Kamakura Corporation

2222 Kalakaua Avenue
Suite 1400
Honolulu, Hawaii 96815, USA
phone 1 808 791 9888
facsimile 1 808 791 9898
www.kamakuraco.com

Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

April 2012

Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 4: Risk Management Applications, Instrument by Instrument

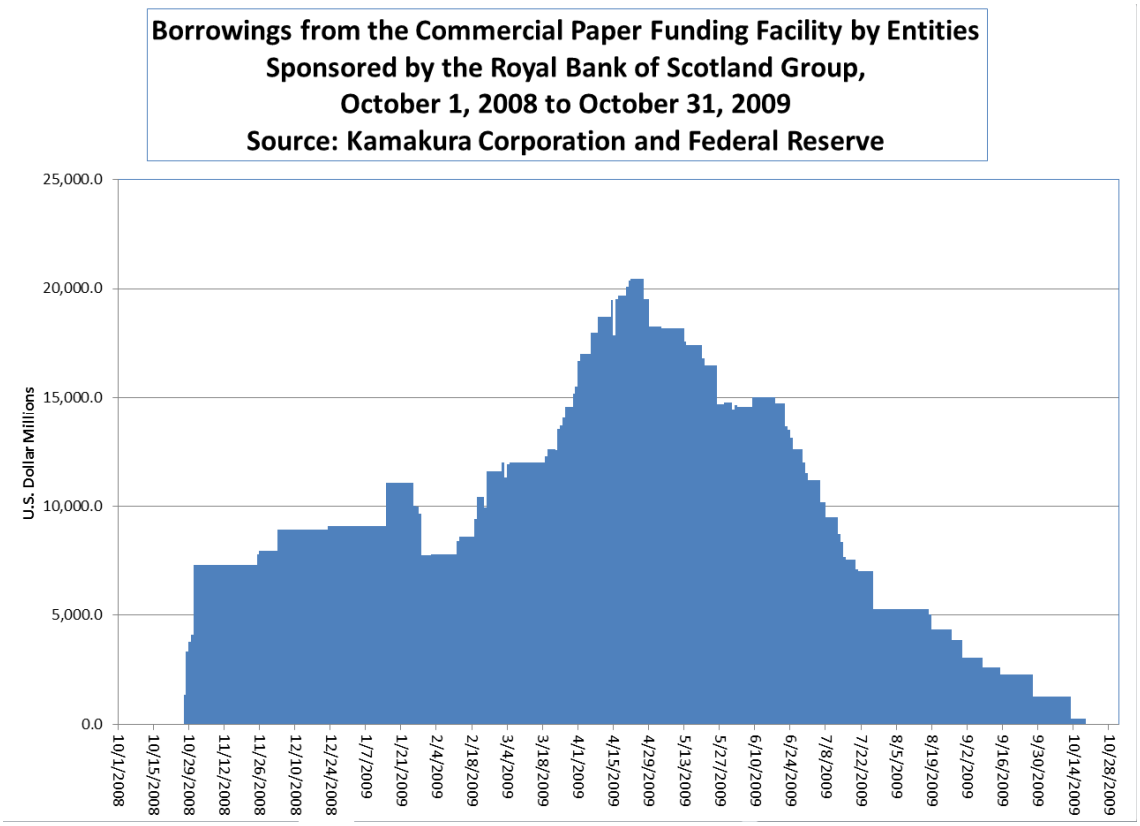
Chapter 33:
Pricing and valuing revolving credit and other facilities

The events of the 2007 to 2010 credit crisis and pervasive regulatory references to 'exposure at default' highlight the need to thoroughly understand what drives the balances outstanding on revolving credit facilities. Once that understanding is gained, we can proceed to valuation and simulation in order to add revolving credit facilities to the list of instruments that can be valued in our ongoing quest to price the Jarrow-Merton put option of Chapter 1 as our primary measure of comprehensive risk.

We turn to that task in this chapter, but we start with a bit of history that we hinted at in the Introduction. Perhaps the most dramatic draw-down of a commercial paper back-up facility was the August 16, 2007 announcement by Countrywide Financial that it was drawing down 100% of its back-up facilities. One of the authors was literally in the head office of Countrywide on that date. The announcement was more than an admission that Countrywide was having trouble rolling over its commercial paper; it was an admission that Countrywide was out of options and this was a borrowing of last resort. As chronicled in the Introduction, Countrywide was acquired shortly thereafter by Bank of America, which in turn received massive injections of capital from the U.S. government in October, 2008.

