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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 26:
Exotic swap and options structures

A difference of opinion makes a market, and that is the major rationale for the development of the market in exotic swaps and exotic options. A second and less happy rationale is the willingness of some market participants to buy and sell securities without much knowledge of their true value. This is particularly dangerous in markets like the Libor market (in which manipulation has been admitted) and the credit default swap market (in which manipulation is suspected and investigations are under way).

In our quest for an integrated measure of interest rate risk and credit risk, we need to be able to handle these exotic structures as they will be found on the balance sheet of many institutions both on the “buy side” and the “sell side.” At the same time, two of the authors are ex-investment bankers. We recognize the endless quest of investment bankers to invent structures that they can value more accurately than their clients. The collateralized debt obligation market, which we discussed in Chapter 20, is a classic example. The real purpose of this chapter is not only to value specific structures but to show that this approach is general enough to apply to new structures as they emerge. That reflects the great power of the Heath Jarrow and Morton framework that we demonstrated in increasingly realistic worked examples in Chapters 6 through 9. In this chapter, we will again employ the 3 factor Heath Jarrow and Morton example from Chapter 9.

In the first sections of this chapter we show how the Heath, Jarrow and Morton framework can be used to value securities that, at least at one point in their life, were considered "exotic". Exotic means both "strange" and "difficult to value," and it is the latter that often motivates the originators of exotic structures. The examples in this chapter are taken from actual deals found in the swap portfolio of some major U.S. financial institutions.

At the end of the chapter, we discuss how to incorporate the potential default of our counterparty so that we can incorporate these kinds of transactions in our valuation of the Jarrow-Merton put option as a comprehensive risk measure. We will return to this discussion in great detail in Chapters 36 through 41.

Arrears Swaps

In most interest rate swaps, the floating rate payment is based on Libor (we use “Libor” until an alternative rate based on actual transactions in a competitive and transparent market replaces Libor as an index) in the relevant currency. If a normal swap calls for six month libor, the libor rate is set on a reference date 6 months before the cash interest payment will normally be made. In an arrears swap, the libor rate and payment amount will be made "in arrears," that is shortly before the cash payment must be made. That is, instead of being known six months in advance and paid at the effective maturity of the underlying index, the rate is set a few days before payment is due at the end of the six month period. How do we value this kind of swap?

As in previous chapters, we can use either simple term structure models (like Vasicek or Hull and White) to get closed form or algebraic answers (MODEL RISK ALERT) or we can use a more accurate approach with many risk factors driving interest rates. Such an approach, which requires numerical methods, is what we do here based on the framework of the 3 factor Heath Jarrow and Morton model. We assume that the fixed rate side of the swap is the same 3% interest rate on $100 notional amount that we analyzed in Chapter 25. For exposition purposes, we shorten the maturity from 4 years (in Chapter 25) to 3 years. We again drop those cash flows into our cash flow table and confirm, after multiplying by the correct probability-weighted discount factors, that the value of the fixed rate side of the swap is 8.8290 in Figure 1:

