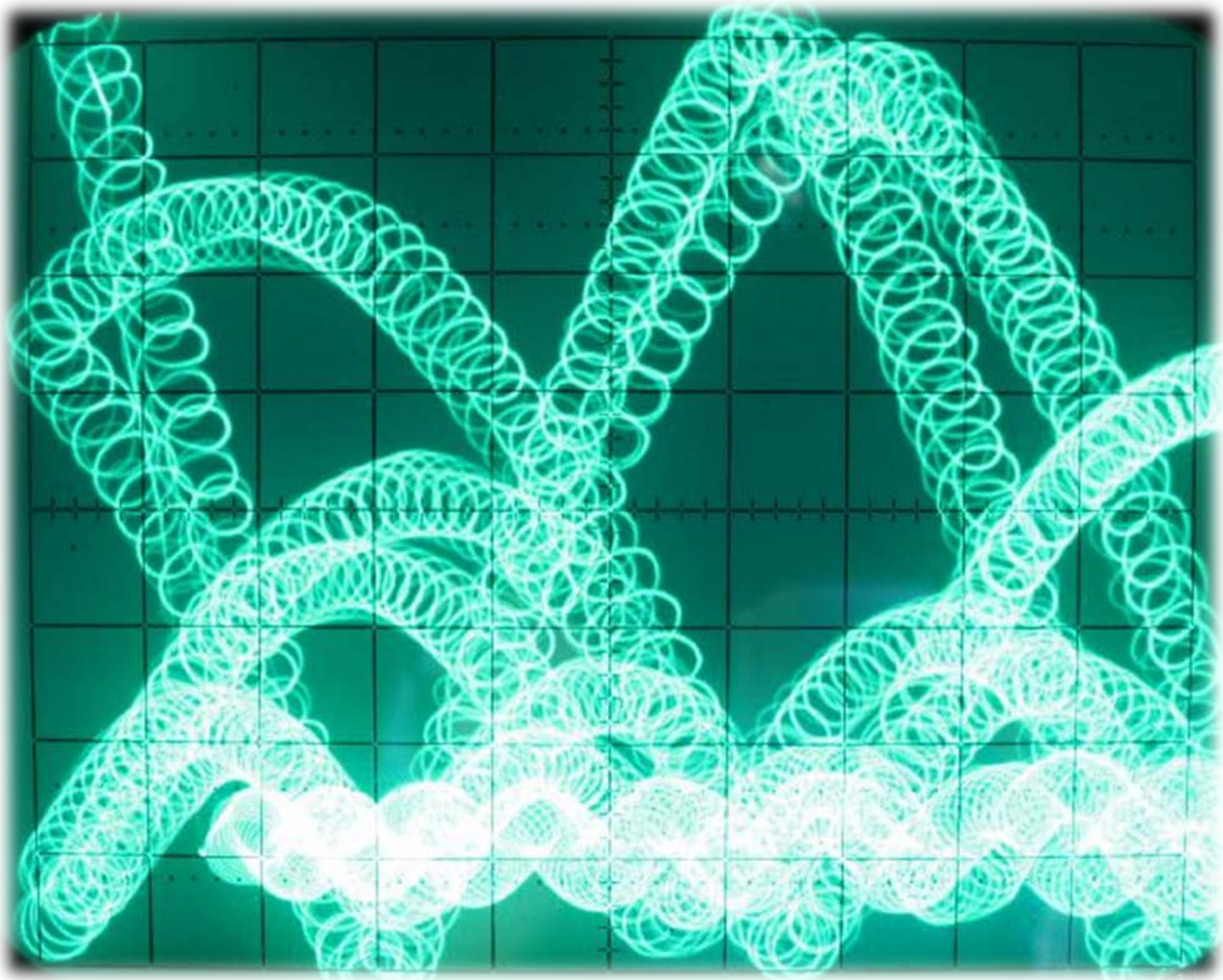
Topic 9

Wave Phenomena



9.1 Simple Harmonic Motion

9.1 Nature of Science

Insights: The equation for simple harmonic motion (SHM) can be solved analytically and numerically. Physicists use such solutions to help them to visualize the behaviour of the oscillator. The use of the equations is very powerful as any oscillation can be described in terms of a combination of harmonic oscillators. Numerical modelling of oscillators is important in the design of electrical circuits

Definitions

Displacement, *x*:

Displacement is distance the object has moved from its rest position in a stated direction. It is a vector.

Amplitude, x_o:

Amplitude is the maximum magnitude of displacement from the equilibrium position. It is a scalar.

Frequency, *f* :

The number of complete oscillations per unit time is called the frequency. SI unit is hertz (Hz). Note: 1 Hz = 1 cycle per second .

Period, *T* :

The period T is the time for one complete oscillation. Note: $T = 1/f$.

Definitions

Angular frequency, ω :

The angular frequency is defined as $2\pi f$. SI unit is radians per second (rad s^{-1}).

Hence, $\omega = 2\pi f = \dfrac{2\pi}{T}$.

As ω is a constant, T is a constant and is independent of the amplitude x_0 of the oscillation . This is an important characteristic of SHM.

Definitions

Simple harmonic motion may be defined as an oscillatory motion of a particle whose acceleration is directly proportional to its displacement from the equilibrium position and this acceleration is always directed towards that position.

$$a \propto -x$$

where *a* is the acceleration,
x is the displacement from equilibrium
and the negative sign implies that acceleration points in the opposite direction of the displacement vector.

Written in equation form, this is

$$\mathbf{a = -\omega^2 x}\ , \quad \textit{where } \omega^2 \textit{ is a constant.}$$

Graphs for SHM

(A) Variation of *x*, *v*, *a* with time *t*

Any SHM can be described in terms of a sinusoidal function. For instance, the value of the displacement *x* of an object can be given by this equation:

$$x = x_o \sin\omega t \,,$$

which means the object is at the equilibrium position at *t* = 0,

then the velocity of the object will be

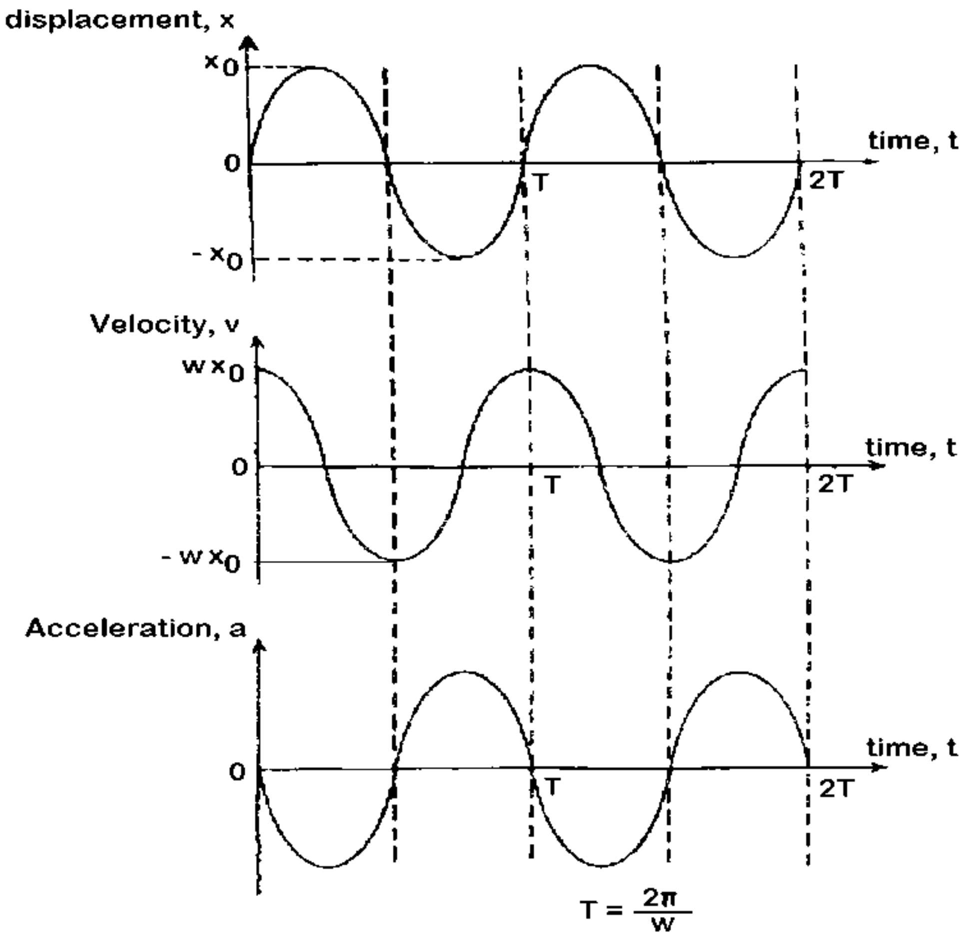
$$v = x_o\omega \cos\omega t$$

and the object's acceleration will be

$$a = -x_o\omega^2 \sin\omega t$$

i.e. $a = -\omega^2 x$

which agrees with what was shown in the previous section.



If $x = x_o \cos \omega t$,

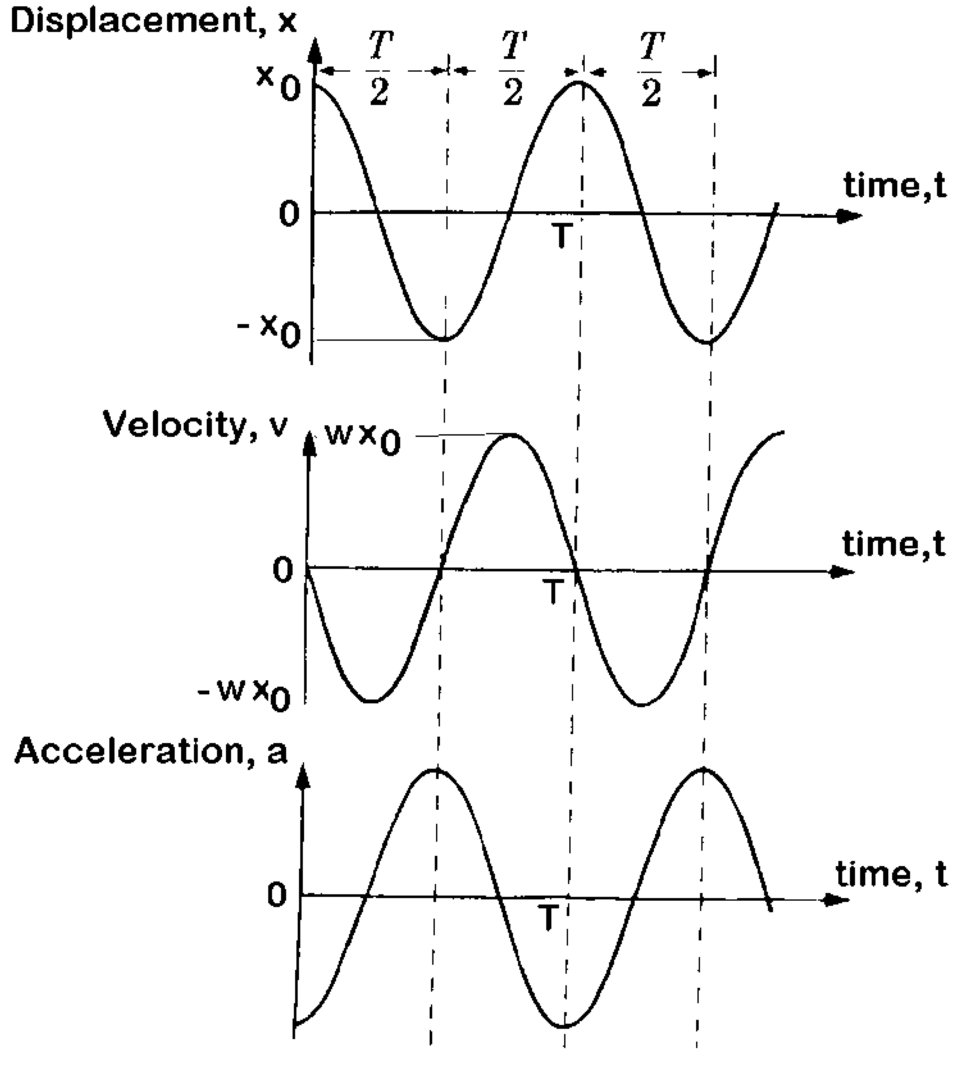
which is for the object starting at the amplitude when $t = 0$,

then

$$v = -x_o \omega \sin \omega t$$

and

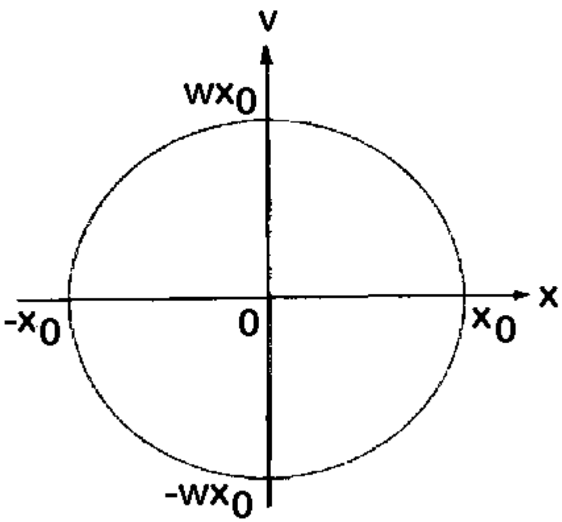
$$a = -x_o \omega^2 \cos \omega t = -\omega^2 x$$



(B) Variation of *v* and *a* with respect to *x*

It can be shown that $v = \pm\omega\sqrt{x_o^2 - x^2}$ and of course, $a = -\omega^2 x$

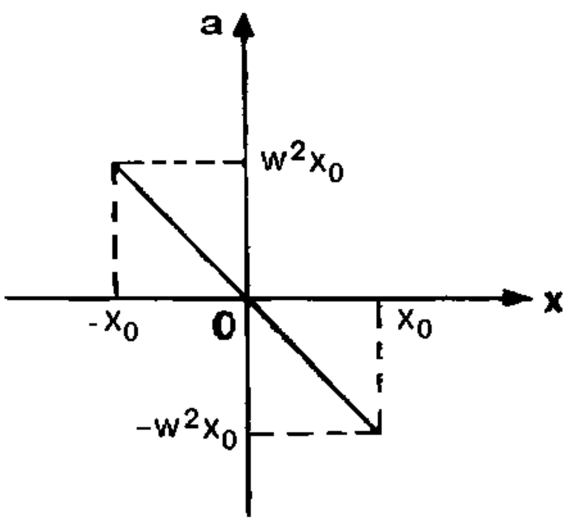
From the v - x plot, the velocity of the particle will be maximum only at x = 0, i.e., when it is passing through the equilibrium position.



Then $v_{max} = \pm\omega x_o$

Also, the velocity will be zero when the particle is at the amplitude, i.e., $x = x_o$.

From the *a* - *x* graph, the acceleration is maximum at $x = x_o$, and is given by

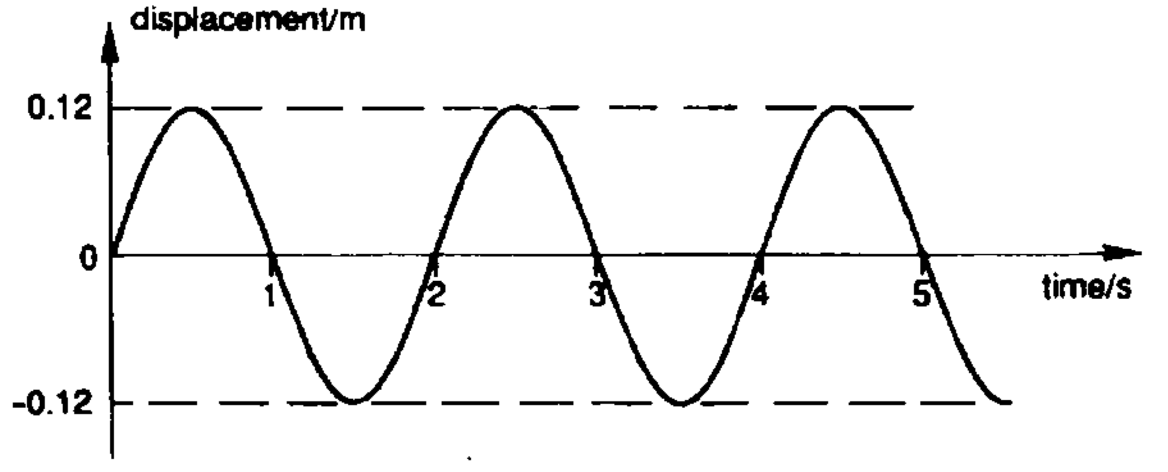


$$a_{max} = -\omega^2 x_o$$

Acceleration is zero when the object is passing through the equilibrium position.

Example 1

The pendulum bob in a particular clock oscillates so that its displacement from a fixed point is as shown:



By taking the necessary readings from the graph, determine for these oscillations,

(a) the amplitude; (b) the period;(c) the frequency; (d) the angular frequency; (e) the acceleration (i) when the displacement is zero; (ii) when the displacement is at its maximum; (f) the maximum velocity of the pendulum bob.

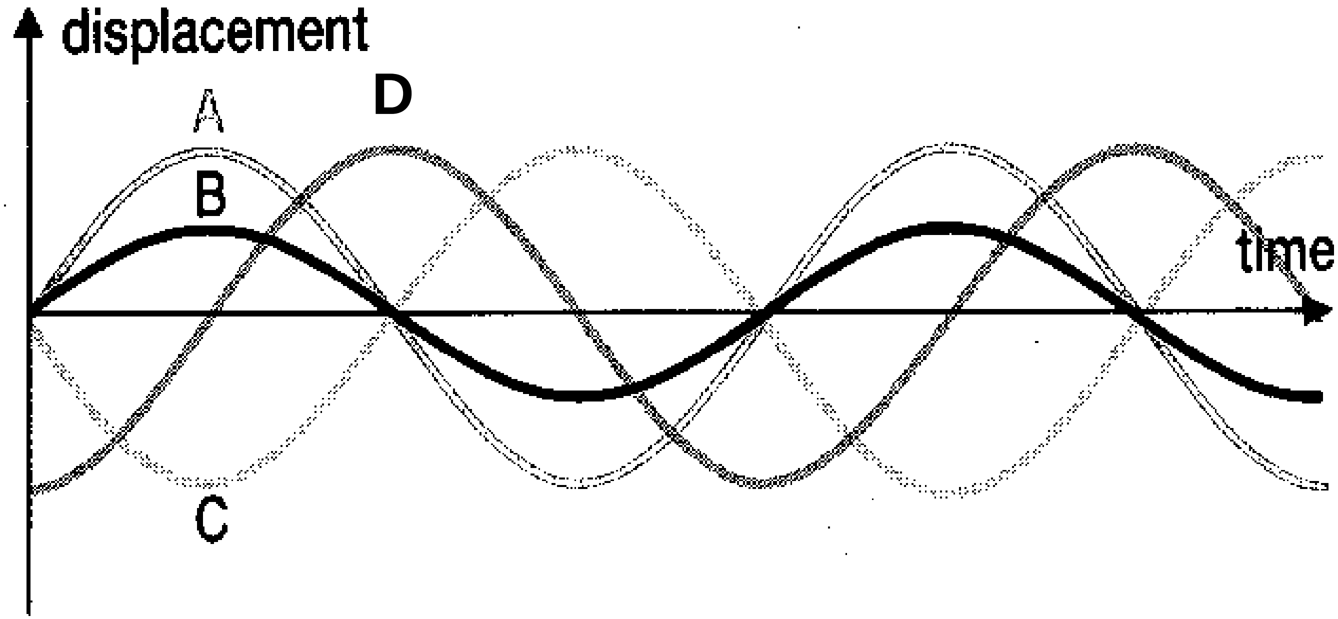
(C) Phase Difference

Consider an oscillation given by the equation $x = x_o \sin \omega t$

and a second oscillation represented by $x = x_o \sin(\omega t + \phi)$.

They are two different oscillations because at any time t, the second oscillation will differ by an angle ϕ . This angle represents the **phase difference** between the two oscillations. Thus, the **phase difference** ϕ is the difference in the phase angle between the two oscillations which have the same frequency.

Example 2



Phase difference between A and B = **<u>0˚</u>** , they are **<u>in phase.</u>**

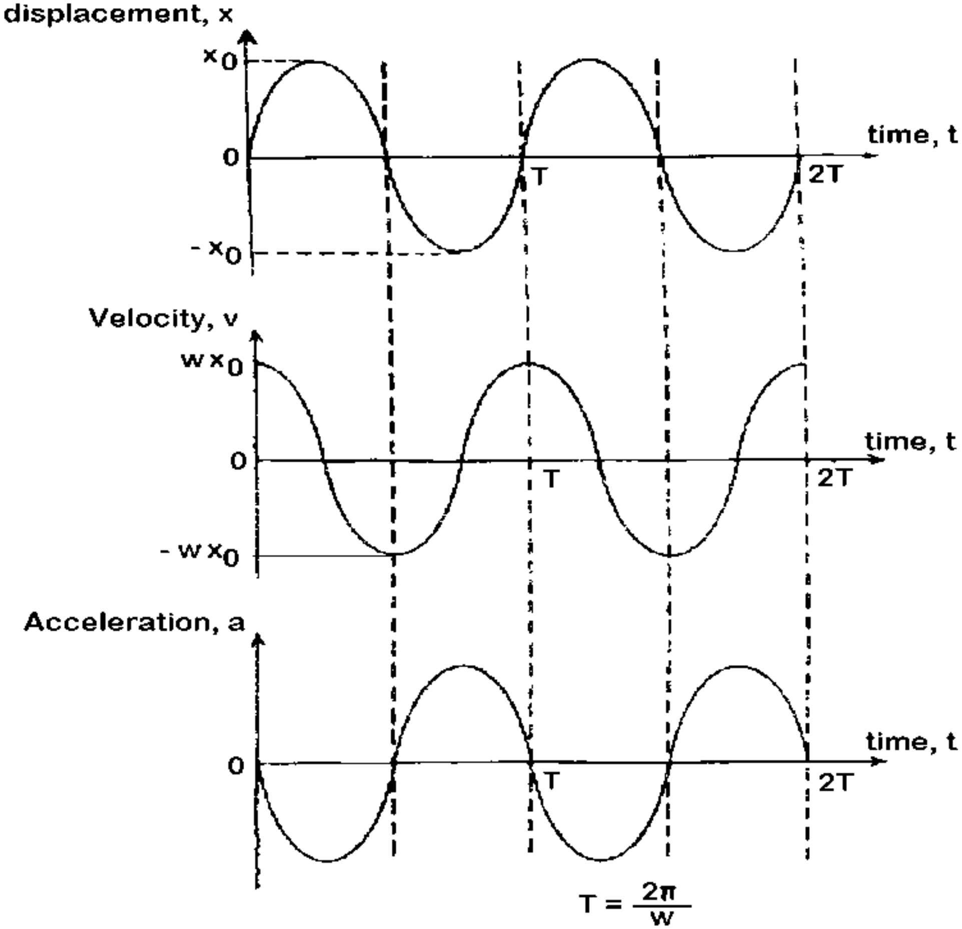
Phase difference between A and C = **<u>180˚</u>** , they are **<u>out of phase.</u>**

Phase difference between A and D = **<u>90˚</u>**, they are **<u>out of phase.</u>**

Phase difference between B and D = **<u>90˚</u>** , they are **<u>out of phase.</u>**

Example 3

What are the phase differences between the displacement-time plot, velocity-time plot and acceleration-time plot for a simple harmonic motion?



