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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 32:
Impact of Collateral on valuation models:
The example of home prices in the credit crisis

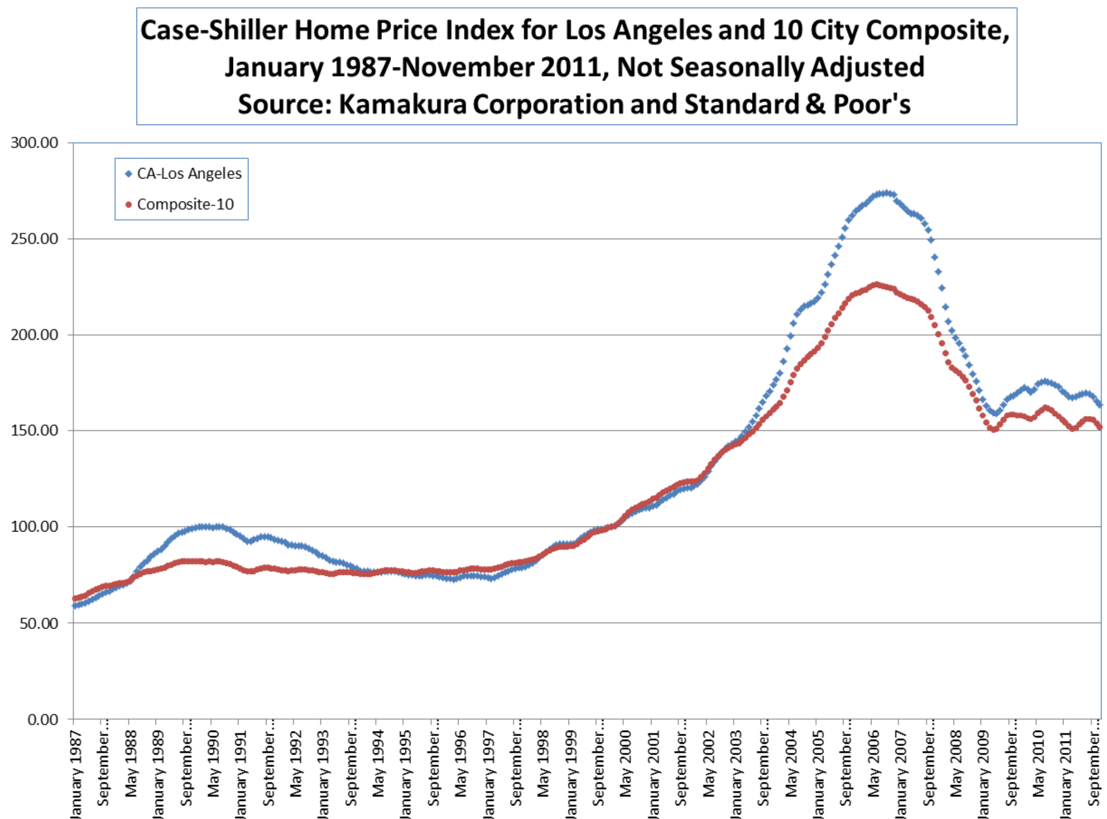
The credit crisis of 2007 to 2010 has re-emphasized what careful bankers have known for decades—collateral can have a significant impact on mitigating the credit risk of a particular loan structure. Lack of such collateral can have the opposite effect, which became obvious when slight declines in home prices triggered almost instantaneous defaults on mortgage loans made at "100% loan to value ratio" levels. That is, those loans for which lenders were willing to lend 100% of the appraised value of the house on the assumption that increases in the home price would soon restore a prudent level of collateral.

In this chapter, we add collateralized transactions to our list of assets and liabilities that can be valued in a total risk management framework. Again, our objective in this effort is to be able to value the Jarrow-Merton put option on the value of the assets (and liabilities) of the firm as a measure of total integrated risk, including interest rate risk, market risk, liquidity risk and credit risk.

