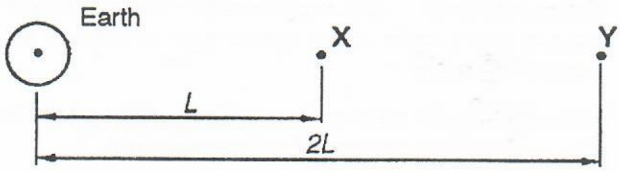
TOPIC 7 Gravitation

1 The diagram shows two points **X** and **Y** at distances L and $2L$, respectively, from the centre of the Earth. The gravitational potential at **X** is –8 kJ kg^{-1}.



What is the gain in gravitational potential energy of a 1 kg mass when it is moved from **X** to **Y**?

A –4 kJ
B –2 kJ
C +4 kJ
D +8 kJ

N02/I/10

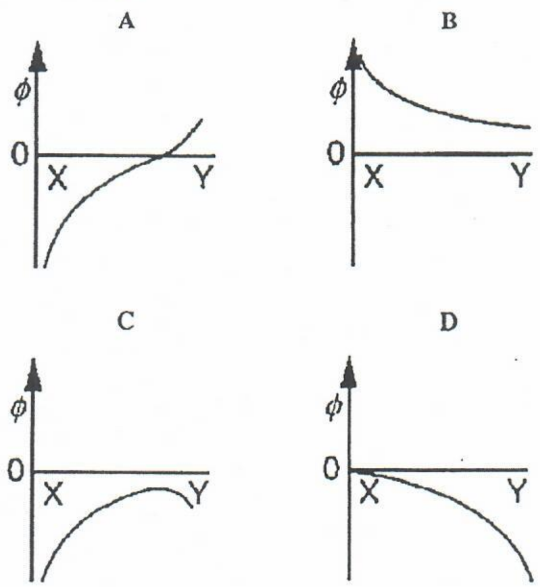
Solution Answer: **C**

The magnitude of the gravitational potential Φ is inversely proportional to the distance r from the centre of the Earth ($\Phi = -GM/r$).
The G.P.E. of a 1 kg mass at X is $U = m\Phi$ = (1 kg)(-8 kJ kg^{-1}) = -8 kJ.
The G.P.E. of a 1 kg mass at Y will be U = (1 kg)(-4 kJ kg^{-1}) = -4 kJ.
The gain in gravitational potential energy = -4 kJ – (-8 kJ) = + 4 kJ.

2 The diagram (not to scale) represents the relative positions of the Earth and the Moon.

The line XY joins the surface of the Earth to the surface of the Moon.

Which graph represents the variation of gravitational potential ϕ along the line XY?



N2000/I/7

Solution Answer: **C**

Gravitational potential Φ is negative at every point in space (except at infinity where it is zero).
The gravitational potential is more negative near the Earth because its mass is greater compared to that of the Moon.
This is evident from the formula $\Phi = -GM/r$.

3 An experimental satellite is found to have a weight W when assembled before launching from a rocket site. It is placed in a circular orbit at a height $h = 6R$ above the surface of the Earth (of radius R).

What is the gravitational force acting on the satellite whilst in orbit?

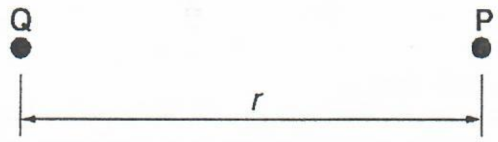
A $\frac{W}{6}$ **B** $\frac{W}{7}$ **C** $\frac{W}{36}$ **D** $\frac{W}{49}$

N97/I/7

Solution Answer: **D**

A height of $6R$ above the surface means that the satellite is $7R$ from the centre of the Earth.
The gravitational force acting on the satellite is inversely proportional to the square of its distance from the centre of the Earth ($F = GMm/R^2$).
When R is increased to 7 times of its ground value, the gravitational force will be reduced to $(1/7)^2$ = 1/49 of its ground value.

4 A mass m is at fixed point **Q**. It produces a gravitational potential at point **P**, distant r from **Q**.



This gravitational potential is equal to the external work done on unit mass in moving it

A from **P** to **Q**.
B from **Q** to **P**.
C from **P** to infinity.
D from infinity to **P**.

N98/I/7

Solution Answer: **D**

By definition, the gravitational potential at a point in a gravitational field is the work done by an external agent in bringing a unit mass from infinity to that point.

5 A satellite of mass 50 kg moves from a point where the gravitational potential due to the Earth is –20 MJ kg^{-1}, to another point where the gravitational potential is –60 MJ kg^{-1}.

In which direction does the satellite move and what is its change in potential energy?

