Kamakura Corporation
2222 Kalakaua Avenue
Suite 1400
Honolulu, Hawaii 96815, USA
phone 1 808 791 9888
facsimile 1 808 791 9898
www.kamakuraco.com

Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

April 2012

Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 2: Risk Management Techniques for Interest Rate Analytics

Chapter 14:
Estimating the Parameters of Interest Rate Models

In this chapter, we return from our theoretical overview of term structure models to practical application. How do we fit the parameters for the term structure model that we believe is most consistent with the assets and liabilities being modeled and the marketplace in which they are traded? That is the subject of this chapter.

Revisiting the Meaning of "No Arbitrage"

As we discussed in prior chapters, the meaning of "no arbitrage" assumptions should be the same both to finance academics and to financial market participants, but sadly that is often not the case. Academics often assume a stylized market of zero coupon bonds in which one can buy or sell them in unlimited amounts without changing the price. Models which produce zero coupon bonds whose theoretical valuations match their input values are judged to be "no arbitrage." This is a basic characteristic of the 1, 2 and 3 factor Heath Jarrow and Morton models implemented in Chapters 6 through 9. This narrow view of the meaning of "no arbitrage" is necessary but not sufficient. We need all securities whose value should stem from the relevant risk free yield curve to have a theoretical valuation which matches observed prices as well. If a perfect match is not possible, the best match one can obtain within the model structure chosen is the objective. We obtain this "best match" by selecting the model parameters that produce the best match.

A Framework for Fitting Term Structure Models

In Chapters 6 through 9, we took the initial zero coupon bond prices with maturities in 1, 2, 3 and 4 years as observable. Most likely, they were obtained from using advanced yield curve smoothing techniques of Chapter 5 on the observable prices of coupon-bearing bonds, which we discuss in Chapter 17. As we saw in each of these chapters, the observable (or derived) zero coupon bonds are not sufficient to determine what interest rate volatility assumptions are best. In fact, we showed in Chapters 6 through 9 that four different assumptions about interest rate volatility can be implemented using the same zero coupon bond prices, and all of them are "no arbitrage" in the academic sense.

Now we add additional securities which provide us with the option of using two different approaches to fitting interest rate volatility:

Historical volatility: Of course we always have the opportunity to use interest rate volatility that is derived from statistical analysis of past interest rate movements. This is what we did in Chapters 6 through 9 using U.S. Treasury data from 1962 to 2011. This exercise need not be repeated here. For an excellent example of historical term structure modeling for a five factor model, see Adrian, Crump and Moench ("Pricing the Term Structure with Linear Regression," Working Paper, Federal Reserve Bank of New York, May 9, 2012).
Implied volatility: In this chapter, we focus on the second alternative in which we use market prices of other securities to "imply" the best fitting interest rate volatility assumptions in order to insure that these other securities are priced in a "no arbitrage" sense as well.

The process for implying the interest rate volatility functions is very similar to that of implying stock return volatility in the Black-Scholes [1973] model for options on common stock. The difference is simple. In the Black-Scholes model, a single formula for options value is assumed to be true, and one equation in one unknown is solved to obtain the stock return volatility consistent with an observed option price. As is well known, the theory implies that the volatility should be the same for all strike prices at a given options maturity, but this is not supported by empirical analysis in which one minimizes the sum of squared prices errors on N options with a single stock return volatility. In optimizing interest rate volatility, however, we can employ whatever interest rate volatility function we choose as long as the volatility functions are consistent with no arbitrage in the Heath-Jarrow-Morton sense as we discussed in Chapter 6.

This allows interest rate volatility take on any number of forms: to be a constant, to take on a specific functional form like most of the one factor models in Chapter 13, to be a function of multiple parameters, and to vary with time to maturity and the level of interest rates in a complex way. We illustrate the process of parameter fitting in a series of increasingly complex examples, with more observable securities which are priced off the zero coupon yield curve.

Fitting Zero Coupon Bond Prices and Volatility Parameters Jointly

In many markets, like the market for mortgage-backed securities in the United States, there are no observable bonds that are non-callable from which zero coupon bond prices can be derived using the smoothing techniques in Chapter 5. Instead, the observable securities are either individual mortgage loans which exhibit complex prepayment behavior or packages of these individual mortgage securities. In this case, it is necessary and desirable to optimize best fitting zero coupon bond prices, prepayment parameters, default parameters, and interest rate volatility simultaneously. This is easy to do even in common spreadsheet software and the process by which one does so is the subject of the rest of this book. In the remainder of this chapter, we take the zero coupon bond prices as given and derive the relevant volatility functions from observable prices of other securities.