ANS:

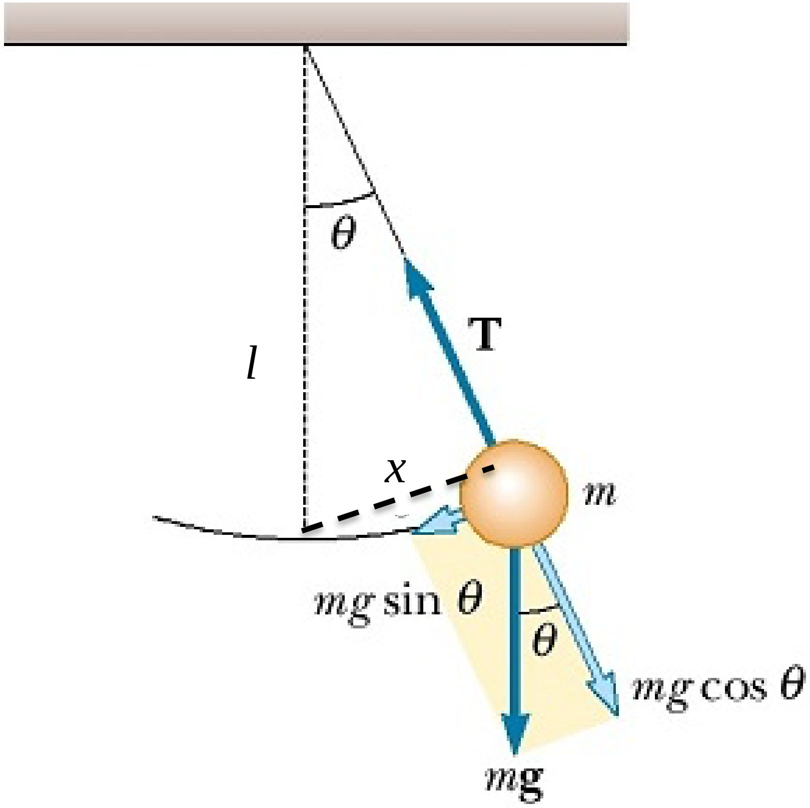
Phase difference between x-t and v-t is = **<u>90˚</u>.**

Phase difference between x-t and a-t is = **<u>180˚</u>.**

Simple Harmonic Systems

1. Simple Pendulum

A simple pendulum can be said to be undergoing simple harmonic motion because it experiences a restoring force that is proportional to its displacement from equilibrium.



The tension, T, in the string is equal to the component of the weight of the bob, $mg\cos\theta$. The restoring force is $mg\sin\theta$.

$$mg\sin\theta = ma$$

For small angles, $\sin\theta \approx \theta \approx \frac{x}{l}$

$$-m\frac{g}{l}x = ma$$

$$\rightarrow \quad -\frac{g}{l}x = a$$

Since $a = -\omega^2 x$

Simple Harmonic Systems

1. Simple Pendulum

Since $a = -\omega^2 x$

$$\omega^2 = \frac{g}{l}$$

The periodic time for SHM is given by $T = \dfrac{2\pi}{\omega}$

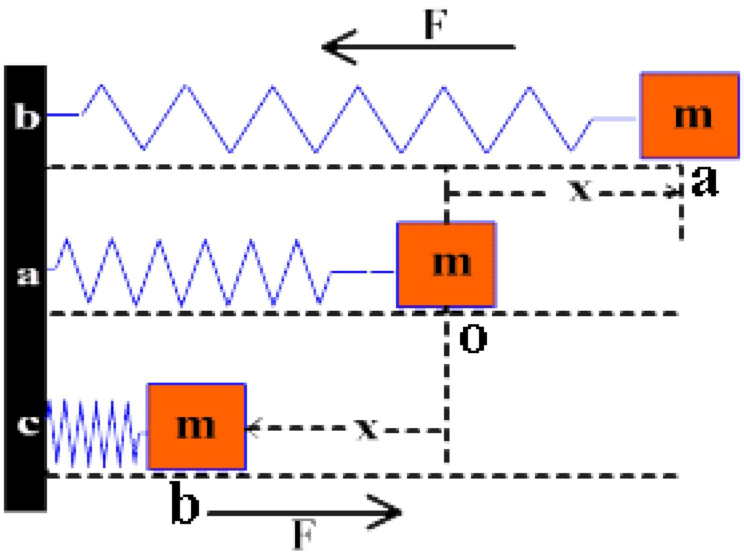
Therefore the period of a pendulum is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

Simple Harmonic Systems

2. Mass-Spring System

A mass-spring system can also be considered to be a good approximation to SHM.



The restoring force, F, is always in the opposite direction to the displacement of the mass from equilibrium.

Since, $F = -kx$

$ma = -kx$

Since $a = -\omega^2 x$

$$\omega^2 = -\frac{k}{m}$$

Simple Harmonic Systems

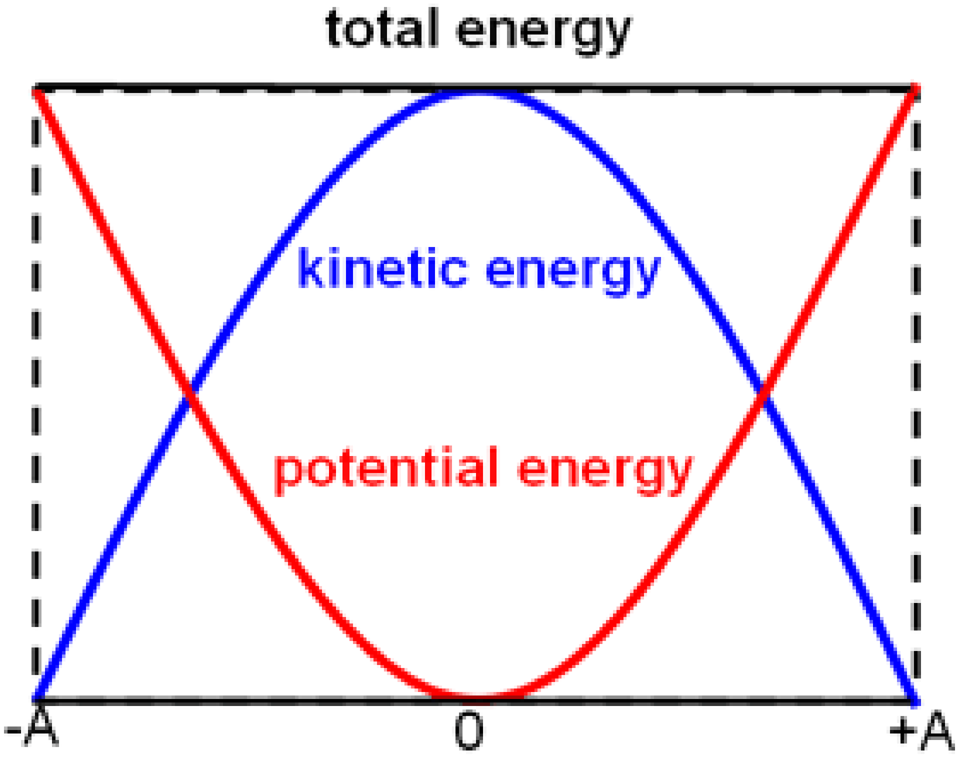
2. SiMass-Spring System

The periodic time for SHM is given by $T = \frac{2\pi}{\omega}$

Therefore the period of a mass-spring system is given by

$$T = 2\pi\sqrt{\frac{m}{k}}$$

Energy changes during SHM



Energy changes during simple harmonic motion (SHM)

In the absence of external and dissipative forces, there will be no energy loss to a system in SHM. Hence, the total energy of the oscillating system is a constant and there is a constant interchange of kinetic and potential energies.

(A) Kinetic Energy

The K.E. of a particle in SHM is given by $\frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(x_o^2 - x^2)$

As mentioned, velocity is *maximum at the equilibrium position*, $x = 0$.
$\therefore$ maximum K.E. also occurs at $x = 0$ and is given by $\frac{1}{2} m\omega^2 x_o^2$
Obviously, *minimum K.E. is zero at the amplitudes*, $x = x_o$

(B) Potential Energy

The P.E. of an oscillating mass is given by $\frac{1}{2}m\omega^2 x^2$

Naturally, *P.E. = 0 at* $x = 0$
and *P.E. is maximum at the amplitudes* $x = x_o$, and is given by $\frac{1}{2}m\omega^2 x_o^2$

(C) Total Energy

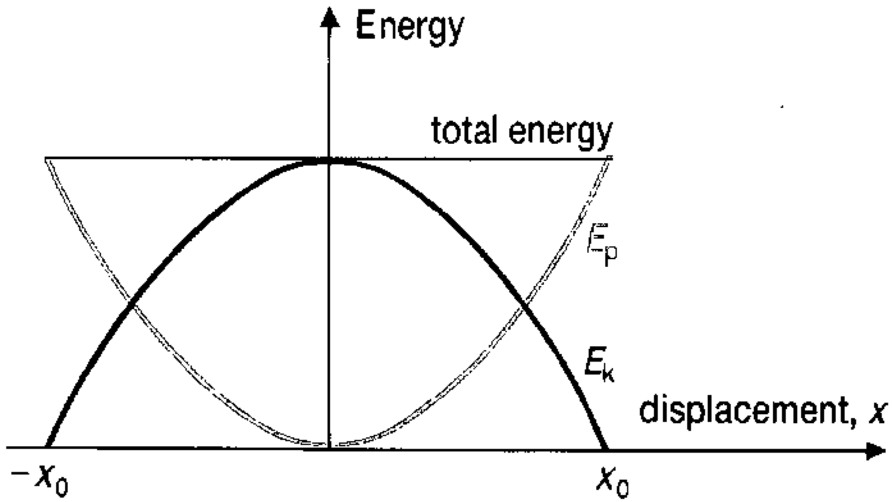
Total energy = P.E. + K.E.

$$= \tfrac{1}{2} m\omega^2(x_o^2 - x^2) + \tfrac{1}{2} m\omega^2 x^2$$

$$= \tfrac{1}{2} m\omega^2 x_o^2$$

This total energy is a constant and does not depend on x.

The graph on the right shows the variation of K.E., P.E. and total energy with displacement. Both are parabolic curves and the sum of the energies at any point is a constant value.

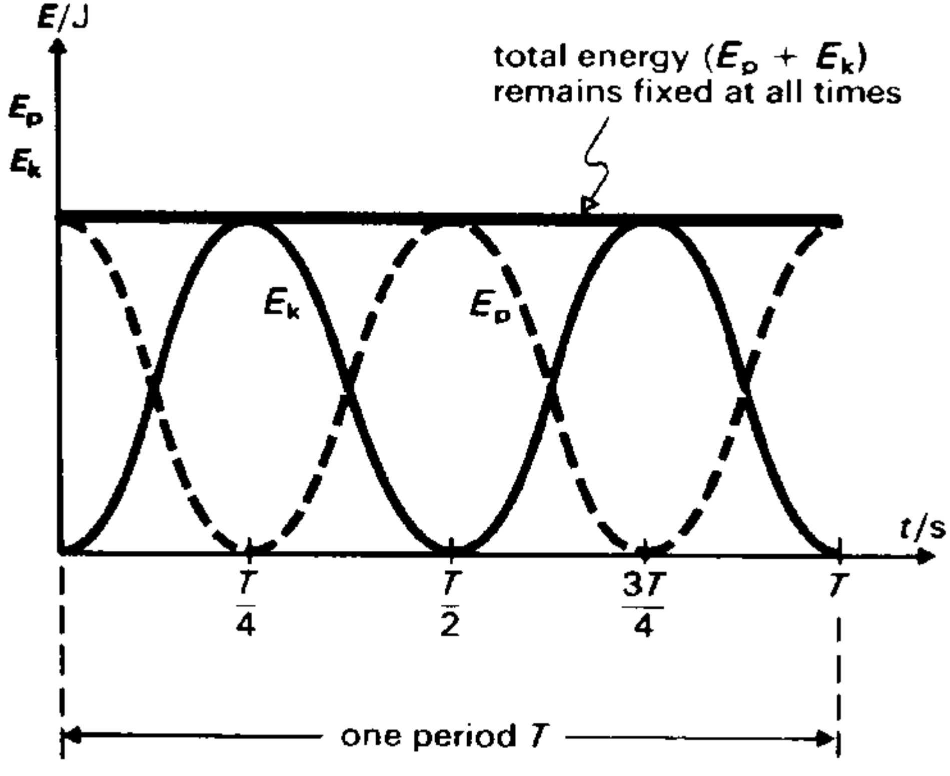


In a simple pendulum, all the energy is kinetic as the bob swings through the equilibrium position and at the top of the swing it is purely potential.

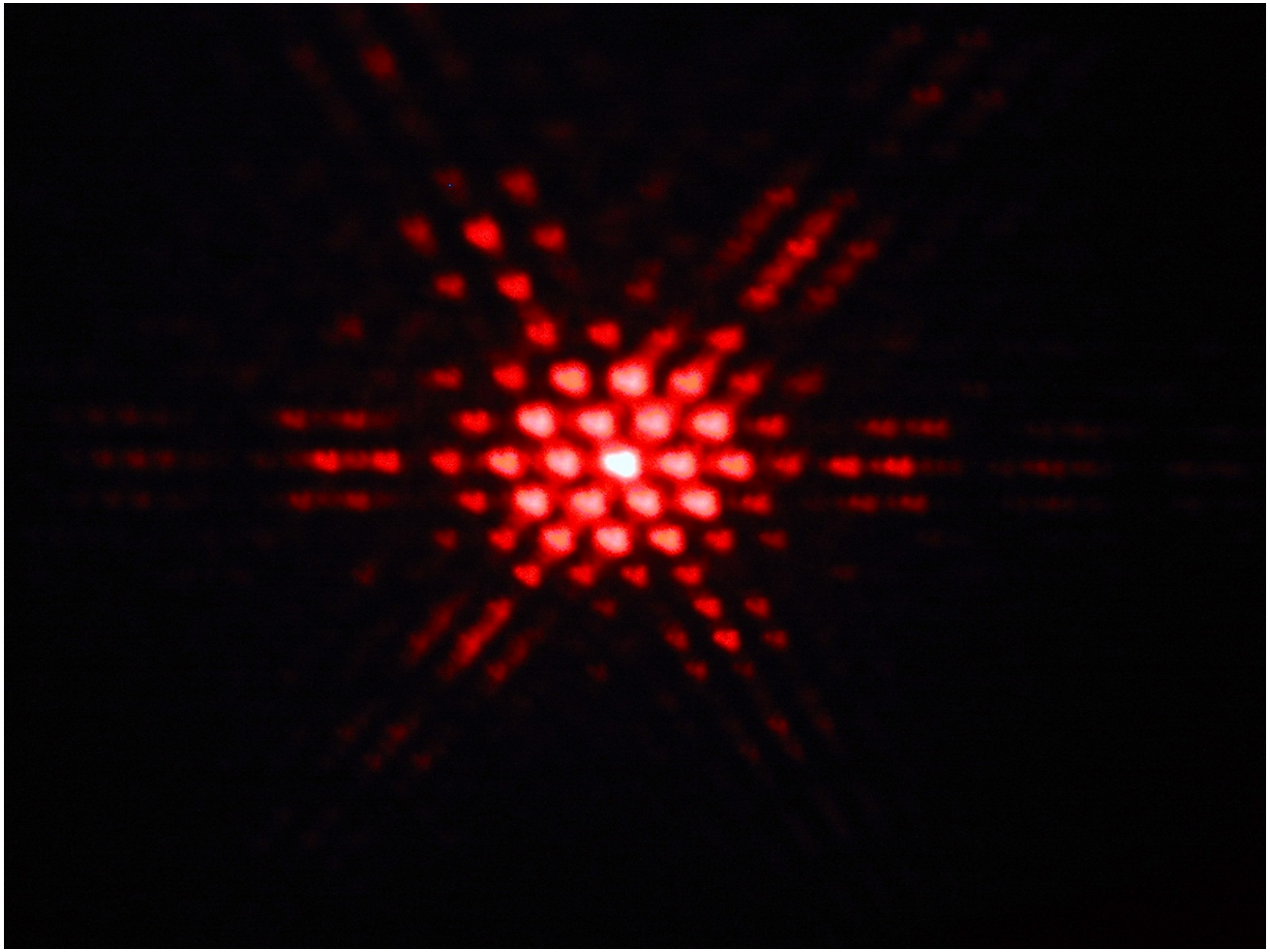
(D) Energy variation with time *t*

It can be shown that for an SHM of equation $x = x_o \cos\omega t$

But total energy is still a constant, given by $\frac{1}{2} m\omega^2 x_o^2$



9.2 Single Slit Diffraction



9.2 Nature of Science

Development of theories: When light passes through an aperture the summation of all parts of the wave leads to an intensity pattern that is far removed from the geometrical shadow that simple theory predicts

Diffraction

Any wave that is constrained to pass through a narrow aperture will spread out. This bending of a wave is called diffraction.

Diffraction effects become significant only when the wavelength λ of the wave is comparable to or bigger than the aperture size b.

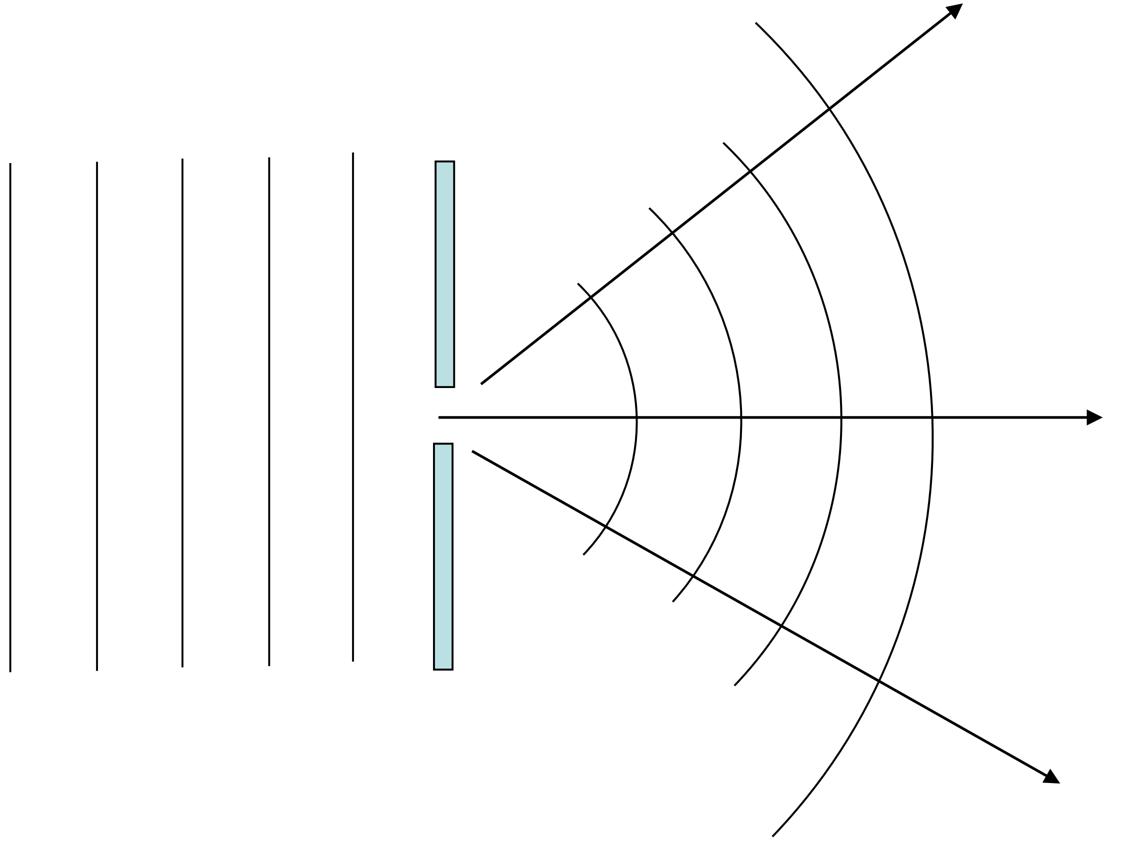
This explains why we can hear, but not see, around corners. For example, a person can hear through an open door because sound diffracts around the opening of the door.

Typical dimension of door openings ≈ 1- 100 cm (0.01 –1m)

λof sound = v / f = 300 / 440 = 0.68 m

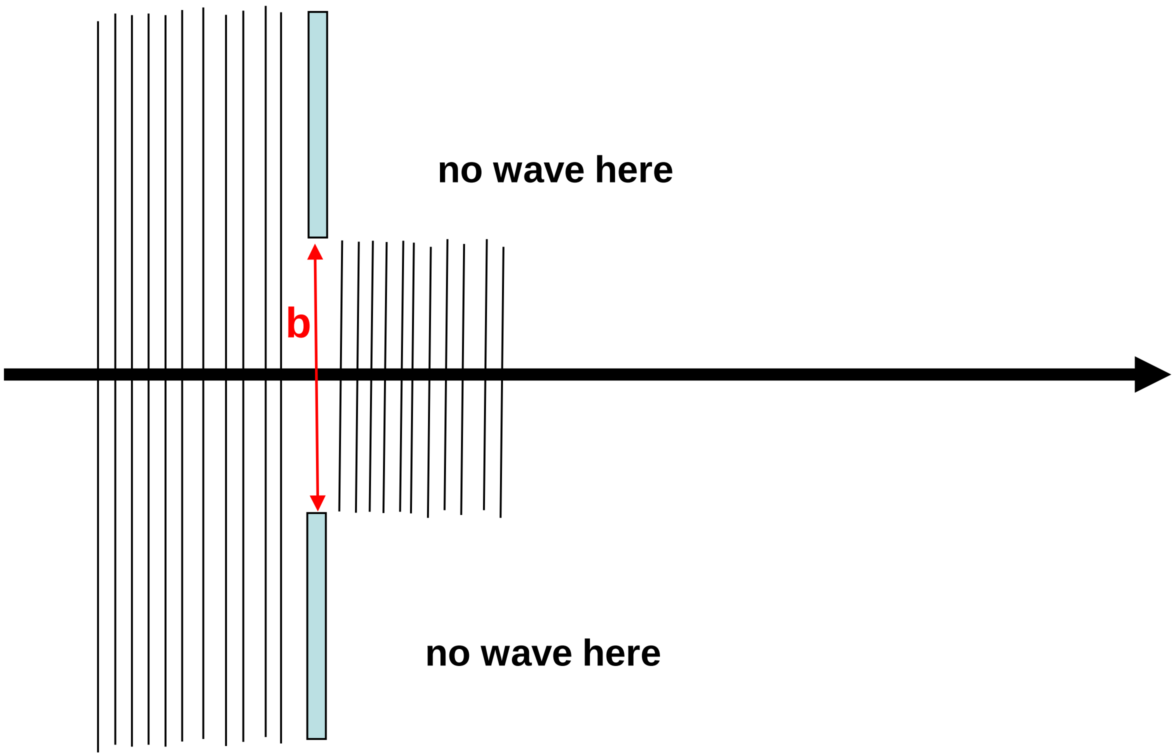
λ of light = 4 – 7.5 x 10^{-7} m (visible region)

