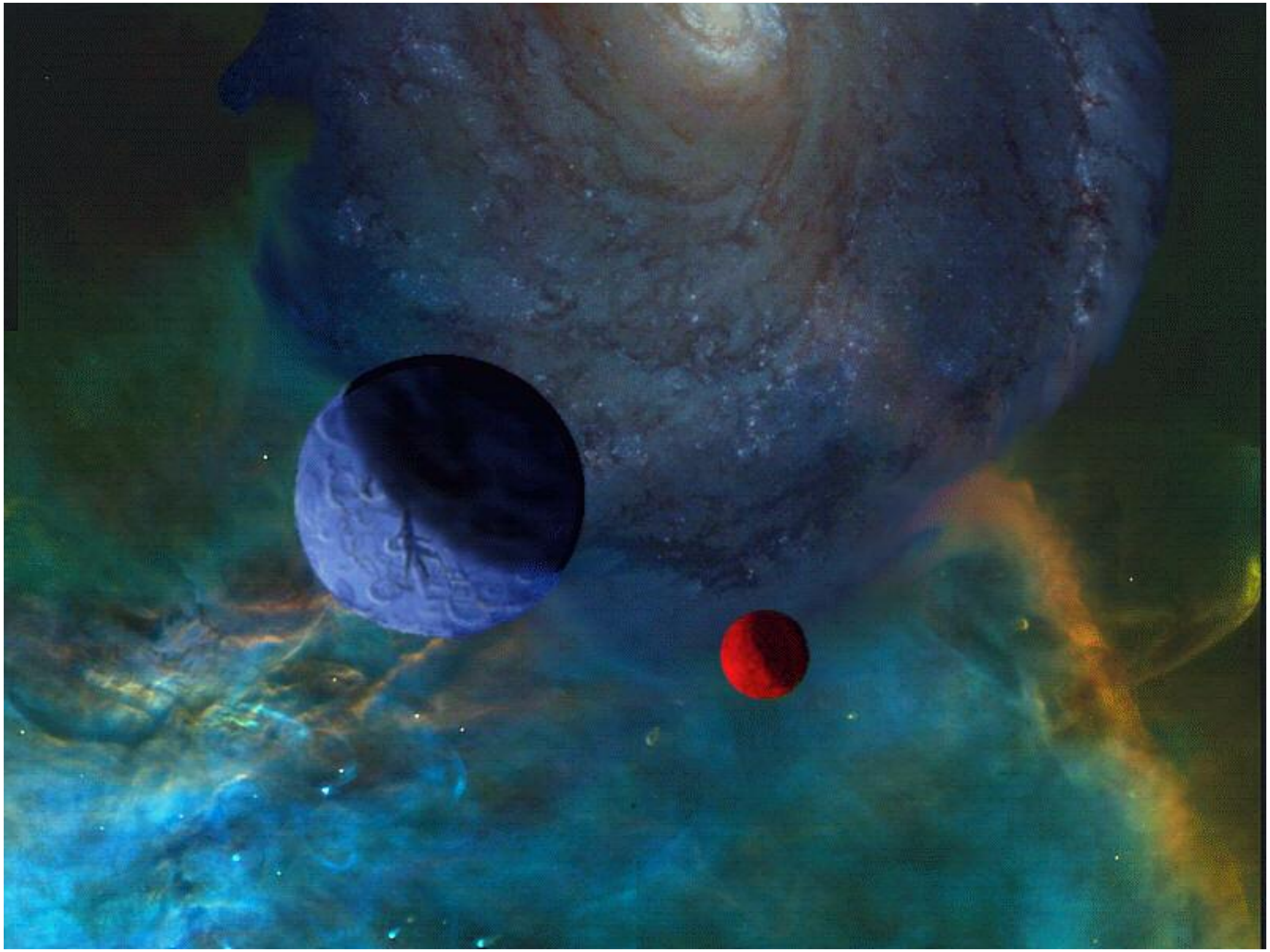
Topic 6 Circular Motion and gravitation



6.1 Circular Motion

Parts adapted from Giancoli Lecture Powerpoint

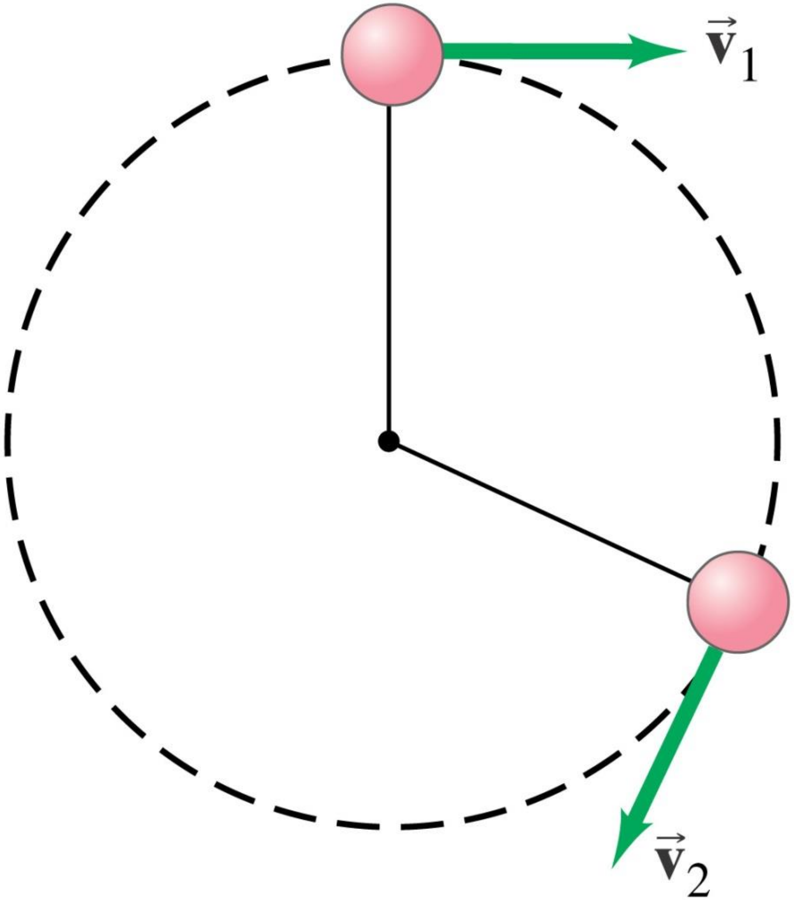
6.1Circular Motion

- **Period , frequency , angular displacement, angular velocity**
- **Vector diagram showing the direction of centripetal acceleration**
- **Expression for centripetal acceleration**
- **forces providing Centripetal force**
- **Dynamics of Circular Motion**
- **Highway Curves, Banked and Unbanked**
- **Examples of circular motion including cases of vertical and horizontal circular motion**

Kinematics of Uniform Circular Motion

Uniform circular motion: motion in a circle of constant radius at constant speed

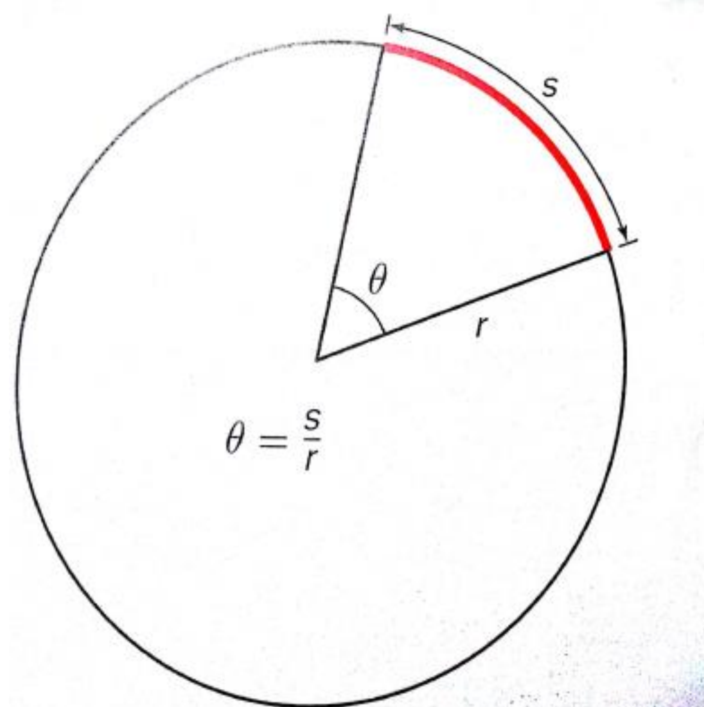
Instantaneous velocity is always tangent to circle.



Angular displacement

- Angular displacement is the angle through which the object moves (when in circular motion) and it can be measured in degrees(^{0}) or in radian (rad)

$$\theta = \frac{arc\ length}{radius} = \frac{s}{r}$$



- Units of θ will be m/m

hence θ is measured in radian.

Relation between radian and degrees

- 1^0 (degree) is defined to be $\frac{1}{360}$ th of the way around a circle.
- One radian is the angle, equal to the circumference of an arc of an circle divided by the radius of arc

$$\theta = s/r$$

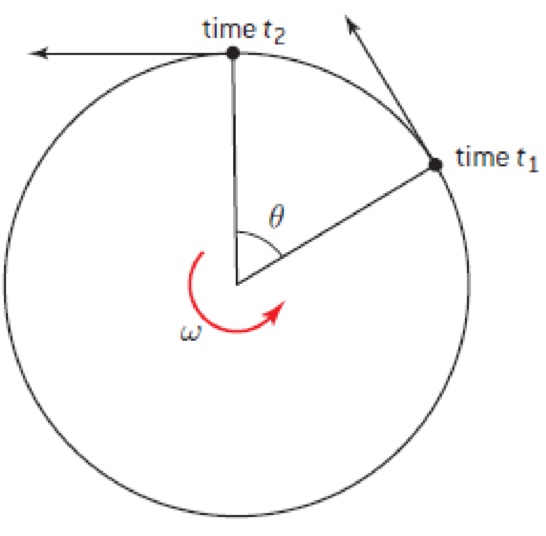
Angular speed (ω)

- Angular speed is rate of change of angular displacement.

$$\text{average angular speed} = \frac{\text{angular displacement}}{\text{time for the angular displacement to take place}}$$

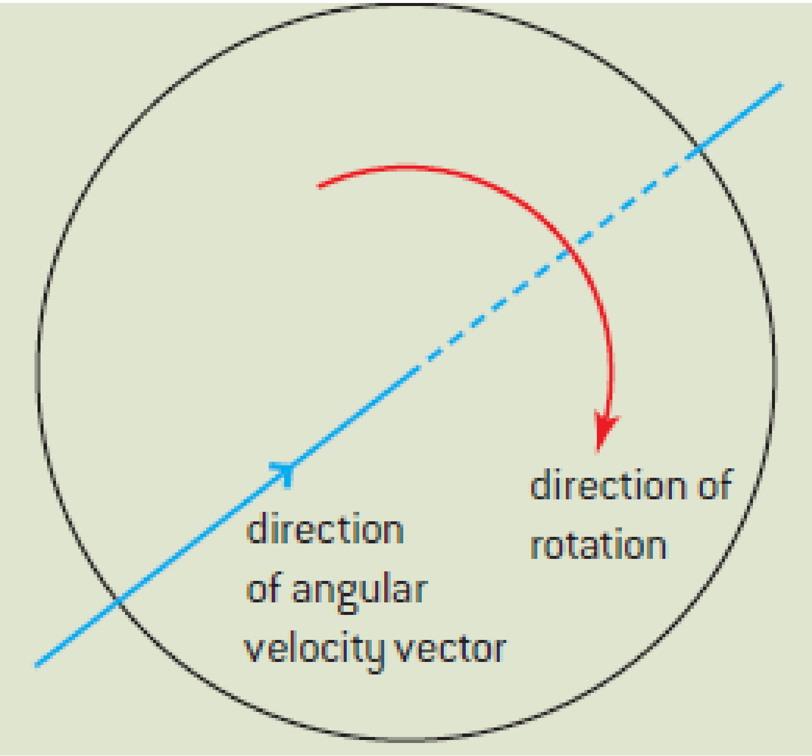
$$\omega\text{, angular speed} = \frac{\theta}{t_2 - t_1} = \frac{\theta}{t}$$

- $\omega = \dfrac{\theta}{t}$



Angular speed or angular velocity?