Figure 01

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	3.0000	3.0000	3.0000	0.0000
2		3.0000	3.0000	3.0000	0.0000
3		3.0000	3.0000	3.0000	0.0000
4		3.0000	3.0000	3.0000	0.0000
5			3.0000	3.0000	0.0000
6			3.0000	3.0000	0.0000
7			3.0000	3.0000	0.0000
8			3.0000	3.0000	0.0000
9			3.0000	3.0000	0.0000
10			3.0000	3.0000	0.0000
11			3.0000	3.0000	0.0000
12			3.0000	3.0000	0.0000
13			3.0000	3.0000	0.0000
14			3.0000	3.0000	0.0000
15			3.0000	3.0000	0.0000
16			3.0000	3.0000	0.0000
17				3.0000	0.0000
18				3.0000	0.0000
19				3.0000	0.0000
20				3.0000	0.0000
21				3.0000	0.0000
22				3.0000	0.0000
23				3.0000	0.0000
24				3.0000	0.0000
25				3.0000	0.0000
26				3.0000	0.0000
27				3.0000	0.0000
28				3.0000	0.0000
29				3.0000	0.0000
30				3.0000	0.0000
31				3.0000	0.0000
32				3.0000	0.0000
33				3.0000	0.0000
34				3.0000	0.0000
35				3.0000	0.0000
36				3.0000	0.0000
37				3.0000	0.0000
38				3.0000	0.0000
39				3.0000	0.0000
40				3.0000	0.0000
41				3.0000	0.0000
42				3.0000	0.0000
43				3.0000	0.0000
44				3.0000	0.0000
45				3.0000	0.0000
46				3.0000	0.0000
47				3.0000	0.0000
48				3.0000	0.0000
49				3.0000	0.0000
50				3.0000	0.0000
51				3.0000	0.0000
52				3.0000	0.0000
53				3.0000	0.0000
54				3.0000	0.0000
55				3.0000	0.0000
56				3.0000	0.0000
57				3.0000	0.0000
58				3.0000	0.0000
59				3.0000	0.0000
60				3.0000	0.0000
61				3.0000	0.0000
62				3.0000	0.0000
63				3.0000	0.0000
64				3.0000	0.0000
Risk Neutral Value =					8.829025

As always, we could have calculated this value using the present value factors for times 1, 2 and 3 instead of the bushy tree. In the normal swap structure, the floating rate cash flow at time T=1 is determined by the relevant reference rate prevailing at time T=0. In this example, however, the reference rate is effectively determined at time T=1 for payment (ignoring a few days lag) at time T=1. Because we have a 3 factor bushy tree from Chapter 9, we have 4 different reference rate values at time T=1, 16 at time T=2, and 64 at time T=3. The floating rate interest payments are simply $100 in notional principal times the prevailing short rate at each node, paid immediately, not 1 period later. These cash flows are shown in Figure 2 and their risk-neutral value is 6.8243.