Steps in Fitting the Interest Rate Volatility Assumptions

In each of the interest rate volatility fitting examples which follow, we pursue the same process:

a. We select the number of risk factors driving the risk free yield curve
b. We select the functional form which describes how volatility varies (if it varies) by interest rate level and maturity
c. We obtain the prices of “other securities” in addition to the four observable zero coupon bond prices used in Chapters 6 through 9.
d. We iteratively solve for the interest rate volatility parameters which minimize the sum of squared pricing errors of the “other securities” in order to ensure that we come as close as possible to a no arbitrage interest rate framework.

In this chapter, we restrict the nature of “other securities” to a simple type: bonds which pay a fixed annual interest rate with a single option to prepay prior to maturity, i.e. bonds with an embedded European option to prepay. In later chapters, we show how to use the Heath, Jarrow and Morton framework for a wide variety of security types. All of them can be used to derive interest rate volatility in a manner similar to what we do in this chapter on similar securities. In each of the examples which follow, we use the three factor Heath Jarrow and Morton example of Chapter 9 as our base case.

Example 1: Fitting Interest Rate Volatility When Six Callable Bonds Are Observable

In this example and in Examples 2 and 3, we use Chapter 9’s three factor Heath Jarrow and Morton example as a base case. For Example 1, we make these assumptions:

1. There are three factors driving random interest rates.
2. These risk factors take on various values at different levels of interest rates

    Risk factor 1 is a constant (“sigma 1”) for all maturities of forward rates and all interest rate levels

    Risk factor 2 takes on three values. The first value (“sigma 2 low”) is for all levels of the 1 period spot rate equal to 2.00% or below. The second value (“sigma 2 mid”) is for all levels of the 1 period spot rate from 2.01% to 3.99%. The third value ("sigma 2 high") is for all levels of the 1 period spot rate of 4.00% or higher.

Risk factor 3 takes on one value ("sigma 3 low") for all levels of the 1 period spot rate equal to 2.99% or below and a second value ("sigma 3 high") for all levels of the 1 period spot rate above that level.

3. There are six observable callable bonds outstanding. They all pay annual coupons of 2%, 3%, or 4% and they are callable either at year 2 only or at year 3 only. In the case of the call dates, it is assumed that the bond holders receive the coupon scheduled for the call date. The assumed observable prices are given here:

Figure 1

Name	Bonds Maturing in Year 4	Call Provisions	Actual Price
Base	Implied Value of $100 Bond with Annual $3 Coupon	Non-Call	104.707100
Base	Implied Value of $100 Bond with Annual $2 Coupon	Non-Call	100.833236
Base	Implied Value of $100 Bond with Annual $4 Coupon	Non-Call	108.580963
Security 1	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 3	104.350000
Security 2	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 3	100.750000
Security 3	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 3	107.600000
Security 4	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 2	103.950000
Security 5	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 2	100.750000
Security 6	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 2	106.250000

Note that we have included for the reader's convenience the actual values of non-callable bonds with annual coupons of 2%, 3%, and 4%. These values are simple to calculate using the formulas of Chapter 4. The zero coupon bond prices that we used as inputs in Chapters 6 through 9 will remain unchanged, and the bushy trees that will be created in this example and examples 2 and 3 preserve these values. We reproduce here the cash flow table for the non-callable bond with a 3% coupon. For any interest rate volatility assumptions, that bond continues to have the same price we found in Chapters 6 through 9:

Figure 2

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.00	3.00	3.00	3.00	103.00
2		3.00	3.00	3.00	103.00
3		3.00	3.00	3.00	103.00
4		3.00	3.00	3.00	103.00
5			3.00	3.00	103.00
6			3.00	3.00	103.00
7			3.00	3.00	103.00
8			3.00	3.00	103.00
9			3.00	3.00	103.00
10			3.00	3.00	103.00
11			3.00	3.00	103.00
12			3.00	3.00	103.00
13			3.00	3.00	103.00
14			3.00	3.00	103.00
15			3.00	3.00	103.00
16			3.00	3.00	103.00
17				3.00	103.00
18				3.00	103.00
19				3.00	103.00
20				3.00	103.00
21				3.00	103.00
22				3.00	103.00
23				3.00	103.00
24				3.00	103.00
25				3.00	103.00
26				3.00	103.00
27				3.00	103.00
28				3.00	103.00
29				3.00	103.00
30				3.00	103.00
31				3.00	103.00
32				3.00	103.00
33				3.00	103.00
34				3.00	103.00
35				3.00	103.00
36				3.00	103.00
37				3.00	103.00
38				3.00	103.00
39				3.00	103.00
40				3.00	103.00
41				3.00	103.00
42				3.00	103.00
43				3.00	103.00
44				3.00	103.00
45				3.00	103.00
46				3.00	103.00
47				3.00	103.00
48				3.00	103.00
49				3.00	103.00
50				3.00	103.00
51				3.00	103.00
52				3.00	103.00
53				3.00	103.00
54				3.00	103.00
55				3.00	103.00
56				3.00	103.00
57				3.00	103.00
58				3.00	103.00
59				3.00	103.00
60				3.00	103.00
61				3.00	103.00
62				3.00	103.00
63				3.00	103.00
64				3.00	103.00