- Angular velocity is a vector with magnitude equal to angular speed and direction is along the axis of rotation through the centre of circle and perpendicular to plane of rotation(as shown in figure).
- In IB course only the angular speed , the scalar quantity is used.



Period and Frequency

In one period , the angular distance travelled is 2π rad. So,

$$T = \frac{2\pi}{\omega}$$

When T is in seconds and the units of ω are radians per second, (rad s^{-1})

But $T = \frac{1}{f}$

Thus $\omega = 2\pi f$

Relation between linear and angular speed

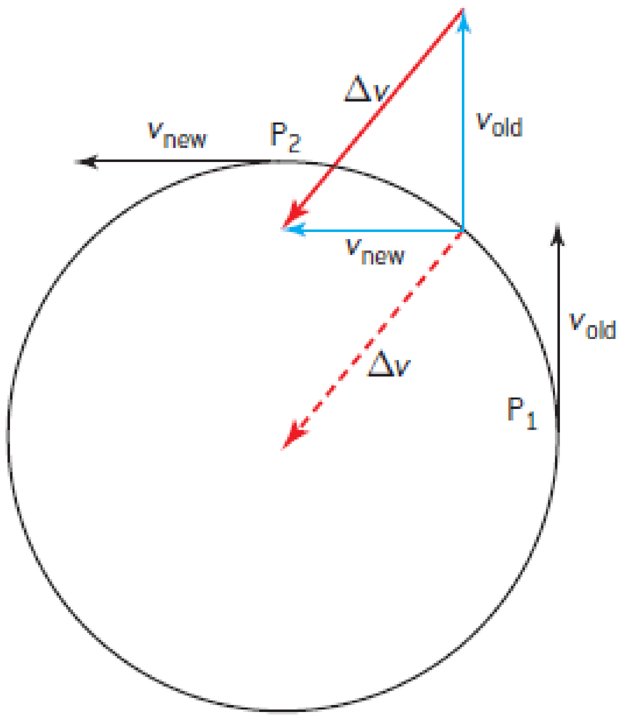
- For a circle having radius r ,the circumference is $2\pi r$ and the time taken to complete one revolution T, then the linear speed is v is given by
- $V = \frac{2\pi r}{T} = 2\pi f$
- Rearranging $T = \frac{2\pi r}{v}$ and by definition $T = \frac{2\pi}{\omega}$ we get $v = r\omega$

Centripetal acceleration

- An object undergoing circular motion at constant speed is being accelerated. So according to Newton's first law there must be an external force acting on it.

The diagram shows two points P_1 and P_2 on the circle together with velocity vectors

v_{old} and v_{new} at these points.

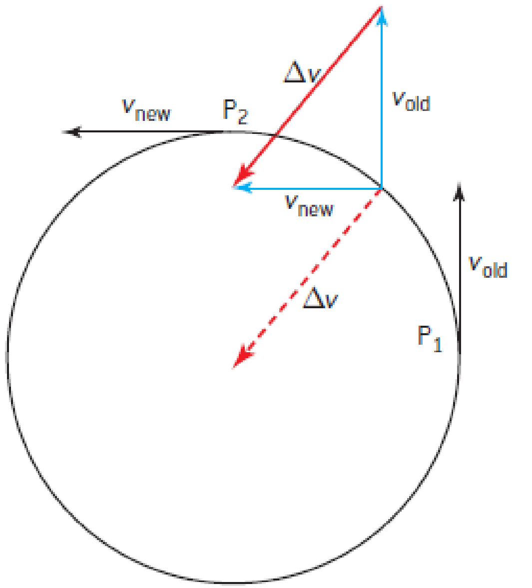


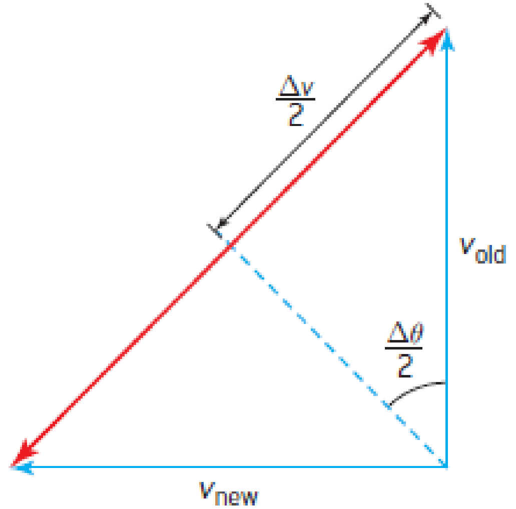
Centripetal Acceleration

The vectors have same length as the speed is constant. The vectors point in different directions as the object moves by angular distance Δθ between points P_1 and P_2

Since acceleration = $\frac{\Delta v}{\Delta t}$

The change in velocity Δv is as shown in the diagram

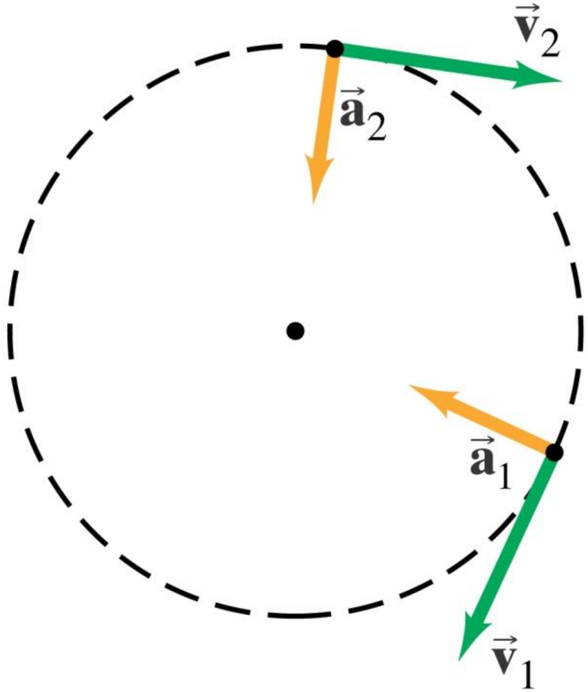




Centripetal acceleration

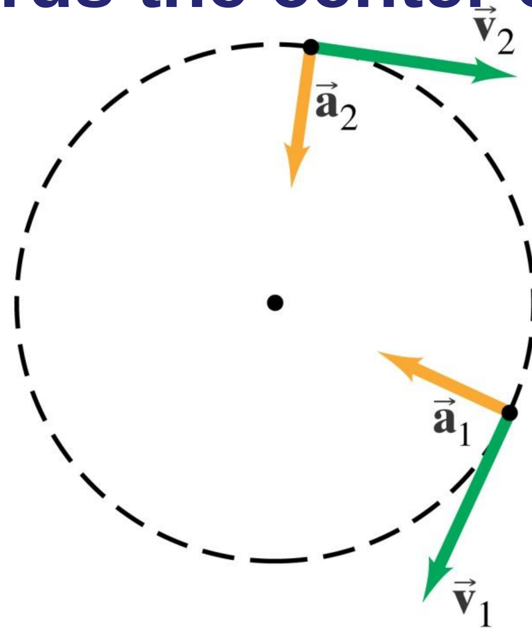
Since, a = $\frac{\Delta v}{\Delta t}$

- The direction of acceleration must be same as direction of Δv
- Therefore the acceleration of a particle with uniform speed in a circle is directed towards the centre of the circle.



Centripetal Acceleration

This acceleration is called the centripetal acceleration (or radial acceleration) and it points towards the center of the circle.



If the object is revolving at constant speed, why is there an acceleration?

Expression for Centripetal Acceleration

For a mass ***m*** moving at a speed ***v*** in a uniform circular motion of radius ***r*** and period ***T***, the centripetal acceleration of the object is given by the following two equations:

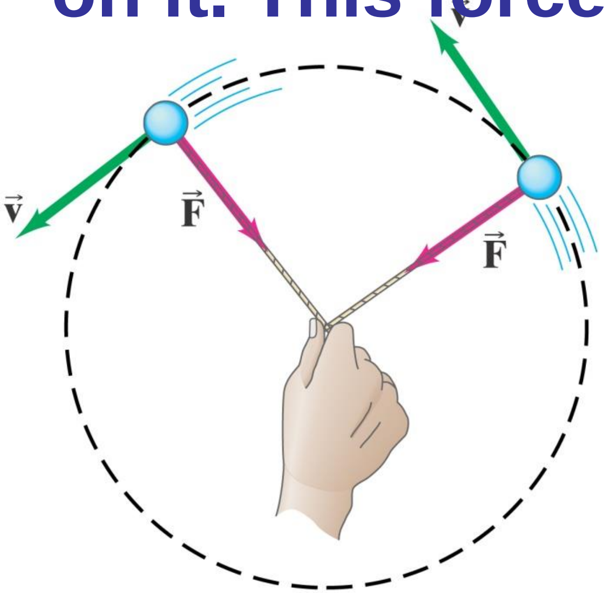
$$a_c = \frac{v^2}{r}$$

$$a = \frac{4\pi^2 r}{T^2}$$

If there is an acceleration, must there be a resultant force present? If yes, why?

Dynamics of Uniform Circular Motion

For an object experiencing centripetal acceleration, there must be a net force acting on it. This force is called centripetal force.



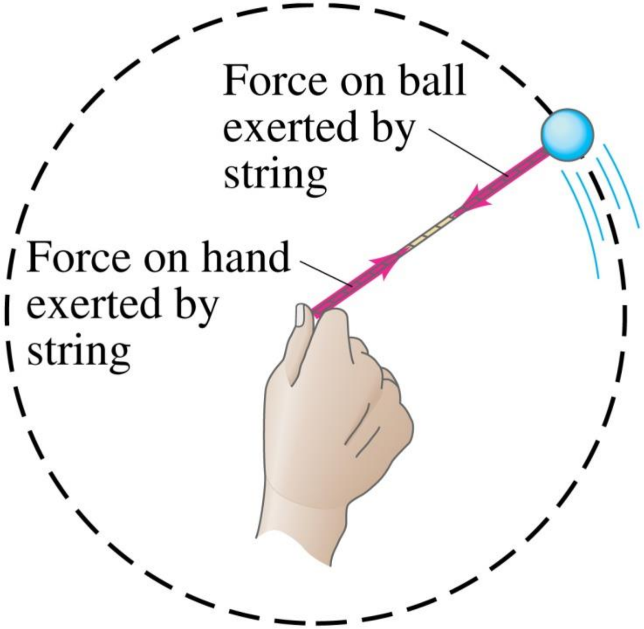
From Newton's second law, we can write:

$$F_c = ma_c = \frac{mv^2}{r} = m\omega^2 r$$

The centripetal force may be provided by friction, gravity, tension, the normal force, or others.

Dynamics of Uniform Circular Motion

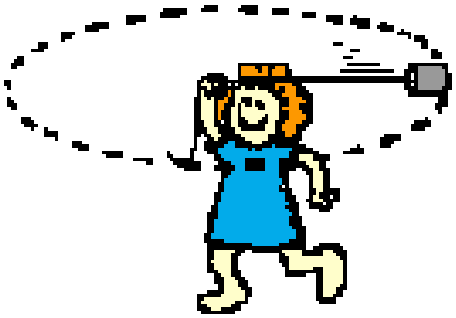
We can see that the centripetal force must be inward by thinking about a ball on a string:

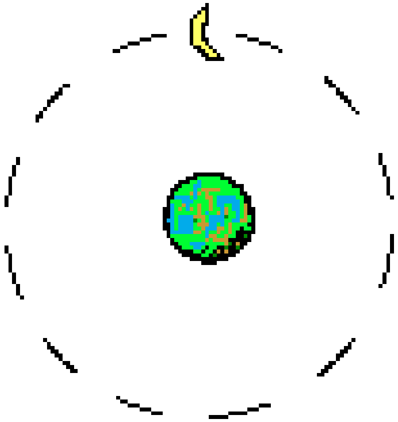


Centripetal force is not a new kind of force. The term merely describes the direction of the net force needed to provide a circular path. Centripetal force must be applied by other objects.