Diffraction



λ ≈ b   or λ > b

Diffraction

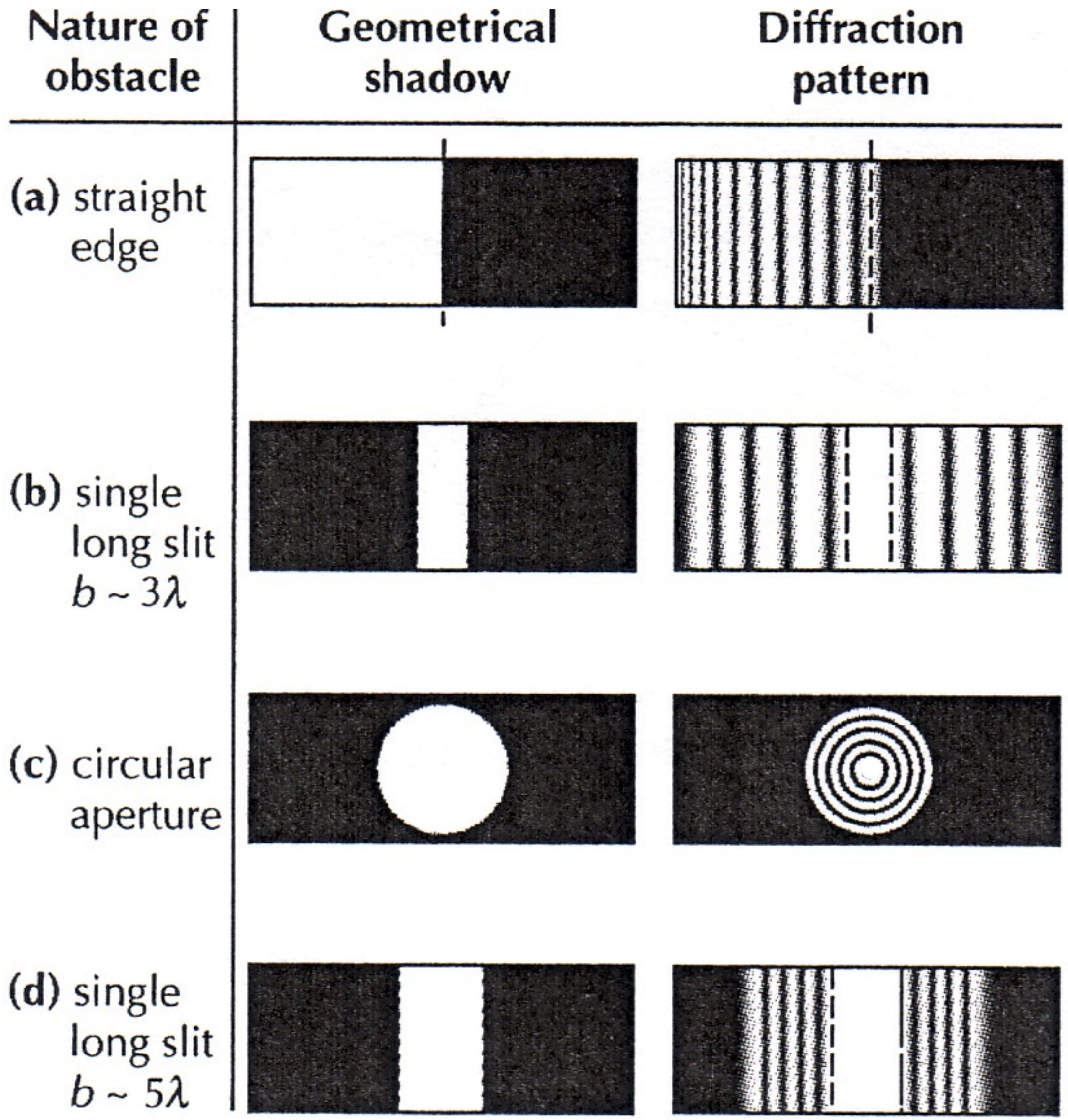


λ << b , where b is the aperture

Diffraction by a Single Edge, Narrow Slit & Circular Aperture

Narrow slit

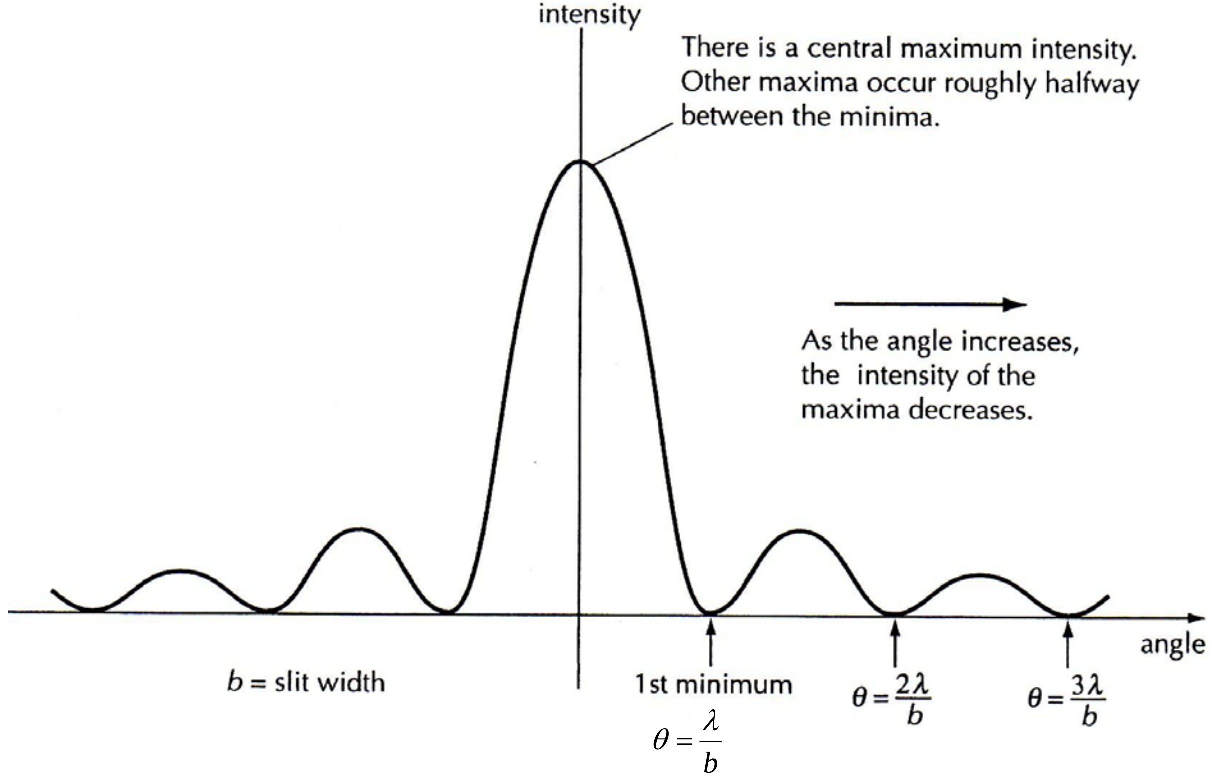
Wide slit



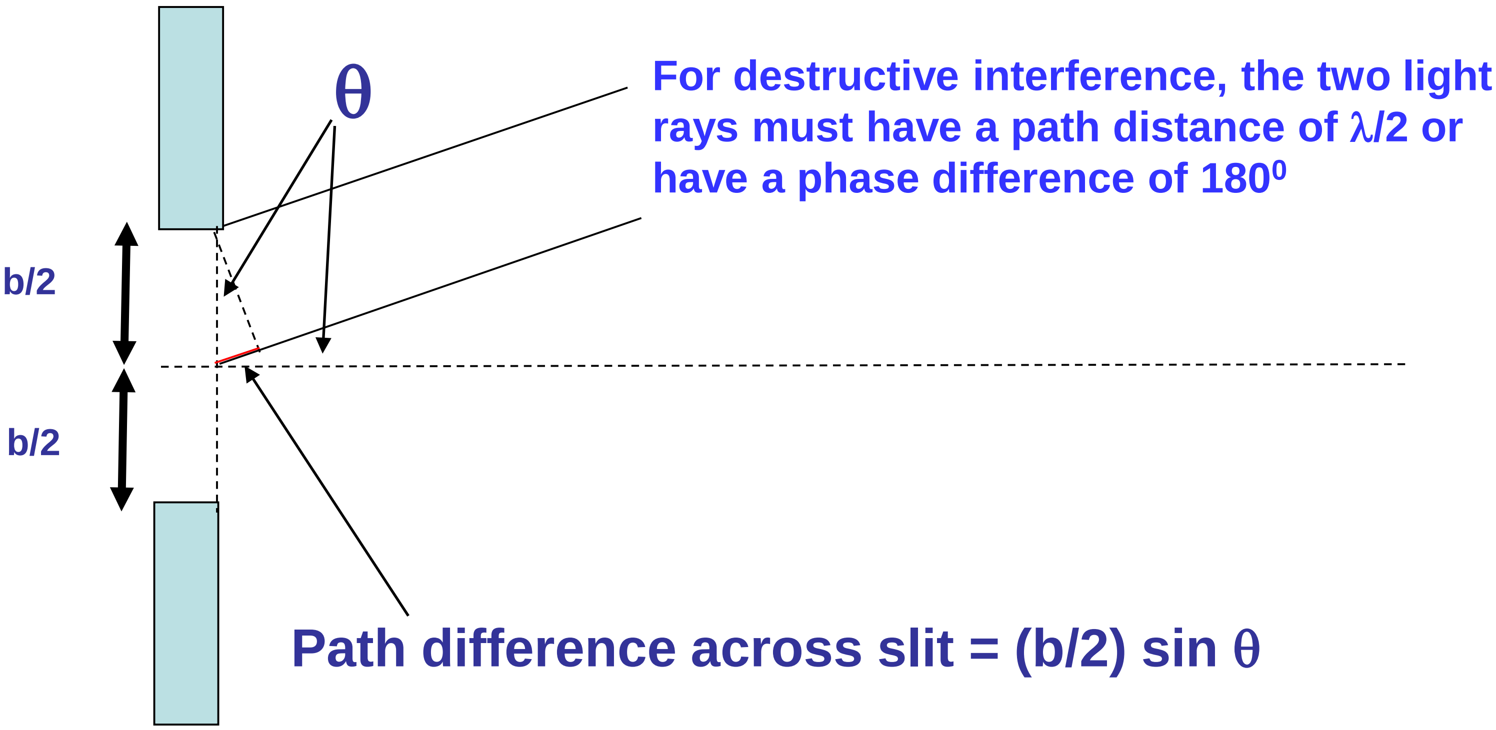
Diffraction by a Single Slit

The resulting pattern of light & dark bands is called a diffraction pattern.

This pattern arises because different points along a slit create wavelets that interfere with each other just as a double slit would.



Imagine the single slit to be divided into two halves, each ray coming from each halves of the slit and interfere, hence effective slit width is only b/2.



$$\text{Path difference} = \frac{b}{2}\sin\theta$$

$$\text{For first minimum,}\ \frac{b}{2}\sin\theta = \frac{\lambda}{2}$$

$$\text{Therefore,}\ \sin\theta = \frac{\lambda}{\mathrm{b}}$$

$$\text{For small angles,}\ \sin\theta = \theta$$

$$\boxed{\theta = \frac{\lambda}{b}}$$

Diffraction by a Single Slit

Generally, the minima of the single slit diffraction pattern occur when

$b \sin \theta = m \lambda$ $\quad$ $m = 1, 2, 3, \ldots$ $\quad$ (b = slit width)

For the 1st minimum, m=1:

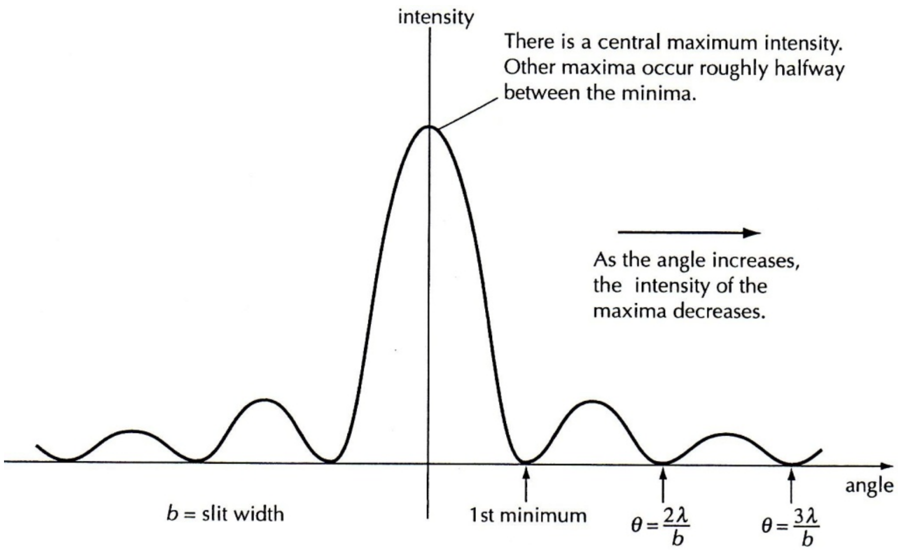
$b \sin \theta = \lambda$

$\sin \theta = \lambda / b$

For **small angles**, the **1st minimum** occurs at $\theta = \dfrac{\lambda}{b}$

Examples

1. A single slit of width 1.50 μm is illuminated with light of wavelength 500 nm. Find the angular width of the central maximum. (38.9^0)



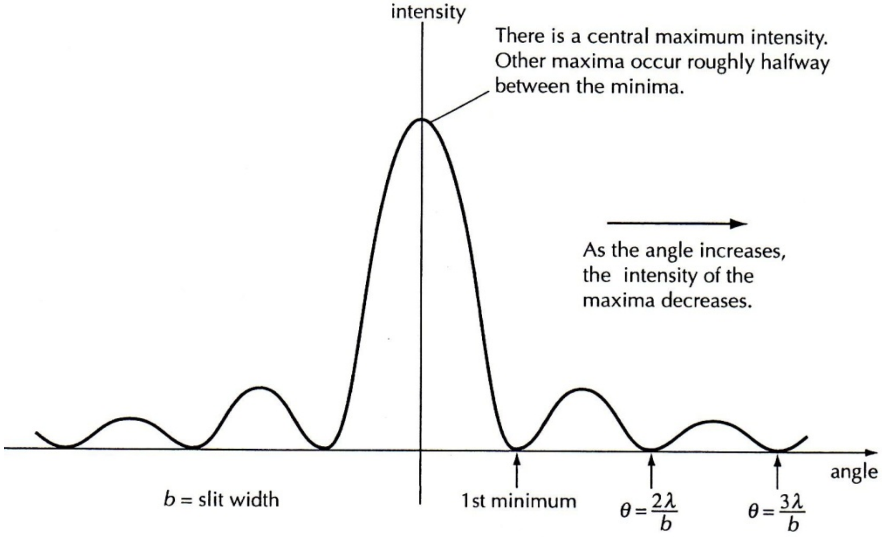
Angular position for first minimum $\sin\theta = \lambda/b$

$= (500\times10^{-9}) / (1.50 \times 10^{-6}) = 0.3333$

Hence $\theta = \sin^{-1}(0.3333) = 19.47^o$

Angular width of the central maximum = $2\theta = 2 \times 19.47^0 = 38.9^0$

2. Microwaves of wavelength 2.80 cm fall on a slit and the central maximum at a distance of 1.0 m from the slit is found to have a half-width (i.e. distance from the middle of central maximum to first minimum) of 0.67 m. Find the width of the slit. (5.03 cm)



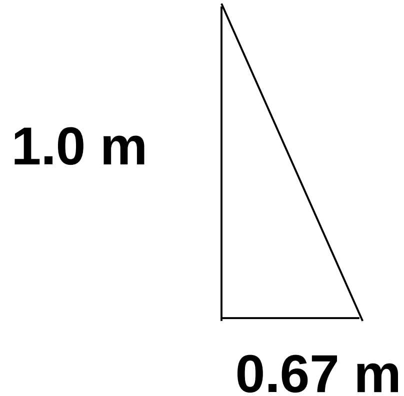
$\tan\theta = 0.67 / 1.00 = 0.67$

$\theta = 33.8^\circ$

Since $b \sin\theta = \lambda$

Hence $b = \lambda / \sin\theta = 2.80 / \sin 33.8$

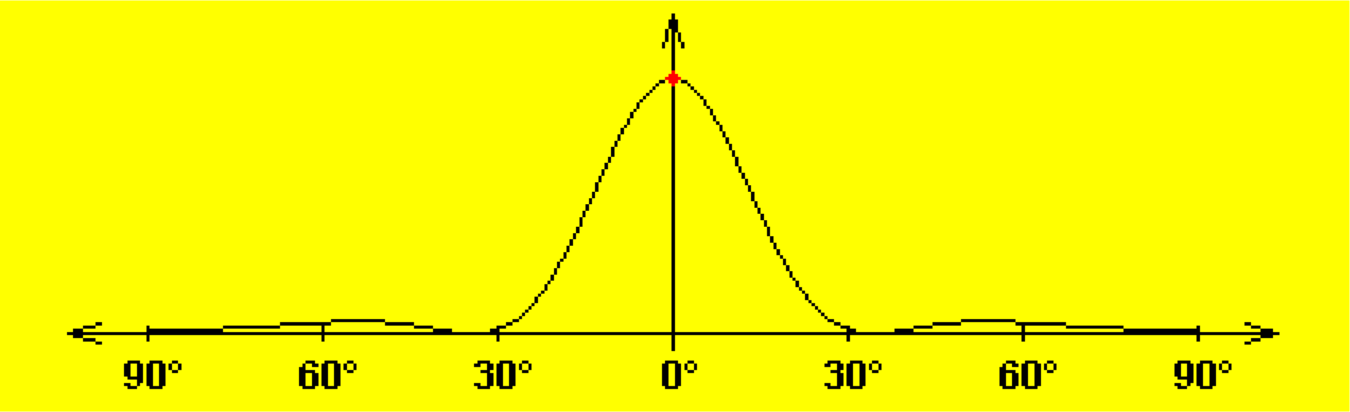
$= 5.03$ cm



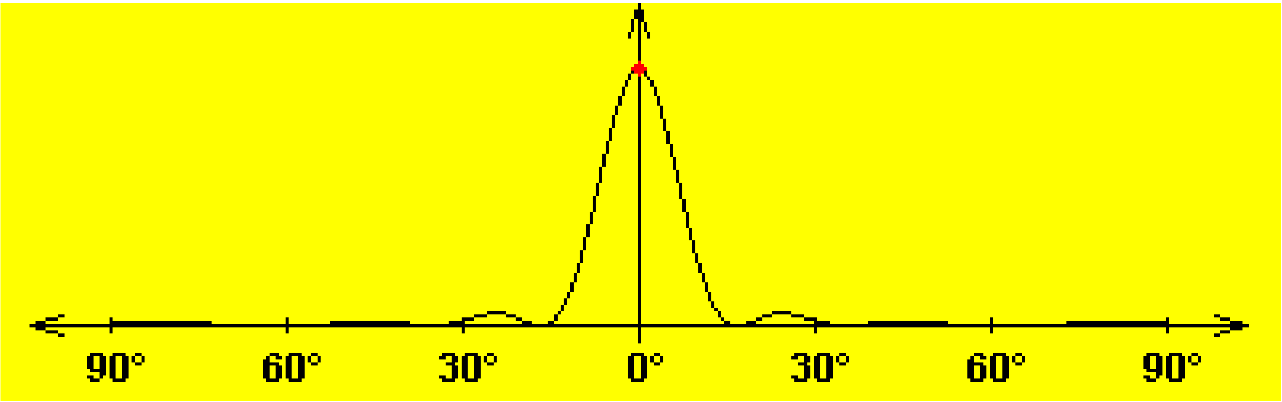
Intensity distribution of a single slit

By using a narrower slit (small aperture b), the central maximum becomes wider and the intensity decreases as less light gets through.

Conversely, using a wider slit will result in the central maximum becoming narrower and an increase in intensity as more light gets through.



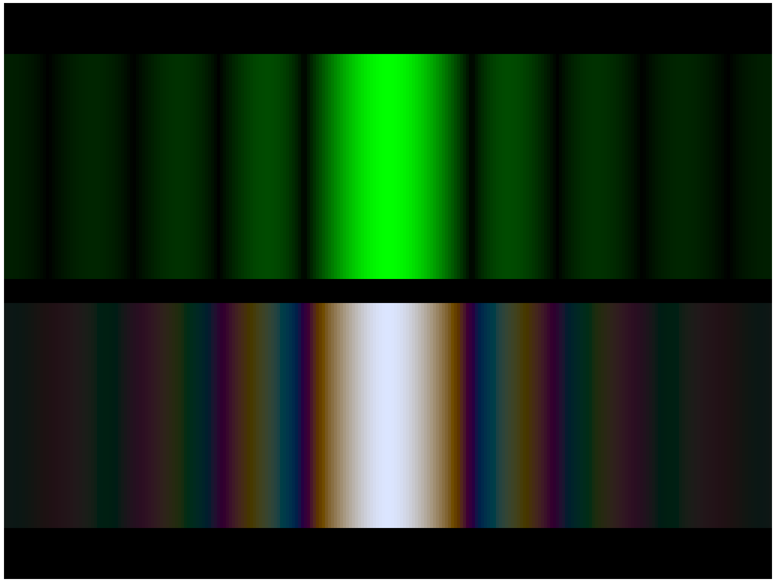
Narrow slit



Wide slit

Monochromatic & White Light

The figure below shows two images of light emerging from a single slit. The upper image is of green light while the lower image is one of white light.



Angular width of central maxima and separation of successive secondary maxima depend on the wavelength of light.

Thus, white light pattern shows coloured secondary maxima.