Risk Neutral Value = 104.7070997370

What happens if this bond is callable? The cash flows at time T=1 and time T=2 are unchanged. At time T=3, we are certain to receive the interest payment of $3. The only question is whether or not to call the bond and pay $100 at that time instead of paying $103 at time T=4. A rational bond issuer will call the bond whenever the present value of paying $103 at time T=4 is more than $100 paid at time T=3 from the perspective of time t=3. We will revisit this issue in detail in Chapters 21 (for European call provisions) and Chapter 27 (for American call provisions). For now, we remind the reader that this decision depends on the one period spot rate that prevails at time t=3 on all 64 nodes of the 3 factor bushy tree that we build using the three risk factors above, exactly as we did in Chapter 9. Here is an example evaluating the decision to call or not call the bond at each of these 64 scenarios using the "best fitting" values for risk factor volatility that we solve for below:

Figure 3

Security 1:		Coupon: 3%		Call Year 3					
	Maturity of Cash Flow Received								
Row Number	0	1	2	3	4	Call Flag (1 - yes)	NPV Factor	NPV of Year 4 Payment	NPV Benefit If Called
1	0	3	3	3	103	0	0.8538	87.9449	
2	0	3	3	3	103	0	0.8905	91.7266	
3	0	3	3	103	0	1	1.0065	103.6657	3.6657
4	0	3	3	3	103	0	0.9363	96.4397	
5	0	0	3	3	103	0	0.8591	88.4844	
6	0	0	3	3	103	0	0.8960	92.2892	
7	0	0	3	103	0	1	1.0126	104.3016	4.3016
8	0	0	3	3	103	0	0.9421	97.0313	
9	0	0	3	3	103	0	0.9696	99.8646	
10	0	0	3	103	0	1	0.9755	100.4771	0.4771
11	0	0	3	103	0	1	0.9842	101.3774	1.3774
12	0	0	3	103	0	1	0.9778	100.7169	0.7169
13	0	0	3	3	103	0	0.9207	94.8326	
14	0	0	3	3	103	0	0.9603	98.9103	
15	0	0	3	103	0	1	0.9777	100.7028	0.7028
16	0	0	3	3	103	0	0.9583	98.7035	
17	0	0	0	3	103	0	0.9188	94.6313	
18	0	0	0	3	103	0	0.9583	98.7004	
19	0	0	0	103	0	1	0.9756	100.4891	0.4891
20	0	0	0	3	103	0	0.9563	98.4940	
21	0	0	0	3	103	0	0.9412	96.9455	
22	0	0	0	3	103	0	0.9470	97.5401	
23	0	0	0	103	0	1	0.9817	101.1108	1.1108
24	0	0	0	3	103	0	0.9622	99.1034	
25	0	0	0	3	103	0	0.9627	99.1538	
26	0	0	0	3	103	0	0.9686	99.7619	
27	0	0	0	103	0	1	0.9772	100.6559	0.6559
28	0	0	0	3	103	0	0.9709	100.0000	
29	0	0	0	3	103	0	0.9435	97.1768	
30	0	0	0	3	103	0	0.9493	97.7729	
31	0	0	0	103	0	1	0.9840	101.3521	1.3521
32	0	0	0	3	103	0	0.9645	99.3399	
33	0	0	0	3	103	0	0.9439	97.2179	
34	0	0	0	3	103	0	0.9497	97.8142	
35	0	0	0	103	0	1	0.9844	101.3949	1.3949
36	0	0	0	3	103	0	0.9649	99.3818	
37	0	0	0	3	103	0	0.9627	99.1538	
38	0	0	0	3	103	0	0.9686	99.7619	
39	0	0	0	103	0	1	0.9772	100.6559	0.6559
40	0	0	0	3	103	0	0.9709	100.0000	
41	0	0	0	103	0	1	0.9713	100.0423	0.0423
42	0	0	0	103	0	1	0.9772	100.6559	0.6559
43	0	0	0	103	0	1	0.9860	101.5578	1.5578
44	0	0	0	103	0	1	0.9796	100.8961	0.8961
45	0	0	0	3	103	0	0.9650	99.3904	
46	0	0	0	3	103	0	0.9709	100.0000	
47	0	0	0	103	0	1	0.9796	100.8961	0.8961
48	0	0	0	103	0	1	0.9732	100.2386	0.2386
49	0	0	0	3	103	0	0.9209	94.8571	
50	0	0	0	3	103	0	0.9605	98.9359	
51	0	0	0	103	0	1	0.9780	100.7289	0.7289
52	0	0	0	3	103	0	0.9585	98.7290	
53	0	0	0	3	103	0	0.9435	97.1768	
54	0	0	0	3	103	0	0.9493	97.7729	
55	0	0	0	103	0	1	0.9840	101.3521	1.3521
56	0	0	0	3	103	0	0.9645	99.3399	
57	0	0	0	3	103	0	0.9650	99.3904	
58	0	0	0	3	103	0	0.9709	100.0000	
59	0	0	0	103	0	1	0.9796	100.8961	0.8961
60	0	0	0	103	0	1	0.9732	100.2386	0.2386
61	0	0	0	3	103	0	0.9457	97.4087	
62	0	0	0	3	103	0	0.9515	98.0062	
63	0	0	0	103	0	1	0.9863	101.5939	1.5939
64	0	0	0	3	103	0	0.9668	99.5769	
Risk Neutral Value =			104.33151						