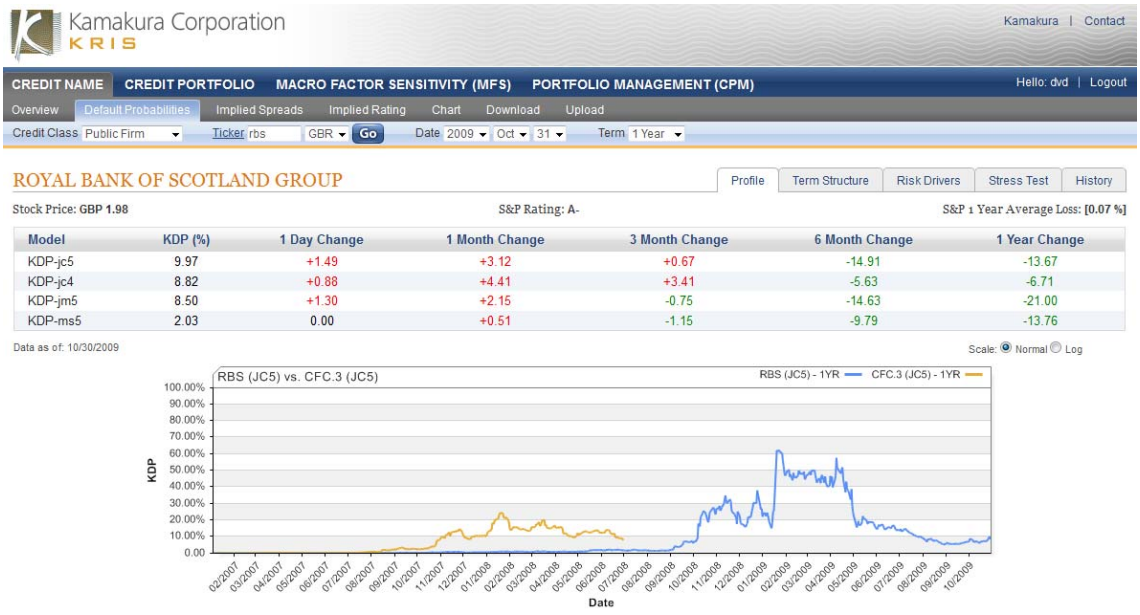
U.S. institutions were not the only firms that had to draw on back-up lines of credit during the credit crisis. As part of its support for troubled firms, the Federal Reserve established the Commercial Paper Funding Facility. Royal Bank of Scotland was a major borrower, taking down more than $21 billion under this facility, as shown in this graph provided by Kamakura Corporation:

Figure 1



Both Countrywide and Royal Bank of Scotland were forced to drawn on these facilities because their credit risk had risen so high that the supply of funds to them from traditional sources was largely cut off. Figure 2, provided by Kamakura Corporation, shows the 1 year default probabilities for Royal Bank of Scotland and Countrywide from January 1, 2007 through October 31, 2009. The Countrywide graph ends with its January 2008 acquisition by Bank of American. The one year default probability for RBS exceeded 60% at its peak:

Figure 2



With this history as background, we plunge into our evaluation of loan commitments and revolving credits. As we will see, financial distress is just one of the many reasons why such a facility may be drawn down.

Analyzing Revolving Credit and Other Facilities

In Chapter 30, we dealt with the risk measurement and valuation of non-maturity deposits in detail. Those instruments (and their asset side counterparty, retail charge card balances) have balances that fluctuate up and down randomly at the option of the depositor. The accounts have no explicit maturity, although the Jarrow-van Deventer methodology [1998] implicitly assumes that the deposit franchise has a finite ending date. On both the balance side and the interest rate side, the non-maturity deposit formulation of Jarrow and van Deventer [1998] explicitly incorporates open market interest rates as a major determinant of deposit balances and deposit interest rates.

Can we use this kind of analysis to value revolving credits? On the surface, there are many parallels. Deposit balances and revolving credit balances can be zero for an extended period of time and then suddenly spring into use. Similarly, other deposits will show a randomly fluctuating balance over a long period of time just like an 'evergreen' line of credit.

