Kamakura Corporation
2222 Kalakaua Avenue
Suite 1400
Honolulu, Hawaii 96815, USA
phone 1 808 791 9888
facsimile 1 808 791 9898
www.kamakuraco.com

Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

April 2012

Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 2: Risk Management Techniques for Interest Rate Analytics

Chapter 4:
Fixed income mathematics, the basic tools

In the first three chapters of this book, we have talked about measures of risk and return in a general way. We have discussed the Jarrow-Merton put option as a measure of risk along with more traditional measures of risk like the sensitivity of net income to changes in risk factors and the sensitivity of the net market value of a portfolio (i.e. the value of equity in the portfolio) to changes in risk factors.
In the rest of this book, we talk about implementation of these concepts in a very detailed and practical way. Implementation requires accurate and efficient modeling of market values of every transaction in the portfolio both at the current time and at any date in the future. Valuation and multi-period simulation also require exact knowledge of cash flow dates and amounts, recognizing that these amounts may be random (like an interest rate on a floating rate mortgage, early prepayment on a callable bond, or payment on a first to default swap). We turn to that task now.

Modern Implications of Present Value

The concept of present value is at the very heart of finance, and yet it can seem like the most mysterious and difficult element of risk management analytics even eight decades after introduction of Macauley's duration in 1938. It is safe to say, though, that no self-respecting finance person in a large financial institution should look forward to a pleasant stay in the finance area if they are uncomfortable with the present value concept and the basics of traditional fixed income mathematics. At the same time, the basic principles of present value have so many applications that a good understanding of them would be very beneficial to a wide variety of finance professionals. In this chapter, we present an overview of present value and fixed income mathematics. We will touch on a wide variety of topics and leave a few to be covered in more detail in later chapters. Yield curve smoothing is covered in Chapter 5, modern simulation of random interest rates in Chapters 6 to 11. Finally, we cover "legacy" approaches to interest rate risk management and contrast them to more modern approaches in Chapters 12 and 14. We integrate this pure interest rate risk-focused section with credit modeling in Chapters 15 to 17.

The present value concept and related issues like yield to maturity and forward interest rates provide the building blocks for these more complex issues. These concepts will be old hat to many readers, but they have great implications for the measurement of risk and return we've discussed in Chapters 1 to 3.

Price, Accrued Interest, and Value

The accounting profession and the economics profession have engaged in many wars during their history. Occasionally, there have been periods of peaceful coexistence and general agreement on what's important in life. That seems to be happening now with the implementation of the market value-based Financial Accounting Standards 133, FAS 157 and similar IFRS pronouncements.

There was one important battle lost by the economics profession that still causes finance professionals pain, however. That battle was fought over three concepts: "price", "accrued interest", and "value."

In this chapter we will focus consistently on the concept of value--what a security is truly worth in the market place. In a rational world, a security's value and the risk-adjusted present value of future cash flows had better well be close to the same thing or the reader is wasting a lot of valuable time reading this book when he could be arbitraging the market. For the purposes of this book, when we say value, we really mean "present value": what a rational person would pay today for the risk-adjusted cash flows to be received in the future.

Unfortunately, this simple concept has been complicated by the noble idea that one "earns" interest on a bond even if you buy it just after the most recent coupon payment was paid and sell it before the next coupon payment is paid. This idea isn't harmful in and of itself, but, in the form the idea has been implemented in many markets, nothing could be farther from economic reality. The person who receives the interest on a bond is the person who is the owner of record on the record date that determines who receives the interest. For accounting (not economic) purposes, the accounting profession has decided that the value of a bond has to be split into two pieces: the "price", which is intended by the accountants to be relatively stable, and "accrued interest", which is an arbitrary calculation that determines who "earned" interest on the bond even if they received no cash from the issuer of the bond.

The basic accounting rule for splitting value isn't harmful on the surface:

Value = price plus accrued interest

What causes the harm is the formula for accrued interest. In a calculation left over from the BC (before calculators) era, accrued interest is calculated as a proportion of the next coupon on a bond. While there are many variations on the calculation of this proportion, the simplest one divides the actual number of days between the settlement date and the last bond coupon payment date by the total number of days between coupon payments.

Calculation of Accrued Interest

ABC Bank sells a 10 year 10 percent coupon bond for an amount of money equal to 102 percent of par value, or $ 1020.00 per bond. The next semiannual coupon of $ 50.00 will be paid in 122 days, and it has been 60 days since the last coupon payment.