A closer to the Earth and a loss of 2000 MJ of potential energy.

B closer to the Earth and a loss of 40 MJ of potential energy.

C further from the Earth and a gain of 2000 MJ of potential energy.

D further from the Earth and a gain of 40 MJ of potential energy.

N99/I/7

Solution Answer: **A**

The gravitational potential is lower (more negative) as the satellite approaches the Earth.
The gravitational potential difference is 40 MJ kg^{-1} hence the loss in gravitational potential energy is (50 kg)(40 MJ kg^{-1}) = 2000 MJ.

6 Which statement about geostationary orbits is **false**?

A A geostationary orbit must be directly above the equator.

B All satellites in a geostationary orbit must have the same mass.

C The period of a geostationary orbit must be 24 hours.

D There is only one possible radius for a geostationary orbit.

N2000/I/8

Solution Answer: **B**

Statement B is **false.**
Based on Kepler's third law $T^2 \alpha R^3$, if the period is the same (24 hours in the case of the Earth's geostationary orbit) then the radius of the orbit will be the same (42000 km), regardless of the mass of the orbiting body.

7 For points outside a uniform sphere of mass M, the gravitational field is the same as that of a point mass M at the centre of the sphere. The Earth may be taken to be a uniform sphere of radius r and density ρ.

How is the gravitational field strength g at its surface related to these quantities and the gravitational constant G?

A $g = \dfrac{G\rho}{r^2}$

B $g = \dfrac{3G}{4\pi r\rho}$

C $g = \dfrac{4\pi r\rho G}{3}$

D $g = \dfrac{4\pi r^2\rho G}{3}$

N01/I/7

Solution Answer: **C**

The gravitational field strength at the surface $g = GM/r^2$ where $M = V\rho = 4/3\ \pi\ r^3\ \rho$.
Substituting M into the equation for g, the gravitational field strength at the surface $g = G(4/3)\ \pi\ r^3\ \rho\ /\ r^2 = 4\pi\ r\rho G/3$.

8 The gravitational field strength outside a uniform sphere of mass M is the same as that due to a point mass M at the centre of the sphere.

The Earth may be taken to be a uniform sphere of radius r. The gravitational field strength at its surface is g.

What is the gravitational field strength at a height h above the surface?

A $\dfrac{gr^2}{(r+h)^2}$

B $\dfrac{gr}{(r+h)}$

C $\dfrac{g(r-h)}{r}$

D $\dfrac{g(r-h)^2}{r^2}$

N04/I/11

Solution Answer: **A**

A height of h above the surface means a distance of $r + h$ from the centre of the Earth.
Since the gravitational field strength is inversely proportional to the square of the distance from the centre of the Earth ($g = GM/r^2$), the gravitational field strength at height h above the surface is $gr^2/(r + h)^2$.

9 *(a)* Define gravitational potential ϕ. [2]

(b) Fig. 2.1 shows part of the orbit of a satellite round the Earth.

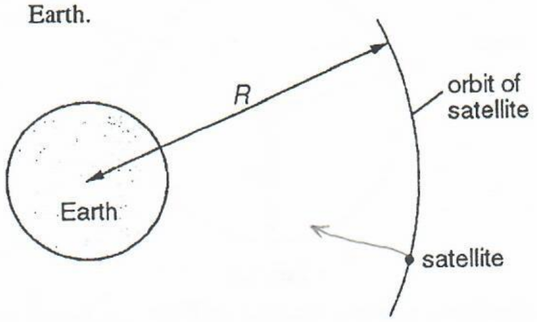


Fig. 2.1

The mass M of the Earth is 6.0×10^{24} kg. It may be assumed that the gravitational field of the Earth is the same as that of a point mass M situated at the centre of the Earth.

(i) On Fig. 2.1 show, by means of an arrow, the direction of the gravitational force on the satellite.

(ii) Explain why the satellite does not move in the direction of the gravitational force.

(iii) Show that v, the linear speed of the satellite in its orbit of radius R, is given by the expression

$$v = \sqrt{\frac{GM}{R}},$$

where G is the gravitational constant. [5]

(c) The satellite is orbiting the Earth with a radius R of 6610 km at a speed v of 7780 m s^{-1}. The satellite is boosted into a higher orbit of radius 6890 km. Show that the speed of the satellite in the new orbit is 7620 m s^{-1}. [1]

(d) (i) In *(c)*, the satellite, of mass 120 kg, moves from one orbit to another. Using the data in *(c)*, calculate, for this satellite, the change in

1. kinetic energy,

change in kinetic energy = ... J

2. gravitational potential energy,

change in potential energy = ... J

3. total energy.

change in total energy = ... J[6]

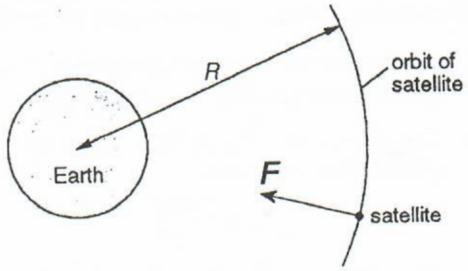
(ii) State whether this change in total energy is an increase or a decrease. [1]

N01/II/2

Solution

(a) The gravitational potential at a point in a gravitational field is the work done by an external agent in bringing a unit mass from infinity to the point, without changing its kinetic energy.

