}{dt} = \frac{d(mv)}{dt} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

For constant mass,

$$\frac{dm}{dt} = 0$$

$$F = m\frac{dv}{dt} = ma$$

For constant velocity,

$$\frac{dv}{dt} = 0$$

$$F = v\frac{dm}{dt}$$

Newton's second law of motion

For constant velocity,

$$F = v\frac{dm}{dt}$$

Example 1

A conveyor belt is used to transfer luggage at an airport. It consists of a horizontal, endless belt running over driving rollers, moving at a constant speed of 1.5 m s^{-1}. To keep the belt moving when it is transporting luggage requires a greater driving force than for an empty belt.

On average, the rate at which luggage is placed on one end of the belt and lifted at the other is 20 kg s^{-1}.

Why is an additional force required, and what is its value?

An additional force is required to overcome the retarding force exerted by the luggage on the belt during impact.

F = v (dm/dt) = 1.5 x (20) = 30 N

Newton's second law of motion

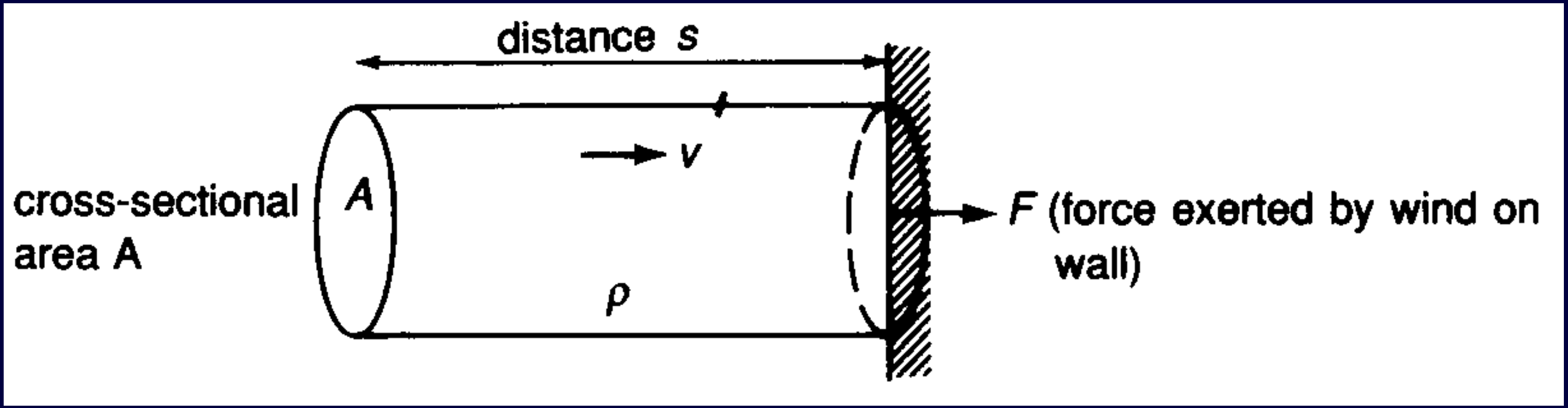
For constant velocity,

$$F = v\frac{dm}{dt}$$

Example 2

A steady wind of 50 ms^{-1} hits against a rigid wall which is in a plane perpendicular to the wind direction. Estimate the pressure exerted by the wind on the wall.

(Density of air = 1.25 kgm^{-3})



P = F / A = (v dm/dt) / A = [v x A x (s/ t) x ρ] / A = v^2 ρ

P = v^2 ρ = 50^2 x 1.25 = 3125 = 3100 N (to 2 s.fs)

Law of Conservation of Momentum

The total momentum of a closed system is constant, provided no external resultant force act on it.

(No external force is only possible if we assume that the surface in which the objects are moving on is frictionless.)

- An extension of Newton's Second & Third Laws of Motion
- For a closed system of two colliding bodies,
  - Newton's Third Law ensures the condition required for the Principle of Conservation of Momentum

Types of Collisions

- Elastic Collision

- After the collision, the two bodies move separately

- Total Kinetic energy is conserved

- Total Momentum is conserved

- Perfectly Inelastic collision

- After the collision, the two colliding bodies move together as one

- Total Kinetic energy is NOT conserved, as some of it is lost in the form of heat and sound energies

- Total Momentum is conserved

- Inelastic collision

- Same as in perfectly inelastic collision, but the two colliding bodies do not move together as one.