Value = $1020.00

Accrued Interest = 60 ($50)/[60+122] =$16.48

Price= Value - Accrued Interest= $1020.00-16.48 = $1003.52 or 100.352 percent of par value

There is one fundamental flaw in these simple rules. In economic terms, the amount of accrued interest is an arbitrary calculation that has no economic meaning.[1] Why? The amount of accrued interest bears no relationship to the current level of interest rates. The "accrued interest" on the bond above is calculated to be the same in an environment when interest rates are 3% as one where interest rates are 30%. The amount of accrued interest depends solely on the coupon on the bond, which reflects interest rate levels at the time of issue, not interest rates today. Since "price" is calculated as value minus an economically meaningless number, then price is an economically meaningless number as well. Unfortunately, the number referred to most often on a day to day basis in the financial industry is "price," so those of us who focus on good numbers have to work backwards to remain focused on what's important: the value of the transaction, $1020.00.[2] Market slang labels true present value as "dirty price," meaning price plus accrued interest. "Clean price" means true present value minus accrued interest, i.e. the price quoted in the market. In our minds, the market has applied the words "dirty" and "clean" to the wrong definitions of price.

From now on, when we say "value" we mean true present value, "dirty price," and price plus accrued interest. To avoid confusion, we will always specify the concept of price we intend to avoid confusion between "clean price" and "dirty price."

[1] One exception to this comment is the differential tax impact of "accrued interest" and "price."

[2] There are exceptions to this concept of accrued interest. Australia and Korea are two markets where there is no artificial division of "value" into "accrued interest" and "price."

Present Value

A dollar received in the future is almost always[3] worth less than a dollar received today. This is more than just a reflection of inflation; it's a recognition that market requires someone using the resources of others to pay "rent" until those resources are returned. The implications of this simple fact are very important but straightforward.

The Basic Present Value Calculation

If the value of one dollar received at time t_i is written $P(t_i)$, then the present value of an investment that generates n varying cash flows $C(t_i)$ at n times t_i in the future (for i equal to 1 through n) can be written as follows:

$$Present\ value = \sum_{i=1}^{n} P(t_i)C(t_i)$$

Example

Cash Flow 1:	$100.00
Date Received:	1 year from now
Cash Flow 2:	$200.00
Date Received:	10 years from now

Value of $1 dollar received in the future

Received in 1 year:	$0.90
Received in 10 years:	$0.50

Value	= 100(.9) + 200(.5) = 190.00

This simple formula is completely general. It is the heart of most banking calculations and much of finance. Note that the present value formula has the following features:

[3] There have been a number of transactions in Japan at negative nominal interest rates in the late 1990s and early part of this decade, and the credit crisis of 2006-2010 has generated numerous examples of negative nominal and real interest rates. We return to this topic in later chapters.

- it is independent of the number of cash flows
- it is independent of the method of interest compounding
- it assumes that cash flows are known with certainty

The $P(t_i)$ values, the value of one dollar received at time t_i, are called discount factors. They provide the basis for all yield curve calculations: bond valuation, forward rates, yield to maturity, forward bond prices, and so on. How they are determined and how they link with other concepts is the topic that makes up the heart of the rest of the book, and in particular we focus on them in Chapters 5 and 17. For the time being, we assume that these discount factors are known. That allows us to write down immediately a number of other formulas for valuation of securities where the contractual cash flows are known with certainty.

Calculating the Value of a

Fixed Coupon Bond with Principal Paid at Maturity

If the actual dollar amount of the coupon payment on a fixed coupon bond is C and principal is repaid at time t_n, the value (price plus accrued interest) of the bond is

$$Value\ of\ \ Fixed\ Coupon\ Bond = C\sum_{i=1}^{n} P(t_i) + P(t_n)Principal$$

Note that this formula applies regardless of how often coupon payments are made and regardless of whether the first coupon period is a full period in length. The calculation is nearly identical to the example given above.