Figure 2

		Time Period				
State	Row	0	1	2	3	4
S-1, S-1, S-1	1		2.1653	4.6388	6.1106	0.0000
S-1, S-1, S-2	2		0.6911	1.4142	5.5667	0.0000
S-1, S-1, S-3	3		1.0972	2.6089	5.1710	0.0000
S-1, S-1, S-4	4		1.3611	4.9179	8.0375	0.0000
S-1, S-2, S-1	5			2.2496	3.1041	0.0000
S-1, S-2, S-2	6			0.5984	2.5913	0.0000
S-1, S-2, S-3	7			0.8121	2.0291	0.0000
S-1, S-2, S-4	8			1.2459	4.0628	0.0000
S-1, S-3, S-1	9			1.9217	5.2210	0.0000
S-1, S-3, S-2	10			1.4149	1.9784	0.0000
S-1, S-3, S-3	11			0.8591	3.1798	0.0000
S-1, S-3, S-4	12			2.8695	5.5017	0.0000
S-1, S-4, S-1	13			2.2088	5.4252	0.0000
S-1, S-4, S-2	14			1.7005	4.8848	0.0000
S-1, S-4, S-3	15			1.1432	4.4917	0.0000
S-1, S-4, S-4	16			3.1593	7.3396	0.0000
S-2, S-1, S-1	17				4.1517	0.0000
S-2, S-1, S-2	18				0.9421	0.0000
S-2, S-1, S-3	19				2.1313	0.0000
S-2, S-1, S-4	20				4.4295	0.0000
S-2, S-2, S-1	21				2.2036	0.0000
S-2, S-2, S-2	22				0.5531	0.0000
S-2, S-2, S-3	23				0.7667	0.0000
S-2, S-2, S-4	24				1.2004	0.0000
S-2, S-3, S-1	25				2.6231	0.0000
S-2, S-3, S-2	26				-0.3734	0.0000
S-2, S-3, S-3	27				0.2593	0.0000
S-2, S-3, S-4	28				2.1679	0.0000
S-2, S-4, S-1	29				1.7764	0.0000
S-2, S-4, S-2	30				1.2703	0.0000
S-2, S-4, S-3	31				0.7153	0.0000
S-2, S-4, S-4	32				2.7228	0.0000
S-3, S-1, S-1	33				2.2804	0.0000
S-3, S-1, S-2	34				1.7718	0.0000
S-3, S-1, S-3	35				1.2141	0.0000
S-3, S-1, S-4	36				3.2315	0.0000
S-3, S-2, S-1	37				2.2804	0.0000
S-3, S-2, S-2	38				1.7718	0.0000
S-3, S-2, S-3	39				1.2141	0.0000
S-3, S-2, S-4	40				3.2315	0.0000
S-3, S-3, S-1	41				3.6354	0.0000
S-3, S-3, S-2	42				0.6093	0.0000
S-3, S-3, S-3	43				1.2483	0.0000
S-3, S-3, S-4	44				3.1757	0.0000
S-3, S-4, S-1	45				4.5070	0.0000
S-3, S-4, S-2	46				1.2864	0.0000
S-3, S-4, S-3	47				2.4797	0.0000
S-3, S-4, S-4	48				4.7858	0.0000
S-4, S-1, S-1	49				3.5013	0.0000
S-4, S-1, S-2	50				0.3117	0.0000
S-4, S-1, S-3	51				1.4934	0.0000
S-4, S-1, S-4	52				3.7773	0.0000
S-4, S-2, S-1	53				2.5831	0.0000
S-4, S-2, S-2	54				2.0730	0.0000
S-4, S-2, S-3	55				1.5136	0.0000
S-4, S-2, S-4	56				3.5370	0.0000
S-4, S-3, S-1	57				2.5831	0.0000
S-4, S-3, S-2	58				2.0730	0.0000
S-4, S-3, S-3	59				1.5136	0.0000
S-4, S-3, S-4	60				3.5370	0.0000
S-4, S-4, S-1	61				4.5061	0.0000
S-4, S-4, S-2	62				2.2686	0.0000
S-4, S-4, S-3	63				2.7252	0.0000
S-4, S-4, S-4	64				5.0230	0.0000
	Risk Neutral Value =					6.82433

If both parties to the arrears swap are default free, the arbitrage free value of the interest rate swap is easy to calculate without using the cash flow table. If our swap counterparty receives the fixed rate side and pays the floating rate side, the value of the swap is 8.8290 - 6.8243 = 2.0047. Alternatively, we can calculate the net cash flows and insert them into the cash flow table. We calculate the risk-neutral value of these net cash flows and get the same value, 2.0047, as shown in Figure 3:

Figure 3

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.8347	-1.6388	-3.1106	0.0000
2		2.3089	1.5858	-2.5667	0.0000
3		1.9028	0.3911	-2.1710	0.0000
4		1.6389	-1.9179	-5.0375	0.0000
5			0.7504	-0.1041	0.0000
6			2.4016	0.4087	0.0000
7			2.1879	0.9709	0.0000
8			1.7541	-1.0628	0.0000
9			1.0783	-2.2210	0.0000
10			1.5851	1.0216	0.0000
11			2.1409	-0.1798	0.0000
12			0.1305	-2.5017	0.0000
13			0.7912	-2.4252	0.0000
14			1.2995	-1.8848	0.0000
15			1.8568	-1.4917	0.0000
16			-0.1593	-4.3396	0.0000
17				-1.1517	0.0000
18				2.0579	0.0000
19				0.8687	0.0000
20				-1.4295	0.0000
21				0.7964	0.0000
22				2.4469	0.0000
23				2.2333	0.0000
24				1.7996	0.0000
25				0.3769	0.0000
26				3.3734	0.0000
27				2.7407	0.0000
28				0.8321	0.0000
29				1.2236	0.0000
30				1.7297	0.0000
31				2.2847	0.0000
32				0.2772	0.0000
33				0.7196	0.0000
34				1.2282	0.0000
35				1.7859	0.0000
36				-0.2315	0.0000
37				0.7196	0.0000
38				1.2282	0.0000
39				1.7859	0.0000
40				-0.2315	0.0000
41				-0.6354	0.0000
42				2.3907	0.0000
43				1.7517	0.0000
44				-0.1757	0.0000
45				-1.5070	0.0000
46				1.7136	0.0000
47				0.5203	0.0000
48				-1.7858	0.0000
49				-0.5013	0.0000
50				2.6883	0.0000
51				1.5066	0.0000
52				-0.7773	0.0000
53				0.4169	0.0000
54				0.9270	0.0000
55				1.4864	0.0000
56				-0.5370	0.0000
57				0.4169	0.0000
58				0.9270	0.0000
59				1.4864	0.0000
60				-0.5370	0.0000
61				-1.5061	0.0000
62				0.7314	0.0000
63				0.2748	0.0000
64				-2.0230	0.0000
Risk Neutral Value =					2.0047

The equilibrium spread s for which an arrears swap has an [efficient] market value of zero is easy to calculate. We can either vary the fixed rate on the swap or add (or subtract) a constant spread from the floating reference rate such that the market value of the swap is zero. This is easy to do in common spreadsheet software. We let the fixed rate vary and solve for the fixed rate which produces zero squared error. The fixed rate which produces a net value for the swap is 2.3188%, a dollar coupon of $2.3188 instead of $3. When we drop the new net cash flows into the cash flow table and multiply by the probability-weighted cash flows, we get the confirmation in Figure 4 that the risk-neutral value is 0:

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.1536	-2.3200	-3.7918	0.0000
2		1.6278	0.9046	-3.2479	0.0000
3		1.2216	-0.2901	-2.8522	0.0000
4		0.9578	-2.5991	-5.7186	0.0000
5			0.0692	-0.7852	0.0000
6			1.7205	-0.2725	0.0000
7			1.5067	0.2897	0.0000
8			1.0729	-1.7440	0.0000
9			0.3971	-2.9022	0.0000
10			0.9040	0.3404	0.0000
11			1.4597	-0.8609	0.0000
12			-0.5507	-3.1828	0.0000
13			0.1100	-3.1064	0.0000
14			0.6183	-2.5660	0.0000
15			1.1756	-2.1729	0.0000
16			-0.8404	-5.0208	0.0000
17				-1.8329	0.0000
18				1.3767	0.0000
19				0.1876	0.0000
20				-2.1107	0.0000
21				0.1152	0.0000
22				1.7657	0.0000
23				1.5521	0.0000
24				1.1185	0.0000
25				-0.3043	0.0000
26				2.6922	0.0000
27				2.0595	0.0000
28				0.1510	0.0000
29				0.5424	0.0000
30				1.0486	0.0000
31				1.6035	0.0000
32				-0.4040	0.0000
33				0.0384	0.0000
34				0.5470	0.0000
35				1.1048	0.0000
36				-0.9127	0.0000
37				0.0384	0.0000
38				0.5470	0.0000
39				1.1048	0.0000
40				-0.9127	0.0000
41				-1.3166	0.0000
42				1.7095	0.0000
43				1.0705	0.0000
44				-0.8568	0.0000
45				-2.1882	0.0000
46				1.0324	0.0000
47				-0.1608	0.0000
48				-2.4670	0.0000
49				-1.1825	0.0000
50				2.0071	0.0000
51				0.8254	0.0000
52				-1.4585	0.0000
53				-0.2643	0.0000
54				0.2459	0.0000
55				0.8052	0.0000
56				-1.2182	0.0000
57				-0.2643	0.0000
58				0.2459	0.0000
59				0.8052	0.0000
60				-1.2182	0.0000
61				-2.1873	0.0000
62				0.0502	0.0000
63				-0.4064	0.0000
64				-2.7042	0.0000
Risk Neutral Value =					0.0000