Diffraction by a Single Slit

Generally, the minima of the single slit diffraction pattern occur when

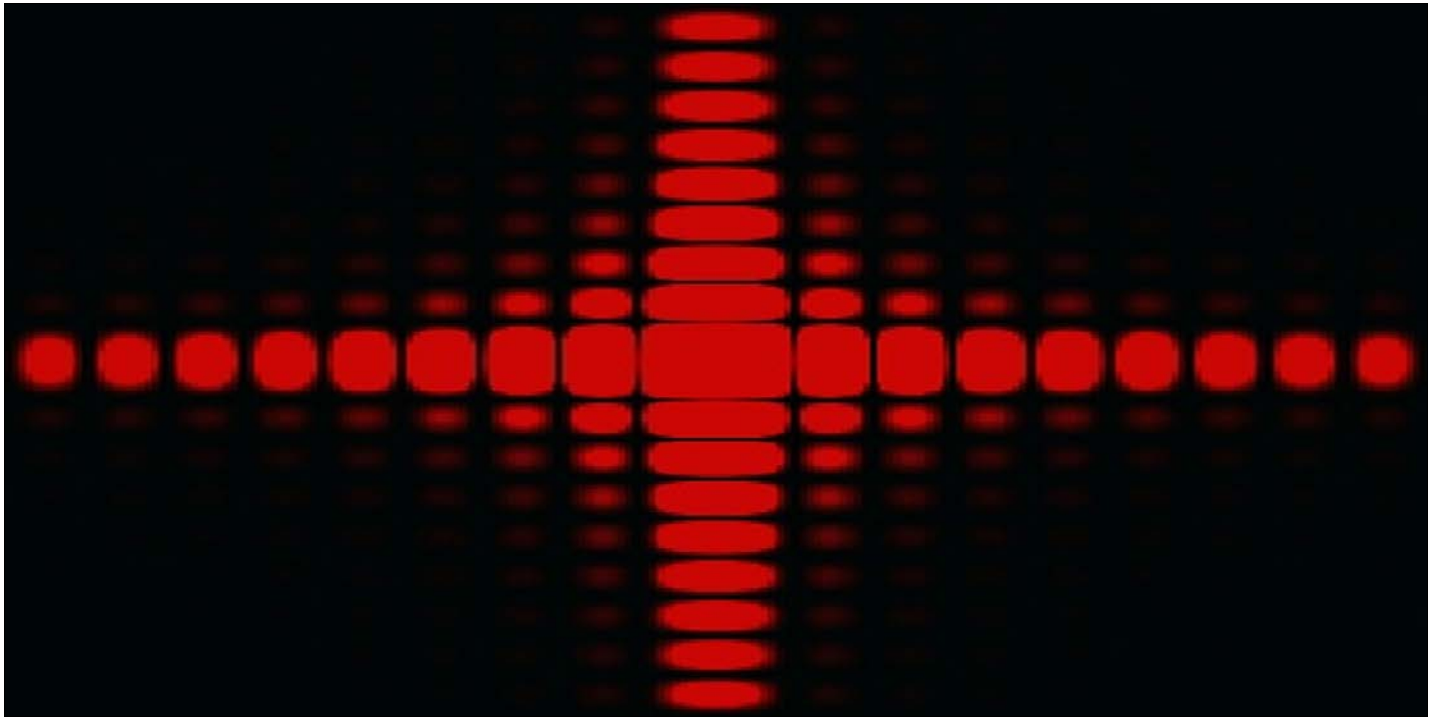
b sin θ = m λ   m = 1, 2, 3,…   (b = slit width)

For the 1st minimum, m=1:

b sin θ = λ
sin θ = λ / b

For small angles, the 1st minimum occurs at $\theta = \frac{\lambda}{b}$

9.3 Two Source Interference



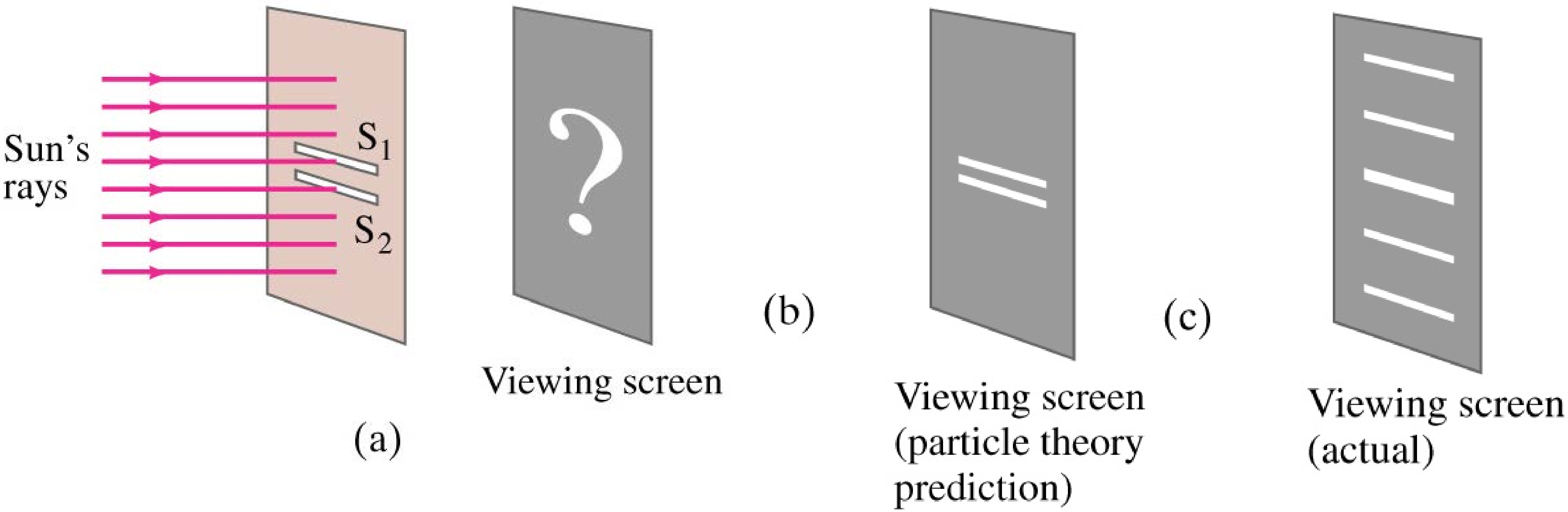
9.3 Nature of Science

Curiosity: Observed patterns of iridescence in animals, such as the shimmer of peacock feathers, led scientists to develop the theory of thin film interference.

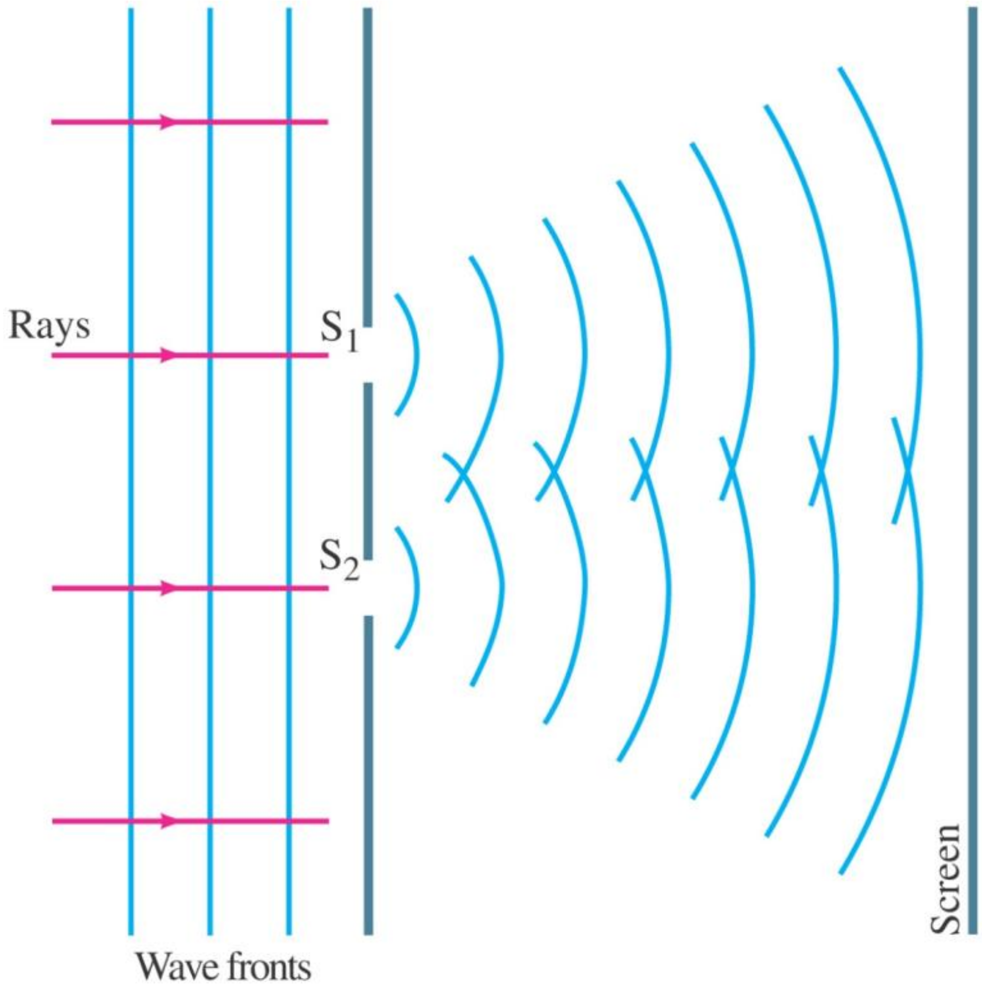
Serendipity: The first laboratory production of thin films was accidental.

Young's Double-Slit Experiment

If light is a wave, interference effects will be seen, where one part of a wave front can interact with another part.



Young's Double-Slit Experiment

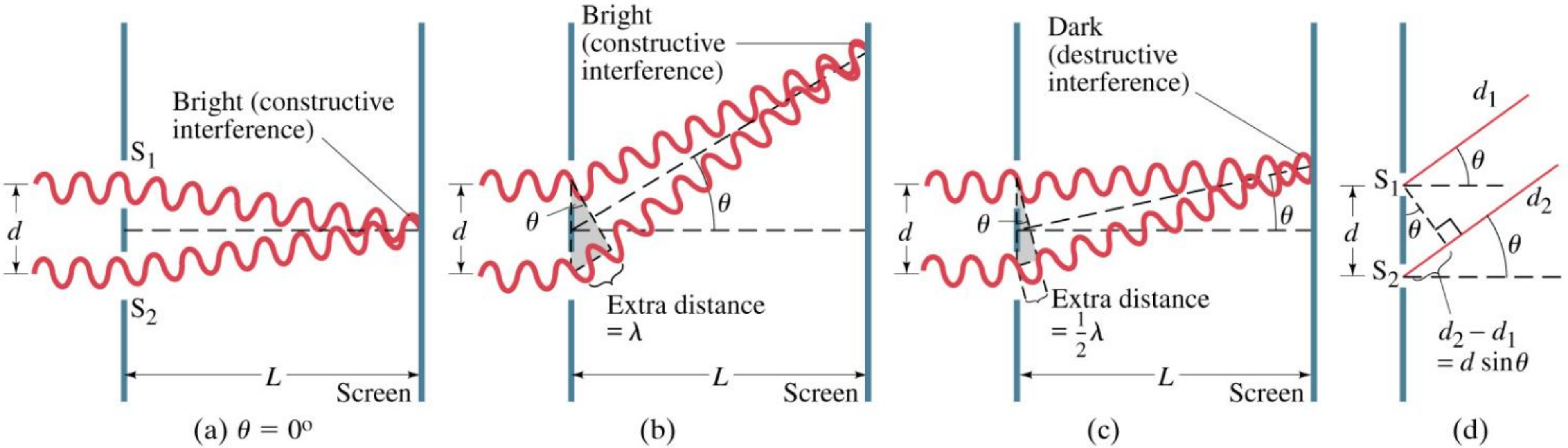


Conditions:

-Coherent sources (constant phase difference)

Interference – Young's Double-Slit Experiment

The interference occurs because each point on the screen is not the same distance from both slits. Depending on the path length difference, the wave can interfere constructively (bright spot) or destructively (dark spot).



Interference – Young's Double-Slit Experiment

We can use geometry to find the conditions for constructive and destructive interference:

$d\sin\theta = n\lambda$, n = 0,1,2,……. Constructive Interference

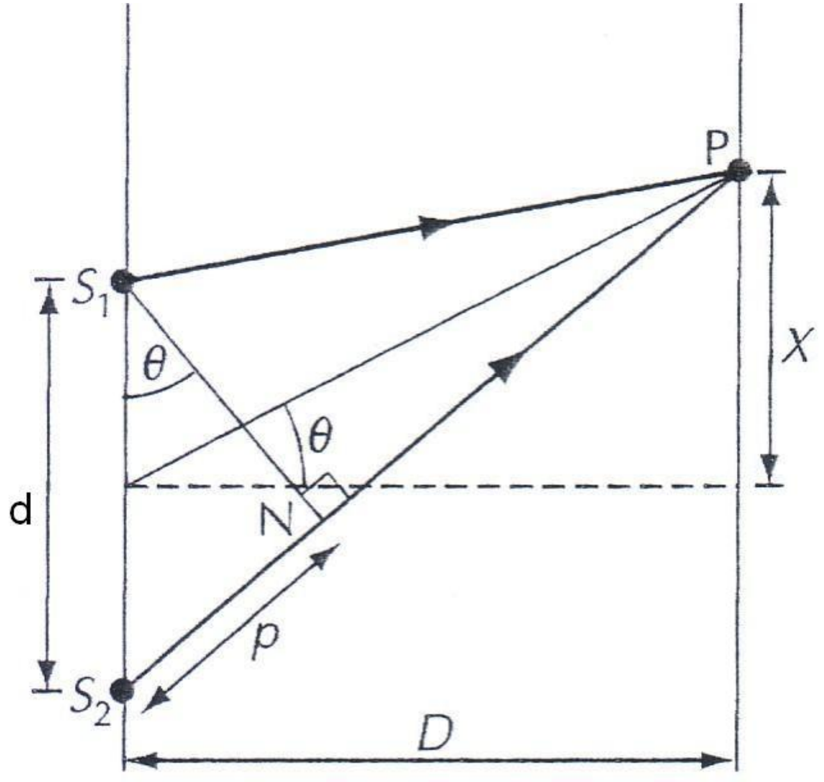
$d\sin\theta = (n+1/2)\lambda$, n = 0,1,2,……. Destructive Interference

Interference – Young's Double-Slit Experiment

$\sin\theta = p/d = n\lambda/d$
$\tan\theta = X_n/D$

Since θ is usually very small,
$\tan\theta \approx \sin\theta$
$X_n/D = n\lambda/d$
$X_n = n\lambda D/d$
$X_{n+1} = (n+1)\lambda D/d$

Fringe width
$s = X_{n+1} - X_n = \lambda D/d$



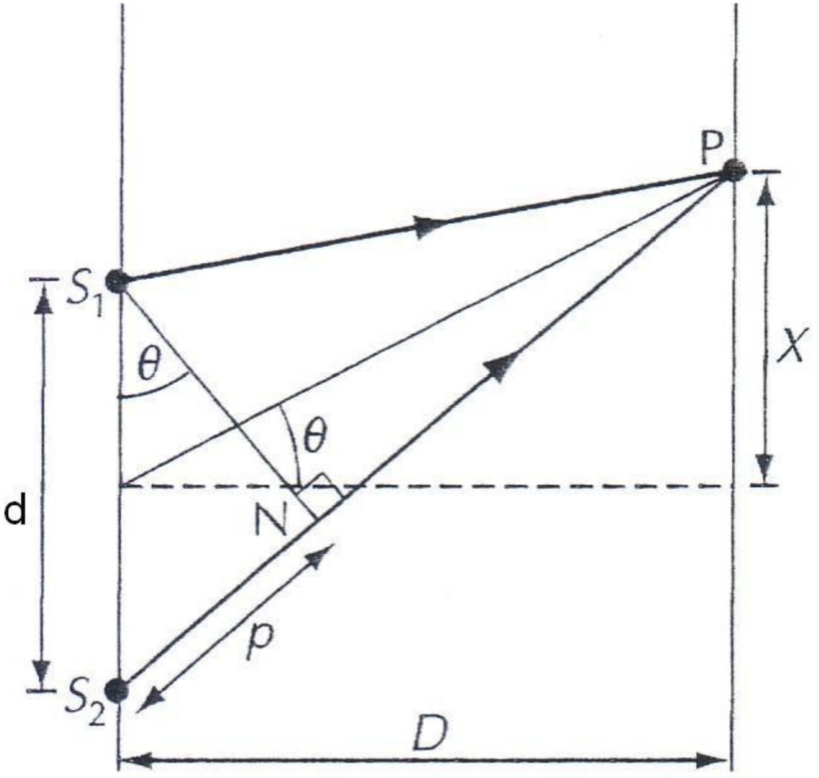
Interference – Young's Double-Slit Experiment

Distance x to the nth bright order fringe

$$\frac{x}{D} = \frac{n\lambda}{d}$$

Distance x to the nth dark order fringe

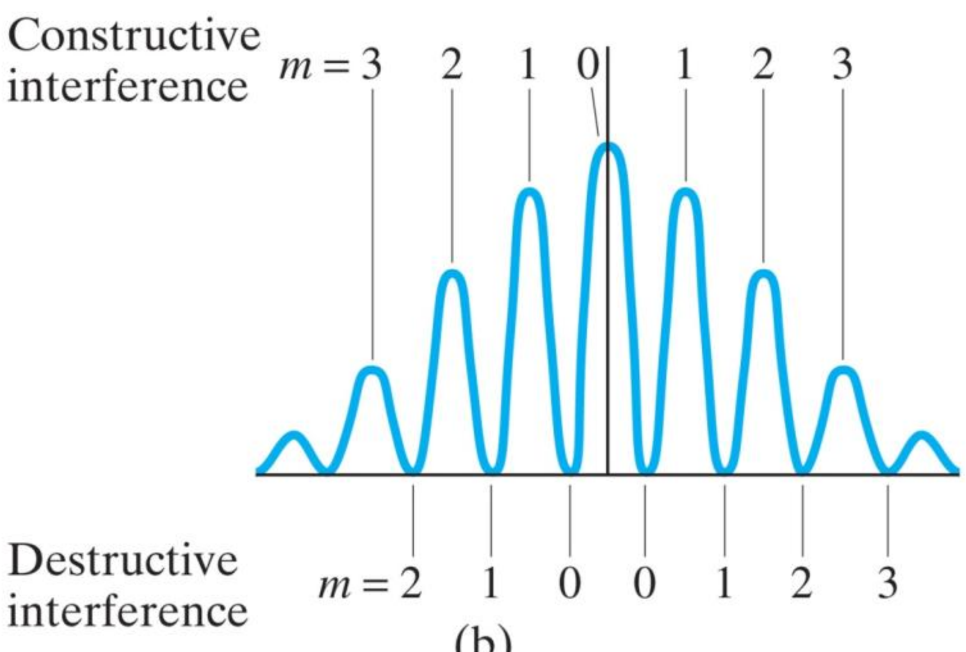
$$\frac{x}{D} = \left(n + \frac{1}{2}\right)\frac{\lambda}{d}$$

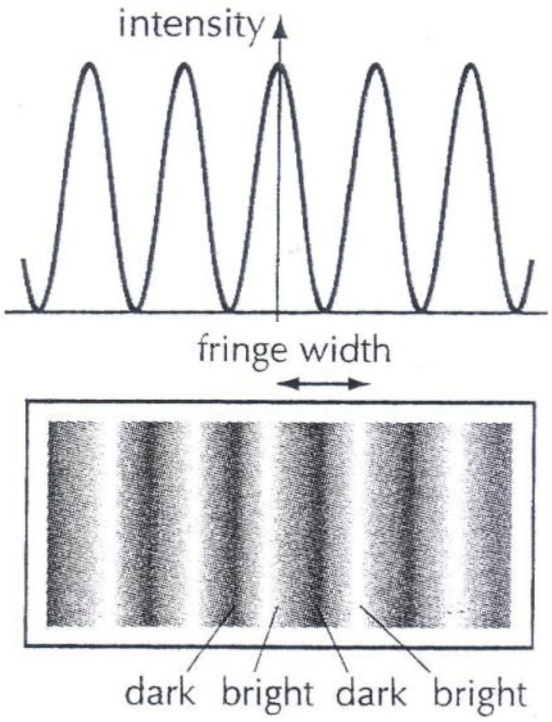


Interference – Young's Double-Slit Experiment

Fringe width/separation s = λD/d

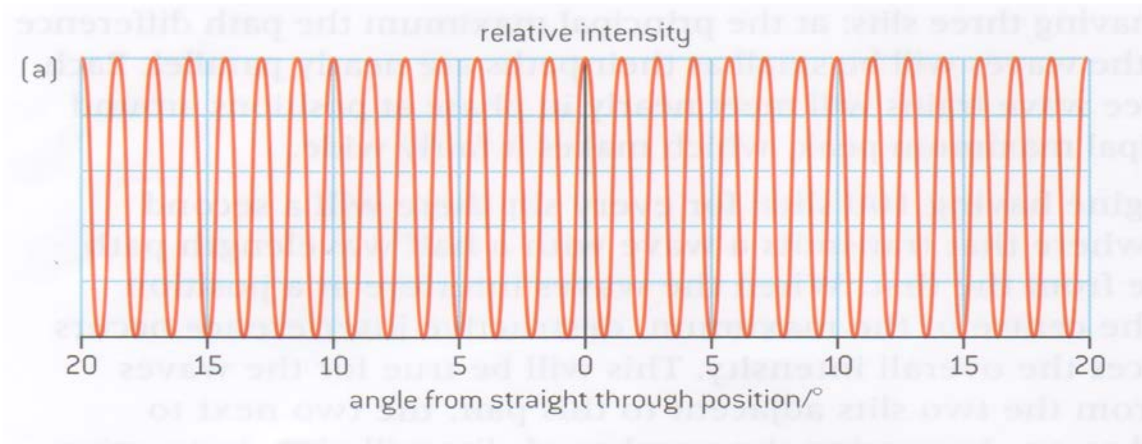
If the slits are sufficiently narrow, the bright fringes are equally bright.





Interference – Young's Double-Slit Experiment

Intensity Variation With Double Slit



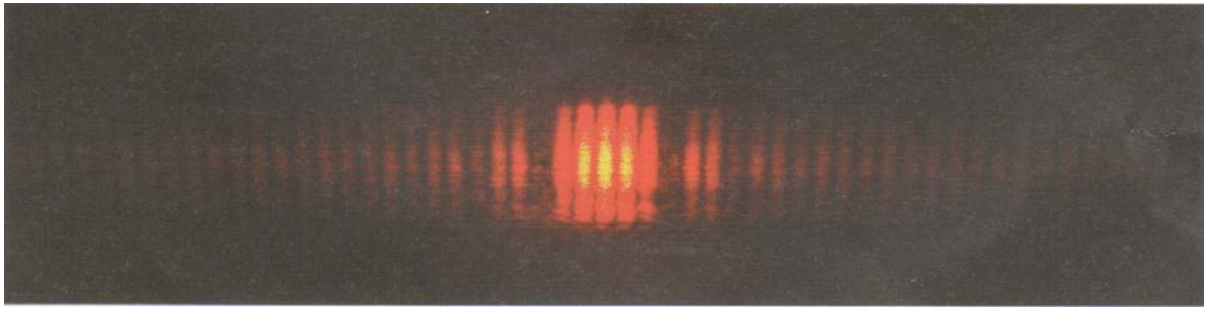
Interference pattern if slits are sufficiently narrow and if modification or ***modulation*** effect is ignored.

In other words, an *ideal* double slit.

Interference – Young's Double-Slit Experiment

Intensity Variation With Double Slit

However, in actual applications, we get an effect like this



The intensities of the secondary maxima are not even and are decreasing the further they are from the central maximum.

This happens due to **modulation**, in which the intensities of the maxima of the double slit pattern is enveloped and 'bounded' by the intensity distribution of the single slit pattern.

