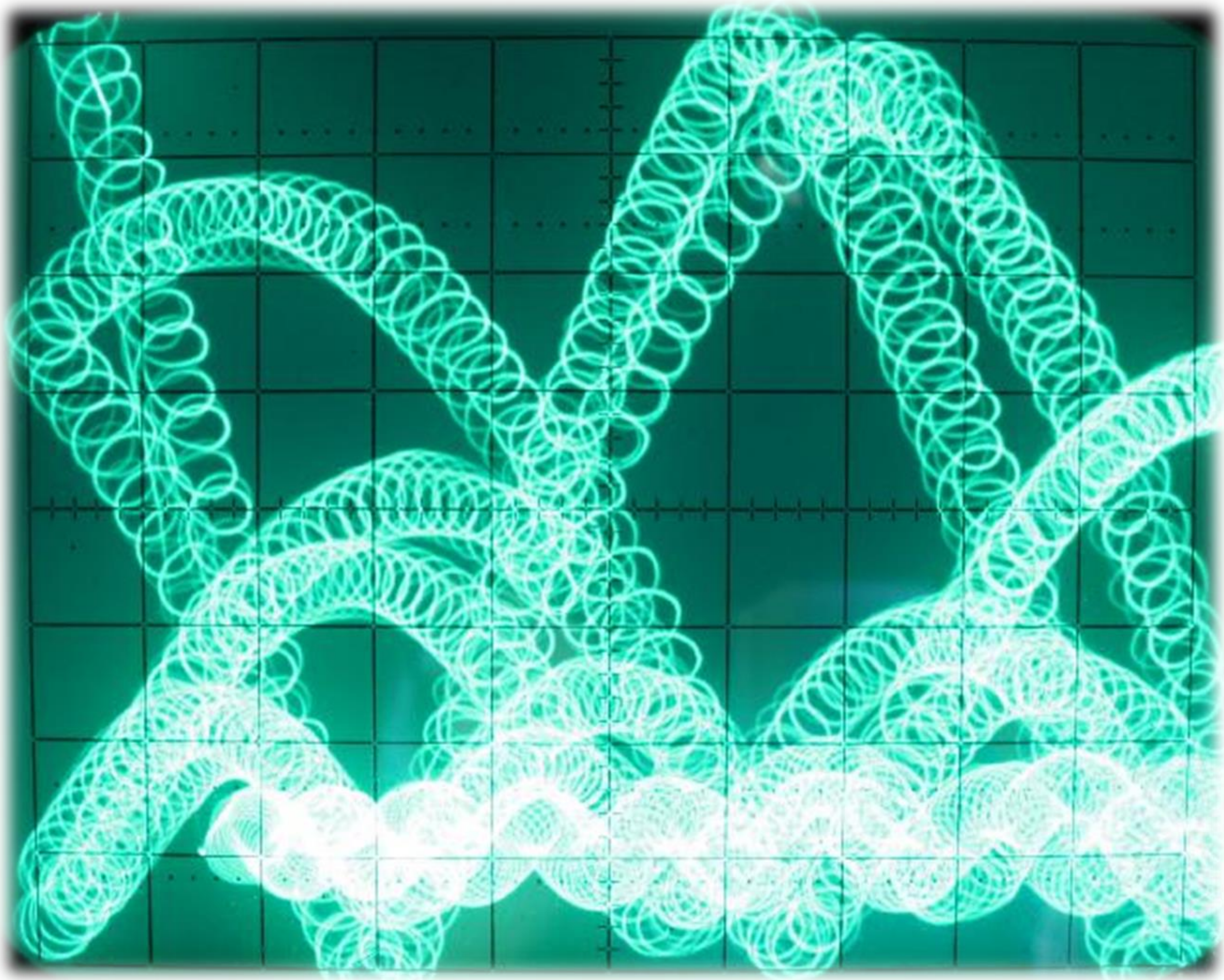
Topic 4:

Waves



4.1

- Oscillations

4.2

- Travelling waves

4.3

- Wave characteristics

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- Wave behaviour

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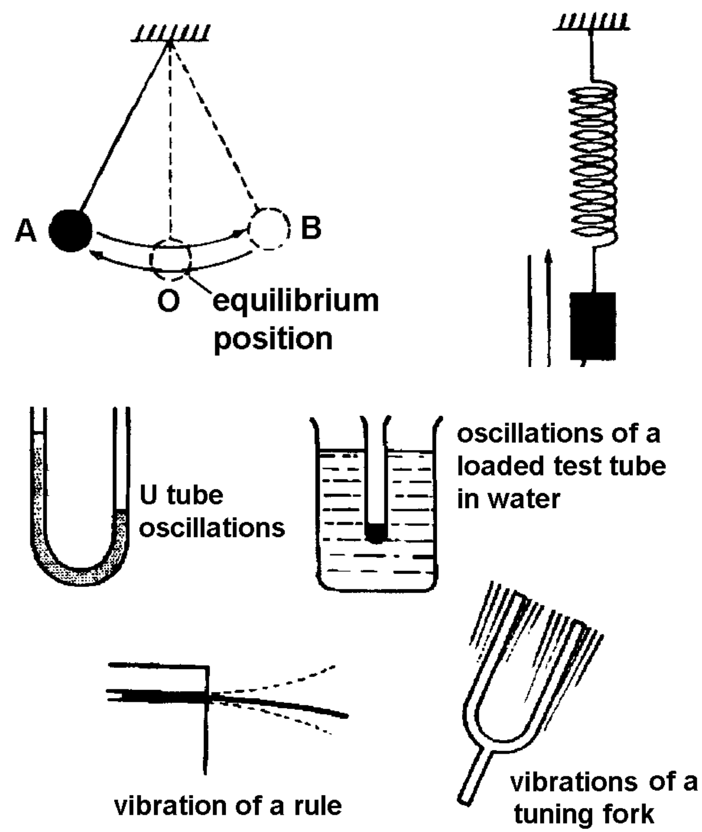
- Standing waves

4.1
Oscillations

4.1.1 Kinematics of simple harmonic motion (SHM)

Oscillation is a type of motion in which an object moves back and forth over the same path repeatedly in a regular manner. Such a motion is also known as a *periodic motion* or a *vibrational motion*.

Examples:
a swinging pendulum
a mass attached to a helical spring
a cantilever
a violin string
atoms or molecules in a solid lattice
air molecules as a sound wave passes by



SHM is the simplest type of oscillatory motion, where the object oscillates between two positions with no loss in energy. Such an oscillation is also known as a *free* oscillation.

Definitions

Displacement, *x*:

Displacement is distance the object has moved from its rest position in a stated direction. It is a vector.

Amplitude, x_o:

Amplitude is the maximum magnitude of displacement from the equilibrium position. It is a scalar.

Frequency, *f* :

The number of complete oscillations per unit time is called the frequency. SI unit is hertz (Hz). Note: 1 Hz = 1 cycle per second .

Period, *T* :

The period T is the time for one complete oscillation. Note: $T = 1/f$.

Definitions

Angular frequency, ω :

The angular frequency is defined as $2\pi f$. SI unit is radians per second (rad s^{-1}).

Hence, $\omega = 2\pi f = \dfrac{2\pi}{T}$.

As ω is a constant, T is a constant and is independent of the amplitude x_0 of the oscillation . This is an important characteristic of SHM.

Definitions

Simple harmonic motion may be defined as an oscillatory motion of a particle whose acceleration is directly proportional to its displacement from the equilibrium position and this acceleration is always directed towards that position.

$$a \propto -x$$

where a is the acceleration,
x is the displacement from equilibrium
and the negative sign implies that acceleration points in the opposite direction of the displacement vector.

Written in equation form, this is

$$\mathbf{a = -\omega^2 x}\ , \quad \textit{where } \omega^2 \textit{ is a constant.}$$

Graphs for SHM

(A) Variation of *x*, *v*, *a* with time *t*

Any SHM can be described in terms of a sinusoidal function. For instance, the value of the displacement *x* of an object can be given by this equation:

$$x = x_o \sin\omega t ,$$

which means the object is at the equilibrium position at $t = 0$,

then the velocity of the object will be

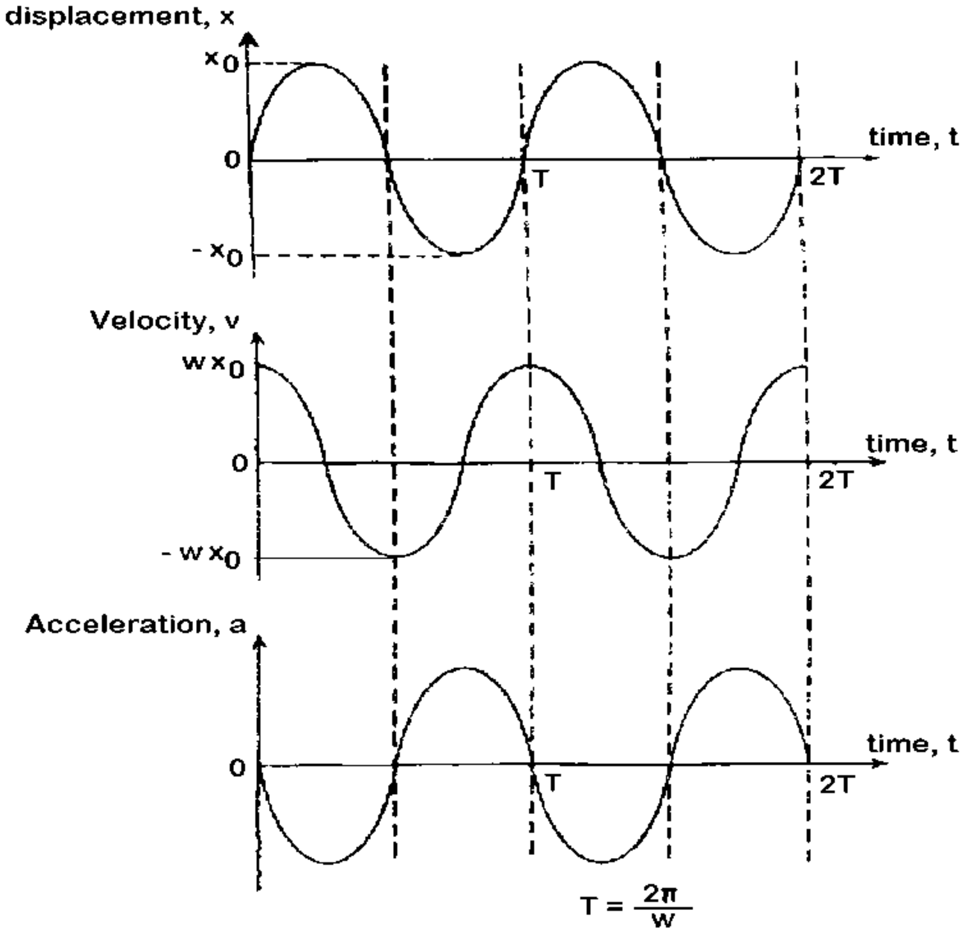
$$v = x_o \omega \cos\omega t$$

and the object's acceleration will be

$$a = -x_o \omega^2 \sin\omega t$$

i.e. $a = -\omega^2 x$

which agrees with what was shown in the previous section.



If $x = x_o \cos \omega t$,

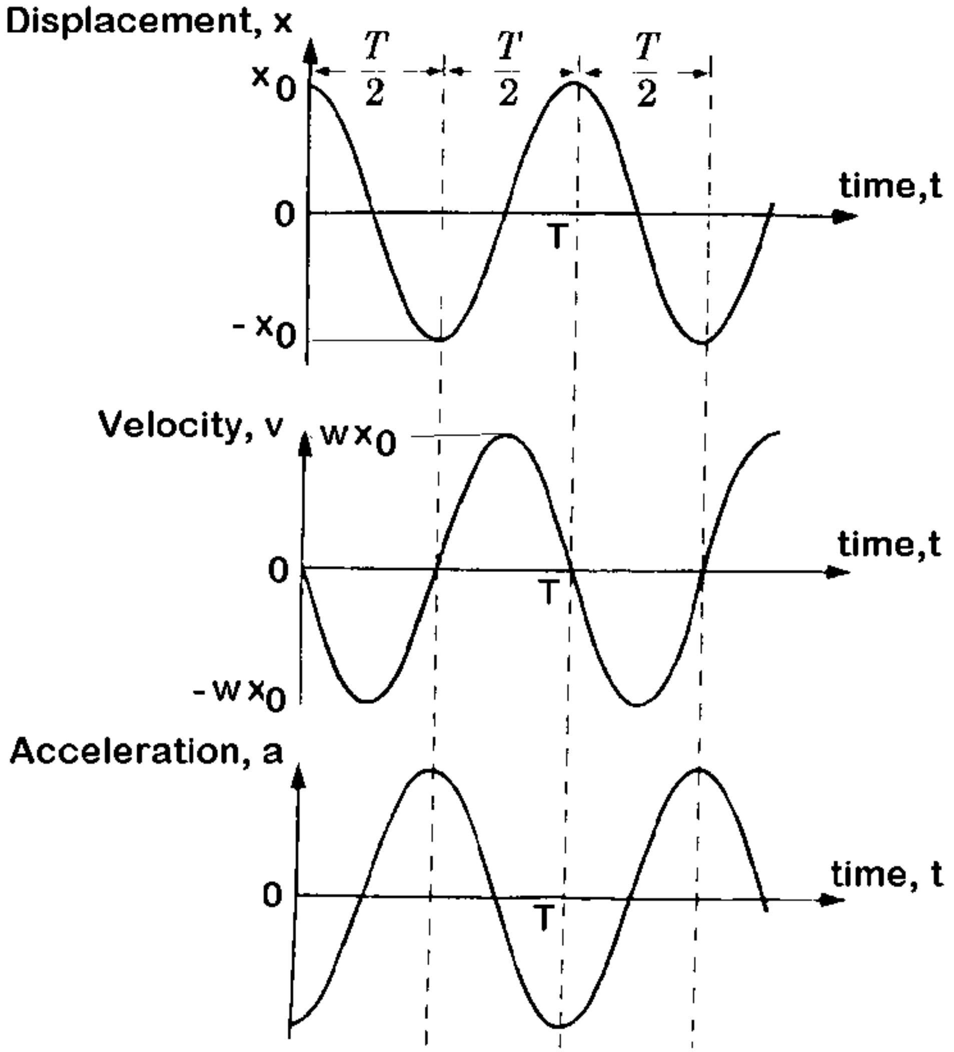
which is for the object starting at the amplitude when $t = 0$,

then

$$v = -x_o \omega \sin \omega t$$

and

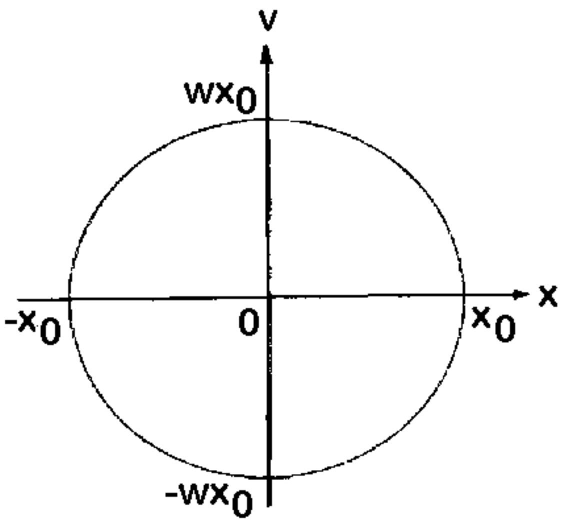
$$a = -x_o \omega^2 \cos \omega t = -\omega^2 x$$



(B) Variation of *v* and *a* with respect to *x*

It can be shown that $v = \pm\omega\sqrt{x_o^2 - x^2}$ and of course, $a = -\omega^2 x$

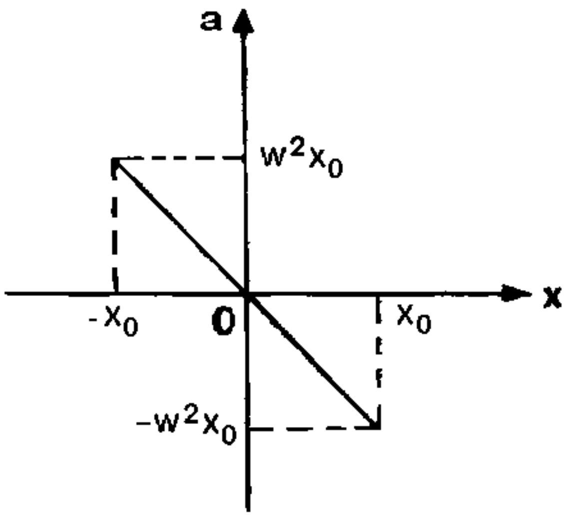
From the v - x plot, the velocity of the particle will be maximum only at x = 0, i.e., when it is passing through the equilibrium position.



Then $v_{max} = \pm\omega x_o$

Also, the velocity will be zero when the particle is at the amplitude, i.e., $x = x_o$.

From the *a* - *x* graph, the acceleration is maximum at $x = x_o$, and is given by

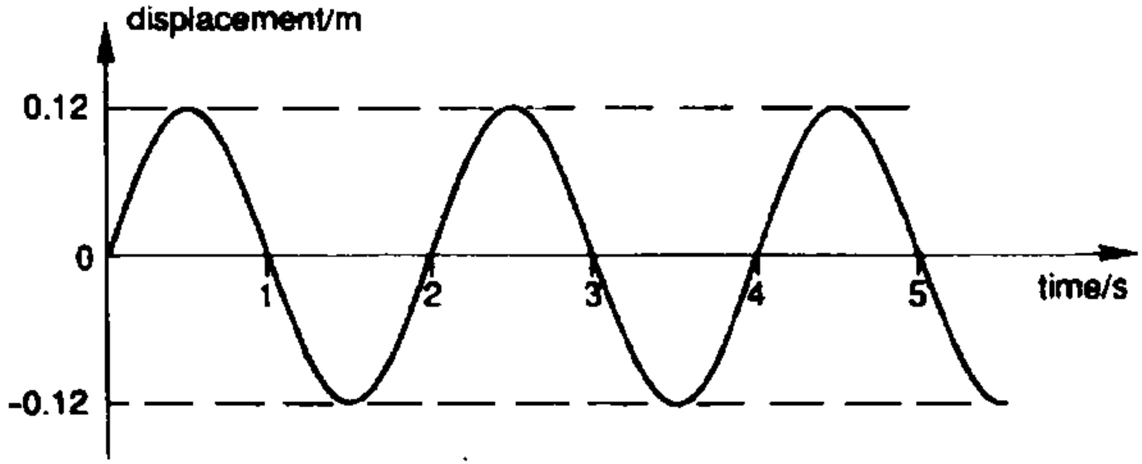


$$a_{max} = -\omega^2 x_o$$

Acceleration is zero when the object is passing through the equilibrium position.

Example 2

The pendulum bob in a particular clock oscillates so that its displacement from a fixed point is as shown:



By taking the necessary readings from the graph, determine for these oscillations,

(a) the amplitude; (b) the period;(c) the frequency; (d) the angular frequency; (e) the acceleration (i) when the displacement is zero; (ii) when the displacement is at its maximum; (f) the maximum velocity of the pendulum bob.

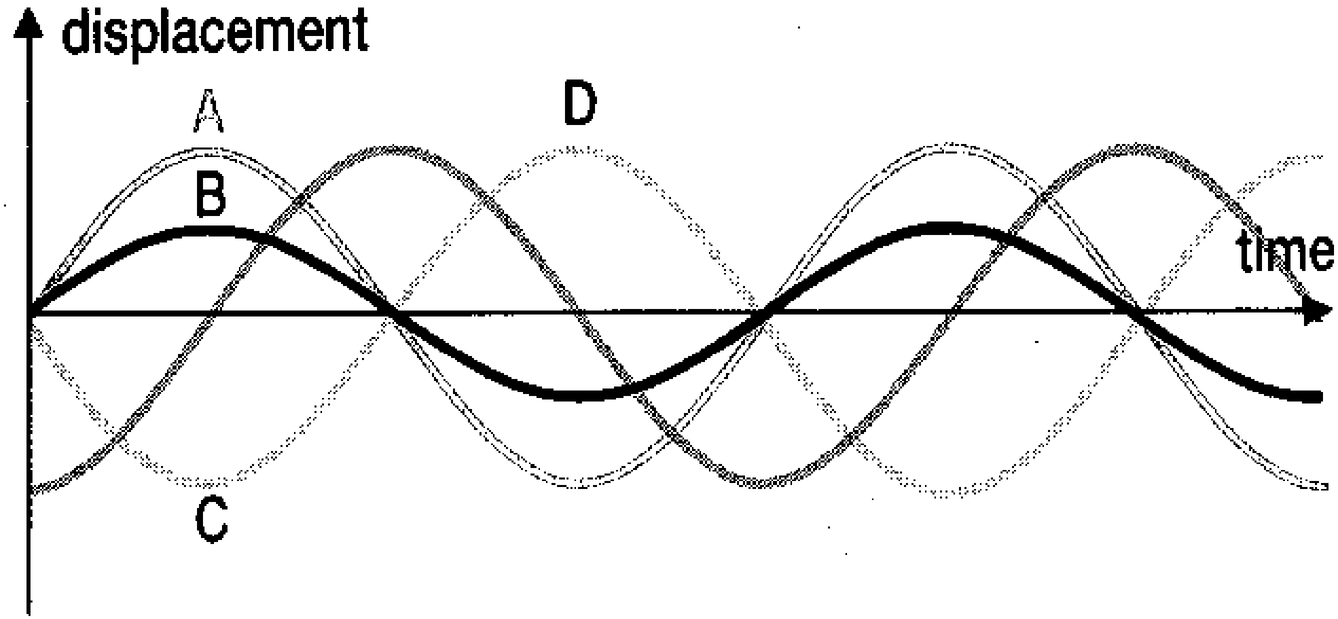
(C) Phase Difference

Consider an oscillation given by the equation $x = x_o \sin \omega t$

and a second oscillation represented by $x = x_o \sin(\omega t + \phi)$.

They are two different oscillations because at any time *t*, the second oscillation will differ by an angle ϕ . This angle represents the **phase difference** between the two oscillations. Thus, the **phase difference** ϕ is the difference in the phase angle between the two oscillations which have the same frequency.

Example 4



Phase difference between A and B = **0°** , they are **in phase.**

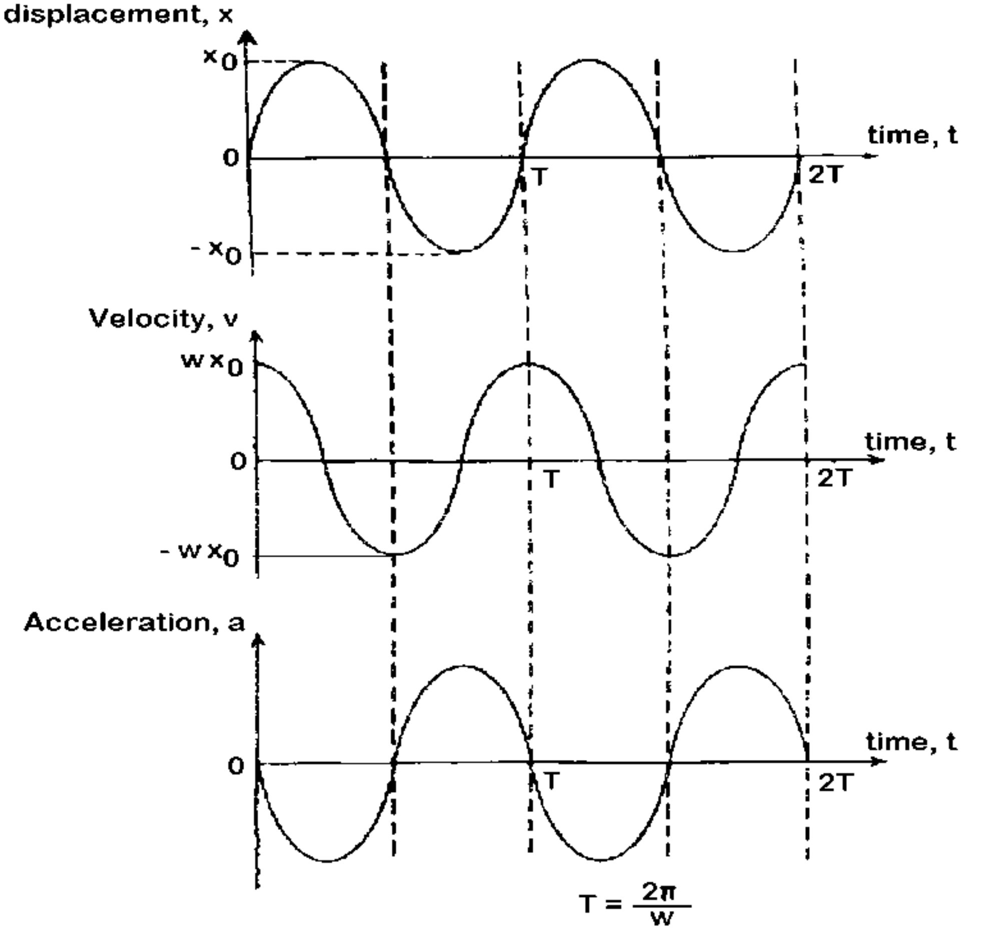
Phase difference between A and C = **180°** , they are **out of phase.**

Phase difference between A and D = **90°**, they are **out of phase.**

Phase difference between B and D = **90°** , they are **out of phase.**

Example 5

What are the phase differences between the displacement-time plot, velocity-time plot and acceleration-time plot for a simple harmonic motion?