The Impact of Changing Home Prices on Collateral Values in the Credit Crisis

Up until 2006 in the United States and until the peak of the Japanese bubble in December, 1989, bankers were willing to lend as much as 100% of the value of the home in the form of a first mortgage loan. This compares to a much more prudent, long term "normal" level of an 80% "loan to value" ratio. What happened in the United States was a crushing collapse in home prices and an immediate response by mortgage loan borrowers. Figure 1 shows the change in the Case-Shiller home price index for Los Angeles and for a ten city composite from 1987 to 2011:

Figure 1



As home prices fell, lenders began scrambling to check collateral values in anticipation of defaults and the default and prepayment behavior of home owners immediately changed to reflect the "new reality" of falling home prices. In Chapters 27 and 28, we analyzed prepayment and default modeling where behavior by consumers was potentially "irrational." In this chapter, we return to the amortizing loan structure with rational prepayment from Chapter 28 and revisit our conclusions in the light of changing collateral values, namely the price of the home. While we discuss mortgages and home prices exclusively in this chapter, the analysis is nearly identical for any form of lending where collateral is part of the overall lending arrangement. The key question we need to address: how does the value of the collateral vary with the changes in prepayment and default experience on the loan? We address that issue in the next section.

Modeling Variations in Collateral Values

In Chapter 31, we used Amin and Jarrow [1991] to study FX derivatives modeling in a Heath Jarrow and Morton context. In this chapter, we turn to another critical paper by Amin and Jarrow [1992, "Pricing Options on Risky Assets in a Stochastic Interest Rate Economy"]. This paper presents a general framework for modeling the value of any asset under no arbitrage assumptions in a Heath Jarrow and Morton interest rate environment like Chapters 6 through 9. The authors show that the expected return on the asset will be a premium to the spot risk free rate of interest that depends on a number of factors:

The risk factors which affect the value of the risk asset

The volatility of the risky asset's returns as a function of these risk factors

The "market price of risk" for each of these risk factors

The level of the spot rate of the risk free term structure

What does this mean for our mortgage loan example? It means that we need to tie the price of the home to each of these factors, most notably to the level of the spot rate of the risk free rate of interest. Amin and Jarrow [1992, equation 3.7, page 220] specify the expected return on the risky asset, in this case the underlying home which is collateral on a mortgage loan, in this way:

$$\mu(t,x) - r(t) = -\sum_{i=1}^{d} \delta_i(t,x) n_i(t)$$

The μ term refers to the expected return on the risky asset, our home used as collateral. The r(t) term refers to the risk-free rate. The δ terms are the volatilities with respect to the d risk factors impacting the value of the home via home price returns. The first n of these d terms will be related to the random interest rate economy. The n(t) terms refer to the market prices of risk for each factor. We captured these terms in our Heath Jarrow and Morton "bushy tree" in Chapters 6 through 9. We now use these insights of Amin and Jarrow on the rationally prepaid mortgage loan of Chapter 28.

The Impact of Collateral Values on a Rationally Prepaid Mortgage

In Chapter 28, we analyzed rational prepayment behavior by the borrower on an amortizing mortgage with a principal of $100, annual amortization, a 4 period maturity, and a coupon rate of 2.00%. The value of this mortgage and the resulting cash flows are reproduced in Figure 2. We derived a mark to market value using the 3 factor Heath, Jarrow and Morton framework of Chapter 9. That value was 101.4234:

Figure 2

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	26.2624	26.2624	26.2624	26.2624
2		102.0000	26.2624	26.2624	26.2624
3		102.0000	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			0.0000	26.2624	26.2624
6			0.0000	26.2624	26.2624
7			0.0000	26.2624	26.2624
8			0.0000	26.2624	26.2624
9			0.0000	26.2624	26.2624
10			0.0000	52.0098	0.0000
11			0.0000	26.2624	26.2624
12			0.0000	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			77.2524	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				0.0000	0.0000
18				0.0000	0.0000
19				0.0000	0.0000
20				0.0000	0.0000
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				0.0000	0.0000
34				0.0000	0.0000
35				0.0000	0.0000
36				0.0000	0.0000
37				0.0000	0.0000
38				0.0000	0.0000
39				0.0000	0.0000
40				0.0000	0.0000
41				0.0000	0.0000
42				0.0000	0.0000
43				0.0000	0.0000
44				0.0000	0.0000
45				0.0000	0.0000
46				0.0000	0.0000
47				0.0000	0.0000
48				0.0000	0.0000
49				26.2624	26.2624
50				52.0098	0.0000
51				52.0098	0.0000
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				52.0098	0.0000
56				26.2624	26.2624
57				0.0000	0.0000
58				0.0000	0.0000
59				0.0000	0.0000
60				0.0000	0.0000
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624
Risk Neutral Value =					101.4234

In that chapter, we ignored the value of the home and talked about the various ways in which borrower default and prepayment behavior can be jointly modeled. Best practice in that regard is multinomial logit. In this chapter, we deal with a special case that assumes a special type of rationality on the part of the borrower: (a) the borrower will always prepay if (starting at time T=1) the risk-neutral value is greater than the principal value on the loan and (b) the borrower will default (a so-called "strategic default") if the value of the home is "too low" relative to the terms on the loan. We discuss the possible meanings of the term "too low" in the context of our example.

Step 1 in this process is to use the Amin and Jarrow [1992] insights to model the price of the home. For ease of exposition, we assume that the market is completely indifferent to variations in home prices. This means that the market price of risk is zero and that the expected return on the home should be equal to the risk-free rate on each of the nodes of the bushy tree used in Chapters 9 and 28. Recall that the spot rates on each of the nodes on the bushy tree take on the values in Figure 3:

Figure 3

State	Spot Rate Process Row Numb	0	1	2	3
S-1, S-1, S-1	1	0.3003%	2.1653%	4.6388%	6.1106%
S-1, S-1, S-2	2		0.6911%	1.4142%	5.5667%
S-1, S-1, S-3	3		1.0972%	2.6089%	5.1710%
S-1, S-1, S-4	4		1.3611%	4.9179%	8.0375%
S-1, S-2, S-1	5			2.2496%	3.1041%
S-1, S-2, S-2	6			0.5984%	2.5913%
S-1, S-2, S-3	7			0.8121%	2.0291%
S-1, S-2, S-4	8			1.2459%	4.0628%
S-1, S-3, S-1	9			1.9217%	5.2210%
S-1, S-3, S-2	10			1.4149%	1.9784%
S-1, S-3, S-3	11			0.8591%	3.1798%
S-1, S-3, S-4	12			2.8695%	5.5017%
S-1, S-4, S-1	13			2.2088%	5.4252%
S-1, S-4, S-2	14			1.7005%	4.8848%
S-1, S-4, S-3	15			1.1432%	4.4917%
S-1, S-4, S-4	16			3.1593%	7.3396%
S-2, S-1, S-1	17				4.1517%
S-2, S-1, S-2	18				0.9421%
S-2, S-1, S-3	19				2.1313%
S-2, S-1, S-4	20				4.4295%
S-2, S-2, S-1	21				2.2036%
S-2, S-2, S-2	22				0.5531%
S-2, S-2, S-3	23				0.7667%
S-2, S-2, S-4	24				1.2004%
S-2, S-3, S-1	25				2.6231%
S-2, S-3, S-2	26				-0.3734%
S-2, S-3, S-3	27				0.2593%
S-2, S-3, S-4	28				2.1679%
S-2, S-4, S-1	29				1.7764%
S-2, S-4, S-2	30				1.2703%
S-2, S-4, S-3	31				0.7153%
S-2, S-4, S-4	32				2.7228%
S-3, S-1, S-1	33				2.2804%
S-3, S-1, S-2	34				1.7718%
S-3, S-1, S-3	35				1.2141%
S-3, S-1, S-4	36				3.2315%
S-3, S-2, S-1	37				2.2804%
S-3, S-2, S-2	38				1.7718%
S-3, S-2, S-3	39				1.2141%
S-3, S-2, S-4	40				3.2315%
S-3, S-3, S-1	41				3.6354%
S-3, S-3, S-2	42				0.6093%
S-3, S-3, S-3	43				1.2483%
S-3, S-3, S-4	44				3.1757%
S-3, S-4, S-1	45				4.5070%
S-3, S-4, S-2	46				1.2864%
S-3, S-4, S-3	47				2.4797%
S-3, S-4, S-4	48				4.7858%
S-4, S-1, S-1	49				3.5013%
S-4, S-1, S-2	50				0.3117%
S-4, S-1, S-3	51				1.4934%
S-4, S-1, S-4	52				3.7773%
S-4, S-2, S-1	53				2.5831%
S-4, S-2, S-2	54				2.0730%
S-4, S-2, S-3	55				1.5136%
S-4, S-2, S-4	56				3.5370%
S-4, S-3, S-1	57				2.5831%
S-4, S-3, S-2	58				2.0730%
S-4, S-3, S-3	59				1.5136%
S-4, S-3, S-4	60				3.5370%
S-4, S-4, S-1	61				4.5061%
S-4, S-4, S-2	62				2.2686%
S-4, S-4, S-3	63				2.7252%
S-4, S-4, S-4	64				5.0230%