Interference – Young’s Double-Slit Experiment

Modulation

Figure 1

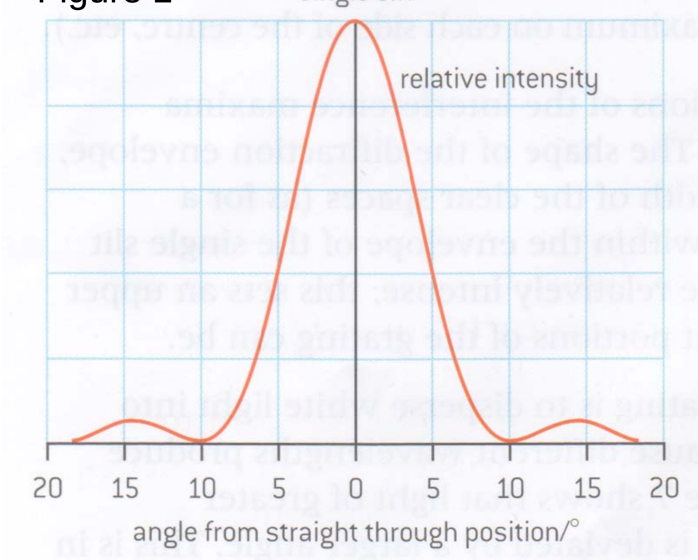


Figure 2

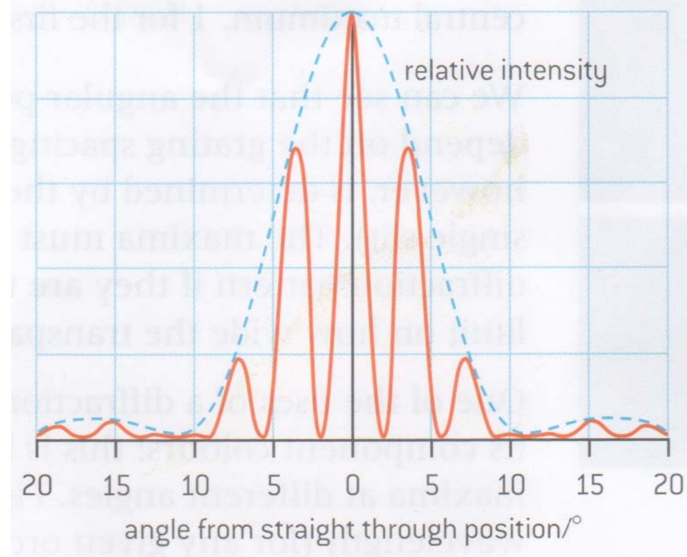


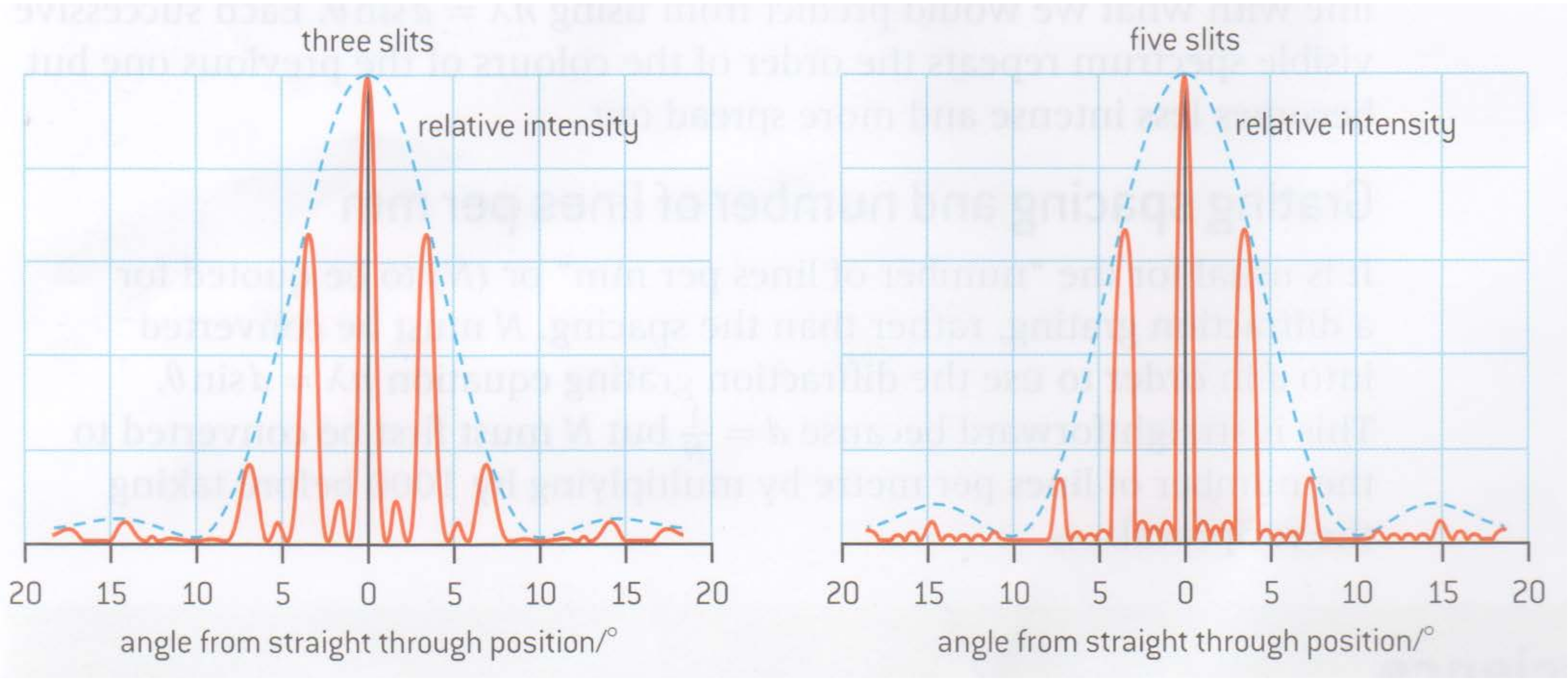
Figure 1 shows the interference pattern or variation of intensity for single slit diffraction.

Figure 2 shows the modulated interference pattern for double slit diffraction.

Interference – Young's Double-Slit Experiment

Modulation

Effects of modulation on interference patterns produced by an increasing number of slits.



Summary

- In Young's double-slit experiment, constructive interference occurs when

$$\sin\theta = m\frac{\lambda}{d}$$

- and destructive interference when

$$\sin\theta = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{d}$$

- Fringe width/separation s = λD/d

- Two sources of light are coherent if they maintain a constant phase difference.

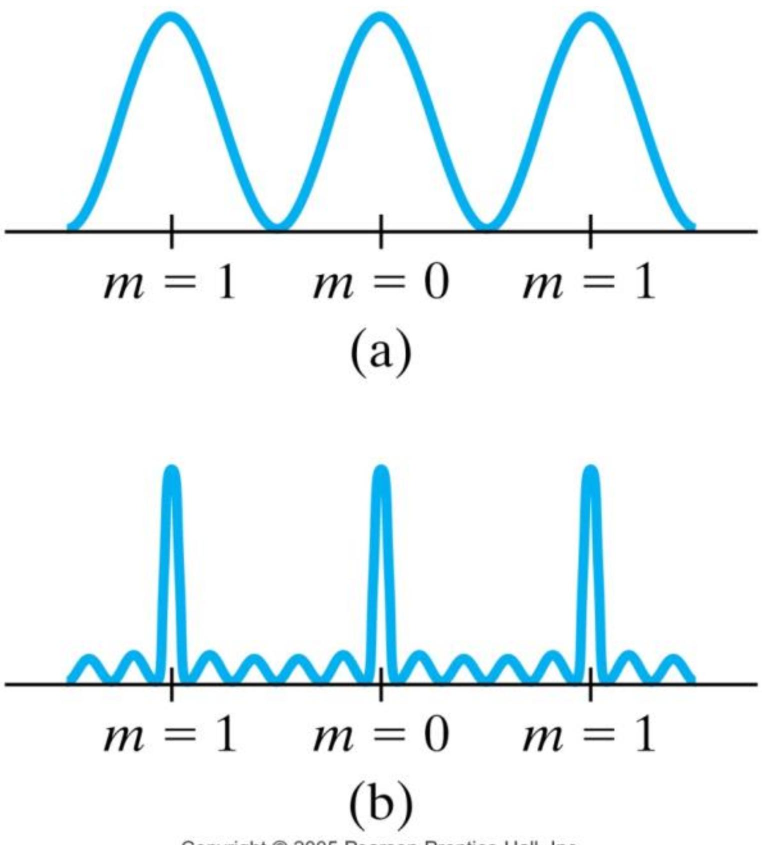
- Modulation affects the intensity of the maxima of an interference pattern.

Multiple Slit Diffraction: Diffraction Grating

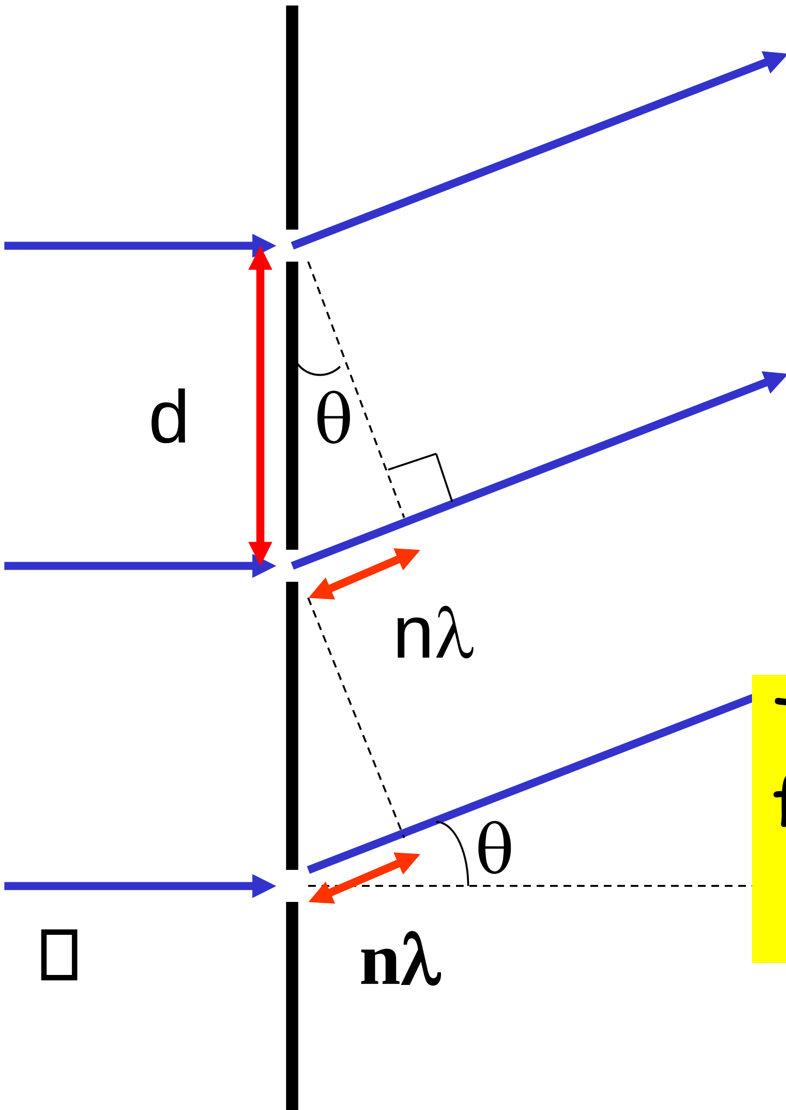
A diffraction grating consists of a large number of **parallel equally spaced narrow slits**.

The more slits there are, the narrower the peaks.

Diffraction at a slit affects the overall appearance of the fringes (final interference pattern).



Suppose monochromatic light is directed at the grating, each slit behaves as a source producing secondary wavelets that superpose with each other.



nth order bright fringe

The condition for the nth bright fringe is

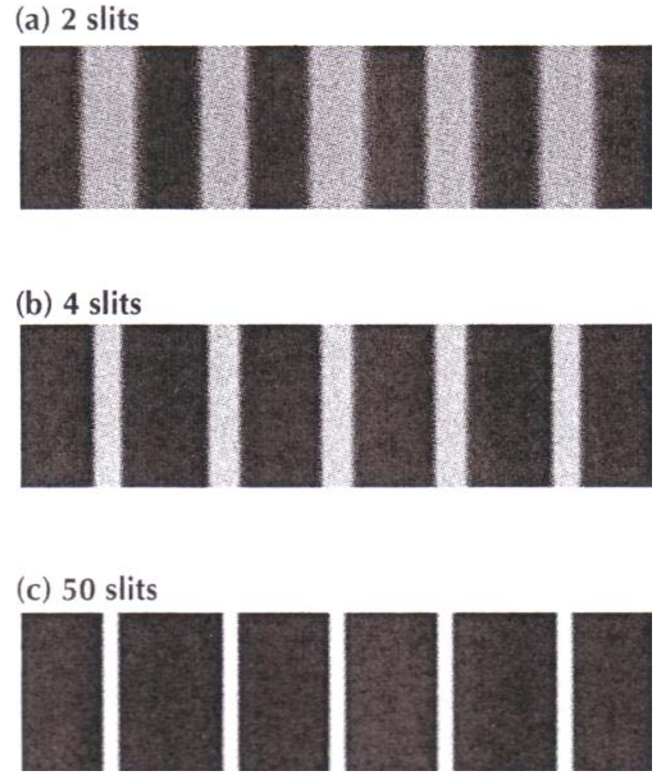
$d \sin \theta = n \lambda$ where n = 0, 1,2, 3..

Diffraction Grating

For constructive interference, path difference between slits $d\sin\theta = n\lambda$

-principal maxima has same separation
- principal maxima becomes much sharper
- pattern increases in intensity (more light through slits)

Uses:
- accurate measurement of wavelength
- investigate spectra



Diffraction Grating

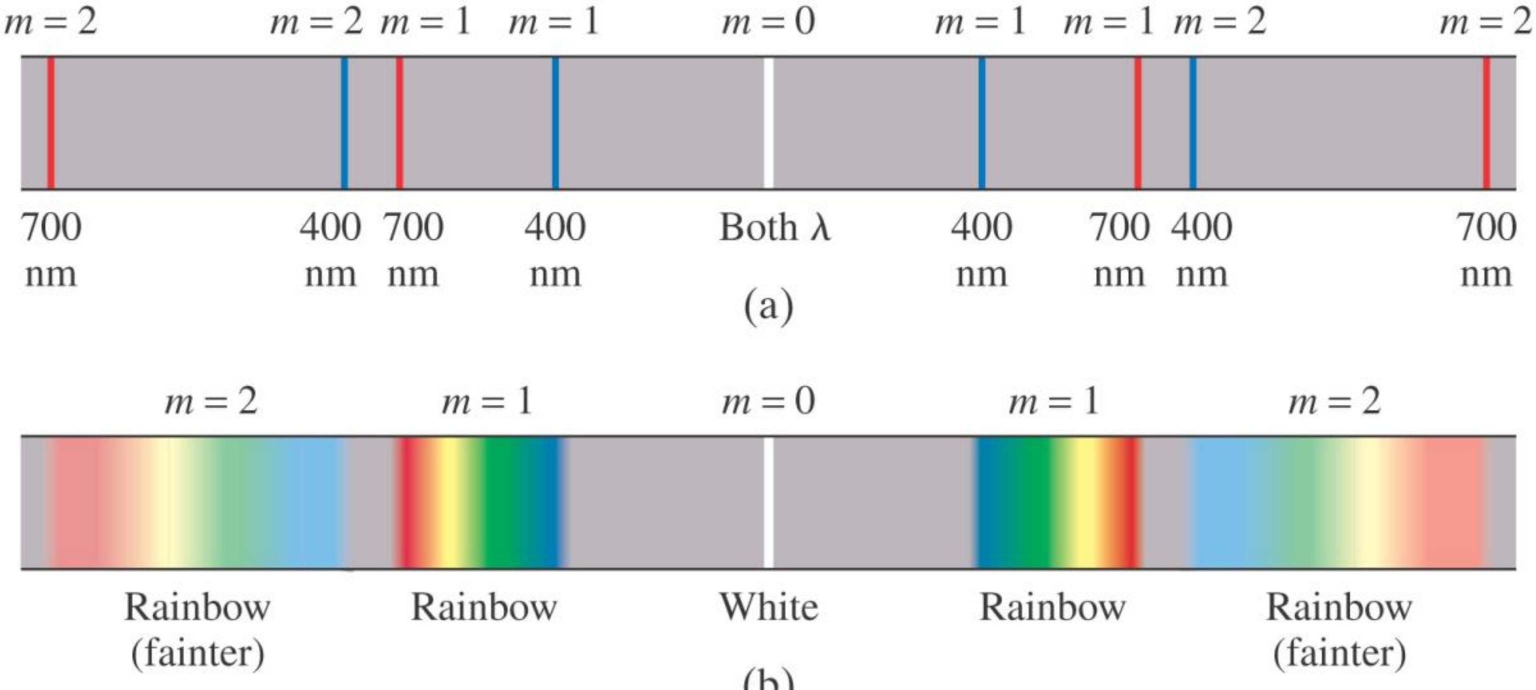
Reason for narrower width of the maxima:

For 2 slit interference, the maxima are wider because there are more rays coming through the two slits from various angles which will undergo "partial" constructive interference on each side of the mid-point of each maxima.

If there are more slits, interference from the rays coming from the many different slits at various angles of entry do not give rise to constructive interference on the screen outside of the optimal angles that give the maximas (when the path difference between each ray is exactly $n\lambda$).

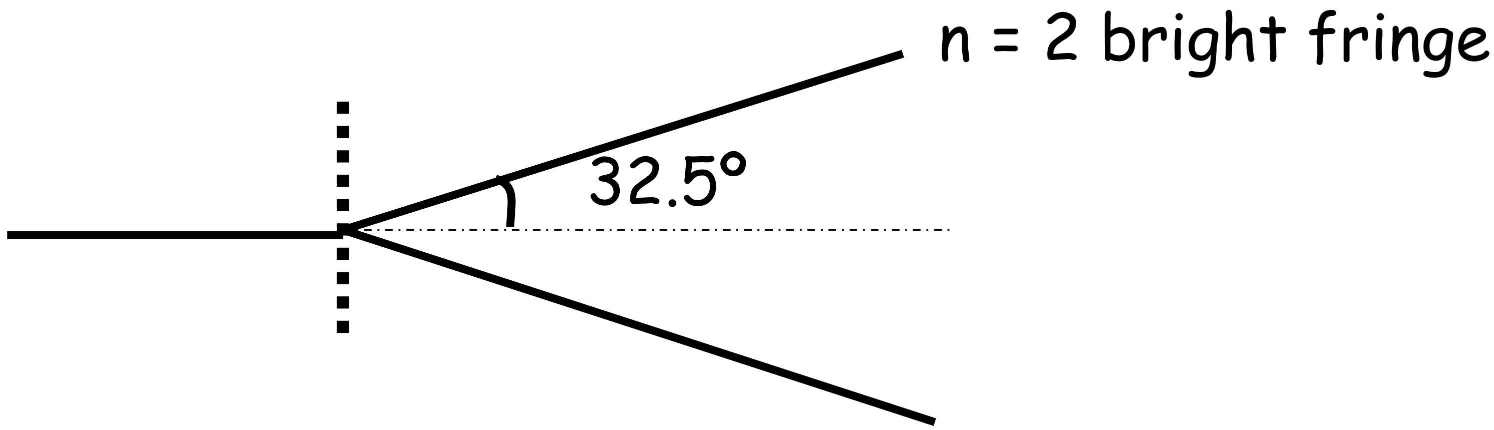
Diffraction Grating

Since the position of the maxima (except the central one) depends on wavelength, the fringes contain a spectrum of colours.



Worked Example

Monochromatic light is incident on a grating that is 75 mm wide and ruled with 50000 lines(slits). The second order maxima is observed at an angle of 32.5 º to the central maximum. Determine the wavelength of the incident light.



□

$$d \sin \theta = n \lambda$$

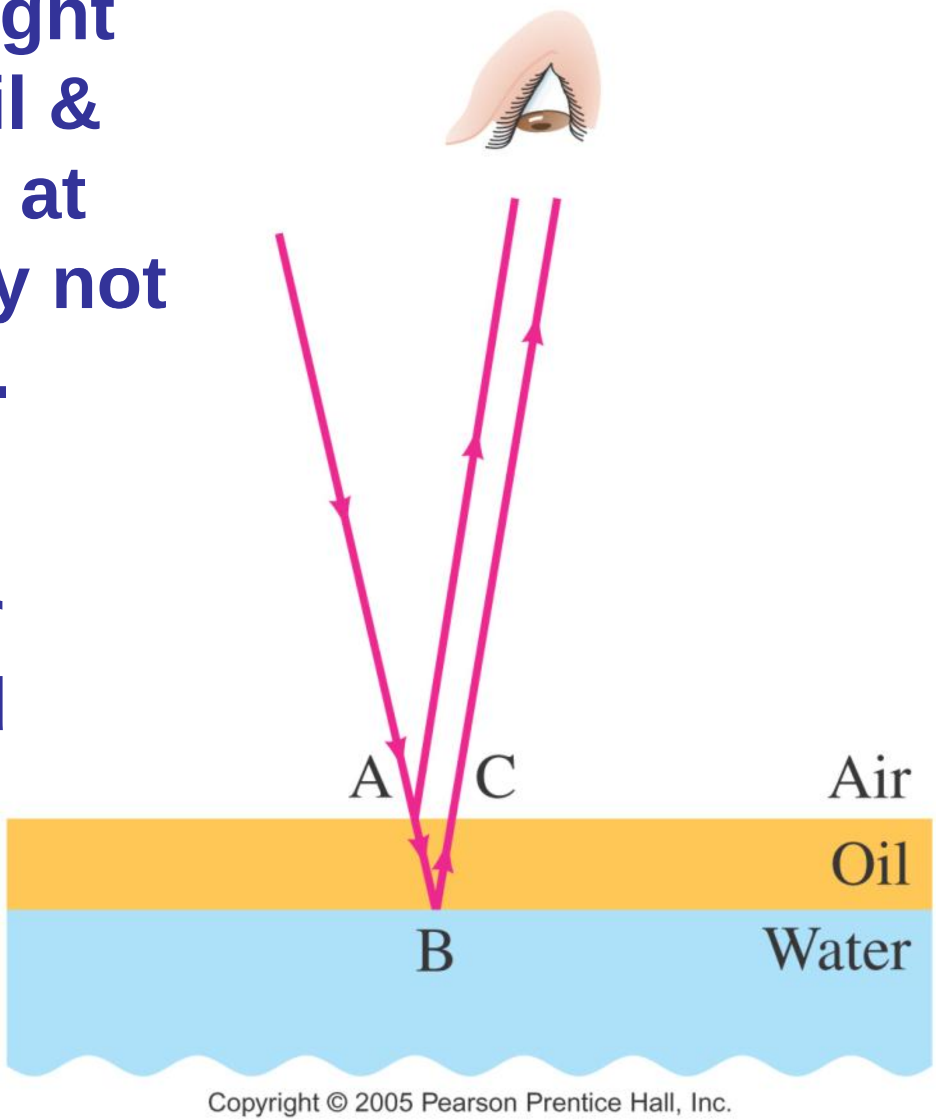
$$\frac{75 \times 10^{-3}}{50000} \sin 32.5^{o} = 2 \lambda$$

$$\Rightarrow \lambda = 4.1 \times 10^{-7} \text{ m}$$

Thin Film Interference

Thin Film Interference: Parallel Films

The wavelength of the light will be different in the oil & the air, & the reflections at points A & B may or may not involve a phase change.



The two reflected rays interfere with each other to produce the observed interference patterns.

Thin Film Interference

When light encounters an interface, a portion of it will get reflected, and the rest will pass into the incident medium and get refracted. This is repeated at every interface within the thin film stack. When the film thickness is on the same order of magnitude as the incident wavelength, the phases of rays reflecting off different interfaces will interact with each other. Depending on the wavelength, film thickness, and refractive indices of films, the rays will undergo one of the three processes: 1) constructive interference, 2) destructive interference, or 3) somewhere in between.

Optical Path

When light travels through a medium, it will slow down. In vacuum, light travels at the speed of light c. The medium has a length L and a refractive index n.

In the same time t light traveled a distance L in the medium while light in vacuum has traveled a distance nL.

$$\frac{n_1}{n_2} = \frac{v_2}{v_1}$$

$$\frac{1}{n} = \frac{v}{c}$$

$$\therefore n = \frac{c}{v}$$

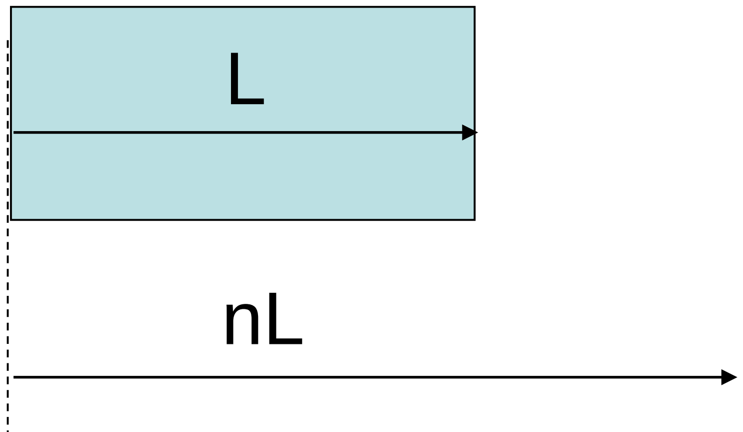
$$t = \frac{L}{v}$$

$$t = \frac{L_{vacuum}}{c}$$

$$\frac{L}{v} = \frac{L_{vacuum}}{c}$$

$$L_{vacuum} = L\frac{c}{v}$$

$$L_{vacuum} = nL$$



Thin Film Interference: Parallel Films

Path lengths can differ, & waves interfere, if waves travel through different media.

Interference can occur between reflections from the front & back surfaces of a thin film.

Light reflects from optically denser medium
- phase change of π rad ($\lambda/2$ path difference).

Light reflects from optically less dense medium
- no phase change.

This can be seen in soap bubbles & oil films
- one or two colours reinforce along a direction in which others cancel.