This change in coupon reduces the value of the fixed rate cash flows to 6.8243, the same as the (unchanged) value of the floating rate cash flows. Alternatively, we could have increased the value of the floating rate cash flows by a present value amount of 2.0047 (generated by adding a fixed amount of $3 - 2.3188 to each floating rate cash flow), while leaving the fixed rate coupon unchanged.

Digital Option

Another category is the "digital" category of derivatives that pays either "1" or "0" depending on the level of a random variable. Given our focus on fixed income derivatives, consider the value of a derivative security which pays $1 at time T_0 if a short term reference rate (such as Libor) is greater than or equal to a critical level K. We demonstrated the valuation of such instruments in the examples of Chapters 6 through 9.

Digital Range Notes

What if the security only pays a principal amount of $100 and coupon rate of $1 if the short term rate is between critical levels K_1 and K_2? Sticking with our prior example, let's assume that the short rate reference rate is measured "in arrears" and that the bond has a 3 year maturity. We assume the critical levels for the reference rate are 2% and 4%. The short term spot rate, measured on an arrears basis, is only between 2% and 4% in the shaded scenarios given in Figure 5:

Figure 5

		Time Period				
State	Row	0	1	2	3	4
S-1, S-1, S-1	1		2.1653	4.6388	6.1106	0.0000
S-1, S-1, S-2	2		0.6911	1.4142	5.5667	0.0000
S-1, S-1, S-3	3		1.0972	2.6089	5.1710	0.0000
S-1, S-1, S-4	4		1.3611	4.9179	8.0375	0.0000
S-1, S-2, S-1	5			2.2496	3.1041	0.0000
S-1, S-2, S-2	6			0.5984	2.5913	0.0000
S-1, S-2, S-3	7			0.8121	2.0291	0.0000
S-1, S-2, S-4	8			1.2459	4.0628	0.0000
S-1, S-3, S-1	9			1.9217	5.2210	0.0000
S-1, S-3, S-2	10			1.4149	1.9784	0.0000
S-1, S-3, S-3	11			0.8591	3.1798	0.0000
S-1, S-3, S-4	12			2.8695	5.5017	0.0000
S-1, S-4, S-1	13			2.2088	5.4252	0.0000
S-1, S-4, S-2	14			1.7005	4.8848	0.0000
S-1, S-4, S-3	15			1.1432	4.4917	0.0000
S-1, S-4, S-4	16			3.1593	7.3396	0.0000
S-2, S-1, S-1	17				4.1517	0.0000
S-2, S-1, S-2	18				0.9421	0.0000
S-2, S-1, S-3	19				2.1313	0.0000
S-2, S-1, S-4	20				4.4295	0.0000
S-2, S-2, S-1	21				2.2036	0.0000
S-2, S-2, S-2	22				0.5531	0.0000
S-2, S-2, S-3	23				0.7667	0.0000
S-2, S-2, S-4	24				1.2004	0.0000
S-2, S-3, S-1	25				2.6231	0.0000
S-2, S-3, S-2	26				-0.3734	0.0000
S-2, S-3, S-3	27				0.2593	0.0000
S-2, S-3, S-4	28				2.1679	0.0000
S-2, S-4, S-1	29				1.7764	0.0000
S-2, S-4, S-2	30				1.2703	0.0000
S-2, S-4, S-3	31				0.7153	0.0000
S-2, S-4, S-4	32				2.7228	0.0000
S-3, S-1, S-1	33				2.2804	0.0000
S-3, S-1, S-2	34				1.7718	0.0000
S-3, S-1, S-3	35				1.2141	0.0000
S-3, S-1, S-4	36				3.2315	0.0000
S-3, S-2, S-1	37				2.2804	0.0000
S-3, S-2, S-2	38				1.7718	0.0000
S-3, S-2, S-3	39				1.2141	0.0000
S-3, S-2, S-4	40				3.2315	0.0000
S-3, S-3, S-1	41				3.6354	0.0000
S-3, S-3, S-2	42				0.6093	0.0000
S-3, S-3, S-3	43				1.2483	0.0000
S-3, S-3, S-4	44				3.1757	0.0000
S-3, S-4, S-1	45				4.5070	0.0000
S-3, S-4, S-2	46				1.2864	0.0000
S-3, S-4, S-3	47				2.4797	0.0000
S-3, S-4, S-4	48				4.7858	0.0000
S-4, S-1, S-1	49				3.5013	0.0000
S-4, S-1, S-2	50				0.3117	0.0000
S-4, S-1, S-3	51				1.4934	0.0000
S-4, S-1, S-4	52				3.7773	0.0000
S-4, S-2, S-1	53				2.5831	0.0000
S-4, S-2, S-2	54				2.0730	0.0000
S-4, S-2, S-3	55				1.5136	0.0000
S-4, S-2, S-4	56				3.5370	0.0000
S-4, S-3, S-1	57				2.5831	0.0000
S-4, S-3, S-2	58				2.0730	0.0000
S-4, S-3, S-3	59				1.5136	0.0000
S-4, S-3, S-4	60				3.5370	0.0000
S-4, S-4, S-1	61				4.5061	0.0000
S-4, S-4, S-2	62				2.2686	0.0000
S-4, S-4, S-3	63				2.7252	0.0000
S-4, S-4, S-4	64				5.0230	0.0000

