Compactness Effects on Partisan Outcomes in Redistricting

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Gerrymander



Gerrymandered districts?



Does compact mean fair?



MGGG ReCom Metropolis-Hastings Algorithm

- 1. Start with a seed plan
- 2. Recombine two districts
- 3. Split the districts into two using minimum spanning trees
- 4. Accept or reject new plan with certain probability
- 5. Repeat steps 2-4



Compactness Metrics

- Polsby-Popper and Schwartzberg
 - **PP = 4πA/P^2**
 - S = 1/√PP
- Reock
 - Divide area of district by the area of the minimum bounding circle
- Convex Hull
 - Divide area of district by the area of its convex hull
- Cut edges
 - Number of edges "cut" to create the district
- Border vs. Whole
 - Density of border precincts against density of the district
- K3
 - Corresponds to how people perceive compactness

Partisan Fairness Metrics

- Least Republican Vote Share (LRVS)
 - The share of Republican voters in the least Republican district
- Partisan Bias
 - How many seats would a party win if it had 50% vote share
- Partisan Gini
 - **TBD**
- Mean-Median
 - Party's median vote share minus its mean vote share
- Efficiency Gap
 - Any vote more than 50% is "wasted".
 - Take difference between two parties' wasted votes divided by total number of votes
- Percentage of Seats vs Vote Share
 - Difference between seat share and vote share
- Competitive Seats
 - Number of seats with party vote share between 44% and 56%







popper -

k3 -

reock log_convex border_vs_whole -16President_mm -G16Senate_mm adoAt-large_mm -18Governor_mm taryofState_mm -18Treasurer_mm adoAt-large_mm -20President_mm -G20Senate_mm -G16All_mm -

num_cut_edges -

ut_edge_density -

- -

Colorado_Senate mm 1 Colorado_Congressional mm 1	
1 0.59 0.94 0.12 0.53 0.55 0.2 0.08 0.03 0.07 0.05 0.03 0.04 0.05 0.06 0.02 popper - 1 -0.4 -0.91 0.1 0.48 0.46 0.05 0.09 0.05 -0.01 0.12 0.09 0.07 0.1 0.1 0.11 0.06	- 1.00
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opper edges reock ka reock k whole e-mm tr_mm k_mm k_mm k_mm k_mm k_mm k_mm k_mm	
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Maryland_Congressional pb 1													
popper -	1	-0.68	-0.93	-0.02		0.39	0.09	-0.01	-0.01	-0.01	0.02	-0.01	-0.01
num_cut_edges -	-0.68	1	0.64	-0.04	-0.3	-0.17	0.25	-0.11	-0.11	-0.07	-0.13	-0.1	-0.11
k3 -	-0.93	0.64		0.04	-0.61	-0.44	-0.11	-0.03	-0.03	-0.02	-0.06	-0.02	-0.03
ut_edge_density -	0.02	-0.04	0.04	1	-0.15	-0.11	-0.16	0.19	0.16	0.15	0.19	0.19	0.19
reock -	0.48	-0.3	-0.61	-0.15		0.35	0.1	0.05	0.05	0.04	0.08	0.04	0.05
log_convex -	0.39	-0.17	-0.44	-0.11	0.35	1	0.05	0.13	0.15	0.16	0.14	0.13	0.13
border_vs_whole -	- 0.09	0.25	-0.11	-0.16	0.1	0.05	1	-0.1	-0.1	-0.09	-0.08	-0.11	-0.1
G16President_pb -	0.01	-0.11	-0.03	0.19	0.05	0.13	-0.1	1	0.89	0.79	0.93	0.96	0.96
G16Senate_pb -	0.01	-0.11	-0.03	0.16	0.05	0.15	-0.1	0.89		0.89	0.86	0.92	0.88
G18Governor_pb -	0.01	-0.07	-0.02	0.15	0.04	0.16	-0.09	0.79	0.89		0.78	0.83	0.79
8Comptroller_pb -	- 0.02	-0.13	-0.06	0.19	0.08	0.14	-0.08	0.93	0.86	0.78		0.92	0.94
orneyGeneral_pb -	0.01	-0.1	-0.02	0.19	0.04	0.13	-0.11	0.96	0.92	0.83	0.92		0.95
G18Senate_pb -	0.01	-0.11	-0.03	0.19	0.05	0.13	-0.1	0.96	0.88	0.79	0.94	0.95	
	popper -	um_cut_edges -	К	_edge_density -	reock -	log_convex -	der_vs_whole -	6President_pb -	316Senate_pb -	8Governor_pb -	omptroller_pb -	eyGeneral_pb -	518Senate_pb -