ANS:

Phase difference between x-t and v-t is = **<u>90˚</u>.**

Phase difference between x-t and a-t is = **<u>180˚</u>.**

4.1.2 Energy changes during simple harmonic motion (SHM)

In the absence of external and dissipative forces, there will be no energy loss to a system in SHM. Hence, the total energy of the oscillating system is a constant and there is a constant interchange of kinetic and potential energies.

(A) Kinetic Energy

The K.E. of a particle in SHM is given by $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_o^2 - x^2)$

As mentioned, velocity is *maximum at the equilibrium position*, $x = 0$.
$\therefore$ maximum K.E. also occurs at $x = 0$ and is given by $\frac{1}{2} m\omega^2 x_o^2$
Obviously, *minimum K.E. is zero at the amplitudes*, $x = x_o$

(B) Potential Energy

The P.E. of an oscillating mass is given by $\frac{1}{2}m\omega^2x^2$

Naturally, *P.E. = 0 at x = 0*
and *P.E. is maximum at the amplitudes* $x = x_o$, and is given by $\frac{1}{2}m\omega^2x_o^2$

(C) Total Energy

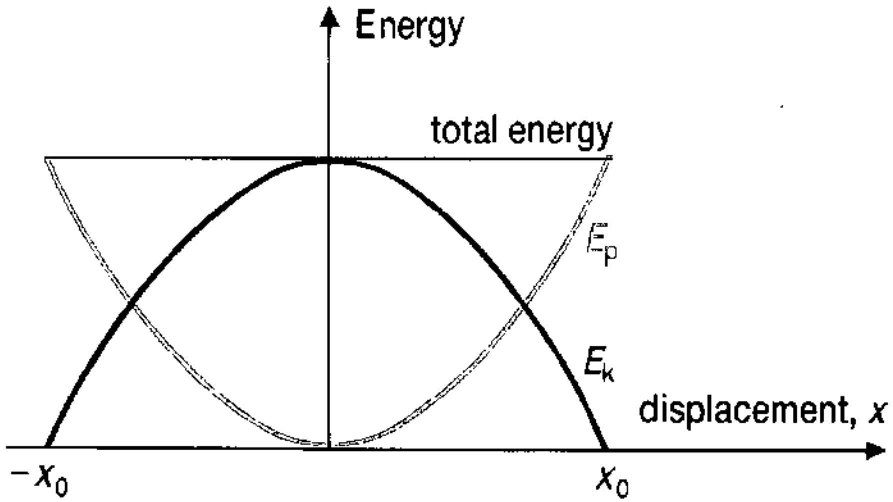
Total energy $= P.E. + K.E.$

$= \frac{1}{2} m\omega^2(x_o^2 - x^2) + \frac{1}{2} m\omega^2 x^2$

$= \frac{1}{2} m\omega^2 x_o^2$

This total energy is a constant and does not depend on x.

The graph on the right shows the variation of K.E., P.E. and total energy with displacement. Both are parabolic curves and the sum of the energies at any point is a constant value.

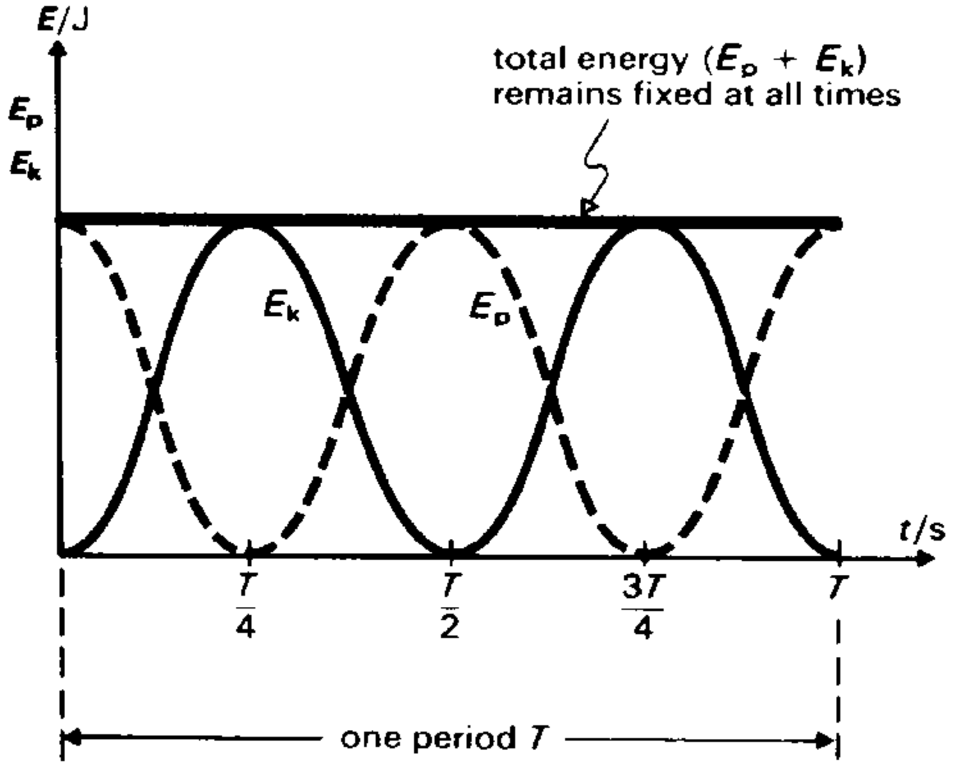


In a simple pendulum, all the energy is kinetic as the bob swings through the equilibrium position and at the top of the swing it is purely potential.

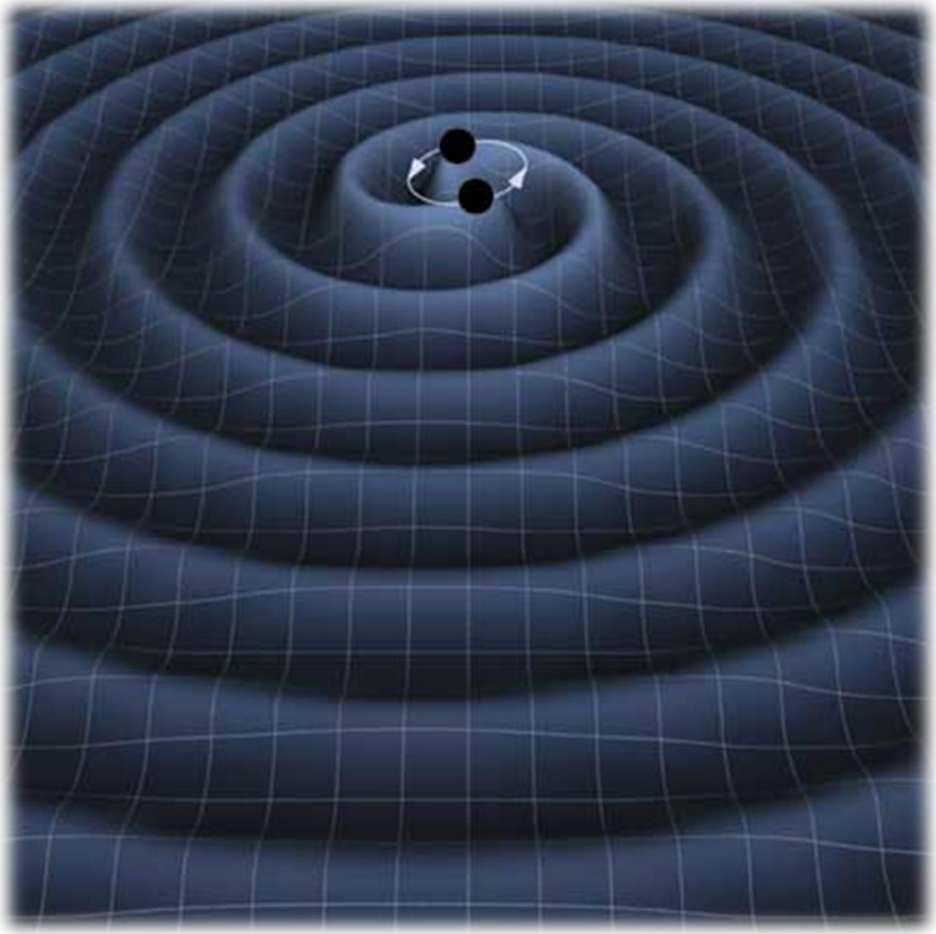
(D) Energy variation with time *t*

It can be shown that for an SHM of equation $x = x_o \cos \omega t$

But total energy is still a constant, given by $\frac{1}{2} m\omega^2 x_o^2$



4.2 Travelling Waves



4.2.1 Travelling waves

All waves are caused by disturbances, which results in some sort of oscillation. These oscillations then spread out as waves.

When a wave travels through a medium, the particles of the medium are set into oscillations about their equilibrium positions. These oscillations make up the disturbance which is transmitted to other particles in the medium, but the *particles do not move along with the disturbance*. The wave propagates as the disturbance is moved through the medium and together with this disturbance, energy is also transported.

An isolated disturbance, traveling through an otherwise undisturbed medium is a *wave pulse*. A pulse occurs when a medium is disturbed only briefly.

A *continuous wave* is produced when a medium is disturbed in a regular, periodic way. A continuous wave repeats itself in space and time.

4.2.2 Wavelength, frequency, period and waves speed

Displacement (*y*):

Displacement is distance of an oscillating particle from its equilibrium position in a specified direction

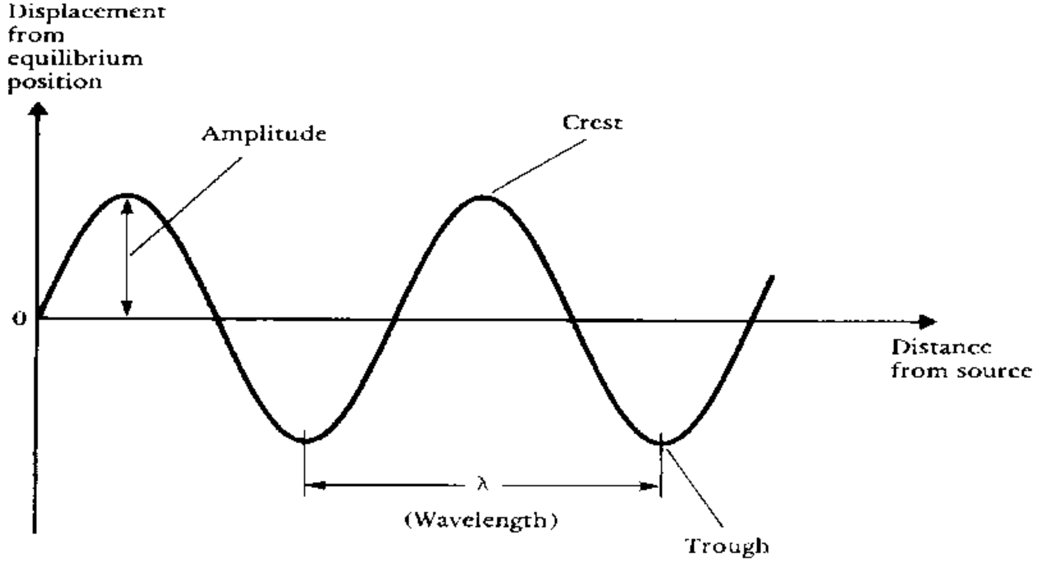
Amplitude (*A*):

Amplitude is the maximum displacement of an oscillating particle from its equilibrium position.

Wavelength (λ):

Wavelength is the distance between any two *successive* points which are in phase.

(It is thus the distance between two successive crests or troughs)



Period (T):

Period is the time taken for a particle to undergo one complete cycle of oscillation.

(It is also the time for the wave to travel through one wavelength)

Frequency (f):

Frequency is the number of complete cycles performed by a particle per unit time .

(It is also the number of wavelengths that pass a given point per unit time)

Unit: Hertz (Hz)

$$f = 1/T$$

Wave speed (v):

Wave speed is the distance the wave profile moves per unit time.

(*Do not confuse this speed with the speed of the oscillating particles within the wave!*)

Derivation (to memorize)

By definition, speed = distance / time

For one cycle, the time taken is one period (T) and the distance covered is one wavelength (λ), thus speed = λ / T .

Since frequency $f = 1/T$, $\therefore$ speed v :

$$v = f\lambda$$

4.2.3 Transverse and longintudinal waves

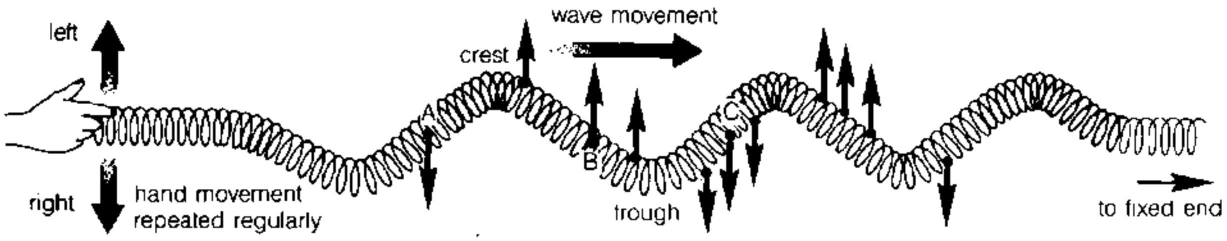
A *progressive* or *travelling* wave is the movement of a disturbance from a source which transfers energy but not material to places around it. Mechanical waves (require a medium for propagation) and electromagnetic waves (do not require a medium) are progressive waves by nature.

Progressive waves can be further classified into *transverse* and *longitudinal* waves.

(a) Transverse Waves

Transverse waves are waves in which the displacement of the particles of the medium is **perpendicular** to the direction of the wave motion.

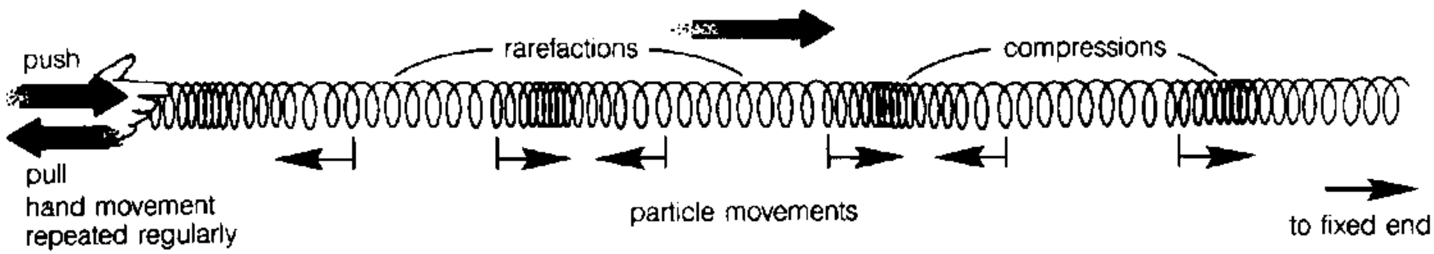
All electromagnetic waves are transverse.



(b) Longitudinal Waves

Longitudinal waves are waves in which the displacement of the particles of the medium is **parallel** to the direction of the propagation of the wave.

All sound waves are longitudinal.



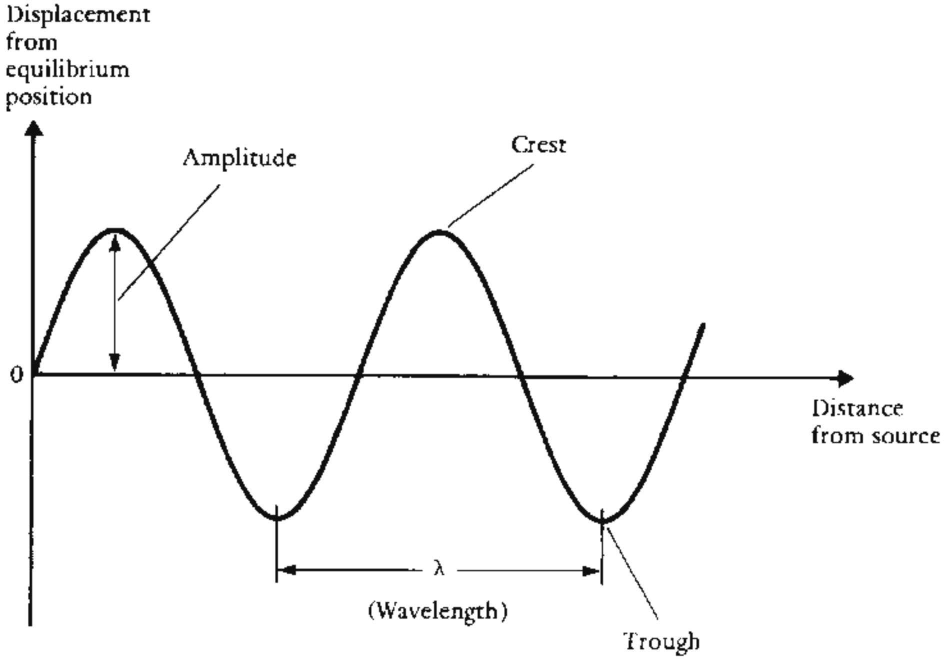
If a spring is repeatedly given a push and a pull, a longitudinal wave is created. The individual parts of the spring move back and forth about their equilibrium positions, causing a series of **compressions** and **rarefactions**. Compressions occur where the loops of the spring are closer together than at equilibrium while rarefactions appear where the loops are farther apart. All movements are parallel to the direction of the wave motion as seen in the diagram:

Graphical Representations of Waves

(a) Displacement-distance graph

This shows how the displacements of particles vary with the distance from the source *at a particular moment in time*.

This graph can be used to determine the **wavelength** (distance between successive crests or successive troughs).



Graphical Representations of Waves

Longitudinal Wave

In the diagram (a), the particles of the medium are shown evenly-spaced in the undisturbed positions.

In diagram (b) The passage of the wave displaces the particles to their new positions.

In diagram (c) The displacement of the particles are represented by the same displacement-time graph as we used for the transverse wave. However, displacements to the right are given positive values and displacements to the left are given negative values.

Diagram (a) :

Diagram (b) :

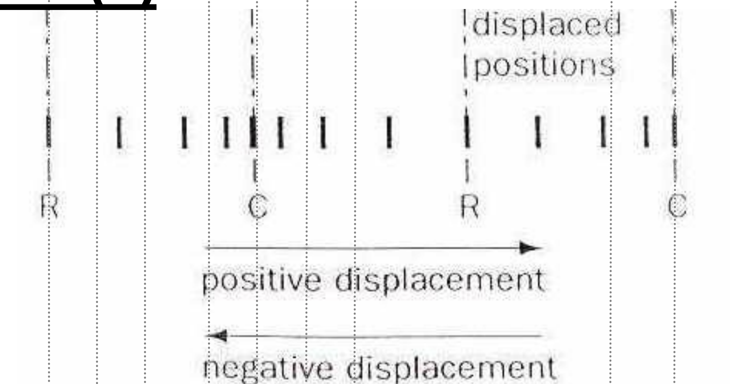
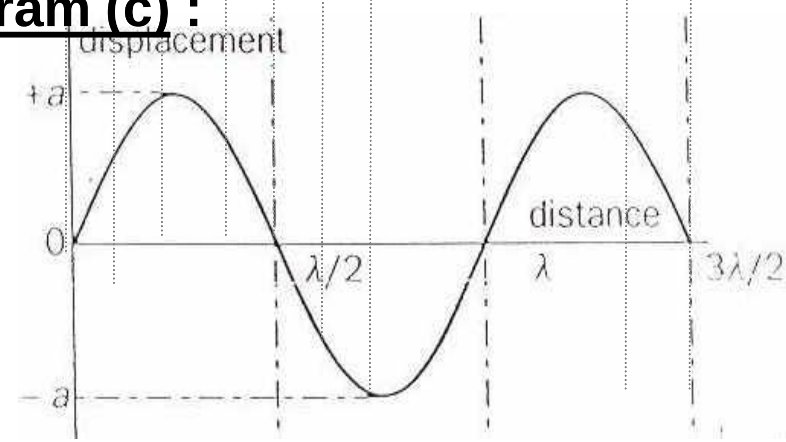


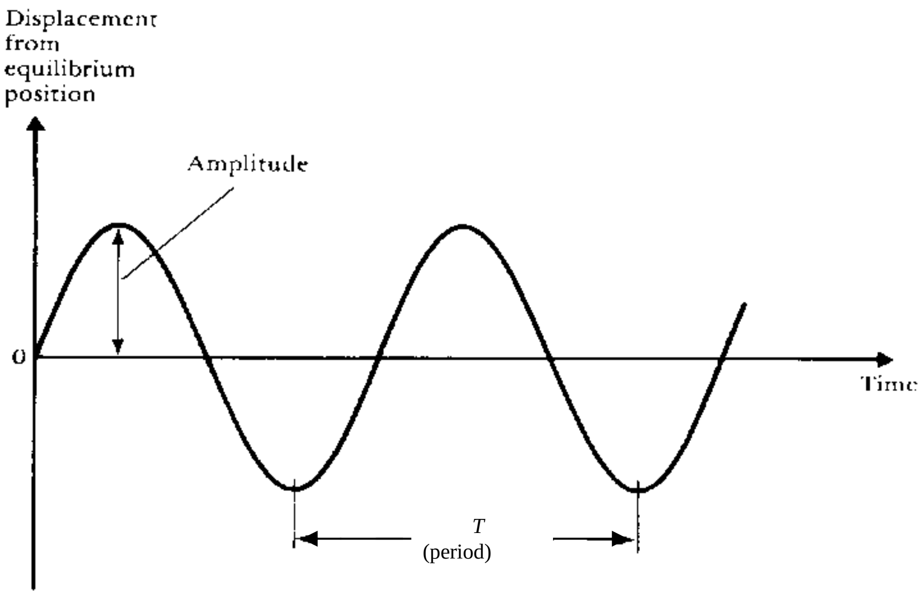
Diagram (c) :



Graphical Representations of Waves

(b) Displacement-time graph

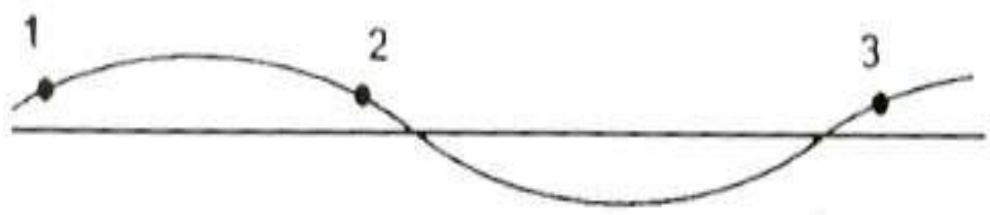
This shows how the displacement of a *single particle* in the wave varies with time.



This graph can be used to determine the **period** (distance between successive crests or successive troughs).

Example 7

The diagram below shows an instantaneous position of a string as a transverse progressive wave travels along it from left to right. Which one of the following correctly shows the directions of the velocities of the points 1, 2 and 3 on the string?

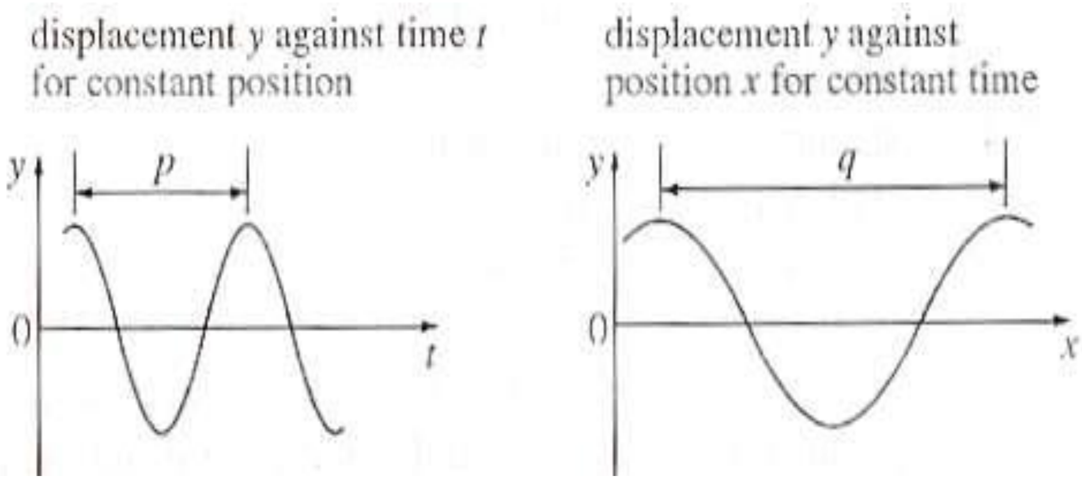


	Point 1	Point 2	Point 3
A	→	→	→
B	→	←	→
C	↓	↓	↓
D	↓	↑	↓
E	↑	↓	↑

Ans : D

Example 8

The same progressive wave is represented by the following graphs.



Which of the following gives the speed of the wave?