What volatility should we use for home price movements? For ease of exposition, we make a false assumption that was very commonly made in the credit crisis and hereby invoke a MODEL RISK ALERT: we assume that the returns on home prices are independent from period to period (they are not). If we make this assumption and run a regression to predict the annual returns of home prices in Los Angeles as a function of the 1 year risk free rate, we find an r-squared of zero and a standard error of the regression equal to 0.13271528. We use this value for the sigma of home price returns. For simplicity, we want to model home prices with an "up shift" and "down shift" like we did for interest rates in Chapter 6. We need the mean return on the price of the house to equal the 1 period risk free rate of interest r and we need the variance of home price returns to equal σ = 0.13271528. The upshifts and down shifts which satisfy these constraints, given that the length of the period is Δ = 1, are the following:

Upshift = r + σ

Downshift = r - σ

How do we calculate these shifts in the context of our bushy interest rate tree in Chapter 9? Recall that we have 4 nodes on the bushy tree at time T=1, 16 at T=2, and 64 at T=3. Our first shift is from Time 0 to Time T=1. There is only one spot rate at time 0, so we have two values of home prices that can prevail at time T=1, the upshift value and the downshift value. The upshift value is the spot rate 0.00300321 + σ and the downshift value is the spot rate 0.00300321 - σ. We show the resulting percentage shift in home prices in Figure 4:

Figure 4

Period Time T=	
1	
Up Shift	Down Shift
0.135718489	-0.129712077

What do we do to model home price movements from time T=1 to time T=2? We have two potential home price values as of time 1, homereturn(1, up) and homereturn(1, down). We have 4 different interest rate shifts from time 0 to time 1, though, so there are 4 potential "mean" shift values and therefore 4 upshifts in home prices and 4 downshifts in home prices from time T=1 to time T=2. We label them homereturn(2, shift 1, up), homereturn(2, shift 1, down), homereturn(2, shift 2, up), homereturn(2, shift 2, down), homereturn(2, shift 3, up), homereturn(2, shift 3, down), homereturn(2, shift 4, up), homereturn(2, shift 4, down). Using the 4 values for the 1 period spot rate at time T=1 and the same home price σ gives these 8 potential changes in home prices from time T=1 to time T=2 in Figure 5:

Figure 5

Returns on Collateral by State and Collateral Value Shift			
Period Time T=			
1		2	
Up Shift	Down Shift	Up Shift	Down Shift
0.135718489	-0.129712077	0.154368006	-0.111062561
		0.139625806	-0.12580476
		0.143687587	-0.121742979
		0.146325938	-0.119104629

Let's assume that the home price was $125 when the loan of $100 was initially made, a loan to value ratio of 80%. What possible home values prevail at time T=2? We apply the up shift and down shift to get two home price values at time T=1. When we apply the 8 shift percentages in Figure 5 to both of the potential T=1 home prices, we have 16 possible values for home prices at time T=2. Those values are shown in Figure 6:

Figure 6

	Returns on Collateral by State and Collateral Value Shift					
	Period Time T=					
	1			2		
Row Number	Up Shift	Down Shift	Shift Scenario	Up Shift	Shift Scenario	Down Shift
1	141.9648111	108.7859903	Up-Shift 1-Up	163.8796359	Up-Shift 1-Down	126.1978357
2			Up-Shift 2-Up	161.7867623	Up-Shift 2-Down	124.1049621
3			Up-Shift 3-Up	162.3633923	Up-Shift 3-Down	124.6815921
4			Up-Shift 4-Up	162.7379452	Up-Shift 4-Down	125.056145
5			Down-Shift 1-Up	125.5790667	Down-Shift 1-Down	96.70393968
6			Down-Shift 2-Up	123.9753219	Down-Shift 2-Down	95.10019488
7			Down-Shift 3-Up	124.4171868	Down-Shift 3-Down	95.54205978
8			Down-Shift 4-Up	124.7042024	Down-Shift 4-Down	95.82907535

The lowest home price value prevailing at time T=1 is $108.79 and at time T=2 is $95.10. Neither of these home price values is low enough to block rational prepayment or to cause strategic default. That is because the lender was prudent and would only lend $100 against a $125 house. You can see the scheduled loan principal balances in Figure 7, which is reproduced from Chapter 28:

Figure 7

Scheduled Amortization	
Coupon Rate:	2.0000%
Time 0 Book Value	100
Periods to Maturity	4
Periodic Payment Amount	26.2624

Period Number	Initial Principal	Amount Paid	Interest	Principal	Ending Principal
0	100.0000				100.0000
1	100.0000	26.2624	2.0000	24.2624	75.7376
2	75.7376	26.2624	1.5148	24.7476	50.9900
3	50.9900	26.2624	1.0198	25.2426	25.7474
4	25.7474	26.2624	0.5149	25.7474	0.0000

What if the initial home price had been $100 instead? The simulated home prices would have been 20% lower, at the levels shown in Figure 8:

Figure 8

Returns on Collateral by State and Collateral Value Shift

Period Time T=

	1			2		
Row Number	Up Shift	Down Shift	Shift Scenario	Up Shift	Shift Scenario	Down Shift
1	113.5718489	87.02879226	Up-Shift 1-Up	131.1037087	Up-Shift 1-Down	100.9582685
2			Up-Shift 2-Up	129.4294098	Up-Shift 2-Down	99.28396966
3			Up-Shift 3-Up	129.8907138	Up-Shift 3-Down	99.74527366
4			Up-Shift 4-Up	130.1903562	Up-Shift 4-Down	100.044916
5			Down-Shift 1-Up	100.4632534	Down-Shift 1-Down	77.36315174
6			Down-Shift 2-Up	99.18025752	Down-Shift 2-Down	76.08015591
7			Down-Shift 3-Up	99.53374944	Down-Shift 3-Down	76.43364782
8			Down-Shift 4-Up	99.76336189	Down-Shift 4-Down	76.66326028

The home prices simulation drop to a low of $87.03 at time T=1 and $76.08 at time T=2. At this point, the analysis of what the borrower will do gets complex, as shown in the shaded cells of Figure 9.