The cash flow on these digital range notes will be a principal payment of $100 at time T=3 and $1 only in the shaded scenarios. The total cash flow on the digital range notes is dropped into the cash flow tables and multiplied by the appropriate probability-weighted discount factors to derive the bond value of 97.2304, as shown in Figure 6:

Figure 6

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	1.0000	0.0000	100.0000	0.0000
2		0.0000	0.0000	100.0000	0.0000
3		0.0000	1.0000	100.0000	0.0000
4		0.0000	0.0000	100.0000	0.0000
5			1.0000	101.0000	0.0000
6			0.0000	101.0000	0.0000
7			0.0000	101.0000	0.0000
8			0.0000	100.0000	0.0000
9			0.0000	100.0000	0.0000
10			0.0000	100.0000	0.0000
11			0.0000	101.0000	0.0000
12			1.0000	100.0000	0.0000
13			1.0000	100.0000	0.0000
14			0.0000	100.0000	0.0000
15			0.0000	100.0000	0.0000
16			1.0000	100.0000	0.0000
17				100.0000	0.0000
18				100.0000	0.0000
19				101.0000	0.0000
20				100.0000	0.0000
21				101.0000	0.0000
22				100.0000	0.0000
23				100.0000	0.0000
24				100.0000	0.0000
25				101.0000	0.0000
26				100.0000	0.0000
27				100.0000	0.0000
28				101.0000	0.0000
29				100.0000	0.0000
30				100.0000	0.0000
31				100.0000	0.0000
32				101.0000	0.0000
33				101.0000	0.0000
34				100.0000	0.0000
35				100.0000	0.0000
36				101.0000	0.0000
37				101.0000	0.0000
38				100.0000	0.0000
39				100.0000	0.0000
40				101.0000	0.0000
41				101.0000	0.0000
42				100.0000	0.0000
43				100.0000	0.0000
44				101.0000	0.0000
45				100.0000	0.0000
46				100.0000	0.0000
47				101.0000	0.0000
48				100.0000	0.0000
49				101.0000	0.0000
50				100.0000	0.0000
51				100.0000	0.0000
52				101.0000	0.0000
53				101.0000	0.0000
54				101.0000	0.0000
55				100.0000	0.0000
56				101.0000	0.0000
57				101.0000	0.0000
58				101.0000	0.0000
59				100.0000	0.0000
60				101.0000	0.0000
61				100.0000	0.0000
62				101.0000	0.0000
63				101.0000	0.0000
64				100.0000	0.0000
Risk Neutral Value =					97.2304