For this example, to swing a ball in a circle, you pull on the string and the string exerts the force on the ball. The force on the ball exerted by the string provides the centripetal force.

Examples of Centripetal Force





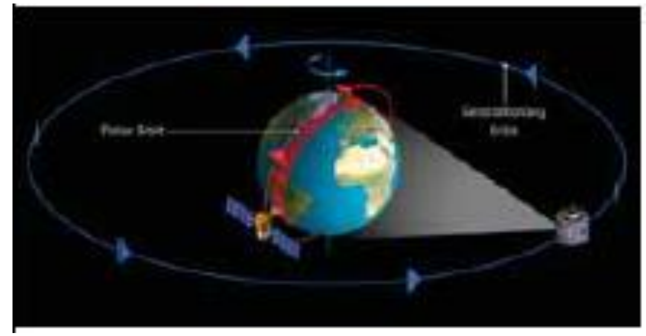
As a car makes a turn, the force of friction acting upon the turned wheels of the car provide the centripetal force required for circular motion.

As a bucket of water is tied to a string and spun in a circle, the force of tension acting upon the bucket provides the centripetal force required for circular motion.

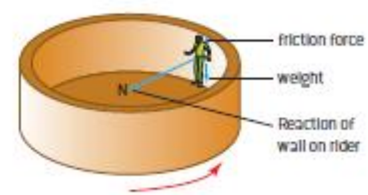
As the moon orbits the Earth, the force of gravity acting upon the moon provides the centripetal force required for circular motion.

Examples of centripetal force

- **Satellites in orbit**



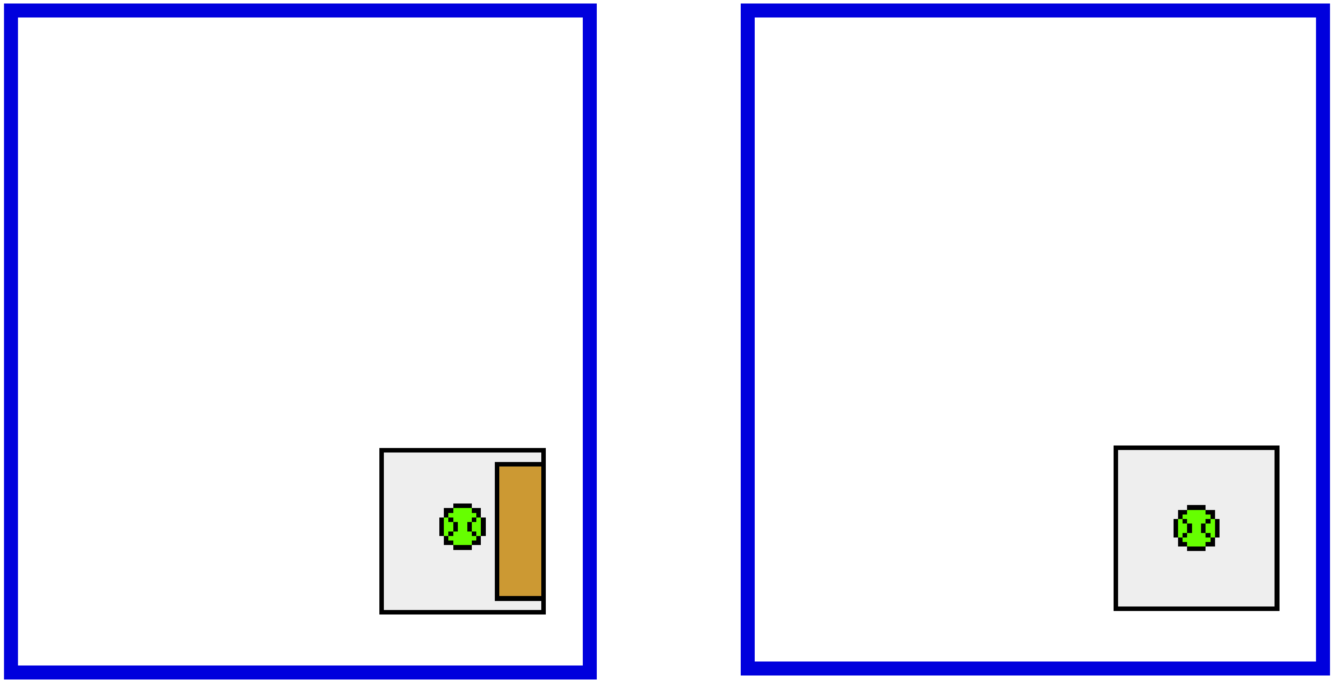
- **Amusement park rides**



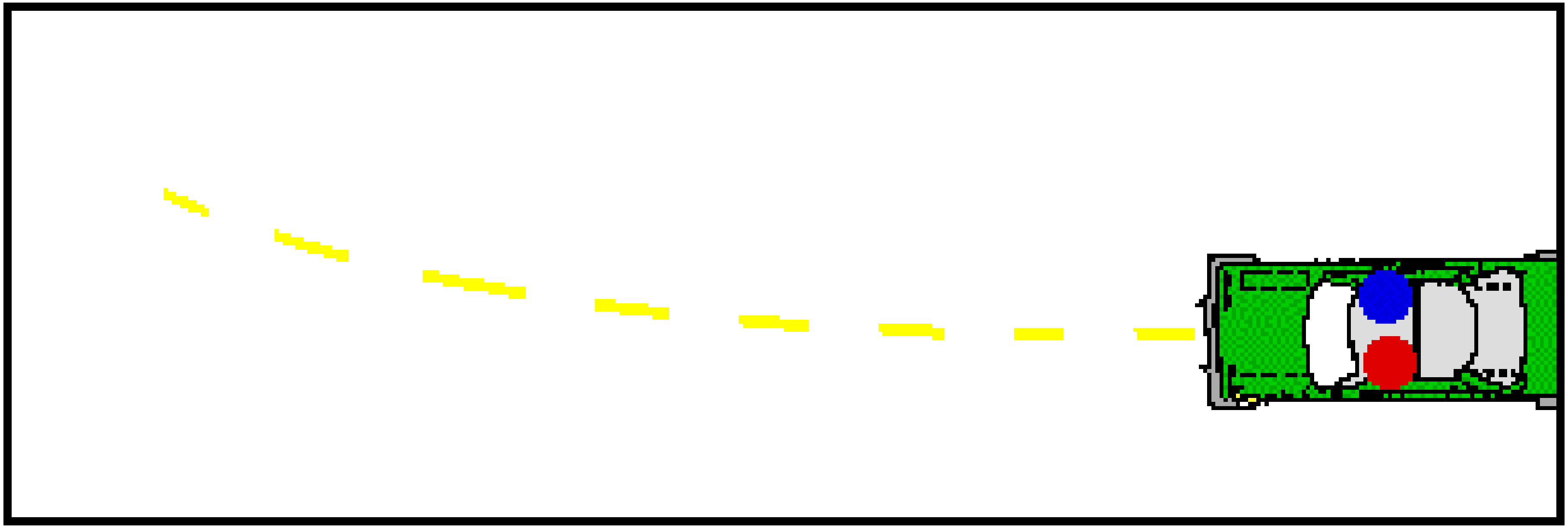
Images : Physics Course Companion –David Homer and Michael Bowen Jones _Oxford 2014

Dynamics of Uniform Circular Motion

If the centripetal force vanishes, the object flies off tangent to the circle.

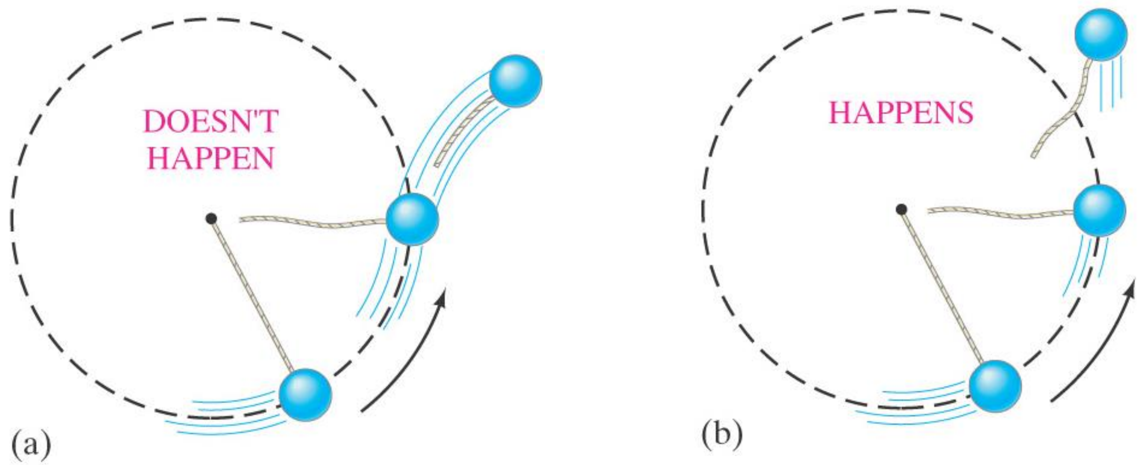


Inertia and the right hand turn



No such thing as a Centrifugal "Force"

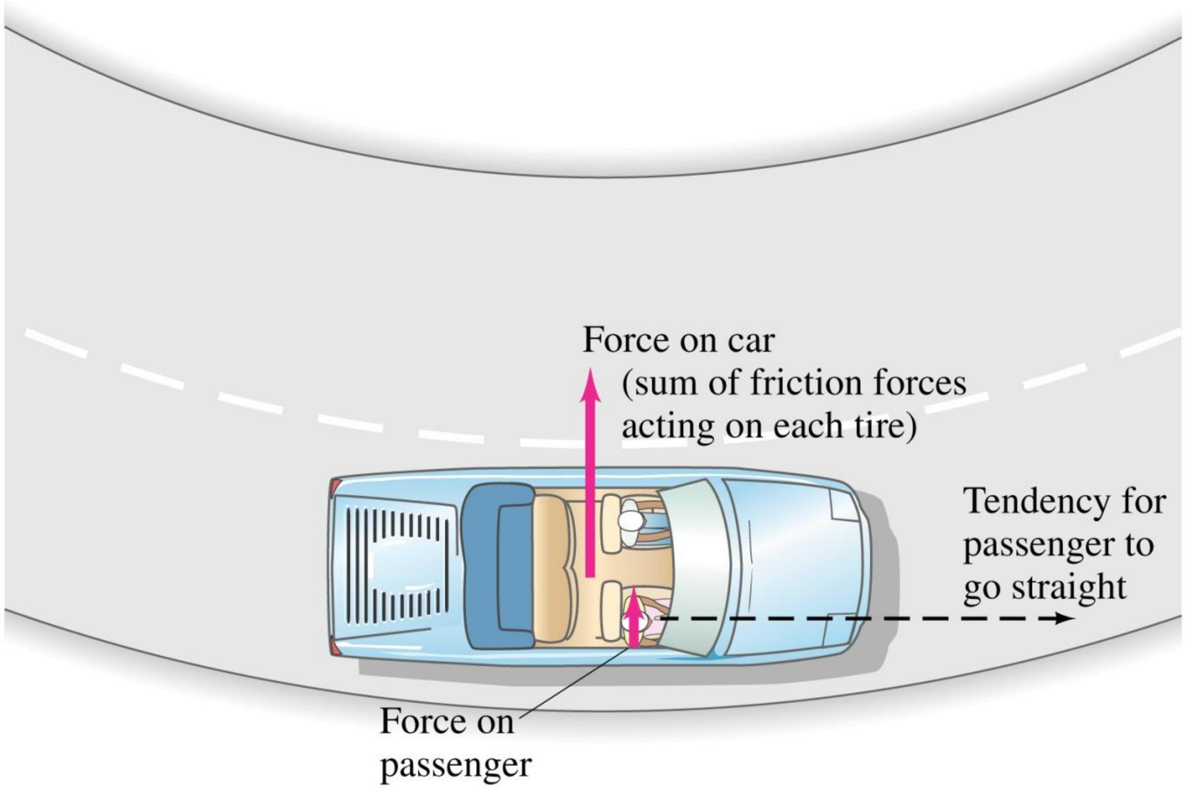
If centrifugal force existed, the revolving ball would fly outward as in (a) when released. In fact, it flies off tangentially as in (b).