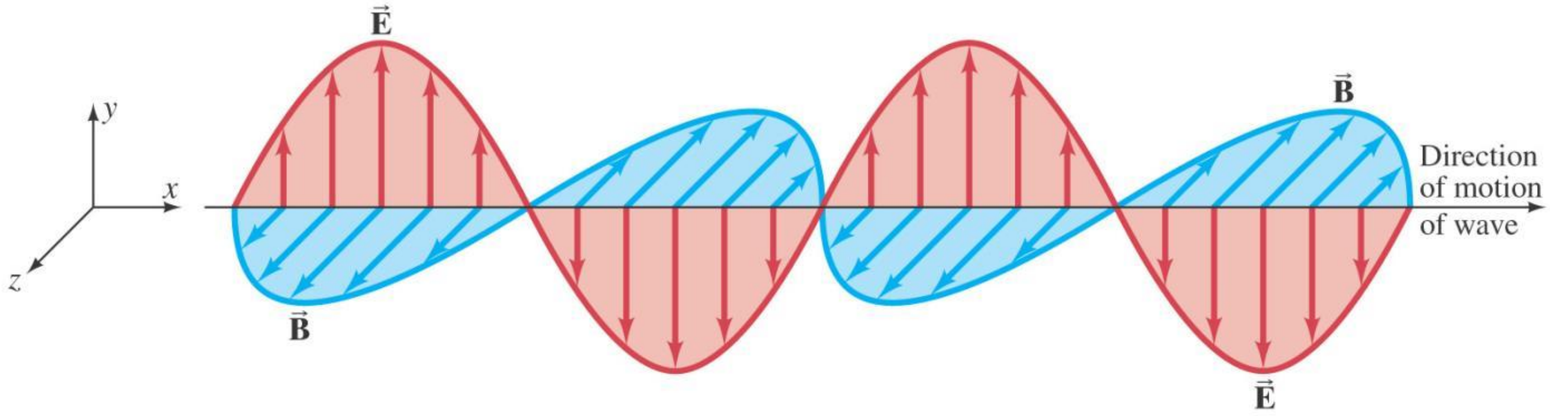
A. *pq* B. p/q C. q/p D. 1/pq

Ans : C

4.2.4 The nature of electromagnetic waves

An **oscillating electric charge** produces sinusoidally varying **electric & magnetic fields** which are perpendicular to each other, & to the direction of propagation.

These propagating fields are called electromagnetic waves.



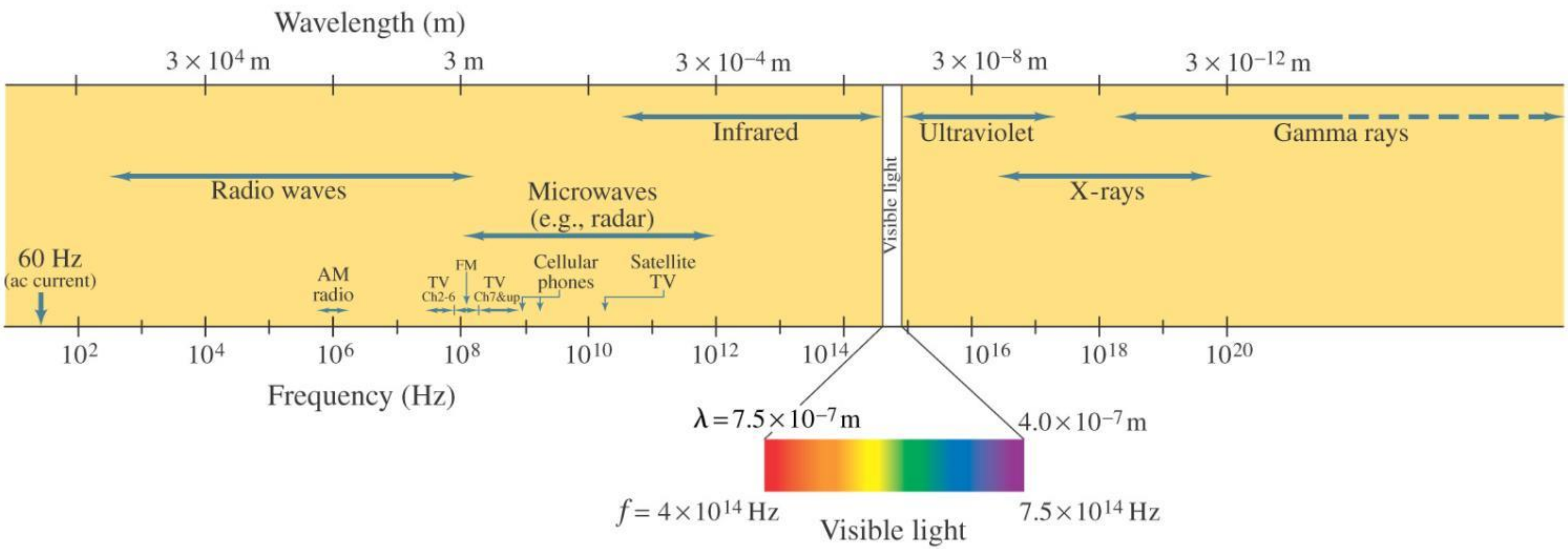
Electromagnetic waves
- are **transverse** waves
- can **travel in a vacuum**

The frequency of an electromagnetic wave is related to its wavelength:

$$v = f\lambda$$

Electromagnetic Spectrum

7 frequency regions of the spectrum are given below:



Electromagnetic Spectrum

Region	Frequency/Hz	Source/s
radio waves	10^{2} - 10^{8}	oscillating electrons - oscillators
micro waves	10^{8} - 10^{12}	oscillating electrons - vacuum tubes (klystrons)
infra red radiation	10^{10} - 10^{14}	oscillating molecules - hot objects
visible light	(4-7.5) x 10^{14}	movement of outermost electrons - fluorescent tubes
ultra violet radiation	10^{15} - 10^{17}	outermost electrons changing energy levels - sun, sun beds
X rays	10^{16} - 10^{20}	very fast moving electrons bombarding heavy metals
gamma rays	> 10^{18}	radioactive decay

Electromagnetic waves

EM waves exhibit the following properties:

1. They consist of two sinusoidal fields – the *E*-field and *B*-field, which are oscillating in phase and at right angles to each other.
2. They are transverse waves.
3. All electromagnetic waves can travel through vacuum(or free space).
4. In vacuum, they travel with the same speed $c = 3.00 \times 10^{8}$ ms^{-1}.
5. All em waves undergo *reflection, refraction, interference, diffraction* and *polarization*.

The Electromagnetic Spectrum:

Radiation	Approximate Range of λ
Gamma Rays	**10^{-14}m - 10^{-11} m**
X-rays	**10^{-11} m - 10^{-9} m**
Ultra-violet	**1 nm - 0.4 μm**
Visible Light	**400 nm - 700 nm**
Infra-red	**0.7 μm - 1 mm**
Microwaves	**1 mm - 0.1 m**
Radio waves	**0.1 m - 10 km**

4.2.4 The nature of sound waves

What is sound?

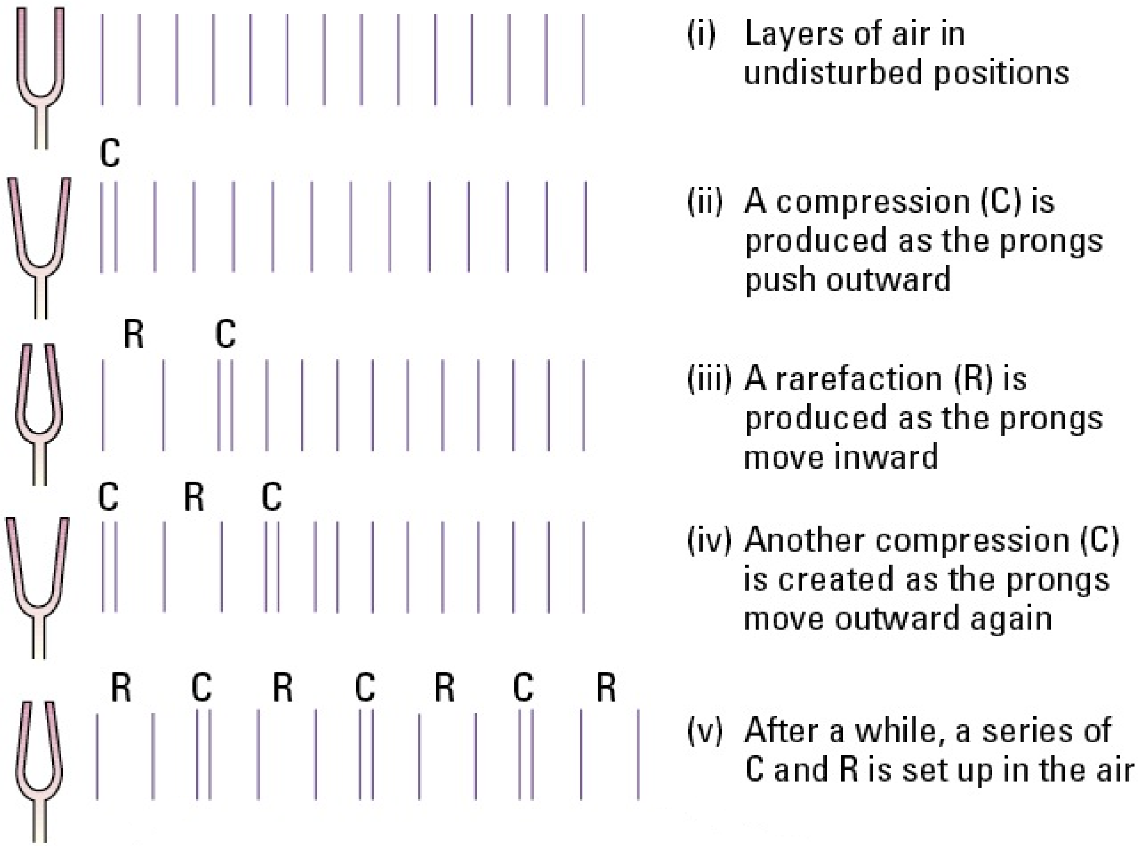
- Sound is a form of energy
- It is passed from one point to another as a wave.
- It is an example of a longitudinal wave and comprises of a series of compressions and rarefactions in the medium.

How is sound produced?

- Sound is produced by vibrating sources placed in a medium.
- A vibrating object in air causes shifting of layers of air particles.
- Longitudinal sound waves are produced.

How does sound travel?

- The direction of vibration of air molecules is parallel to the direction of wave motion.



As a tuning fork vibrates, it shifts layers of air inward and outward, creating a series of compression (C) and rarefactions (R).

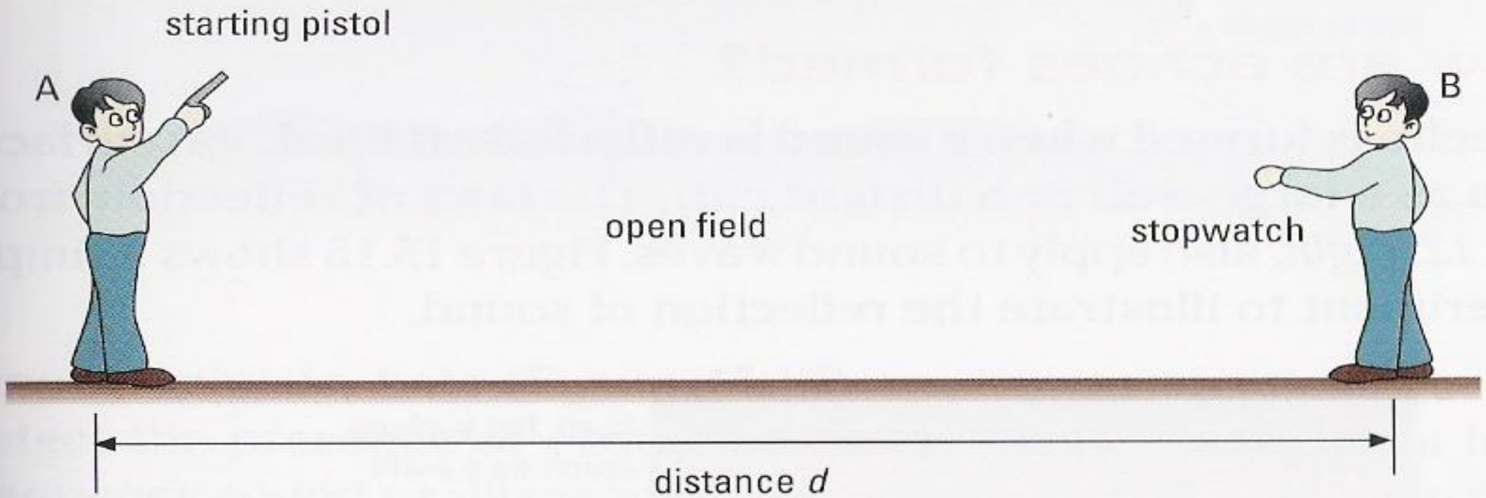
Measuring the speed of sound in air

Objective

To measure the speed of sound in air by a direct method

Apparatus

starting pistol, stopwatch, measuring tape



The direct method of measuring the speed of sound

Procedure

1. Using a measuring tape, observers A and B are positioned at a known distance d apart in an open field.
2. Observer A fires a starting pistol.
3. Observer B, on seeing the flash of the starting pistol, starts the stopwatch and then stops it when he hears the sound. The time interval t is then recorded.

Results

A typical set of data for d and t is d = 800 m and t = 2.4 s. The speed of sound v in air is then given by:

$$v = \frac{\text{distance travelled by sound}}{\text{time taken}} = \frac{d}{t} = \frac{800}{2.4} = 333 \text{ m s}^{-1}$$

The result of the speed of sound in air v can be improved in two ways:

1. Repeat the experiment a few times and compute the values of the speed of sound for each experiment. Find the average value. Taking the average will minimise the random errors that may occur while finding the time interval between seeing the flash and hearing the sound.
2. Observers A and B should exchange positions and repeat the experiment. This will cancel the effect of wind on the speed of sound in air.

4.3
Wave Characteristics

4.3.1 Wavefront and rays

Wavefront:

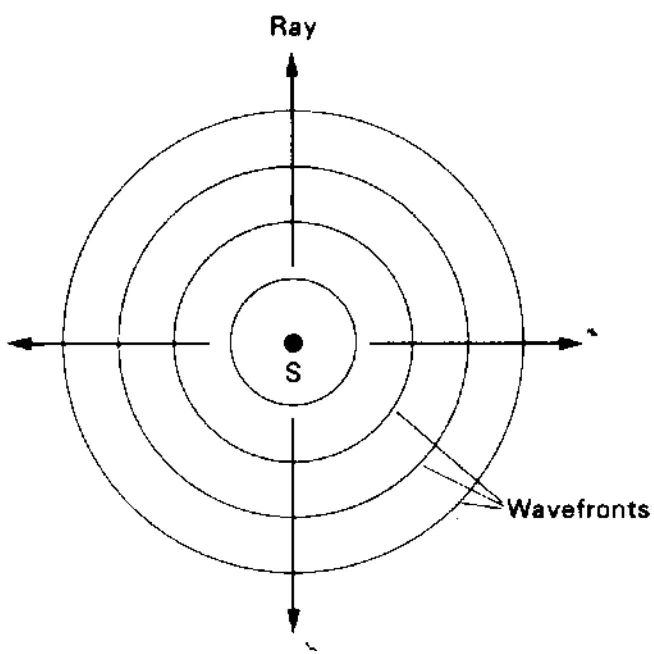
Wavefront is a line or surface joining points of a wave that are in phase, e.g. a line joining crest to crest in a wave.

The distance between 2 successive wavefronts is one wavelength.

Ray :

Ray is the path taken by the wave and is used to indicate the direction of wave propagation.

Rays are always at right angles to the wavefronts



4.3.2 Amplitude and intensity

Intensity (I):

Intensity of a wave is defined as **the rate of energy flow per unit cross-sectional area perpendicular to the direction of wave propagation.**

Unit : W m^{-2}.

$$I = \frac{P}{S} = \frac{E}{tS}$$

where P is the incident power
and S is the perpendicular cross-sectional area

It can be shown that the energy transported by a wave is proportional to the *square* of its amplitude,

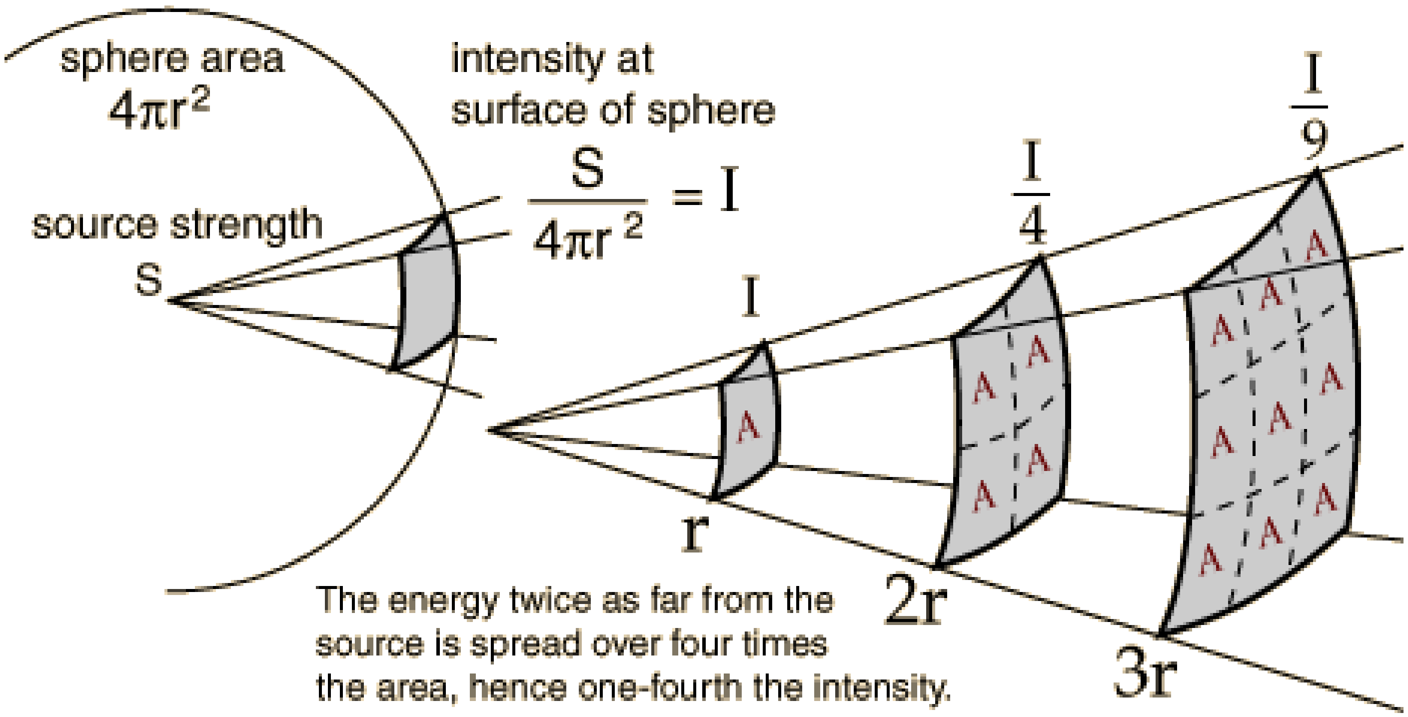
i.e. $E \propto A^2$

From the above energy relation, the intensity of a wave is also proportional to the *square* of its amplitude.

$$\boldsymbol{I \propto A^2}$$

Inverse square law

- As the distance of an observer from a point source (light/sound) increases, the power received by the observer will decreases as energy spreads out over a larger area.



Inverse square law

- The surface area *A* of a sphere of radius *r* is calculated using $A = 4\pi r^2$
- If the point source radiates a total power *P* in all directions, then the power received per unit area (the intensity, I) at a distance *x* away is

$$I = \frac{P}{4\pi x^2}$$

For a constant P, $I \propto x^{-2}$

- The inverse square law can be used to calculate the intensity of the Sun’s radiation incident on a planet.

Example 9

A sound wave of amplitude 0.20 mm has an intensity of 3.0 W m^{-2}. What will be the intensity of a sound wave of the same frequency which has an amplitude of 0.40 mm?

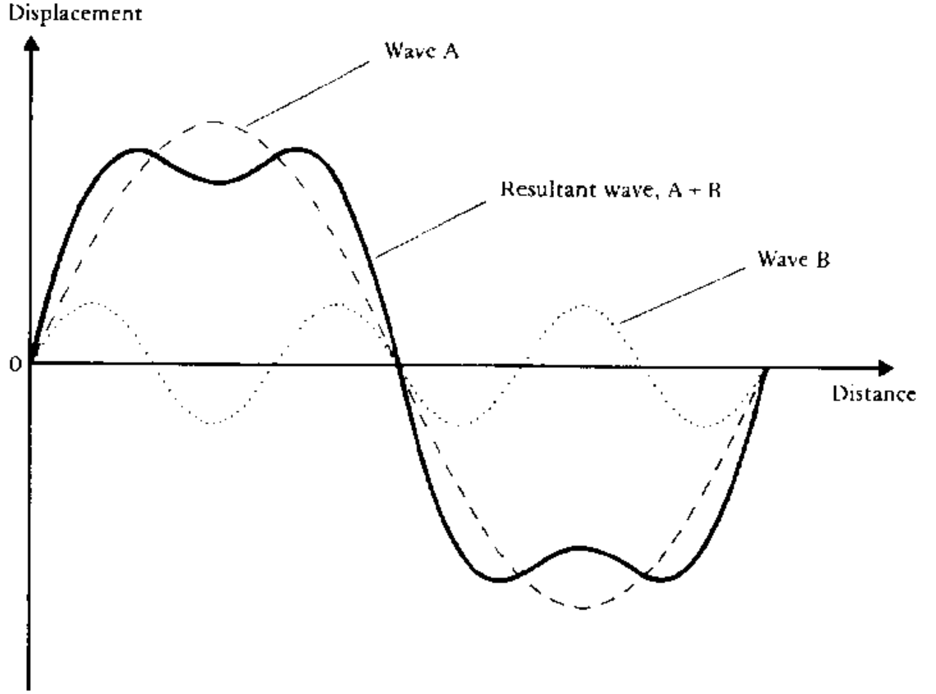
Since $I \alpha A^2$

$$\frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} \Rightarrow \frac{3.0}{I_2} = \frac{0.20^2}{0.40^2} \Rightarrow I_2 = 12\ \text{Wm}^{-2}$$

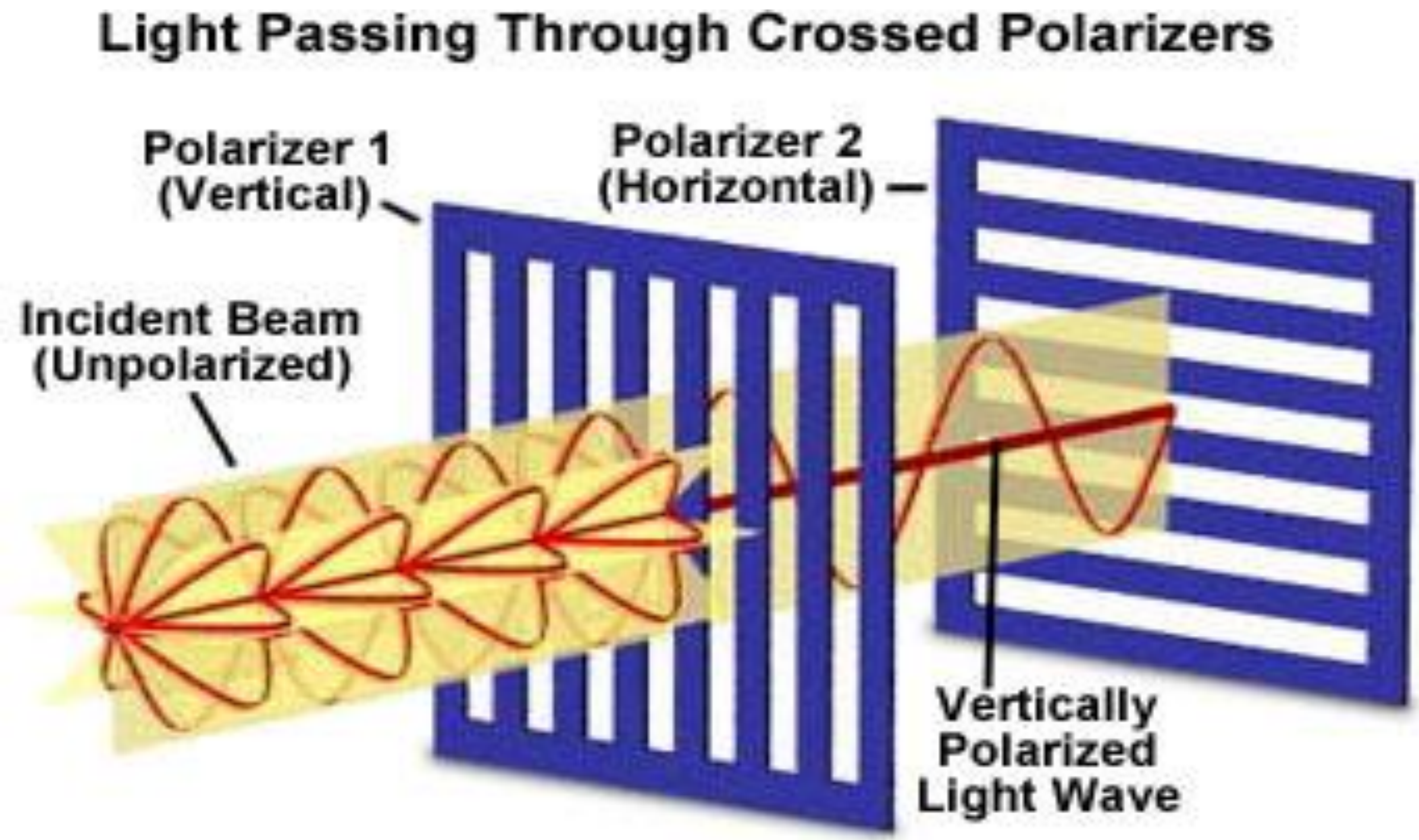
4.3.3 Principle of Superposition

When two or more waves of the same kind exist simultaneously at a point in a medium, the resultant displacement of waves at a given point in time and space is the vector sum of the displacement due to each wave acting independently.