Range Floater

Another derivative floating rate structure is the so-called "range floater." Consider a 4 period security which pays the reference rate when the rate is less than a critical level K, and 0 otherwise. Cash flow is determined in the normal method, that is at time T_0 based on the level of an underlying zero coupon bond with maturity T_1, with cash flowing at time T=1. We can use the spot rates above, adjusted for payment with the normal 1 period lag after the interest rate is determined, to calculate the payments on this $100 4 period note. Let's assume that K is 3% and any spot rate value over 3% results in a coupon payment of zero. Figure 7 shows the cash flows on the note at each node of the bushy tree. The risk-neutral value of this structure is 96.5770:

Figure 7

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.3003	2.1653	0.0000	100.0000
2		0.3003	2.1653	0.0000	100.0000
3		0.3003	2.1653	0.0000	100.0000
4		0.3003	2.1653	0.0000	100.0000
5			0.6911	1.4142	100.0000
6			0.6911	1.4142	102.5913
7			0.6911	1.4142	102.0291
8			0.6911	1.4142	100.0000
9			1.0972	2.6089	100.0000
10			1.0972	2.6089	101.9784
11			1.0972	2.6089	100.0000
12			1.0972	2.6089	100.0000
13			1.3611	0.0000	100.0000
14			1.3611	0.0000	100.0000
15			1.3611	0.0000	100.0000
16			1.3611	0.0000	100.0000
17				2.2496	100.0000
18				2.2496	100.9421
19				2.2496	102.1313
20				2.2496	100.0000
21				0.5984	102.2036
22				0.5984	100.5531
23				0.5984	100.7667
24				0.5984	101.2004
25				0.8121	102.6231
26				0.8121	99.6266
27				0.8121	100.2593
28				0.8121	102.1679
29				1.2459	101.7764
30				1.2459	101.2703
31				1.2459	100.7153
32				1.2459	102.7228
33				1.9217	102.2804
34				1.9217	101.7718
35				1.9217	101.2141
36				1.9217	100.0000
37				1.4149	102.2804
38				1.4149	101.7718
39				1.4149	101.2141
40				1.4149	100.0000
41				0.8591	100.0000
42				0.8591	100.6093
43				0.8591	101.2483
44				0.8591	100.0000
45				2.8695	100.0000
46				2.8695	101.2864
47				2.8695	102.4797
48				2.8695	100.0000
49				2.2088	100.0000
50				2.2088	100.3117
51				2.2088	101.4934
52				2.2088	100.0000
53				1.7005	102.5831
54				1.7005	102.0730
55				1.7005	101.5136
56				1.7005	100.0000
57				1.1432	102.5831
58				1.1432	102.0730
59				1.1432	101.5136
60				1.1432	100.0000
61				0.0000	100.0000
62				0.0000	102.2686
63				0.0000	102.7252
64				0.0000	100.0000
Risk Neutral Value =					96.5770