There is <u>no</u> centrifugal "force" pointing outward; what happens is that the natural tendency of the object to move in a straight line must be overcome.

Highway Curves, Banked and Unbanked

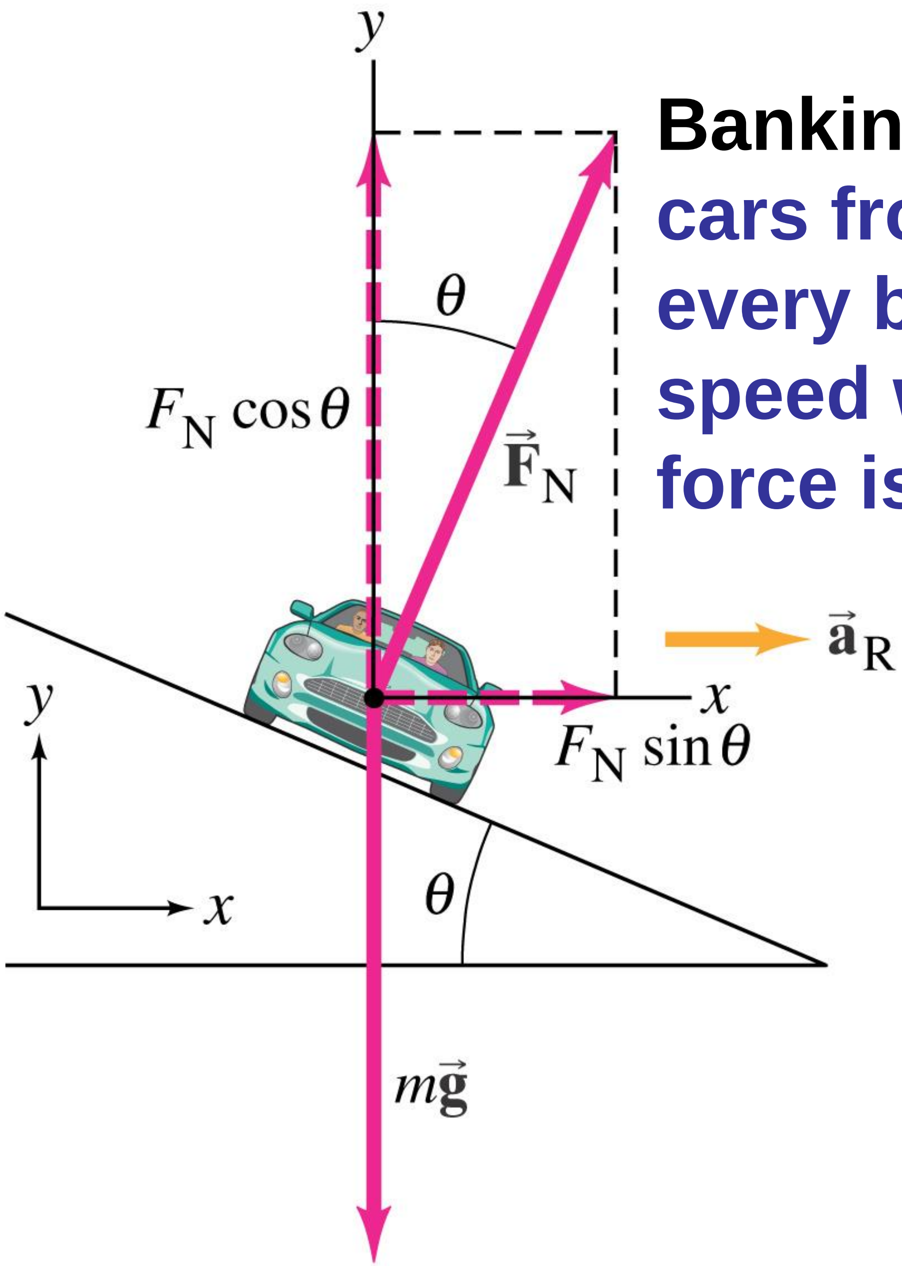
When a car goes around a curve, there must be a net force towards the center of the circle of which the curve is an arc. If the road is flat, that force is supplied by friction.



Highway Curves, Banked and Unbanked

If the frictional force is **insufficient**, the car will tend to move more nearly in a **straight line**, as the skid marks show.

Highway Curves, Banked and Unbanked



Banking the curve can help keep cars from skidding. In fact, for every banked curve, there is one speed where the entire centripetal force is supplied by the horizontal component of the **normal** force, and no friction is required. This occurs when:

$$F_N \sin\theta = m\frac{v^2}{r}$$

Highway and banked roads

$$F_N \sin\theta = m\frac{v^2}{r} \qquad (1)$$

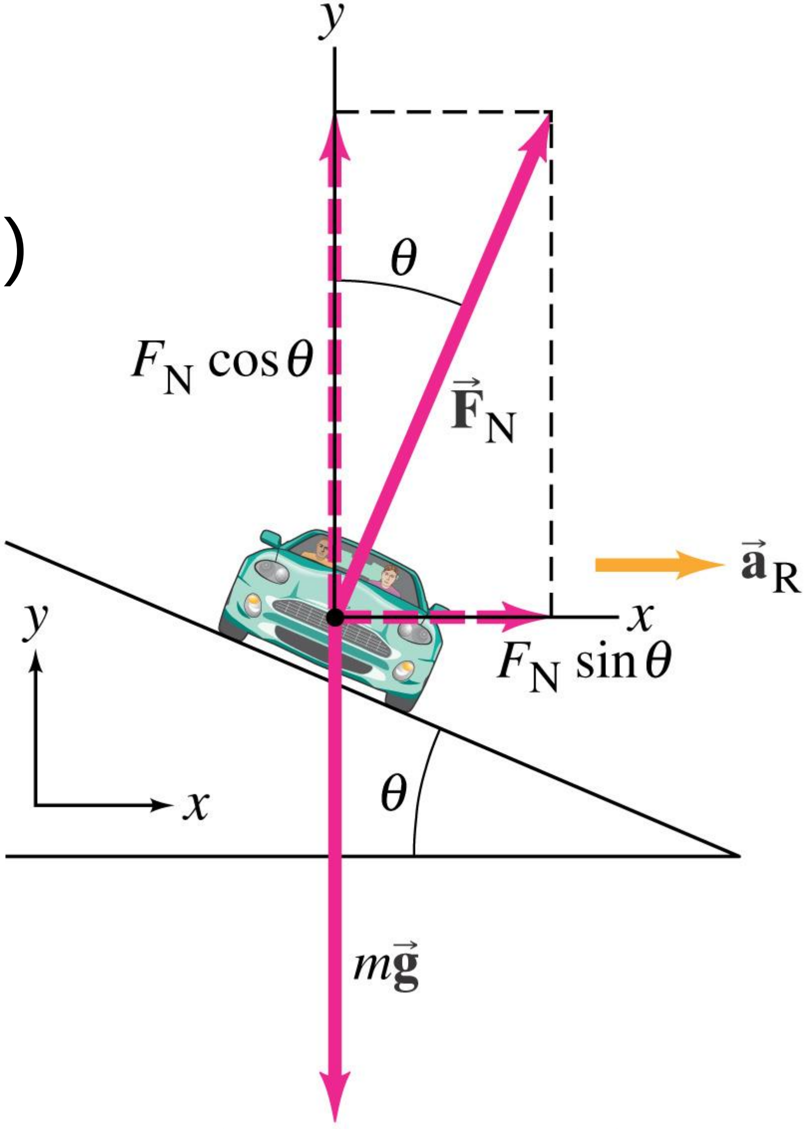
The vertical component balances mg

$F_N \cos\theta = mg$ (2)

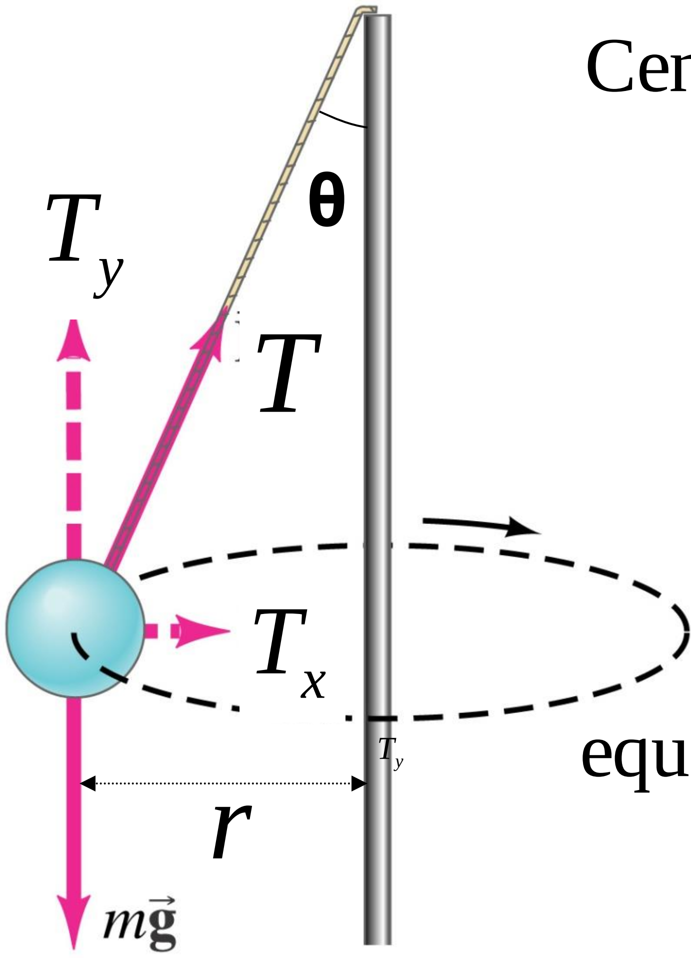
Divide (1) by (2)

$\tan\theta = \frac{v^2}{rg}$

This eqn is independent of mass of vehicle.