In the following diagram, wave A and wave B are superposed together to give the resultant wave. Notice how the individual displacements of A and B are added to give the resultant displacement at various points.

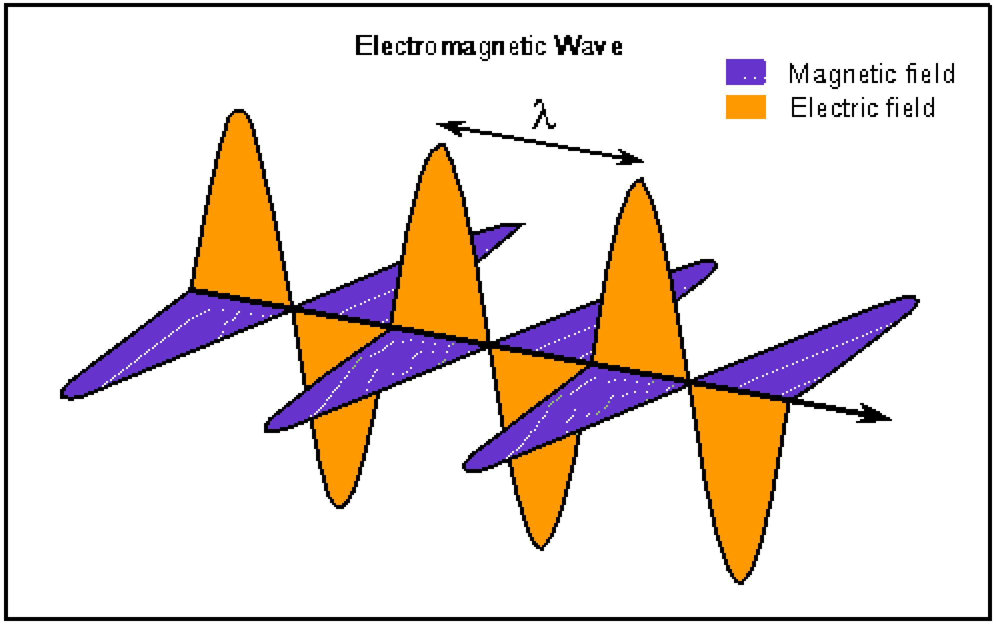


4.3.4 Polarization



Electromagnetic Waves

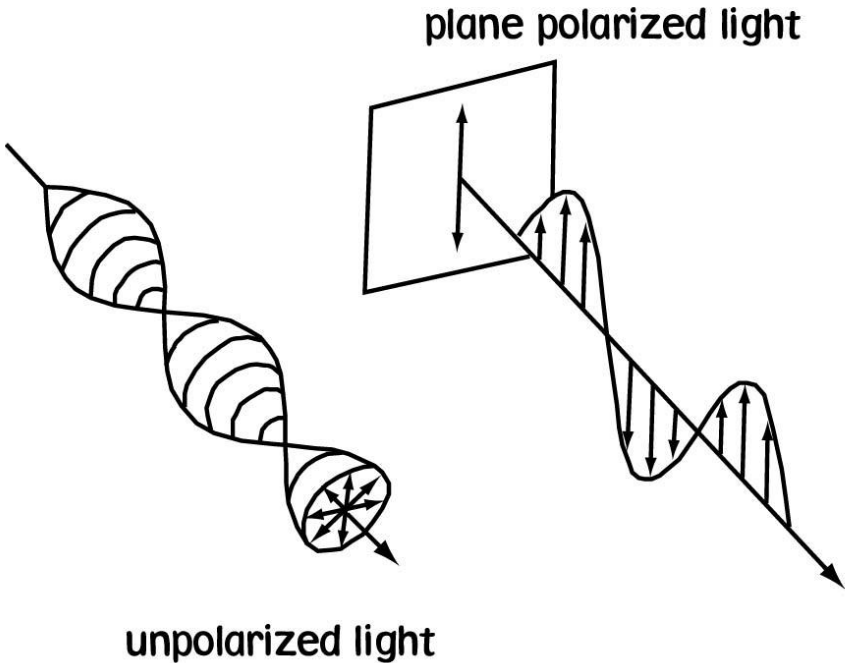
EM waves consist of oscillating electric and magnetic fields. The electric field vector is perpendicular to the magnetic field vector. The oscillations of the fields are confined to the plane of the wavefront. If we consider just the electric field vector, then the angle of vector within this plane can take any value between zero and 360˚. This angle is continually changing as the wave advances.



Light in which the plane of vibration of the electric vector is continually changing is said to be unploarized.

Polarization

Light is polarized when its electric fields oscillate in a single plane, rather than in any direction perpendicular to the direction of propagation.



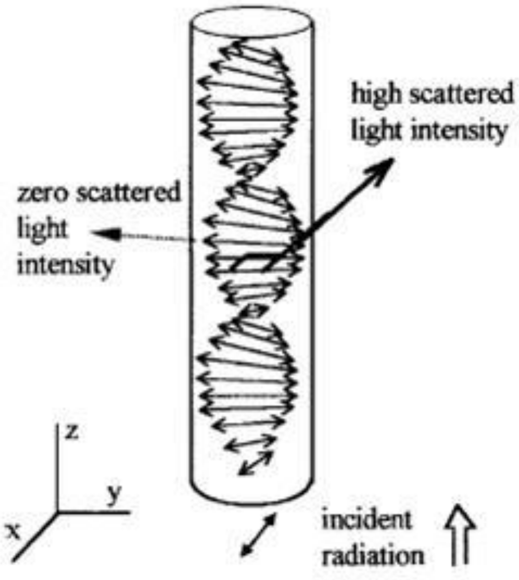
Polarized light will not be transmitted through a polarized film whose axis is perpendicular to the polarization direction.

The Barber Pole Demo

Molecular antennae, called dipoles, constituting an optically active liquid absorb and reradiate light. This process is the result of electric field vibrations acting on electrons within the molecules. Known as scattering, re-radiation occurs most strongly in the plane perpendicular to each dipole.

As polarized, monochromatic light passes through an optically active liquid, its plane of polarization rotates, and with it, the direction of scattering. The figure below shows how the plane of polarization "corkscrews" as it passes through the optically active liquid.

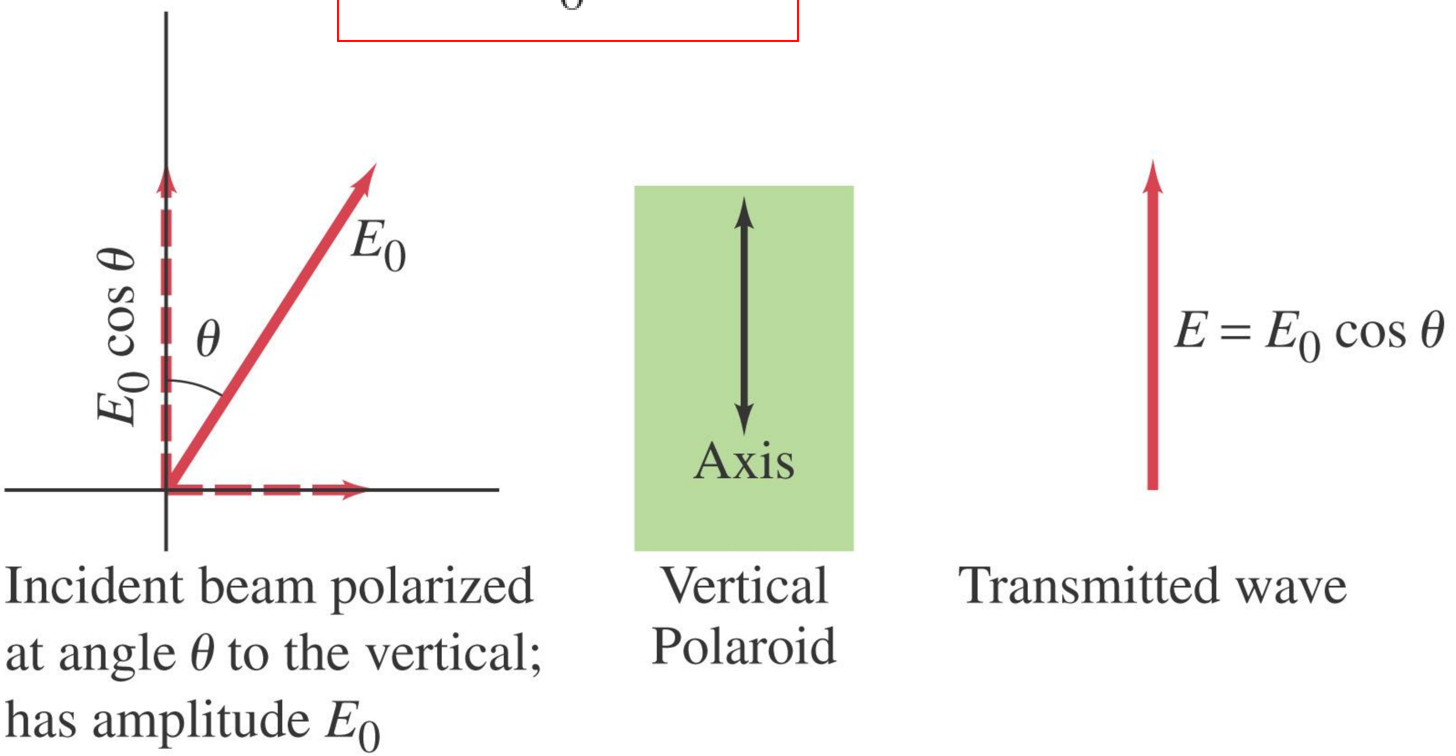
When polarized white light passes through an optically active liquid, the plane of polarization of each of its constituent colors changes by a different amount. Thus each color is scattered in a different direction, producing effect shown in the photo above.



Polarization – Malus's Law

When light passes through a polarizer, only the component parallel to the polarization axis is transmitted. If the incoming light is plane-polarized, the outgoing intensity is:

$$I = I_0 \cos^2 \theta$$



Polarization – Malus's Law

$$I = I_0 \cos^2 \theta$$

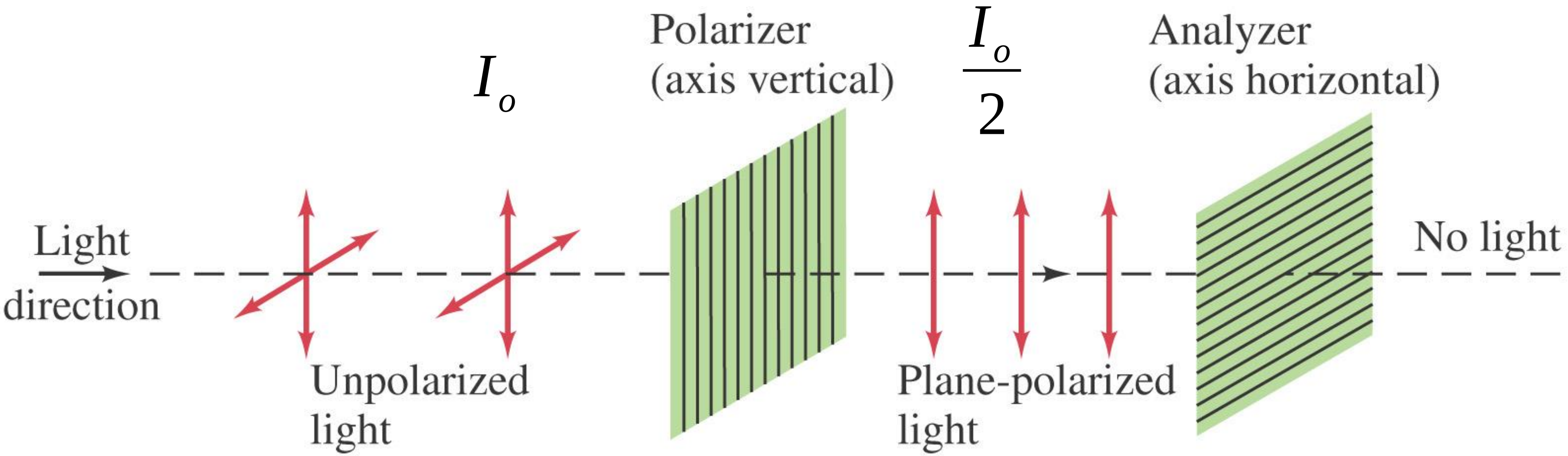
θ : angle between the polarization axis and the Electric Field direction of the incident plane polarized light

I_0 : Intensity of incident plane polarized light beam

I : Intensity of transmitted plane polarized light beam

Polarization

This means that if initially unpolarized light passes through crossed polarizers, no light will get through the second one.



Polarization

If unpolarized light of intensity I_o is incident on a Polaroid sheet, then the resultant intensity is 0.5 I_o.

The intensity of a single plane of the unpolarized light is I_o/π.

The intensity of the resultant light will be to integrate Malus's Law over π.

$$I = \frac{I_o}{\pi}\int_0^{\pi} \cos^2\theta d\theta$$

$$\text{Since } \cos 2\theta = 2\cos^2\theta - 1$$

$$I = \frac{I_o}{\pi}\int_0^{\pi}\left(\frac{\cos 2\theta + 1}{2}\right)d\theta$$

$$I = \frac{I_o}{2\pi}\int_0^{\pi}(\cos 2\theta + 1)d\theta$$

$$I = \frac{I_o}{2\pi}\left[\frac{\sin 2\theta}{2} + \theta\right]_0^{\pi}$$

$$I = \frac{I_o}{2\pi}\pi = \frac{I_o}{2}$$

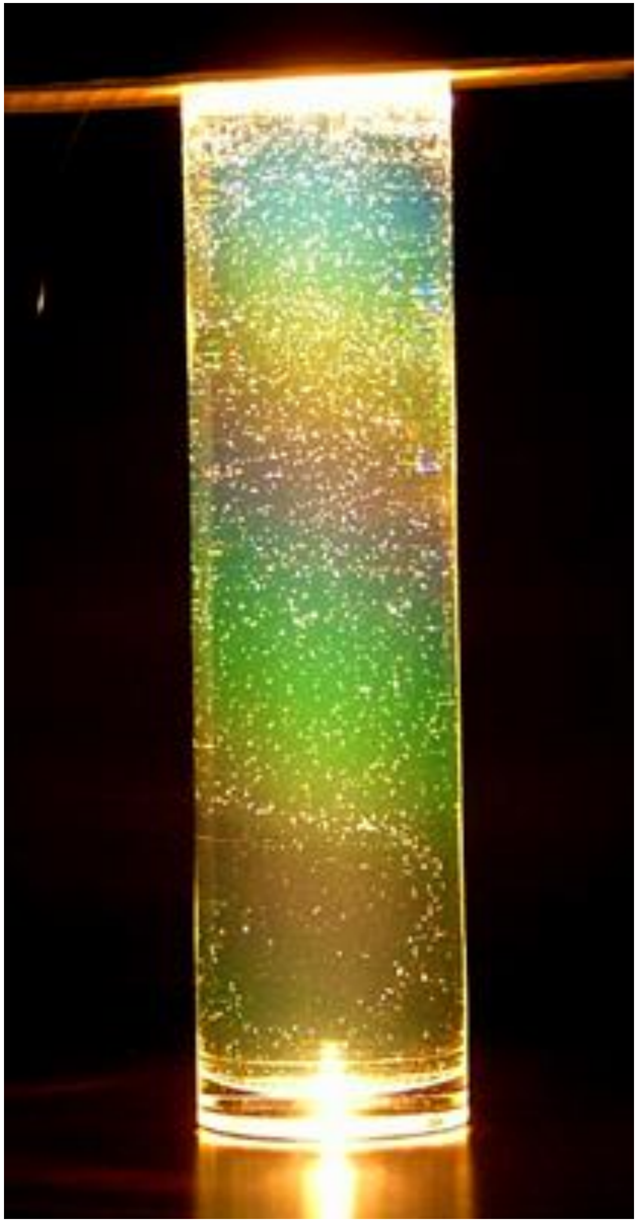
Optically Active Substances

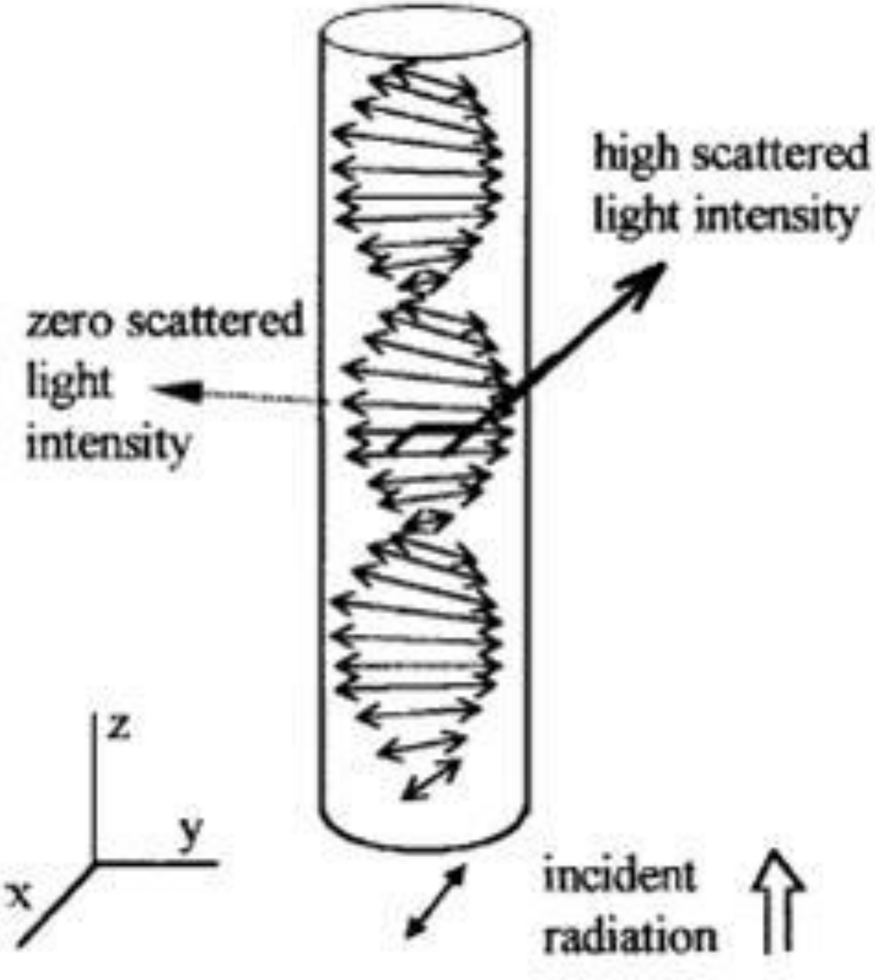
Optically active substances **rotate the plane of polarization** **of a beam of polarized light.**

This means that the plane of polarized light is rotated as it passes through the material. This rotation is a result of the interaction between the molecules in the material and the incident light. This occurs when the molecular structure of the compound is not symmetrical. These materials include sugar solutions, corn syrup, turpentine, amino acids, and some crystals.

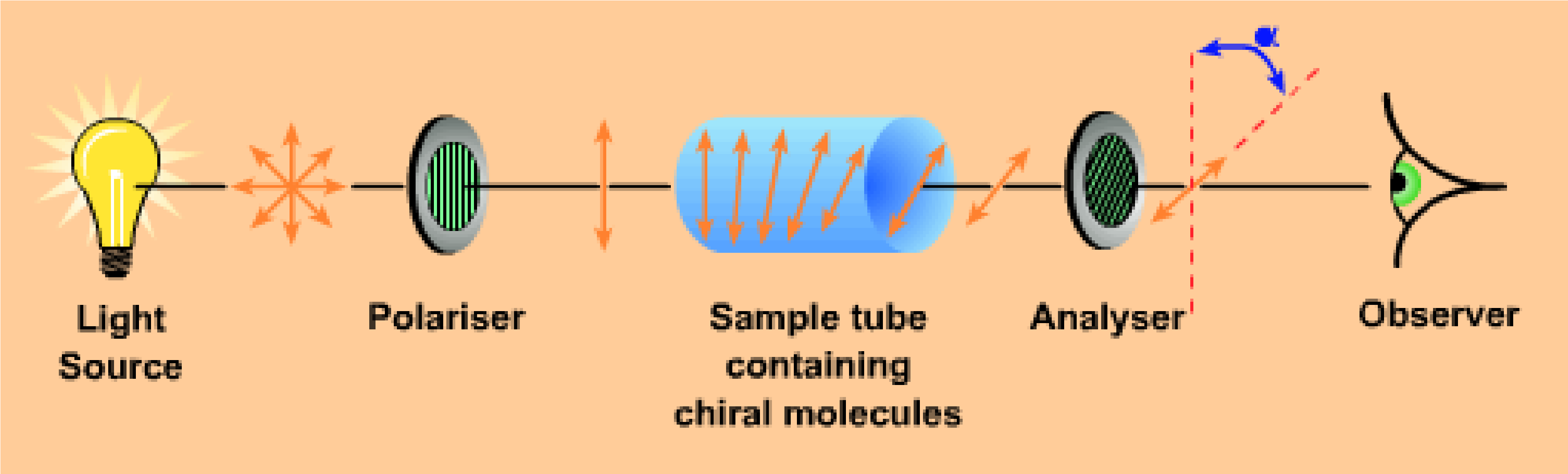
The degree of rotation of the plane of polarization depends on the depth of the liquid. Therefore, different depths of solutions will exhibit different colors when viewed through a stationary polarizing filter. In the photo, pieces of glass placed in Karo syrup create a variety of depths, and hence different colors.

The Barber Pole Demo





Polarimeter



The degree of rotation for a given substance is measured using two polarizers. The first one is called a Polarizer and it produces plane polarized light. A second polarizer is called the Analyzer. The polarizer and analyzer are initially aligned without the sample tube. A sample tube with an optically active substance is then introduced. The analyzer is viewed and rotated an angle α until maximum intensity of transmitted light is seen.

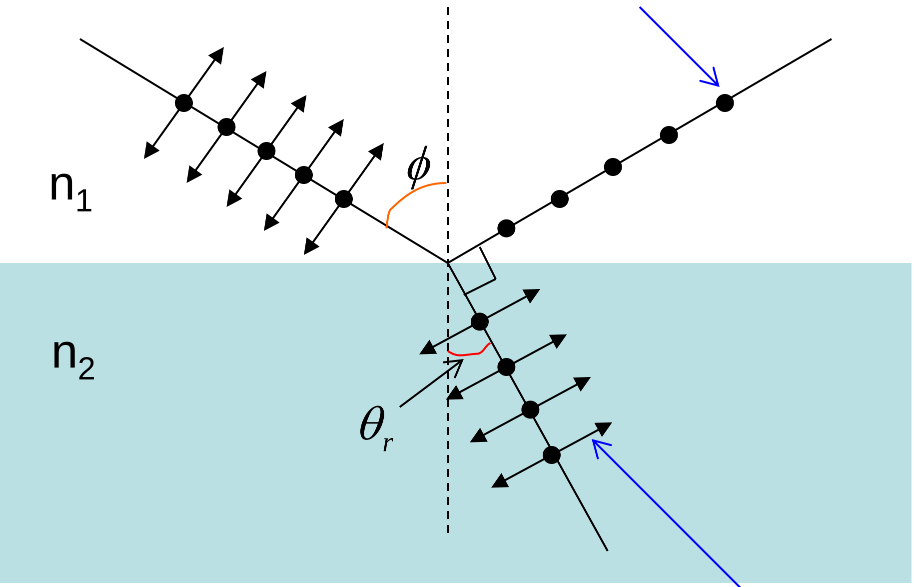
The angle through which the plane of polarization of light is rotated depends on the length of the sample and its concentration.

http://scholar.hw.ac.uk/site/chemistry/activity5.asp

Polarization by reflection – Brewster's Law

A ray of light incident on the boundary between two media will, in general, be reflected and refracted. The reflected ray is always partially plane-polarized preferentially in the plane parallel to the surface. If the reflected ray and the refracted ray are at right angles to one another, then the reflected ray is totally plane-polarized. The angle of incidence for this condition is known as the polarizing angle or Brewster's angle ϕ. This angle is related to the index of refraction of the two materials on either side of the boundary by the equation:

Electric field oscillation into the paper



Electric field oscillation in the plane of the paper

$$\tan\phi = \frac{n_2}{n_1}$$

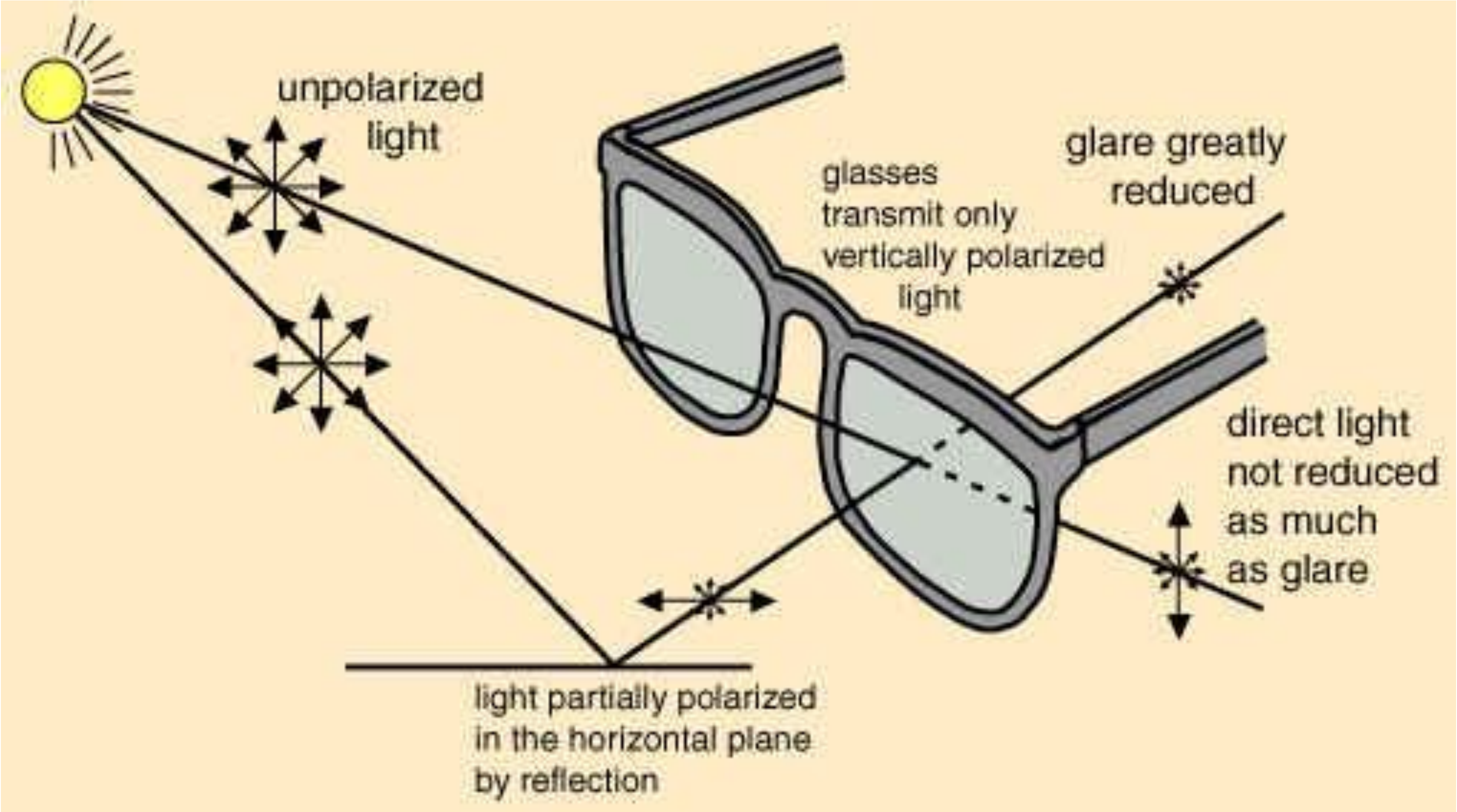
ϕ : Brewster's Angle

If $n_1 = 1$ (air), then

$$\tan\phi = n$$

Polarization by reflection

Polaroid sunglasses are made with the axes vertical to eliminate the more strongly reflected horizontal component, and thus reduce glare.