We construct a modified cash flow table for each of the six callable securities that looks exactly like Figure 3. We now need to solve for the single parameter for risk factor 1, the 3 parameters for risk factor 2 and the 2 parameters for risk factor 3 that provide the best fit. This solution can be achieved using the "solver" function in common spreadsheet software or the advanced non-linear optimization techniques found in Chapter 36 when we discuss optimal capital structure. We take the former approach with example 1. We go through these steps in an automated fashion when using the "solver" functionality:

1. Guess the six parameter values

2. Value each of the callable bonds

3. Calculate the sum of squared pricing errors compared to observable prices

4. If the guess can be improved, go to step 1 and repeat. If the guess cannot be improved, we produce the final interest rate volatility values.

In step 2, valuation of the callable bonds is done by taking the modified cash flow table for that parameter set for each bond and multiplying those cash flows by the probability weighted discount factors like those from Chapter 9. The best fitting parameter values that emerge from this process are as follows:

Figure 4

Input variables	**Sigma 1**	0.0274%
	Sigma 3 Low	0.1529%
	Sigma 3 High	1.0525%
	Sigma 2 Low	0.4235%
	Sigma 2 Mid	1.3792%
	Sigma 2 High	5.0703%

These volatility parameters result in the probability weighted discount factors in this table:

Figure 5

Probability Weighted Discount Factors

	Current Time				
Row	0	1	2	3	4
1	1.000000	0.124626	0.015242	0.001805	0.001541
2		0.124626	0.015242	0.001805	0.001608
3		0.249251	0.030485	0.003610	0.003634
4		0.498503	0.060969	0.007220	0.006760
5			0.015336	0.001816	0.001560
6			0.015336	0.001816	0.001627
7			0.030672	0.003632	0.003678
8			0.061343	0.007265	0.006844
9			0.030946	0.003765	0.003651
10			0.030946	0.003765	0.003673
11			0.061893	0.007531	0.007412
12			0.123785	0.015061	0.014727
13			0.061489	0.007381	0.006796
14			0.061489	0.007381	0.007088
15			0.122979	0.014762	0.014433
16			0.245958	0.029524	0.028293
17				0.001852	0.001702
18				0.001852	0.001775
19				0.003705	0.003614
20				0.007409	0.007085
21				0.001864	0.001754
22				0.001864	0.001765
23				0.003727	0.003659
24				0.007455	0.007173
25				0.003761	0.003620
26				0.003761	0.003643
27				0.007522	0.007350
28				0.015043	0.014605
29				0.007473	0.007050
30				0.007473	0.007093
31				0.014945	0.014706
32				0.029890	0.028828
33				0.003771	0.003560
34				0.003771	0.003581
35				0.007543	0.007425
36				0.015085	0.014555
37				0.003794	0.003653
38				0.003794	0.003675
39				0.007589	0.007416
40				0.015178	0.014736
41				0.007657	0.007437
42				0.007657	0.007483
43				0.015314	0.015099
44				0.030628	0.030002
45				0.015214	0.014681
46				0.015214	0.014771
47				0.030428	0.029807
48				0.060856	0.059225
49				0.007445	0.006856
50				0.007445	0.007151
51				0.014889	0.014561
52				0.029779	0.028544
53				0.007490	0.007067
54				0.007490	0.007110
55				0.014981	0.014741
56				0.029961	0.028897
57				0.015115	0.014585
58				0.015115	0.014675
59				0.030230	0.029612
60				0.060460	0.058839
61				0.030033	0.028403
62				0.030033	0.028577
63				0.060066	0.059246
64				0.120132	0.116139