Thin Film Interference: Parallel Films

Total path difference

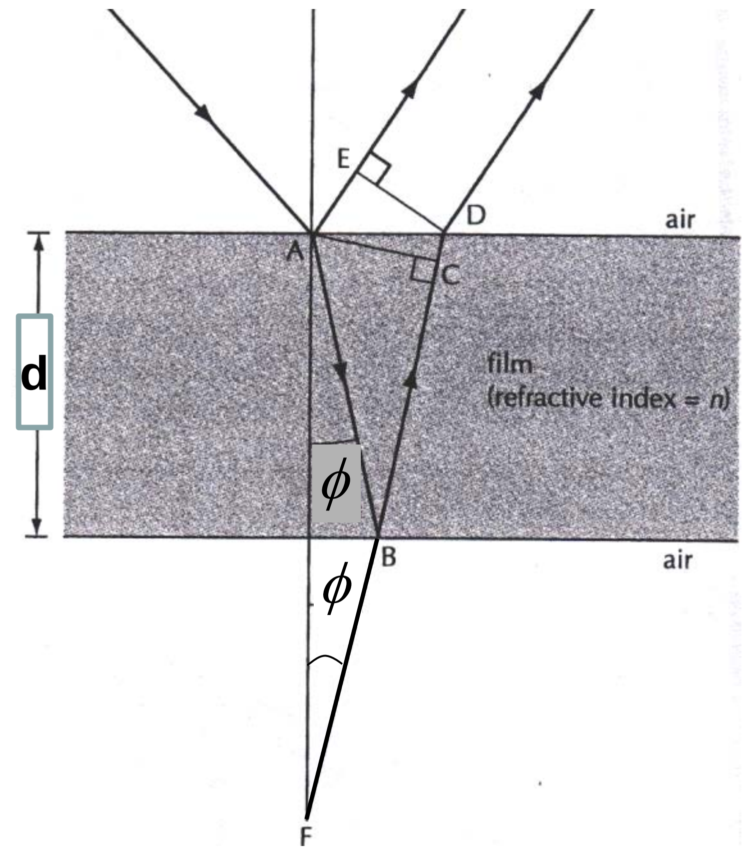
= (AB + BC) in film + λ/2 phase change at A

Note that the path AE in air is equivalent to CD in the film.

= n (AB + BC) + λ/2

= n (FC) + λ/2

= n (2dcosϕ) + λ/2



Path difference = 2ndcosϕ + λ/2

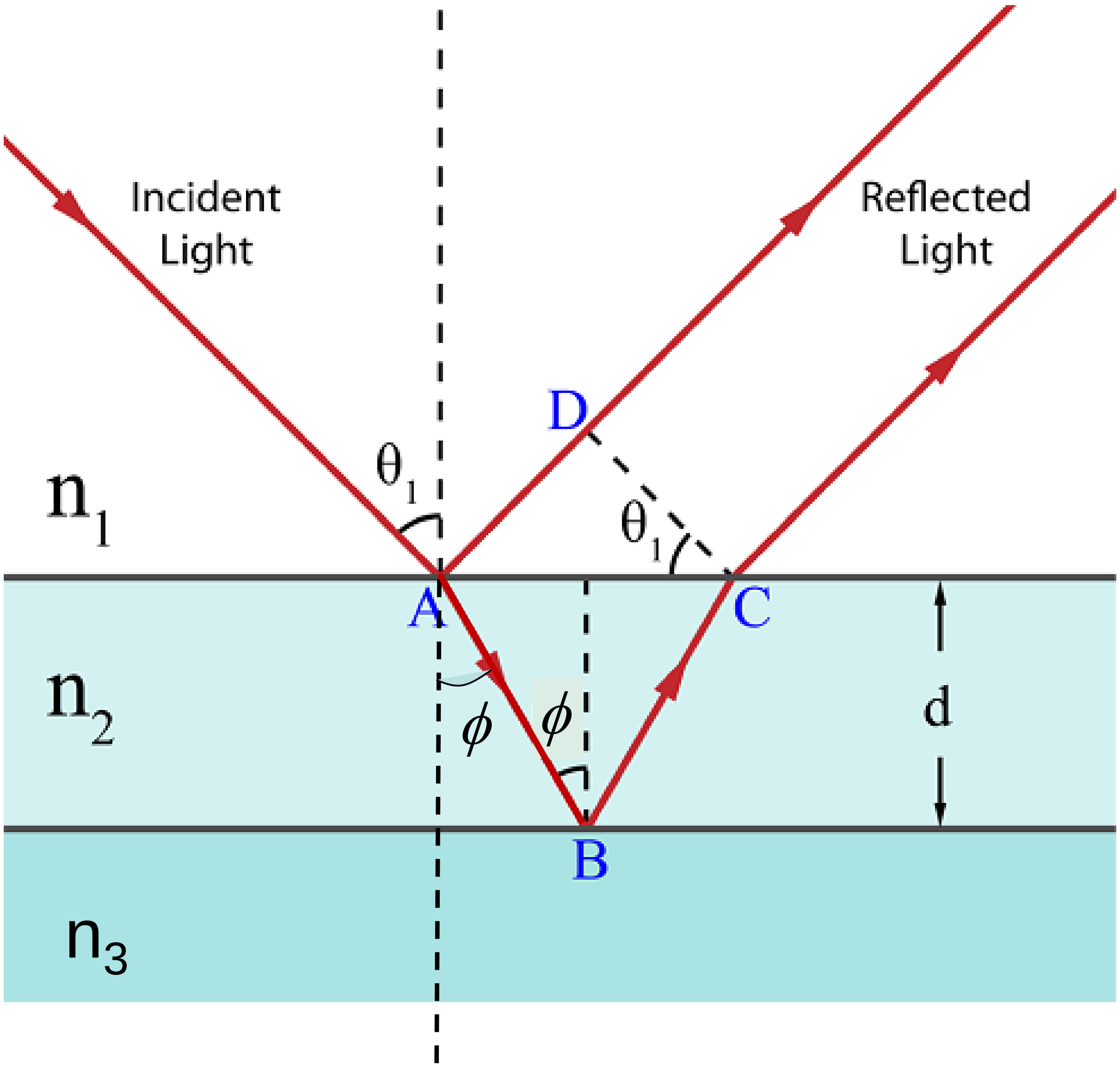
Constructive interference

Path difference = mλ

Destructive interference

Path difference = (m + ½)λ

Thin Film Interference: Parallel Films



Thin Film Interference: Parallel Films

$$2dn\cos\phi = (m+\frac{1}{2})\lambda \quad \text{(A)}$$

$$2dn\cos\phi = m\lambda \quad \text{(B)}$$

The index number m = 0, 1, 2, 3…

The formula used depends on the relative reflective indices.

For air-glass-air, (A) will be used for constructive interference and (B) for destructive interference.

If $n_1 < n_2 < n_3$, then (A) = destructive and

(B) = constructive

Thin Film Interference: Parallel Films

$$\text{Path difference} = 2nt\cos\phi\left(\pm\frac{1}{2}\lambda\right)$$

180º phase change at A	**180º phase change at B**	**Path Difference**
Yes No	Yes No	$2nt\cos\phi$
Yes No	No Yes	$2nt\cos\phi \pm \frac{\lambda}{2}$

Thin Film Interference: Parallel Films

$$2nt\cos\phi = (m + \frac{1}{2})\lambda \quad \text{(Eq 1)}$$

$$2nt\cos\phi = m\lambda \quad \text{(Eq 2)}$$

180º phase change at A	180º phase change at B	Equation for Constructive Interference	Equation for Destructive Interference
Yes No	Yes No	$2nt\cos\phi = m\lambda$	$2nt\cos\phi = (m + \frac{1}{2})\lambda$
Yes No	No Yes	$2nt\cos\phi = (m + \frac{1}{2})\lambda$	$2nt\cos\phi = m\lambda$

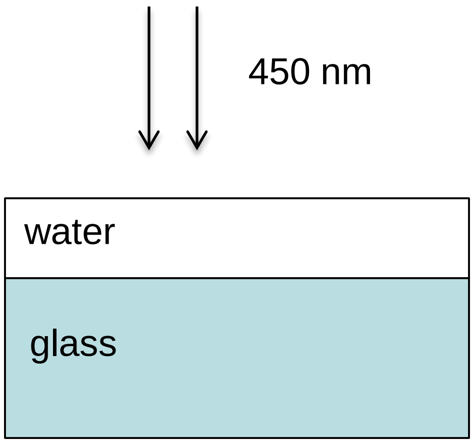
Thin Film Interference: Parallel Films

Example 1

A film of water of refractive index n = 1.33 lies on top of a layer of glass (refractive index n = 1.50) and is illuminated by blue light of wavelength 450 nm as shown below. Determine the ***minimum*** thickness of the water film that will allow for the film, when observed near normal incidence, to

(a) appear blue,

(b) appear dark.



Thin Film Interference: Parallel Films

Example 1

Rays that will interfere with each other:

1. Ray reflected off air-water boundary (phase change of $\frac{\lambda}{2}$)
2. Ray reflected off water-glass boundary (phase change of $\frac{\lambda}{2}$)

(a) Rays must interfere constructively for film to appear blue.

Base condition for constructive interference: $2dn = m\lambda$
**factor in and add phase changes to this*

$$2dn = m\lambda + \frac{\lambda}{2} + \frac{\lambda}{2}$$

$2dn = (m+1)\lambda$ (still an integer multiple of λ) which is the same as $2dn = m\lambda$

Thus criteria for constructive interference is still $2dn = m\lambda$

Thin Film Interference: Parallel Films

Example 1

Thus criteria for constructive interference is still $2dn = m\lambda$

Since minimum thickness is required, $m = 1$

$2(d)(1.33) = (1)(450 \times 10^{-9})$

Minimum thickness, d for film to appear blue

$$= \frac{450 \times 10^{-9}}{2(1.33)}$$

$= 1.69 \times 10^{-7}$ m

Thin Film Interference: Parallel Films

Example 1

(b) For film to appear dark, destructive interference must have occurred.

Repeating steps of factoring in phase changes,
criteria for destructive interference is still $2dn = (m + 0.5)\lambda$

Since minimum thickness is required, $m = 0$

$2(1.33)d = (0.5)(450 \times 10^{-9})$

Minimum thickness, d for film to appear dark

$$= \frac{(0.5)(450 \times 10^{-9})}{2(1.33)}$$

$= 8.46 \times 10^{-8}$ m

Thin Film Interference: Parallel Films

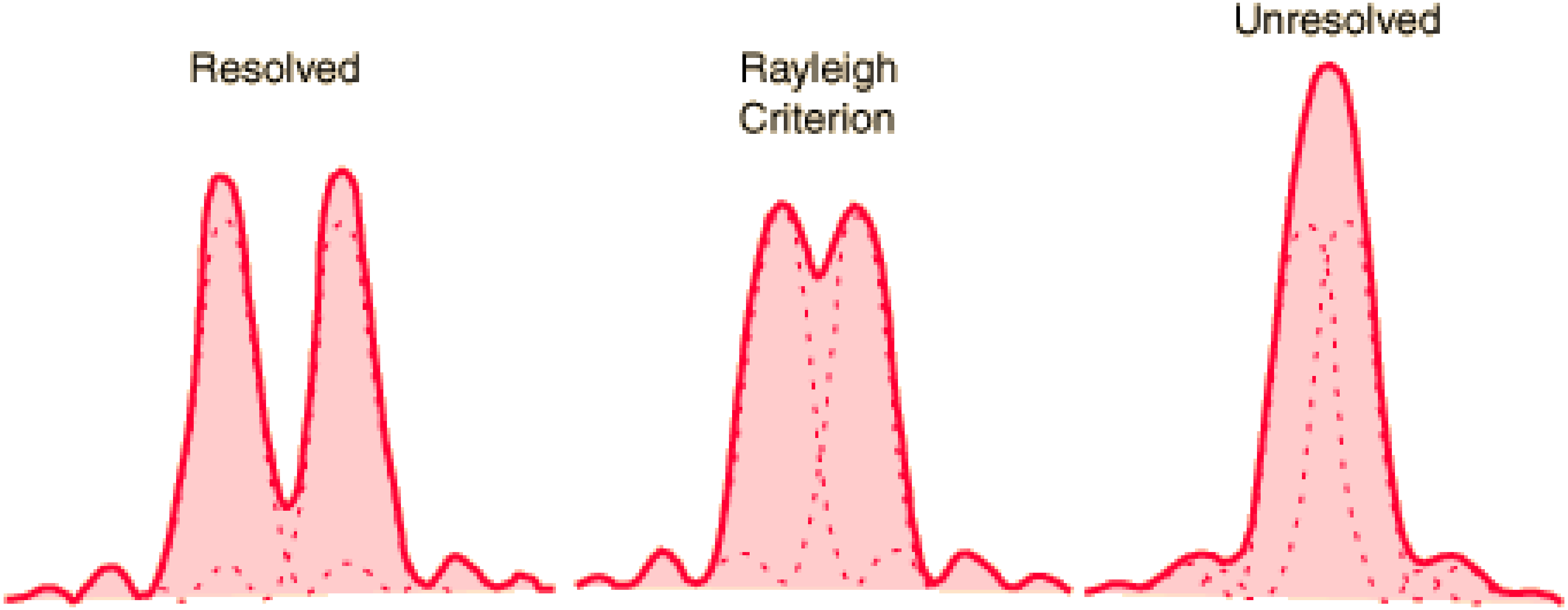
Multicoloured fringes

- When light falls onto a thin film, the different wavelengths will form fringes at different angles thus causing a multi coloured pattern to appear

Applications

- Non reflecting surfaces on military aircraft
- Measurement of thickness of oil spills
- Reduction of reflection from lens

9.4 Resolution



9.4 Nature of Science

Improved technology: The Rayleigh criterion is the limit of resolution. Continuing advancement in technology such as large diameter dishes or lenses or the use of smaller wavelength lasers pushes the limits of what we can resolve.

Resolution

The term resolution or minimum resolvable distance is the minimum distance between distinguishable objects in an image.

Similarly, angular resolution is the smallest angular distance between two light sources that can be resolved. It also describes the resolving power of any image forming device such as an optical or radio telescope, a microscope, a camera, or an eye.

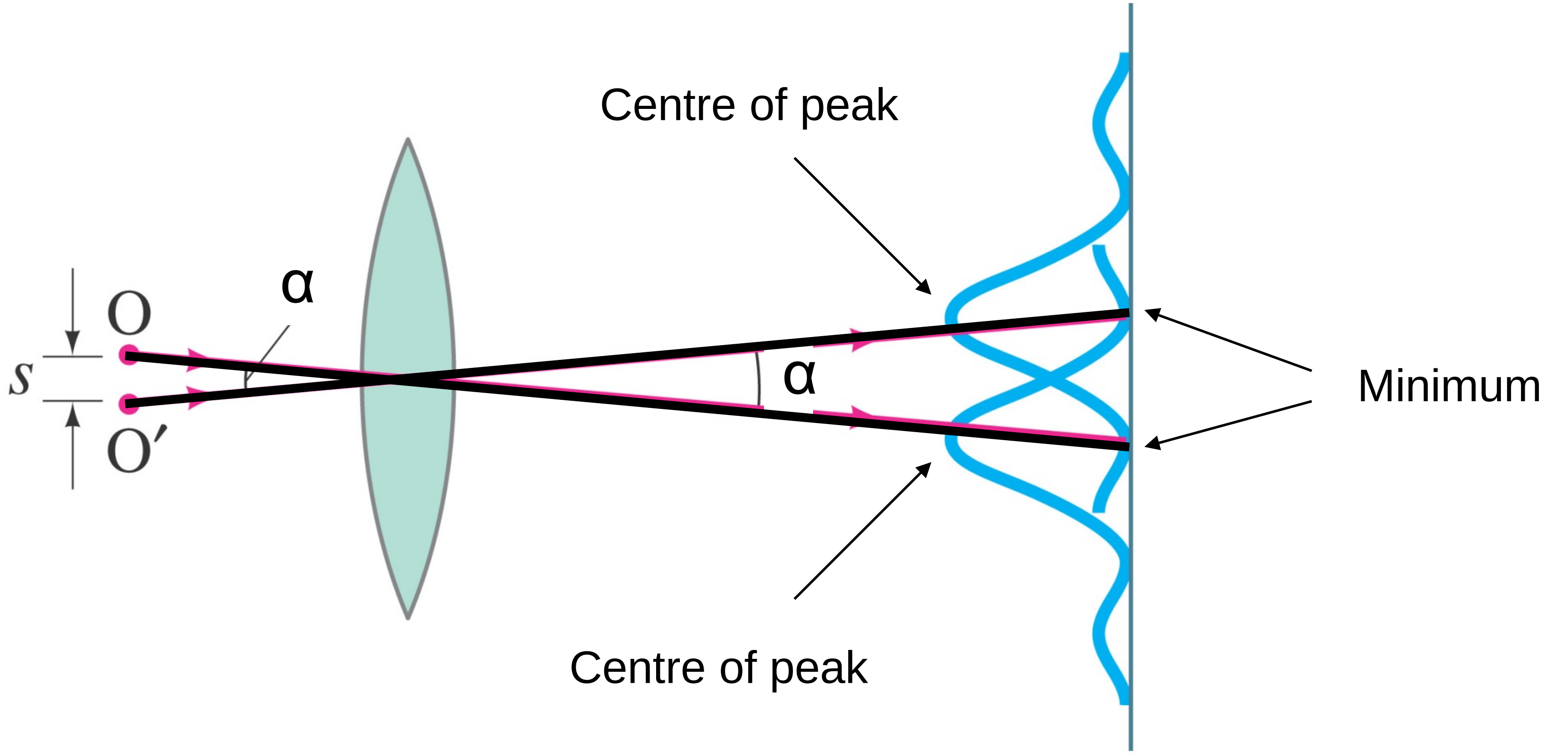
Resolution

When light from two different sources is brought together, the principle of superposition determines the final image.

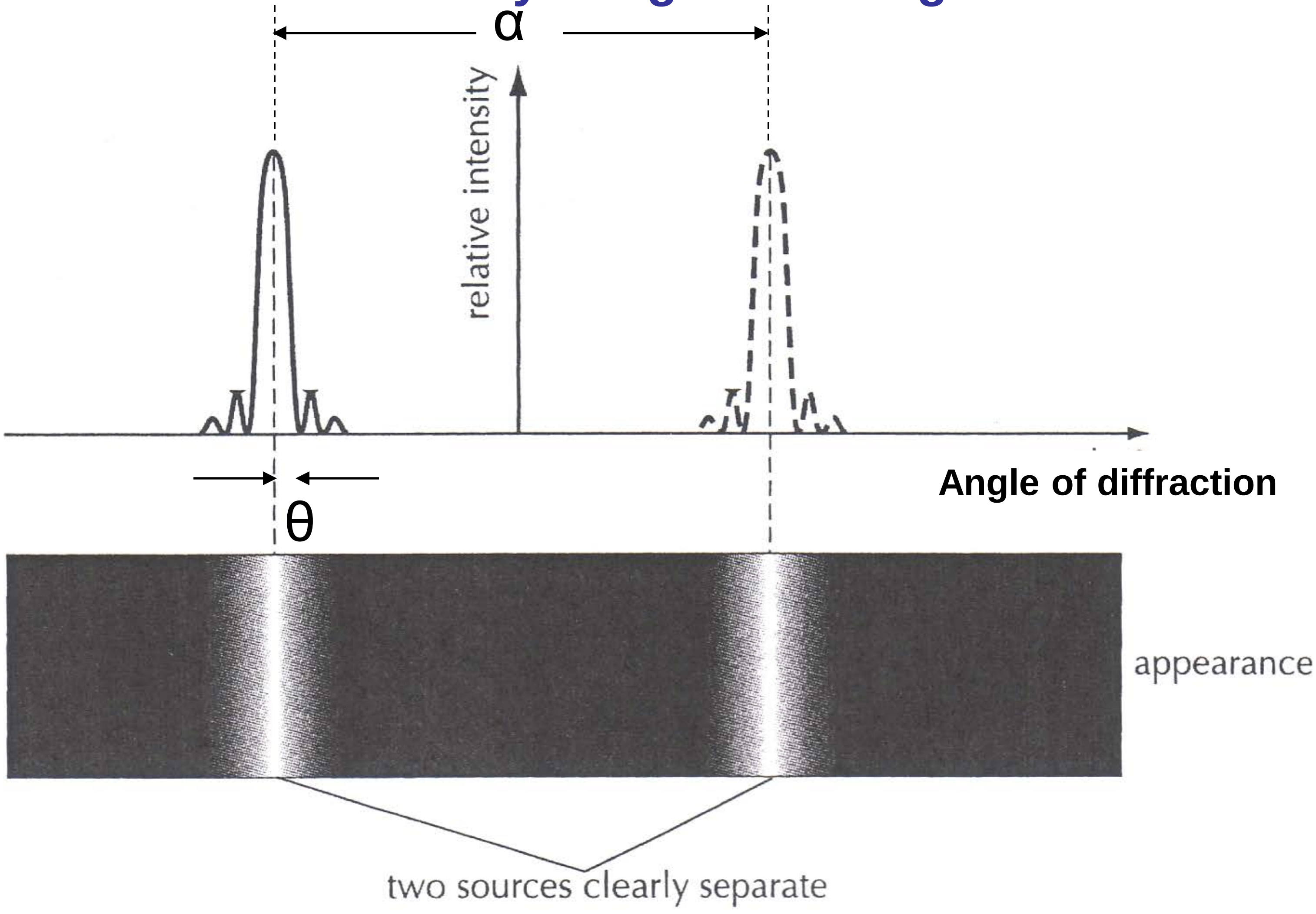
When viewing two sources through an aperture, such as stars through a telescope, the overall result is the addition of individual diffraction patterns (since each source will result in a diffraction pattern through the aperture).

Resolution

Rayleigh criterion: Two sources are just resolvable when the first minimum of the diffraction pattern of one of the sources falls on the central maximum of the diffraction pattern of the other source.