Liquid Crystal Displays (LCD)

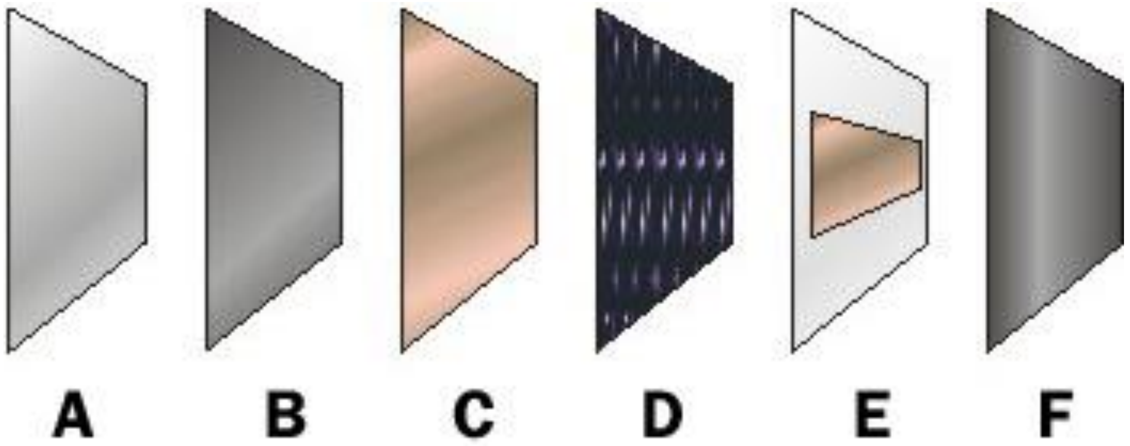
Liquid crystals are unpolarized in the absence of an external voltage, and will easily transmit light. When an external voltage is applied, the crystals become polarized and no longer transmit; they appear dark.

The liquid crystal is sandwiched between two electrodes, which are in turn between two crossed polarizers. Light enters from the front and any light that reaches the reflector at the back is returned to the observer.

The liquid crystal has a twisted structure and, in the absence of a p.d., causes the plane of polarization to rotate through 90°. So much of the light entering the front of the display will be returned back and thus the display will appear light.

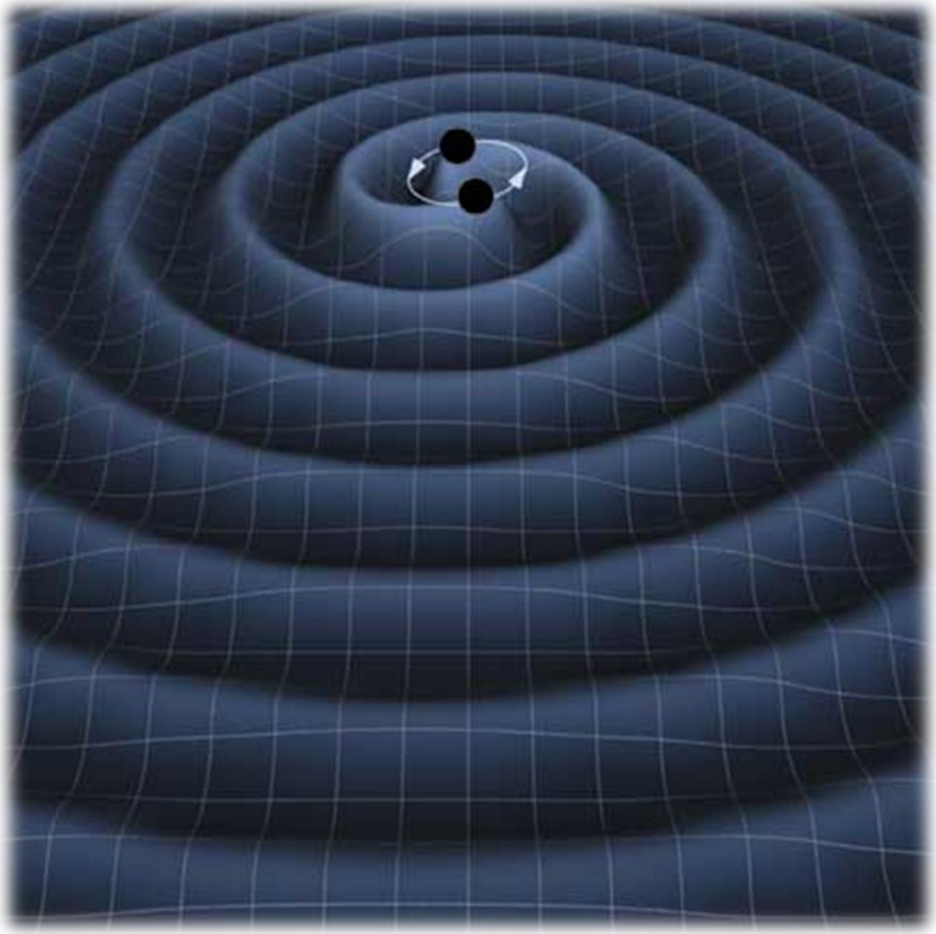
A p.d. across the liquid crystal causes the molecules to align with the electric field. This means less light will be transmitted and this section of the LCD will appear darker. It is possible for the plane of polarizations to be such that the region appears black.

Liquid Crystal Displays (LCD)



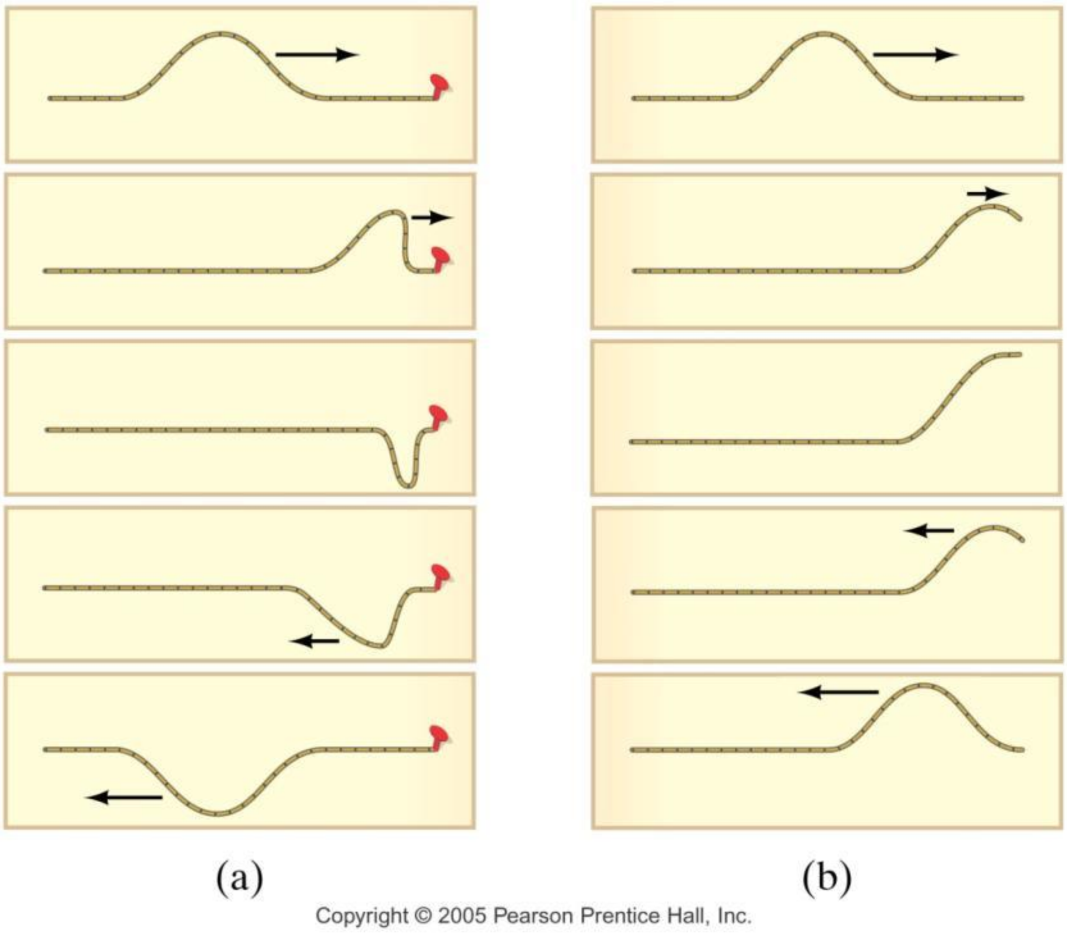
The simplest possible LCD with just a single rectangular electrode on it would look like this: It has a mirror (A) in back, which makes it reflective. Then, we add a piece of glass (B) with a polarizing film on the bottom side, and a common electrode plane (C) made of indium-tin oxide on top. A common electrode plane covers the entire area of the LCD. Above that is the layer of liquid crystal substance (D). Next comes another piece of glass (E) with an electrode in the shape of the rectangle on the bottom and, on top, another polarizing film (F), at a right angle to the first one.

4.4
Wave Behaviour



4.4.1 Reflection and refraction

Reflection and Transmission of Waves

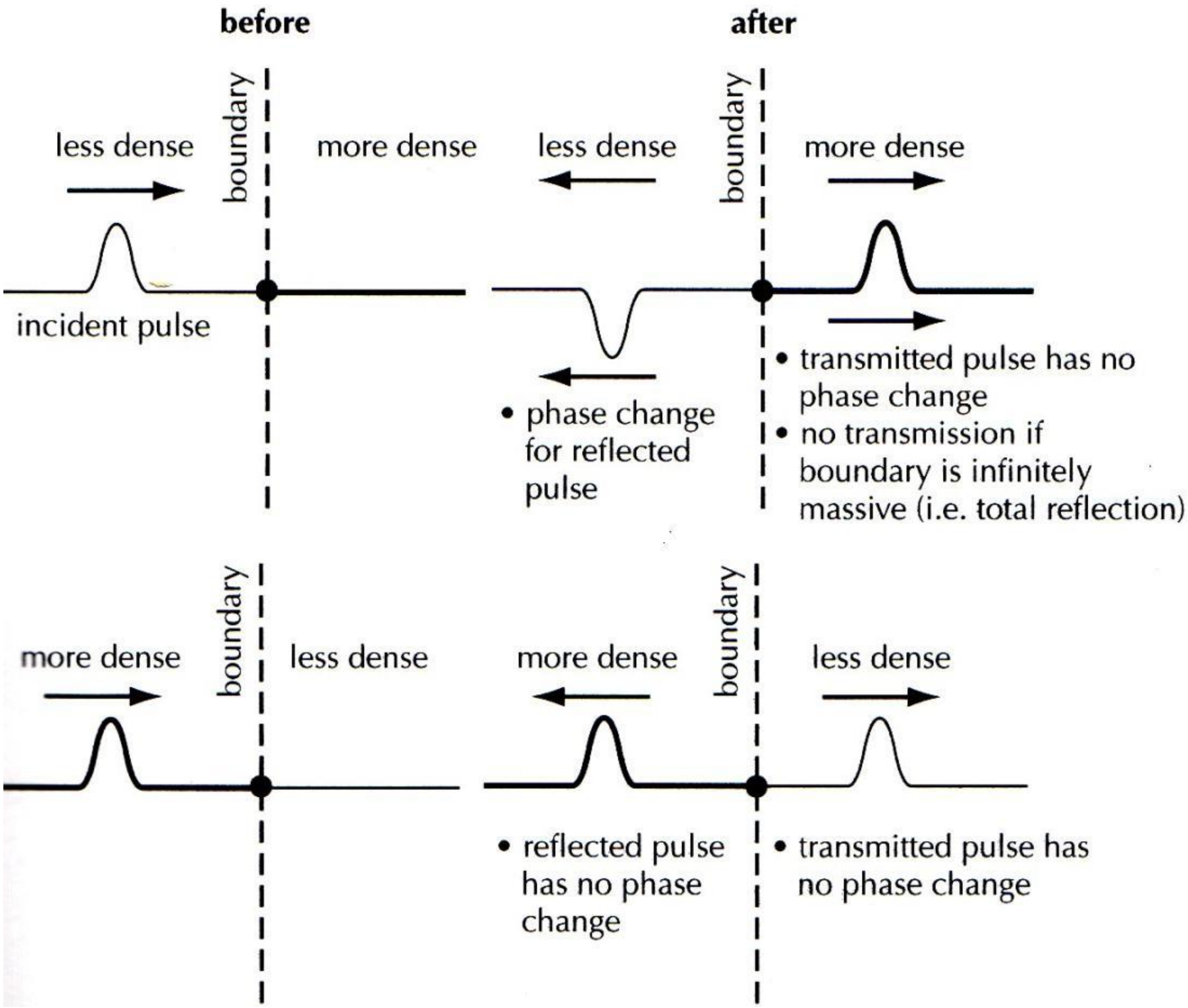


A wave hitting an **obstacle/fixed or rigid end** will be reflected as shown in (a) with an **inverted reflection**.

A wave encountering a **free end** will be reflected as shown in (b), its reflection will be **upright**.

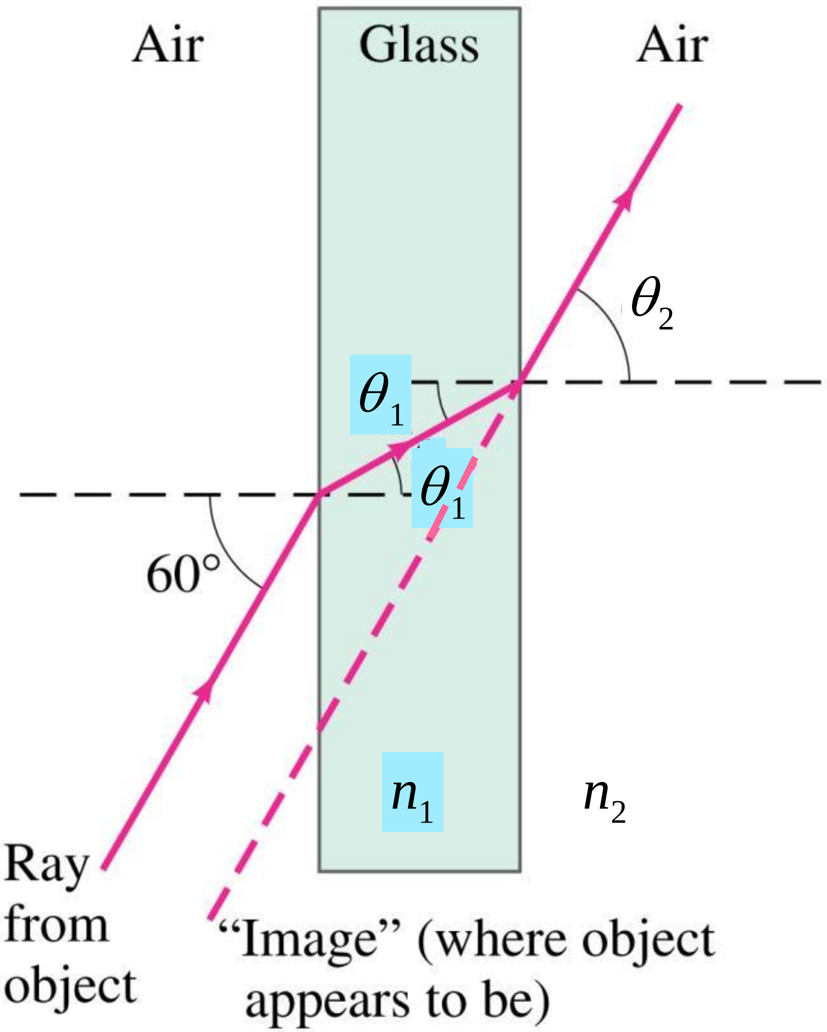
A wave encountering a **boundary between 2 different media** will be **partly reflected** and **partly transmitted**.

Reflection and Transmission of Waves



4.4.2 Snell's law, critical angle and total internal reflection

Refraction: Snell's Law

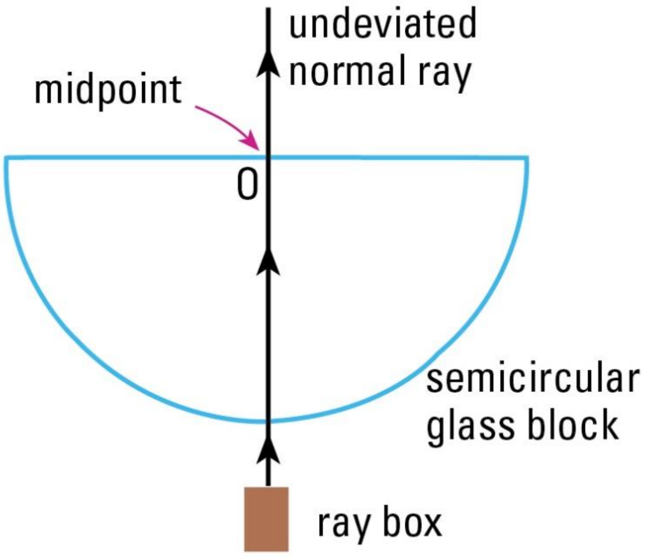


The angle of refraction depends on the refractive index, & is given by Snell's law:

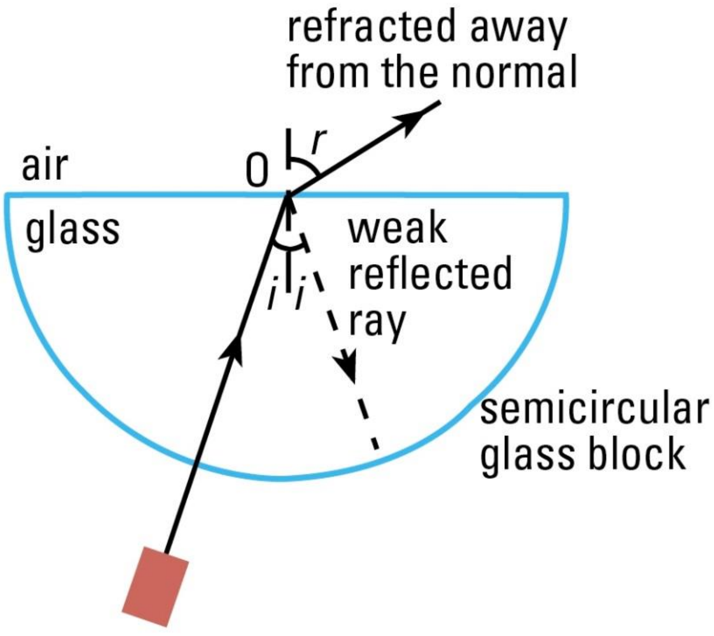
$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1}$$

Critical angle

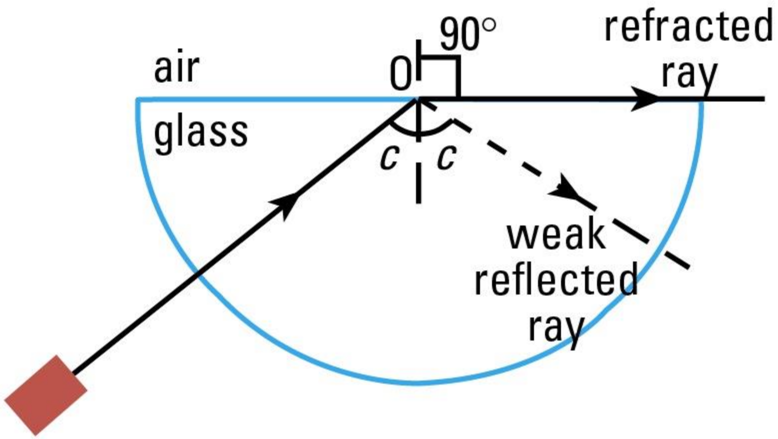
1. When a ray of light is directed perpendicularly through the semicircular glass block, the light ray will pass through without deviation.



2. When the light ray is directed at an angle *i* at point O, the light ray is refracted away from the normal when it travels from the glass into air.

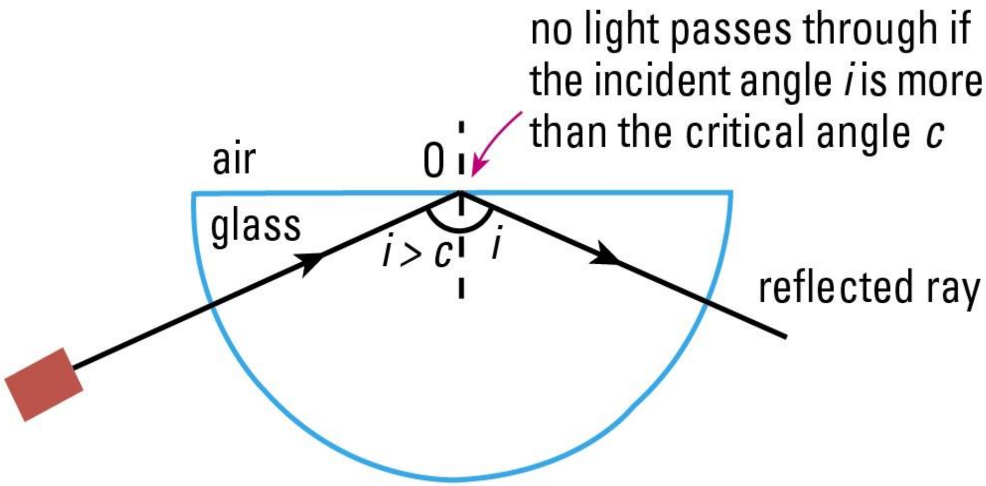


3. As the incident angle is increased, the refracted ray is seen to bend further away from the normal. It comes a point when the refracted ray is 90° to the normal.



Critical angle is the angle of incidence in the optically denser medium for which the angle of refraction in the less dense medium is 90°.

When the angle of incidence is increased beyond the critical angle, the light ray will be reflected back into the glass block. No light ray is refracted through. This is known as **total internal reflection**.



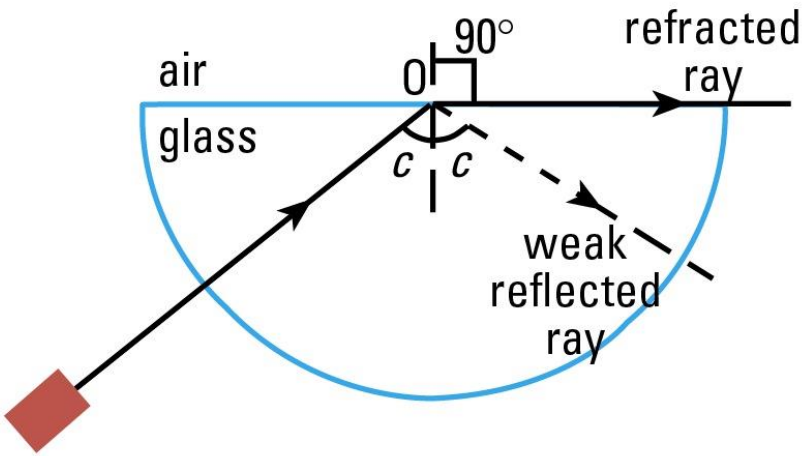
Total Internal Reflection takes place only when

1. a ray of light travels from an optical denser medium to a less dense medium
2. The angle of incidence in the optically denser medium is greater than the critical angle.