(b) (i) The direction of the gravitational force F on the satellite is shown in the figure below.



(ii) The gravitational force exerted by the Earth on the Moon is just sufficient to provide the centripetal force for the Moon's circular orbit about the Earth.
The resulting centripetal acceleration of the Moon acts towards the centre of rotation and is always perpendicular to the Moon's linear velocity.
This centripetal acceleration continuously changes the direction but not the magnitude of the Moon's velocity.
As the Moon follows a circular path due to this acceleration, the Earth's surface also curves in a circular manner, thus ensuring that the Moon maintains a constant distance from the Earth.

(iii) The gravitational force provides the centripetal force.

$$GMm/r^2 = mv^2/r$$

$$v = \sqrt{(GM/r)}.$$

(c) Using the equation $v = \sqrt{(GM/r)}$, it can be seen that speed v is inversely proportional to the square root of orbital radius r. Hence,

$$v_2 / v_1 = \sqrt{r_1} / \sqrt{r_2} = \sqrt{6610} / \sqrt{6890}$$

$$v_2 = (\sqrt{6610} / \sqrt{6890}) \times 7780 \text{ m s}^{-1} = 7620 \text{ m s}^{-1}$$

(d) (i) **1.** Change in kinetic energy
$= \tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2$
$= \tfrac{1}{2}(120 \text{ kg})(7620 \text{ ms}^{-1})^2 - \tfrac{1}{2}(120 \text{ kg})(7780 \text{ ms}^{-1})^2$
$= -1.48 \times 10^8$ J.

2. Change in gravitational potential energy
$= -GMm/r_2 - (-GMm/r_1)$
$= GMm/r_1 - GMm/r_2$
$= mv_1^2 - mv_2^2$
$= (120 \text{ kg})(7780 \text{ m s}^{-1})^2 - (120 \text{ kg})(7620 \text{ m s}^{-1})^2$
$= 2.96 \times 10^8$ J.

3. Change in total energy = change in kinetic energy + change in gravitational potential energy
$= -1.48 \times 10^8 \text{ J} + 2.96 \times 10^8$ kg
$= 1.48 \times 10^8$ J.

(ii) Increase.

10 Towards the end of the eighteenth century, Cavendish successfully measured the force of attraction between two spheres. Hence, he became one of the first scientists to determine a value for the gravitational constant G.

(a) Write down an equation representing Newton's law of gravitation, explaining any symbols used. [3]

(b) The acceleration of free fall g at the Earth's surface is given by the expression

$$g = \frac{GM}{r^2},$$

where M is the mass of the Earth and r is its radius.

Calculate a value for the mass of the Earth, given that $r = 6.40 \times 10^6$ m.

mass = ... kg [2]

(c) Some textbooks describe Cavendish's experiment as a means of 'weighing the Earth'.

Suggest why this statement is incorrect. [2] N02/II/1

Solution

(a) The equation is $F = GMm/r^2$ where G is the gravitational constant.
Newton's law of gravitation states that the gravitational force of attraction F between two masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of the distance r separating their centres of mass.

(b) The gravitational field strength at the surface of the Earth is 9.81 N kg^{-1}.
Using this value for g in the equation,

$$g = GM/r^2$$

$$9.81 \text{ N kg}^{-1} = (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})M/(6.40 \times 10^6 \text{ m})^2$$

$$M = 6.02 \times 10^{24} \text{ kg}$$

(c) In order to weigh an object of mass m, that object must be present in a gravitational field g, so that its weight can calculated from the equation $W = mg$.
The weight of an object thus varies depending on the strength of the external gravitational field in which it is located.
In this case, the Earth is not present in an external gravitational field generated by some other body. Therefore it is not possible to find the weight of the Earth. The value obtained here is in fact its mass, which is a measure of the amount of matter in it and is independent of the strength of the gravitational field in which it is located (which is zero in this case).

