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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 19
Valuing credit risky bonds

In this section of the book, we turn to a very concrete set of tasks. Our ultimate objective is to calculate the value of the Jarrow-Merton put option on the net assets (value of assets less value of liabilities) of the firm, which Jarrow and Merton propose as the best measure of total risk. Total risk combines interest rate risk, market risk, liquidity risk, credit risk, foreign exchange risk, and so on. In order to value this option, we need to be able to value all of the instruments on the financial institution's balance sheet. We derive the value of the option on all assets less all liabilities by performing this analysis on each underlying instrument, one by one. There is no reason to "summarize" or "stratify" the transactions in order to do this calculation. Unless the instruments are literally identical (like a demand deposit or time deposit with identical coupon and maturity date), any summarization introduces an error whose magnitude is unknown without a comparable transaction level calculation. The cost of the hardware to achieve a target calculation time is a lower cost than the cost of human labor to oversee the data summarization and quality control on that process for almost every imaginable risk calculation. If you are hearing otherwise, someone is misinformed.

Now, in addition to valuation, we need to know how to simulate the value that each financial instrument may take on at multiple points in the future and in multiple scenarios. These scenarios may be deterministic (like the regulatory stress tests widely imposed in 2012) or stochastic, produced by a Monte Carlo simulation.

We go through this process in a systematic pattern. We start first with straight bonds, bonds with credit risk and bonds with no credit risk. In the next chapter, we extend this analysis to credit derivatives and the formerly trendy collateralized debt instruments. In Chapter 21, we begin to get to the "option" portion of this exercise by studying options on bonds. We follow with valuation techniques for a string of related financial instruments thereafter, bringing it all together in Chapters 36 to 41.

We turn now to the valuation of bonds with and without credit risk. We then turn to simulation of future values.

The Present Value Formula

In Chapter 4, we reviewed the basic case where the value of a financial instrument was the simple sum of the present value of each cash flow. The expression P(t) is the present value of a dollar received at time t from the perspective of time zero. The expression C(t) is the cash flow to be paid at time t.

$$Present\ value = \sum_{i=1}^{n} P(t_i)C(t_i)$$

Do we need to say any more than this about bond valuation? Surprising, the answer is yes.

Valuing Bonds with No Credit Risk

Fortunately, in the case of bonds with a default probability of zero, valuation is as simple as the formula above. In a no arbitrage economy with frictionless trading, the present value formula above applies. As we noted in Chapter 4, "present value" does not mean "price": it means "price plus accrued interest. Where do we get the values of the discount factors P(t)? We get them from the yield curve smoothing techniques discussed in Chapter 5.

Given these P values and the terms of the bond, its valuation is very simple. What about simulating future values? Chapter 6 through 9 on the Heath Jarrow and Morton approach showed us how this should be done.

Simulating the Future Values of Bonds with No Credit Risk

The Jarrow-Merton put option that is our ultimate valuation objective can be thought of as a European option, valued on only one date, or it can be thought of as an American option with many possible exercise dates. The European option analogy is a way of thinking about risk that is consistent with traditional value at risk concepts—risk matters on only one date, we can ignore intervening cash flows, and analyze the dickens out of the single date we choose as important. Common choices are 10 day horizons, 1 month horizons, and one year horizons. The longer the horizon, the more tenuous this way of thinking becomes. Most analysts, including the authors, believe that a multi-period approach to the Jarrow-Merton put option and risk management in general is essential. How to we calculate the variation in the value of a bond with no credit risk over time?
We use the Heath Jarrow and Morton approaches of Chapters 6 through 9 and the parameter fitting techniques of Chapter 14. In Chapter 3, we presented the statistical evidence that shows why it is necessary to simulate future yield curve movements using multiple risk factors, not a single risk factor like a parallel shift or a change in the short rate that drives the rest of the "single factor" risk free yield curve. The steps in simulating the risk free term structure can be summarized as follows, for N scenarios

covering M time steps and K risk factors:

1. Capture today's continuous zero coupon yield curve using a sophisticated yield curve smoothing technique like the "maximum smoothness forward rate technique." We use the yield curve inputs from Chapter 9 from now on.
2. Choose a term structure model and calculate its parameters. For the rest of this book, we will rely primarily on the 3 factor Heath Jarrow and Morton risk-free yield curve of Chapter 9.
3. Choose a time step to step forward in time, delta t, to arrive at time step 1. For expository purposes, we'll use the same 1 year time step that we employed in Chapter 9.
4. Calculate the random value for scenario 1 for the K risk factors at time step 1. In Chapter 9, the random factors were the change in the year one spot rate, the change in the 1 year forward rate maturing at time T=10, and a third factor that is "everything else." The volatilities of these factors are given in the volatility tables from Chapter 9, and their mean values differ at each forward point in time, conditional on the "state" or level of interest rates in that scenario. As in Chapter 9, we do a table "look up" to get the right values. Alternatively, we could have specified a mathematical function of the level of rates.
5. Re-smooth the yield curve at time step 1. We apply the yield curve smoothing techniques of Chapter 5 to smooth the yield curve between the values of the yields whose simulated values we know (the Heath Jarrow and Morton values at each annual time step) so that we can value cash flows which do not fall exactly on a time step.
6. Move to the next time step and repeat steps 4-5 until you have completed the simulation to M time steps, completing scenario j (and of course for the first scenario j=1)
7. Repeat steps 4-6 for a total of N scenarios.

At each point in time, in each scenario, we can calculate the value of the risk free bond using the formula above. The number of cash flows and the time before they arrive will of course change as time passes.

We now show how to value a "bullet" bond, an amortizing loan, and a floating rate loan using the Heath Jarrow and Morton 3 factor interest rate model from Chapter 9.

Current and Future Values of Fixed Income Instruments:

HJM Background and a Straight Bond Example

We will use the risk free yield curve and the bushy tree from the 3 factor Heath Jarrow and Morton model in Chapter 9 for the examples in this chapter. We create a busy tree for 4 periods and three risk factors, so there is the same number of nodes in the tree as in Chapter 9: 4 in period 1, 16, in period 2, and 64 in period 3, which we can use to evaluate cash flows in period 4. The zero coupon bond prices prevailing at time zero are given in Figure 1, taken directly from Chapter 9:

Figure 1

Key Assumptions		**Period of Maturity**			
		1`	**2**	**3**	**4**
Input Zero Coupon Bond Prices		0.9970057865	0.9841101497	0.9618922376	0.9308550992

The probability of each node in the tree is given in Figure 2:

Figure 2

	Probability of Each State				
	Current Time				
Row	0	1	2	3	4
1	100.0000%	12.5000%	1.5625%	0.1953%	0.1953%
2		12.5000%	1.5625%	0.1953%	0.1953%
3		25.0000%	3.1250%	0.3906%	0.3906%
4		50.0000%	6.2500%	0.7813%	0.7813%
5			1.5625%	0.1953%	0.1953%
6			1.5625%	0.1953%	0.1953%
7			3.1250%	0.3906%	0.3906%
8			6.2500%	0.7813%	0.7813%
9			3.1250%	0.3906%	0.3906%
10			3.1250%	0.3906%	0.3906%
11			6.2500%	0.7813%	0.7813%
12			12.5000%	1.5625%	1.5625%
13			6.2500%	0.7813%	0.7813%
14			6.2500%	0.7813%	0.7813%
15			12.5000%	1.5625%	1.5625%
16			25.0000%	3.1250%	3.1250%
17				0.1953%	0.1953%
18				0.1953%	0.1953%
19				0.3906%	0.3906%
20				0.7813%	0.7813%
21				0.1953%	0.1953%
22				0.1953%	0.1953%
23				0.3906%	0.3906%
24				0.7813%	0.7813%
25				0.3906%	0.3906%
26				0.3906%	0.3906%
27				0.7813%	0.7813%
28				1.5625%	1.5625%
29				0.7813%	0.7813%
30				0.7813%	0.7813%
31				1.5625%	1.5625%
32				3.1250%	3.1250%
33				0.3906%	0.3906%
34				0.3906%	0.3906%
35				0.7813%	0.7813%
36				1.5625%	1.5625%
37				0.3906%	0.3906%
38				0.3906%	0.3906%
39				0.7813%	0.7813%
40				1.5625%	1.5625%
41				0.7813%	0.7813%
42				0.7813%	0.7813%
43				1.5625%	1.5625%
44				3.1250%	3.1250%
45				1.5625%	1.5625%
46				1.5625%	1.5625%
47				3.1250%	3.1250%
48				6.2500%	6.2500%
49				0.7813%	0.7813%
50				0.7813%	0.7813%
51				1.5625%	1.5625%
52				3.1250%	3.1250%
53				0.7813%	0.7813%
54				0.7813%	0.7813%
55				1.5625%	1.5625%
56				3.1250%	3.1250%
57				1.5625%	1.5625%
58				1.5625%	1.5625%
59				3.1250%	3.1250%
60				6.2500%	6.2500%
61				3.1250%	3.1250%
62				3.1250%	3.1250%
63				6.2500%	6.2500%
64				12.5000%	12.5000%
Total	100.0000%	100.0000%	100.0000%	100.0000%	100.0000%