To determine the critical angle of a medium?

When the angle of incidence is at the critical angle $\angle c$, then the angle of refraction $\angle r = 90^o$. Since the light ray is from the medium into air, we use the principle of reversibility. Hence

$$n = \frac{\sin 90^o}{\sin c} = \frac{1}{\sin c}$$

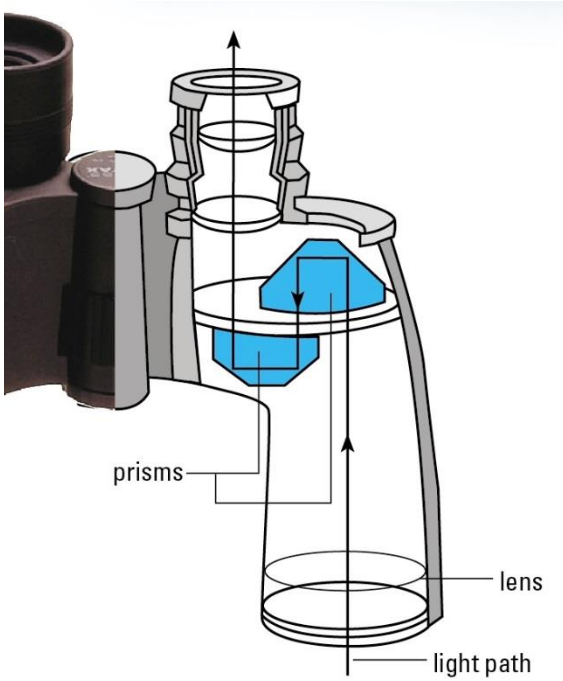


$$\sin c = \frac{1}{n}$$

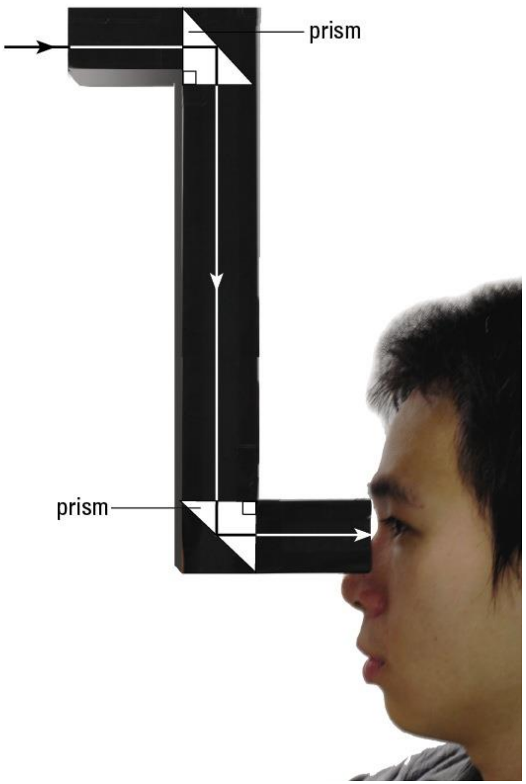
Applications of Total Internal Reflection

1. Total internal reflection in glass prism. These glass prisms can be found in the following optical instruments:

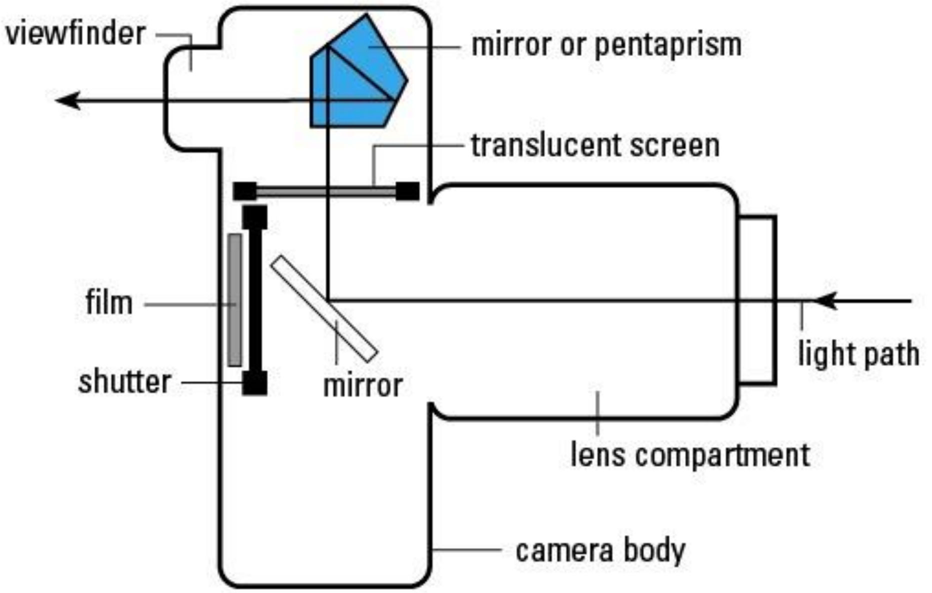
a. Binoculars



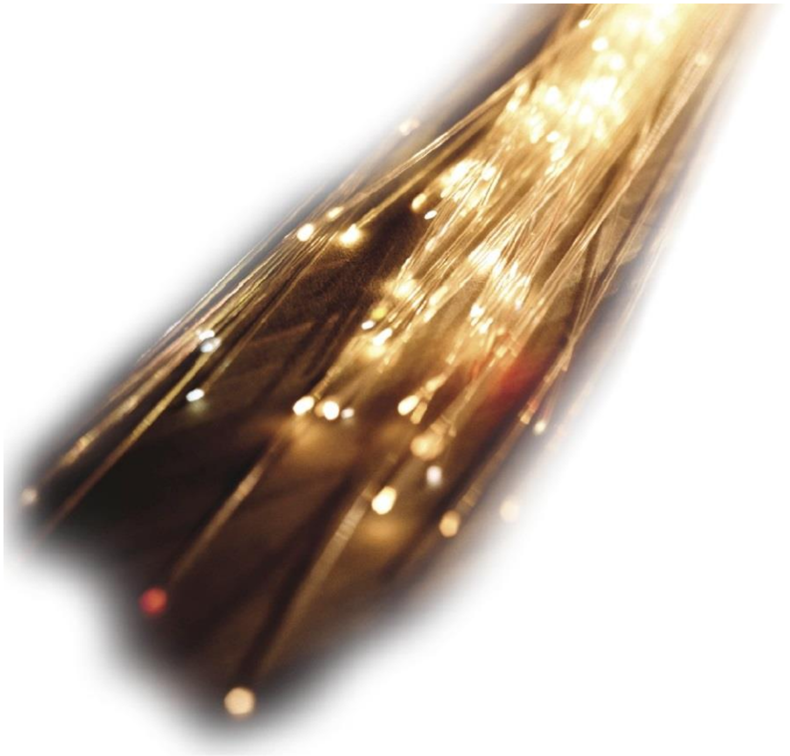
b. Periscope



c. Single lens reflex (SLR) camera

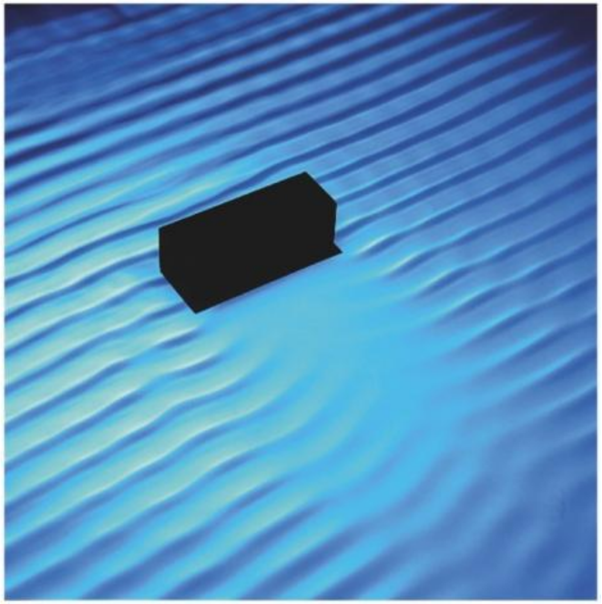


2. Total internal reflection in **optical fibres** can transmit light over long distances. These optical fibres are important in telecommunications and in endoscopy.



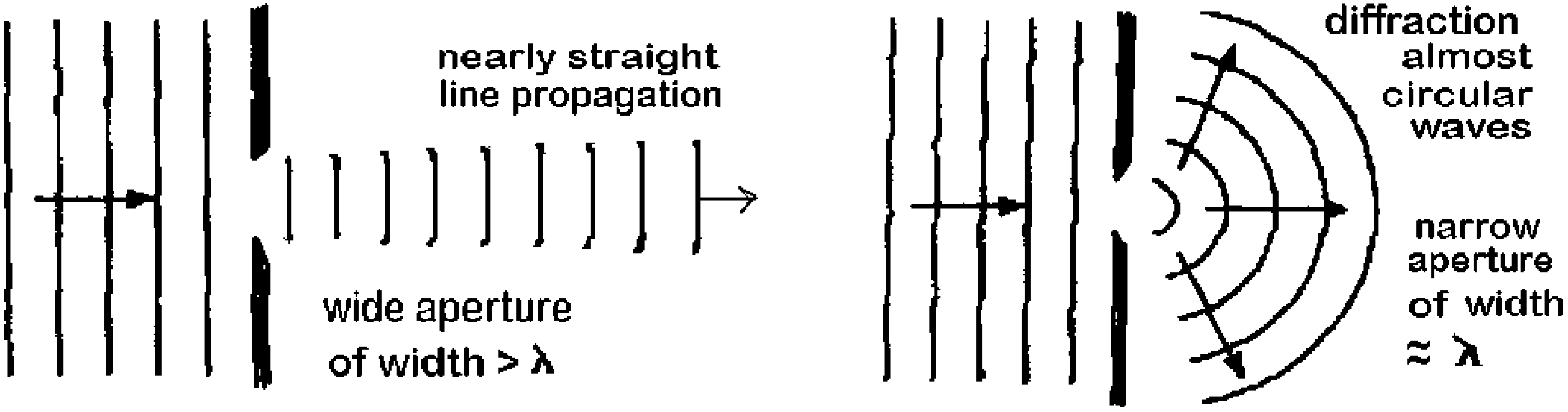
4.4.3 Diffraction through a single-slit and around objects

Diffraction

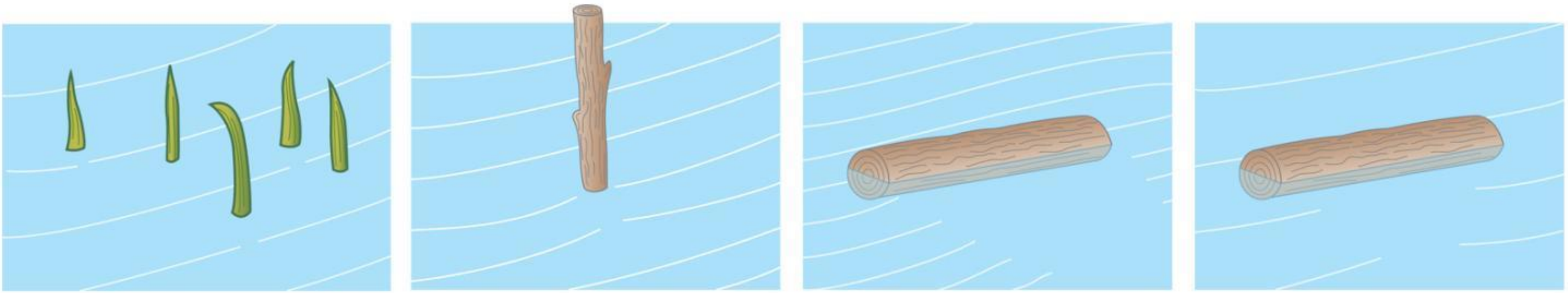


Diffraction refers to the phenomenon of bending or spreading of waves when they pass an obstacle, or pass through an aperture.

Diffraction is relatively more important when the wavelength is comparable to the size of the aperture or obstacle.



Diffraction

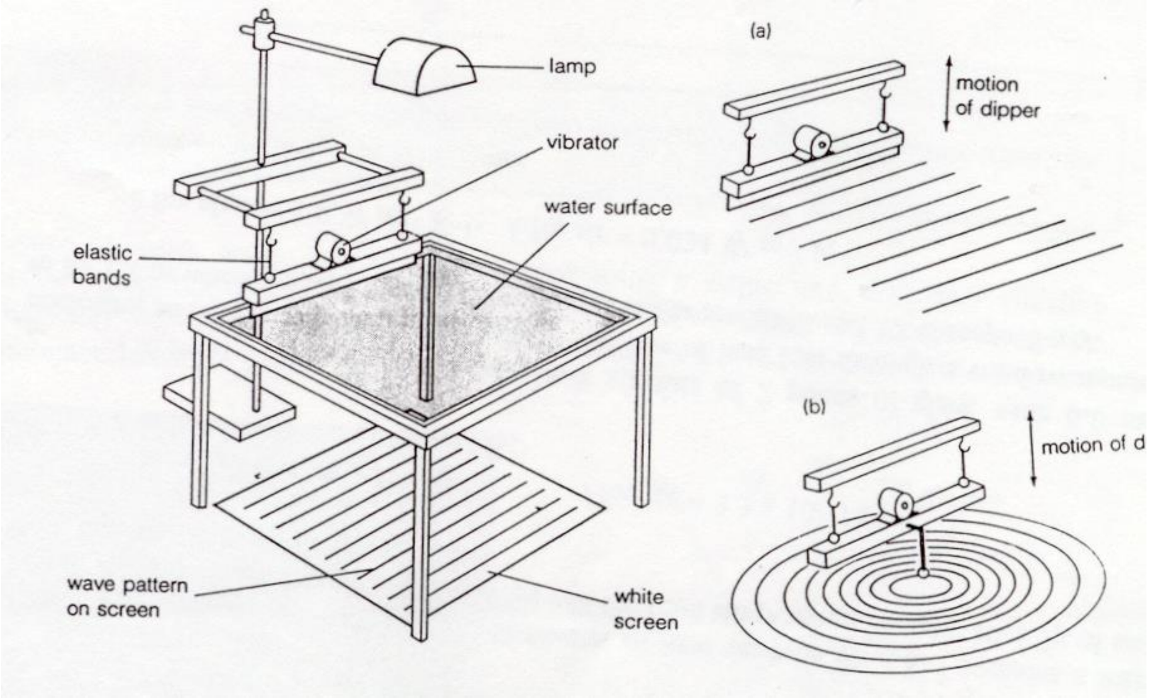


(a) Water waves passing blades of grass (b) Stick in water (c) Short-wavelength waves passing log (d) Long-wavelength waves passing log

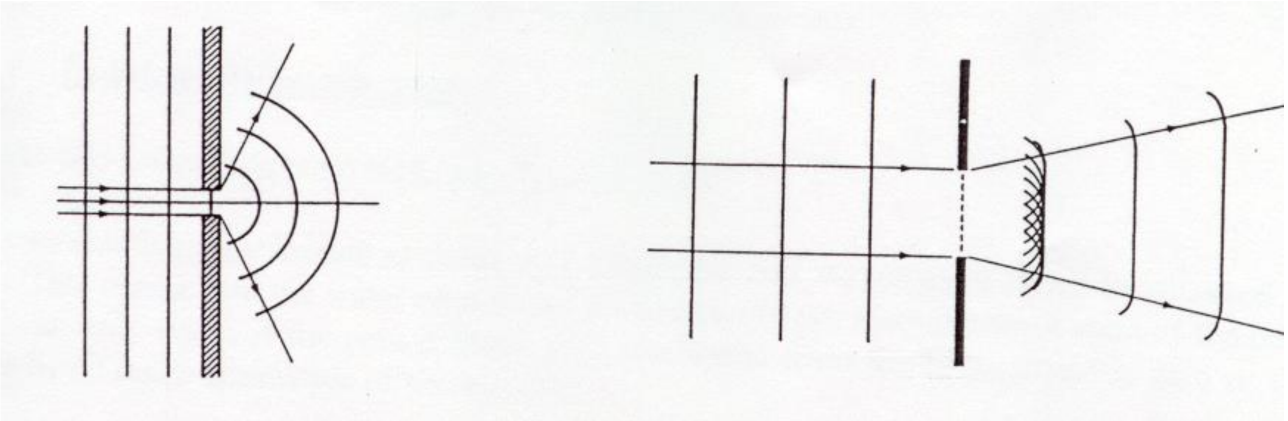
Sound has long wavelengths and can diffract after passing through doorways about a metre wide. A person talking loudly in a room can be heard round a corner without being seen.

Light has a very short wavelength such as 600 nm and so light waves are diffracted appreciably only through very small openings. When the light source is far away and the openings are small like pinholes and slits, diffraction effect is noticeable. Eg. view a distant street lamp through a pinhole in a card, or through a fine silk handkerchief, or through a slit between two fingers.

Diffraction of waves can be demonstrated with the help of a ripple tank



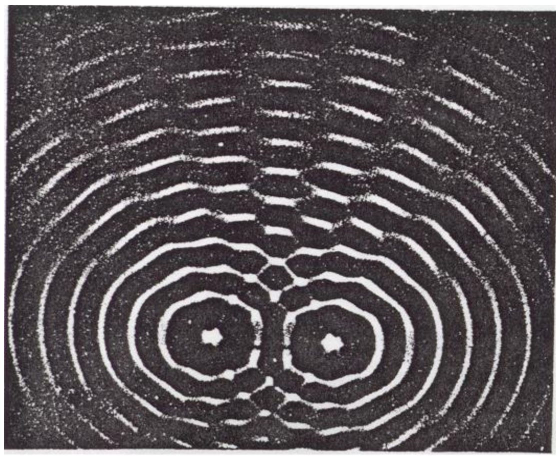
If a plane wave produced in a ripple tank is allowed to pass through a slit (see figure below), the following are observed:

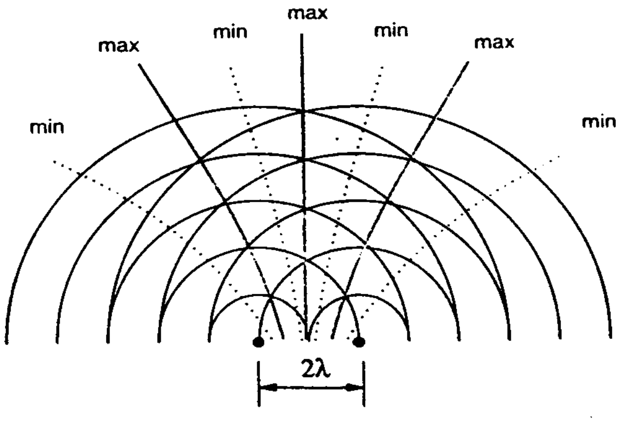


Interference

Interference is said to occur when waves from two or more coherent sources superpose with one another producing a resultant wave.

Interference of water waves can be demonstrated using a ripple tank with two vibrating dippers.(see figures below) The two dippers send our circular waves that are *in phase* and of the *same frequency. An interference pattern consisting of easily observed lines of constructive and destructive interference is seen.*

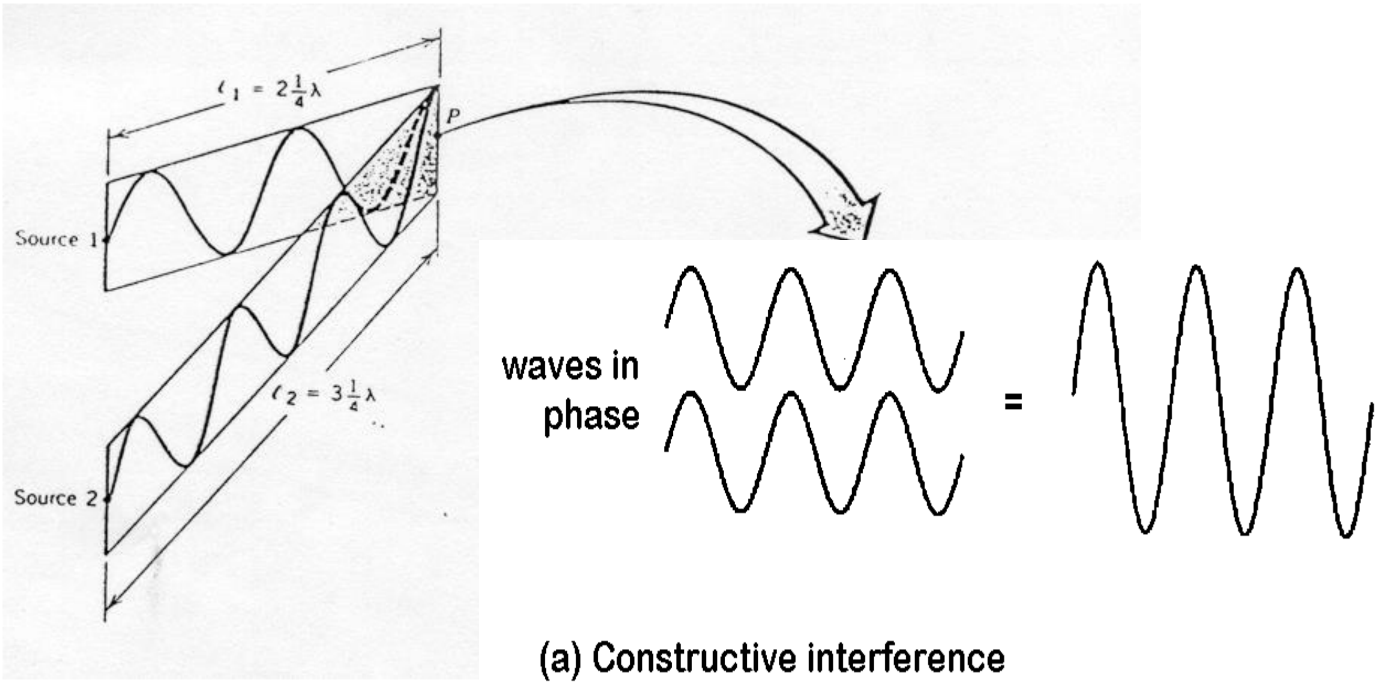




The lines of ***constructive*** interference (**maxima**) are places where the **resultant amplitude is double** the amplitude of one wave. The lines of ***destructive*** interference **(minima**) are places where the waves cancel out and the **resultant amplitude is zero.**

(a) Constructive Interference

For constructive interference to occur at a point, the two waves must arrive **in phase** at the point.This means that the wave crest from one source always meet the wave crest of the other. This can only occur if the **path difference** of the waves from the two sources **is zero or they differ by an integral multiple of wavelength**.

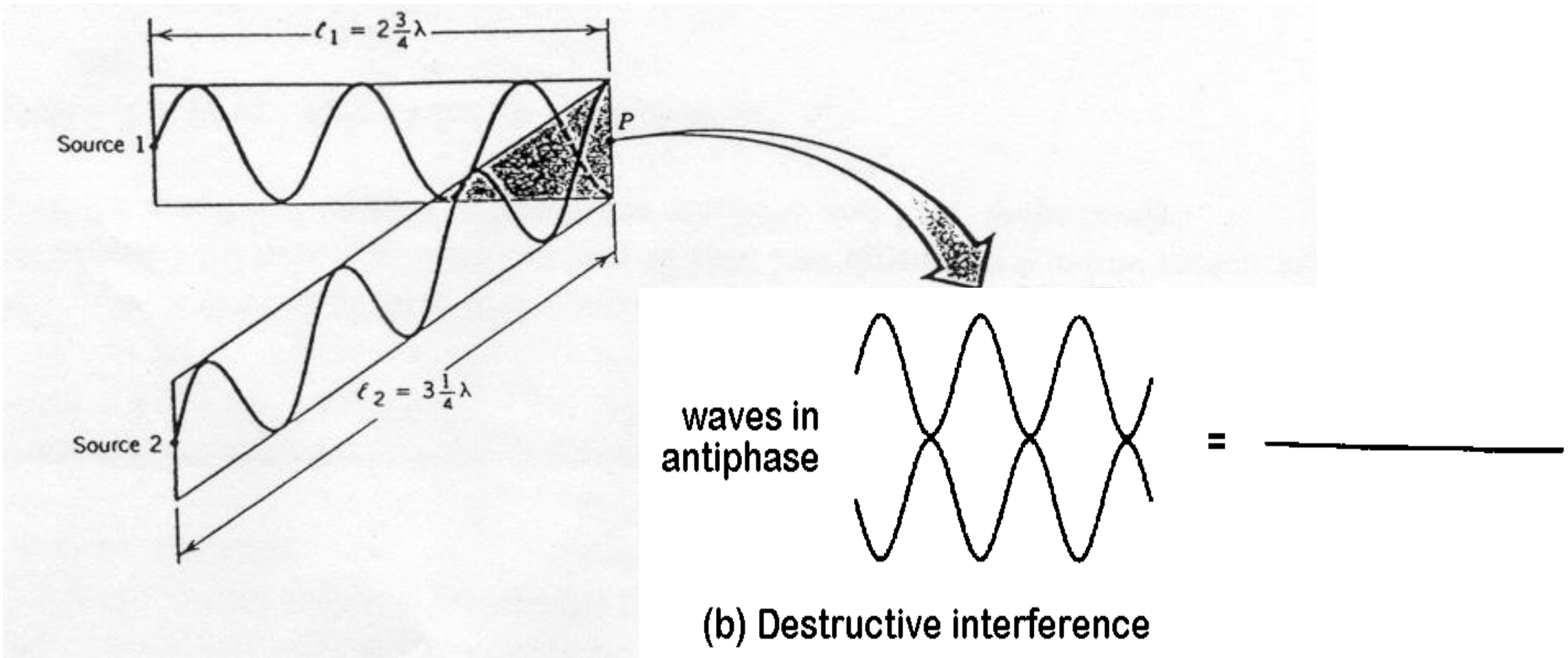


(a) Constructive interference

Path difference $S_2 P - S_1 P = n\lambda$ where n = 0,1,2,3…….