11 A satellite orbits the Earth in a circular path, as illustrated in Fig. 3.1.

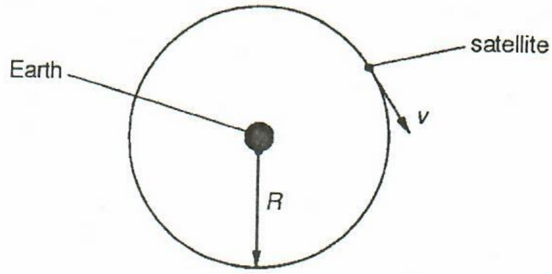


Fig. 3.1

Both the Earth and the satellite may be considered to be point masses with their masses concentrated at their centres.

The satellite has speed v and the radius of its orbit about the Earth is R.

(a) (i) Show that speed v is given by the expression

$$v^2 = \frac{GM}{R},$$

where M is the mass of the Earth and G is the gravitational constant. [2]

(ii) The mass of the satellite is m. Determine an expression for the kinetic energy E_k of the satellite in terms of G, M, m and R. [2]

(b) (i) State an expression, in terms of G, M, m and R, for the gravitational potential energy E_p of the satellite. [1]

(ii) Hence show that the total energy E_t of the satellite is given by

$$E_t = -\frac{GMm}{2R}.$$

[2]

(c) As the satellite orbits the Earth, it gradually loses energy because of air resistance.

(i) State whether the total energy E_t becomes more or less negative. [1]

(ii) Hence state and explain the effect of this change on

1. the radius of the orbit, [2]

2. the speed of the satellite. [2]

N04/II/3

Solution

(a) (i) The gravitational force provides the centripetal force.

$$GMm/R^2 = mv^2/R$$

$$v^2 = GM/R$$

(ii) Using the same equation $GMm/R^2 = mv^2/R$,
Kinetic energy $E_k = \frac{1}{2}mv^2 = \frac{1}{2}GMm/R$.

(b) (i) Gravitational potential energy $E_p = -GMm/R$.

(ii) Total energy
$E_t = E_k + E_p = \frac{1}{2}GMm/R + (-GMm/R) = -\frac{1}{2}GMm/R$.

(c) (i) More negative (since E_t decreases).

(ii) 1. Decreases.
As E_t becomes more negative, the magnitude of the total energy $|\frac{1}{2}GMm/R|$ increases implying that the radius of orbit R decreases.

2. Increases.
As speed v is related to orbital radius R by the equation $v^2 = GM / R$, v is inversely proportional to $\sqrt{R}$ hence as R decreases, v increases. The speed becomes greater.

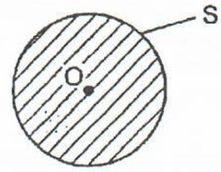
12 Which statement about a geostationary satellite is true?

- **A** It can remain vertically above any chosen fixed point on the Earth.
- **B** Its linear speed is equal to the speed of a point on the Earth's equator.
- **C** It has the same angular velocity as the Earth's rotation on its axis.
- **D** It is always travelling from east to west. N05/I/11

Solution Answer: **C**

Since a geostationary satellite is always vertically above a fixed point on the equator, it follows the Earth's rotation with the same period (24 hours) and hence the same angular velocity (Answer C).
Answer A is wrong because the geostationary satellite must be above the equator and not any chosen fixed point.
Answer B is wrong because linear speed $v = r\omega$. The satellite, which has a larger radius of orbit than a point on the Earth's surface, will have a larger linear speed (although its angular speed ω is the same as that of a point on the Earth's surface).
Answer D is wrong because geostationary satellites follow the Earth's rotation from west to east, and not from east to west.

13 An astronomical gas cloud has mass M and radius R. The gravitational potential on its surface S is $-\frac{GM}{R}$ and at its centre O it is $-\frac{3GM}{2R}$



A unit mass is moved slowly by means of an external force from the surface S to the centre O.

What is the work done on the mass by the external force?

A $-\frac{5GM}{2R}$ **B** $-\frac{GM}{2R}$ **C** $\frac{GM}{2R}$ **D** $\frac{5GM}{2R}$

N06/I/10

Solution Answer: **B**

Work done by external force = change in gravitational potential energy
$= (-3GM / 2R) - (-GM / R) = -GM / 2R$.

Due to the attractive nature of the gravitational force, the work done by the external force is negative as positive work is done by the gravitational force.

14 *(a)* Explain how an object travelling in a circle with constant speed has an acceleration. In what direction is this acceleration? [4]

(b) A satellite P of mass 2400 kg is placed in a geostationary orbit at a distance of 4.23×10^7 m from the centre of the Earth.