It is tempting to use the Jarrow-van Deventer methodology for non-maturity deposits if it weren't for one major assumption they make that doesn't hold in this case—they assume that the borrower of the non-maturity deposits (the bank) is riskless because of government deposit insurance. In a parallel way, they assume that the credit risk of charge card borrowers is non-zero, but known with certainty. Unfortunately, nothing could be further from the truth in the case of revolving credits. In what follows, we draw on the insights of Chava and Jarrow [2003, 2008].

Fluctuating Credit Risk and Revolving Credit Draw-downs

The changing credit quality of a borrower, along with randomly fluctuating cash needs that are linked in part to credit quality, is at the heart of the analysis of revolving credits. Cash needs are fluctuating for various reasons, above and beyond the credit distress we discussed above:

- The fundamental business of the borrower may have highly seasonal revenues (as in retailing, agriculture, and tourism) but expenses that are non-seasonal.
- The fundamental business of the borrower may have very steady revenues but highly seasonal cash needs (like farming).
- Cash flows of the business are not perfectly predictable due to normal uncertainties of client payments on the asset side and payments to suppliers on the liability side.
- Scheduled payments may not occur due to operational issues (a wire transfer failure) or credit quality problems on the part of either the company or its clients.
- The deteriorating credit quality of the borrower may shut off some sources of funding that may need to be replaced with a bank line of credit.
- Conversely, improving credit quality on the part of the borrower may open up new sources of funding that decrease the need for usage on the line of credit.

All of these factors contribute greatly to the need for a special understanding of revolving lines of credit, but it is the last three factors that are most important. If the only factors driving line of credit usage were the first three factors, we could use the Jarrow-van Deventer approach to non-maturity deposit valuation in this context without hesitation. Alternatively, we could simply project the draw-down dates and implied forward interest rates and use the Heath Jarrow and Morton discounting of Chapter 9 and 19 to calculate valuation directly.

Incorporating Links between Credit Quality and Line Usage

In van Deventer and Imai [2003] there is an extensive discussion of the J.P. Morgan-SK Securities incident of 1998 where the amount of money owed to Morgan by SK Securities and the default probability of SK's special purpose vehicle borrowing the money were strongly correlated. The very nature of the revolving line of credit has this issue at its heart— one of the primary 'use of proceeds' and one of the primary motivations of line of credit usage is for 'commercial paper back up lines'. These lines are intended to provide an alternative source of funding in the event of a disruption in the commercial paper market in general, or in case the borrower for some reason is unable to issue commercial paper, presumably because of credit quality problems. The Countrywide and Royal Bank of Scotland examples are just two of very many such incidents in the credit crisis of 2007 to 2010.

Providers of revolving lines of credit walk a fine line—they know that this change in credit quality is the primary motivator of the line and the lender guesses that the potential deterioration in credit quality will not be so severe that the lender is not fully compensated for the risk they are taking. It goes without saying that the default probabilities of all borrowers with lines of credit and the usage under the lines of credit are driven by a set of macro-economic factors common to many of the borrowers. We discussed this issue extensively in Chapters 19 and 20. In both the Countrywide and Royal Bank of Scotland cases, it was home price declines that triggered a severe cash need by both firms.

The cash needs under the line of credit are the sum of a number of influences:

$$C(t)=C_1(t) + C_2(\lambda) + C_3(\lambda_1,\lambda_2,\ldots\lambda_n)+Z(t)$$

Cash needs at any point in time consist of a time-dependent expected amount, which can vary seasonally, plus an amount that depends on the default intensity of the borrower λ, plus a third amount that depends on the default intensities of the major counterparties of the business, which we write $\lambda_1,\lambda_2,\ldots\lambda_n$. The final term is a random term. We don't mean to suggest any particularly functional form for these cash flow needs. In Chapters 36 and 47, we suggest precisely how these cash flow needs can be modeled and forecast.

Unfortunately, the multi-period nature of the usage on a line of credit means that the Merton model we reviewed in Chapter 18 is completely unsuited as a framework for line of credit valuation and estimating exposure at default. What we need instead is the kind of continuous time variation that is described by a function like that above or in the Jarrow-van Deventer non-maturity deposit model.