Conical Pendulum



Centripetal force is provided by the horizonal component of tension.

$$T_x = T\sin\theta = \frac{mv^2}{r}$$

If the pendulum is at equilibrium in the vertical axis, then

$$T_y = mg$$

Problem Solving Strategies

1. Identify the particle or object moving in circular motion
2. Identify all the forces acting on the particle or object of interest and draw a free-body diagram
3. Determine which of the forces, or which of their components, act to provide the centripetal force.
4. The sum of forces or components of forces that act toward or away from the center of the circular path is the resultant force.
5. Equate this resultant force (which is the centripetal force) to mass times centripetal acceleration

$$\sum F = F_c = ma_c = \frac{mv^2}{r}$$

Summary of Uniform Circular Motion

- **An object moving in a circle at constant speed is in uniform circular motion.**
- **It has a centripetal acceleration**

$$a_c = \frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

- **There is a centripetal force given by**

$$F_c = ma_c = \frac{mv^2}{r} = \frac{4\pi^2 rm}{T^2}$$

- **The centripetal force may be provided by friction, gravity, tension, the normal force, or others.**
- **Other equations**

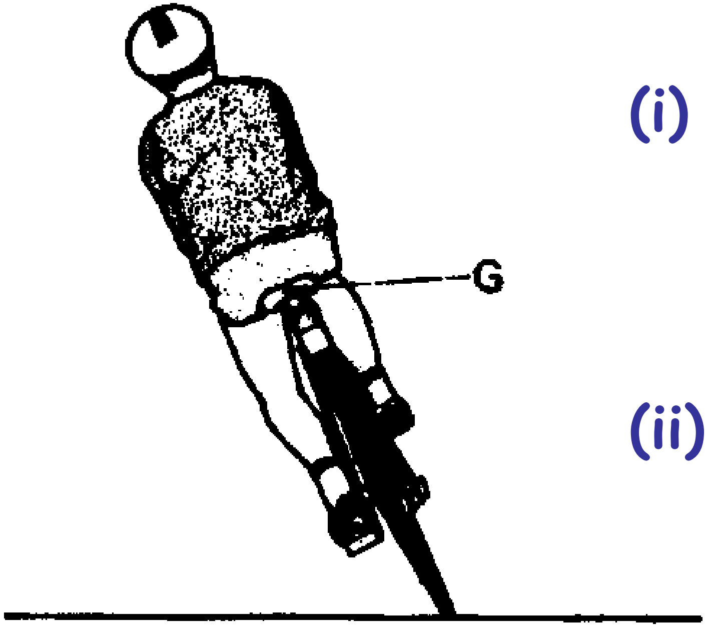
$$T = \frac{1}{f}$$

$$v = \frac{2\pi r}{T} = 2\pi r f$$

Problem Solving (Horizontal Circular Motion)

Question 1

The diagram represents a cylist making a left turn on a rough road surface at constant speed *v*, as viewed from behind. The total mass of the bicycle and rider is *m* and their combined centre of gravity is at *G*.



(i) Draw a vector diagram representing the directions of the forces acting on the bicycle and the rider

(ii) Indicate the centripetal force causing the motion

Problem Solving (Horizontal Circular Motion)

Question 2

The maximum frictional force between the road surface and the wheels of a certain car at a particular unbanked bend is halved when the road is wet. If the maximum safe speed for rounding the bend is 20 ms^{-1} when the road is dry, what is the maximum safe speed when the road is wet?

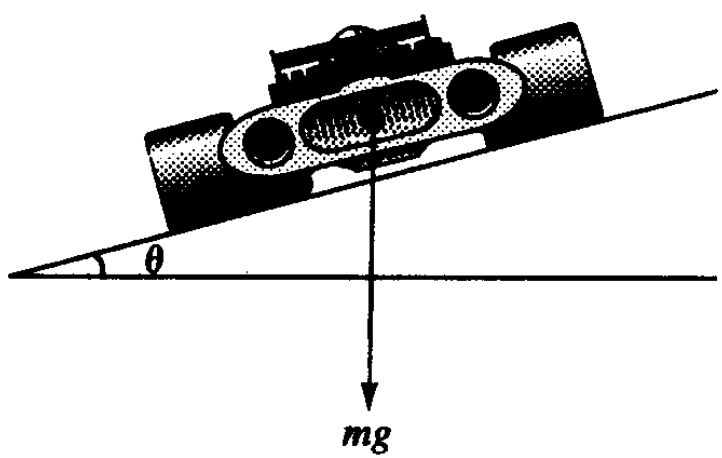
Ans: 14 ms^{-1}

Problem Solving (Angled Circular Motion)

Question 3

You cannot always count on friction to get your car around a curve, especially if the road is icy or wet. That is why highways are banked.

A curve of radius 30m is to be banked so that a car may make the turn at a speed of 13 ms^{-1} without depending on friction. What must be the slope of the curve?



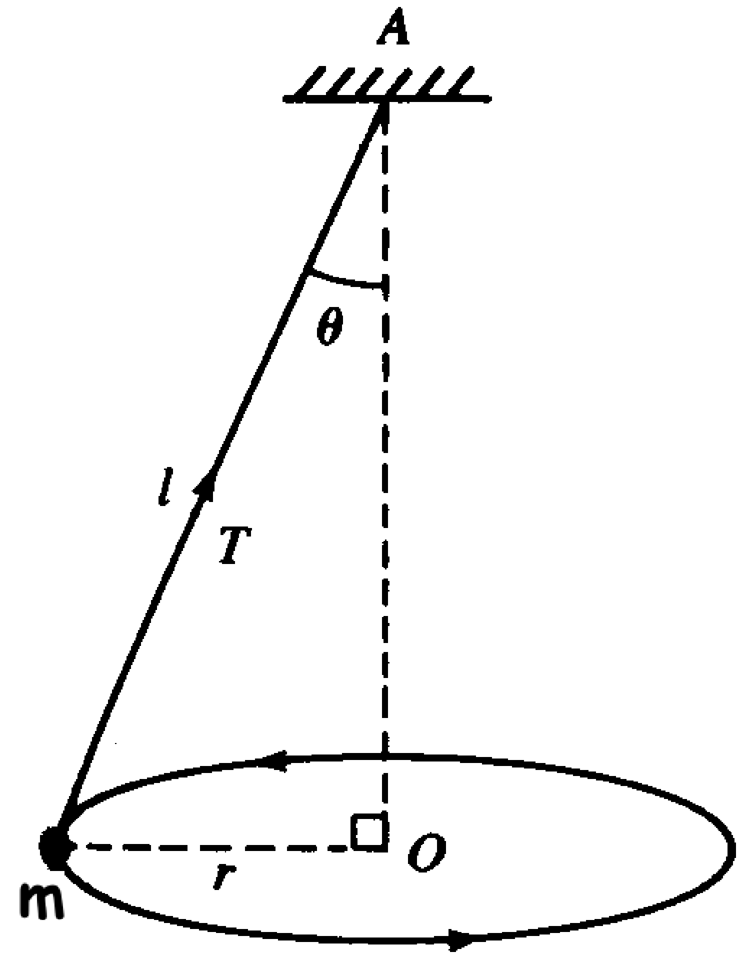
Ans: 30^{o}

Problem Solving (Conical Pendulum)

Question 4

A bob, mass m 1.5 kg, whirls around in a horizontal circle at constant speed v at the end of a cord whose length L, 1.7m. It makes an angle θ of 37° with the vertical. Find the period of the pendulum.

Ans: 2.3 sec



Problem Solving (Vertical Motion)

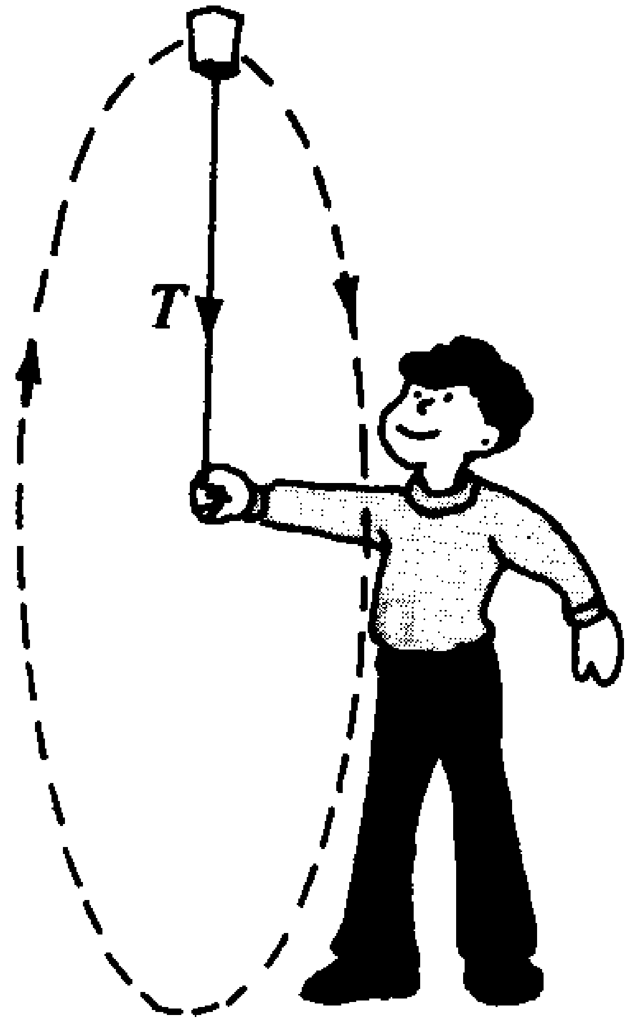
Question 5*

A stone of mass m is attached to a string of length r, which will break if the tension in it exceed Tmax.

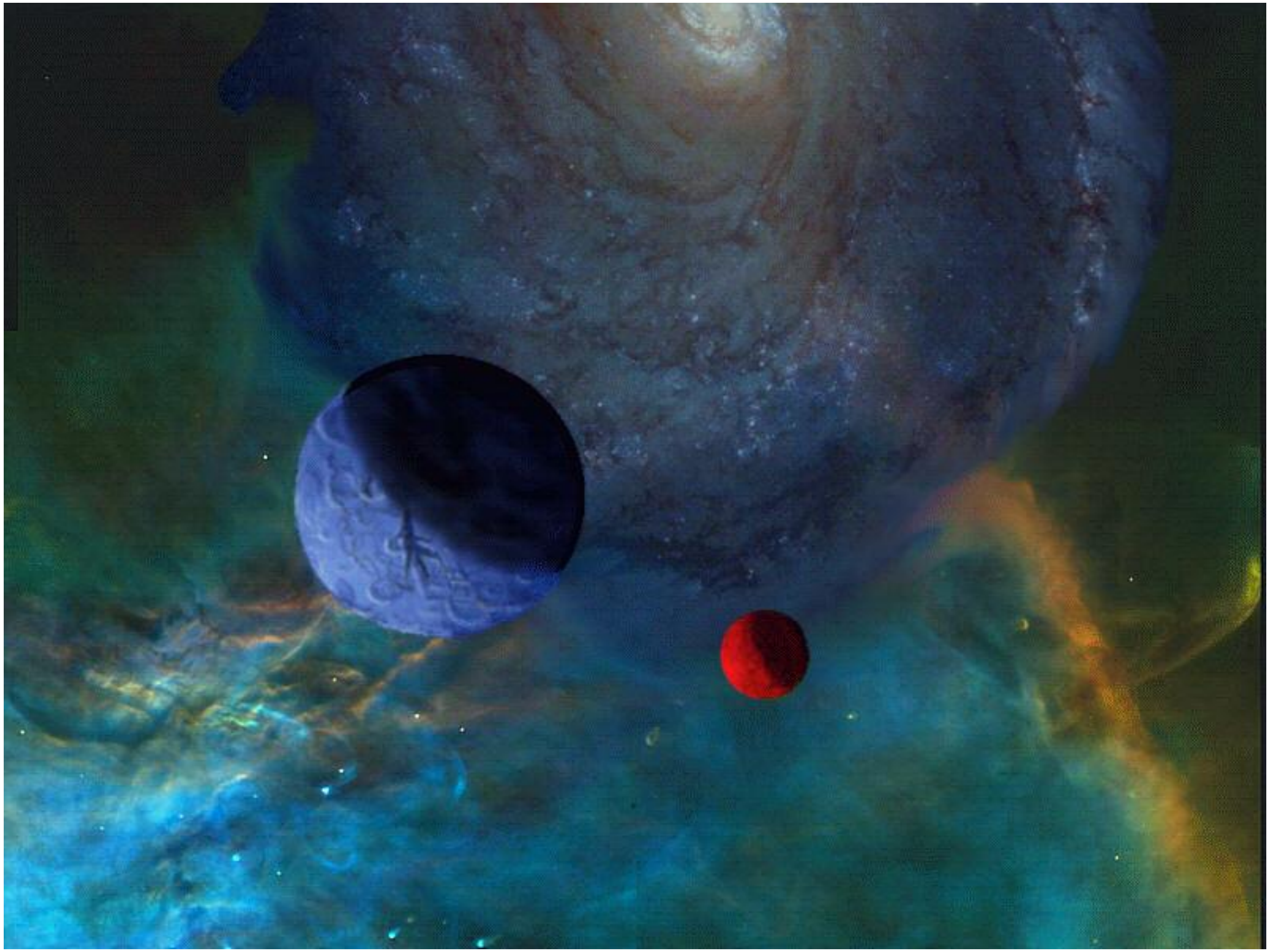
(a) Draw force diagrams showing forces acting on the stone when it is at

(i) top, (ii) bottom of circle

(b) For what position of the stone is the string most likely to break?