(i) Explain what is meant by the term *geostationary orbit*. [1]

(ii) Calculate

1. the angular velocity of the satellite,
2. the speed of the satellite,
3. the acceleration of the satellite,
4. the force of attraction between the Earth and the satellite,
5. the mass of the Earth. [10]

(c) Explain why a geostationary satellite

(i) must be placed vertically above the equator,

(ii) must move from west to east. [4]

(d) Why is a satellite in a geostationary orbit often used for telecommunications? [1]

N99/III/2

Solution

(a) Acceleration is defined as the rate of change of velocity.
Velocity is a vector quantity which has both magnitude and direction, there is a change in velocity when there is a change in its direction.
An object travelling in a circle is constantly changing its direction.
Since its direction changes, there is a change in its velocity and hence it experiences an acceleration.
This acceleration, known as the centripetal acceleration, is directed towards the centre of the circle.
As the centripetal acceleration is perpendicular to the object's velocity, it changes only the direction but not the magnitude of the velocity.

(b) (i) A *geostationary orbit* is an orbit in which objects placed within it will always be directly above the same location on the Earth's equator.
Such orbiting objects will thus have a period equal to the Earth's period of rotation about its own axis, i.e. 24 hours.

(ii) 1. Angular velocity
$\omega = 2\pi/T$
$= 2\pi/(24 \times 60 \times 60\text{s})$
$= 7.2722 \times 10^{-5}$
$= 7.27 \times 10^{-5}$ rad s^{-1}.

2. Linear speed
$v = r\omega$
$= (4.23 \times 10^{7}\text{ m})(7.2722 \times 10^{-5}\text{ rad s}^{-1})$
$= 3076.1$
$= 3080$ m s^{-1}.

3. Acceleration
$a = r\omega^2$
$= (4.23 \times 10^{7}\text{ m})(7.2722 \times 10^{-5}\text{ rad s}^{-1})^2$
$= 0.22370$
$= 0.224$ m s^{-2}.

4. Force
$F = ma$
$= (2400\text{ kg})(0.22370\text{ m s}^{-2})$
$= 536.88$
$= 537$ N.

5. $g = 0.22370$ N kg^{-1}

$GM/r^2 = 0.022370$ N kg^{-1}

$(6.67 \times 10^{-11}\text{ N m}^2\text{ kg}^{-2})M / (4.23 \times 10^{7}\text{ m})^2 = 0.022370$ N kg^{-1}

$M = 6.00 \times 10^{24}$ kg.

(c) (i) The gravitational force acting on the satellite provides the centripetal force for its circular orbit.
Since the gravitational force acts towards the centre of the Earth, the plane of the satellite's orbit must include the centre of the Earth.
Additionally, since the satellite must be permanently above a fixed point on the Earth's equator and follow the Earth's rotation, its plane of orbit must also include the equator.

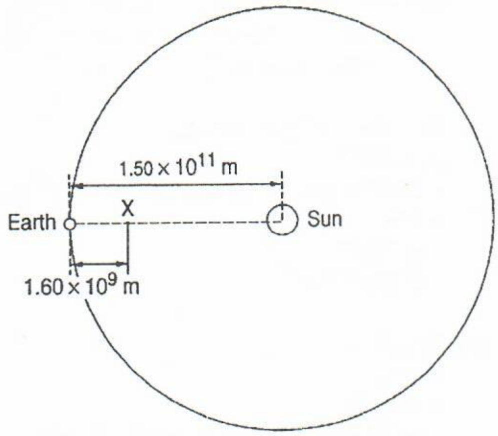
(ii) The Earth rotates from west to east about its north-south axis.
Since the geostationary satellite must be permanently above a fixed location on the Earth's equator and follow the Earth's rotation, it must also rotate from west to east.

(d) Since a geostationary satellite is permanently above a fixed location on the Earth's equator, it will always be at a fixed location in the sky with respect to the ground.
This allows uninterrupted telecommunications transmission between the ground stations and the satellite.

15 *(a)* (i) Define *angular velocity* for an object travelling in a circle.

(ii) Calculate the angular velocity of the Earth in its orbit around the Sun. Assume that the orbit is circular and give your answer in terms of the SI unit for angular velocity. [4]

(b) In order to observe the Sun continuously, a satellite of mass 425 kg is at point X, a distance of 1.60×10^9 m from the centre of the Earth, as shown in Fig. 8.



mass of Sun = 1.99×10^{30} kg
mass of Earth = 5.98×10^{24} kg
Earth-Sun distance = 1.50×10^{11} m

Fig. 8

(i) Calculate, using the data given,

1. the pull of the Earth on the satellite,
2. the pull of the Sun on the satellite. [3]

(ii) Using Fig. 8 as a guide, draw a sketch to show the relative positions of the Earth, the Sun and the satellite. On your sketch draw arrows to represent the two forces acting on the satellite. Label the arrows with the magnitude of the forces. [2]