Once we specify how the cash needs of a company vary with its own default risk and the relevant macro-economic factors, we can proceed directly to valuation of the line of credit, simulation of cash needs, expected exposure at default, and the full probability distribution over time and in dollar terms of exposure of default. This analysis is completely analogous of the 'own firm' risk assessment that we explore in Chapters 36 and 37.

Is a Line of Credit A Put Option on the Debt of the Issuer?

Lines of credit are often thought of as a put option on the floating-rate debt of the borrower. This view is almost, but not exactly, correct. Using the analogy with the non-maturity deposit modeling of Chapter 30, part of the cash needs listed in the previous section has nothing to do with credit risk. They typically reflect the seasonality of the business. Given random interest rates and a constant default intensity of the borrower λ, this non-credit risk related cash need can be valued directly in a closed form solution using the Heath Jarrow and Morton examples in Chapters 9 and 19. The second component of cash need comes from potential changes in the credit quality of the borrower's major business counterparties. In a sense, the portfolio of accounts receivable that are held by the borrower are a collection of debt instruments like those we modeled in Chapter 19 on credit risky bonds. These too have a known solution in the reduced form modeling framework; see the worked examples in Chapter 19.

In some cases, the credit quality of the business counterparties of the borrower affects cash flow in a more complex way than a default/no default kind of model. A rise in default probability of ABC Company, for example, may lengthen the terms of payment, for example, without triggering default. These kinds of influences of the default probability of ABC Company can be modeled like small portfolios of digital default swaps that are triggered, not so much by the event of default, but by the level of the default probability reaching a certain range. The graph of the default probabilities of Countrywide and Royal Bank of Scotland, in fact, could be used to (with hindsight) do exactly this.

The final default intensity-related component of cash needs of the company is the term $C_2(\lambda)$ in the cash need function above. This function is undoubtedly a complex non-linear function of the default probability that reflects sources of funding like the commercial paper market. As we discuss in Chapter 37, issuers of commercial paper rated A3/P3 have dramatically less access to commercial paper than more highly rated issuers. As the Countrywide and RBS examples show, market participants don't need to wait for a ratings change to withdraw their interest in the commercial paper of a given issuer.

Similarly, a much larger universe of institutions can buy investment grade bonds (we prefer the term "low default probability bonds" than can buy non-investment grade ("high default probability") bonds. All of these factors affect the function $C_2(\lambda)$. Another factor comes into play as well. Even if the company holds relatively liquid assets, it may become obvious to other market participants that the company is in trouble if the default probability exceeds a certain level. The famous Long Term Capital Management incident in 1998 illustrates the result—in a market that is not perfectly liquid, other market participants will lower their bid prices for the assets held by the company because they know credit problems will force the company to take any price. We return to this theme in detail in Chapter 37. In summary, the cash flows generated by the $C_2(\lambda)$ term have a nature much like that of a 'range' floater in credit derivatives terms. For a given range of default intensity, cash flows will take on a particular value. In the Jarrow model, the straightforward nature of the default intensity function gives us a good chance of a closed form valuation for this component of draws on the line of credit. Chava and Jarrow [2003, 2008] use nested logistic regressions to model the probability of a credit crisis for ABC Company and, given that the liquidity crisis has occurred, the probability of a default in their valuation of lines of credit. The analysis parallels the analysis of "irrational exercise" of fixed income options in Chapter 28. Instead of modeling partial prepayments, however, we are modeling partial draw-downs of the line of credit. Multinomial logistic regression is the best practice approach to modeling events like these where there are n possible outcomes, not just two outcomes (i.e. no drawdown or 100% drawdown).

The final component of cash flow needs is the random term $Z(t)$. If we have properly included all of the macro factors driving cash flows in the other terms of the cash flow function, then we can make an argument that $Z(t)$ has mean zero and that diversification arguments with respect to the $Z(t)$ term's impact on valuation will prevail. This is an insight of Jarrow, Lando and Yu [2005].

The upshot of this micro-focused discussion of the determinants of cash needs on a line of credit is that a finely detailed analysis of cash needs will allow us to do a high quality valuation and simulation of lines of credit and exposure at default. Most of what we need to do has already been discussed in other chapters in this book. What little we need to add to do a complete job on lines of credit risk analysis will be discussed in detail in Chapters 36 and 37. We next turn to equity-linked instruments in our quest to value the Jarrow-Merton put option.