Other Derivative Securities

As we discussed in Chapter 25, literally almost any fixed income derivative structure can be analyzed in this framework due to the richness of the Heath Jarrow and Morton term structure framework. "Basis swaps" which exchange one floating rate structure for another, CMT "constant maturity" swaps which pay the par coupon bond yield at each point in time versus a more traditional floating rate index, amortizing interest rate swaps, and many other structures fit into the same basis structure. The only task is fairly mechanical: to correctly calculate the cash flows at each node of the HJM bushy tree or at each point in time for N Monte Carlo scenarios in the HJM framework. All of these numerical solutions have related interest rate sensitivity statistics. Hedging portfolios can be derived for each individual transaction or (more efficiently from an execution point of view) for the aggregated interest rate risk position of the organization. Shifts in the zero coupon bond prices input at any of the maturities (in our example, that is at times 1, 2, 3 and 4) produce "deltas" in value with respect to that particular point on the yield curve. Shifts in interest rate volatility assumptions produce the multi-factor "vega" equivalents of vega in the one factor Black-Scholes options model. The class of securities that we have not yet valued is that class which involves an American option. We turn to that problem in the next chapter. Before doing so, however, we turn as usual to the impact of credit risk on the valuation framework above.

Credit Risk and Exotic Derivatives Structures

For all of the instruments we have analyzed in Chapters 19 through 26, we face a common dilemma. If we assume that the default probability of our counterparty is independent of the factors which drive pay-offs on the instrument, then we make the same (potentially huge) mistake which J.P. Morgan made in the famous 1998 incident with SK Securities discussed in depth by van Deventer and Imai [2003]. This mistake was repeated by a wave of financial institutions in the 2007 to 2010 credit crisis when they ignored home price risk and its correlated impact on the value of RMBS, CDOs and credit default swaps of both individual firms and individual securities. This "default risk independence" assumption is very seductive, because as Jarrow and Turnbull show, for many structures the impact of credit risk on the value of a structure just becomes a simple multiplier of the original valuation formula. As dangerous as this simplification can be, it's a better approximation than doing nothing. "Doing nothing" was one of the most common risk management techniques prior to the most recent credit crisis, it's sad to say.

As is the case with the other structures discussed in the last few chapters, the authors believe that such an "independence" assumption is particularly dangerous in the case of derivative instrument credit risk (even in the presence of AAA-rated special purpose vehicles) for two reasons. First, interest rates clearly have an impact, and potentially a substantial impact, on the default probabilities of financial institutions as we have learned time and time again in incidents ranging from the savings and loan crisis to the Asia crisis. This increased default risk comes not necessarily from interest rate mismatches but from the increase in the default probabilities of almost every counterparty on the balance sheet of the financial institution. Second, there is a tremendous concentration of market share in the derivatives business, and the default of any one of those institutions has a double impact, as we saw with the failures of Lehman Brothers and Bear Stearns in 2008 and the effective failures of firms like AGI and Merrill Lynch. Just one default would adversely affect 10-20% of the derivatives transactions on the books of an institution (since market shares of the major derivatives players are typically in the 10-20% range) and it would cause an important market disruption. It would require a huge wave of derivatives to be analyzed and replaced with another counterparty, one whose default probability is most likely highly correlated with the institution that has defaulted. One bankruptcy lawyer told us that “the few remaining staff of Lehman Brothers are being less than helpful in calculating the net exposure my client has with Lehman.” This macro factor risk is much broader than the impact of interest rates along. Home prices (remember Japan and 2007-2010 in the USA and Europe), commercial real estate, exchange rates, oil prices, and a very wide array of other factors can have similar massive correlated impacts on risk.

For that reason, we think the valuation and simulation of interest rate exotic structures have to be analyzed using the same process outlined beginning in Chapters 19 and 20,:

1. Default probabilities of all counterparties should be explicitly linked to key macro factors

2. Interest rate and other macro factor simulation will impact both the payoffs due on the structure and the default probabilities and zero coupon bond prices of each risky counterparty

3. The true credit adjusted value and cash flow can then be accurately derived.

Any other strategy is the financial equivalent of sticking your head in the sand and ignoring the problem because you don’t want to face the reality of the problem. Fortunately for shareholders, depositors and deposit insurance funds of various nations, regulators from the Basel Committee on Banking Supervision, the Financial Services Authority in the UK, and the array of banking regulators in the United States are focusing on this issue with the intensity it deserves. We do the same in Chapters 36 through 41.