6.2 Gravitational force and field



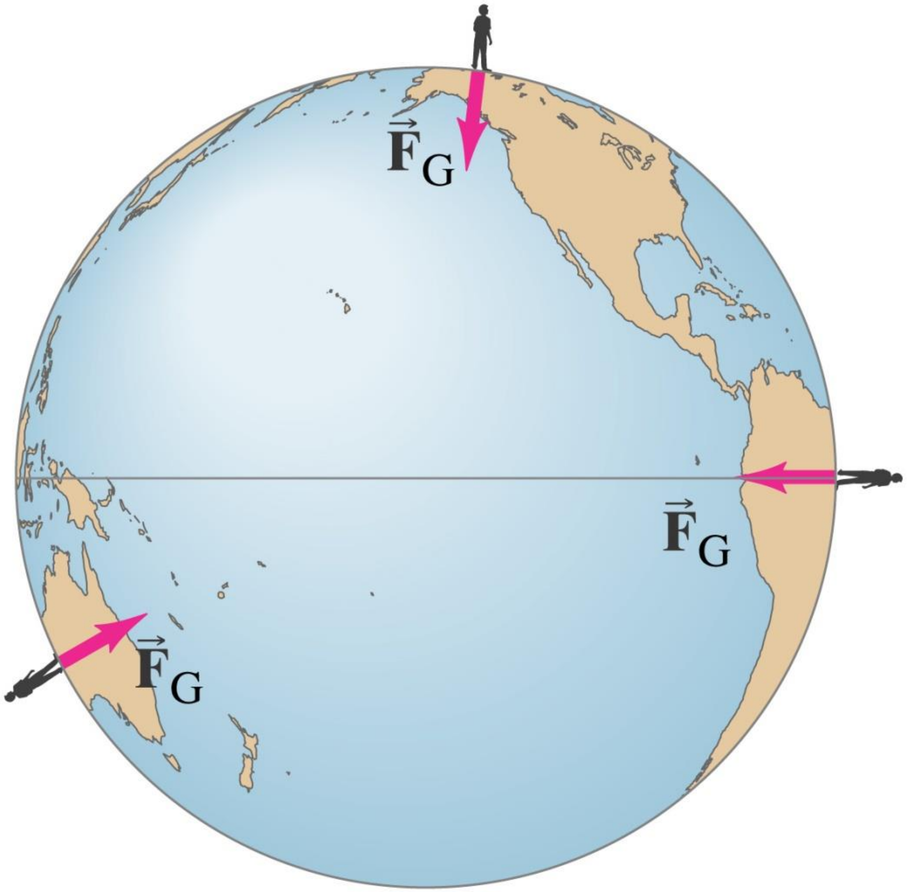
Parts adapted from Giancoli Lecture Powerpoint, Chapter 5, EW

6.2 Gravitational force and field

- **Newton's Law of Universal Gravitation**
- **Gravitational Field Strength**
- **Gravitational field due to one or more point masses**
- **Derivation of Expression for Gravitational Field Strength at the surface of a planet**

Newton's Law of Universal Gravitation

If the force of gravity is being exerted on objects on Earth, what is the **origin** of that force?



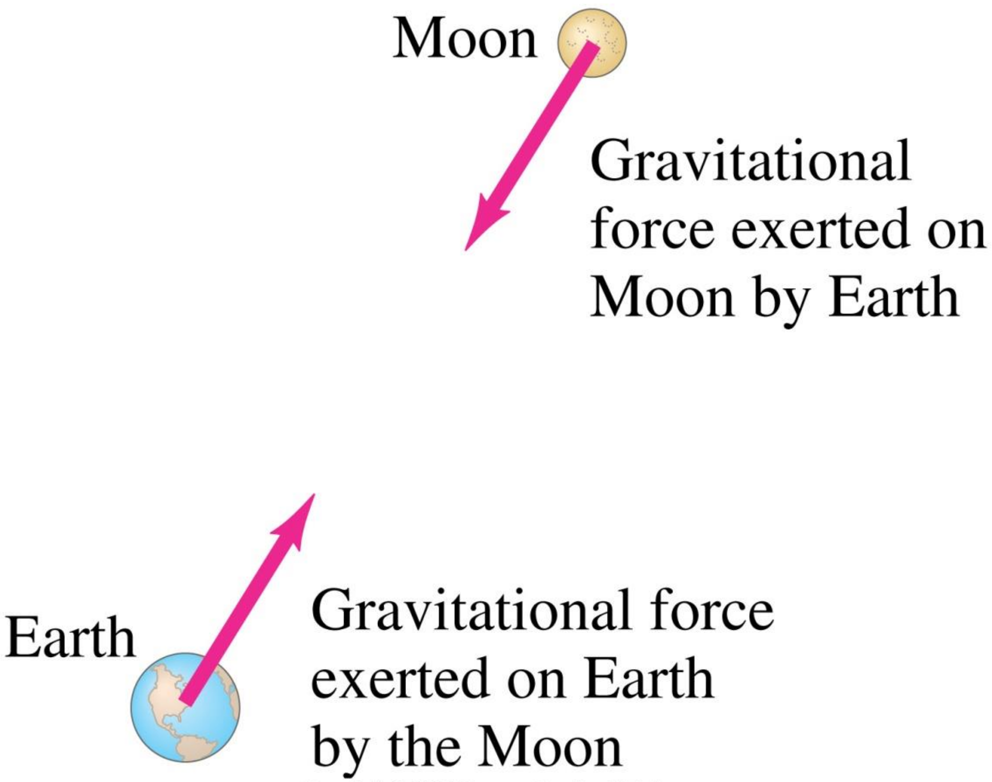
Newton's realization was that the force must come from the **Earth.**

He further realized that this force must be what keeps the **Moon** in its orbit.

Newton's Law of Universal Gravitation

The gravitational force on you is one-half of a Third Law pair: the **Earth** exerts a **downward** force on you, and you exert an **upward** force on the Earth.

When there is such a **disparity** in masses, the reaction force is undetectable, but for bodies more equal in mass it can be **significant**.



Newton's Law of Universal Gravitation

Therefore, the gravitational force must be proportional to both masses.

By observing planetary orbits, Newton also concluded that the gravitational force must decrease as the inverse of the square of the distance between the masses.

In its final form, the Law of Universal Gravitation reads:

$$F = G\frac{m_1 m_2}{r^2}$$

where

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Statement of Newton's Law of Universal Gravitation

Newton's Universal Law of Gravitation states every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

If the particles have masses m_1 and m_2 and are separated by a distance r (from their centers of gravity), the magnitude of this gravitational force is:

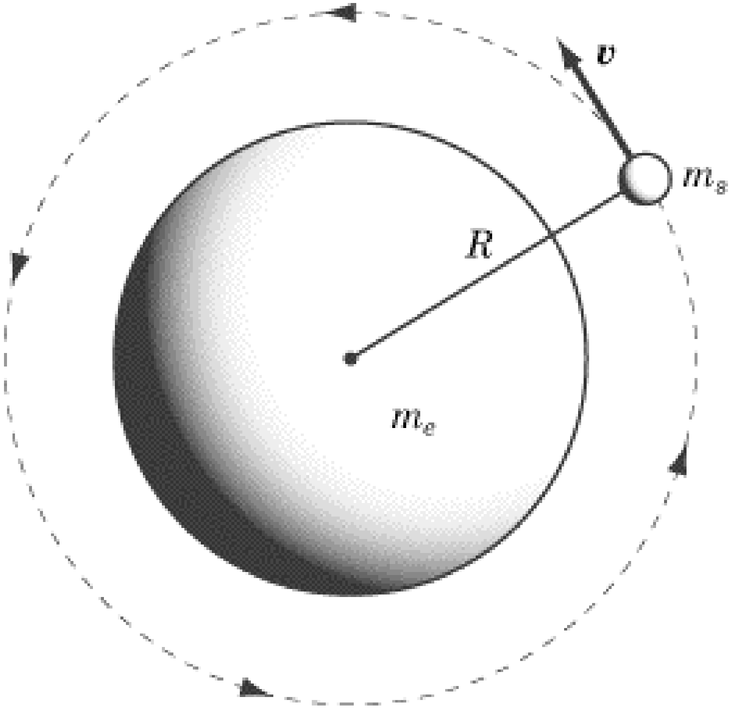
$$F = G\frac{m_1 m_2}{r^2}$$

where G is a universal constant called the gravitational constant.

Points to note about Newton's Law of Universal Gravitation

1. **The law only deals with point masses.**
2. **The interaction between two spherical masses turns out to be the same as if the masses were concentrated at the centers of the spheres. This is true if the separation between the two spheres is large compared to their radii.**
3. **There is a force acting on each of the masses. These forces are EQUAL and OPPOSITE (even if the masses are not equal).**
4. **The forces are always attractive**
5. **Gravitation forces act between ALL objects in the Universe. The forces only become significant if one (or both) of the objects involved are massive, but they are there nonetheless.**

Gravitational Force and Centripetal Force



Consider a satellite of mass m_s revolving in a circular orbit about the earth of mass m_e .

The point-to-point distance between the centres of earth and the satellite is R.

The gravitational force between the masses provided the centripetal force for the satellite to stay in its orbit.

Gravitational Force and Centripetal Force

Mathematically for the satellite;

$$\frac{Gm_s m_e}{R^2} = \frac{m_s v^2}{R}$$

$$v = \sqrt{\frac{Gm_e}{R}}$$

where v is the orbital speed

The orbital period T of the satellite can be found using:

$$T = \frac{2\pi R}{v}$$

Gravitational Field Strength

Gravitational Field Strength at a point is defined as the force per unit mass exerted on a mass placed at that point.

Alternatively: Gravitational Field Strength at a point is defined as the force acting on a 1kg mass placed at that point.

Gravitational Field Strength is a vector.

The SI unit for Gravitational Field Strength is N kg^{-1}.

Expression for Gravitational Field Strength from a point mass

Gravitational Field Strength at a point is defined as the force per unit mass exerted on a mass placed at that point.

From Newton's Law of Gravitation :

$$F = \frac{GMm}{r^2}$$

rearranging we get $g = \frac{F}{m} = \frac{GM}{r^2}$

$$\text{Gravitational Field Strength, } g = \frac{GM}{r^2}$$

Derivation of Expression for Gravitational Field Strength at the surface of a planet

At the surface of a planet of radius R,

$$\text{Gravitational Field Strength} = \frac{F}{m} = \frac{GM}{R^2}$$

$$\text{From Newton's 2nd Law, } \frac{F}{m} = a = g$$

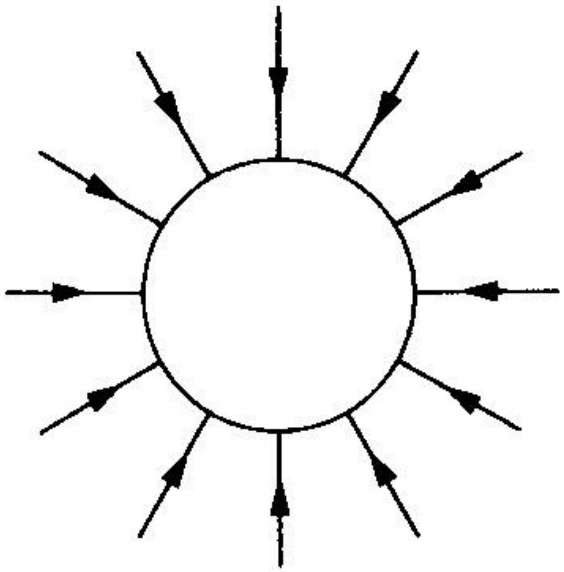
$$\text{Therefore } g = \frac{GM}{R^2}$$

Therefore gravitational field strength at the surface of a planet must be the same as the acceleration due to gravity at the surface.