Comparison Table for Collisions & Explosion

Collision & Explosion	Momentum Conserved	Kinetic Energy Conserved	Colliding bodies stick together after collision
Elastic Collision	YES	YES	NO
Perfectly Inelastic Collision	YES	NO	YES
Inelastic Collision	YES	NO	NO
Explosion	YES	NO	[opposite of perfectly inelastic collision]

Elastic Collision Equation

For elastic collisions, both momentum and kinetic energy are conserved. It can be shown that for Elastic Collisions, the following formula can be derived (for one dimension only):

See Giancoli page 176…

Equation 7-7:

$$u_A - u_B = -(v_A - v_B)$$

Example: Elastic collision

A 0.060-kg tennis ball, moving with a speed of 2.50 m s^{-1} collides head-on with a 0.090 kg ball initially moving away from it at a speed of 1.15 m s^{-1}. Assuming a perfectly elastic collision, what are the speed and direction of each ball after the collision? (Giancoli, Qn 25)

Let A represent the 0.060-kg tennis ball, and let B represent the 0.090-kg ball. The initial direction of the balls is the positive direction. We have $v_A = 2.50\,\mathrm{m/s}$ and $v_B = 1.15\,\mathrm{m/s}$. Use Eq. 7-7 to obtain a relationship between the velocities.

$$v_A - v_B = -(v'_A - v'_B) \quad \rightarrow \quad v'_B = 1.35\,\mathrm{m/s} + v'_A$$

Substitute this relationship into the momentum conservation equation for the collision.

$$m_A v_A + m_B v_B = m_A v'_A + m_B v'_B \quad \rightarrow \quad m_A v_A + m_B v_B = m_A v'_A + m_B (1.35\,\mathrm{m/s} + v'_A) \quad \rightarrow$$

$$v'_A = \frac{m_A v_A + m_B (v_B - 1.35\,\mathrm{m/s})}{m_A + m_B} = \frac{(0.060\text{ kg})(2.50\,\mathrm{m/s}) + (0.090\text{ kg})(1.15\,\mathrm{m/s} - 1.35\,\mathrm{m/s})}{0.150\text{ kg}}$$

$$= \boxed{0.88\,\mathrm{m/s}}$$

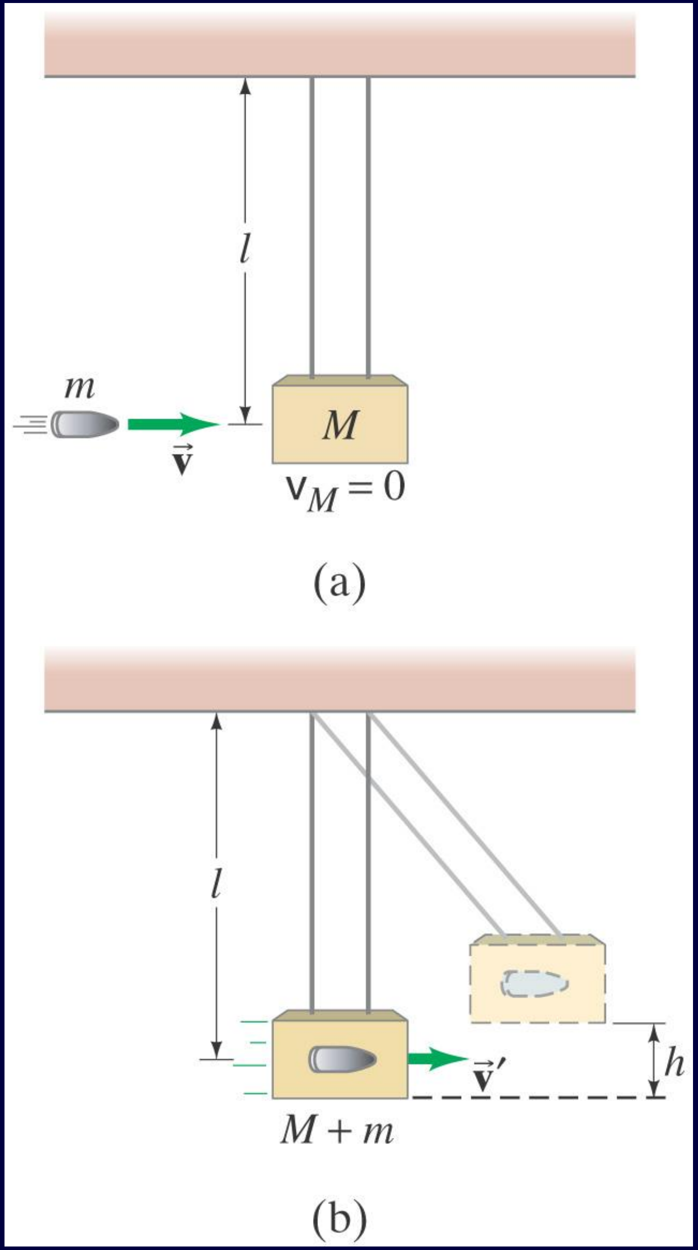
$$v'_B = 1.35\,\mathrm{m/s} + v'_A = \boxed{2.23\,\mathrm{m/s}}$$