Calculating the Coupon of a

Fixed Coupon Bond with Principal Paid at Maturity

when Value is Known

If the value (price plus accrued interest) of a fixed coupon bond is known and the discount factors are known, the dollar coupon payment which leads a bond to have such a value is calculated by rearranging the formulas above:

$$\text{Dollar Coupon Amount of Fixed Coupon Bond} = \frac{\text{Value - } P(t_n)\text{Principal}}{\sum_{i=1}^{n} P(t_i)}$$

Example

Principal Amount:	$1000.00
Interest Paid:	Semiannually
Value:	$1150.00
Periods to Maturity:	4 semiannual periods
Days to Next Coupon:	40 days
Value of $1 dollar received in the future	
Received in 40 days:	$0.99
Received in 6 months plus 40 days:	$0.94
Received in 1 year plus 40 days:	$0.89
Received in 1.5 years plus 40 days:	$0.83

Coupon Amount	= (1150-.83*1000)/(.99+.94+.89+.83)
	= 87.67
Coupon Rate	= [2 payments*Amount]/1000.00
	= 17.53%

The Value of an Amortizing Loan

The value of an amortizing loan is the same calculation as that for a fixed coupon bond except that the variable C represents the constant level payment received in each period. There is no explicit principal amount at the end since all principal is retired in period by period payments included in the amount C. When the periodic payment dollar amount is known, the value of the loan is

$$Value\ of\ \ Amortizing\ Bond = C\left[\sum_{i=1}^{n} P(t_i)\right]$$

Note again that this formula holds even for a short first period and for any frequency of payments, be they daily, weekly, monthly, quarterly, semiannually, annually or on odd calendar dates.

Calculating the Payment Amount of an

Amortizing Bond when Value is Known

Like the fixed coupon bond case, the payment amount on an amortizing loan can be calculated using the known amount for value (principal plus accrued interest) and rearranging the equation above for amortizing loans.

$$C = Payment\ Amount\ on\ Amortizing\ Bond = \frac{Value}{\sum_{i=1}^{n} P(t_i)}$$

Risk Management Implications

In Chapters 3, we talked about the fact that the interest rate risk of a financial institution is simply the sum of the present value of its assets less the present value of its liabilities. We also noted that the width of the interest rate risk safety zone is at least consistent with the capital of the organization being invested in any maturity from 1 day to infinitely long, provided all other assets are match funded. We now turn to the use of basis present value concepts for valuation of floating rate securities.

Calculating the Value of a

Floating Rate Bond or Loan with Principal Paid at Maturity

The calculation of the value of a floating rate bond is more complicated than the valuation of a fixed rate bond because future coupon payments are unknown. In order to value floating rate bonds, we divide the formula for determining the coupon on the bond into two pieces: an index plus a fixed dollar spread over the index. We assume the index is set at a level such that the value of a bond with one period (equal in length to 1/m years) to maturity and an interest rate equal to the index would be par value. This is equivalent to assuming that the yield curve (and its movements) which determine future values of the index is the same as the yield curve we should use for valuing the floating rate security. For example, we assume that the right yield curve for valuing all securities whose rate is reset based on the London Interbank Offered Rate should be valued at the “Libor” yield curve. We will relax this assumption below and in later chapters when we have more powerful tools to analyze more realistic assumptions about the value of a bond whose rate floats at the index plus zero spread. Using our simple assumptions for the time being, the present value of a floating rate bond priced at the index level plus a fixed dollar amount (the "spread") per period is equal to par value (the value of all future payments at an interest rate equal to the floating index plus the return of principal) plus the present value of the stream of payments equal to the spread received each period. The value of the stream of payments equal to the spread can be valued using equation above for the value of an amortizing bond. Therefore the value of a floating rate bond is

$$Value\ of\ Floating\ Rate\ Bond$$
$$= P(t_1)\left[\left(1+\frac{index}{m}\right)principal\right] + spread\left[\sum_{i=1}^{n} P(t_i)\right]$$

Note that the first term will be equal to the principal value, given our definition of the index, if the time to the first payment t_1 is equal to the normal length of the period between coupons. The first term will generally not be equal to the principal value if the length of the first period is shorter that its normal length, such as 17 days when the interval between interest rate resets is 6 months. Note also that the index may not be the same as the interest rate (which we will call the "formula rate" from here on) used as the formula for setting the coupon on the bond. See the examples for valuation of bonds where the index rate and the formula rate are different.

Example

In this example, the coupon formula is based on the London Interbank Offered Rate (Libor), which is a yield curve that was intended to be consistent with the credit risk of major international banks. Lately, this consistency has eroded but we ignore that fact in this chapter. In this example, our counterparty has a higher level of credit risk than a major international bank because the "index value" which would cause a floating rate note to have a present value equal to par value is Libor plus 1%, so the difference between the coupon rate on the floating rate note and "par pricing" is 2.00% - 1.00%.