(iii) Calculate

1. the magnitude and direction of the resultant force on the satellite,
2. the acceleration of the satellite. [3]

(iv) The satellite is in a circular orbit around *the Sun*. Calculate the angular velocity of the satellite. [3]

(v) Using your answer to *(a)* (ii) describe the motion of the satellite relative to the Earth. Suggest why this orbit around the Sun is preferable to a satellite orbit around the Earth. [3]

(vi) Suggest **two** disadvantages of having a satellite in this orbit. [2]

N2000/III/2

Solution

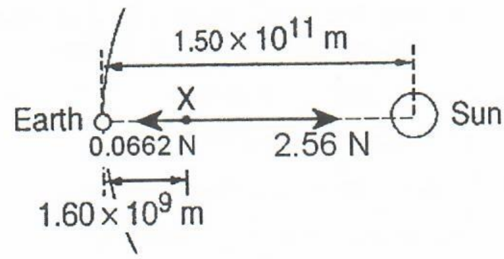
(a) (i) *Angular velocity* is the rate of change of angular displacement of the object about the centre of the circle.
Its units are rad s^{-1}.

(ii) Angular velocity $\omega = 2\pi/T$ where T is the period of the Earth about the Sun (365 days = 3.1536×10^7 s).
Therefore, $\omega = 2\pi/(3.1536 \times 10^7 \text{ s})$
$= 1.9924 \times 10^{-7}$
$= 1.99 \times 10^{-7}$ rad s^{-1}.

(b) (i) 1. $F = GM_e m/r^2$
$= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(5.98 \times 10^{24} \text{ kg})(425 \text{ kg})/(1.60 \times 10^9 \text{ m})^2$
$= 0.066218$
$= 0.0662$ N.

2. $F = GM_s m/r^2$
$= (6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2})(1.99 \times 10^{30} \text{ kg})(425 \text{ kg})/(1.50 \times 10^{11} \text{ m} - 1.60 \times 10^9 \text{ m})^2$
$= 2.5615$
$= 2.56$ N.

(ii) The sketch is shown in the figure below.



(iii) 1. Resultant force = 2.56 N – 0.0662 N
$= 2.4938$
$= 2.49$ N.

The direction is towards the Sun.

2. Acceleration
$a = F/m$
$= 2.4938 \text{ N} / 425 \text{ kg}$
$= 5.8678 \times 10^{-3}$
$= 5.87 \times 10^{-3}$ m s^{-2}.

(iv) Centripetal acceleration $a = r\omega^2$.

$$5.8678 \times 10^{-3} \text{ m s}^{-2} = (1.484 \times 10^{11} \text{ m})\, \omega^2$$

$$\omega = 1.9885 \times 10^{-7} = 1.99 \times 10^{-7} \text{ rad s}^{-1}$$

(v) Since the angular velocity of the satellite is the same as that of the Earth, the satellite will orbit the Sun with the same period as the Earth.
As a result, the satellite will always be located between the Earth and the Sun, on the straight line joining the Earth and the Sun.

Since the satellite is permanently between the Earth and the Sun, there will never be an instance when the Earth will come between the satellite and the Sun.
Therefore, the satellite will be able to observe the Sun continuously – the purpose of its mission as stated in the question.

16 *(a)* Write down an equation expressing Newton's law of gravitation. Define your symbols. [2]

(b) Use the equation in *(a)* to derive a value for g, the acceleration due to gravity, at the Earth's surface.

mass of Earth = 5.98×10^{24} kg

mean radius of Earth = 6.37×10^{6} m [3]

(c) A geostationary satellite has to be placed above the equator.

(i) State what is meant by *geostationary*. [1]

(ii) State the direction of rotation of the satellite around the Earth's axis. [1]

(iii) Explain why the satellite must be above the equator. [2]

(d) A geostationary satellite is in orbit at a distance of 4.23×10^{7} m from the centre of the Earth. Calculate

(i) the Earth's gravitational field strength at this distance from the centre of the Earth, [1]

(ii) the speed of the satellite, [3]

(iii) the acceleration of the satellite. [2]

(e) Under the heading *Data* there is the entry

acceleration of free fall, $g = 9.81$ m s^{-2}.

Compare and comment on small differences between this value, the value you obtained in part *(b)*, and the value of 9.79 m s^{-2} (which is the value obtained by making accurate measurements in Singapore, near the equator). [5]

N03/III/2

Solution

(a) Newton's law of Gravitation states that the gravitational force of attraction F between two masses M and m is directly proportional to the product of their masses, and inversely proportional to the square of the distance r separating their centres of mass.
The expression is $F = GMm/r^2$ where G is the gravitational constant.