(b) Destructive Interference

Destructive interference occurs if the waves from the two sources arrive exactly **out of phase**, i.e. the wave crest always meets the trough. This happens when the **path difference of the waves is (n + ½)λ**.



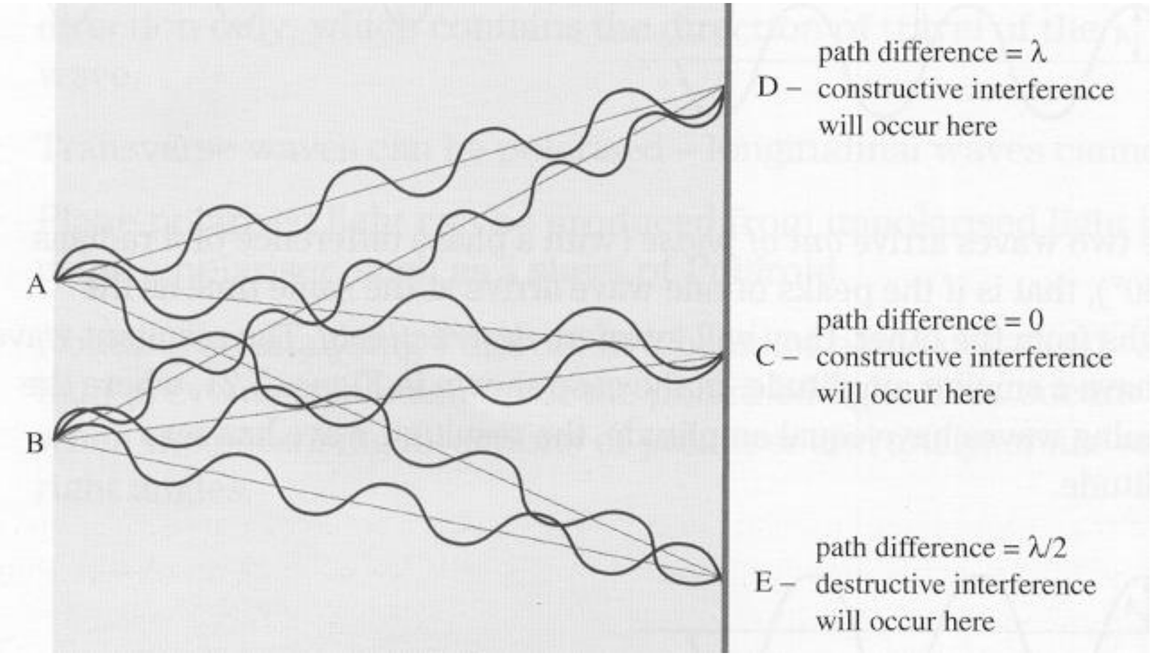
(b) Destructive interference

Path difference $S_2P - S_1P = (n + \frac{1}{2})\lambda$. where n = 0,1,2,3…….

The figure below illustrates the production of an interference pattern by two point sources A and B. The point C is equidistant from A and B. A wave travelling from A to C will cover the same distance as the wave from B to C. If the waves started out in phase at A and B, they will arrive in phase at C and interfere constructively. A maxima is obtained at C.

At other places such as D, the waves will have travelled different distances from the two sources. There is a path difference between the waves arriving at D. If this path difference is a whole number of wavelengths (λ., 2λ., 3λ.,..or nλ.) the waves will arrive in phase and interfere constructively, producing regions of maxima.

However at places such as E, the path difference is odd number of half-wavelengths (λ./2, 3λ./2, 5λ./2…or (n+½) λ.). The waves arrive at E out of phase, and interference is destructive, producing regions of minima. The collection of maxima and minima is called an interference pattern.



Conditions to produce an observable Interference Pattern

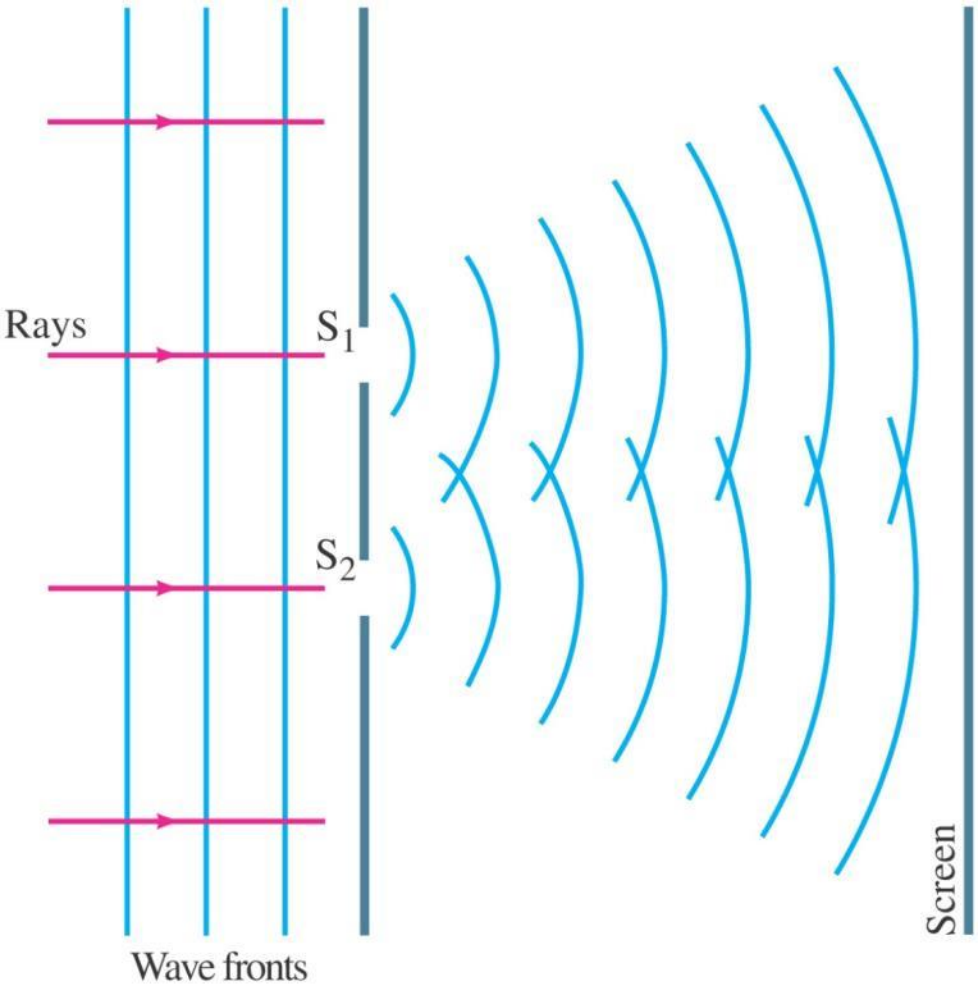
For an interference pattern to be observable,

1. The sources must be **coherent** i.e. *the waves from each source have a constant phase difference.*

2. The sources must have roughly the *same amplitude*.

3. The sources must be either unpolarised or have the same plane of polarisation.

Young's Double-Slit Experiment

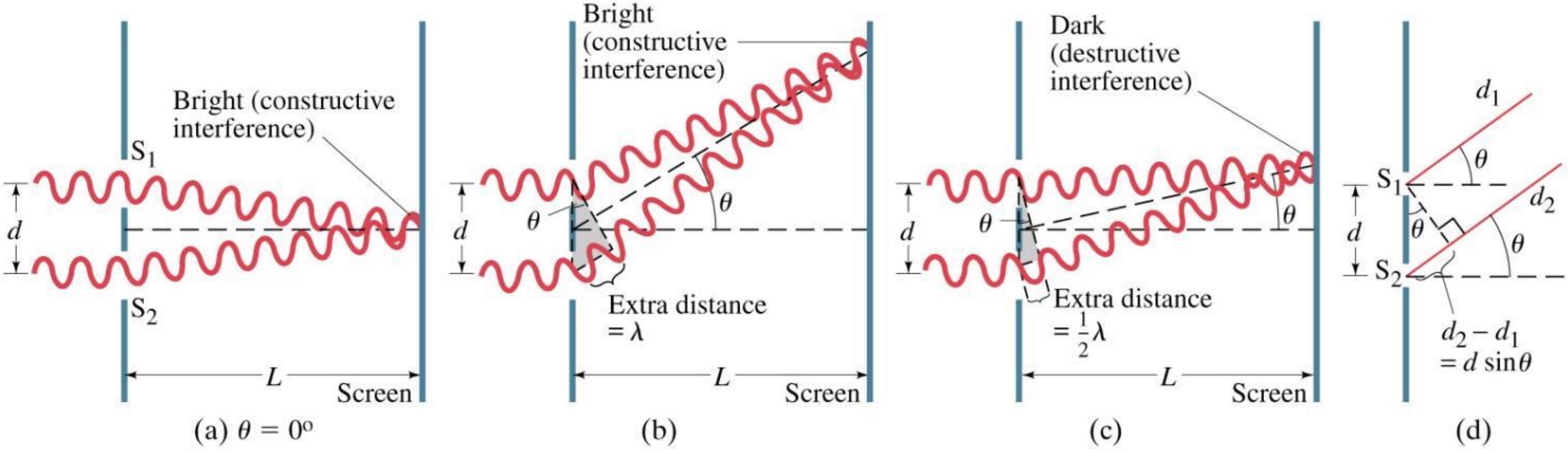


Conditions:

-Coherent sources (constant phase difference)

Interference – Young's Double-Slit Experiment

The interference occurs because each point on the screen is not the same distance from both slits. Depending on the path length difference, the wave can interfere constructively (bright spot) or destructively (dark spot).



Interference – Young's Double-Slit Experiment

We can use geometry to find the conditions for constructive and destructive interference:

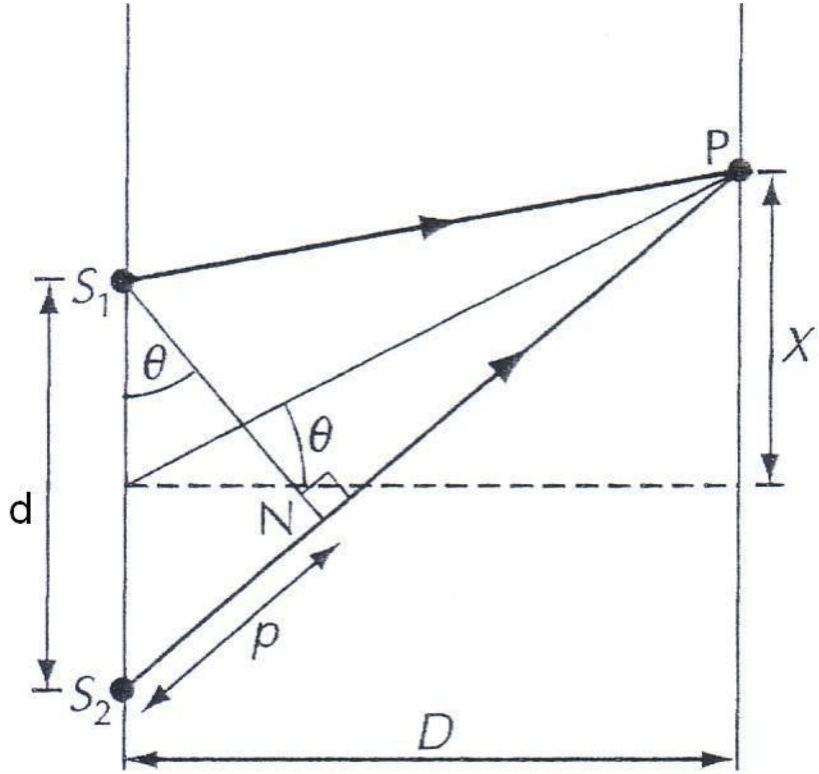
$d\sin\theta = n\lambda$, $n = 0,1,2,\ldots\ldots.$ Constructive Interference

$d\sin\theta = (n+1/2)\lambda$, $n = 0,1,2,\ldots\ldots.$ Destructive Interference

Interference – Young's Double-Slit Experiment

$\sin \theta = p/d = n\lambda/d$
$\tan \theta = X_n/D$

Since θ is usually very small,
$\tan \theta \approx \sin \theta$
$X_n/D = n\lambda/d$
$X_n = n\lambda D/d$
$X_{n+1} = (n+1)\lambda D/d$



Fringe width
$s = X_{n+1} - X_n = \lambda D/d$

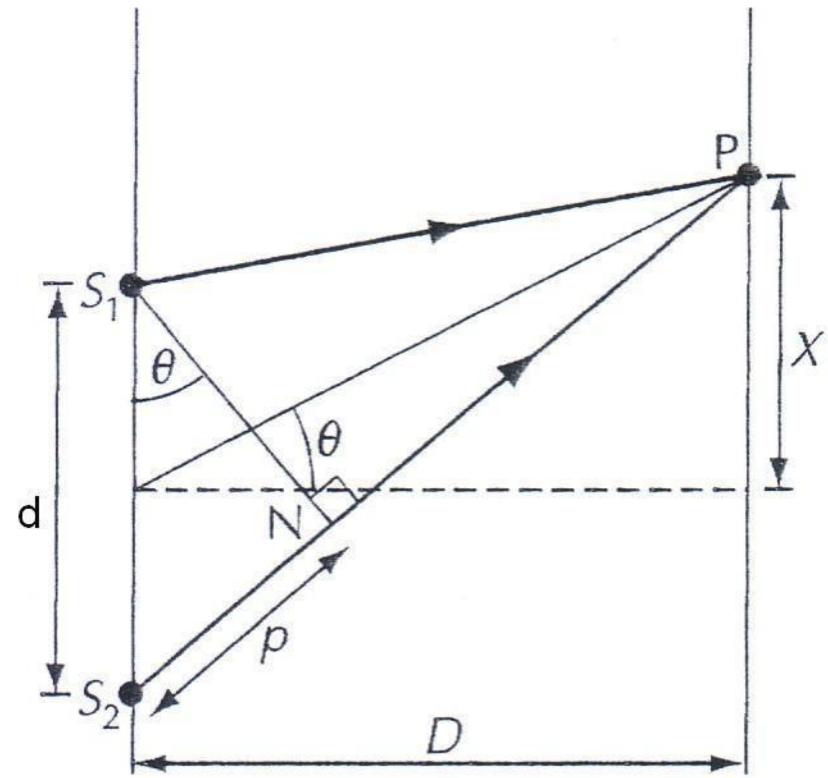
Interference – Young's Double-Slit Experiment

Distance x to the nth bright order fringe

$$\frac{x}{D} = \frac{n\lambda}{d}$$

Distance x to the nth dark order fringe

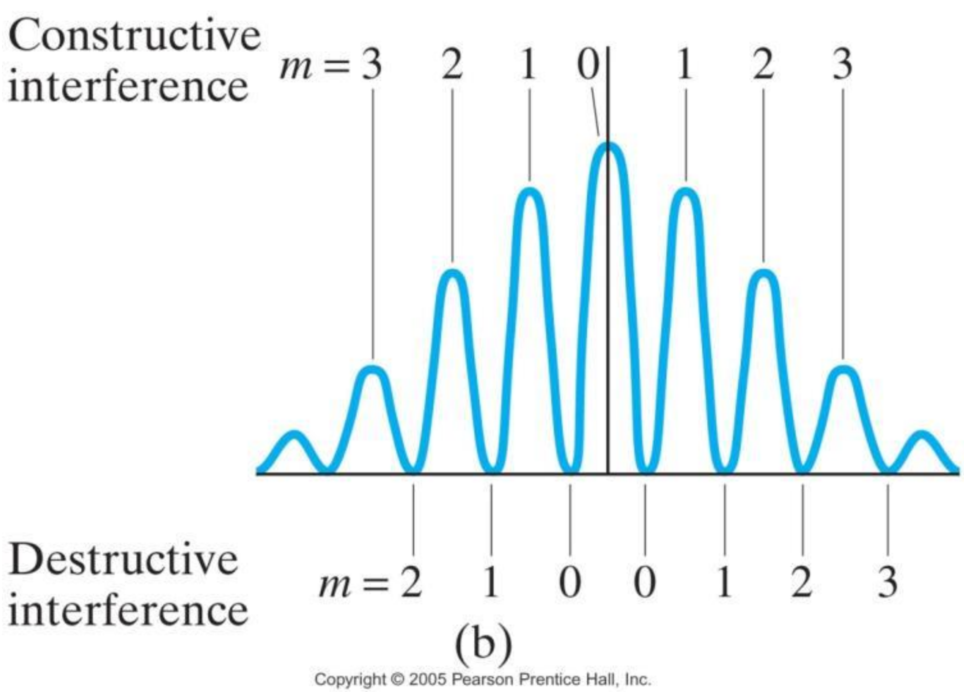
$$\frac{x}{D} = \left(n + \frac{1}{2}\right)\frac{\lambda}{d}$$

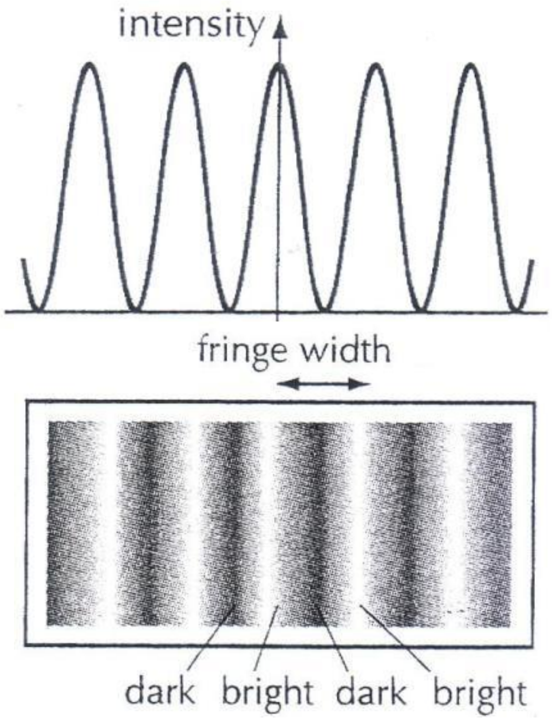


Interference – Young's Double-Slit Experiment

Fringe width/separation **$s = \lambda D/d$**

If the slits are sufficiently narrow, the bright fringes are equally bright.





Summary

- In Young's double-slit experiment, constructive interference occurs when

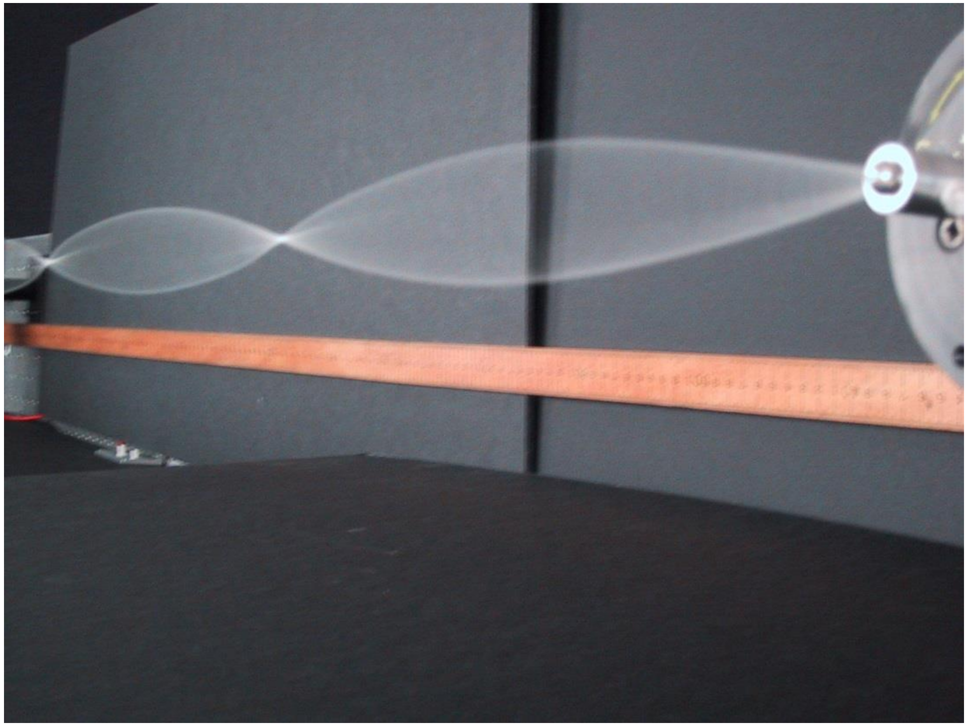
$$\sin\theta = n\frac{\lambda}{d}$$

- and destructive interference when

$$\sin\theta = (n + \frac{1}{2})\frac{\lambda}{d}$$

- Fringe width/separation s = λD/d
- Two sources of light are coherent if they maintain a constant phase difference.

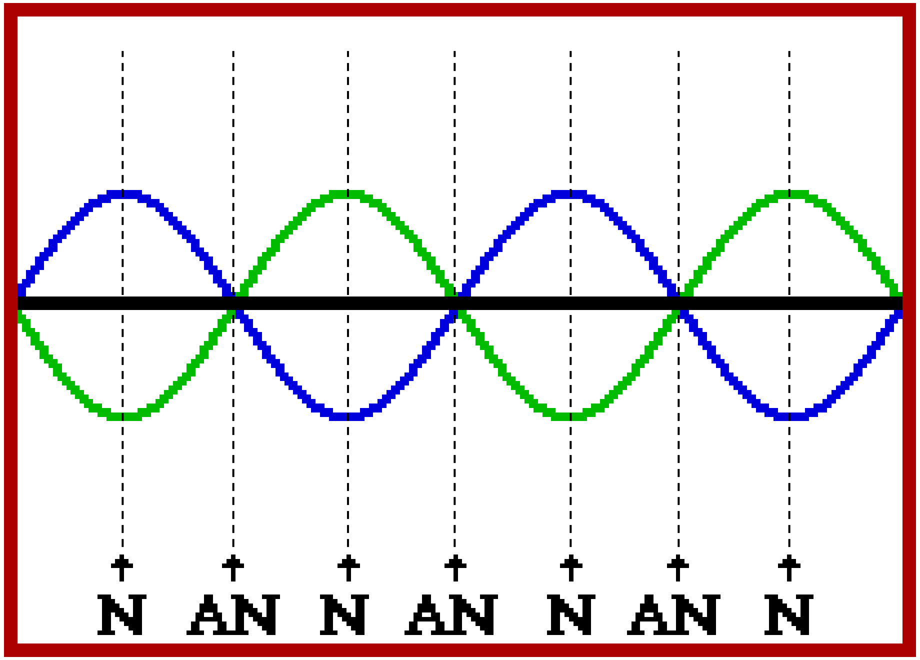
4.5 Standing Waves



Nature of Standing (stationary) waves

A standing wave is formed by the superposition (i.e. vector addition) of two waves that are:

- the same type of wave
- of the same amplitude
- of the same frequency
- travelling in the opposite direction



Nature of Standing (stationary) waves

Although each of the individual waves is still travelling in their separate directions, the result of the superposition of these waves is a new wave. The phase of each point on the new wave remains fixed with time but the amplitude of the resultant wave varies with time. We say that this wave is fixed in place or, in other words, is a standing wave.

The points that always show no displacement are called nodes. The points that reach maximum amplitude of displacement are called antinodes.

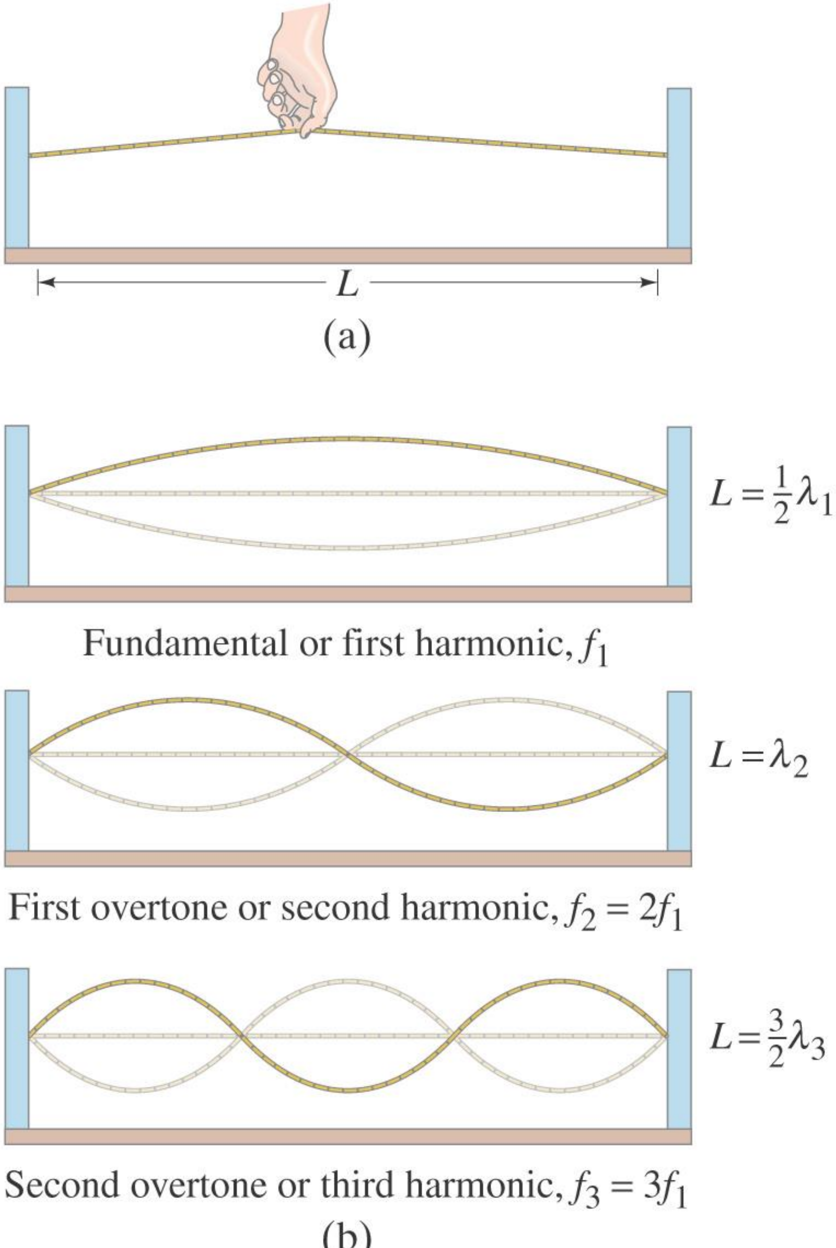
Boundary Conditions

In many situations, one of the waves that is involved in the creation of the standing wave results from a reflection of a travelling wave.