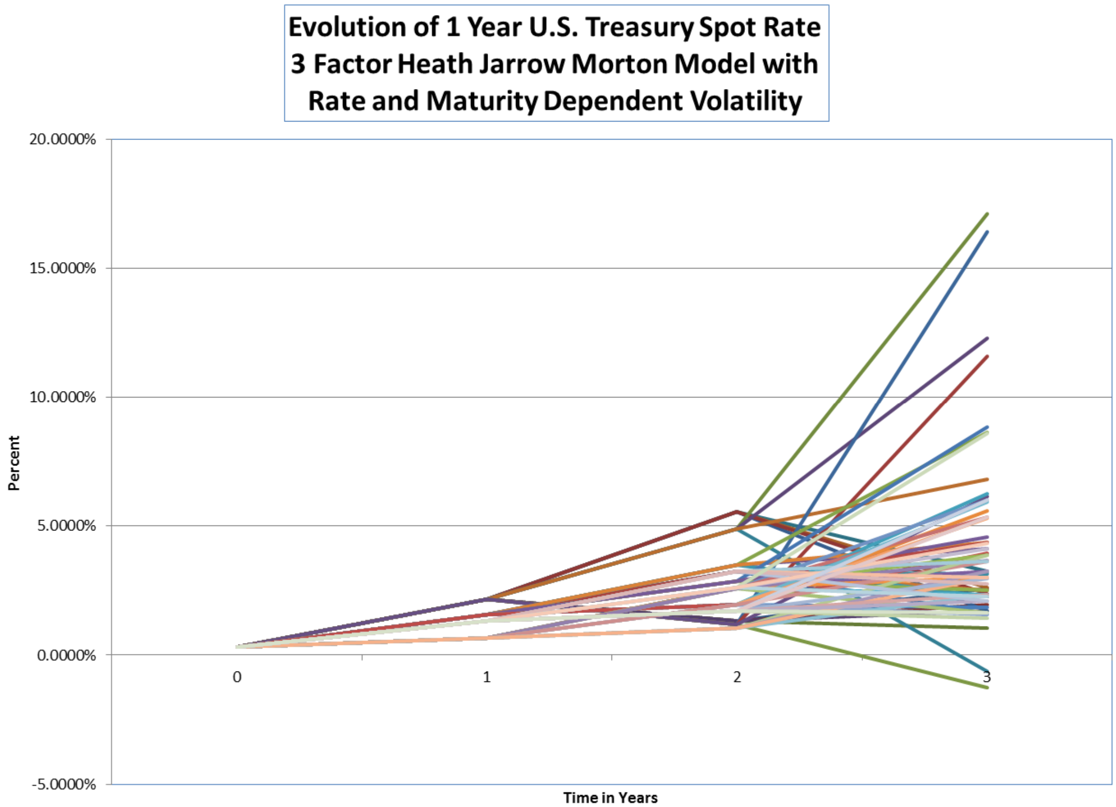
The zero coupon bond prices that we used as input are valued in exactly the same way as in Chapter 9, and the theoretical values and input values match exactly. For the callable bonds that were observable, the fit is very good but not exactly perfect:

Figure 6

Name	Bonds Maturing in Year 4	Call Provisions	Model Price	Actual Price	Squared Error
Base	Implied Value of $100 Bond with Annual $3 Coupon	Non-Call	104.707100	104.707100	
Base	Implied Value of $100 Bond with Annual $2 Coupon	Non-Call	100.833236	100.833236	
Base	Implied Value of $100 Bond with Annual $4 Coupon	Non-Call	108.580963	108.580963	
Security 1	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 3	104.331512	104.350000	0.000342
Security 2	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 3	100.748886	100.750000	0.000001
Security 3	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 3	107.619247	107.600000	0.000370
Security 4	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 2	103.950321	103.950000	0.000000
Security 5	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 2	100.760819	100.750000	0.000117
Security 6	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 2	106.237123	106.250000	0.000166
					0.000996