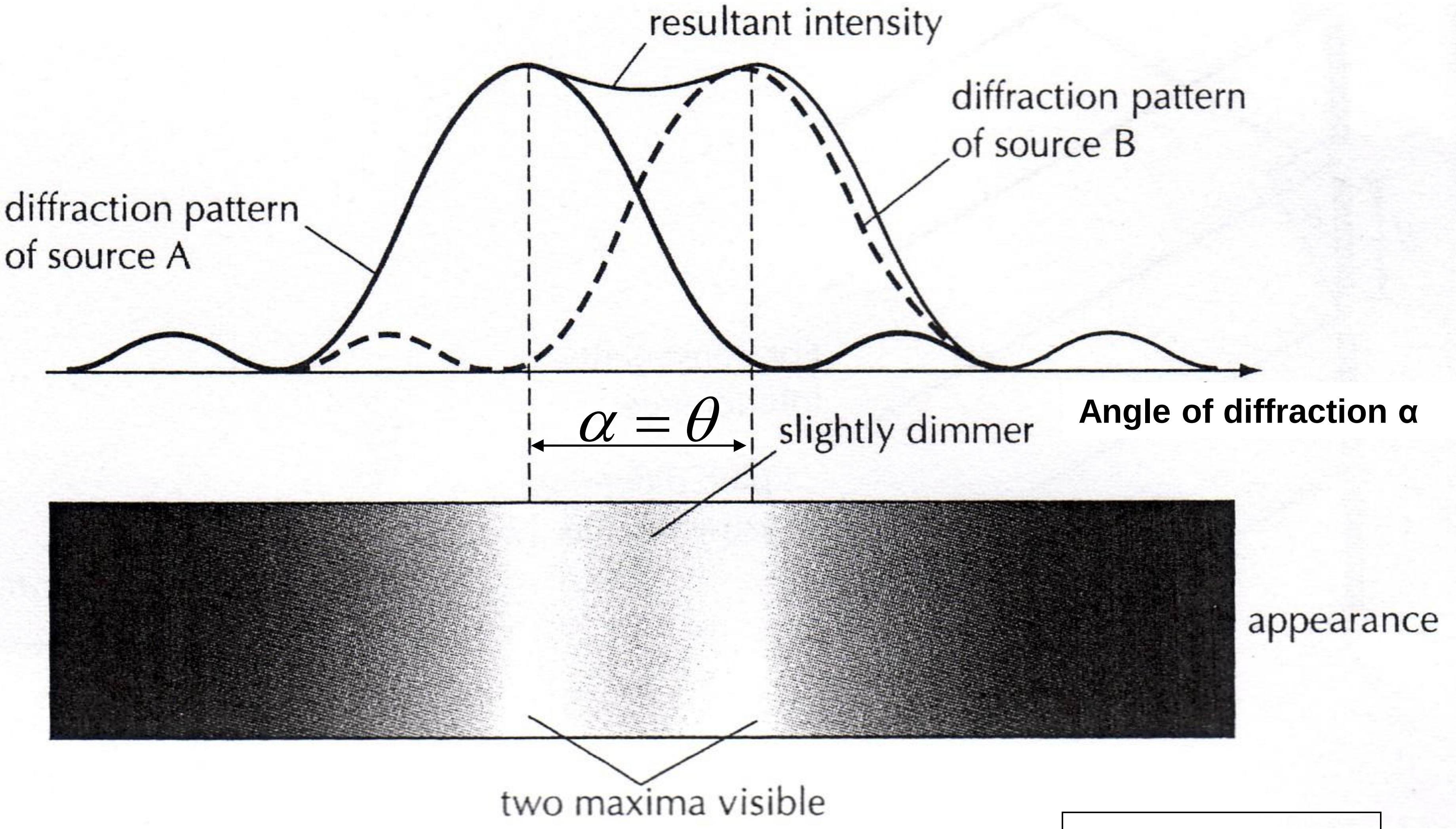
Variation of relative intensity of light with angle of diffraction



Diffraction patterns are well resolved

$$\alpha > \theta$$

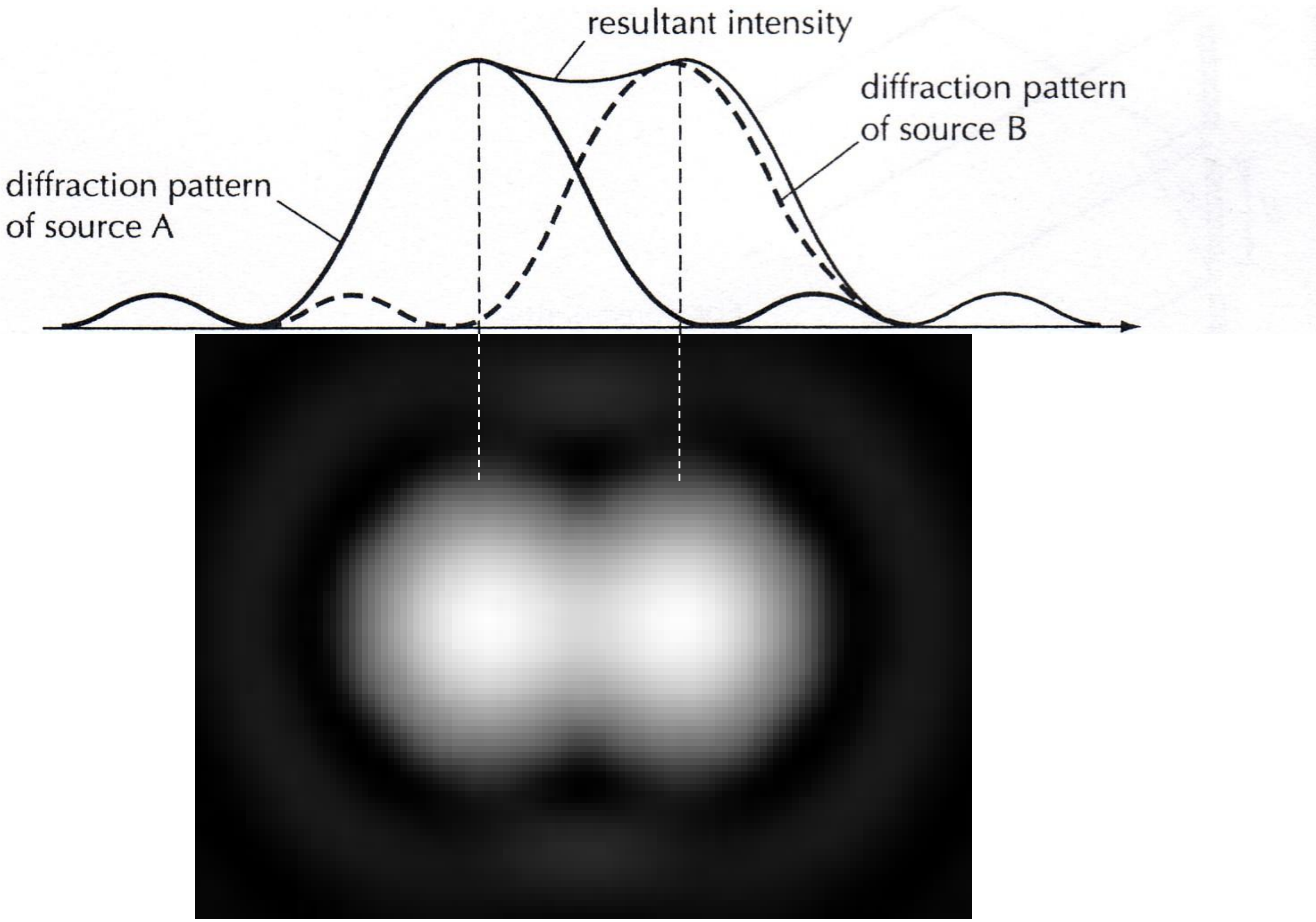
Variation of relative intensity of light with angle of diffraction



Diffraction patterns are just resolved

$$\alpha = \theta$$

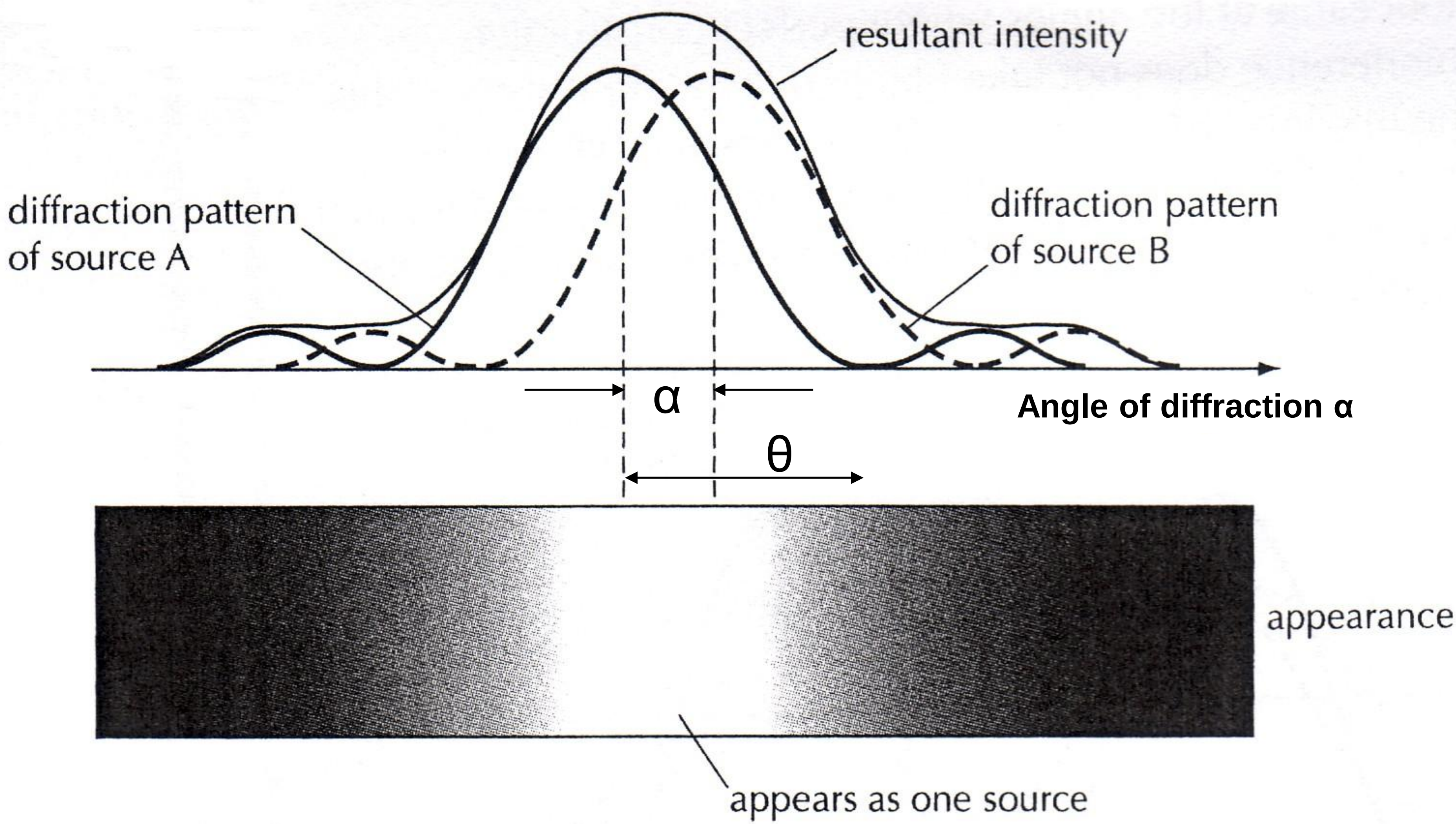
Variation of relative intensity of light with angle of diffraction



Diffraction patterns are just resolved

- for 2 point sources

Variation of relative intensity of light with angle of diffraction



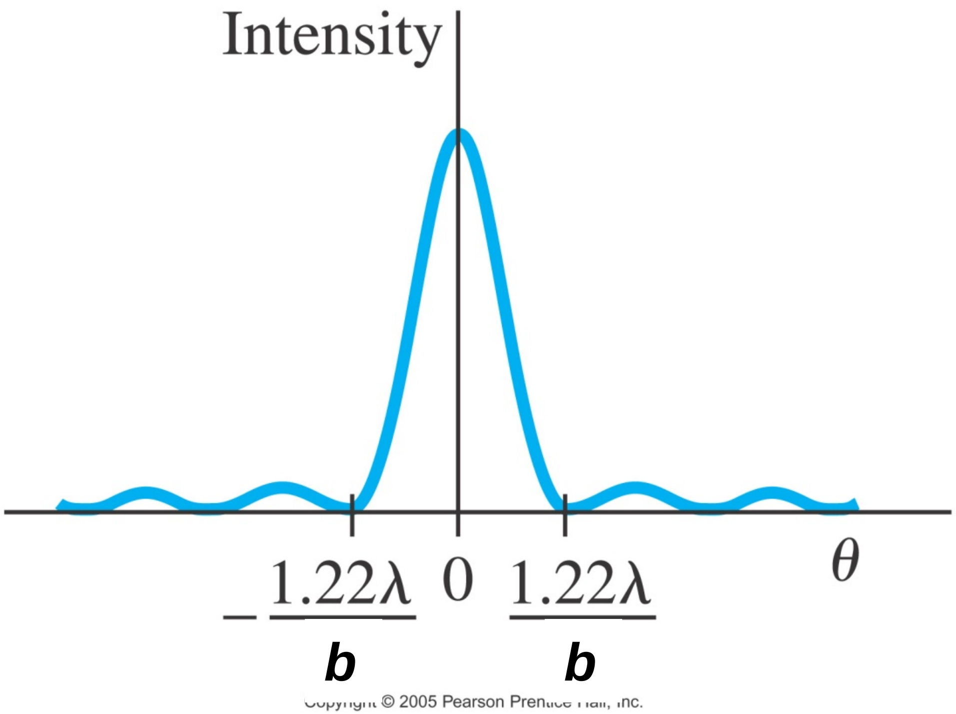
Diffraction patterns are not resolved

$$\alpha < \theta$$

Resolution: Circular Apertures

For a circular aperture of diameter b, the 1st minimum occurs at

$$\theta = \frac{1.22\lambda}{b}$$

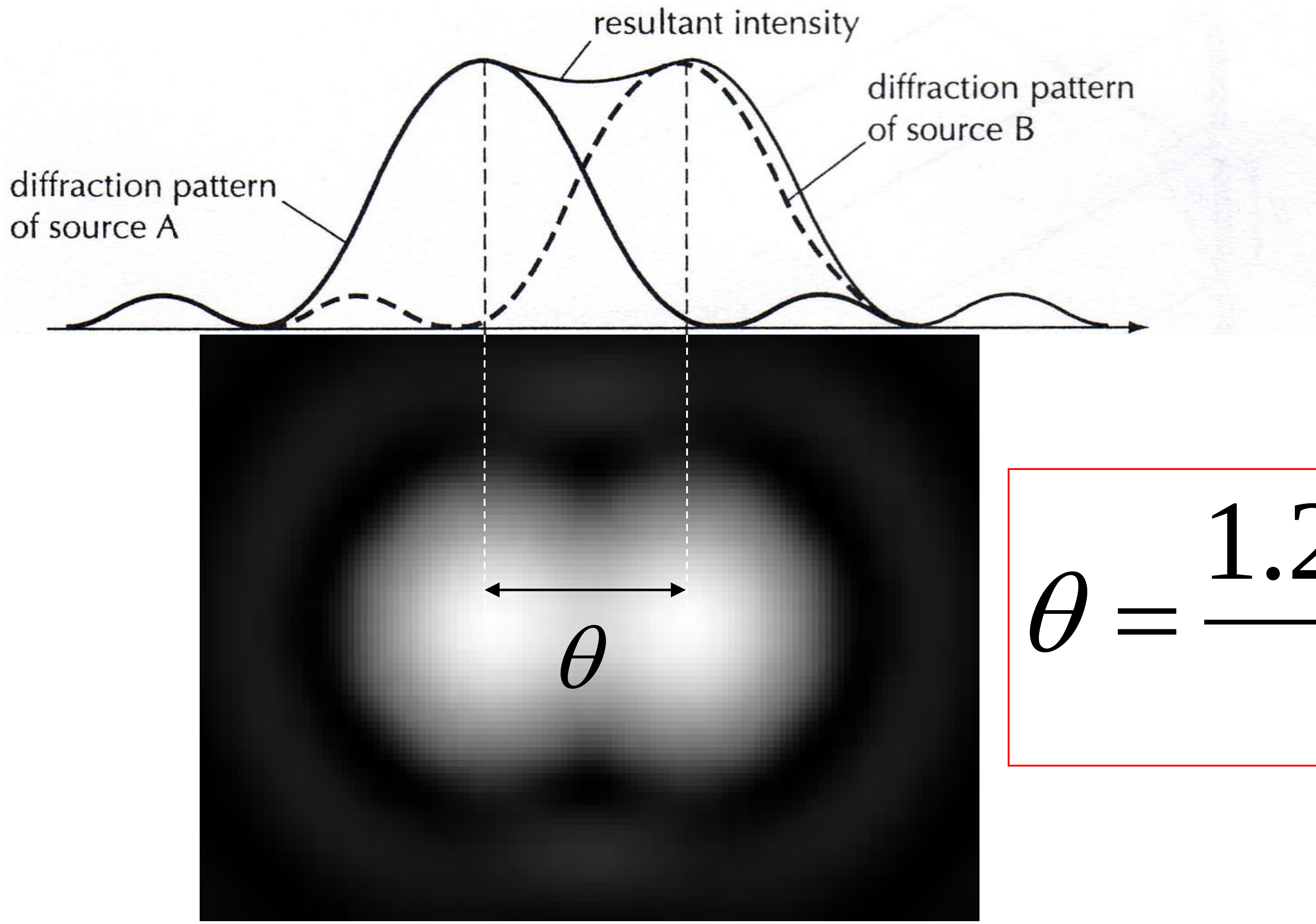


θ : angle between the straight through direction and the first minimum

λ : wavelength

b : diameter of aperture

Rayleigh criterion for circular apertures



$$\theta = \frac{1.22\lambda}{b}$$

What do you think will be the Rayleigh Criterion for a single-slit aperture?

Example 1

The camera of a spy satellite orbiting at 200 km has a diameter of 35 cm. What is the smallest distance this camera can resolve on the surface of the earth? (Assume a wavelength of 500 nm)

Using Rayleigh's criterion and a wavelength of 5.0×10^{-7} m, we find that the distance s that can be resolved is given by

$s = r\theta$ where $\theta \approx (1.22 \times 500 \times 10^{-9}) / 35 \times 10^{-2} \approx 1.74 \times 10^{-6}$ rad

Hence $s = 2 \times 10^{5} \times 1.74 \times 10^{-6} = 0.34$ m

Example 2

The headlights of a car are 2m apart. The human eye has a lens with a diameter of about 2 mm and suppose that light of wavelength 500 nm is being used.

What is the maximum distance at which the two headlights are seen as distinct?

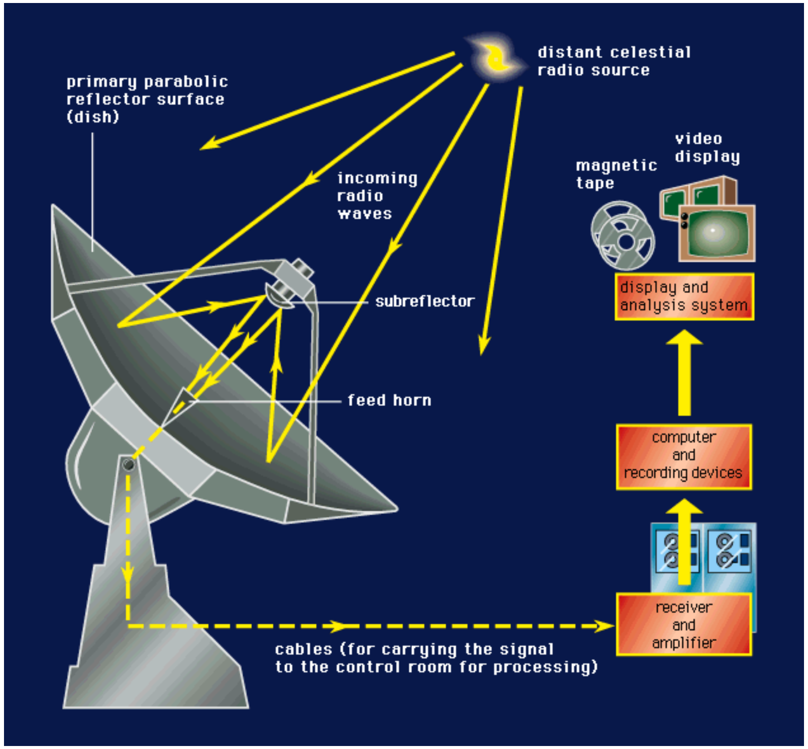
The resolution of the eye is $\theta \approx (1.22 \times 500 \times 10^{-9}) / 2 \times 10^{-3}$

$$\approx 3 \times 10^{-4} \text{ rad}$$

Since $r = s / \theta = 2 / 3 \times 10^{-4} = 0.67 \times 10^{4} \approx 7000$ m

The car should be no more that this distance away.

Significance of resolution – Radio Telescope



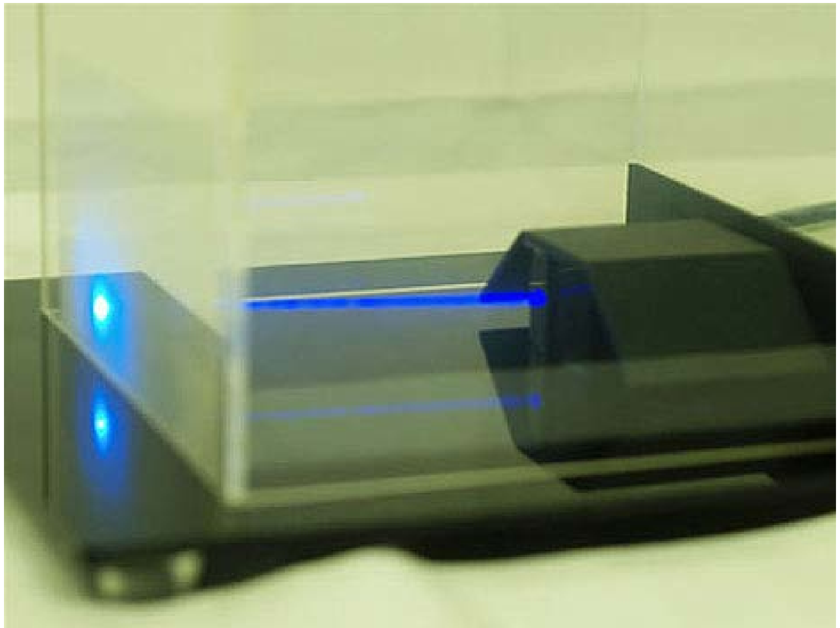
Since θ is inversely proportional to the diameter b, radio telescopes are built very large so that θ is small. Two celestial radio sources with small angular separation can still be resolved.

Significance of resolution – CD & DVD

CD-ROM drives employ a 780 nm (near-infrared) laser diode whereas the standard DVD laser diodes have a wavelength of 650 nm (red).

High-definition Blu-ray drives have a wavelength of 405 nm (blue-violet).

Since θ is proportional to wavelength λ, shorter wavelengths are used so that θ is small. This means more data can be stored per unit area.



Significance of resolution – Electron Microscope

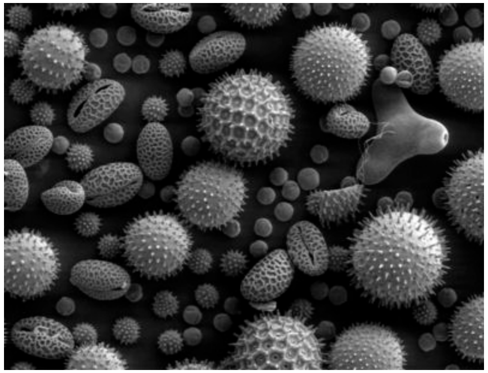
In a transmission electron microscope (TEM), electrons are passed through a wafer thin sample which are then focused by a magnetic field onto a CCD which then captures the image.

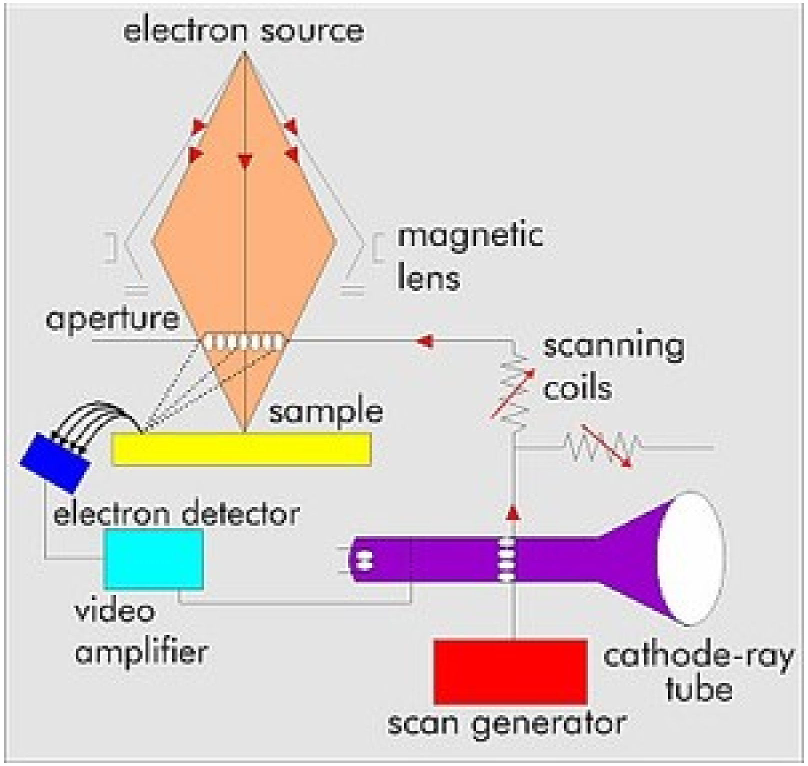
A scanning electron microscope (SEM) uses electrons which are scattered from the surface of the sample which are then focused as in the TEM to form an image of the surface.