The processes involved in causing the reflection at the surface mean that a boundary condition is known to apply to the wave. The boundary conditions of the system specify the conditions that must be met at the edges (the boundaries) of the system when standing waves are taking place.

Any standing wave that meets these boundary conditions will be a possible resonant mode of the system. A boundary condition is any principle known to apply to an end point of the wave.

Standing Waves in stretched strings

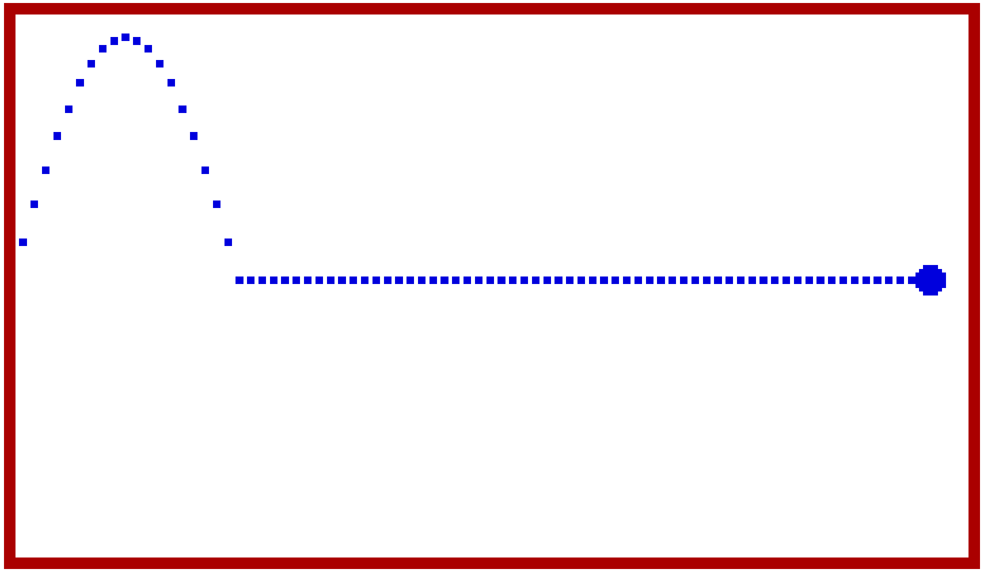


When both ends of the vibrating string are fixed, standing waves will occur.

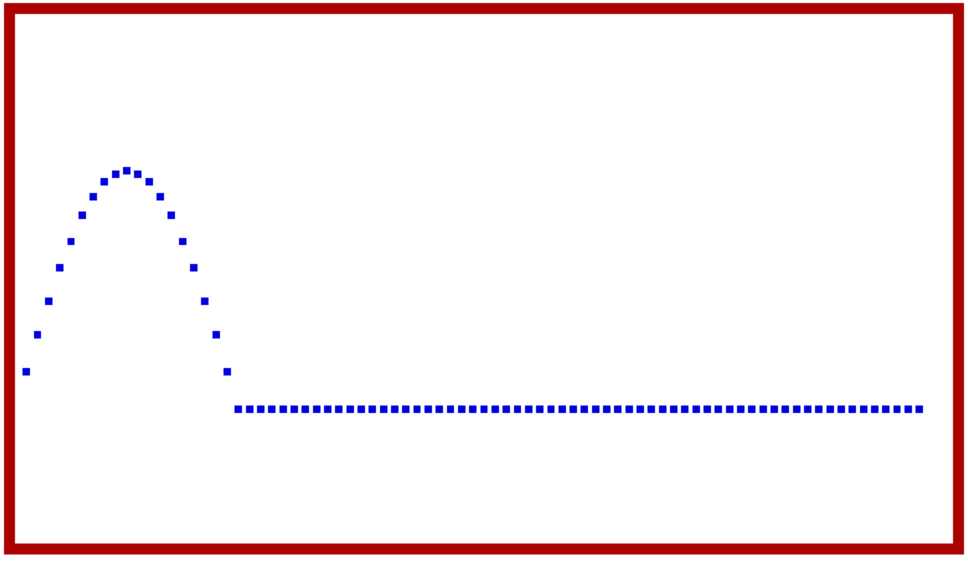
The boundary condition in this case is that each end of the string is fixed and cannot move, so the standing wave that is created must have a node at the fixed end. Another condition is that transverse waves travelling along the string meeting a fixed boundary at each end will be reflected.

Boundary Conditions – Reflection

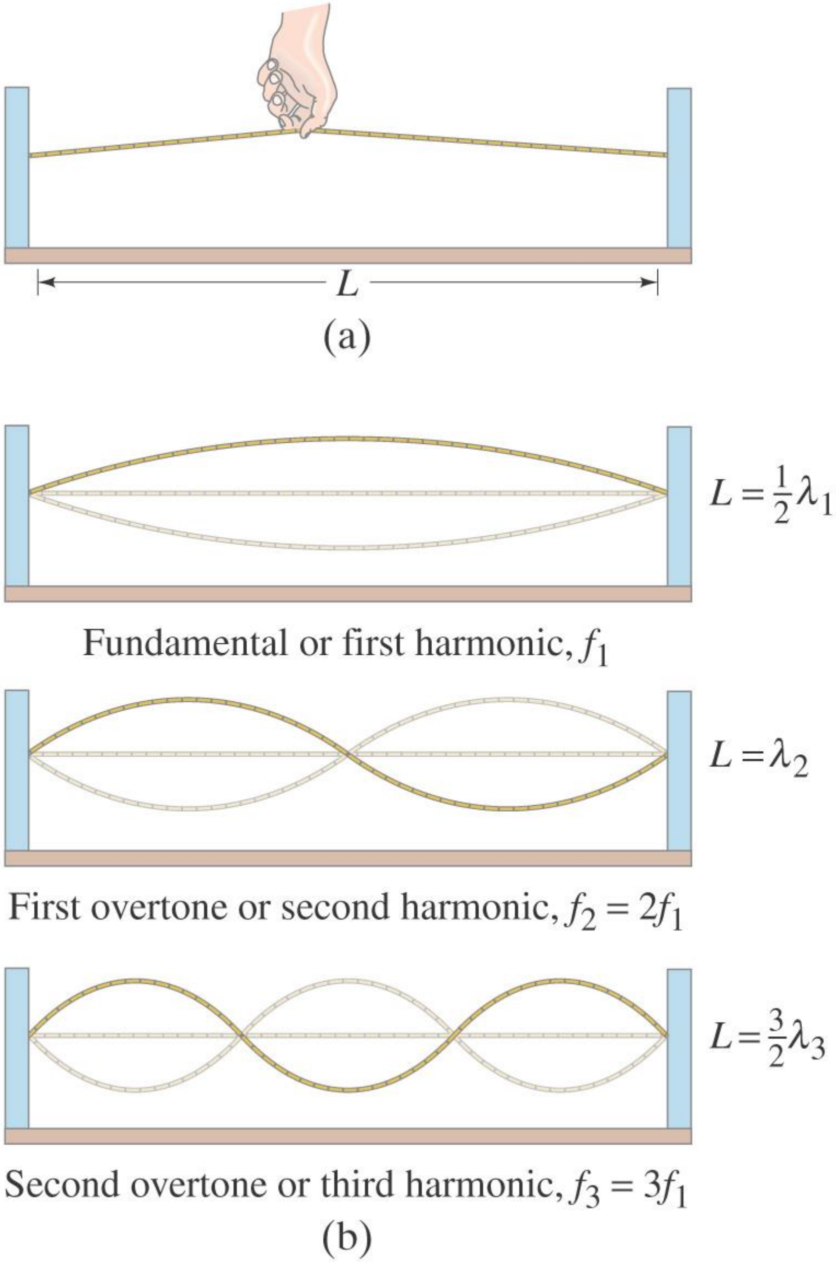
Reflection at fixed end



Reflection at free end



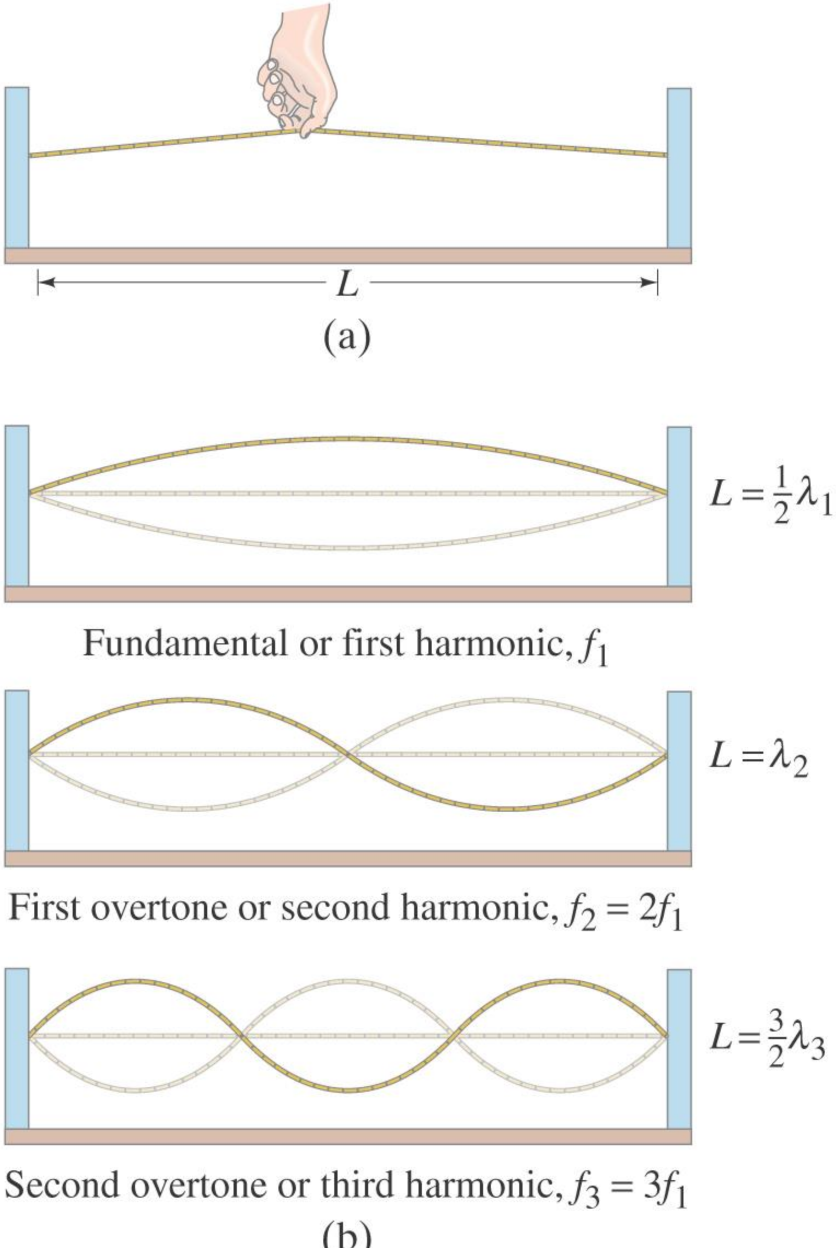
Standing Waves in stretched strings



Points of destructive interference (no vibration) are nodes.

Points of constructive interference (maximum vibration) are antinodes.

Standing Waves in stretched strings



Interference between travelling waves and reflected waves can result in vibration of the string as a whole and the seeming standing still of the waves.

The **frequencies** of the standing waves on a particular string are called **resonant** frequencies.

They are also referred to as the **fundamental** and **harmonics**.

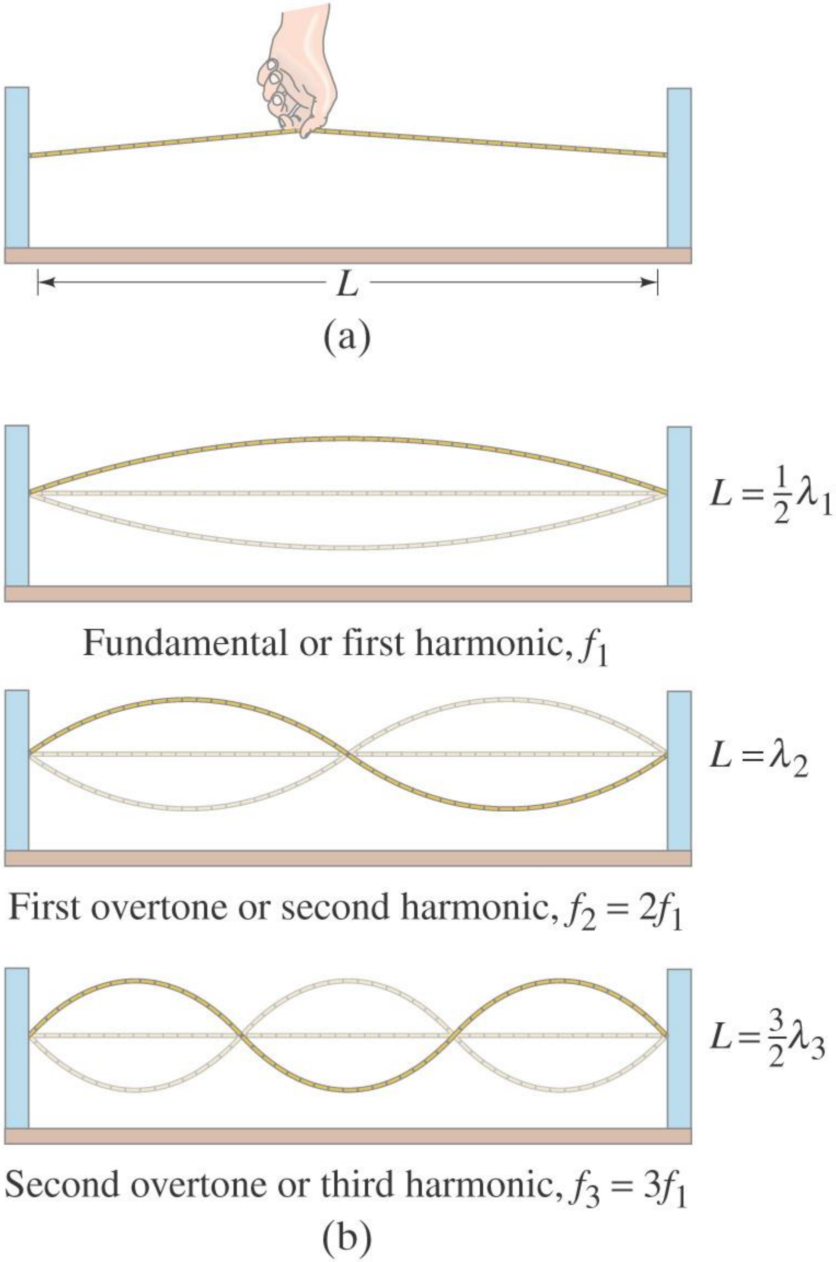
Standing Waves in stretched strings

The standing wave with the lowest possible frequency is called the fundamental frequency or first harmonic.

Higher resonant modes are called harmonics.

The difference between the sound of a guitar and a violin playing the same note, is a result of the relative amplitudes of the different harmonics that are produced by the instrument. This different sound is called the quality or timbre.

Standing Waves in stretched strings



$$f_n = nf_1$$
$$\text{where } n = 1, 2, 3 \ldots$$

Vibrating Air Columns

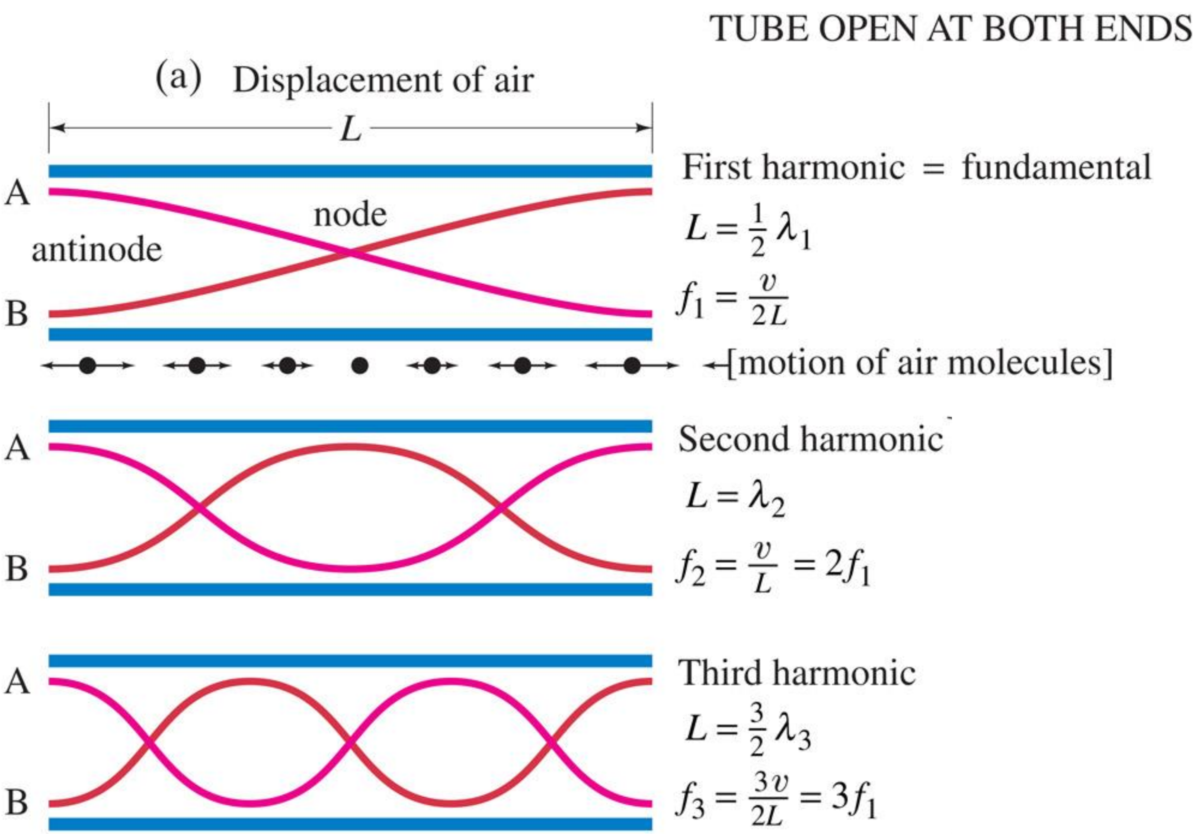
Wind instruments create sound through standing waves in a column of air.

Stationary longitudinal waves are set up in a column of air as a result of interference between sound waves travelling in opposite directions.

In a pipe, air blown across one end cause progressive sound waves that travel to the far end of the pipe where they are reflected and superpose with the incident waves to form stationary longitudinal waves.

Vibrating Air Columns

A tube open at both ends has displacement antinodes at the ends.



Vibrating Air Columns

In a pipe open at both ends, all harmonics are present. Therefore resonant frequencies are integral multiples of the fundamental frequency.

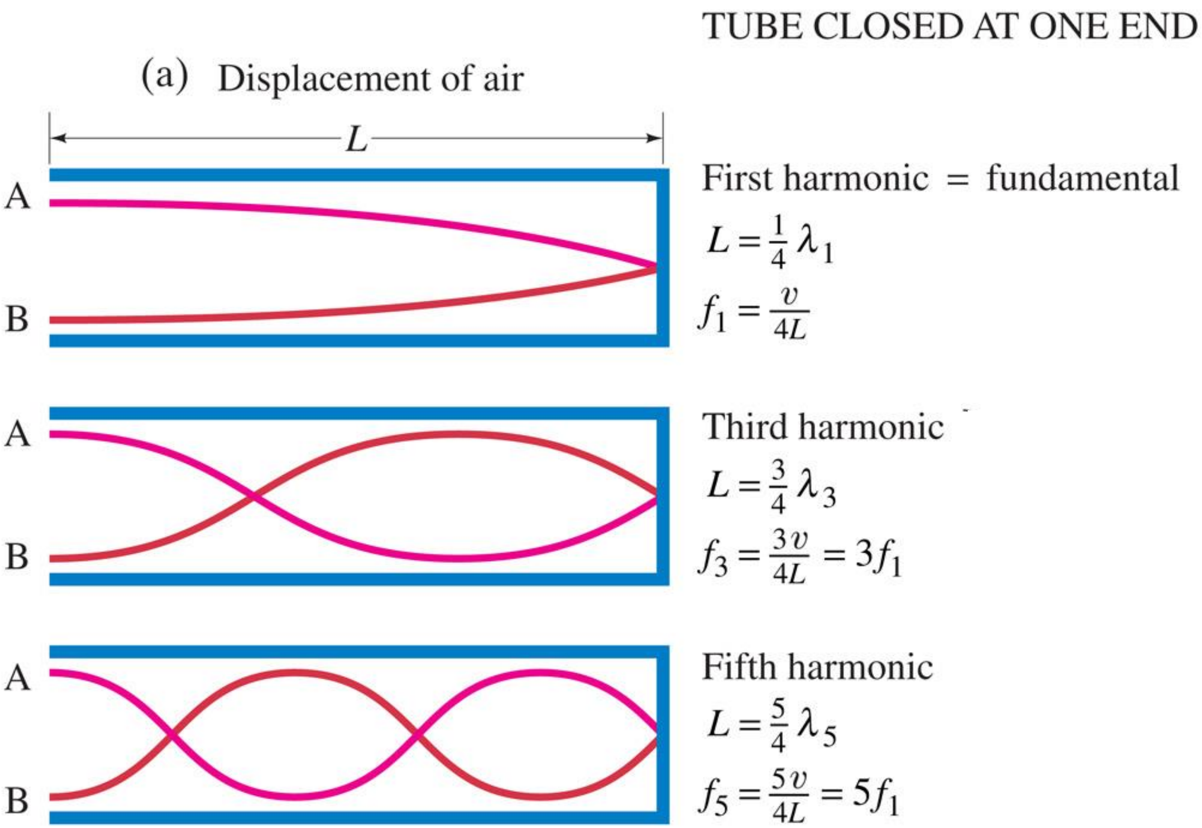
$$f_n = nf_1 \quad \text{where n} = 1, 2, 3 \ldots$$

$$L = \frac{\lambda_1}{2} \Rightarrow \lambda_1 = 2L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L}$$

Vibrating Air Columns

A tube closed at one end has a displacement node at the closed end.



Vibrating Air Columns

In a pipe closed at one end, only **odd harmonics** are present. Resonant frequencies are always odd integers of the fundamental frequency.

$$f_n = nf_1 \quad \text{where n} = 1, 3, 5...$$

$$L = \frac{\lambda_1}{4} \Rightarrow \lambda_1 = 4L$$

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{4L}$$

Vibrating Strings and Air Columns

Resonant frequency	Strings/Open pipes		Closed pipes	
Fundamental frequency/ 1st harmonic	$\lambda_1 = 2L$	$f_1 = v/2L$	$\lambda_1 = 4L$	$f_1 = v/4L$
2nd or 3rd harmonic	$\lambda_2 = L$	$f_2 = v/L$	$\lambda_3 = 4L/3$	$f_3 = 3v/4L$
3rd or 5th harmonic	$\lambda_3 = 2L/3$	$f_3 = 3v/2L$	$\lambda_5 = 4L/5$	$f_5 = 5v/4L$
nth harmonic	$\lambda_n = 2L/n$	$f_n = v/\lambda_n$	$\lambda_n = 4L/n$	$f_n = v/\lambda_n$
	All harmonics	n = 1,2,3…	Odd harmonics	n = 1,3,5…

Comparison between standing waves and travelling waves

	Stationary Wave	Travelling Wave
Amplitude	All points on the wave have different amplitudes. At any particular point on the wave, the maximum amplitude is fixed between zero (at the nodes) to twice the amplitude of the component waves (at the antinodes).	All points on the wave have the same amplitude (provided energy is not dissipated).
Energy	Energy is not transmitted by the wave, but it does have an energy associated with it.	Energy is transmitted by the wave.
Wave pattern	Does not move.	Moves.

Comparison between standing waves and travelling waves

	Stationary Wave	**Travelling Wave**
Frequency	All points oscillate with the same frequency.	All points oscillate with the same frequency.
Wavelength	This is twice the distance from one node (or antinode) to the next node (or antinode).	This is the shortest distance along the wave between two points that are in phase with one another.
Phase	The phase of all points between two nodes are identical.	All particles along a wavelength have difference phases.