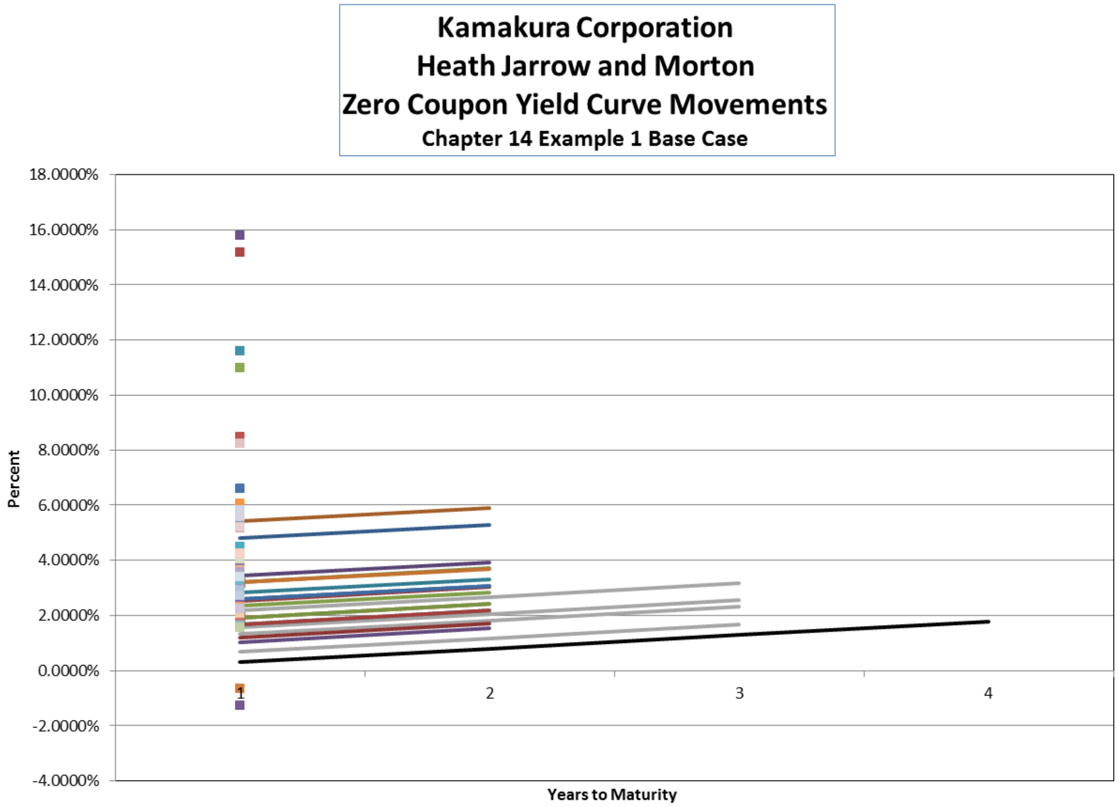
We could make the fit perfect by either decreasing the size of the time step (in which case the value will converge to the theoretical value of a callable European bond that we discuss in Chapter 21) or by decreasing the number of observable bonds or interest rate volatility parameters. The former strategy is the better strategy, because limiting the realism of the volatility model and the number of observable bonds used as inputs decreases the realism of the model, even if the model fits observable prices exactly. We show that in examples 2 and 3. The volatility assumptions that we have derived above produce a realistic "bushy tree" of short rate values as shown here:

Figure 7



The yield curve shifts implied by the interest rate volatility parameters we have derived are given here:

Figure 8



We have 4 options for further improving the realism of projected interest rate movements:

1. Find more observable securities prices to use as input

2. Combine the existing inputs with historical analysis like that of Chapter 7 or Adrian, Crump and Moench (2012).

3. Use judgment

4. Some combination of 1, 2 and 3

A Note on Parameter Fitting in Example 1

Whether one uses advanced non-linear optimization like that discussed in Chapter 36 or common spreadsheet software, it is very common for optimization problems to have multiple local optima. The user can "guide" the solution to be most realistic answer with an educated guess at the starting values used in the optimization. We encourage readers to do so for every volatility model one might employ as a model risk assessment of parameter stability. Working hard to add more observable securities is always a good thing, particularly if the additional data reveals that the assumptions of the model are too simple to provide an accurate fit to the input data. A classic example is the Black-Scholes options model's "volatility smile," which is more aptly named a "volatility frown." The frown is due to the fact that the Black-Scholes model assumes that stock return volatility is constant, but the "frown" proves that it is not. Fitting the frown is not the right solution. The correct solution is to use a volatility model with a richer structure, as the Heath Jarrow and Morton approach permits.