The discount factor for a $1 received at each state is given in Figure 3, as calculated in Chapter 9:

Figure 3

Discount Factors

	Current Time				
Row	0	1	2	3	4
1	1.0000	0.9970	0.9759	0.9326	0.8789
2		0.9970	0.9759	0.9326	0.8834
3		0.9970	0.9759	0.9326	0.8868
4		0.9970	0.9759	0.9326	0.8632
5			0.9902	0.9623	0.9333
6			0.9902	0.9623	0.9380
7			0.9902	0.9623	0.9431
8			0.9902	0.9623	0.9247
9			0.9862	0.9511	0.9039
10			0.9862	0.9511	0.9326
11			0.9862	0.9511	0.9218
12			0.9862	0.9511	0.9015
13			0.9836	0.9301	0.8823
14			0.9836	0.9301	0.8868
15			0.9836	0.9301	0.8901
16			0.9836	0.9301	0.8665
17				0.9684	0.9298
18				0.9684	0.9593
19				0.9684	0.9482
20				0.9684	0.9273
21				0.9843	0.9631
22				0.9843	0.9789
23				0.9843	0.9768
24				0.9843	0.9726
25				0.9822	0.9571
26				0.9822	0.9859
27				0.9822	0.9796
28				0.9822	0.9613
29				0.9780	0.9609
30				0.9780	0.9657
31				0.9780	0.9710
32				0.9780	0.9521
33				0.9676	0.9460
34				0.9676	0.9507
35				0.9676	0.9560
36				0.9676	0.9373
37				0.9724	0.9507
38				0.9724	0.9555
39				0.9724	0.9608
40				0.9724	0.9420
41				0.9778	0.9435
42				0.9778	0.9719
43				0.9778	0.9657
44				0.9778	0.9477
45				0.9587	0.9173
46				0.9587	0.9465
47				0.9587	0.9355
48				0.9587	0.9149
49				0.9624	0.9298
50				0.9624	0.9594
51				0.9624	0.9482
52				0.9624	0.9273
53				0.9672	0.9428
54				0.9672	0.9475
55				0.9672	0.9528
56				0.9672	0.9341
57				0.9725	0.9480
58				0.9725	0.9528
59				0.9725	0.9580
60				0.9725	0.9393
61				0.9535	0.9124
62				0.9535	0.9323
63				0.9535	0.9282
64				0.9535	0.9079

These discount factors are consistent with the term structure of zero coupon bond prices derived in Chapter 9. The probability weighted discount factors for each state are given in Figure 4. For each cash flow pattern we want to value, we simply multiply the cash flow at each period in each state times these probability weighted discount factors to get the valuation at time zero:

Figure 4

Probability Weighted Discount Factors

	Current Time				
Row	**0**	**1**	**2**	**3**	**4**
1	1.0000	0.1246	0.0152	0.0018	0.0017
2		0.1246	0.0152	0.0018	0.0017
3		0.2493	0.0305	0.0036	0.0035
4		0.4985	0.0610	0.0073	0.0067
5			0.0155	0.0019	0.0018
6			0.0155	0.0019	0.0018
7			0.0309	0.0038	0.0037
8			0.0619	0.0075	0.0072
9			0.0308	0.0037	0.0035
10			0.0308	0.0037	0.0036
11			0.0616	0.0074	0.0072
12			0.1233	0.0149	0.0141
13			0.0615	0.0073	0.0069
14			0.0615	0.0073	0.0069
15			0.1230	0.0145	0.0139
16			0.2459	0.0291	0.0271
17				0.0019	0.0018
18				0.0019	0.0019
19				0.0038	0.0037
20				0.0076	0.0072
21				0.0019	0.0019
22				0.0019	0.0019
23				0.0038	0.0038
24				0.0077	0.0076
25				0.0038	0.0037
26				0.0038	0.0039
27				0.0077	0.0077
28				0.0153	0.0150
29				0.0076	0.0075
30				0.0076	0.0075
31				0.0153	0.0152
32				0.0306	0.0298
33				0.0038	0.0037
34				0.0038	0.0037
35				0.0076	0.0075
36				0.0151	0.0146
37				0.0038	0.0037
38				0.0038	0.0037
39				0.0076	0.0075
40				0.0152	0.0147
41				0.0076	0.0074
42				0.0076	0.0076
43				0.0153	0.0151
44				0.0306	0.0296
45				0.0150	0.0143
46				0.0150	0.0148
47				0.0300	0.0292
48				0.0599	0.0572
49				0.0075	0.0073
50				0.0075	0.0075
51				0.0150	0.0148
52				0.0301	0.0290
53				0.0076	0.0074
54				0.0076	0.0074
55				0.0151	0.0149
56				0.0302	0.0292
57				0.0152	0.0148
58				0.0152	0.0149
59				0.0304	0.0299
60				0.0608	0.0587
61				0.0298	0.0285
62				0.0298	0.0291
63				0.0596	0.0580
64				0.1192	0.1135

Valuation of a Straight Bond with a "Bullet" Principal Payment at Maturity

We saw in Chapter 9 that a bond paying an annual coupon of $3 and principal of $100 at time T=4 could be valued directly using the present value formula above. Alternatively, we can do a Monte Carlo simulation or bushy tree using the Heath Jarrow and Morton no-arbitrage framework and get the identical time 0 value. Figure 5 shows that the valuation can also be calculated using the bushy tree by inserting the correct cash flow in the cash flow table and multiplying each cash flow by the probability weighted discount factors in Figure 4. When we do this, we get the same value as in Chapter 9: 104.7071.

Figure 5

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.00	3.00	3.00	3.00	103.00
2		3.00	3.00	3.00	103.00
3		3.00	3.00	3.00	103.00
4		3.00	3.00	3.00	103.00
5			3.00	3.00	103.00
6			3.00	3.00	103.00
7			3.00	3.00	103.00
8			3.00	3.00	103.00
9			3.00	3.00	103.00
10			3.00	3.00	103.00
11			3.00	3.00	103.00
12			3.00	3.00	103.00
13			3.00	3.00	103.00
14			3.00	3.00	103.00
15			3.00	3.00	103.00
16			3.00	3.00	103.00
17				3.00	103.00
18				3.00	103.00
19				3.00	103.00
20				3.00	103.00
21				3.00	103.00
22				3.00	103.00
23				3.00	103.00
24				3.00	103.00
25				3.00	103.00
26				3.00	103.00
27				3.00	103.00
28				3.00	103.00
29				3.00	103.00
30				3.00	103.00
31				3.00	103.00
32				3.00	103.00
33				3.00	103.00
34				3.00	103.00
35				3.00	103.00
36				3.00	103.00
37				3.00	103.00
38				3.00	103.00
39				3.00	103.00
40				3.00	103.00
41				3.00	103.00
42				3.00	103.00
43				3.00	103.00
44				3.00	103.00
45				3.00	103.00
46				3.00	103.00
47				3.00	103.00
48				3.00	103.00
49				3.00	103.00
50				3.00	103.00
51				3.00	103.00
52				3.00	103.00
53				3.00	103.00
54				3.00	103.00
55				3.00	103.00
56				3.00	103.00
57				3.00	103.00
58				3.00	103.00
59				3.00	103.00
60				3.00	103.00
61				3.00	103.00
62				3.00	103.00
63				3.00	103.00
64				3.00	103.00
Risk Neutral Value =				104.7070997370	

We can calculate the future values of this bond, at each node of the bushy tree in each state, by using the term structure of zero coupon bond prices that prevails on the bushy tree, shown in Chapter 9. We now turn to an amortizing structure.