Figure 9

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	26.2624	26.2624	26.2624	26.2624
2		102.0000	26.2624	26.2624	26.2624
3		102.0000	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			0.0000	26.2624	26.2624
6			0.0000	26.2624	26.2624
7			0.0000	26.2624	26.2624
8			0.0000	26.2624	26.2624
9			0.0000	26.2624	26.2624
10			0.0000	52.0098	0.0000
11			0.0000	26.2624	26.2624
12			0.0000	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			77.2524	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				0.0000	0.0000
18				0.0000	0.0000
19				0.0000	0.0000
20				0.0000	0.0000
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				0.0000	0.0000
34				0.0000	0.0000
35				0.0000	0.0000
36				0.0000	0.0000
37				0.0000	0.0000
38				0.0000	0.0000
39				0.0000	0.0000
40				0.0000	0.0000
41				0.0000	0.0000
42				0.0000	0.0000
43				0.0000	0.0000
44				0.0000	0.0000
45				0.0000	0.0000
46				0.0000	0.0000
47				0.0000	0.0000
48				0.0000	0.0000
49				26.2624	26.2624
50				52.0098	0.0000
51				52.0098	0.0000
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				52.0098	0.0000
56				26.2624	26.2624
57				0.0000	0.0000
58				0.0000	0.0000
59				0.0000	0.0000
60				0.0000	0.0000
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624
Risk Neutral Value =					101.4234

At time T=1, would the borrower prepay on the loan, paying $102, to live in a house worth $87.03? Surprisingly, we don't know enough to answer the question. If the borrower is Warren Buffett and the home is his childhood home, the borrower would repay no matter what (because he is cash rich and because he wants to stay in that specific house). If the borrower is living paycheck to paycheck, he may only have enough money to make the scheduled payment of $26.2624. The remaining funds that would be needed to prepay the loan ($75.7376, the principal still outstanding after making the schedule payment), would have to be borrowed. If the house is now worth only $87.03, he would need to get a loan equal to 87% of home value. If lenders will now only lend 80% of home value, the homeowner does not have enough money to prepay.

What about time T=2? The home, at its low point, is only worth $76.08. If the borrower makes the scheduled payment of $26.2624 at time T=2, the remaining principal outstanding is $50.99. If the borrower has the money to pay $26.2624, it's rational to keep the house and make the payment. What if the borrower only has $20? Before making the payment, the principal outstanding is $75.7376. The house is worth only $76.08. The borrower loses almost nothing by walking away from the loan and moving in with his brother. In lots of circumstances, this is what may well happen and that is why the multinomial logit probabilities of default and prepayment are such a critical overlay over "rational prepayment" and "rational strategic default." They help predict very accurately rational acts by borrowers that appear "irrational" simply because we don't have complete information about the borrower (he only has $20 and he can move in with his brother if he needs to).

How do we carry the analysis out to time T=3? From the perspective of time T=2, we know that there are 16 different possible home values. There are 16 different spot rates which could prevail, so there are 16 x 16 =256 mean values of the home that could be simulated. We have an upshift and a downshift in home prices for each of these mean values, for a total number of potential home prices equal to 16 x 16 x 2 = 512. From a computer science perspective, this is a small number and this is exactly what an enterprise wide risk management system is designed to do.

Conclusions about the Impact of Collateral Values

In the mortgage market during the credit crises of 2007 to 2010, lenders and investors overlooked the huge impact on value that a collapse in collateral values can bring about. Others, typically agents originating but not keeping mortgages, knew about the risks but had no reason to care, since they did not keep the mortgages. Home price risk was identified as a key driver of the failure of U.S. banking companies by Jarrow et all (2003, in the FDIC Loss Distribution Model) but this basic fact of banking fundamentals was forgotten in one of the largest acts of mass amnesia in the history of finance.

For all types of loans and all types of collateral, simulating the interaction of collateral values and borrower behavior is critical to getting an accurate assessment of the Jarrow-Merton put option. We turn next to a related issue, the valuation of lines of credit.