Example 2: The Consequences of Fewer Inputs

We now ask this question: what are the consequences of having less input data? Assume that securities 4 through 6, the bonds callable in year 2, are no longer observable. The first implication of this is that we have only 3 "equations," the valuation for the 3 observable bonds, so we are able to unique solve for only 3 unknown parameters. We arbitrarily set the sigma 2 parameters (sigma 2 low, sigma 2 mid and sigma 2 high) to zero and solve for the best fitting values of the other three parameters:

Figure 9

Name	Bonds Maturing in Year 4	Call Provisions	Model Price	Actual Price	Squared Error
Base	Implied Value of $100 Bond with Annual $3 Coupon	Non-Call	**104.707100**	**104.707100**	
Base	Implied Value of $100 Bond with Annual $2 Coupon	Non-Call	100.833236	100.833236	
Base	Implied Value of $100 Bond with Annual $4 Coupon	Non-Call	108.580963	108.580963	
Security 1	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 3	104.326910	104.350000	0.000533
Security 2	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 3	100.720693	100.750000	0.000859
Security 3	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 3	107.651271	107.600000	0.002629
Security 4	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 2	103.871326	103.950000	
Security 5	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 2	100.587339	100.750000	
Security 6	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 2	106.226133	106.250000	
					0.004021

Input variables	**Sigma 1**	0.0000%
	Sigma 3 Low	0.6596%
	Sigma 3 High	2.6493%
	Sigma 2 Low	0.0000%
	Sigma 2 Mid	0.0000%
	Sigma 2 High	0.0000%
1 million times sum of squared errors		4,021

We obtain a good fit to the 3 remaining observable bonds, but the theoretical values for the other three bonds drift away from the values that we could observe in Example 1.

Example 3: The Case of One Input

What if we have only one observable callable bond? In that case, we can solve for only one input. We set all of the sigma 2 and sigma 3 parameters to zero and solve for our only remaining interest volatility parameter. This case is the equivalent of a one factor Ho and Lee [1986] implementation in the Heath, Jarrow and Morton framework. We assume that our only observable callable bond is the bond with a 3% coupon that is callable only at time T=3. Our best fitting parameter values are as follows:

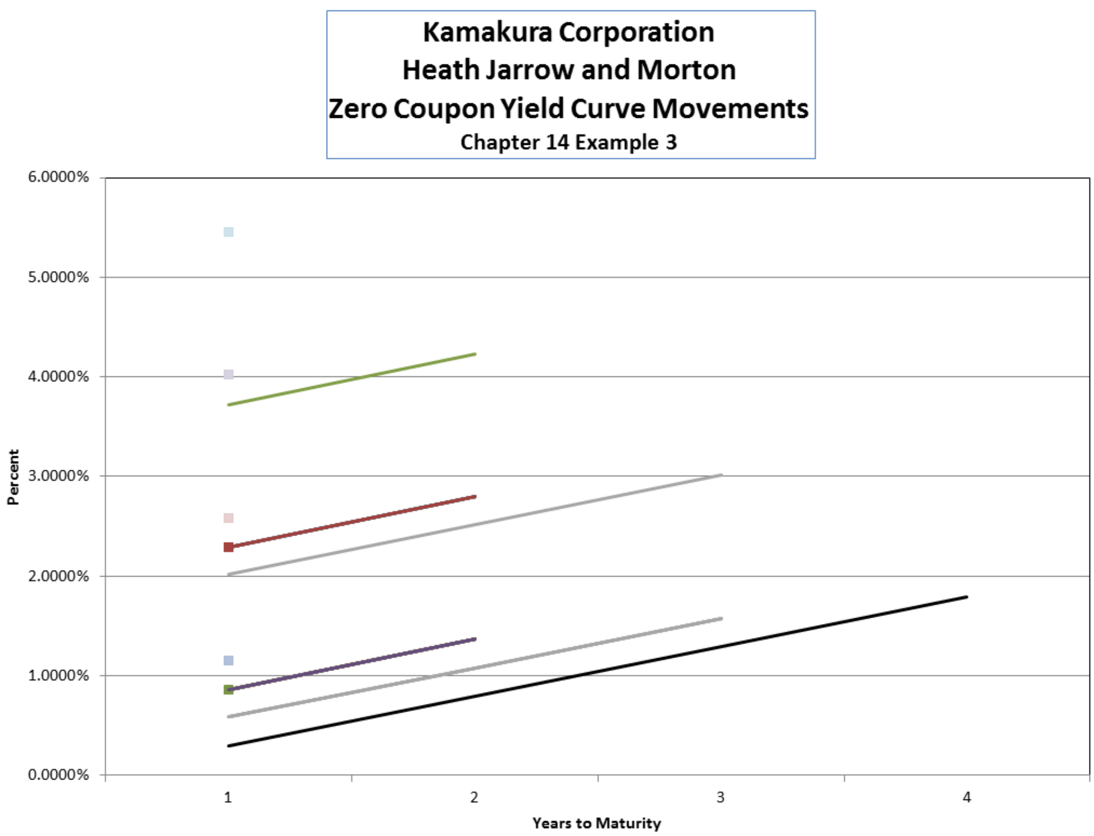
Figure 10

Name	Bonds Maturing in Year 4	Call Provisions	Model Price	Actual Price	Squared Error
Base	Implied Value of $100 Bond with Annual $3 Coupon	Non-Call	104.707100	104.707100	
Base	Implied Value of $100 Bond with Annual $2 Coupon	Non-Call	100.833236	100.833236	
Base	Implied Value of $100 Bond with Annual $4 Coupon	Non-Call	108.580963	108.580963	
Security 1	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 3	104.350000	104.350000	0.000000
Security 2	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 3	100.731641	100.750000	
Security 3	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 3	107.748447	107.600000	
Security 4	Implied Value of $100 Bond with Annual $3 Coupon	Call in Year 2	103.753160	103.950000	
Security 5	Implied Value of $100 Bond with Annual $2 Coupon	Call in Year 2	100.527566	100.750000	
Security 6	Implied Value of $100 Bond with Annual $4 Coupon	Call in Year 2	106.194074	106.250000	
					0.000000

Input variables		
	Sigma 1	0.7155%
	Sigma 3 Low	0.0000%
	Sigma 3 High	0.0000%
	Sigma 2 Low	0.0000%
	Sigma 2 Mid	0.0000%
	Sigma 2 High	0.0000%

While we get an exact fit for the only observable bond, our model values for the other bonds that we could observe in Example 1 are less accurate. Even more important, the simulated yield curve shifts implied by Example 3 are much less realistic and comprehensive than those we found in Example 1:

Figure 11



We conclude with some brief notes on implementation of parameter fitting on a fully automated basis.

Interest Rate Parameter Fitting in Practical Application

In this chapter, we noted that more data, more historical analysis and more judgment are the ingredients necessary to fully exploit the Heath Jarrow and Morton approach (and any other high quality no arbitrage modeling framework) for accurate risk measurement and risk management. In the chapters that remain in this book, we take on that task. There are a number of barriers to accomplishing that objective that we need to overcome:

1. We have been assuming a risk free yield curve, but most yield curves involve an issuer of securities who can default. We begin the introduction to credit risk analytics in Chapters 15 and 16.

2. When we deal with an issuer who can default, we note that the credit spread is the product of supply and demand for the credit of that particularly issue, and yet the default probabilities and recovery rates of that issuer are but a few of the many determinants of what the credit spread will be. We deal with that issue extensively in Chapter 17.

3. Many of the yield curves for credit risky issuers involve observable data from markets like the credit default swap market and the London Interbank Offered Rate arena where lawsuits and investigations of potential market manipulation are underway as we write this chapter

4. Many of the yield curves are composites or indices of risky issuers, like the credit index at the heart of JPMorgan Chase's losses of $2 billion announced on May 8, 2012. The Libor rate is another index. The credit risk of indices because of a number of complications:

a. A potential default of one's counterparty

b. A potential default by a firm that is a component of the index, like a bank that is a member of the U.S. dollar Libor panel

c. A potential re-benchmarking of the index, like a change in the Libor panel, because of credit risk concerns about one of the original panel members or because of concerns by a panel member about the process of setting the index itself.

d. The potential manipulation of the index by market participants

e. A compounding of the four issues above, which is now thought of as the norm by many market participants

We now tackle these challenges with an introduction to credit risk.