(b) The gravitational force acting on a mass m at the Earth's surface is:

$F = GMm/R^2 = (6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2})(5.98 \times 10^{24}\ \text{kg})\text{m} / (6.37 \times 10^{6}\ \text{m})^2 = 9.830m$ N = $9.8m$ N.

This force causes the mass m to acceleration, according to Newton's 2nd law of motion, $F = ma$.

The acceleration due to gravity $a = F/m = 9.830m$ N / m = 9.830 = 9.8 m s^{-2}.

(c) (i) A *geostationary* satellite will always be directly above the same location on the Earth's equator.
Such satellites will thus have a period equal to the Earth's rotational period about its own axis, i.e. 24 hours.

(ii) A geostationary satellite rotates from west to east around the Earth's axis, following the Earth's own direction of rotation.

(iii) The gravitational force acting on the satellite provides the centripetal force for its circular orbit.
Since the gravitational force acts towards the centre of the Earth, the plane of the satellite's orbit must include the centre of the Earth.
Additionally, since the satellite must be permanently above a fixed point on the Earth's equator and follow the Earth's rotation, its plane of orbit must also include the equator.

(d) (i) Gravitational field strength
$g = GM/r^2$
$= (6.67 \times 10^{-11}\ \text{N m}^2\ \text{kg}^{-2})(5.98 \times 10^{24}\ \text{kg})/(4.23 \times 10^{7}\ \text{m})^2$
= 0.22292
= 0.223 N kg^{-1}.

(ii) Speed
$v = r\omega$
$= r(2\pi/T)$
$= (4.23 \times 10^{7}\ \text{m})(2\pi/ (24 \times 60 \times 60\ \text{s}))$
= 3076.1
= 3080 m s^{-1}.

(iii) Acceleration is numerically equal to the gravitational field strength, $a = 0.223$ m s^{-2}.

(e) The value obtained in part (b), 9.83 m s^{-2}, does not take into account the centripetal acceleration due to the Earth's rotation about its own axis.
The value 9.83 m s^{-2} is the acceleration of free fall at the north or south poles, which are not rotating.
At the equator where Singapore is located, due to the Earth's rotation about its own axis, the gravitational field strength has to provide for both a body's acceleration of free fall and its centripetal acceleration.
As a result, the acceleration of free fall is slightly less in magnitude than the gravitational field strength.
The value of 9.81 m s^{-2} is an average between the poles (9.83 m s^{-2}) and the equator (9.79 m s^{-2}).

17 The Earth may be assumed to be a uniform sphere of radius R and mass M. At its surface, the gravitational field strength is g. The gravitational field above the surface is the same as that due to a point mass M situated at the centre of the Earth.

(a) Explain what is mean by a *gravitational field.* [2]

(b) A satellite orbits the Earth at a height 0.30 R above its surface. Show that the gravitational field strength at this height is 0.59 g. [2]

(c) A person in the satellite in *(b)* experiences 'weightlessness' although the gravitational field strength is not zero.

(i) Explain why the person seems to be weightless. [2]

(ii) Show that the angular speed of the satellite about the Earth is approximately 8.3 x 10^{-4} rad s^{-1}. The radius R of the Earth is 6.4 x 10^6 m. [2]

(iii) Calculate the time, in hours, for one complete orbit of the satellite.

time = ... hours [2]

N05/II/3

Solution

(a) A gravitational field is a region in space where a mass experiences a gravitational force due to the presence of another mass.

The strength of the gravitational field is defined as the gravitational force acting on a unit mass placed in the gravitational field, measured in N kg^{-1}.

(b) Gravitational field strength due to the Earth $g = GM/R^2$ where G is the gravitational constant, M the mass of the Earth and R the distance from the centre of the Earth. Hence, g is inversely proportional to R^2.

A distance of 0.3 R from the surface of the Earth is equivalent to a distance of 1.3 R from the centre of the Earth.

Let g' be the gravitational field strength at this height,

$$g'/g = (R / 1.3R)^2$$

$$g' = 0.59\,g$$

(c) (i) The gravitational force acting on the person is just sufficient to provide the centripetal force and cause the centripetal acceleration for his circular motion.

Hence there is no contact force between the person and the floor of the satellite on which he is standing.

The person thus seems to be weightless.

(ii) The radius of orbit
= 1.3 R
= 1.3 x 6.4 x 10^6 m
= 8.32 x 10^6 m.