Coupon Formula:	6 month Libor (adjusted[4])+ 2.00% percent
Index Value:	6-month Libor + 1.00%
Principal Amount:	2000.00
Spread in dollar terms over and above "par pricing":	(.02-.01)*2000/2=10.00
Maturity:	2 years
Time to next coupon	Exactly 6 months

Value of $1 received in the future

[4] Whenever "Libor" (the London Interbank Offered Rate) is mentioned in text that follows, we assume the rate has been converted from the market convention of actual days/360 days per year to actual days/365 days per year by multiplying nominal Libor by 365/360.

Received in 6 months:	0.96
Received in 1 year:	0.92
Received in 1.5 years:	0.87
Received in 2.0 years:	0.82

$$Value = .96\left[\left(1+\frac{Libor+.01}{2}\right)Principal\right]+10.00[0.96+0.92+0.87+0.83]$$
$$= 2035.8$$

Risk Management Implications

We now have enough tools to analyze the interest rate risk embedded in a simple financial institution. Armed with these simple tools and the right set of present value factors, we can

1. Mark the assets and liabilities to market
2. Calculate the implied value of the equity of the financial institution
3. Change the discount factors to "stress test" the calculations in 1 and 2 with respect to interest rate risk

We will illustrate how to do this in more detail in subsequent chapters. We now turn to another set of basic tools which will help us do this for a broad range of assets and liabilities.

Compound Interest Conventions and Formulas

No one who has been in the financial world for long expects interest rates to remain constant. Still, when financial analysts perform traditional yield to maturity calculations, the constancy of interest rates is the standard assumption simply because of the difficulty of making any other choice. Particularly when discussing compound interest, the assumption that interest rates are constant is a very common assumption. This section discusses those conventions as preparation for the yield to maturity discussion later in this chapter. Using the techniques in Chapter 5, we can relax these simple but inaccurate assumptions.

The future value of an invested amount

earning at a simple interest rate of y

compounded m times per year for n periods.

Almost all discussions of compound interest and future value depend on four factors:

- The constancy of interest rates
- The nominal annual rate of interest
- The number of times per year interest is paid
- The number of periods for which interest is compounded

$$Future\ value\ of\ invested\ amount = (invested\ amount)\left(1+\frac{y}{m}\right)^{n}$$

The future value of an invested amount

earning at a simple interest rate of y

compounded continuously for n years.

What happens if the compounding period shrinks smaller and smaller, so that interest is compound every instant, not every second or every day or every month? If one assumes a simple interest rate of y invested for n years, the corresponding formula to the equation above is

$$Future\ amount = invested\ amount(e^{yn})$$

Many financial institutions actually use a continuously compounded interest rate formula for consumer deposits. As we will see in later articles in this series, the continuous compounding assumption is a very convenient mathematical shorthand for compound interest because derivatives of the constant ***e=2.7128...*** to a power are much simpler (once one gets used to them) than the derivatives of a discrete compounding formula. The continuous time assumption also allows the use of the powerful mathematical tools called stochastic processes which we use in Chapter 6 through 11 and discuss explicitly in Chapter 13.

Example

Invested amount:	100.00
Continuously compounded interest rate:	12.00%

Investment period: 2 years and 180 days

$$Future\ value = 100.00\, e^{.12*\left(2+\frac{180}{365}\right)}$$
$$= 134.88$$

The present value of a future amount

if funds are invested at a simple interest rate of y

compounded m times per year for n periods.

A parallel question to the compound interest investment question is this: if one seeks to have 100 dollars n periods in the future and funds had been invested at a constant rate y compounded m periods per year, how much needs to be invested initially? The answer is a rearrangement of our original formula for compounding of interest:

$$Invested\ amount = \frac{Future\ value\ of\ invested\ amount}{\left(1+\frac{y}{m}\right)^{n}}$$

The present value of a future amount

if funds are invested at a simple interest rate of y

compounded continuously for n years.

When interest is assumed to be paid continuously, the investment amount can be calculated by rearranging the equation for continuous compounding of interest:

$$Invested\ amount = \frac{Future\ amount}{e^{yn}}$$
$$= Future\ amount(e^{-yn})$$

Compounding Formulas and Present Value Factors P(t)

In these sections, we have focused on the calculation of present value factors on the assumption that interest rates are constant but compound at various frequencies. We could use this approach to calculate the present value factors that we used in all of the examples in the first part of this chapter, but as President Richard M. Nixon famously said, "That would be wrong." Why? Because the yield curve is almost never absolutely flat. We deal with this issue in Chapter 5. With these compounding formulas behind us, the yield to maturity calculations can be discussed using a combination of the formulas in the second and third section of this chapter. The yield to maturity formula no longer plays an important role in modern interest rate risk management, but it is an important piece of market slang and is a concept that is essential to communicating the "present value" of a wide array of financial instruments. Besides that usage, the yield to maturity concept and the related duration and convexity formulas are "legacy" concepts that are now outmoded except as a means to communicate interest rate risk concepts in a conventional, albeit, dated, way.