*** <u>Assumption</u>: all mass concentrated at its centre**

Gravitational Field Strength

Gravitational Field Strength is a <u>vector quantity</u> and can be represented by the use of field lines.



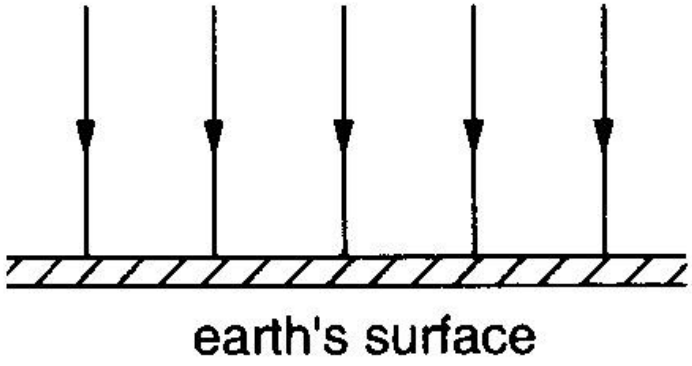
-Represented by imaginary field lines

-For Spherical body, gravitational field lines are directed radially inward towards the centre of body

-Lines closer, strong field

-Lines widely separated, weak field

-Near surface of earth, lines of force almost parallel, hence constant field



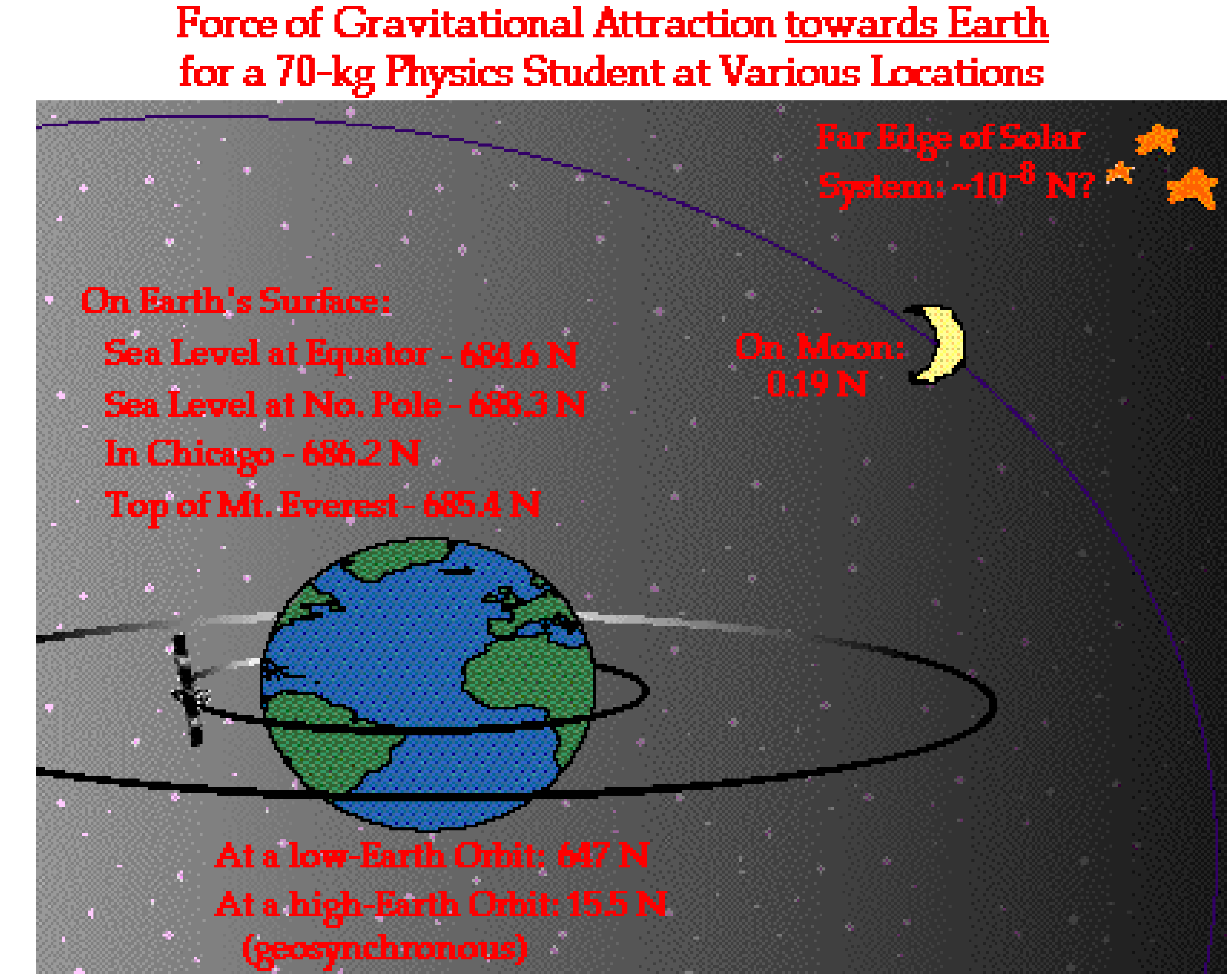
Gravity Near the Earth's Surface; Geophysical Applications

TABLE 5–1
Acceleration Due to Gravity at Various Locations on Earth

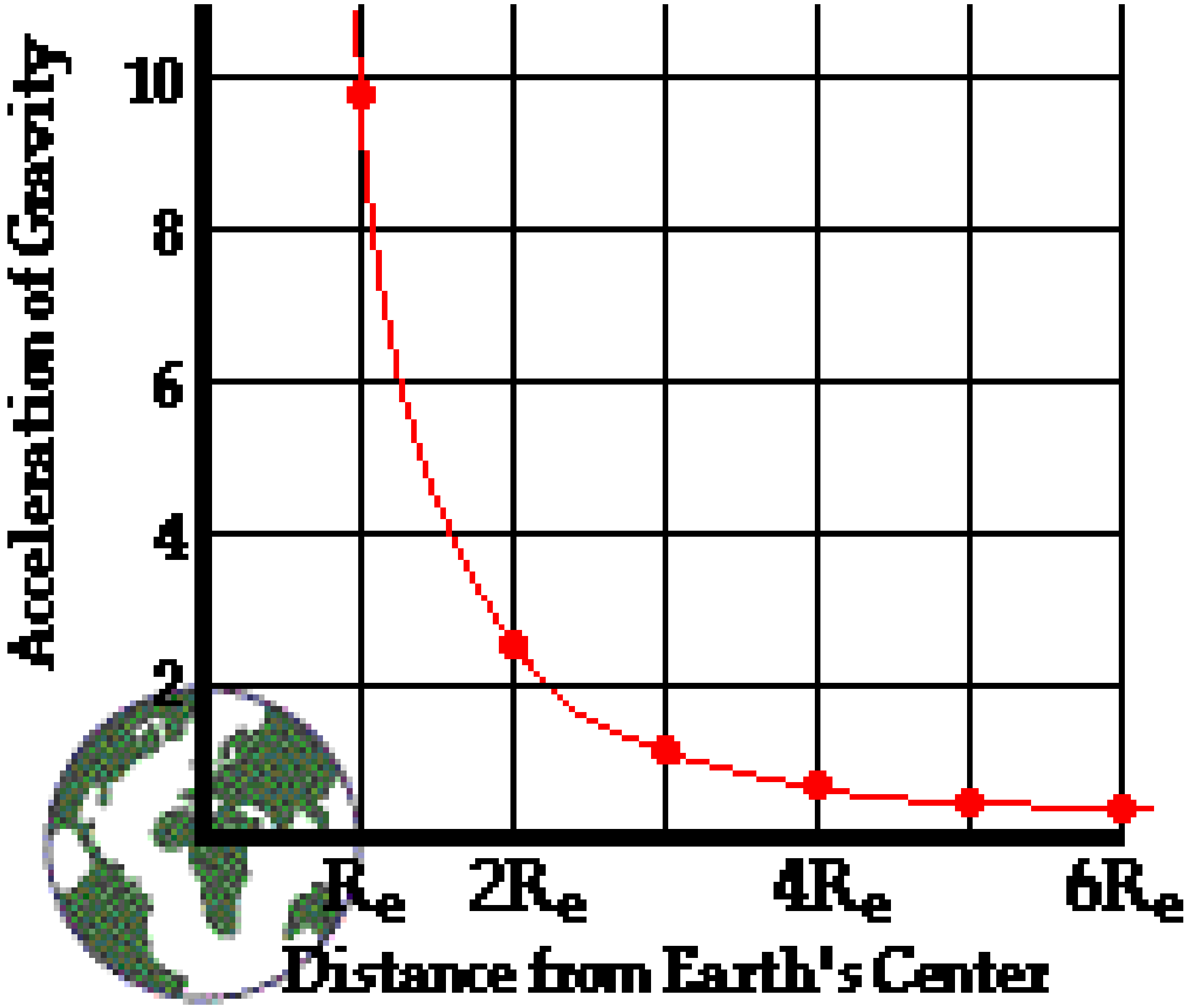
Location	Elevation (m)	g (m/s^2)
New York	0	9.803
San Francisco	0	9.800
Denver	1650	9.796
Pikes Peak	4300	9.789
Sydney, Australia	0	9.798
Equator	0	9.780
North Pole (calculated)	0	9.832

The acceleration due to gravity **varies** over the Earth's surface due to altitude, local geology, and the shape of the Earth, which is not quite spherical.

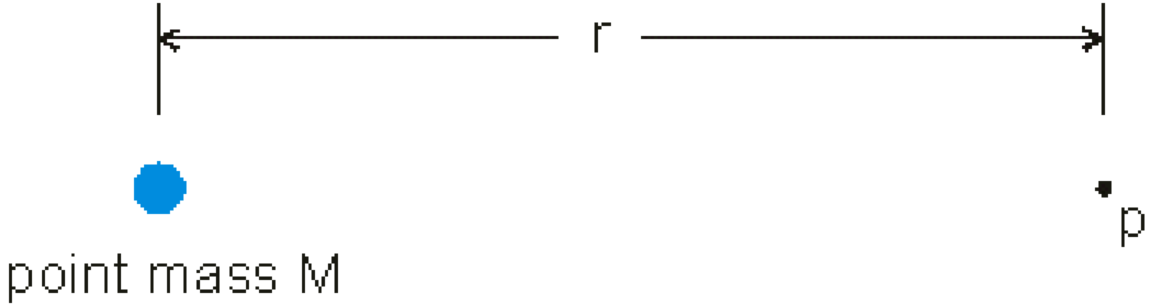
Force of Gravitation at various locations