Since the gravitational force acting on the person is just equal to his centripetal force, the gravitational field strength is just equal to his centripetal acceleration.

$$g' = r\omega^2$$

$$0.59\,g = r\omega^2$$

$$0.59\,(9.81\text{ N kg}^{-1}) = (8.32 \times 10^6\text{ m})\,\omega^2$$

$$\omega = 8.341 \times 10^{-4} = 8.3 \times 10^{-4}\text{ rad s}^{-1}$$

(iii) Angular velocity $\omega = 2\pi / T$ where T is the period.

$$T = 2\pi / \omega$$

$$= 2\pi / 8.341 \times 10^{-4}\text{ rad s}^{-1}$$

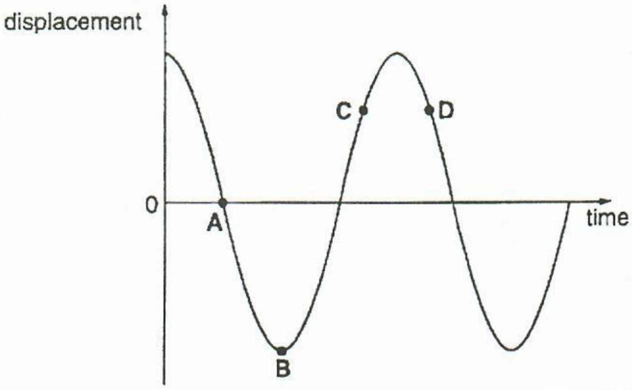
$$= 7533\text{ s}$$

$$= 2.1\text{ hours.}$$

TOPIC 8 Oscillations

1 The diagram shows the graph of displacement against time for a body performing simple harmonic motion.

At which point are the velocity and acceleration in opposite directions?

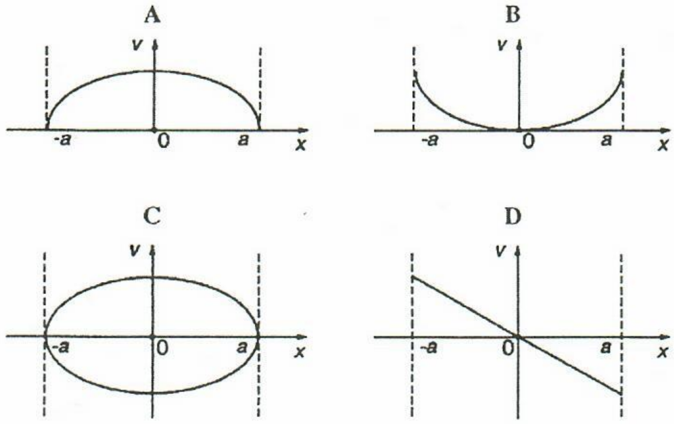


N98/I/9

Solution Answer: **C**

At point C, the displacement is increasing with time in the positive direction hence the velocity is in the positive direction.
In simple harmonic motion, the direction of the acceleration is opposite to that of the displacement.
Hence the acceleration at C is in the negative direction (towards the equilibrium position).

2 Which graph best shows how the velocity v of an object performing simple harmonic motion of amplitude a varies with displacement x for one complete oscillation?



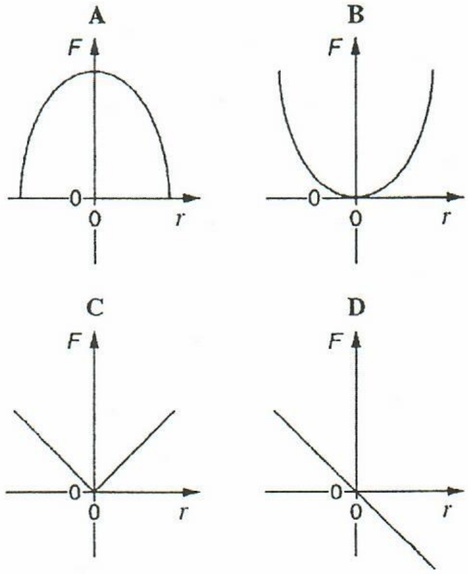
N97/I/9

Solution Answer: **C**

In simple harmonic motion, the velocity v is related to displacement x by the following equation $v = \pm\omega\sqrt{(x_0^2 - x^2)}$.
This is the equation of an ellipse.
The velocity is zero when the object is at the maximum displacement positions $\pm a$.
The displacement is zero when the velocity is at the maximum values $\pm v_{max}$.

3 A resultant force F acts on a particle moving with simple harmonic motion.

Which graph shows the variation with displacement r of force F?



N99/I/9

Solution Answer: **D**

The defining equation of simple harmonic motion is $a = -\omega^2 r$ where a is the acceleration of the particle and r its displacement from the equilibrium position.
The resultant force acting on the particle $F = ma = -m\omega^2 r$.
Therefore a graph of F vs r is a straight line through the origin with a negative gradient of $-m\omega^2$.