Yields and Yield to Maturity Calculations

The concept of yield to maturity is one of the most useful and one of the most deceptive calculations in finance. Take the example of a five year bond with a current value equal to its par value and with coupons paid semiannually. The market is forecasting 10 different forward interest rates (which we discuss in the next section) that have an impact on the valuation of this bond. There are an infinite number of forward rate combinations that are consistent with the bond having a current value equal to its par value. What is the probability that the market is implicitly forecasting all 10 semiannual forward rates to be exactly equal? Except by extraordinary coincidence, the probability is a number only the slightest bit greater than zero. Nonetheless, this is the assumption implied when one asks "What's the yield to maturity on ABC company's bond?" It's an assumption that is no worse than any other--but no better either, in the absence of any other information.

What if ABC company has two bonds outstanding--one of 10 years in maturity and another of five years of maturity? When an analyst calculates the yield to maturity on both bonds, the result is the implicit assumption that rates are constant at one level when analyzing the first bond and constant at a different level when analyzing the second bond. The implications of this are discussed in detail in Chapter 5. In later chapters, we will discuss a number of ways to derive more useful information from a collection of outstanding value information on bonds of a comparable type. As an example, if ABC company has bonds outstanding at all 10 semiannual maturities, it is both possible and very easy to calculate the implied forward rates, zero coupon bond prices (discount factors P[t]), and pricing on all possible combinations of forward bond issues by ABC company. The calculation of yield to maturity is more complex than these calculations, and it yields less information. Nonetheless, it's still the most often quoted bond statistic other than "clean price" (present value less accrued interest).

The Formula for Yield to Maturity

For a settlement date (for either a secondary market bond purchase or a new bond issue) that falls exactly one "period" from the next coupon date, the formula for yield to maturity can be written as follows for a bond which matures in n periods and pays a fixed coupon amount C m times per year:

$$value = C\left[\sum_{i=1}^{n} P(t_i)\right] + P(t_n)[Principal]$$

$$with\ P(t_i) = \frac{1}{\left(1+\frac{y}{m}\right)^{i}}$$

Yield to maturity is the value of y that makes this equation true. This relationship is the present value equation for a fixed coupon bond, with the addition that the discount factors are all calculated by dividing one dollar by the future value of one dollar invested at a constant rate y for i periods with interest compounded m times per year. In terms of the present value concept, y is the internal rate of return on this bond. As recently as 3 decades ago, dedicated calculators were necessary to estimate y. Now it’s easy to solve for y using the “solver” function or other functions in common spreadsheet software.

Yield to Maturity for Long or Short First Coupon Payment Periods

For most outstanding bonds or securities, there is almost always a short (or occasionally a long) interest payment period remaining until the first coupon payment is received. The length of the short first period is equal to the number of days from settlement to the next coupon, divided by the total number of days from the last coupon (or hypothetical last coupon, if the bond is a new issue with a short coupon date) to the next coupon. The method for counting “days” is specified precisely by the interest accrual method appropriate for that bond. We call this ratio of “remaining days” to “days between coupons” the fraction x.[5] Then the yield to maturity can be written

[5] We reemphasize that the calculation of remaining days to days between coupons will vary depending on which of the common interest accrual methods are used to count the days.

$$value = C\left[\sum_{i=1}^{n} P(t_i)\right] + P(t_n)[Principal]$$

$$with\ P(t_i) = \frac{1}{\left(1+\frac{y}{m}\right)^{i-1+x}}$$

where, as above, y is the value that makes this equation true.

Calculating Forward Interest Rates and Bond Prices

What is the 6-month interest rate that the market expects[6] to prevail two years in the future? Straightforward questions like this are the key to funding strategy or investment strategy at many financial institutions. Once discount factors (or, equivalently, zero coupon bond prices) are known, making these calculations is simple.