Both balls move in the direction of the tennis ball’s initial motion.

Example: Inelastic Collision

The ballistic pendulum is a device used to measure the speed of a projectile, such as a bullet. The projectile, of mass m, is fired into a large block of wood of mass M, which is suspended like a pendulum. As a result of this collision, the pendulum and projectile together swing up to a maximum height h. Determine the relationship between the initial horizontal speed of the projectile, v, and the maximum height h.

(Giancoli, Example 7-10)



Answer to Ballistic Pendulum example

Momentum is conserved, therefore:

Total momentum before collision = Total momentum after collision

$mv = (m + M)\, v'$ ------ (1)

Kinetic energy is not conserved in the initial collision. So we cannot use the derived formula Eq 7-7.

However, after the bullet and block becomes "one" object, we can use conservation of energy from kinetic to potential energy.

Answer to Ballistic Pendulum example …cont.

Therefore we can write:

(KE + PE) just after collision =

(KE + PE) at pendulum's maximum height

$\frac{1}{2}(m + M)\, v'^2 = (m + M)gh$ -------- (2)

Solving using both equations (1) and (2) we get the following:

$$v = \frac{m+M}{m}v' = \frac{m+M}{m}\sqrt{2gh}$$

Example: Explosion, an inelastic "collision"

An internal explosion breaks an object, initially at rest, into two pieces, one of which has 1.5 times the mass of the other. If 7500 J were released in the explosion, how much kinetic energy did each piece acquire? (Giancoli, Qn 34)

Note: You can think of an explosion as the opposite of an inelastic collision.

Answer to Explosion example

Use conservation of momentum in one dimension, since the particles will separate and travel in opposite directions. Call the direction of the heavier particle's motion the positive direction. Let A represent the heavier particle, and B represent the lighter particle. We have $m_{\text{A}} = 1.5 m_{\text{B}}$, and $v_{\text{A}} = v_{\text{B}} = 0$.

$$p_{\text{initial}} = p_{\text{final}} \rightarrow 0 = m_{\text{A}} v'_{\text{A}} + m_{\text{B}} v'_{\text{B}} \rightarrow v'_{\text{A}} = -\frac{m_{\text{B}} v'_{\text{B}}}{m_{\text{A}}} = -\tfrac{2}{3} v'_{\text{B}}$$

The negative sign indicates direction.

Since there was no mechanical energy before the explosion, the kinetic energy of the particles after the explosion must equal the energy added.

$$E_{\text{added}} = KE'_A + KE'_B = \tfrac{1}{2} m_A v'^2_A + \tfrac{1}{2} m_B v'^2_B = \tfrac{1}{2}(1.5 m_B)\left(\tfrac{2}{3} v'_B\right)^2 + \tfrac{1}{2} m_B v'^2_B = \tfrac{5}{3}\left(\tfrac{1}{2} m_B v'^2_B\right) = \tfrac{5}{3} KE'_B$$

$$KE'_B = \tfrac{3}{5} E_{\text{added}} = \tfrac{3}{5}(7500\ \text{J}) = 4500\ \text{J} \qquad KE'_A = E_{\text{added}} - KE'_B = 7500\ \text{J} - 4500\ \text{J} = 3000\ \text{J}$$

Thus $\boxed{KE'_A = 3.0 \times 10^3\ \text{J} \quad KE'_B = 4.5 \times 10^3\ \text{J}}$