Electrons used in an electron microscope have a wavelength λ of about 5×10^{-12} m.

This means more data can be stored per unit area.

SEM image of pollen



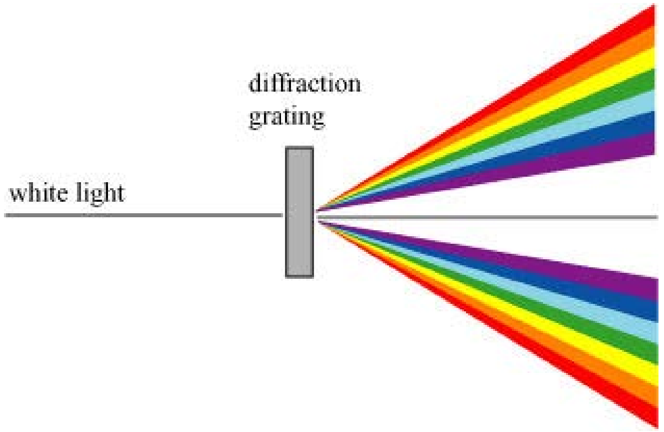


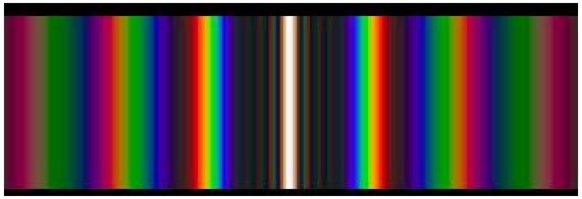
Resolvance

Diffraction gratings can and are used to disperse light of different wavelengths**.**

However, just like how there is a limit to how close two objects can be before their diffraction images are indistinguishable from each other, the same applies to the different wavelengths that can be resolved by a diffraction grating.

The way we gauge the ability of a diffraction grating to resolve these wavelengths is known as **resolvance**.





Interference pattern of white light through a grating

Resolvance

Definition:

The resolvance, R, for a diffraction grating is defined as the ratio of the wavelength λ of the light to the smallest difference in wavelength that can be resolved by the grating $\Delta\lambda$.

The resolvance is also equal to Nm, where N is the total number of slits illuminated by the incident beam of light and m is the order of diffraction.

$$R = \frac{\lambda}{\Delta\lambda} = Nm$$

The greater the resolvance, the better the resolution of the grating.

Example 3

Two lines in the emission spectrum of Caesium have wavelengths of 621.3 nm and 621.7 nm. Determine the number of lines per mm of the diffraction grating for the lines to be resolved in the third order spectrum with a beam width of 0.20 mm.

Resolvance, $R = \frac{\lambda}{\Delta\lambda}$

$$= \frac{621.3 \times 10^{-9}}{(621.7 - 621.3) \times 10^{-9}}$$

$$= 1553$$

*Note that you can also use the largest wavelength as λ.

Example 3 - Continued

$$1553 = Nm$$

$$1553 = 3N$$

$$N = 517.8 \; lines\,/\,slits$$

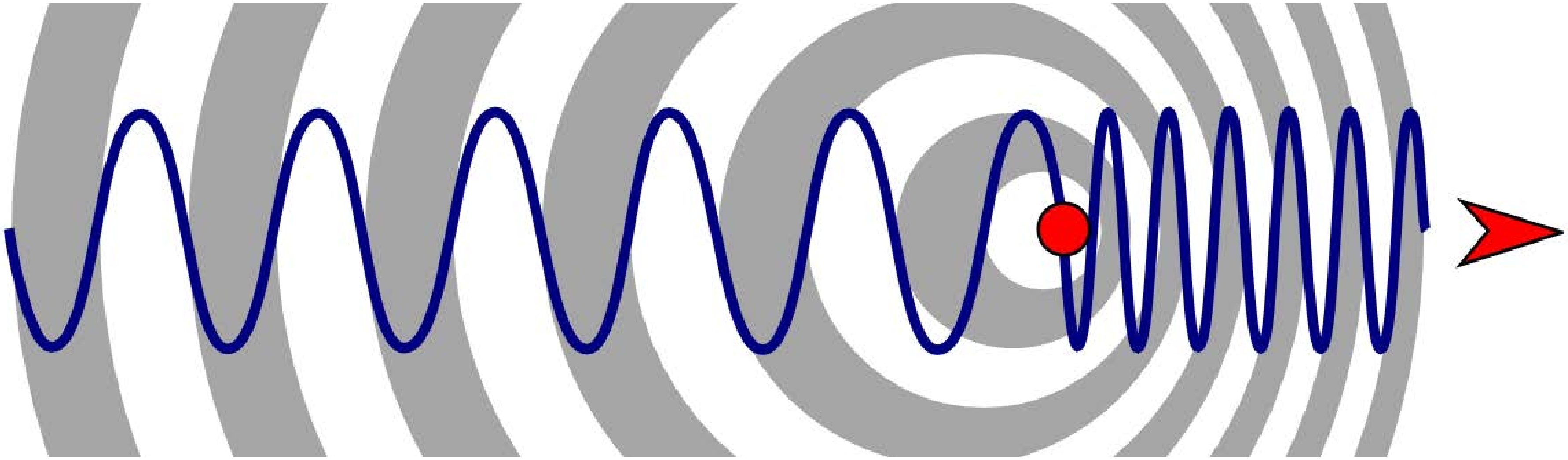
This means that the beam of width 0.20 mm illuminates 517.8 lines/slits on the grating.

Thus, the number of slits on the grating per mm is given by

517.8 x 5 = 2589 ≈ 2600 lines/slits

Gratings are not made to such a specific 2589 lines per mm, thus the sensible choice is to use a grating with 2600 lines per mm.

9.5 Doppler Effect

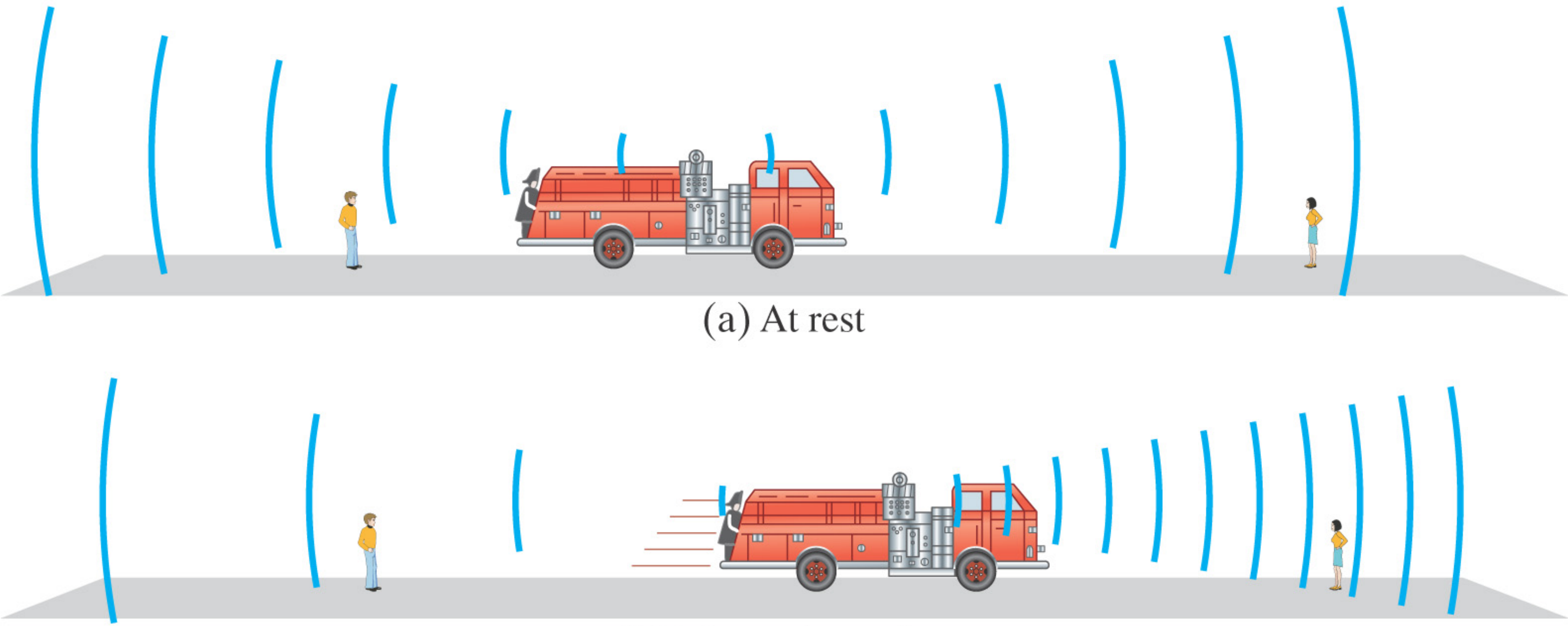


9.5 Nature of Science

Technology: Although originally based on physical observations of the pitch of fast moving sources of sound, the Doppler effect has an important role in many different areas such as evidence for the expansion of the universe and generating images used in weather reports and in medicine..

Doppler Effect

The **Doppler effect** occurs when a **source** of sound is **moving** with respect to an **observer**.



(a) At rest

(b) Firetruck moving

Doppler Effect

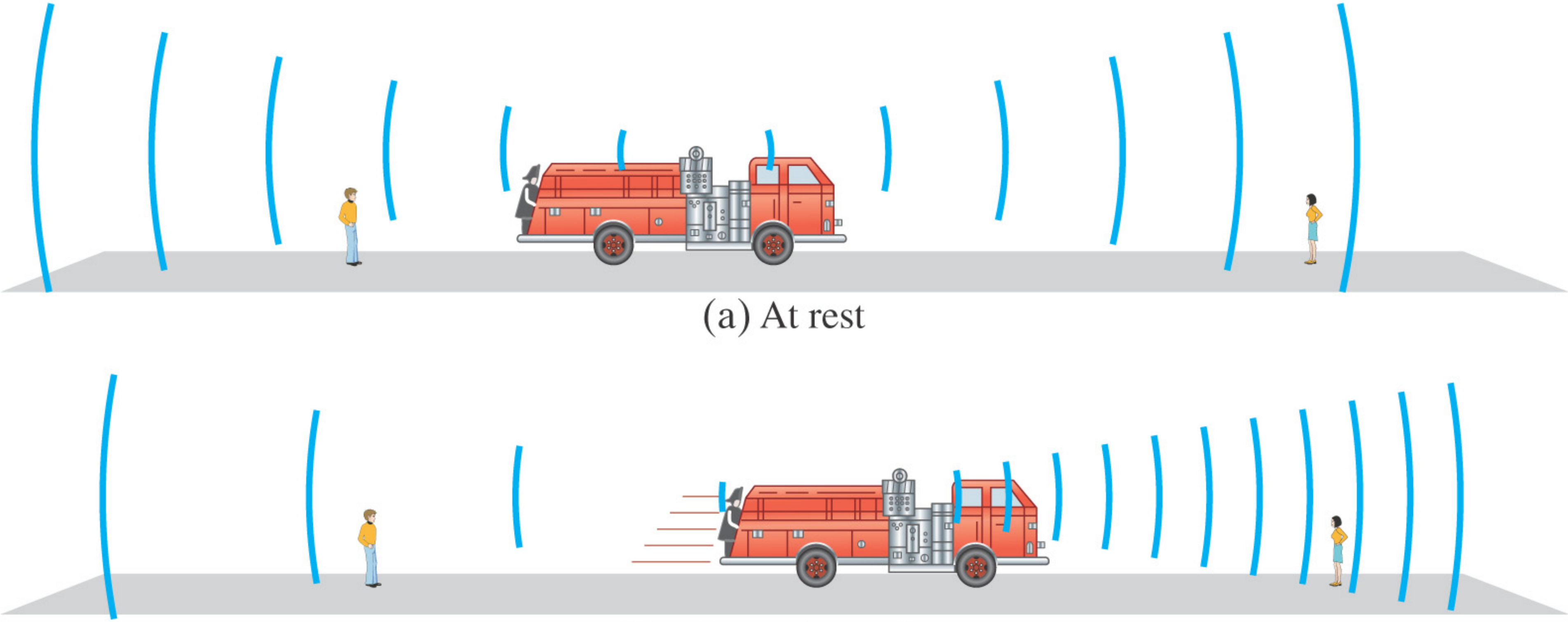
As can be seen in the previous image, a source moving **toward** an observer has a **higher** frequency and **shorter** wavelength.

The opposite is true when a source is moving **away** from an observer.

Definition: The Doppler Effect is the change in the measured frequency of a wave that results from the relative motion of a source and/or an observer relative to the medium in which the wave is propagated.

Doppler Effect – Moving Source

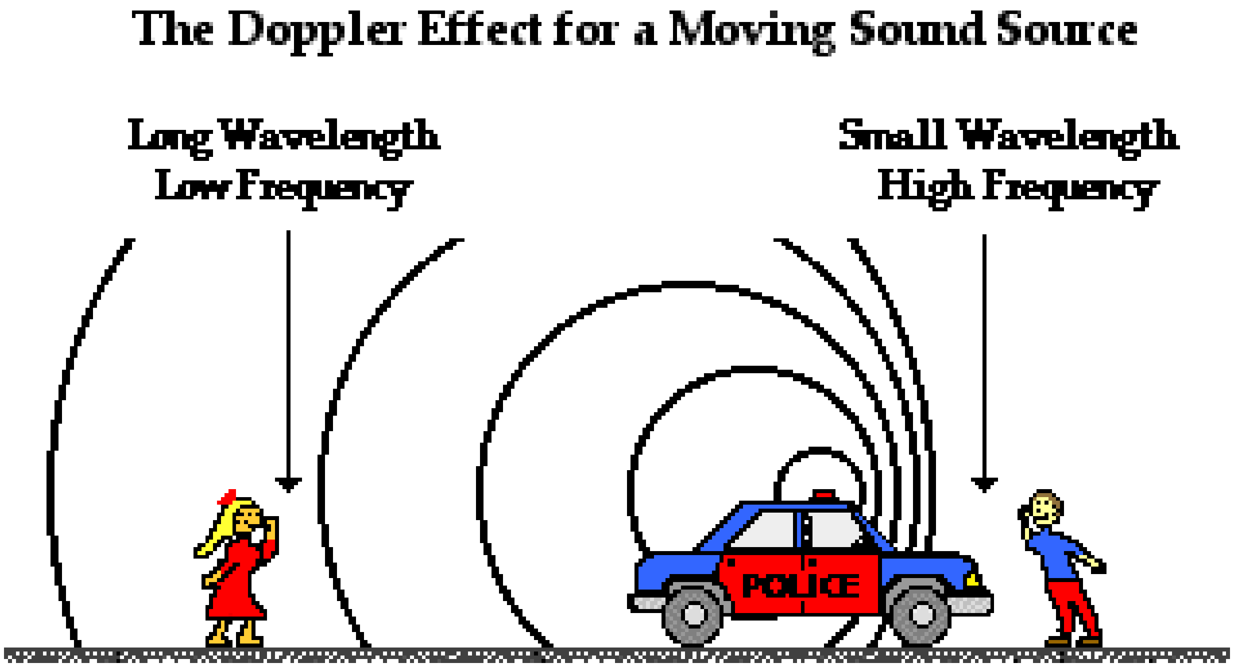
The **Doppler effect** occurs when a **source** of sound is **moving** with respect to an **observer** (**relative motion** between source & observer).



(a) At rest

(b) Firetruck moving

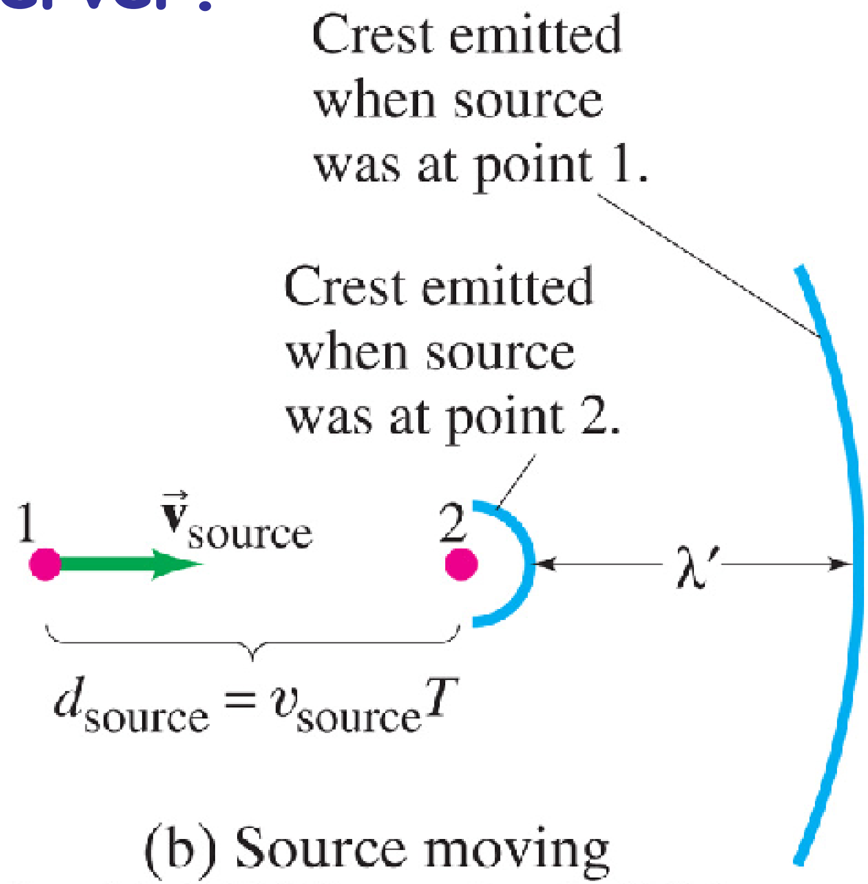
Doppler Effect – Moving Source



The source movement causes a change in the wave speed relative to the source. This means that the waves will "bunch" together in the forward direction and be spread apart in the reverse direction. To the observer, this causes <u>a change of wavelength</u> for the observer while the wave speed relative to the observer remains constant.

Doppler Effect – Moving Source

A source moving **toward** an observer has a **higher** frequency and **shorter** wavelength; the opposite is true when a source is moving **away** from an observer.



(b) Source moving

Doppler Effect - Moving Source

For a source moving **towards** a stationary observer:

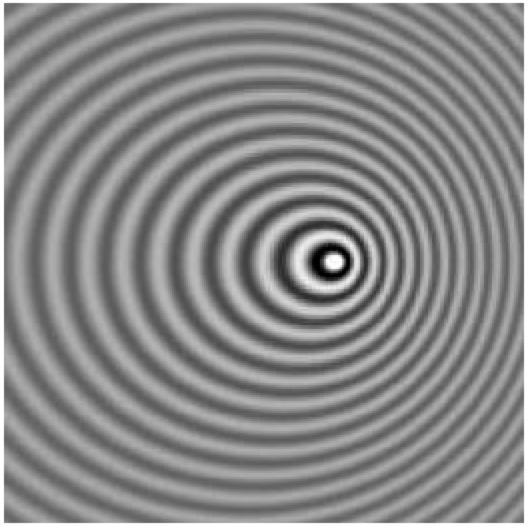
$$f' = f\left(\frac{v}{v - u_s}\right)$$

v : speed of sound in medium

u_s : speed of source

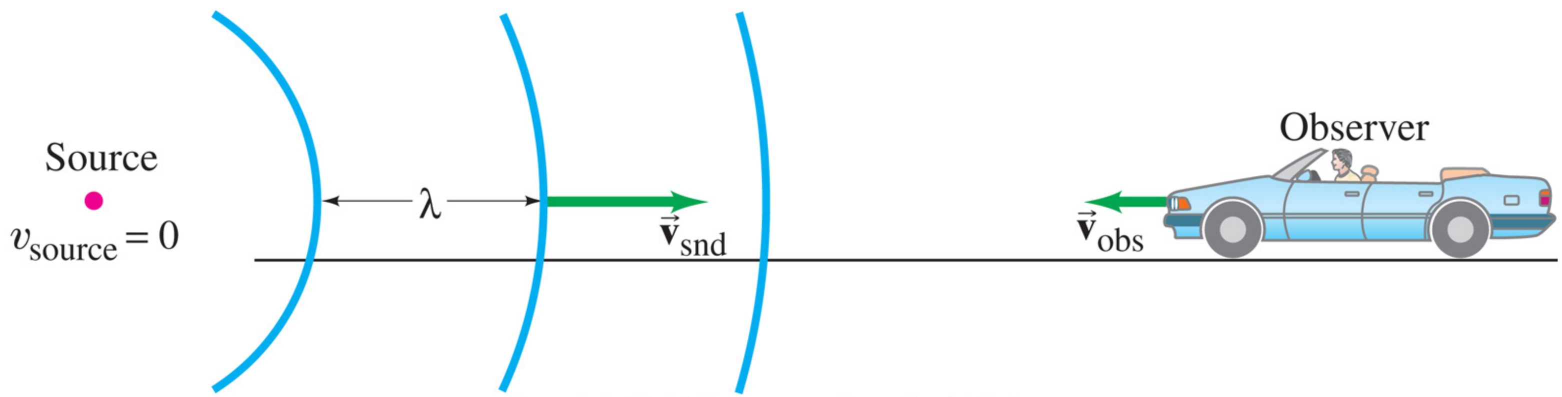
And if the source is moving **away from** a stationary observer :

$$f' = f\left(\frac{v}{v + u_s}\right)$$



Doppler Effect – Moving Observer

If the **observer** is moving with respect to a stationary source, the <u>**wavelength** remains the same with respect to the observer</u>. But the <u>**wave speed in the observer frame of reference** is different.</u>



An observer moving towards a wave source will receive more waves per unit time when compared with a stationary observer.

Doppler Effect – Moving Observer

For an observer moving **towards** a stationary source:

$$f' = f\left(\frac{v + u_0}{v}\right)$$

v : speed of sound in medium

u_o : speed of observer

And if the observer is moving **away from the source**:

$$f' = f\left(\frac{v - u_0}{v}\right)$$

Doppler Effect Compared for Moving Source and Moving Observer

$$v = f\lambda$$

Moving source: For moving source and stationary observer, speed of sound with respect to observer is unchanged. However, wavelength changes leading to a change in frequency.

In other words, since v is fixed, a reduction in λ will result in an increase in f. A increase in λ will result in a decrease in f.

Moving observer: For a moving observer and a stationary source, the speed of sound with respect to the observer is changed. Wavelength remains unchanged.

Since λ remains unchanged, an increase in v will result in an increase in f. A decrease in v will result in a decrease in f.

Doppler Effect

Two things need to be noted:

1) The speed of the wave through the medium is not altered but the received frequency is altered.

2) The Doppler Effect can also be applied to light waves but in this case, no medium is necessary for the propagation of light. It also turns out that the equations are the same form as that of sound if the relative velocity between source and observer is a lot less than that of the speed of light and relative velocity is used in the equation.

Effect of wind

If there is a wind blowing from the direction of a stationary source towards a stationary observer, what is the effect with respect to the observer on

1) the speed of the wave; [Ans: higher]

2) the wavelength; [Ans: longer]

3) the observed frequency of the source [Ans: unchanged]?

Doppler Effect – Electromagnetic Waves

If the velocities of the source or observer, v, is small compared to the speed of the waves, c, all four equations can be reduced to the following equation:

$$\Delta f = \frac{v}{c} f \quad \text{if } v << c$$

$$\frac{\Delta f}{f} = \frac{\Delta \lambda}{\lambda} = \frac{v}{c}$$

Δf : difference in frequency between source and observer

f : frequency of the source

v : relative speed between source and observer

c : speed of the electromagnetic wave

<u>**Note**</u>: There is a change in terminology for v and c in this equation.