Gravity Near the Earth's Surface

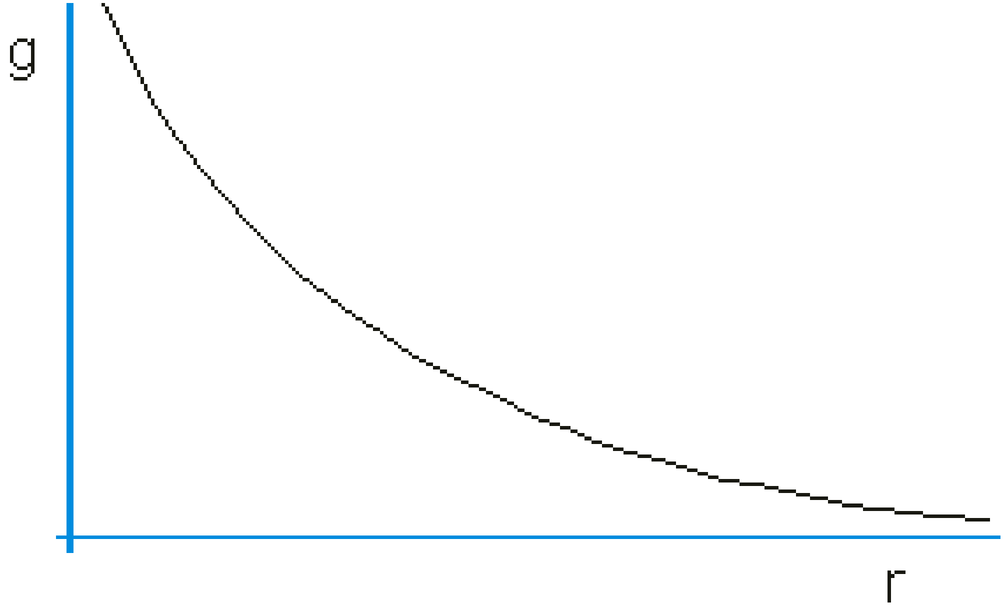


Variation of g with Distance From A Point Mass

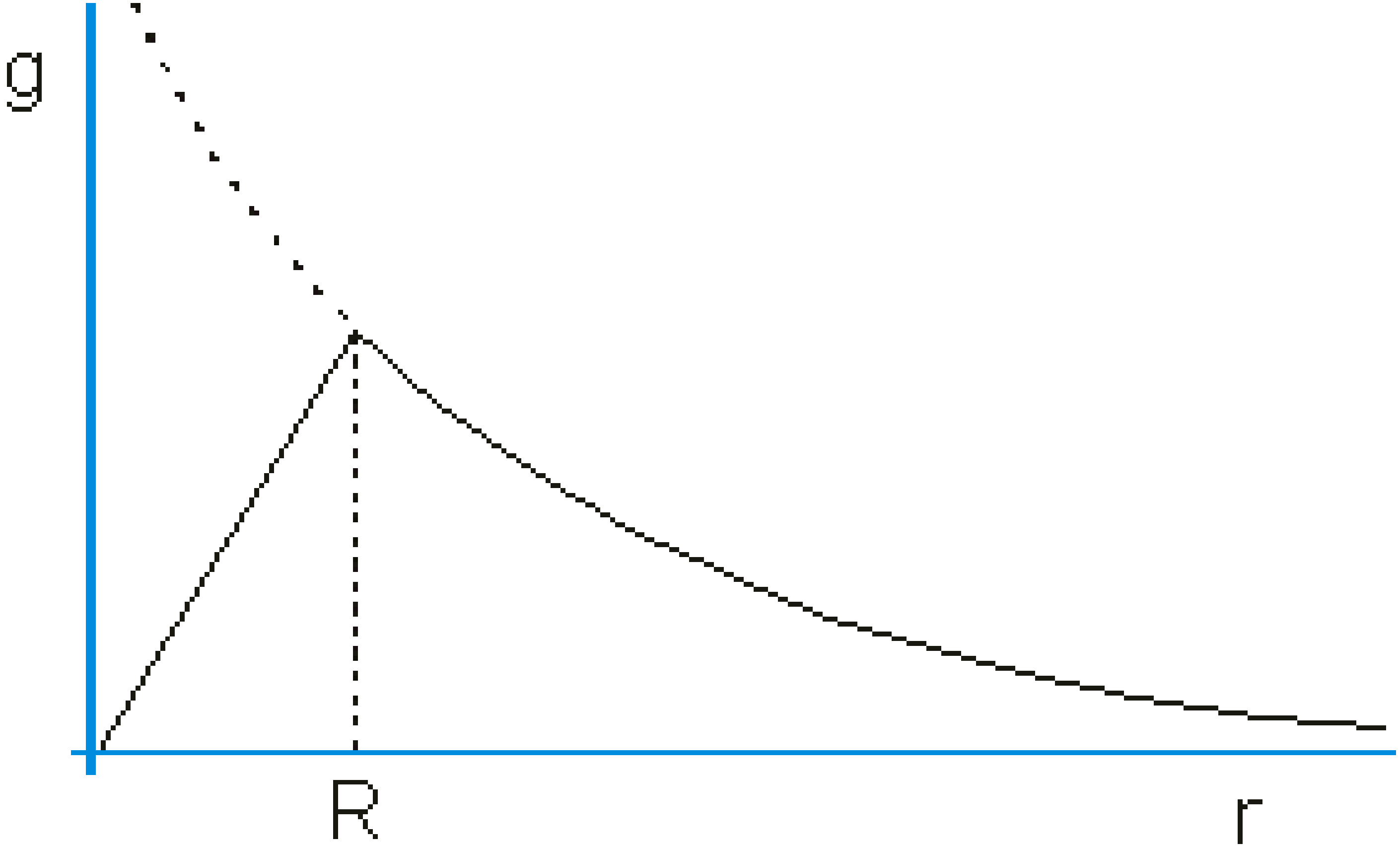


Gravitational Field Strength at point p is given by

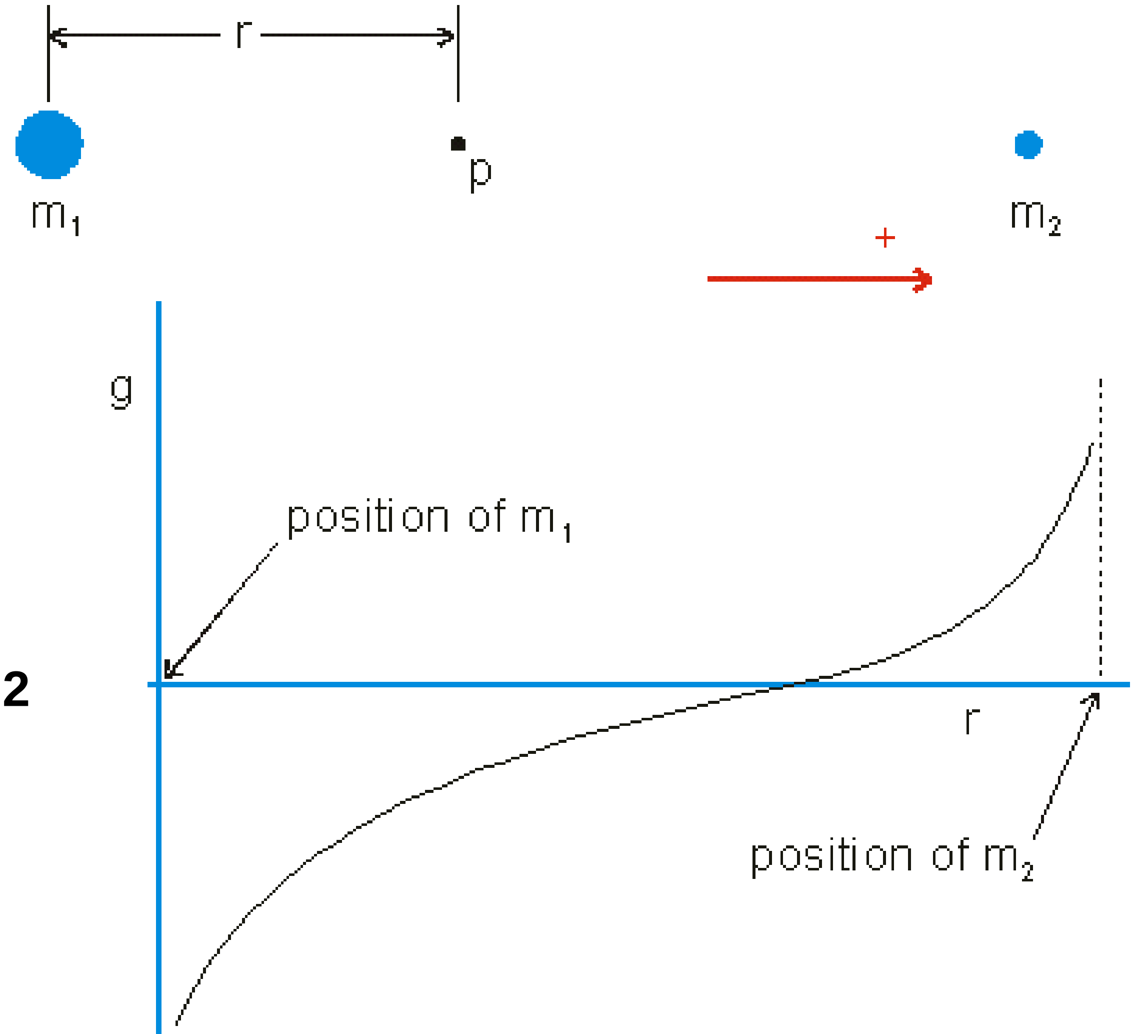
$$g = G\frac{M}{r^2}$$



Variation of g with Distance from the Centre of a Uniform Spherical Mass of radius, R



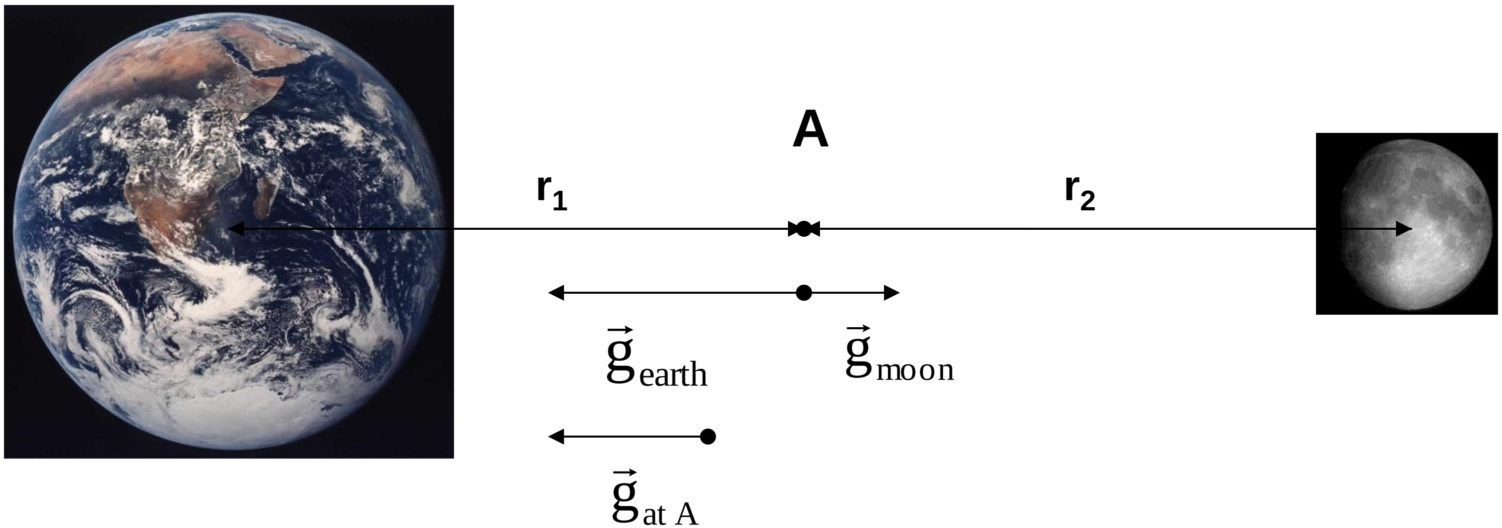
Variation of g on a line joining the Centres of Two Point Masses



$m_1 > m_2$

Gravitational Field Strength is a Vector

The overall Gravitational Field Strength at any point is the resultant of a <u>vector addition</u> of all Gravitational Field Strength vectors at that point.



$$\vec{g}_{at\ A} = \vec{g}_{earth} + \vec{g}_{moon}$$

Is there a point between the Earth and the Moon where the Gravitational Field Strength is zero?