Nebraska_Senate mm 1															- 1.00		
popper -	1	-0.68	-0.93	0.16	0.37	0.24	-0.13	0.04	0.2	0.02	0.09	0.02	-0.07	-0.04			1.00
num_cut_edges -	-0.68		0.66	-0.21	-0.25	-0.21		-0.13	-0.08	0.08	0	-0.05	-0.03	-0.01			0.75
k3 -	-0.93	0.66	1	-0.11	-0.51	-0.31	0.07	-0.03	-0.11	0.04	-0.03	0.05	0.04	0.1			
ut_edge_density -	0.16	-0.21	-0.11	1	-0.02	-0.15	-0.06	0.05	-0.12	-0.09	0.1	-0.04	0.06	0.07			- 0.50
reock -	0.37	-0.25	-0.51	-0.02	1	0.24	0.02	-0.02	-0.14	-0.11	-0.1	-0.14	-0.03	-0.23			
log_convex -	0.24	-0.21	-0.31	-0.15	0.24	1	-0.08	-0.03	0.05	-0.01	-0	0.03	0.08	0.13		-	0.25
border_vs_whole -	-0.13	0.47	0.07	-0.06	0.02	-0.08	1	-0.01	-0.06	0.06	0.07	0.15	0.12	0.02			
16President_mm -	0.04	-0.13	-0.03	0.05	-0.02	-0.03	-0.01	1	0.32	0.52	0.67	0.53	0.56	0.29			- 0.00
G18Senate_mm -	0.2	-0.08	-0.11	-0.12	-0.14	0.05	-0.06	0.32	1	0.59		0.4	0.1	0.37			0.25
18Governor_mm -	0.02	0.08	0.04	-0.09	-0.11	-0.01	0.06	0.52	0.59	1	0.74	0.62	0.24	0.28			
:taryofState_mm -	0.09	0	-0.03	0.1	-0.1	-0	0.07	0.67		0.74	1	0.58	0.3	0.37		-	0.50
18AUDITOR_mm -	0.02	-0.05	0.05	-0.04	-0.14	0.03	0.15	0.53	0.4	0.62	0.58	1	0.33	0.33			
20President_mm -	0.07	-0.03	0.04	0.06	-0.03	0.08	0.12		0.1	0.24	0.3	0.33	1	0.35			-0.75
G20Senate_mm -	0.04	-0.01	0.1	0.07	-0.23	0.13	0.02	0.29	0.37	0.28	0.37	0.33	0.35	1			
	popper -	um_cut_edges -	ka -	_edge_density -	reock -	log_convex -	rder_vs_whole -	President_mm -	18Senate_mm -	Governor_mm -	ryofState_mm -	AUDITOR_mm -	President_mm -	20Senate_mm -			1.00

Nebraska_Congressional mm 1														
popper -	1	-0.81	-0.94	0.16	0.69	0.19	-0.06	-0.36		-0.42	-0.41	-0.3	-0.34	-0.38
num_cut_edges -	-0.81		0.73	-0.13	-0.61	0.21	0.33	0.09	0.24	0.19	0.15	0.09	0.07	0.14
k3 -	-0.94	0.73	1	-0.11	-0.76	-0.24	0.06	0.28	0.41	0.34	0.34	0.24	0.26	0.31
ut_edge_density -	0.16	-0.13	-0.11	1	-0.06	0.06	-0.12	-0.31	-0.23	-0.3	-0.28	-0.34	-0.31	-0.32
reock -	0.69	-0.61	-0.76	-0.06	1	0.1	-0.09	-0.19	-0.33	-0.26	-0.25	-0.16	-0.16	-0.21
log_convex -	0.19	0.21	-0.24	0.06	0.1	1	0.13		-0.44	-0.41	-0.45	-0.41	-0.49	-0.44
border_vs_whole -	-0.06	0.33	0.06	-0.12	-0.09	0.13	1	-0.09	-0.05	-0.04	-0.09	-0.02	-0.1	-0.08
16President_mm -	-0.36	0.09	0.28	-0.31	-0.19		-0.09	1	0.93	0.96	0.98	0.96	0.99	0.97
G18Senate_mm -	-0.48	0.24	0.41	-0.23	-0.33	-0.44	-0.05	0.93		0.97	0.95	0.89	0.91	0.92
18Governor_mm -	-0.42	0.19	0.34	-0.3	-0.26	-0.41	-0.04	0.96	0.97		0.97	0.96	0.95	0.96
staryofState_mm -	-0.41	0.15	0.34	-0.28	-0.25	-0.45	-0.09	0.98	0.95	0.97	1	0.96	0.97	0.99
18AUDITOR_mm -	-0.3	0.09	0.24	-0.34	-0.16	-0.41	-0.02	0.96	0.89	0.96	0.96		0.95	0.95
20President_mm -	-0.34	0.07	0.26	-0.31	-0.16		-0.1	0.99	0.91	0.95	0.97	0.95		0.96
G20Senate_mm -	-0.38	0.14	0.31	-0.32	-0.21	-0.44	-0.08	0.97	0.92	0.96	0.99	0.95	0.96	
	popper -	um_cut_edges -	k3 -	edge_density -	reock -	log_convex -	der_vs_whole -	President_mm -	L8Senate_mm -	Governor_mm -	-yofState_mm	AUDITOR_mm -	President_mm -	20Senate_mm -

- 1.00

- 0.75

- 0.50

- 0.25

- 0.00

- -0.25

- -0.50

- -0.75

- -1.00



Nebraska Vote Distribution Scaled for Population Density

Why Compactness Still Matters

- Ease of travel
- Neighbors vote together
- Regional issues