Implied Forward Interest Rates on Zero Coupon Bonds

The forward simple interest rate t_i years in the future on a security that pays interest and principal at time t_{i+1} is

$$Forward\ Interest\ Rate\ = \frac{100}{t_{i+1} - t_i}\left[\frac{P(t_i)}{P(t_{i+1})} - 1\right]$$

This forward rate is the simple interest rate consistent with $1/(t_{i+1} - t_i)$ interest payments per year. For example, if t_i is two years and t_{i+1} is 2.5 years, then the forward interest rate is expressed on the basis of semiannual interest payments.

Example

Value of $1 dollar received in the future

[6] In later chapters, we will discuss "risk neutral" interest rates and their relationship to the true expected value of future interest rates.

Received in 350 days: $0.88

Received in 1 year plus 350 days: $0.80

$$Forward\ Rate = \frac{100}{1}\left[\frac{.88}{.80} - 1\right]$$
$$= 10.00$$

This is the forward rate the market expects to prevail 350 days in the future on a one year bond. The forward rate is expressed on the basis of annual compounding of interest.

Implied Forward Zero Coupon Bond Prices

A parallel question to the question of the level of forward interest rates is this query: what is the forward price of a six month zero coupon bond two years in the future? The answer is a simple ratio. From the perspective of time 0 (the current time), the forward price at time t_i of a zero coupon bond with maturity at time t_{i+1} is

$$Implied\ Forward\ Zero\ Coupon\ Bond\ Price = \frac{P(t_{i+1})}{P(t_i)}$$

Present Value of Forward Fixed Coupon Bond

What is the present value of a bond to be issued on known terms some time in the future? The answer is a straightforward application of the basic present value formula. If the actual dollar amount of the coupon payment on a fixed coupon bond is C and principal is repaid at time t_n, the value (price plus accrued interest) of the bond is

$$Present\ Value\ of\ Forward\ Fixed\ Coupon\ Bond$$
$$= C\left[\sum_{i=1}^{n} P(t_i)\right] + P(t_n)[Principal]$$

This is exactly the same present value formula used earlier in the chapter.

Implied Forward Price on a Fixed Coupon Bond

There is another logical question to ask about a bond to be issued in the future on known terms. What would be the forward price, as of today, of the "when issued bond" if we know its offering date? The answer is again a straightforward application of the basic present value equation. If the actual dollar amount of the coupon payment on a fixed coupon bond is C and principal is repaid at time t_n, the forward value (price plus accrued interest) of the bond at the issuance date t_0 is

$$\begin{aligned}&\textit{Forward Value of Fixed Coupon Bond}\\&=\frac{C\left[\sum_{i=1}^{n} P(t_i)\right]+P(t_n)[\textit{Principal}]}{P(t_0)}\end{aligned}$$

Implied Forward Coupon on a Fixed Coupon Bond

Finally, a treasurer may often be asked the implied forward issue costs (i.e. forward coupon rates) for a bond to be issued at par (or any other value). This question can be answered using a minor modification of another basic present value equation. For a bond to be issued at time t_0 with n interest payments and principal repaid at time t_n, the dollar amount of the coupon is given by the formula

$$\begin{aligned}&\textit{Dollar Coupon Amount of Fixed Coupon Bond}\\&=\frac{P(t_0)(\textit{Value at Issue})-P(t_n)\textit{Principal}}{\sum_{i=1}^{n} P(t_i)}\end{aligned}$$

Other Forward Calculations

The same types of calculations can be performed for a very wide variety of other instruments and statistics. For instance, the forward yield to maturity can be calculated on a fixed coupon bond. Forward amortizing bond prices and payment amounts can be calculated as well. The present values and forward prices for literally any form of cash flow can be analyzed using the formulas presented above.

Summary

This chapter has summarized the building blocks of all financial market calculations: the concepts of present value, compound interest, yield to maturity, and forward bond yields and prices. All of these concepts can be expressed using algebra and nothing else. There are no mysteries here, yet the power of these simple concepts is great. Risk managers who make full use of present value and its implications truly do have a leg up on the competition. Those who don't bear a heavy burden in a very competitive world. As simple as these concepts are compared to the fixed income derivatives calculations we tackle in later chapters, they are still widely misunderstood and their powerful implications are ignored by many financial institutions managers.

At this point, there are probably more than a few readers thinking "yes, these concepts aren't so difficult as long as someone gives you the discount factors--but where am I supposed to get them?" We give the answers to that question in the next chapter and (on a credit-adjusted basis) in Chapter 17. There are so many potential ways to calculate the "correct" discount factors that we purposely have chosen to speak about them generally in this chapter so that no one technique would obscure the importance of the present value calculations.