Valuing an Amortizing Loan

An amortizing bond or loan pays the same dollar amount in each period, say A. If there is no default risk or prepayment risk, a loan paying this constant amount A per period for n periods can be valued directly using a special case of the present value formula above:

$$Present\ value = A\sum_{i=1}^{n} P(t_i)$$

Keeping with our example from Chapter 9, we can use the zero coupon bond prices to value an amortizing loan that pays \$50 at times 1, 2, 3, or 4. Alternatively, we can get exactly the same valuation by inserting \$50 the cash flow table for every state at all four times 1, 2, 3 and 4, producing the valuation of \$193.6932 as in Figure 6:

Figure 6

Maturity of Cash Flow Received

	Current Time				
Row	0	1	2	3	4
1	0.00	50.00	50.00	50.00	50.00
2		50.00	50.00	50.00	50.00
3		50.00	50.00	50.00	50.00
4		50.00	50.00	50.00	50.00
5			50.00	50.00	50.00
6			50.00	50.00	50.00
7			50.00	50.00	50.00
8			50.00	50.00	50.00
9			50.00	50.00	50.00
10			50.00	50.00	50.00
11			50.00	50.00	50.00
12			50.00	50.00	50.00
13			50.00	50.00	50.00
14			50.00	50.00	50.00
15			50.00	50.00	50.00
16			50.00	50.00	50.00
17				50.00	50.00
18				50.00	50.00
19				50.00	50.00
20				50.00	50.00
21				50.00	50.00
22				50.00	50.00
23				50.00	50.00
24				50.00	50.00
25				50.00	50.00
26				50.00	50.00
27				50.00	50.00
28				50.00	50.00
29				50.00	50.00
30				50.00	50.00
31				50.00	50.00
32				50.00	50.00
33				50.00	50.00
34				50.00	50.00
35				50.00	50.00
36				50.00	50.00
37				50.00	50.00
38				50.00	50.00
39				50.00	50.00
40				50.00	50.00
41				50.00	50.00
42				50.00	50.00
43				50.00	50.00
44				50.00	50.00
45				50.00	50.00
46				50.00	50.00
47				50.00	50.00
48				50.00	50.00
49				50.00	50.00
50				50.00	50.00
51				50.00	50.00
52				50.00	50.00
53				50.00	50.00
54				50.00	50.00
55				50.00	50.00
56				50.00	50.00
57				50.00	50.00
58				50.00	50.00
59				50.00	50.00
60				50.00	50.00
61				50.00	50.00
62				50.00	50.00
63				50.00	50.00
64				50.00	50.00
Risk Neutral Value =				193.6931636452	

We now turn to a floating rate structure.

Valuing Risk Free Floating Rate Loans

In this section, we show how to value a floating rate loan whose payments are tied to the 1 period spot rate of interest that is consistent with the bushy tree from Chapter 9. The spot rate of interest at each point in time from time 0 to time 4 is given in Figure 7:

Figure 7

State	Row	0	1	2	3
		Spot Rate Process			
S-1, S-1, S-1	1	0.3003%	2.1653%	4.6388%	6.1106%
S-1, S-1, S-2	2		0.6911%	1.4142%	5.5667%
S-1, S-1, S-3	3		1.0972%	2.6089%	5.1710%
S-1, S-1, S-4	4		1.3611%	4.9179%	8.0375%
S-1, S-2, S-1	5			2.2496%	3.1041%
S-1, S-2, S-2	6			0.5984%	2.5913%
S-1, S-2, S-3	7			0.8121%	2.0291%
S-1, S-2, S-4	8			1.2459%	4.0628%
S-1, S-3, S-1	9			1.9217%	5.2210%
S-1, S-3, S-2	10			1.4149%	1.9784%
S-1, S-3, S-3	11			0.8591%	3.1798%
S-1, S-3, S-4	12			2.8695%	5.5017%
S-1, S-4, S-1	13			2.2088%	5.4252%
S-1, S-4, S-2	14			1.7005%	4.8848%
S-1, S-4, S-3	15			1.1432%	4.4917%
S-1, S-4, S-4	16			3.1593%	7.3396%
S-2, S-1, S-1	17				4.1517%
S-2, S-1, S-2	18				0.9421%
S-2, S-1, S-3	19				2.1313%
S-2, S-1, S-4	20				4.4295%
S-2, S-2, S-1	21				2.2036%
S-2, S-2, S-2	22				0.5531%
S-2, S-2, S-3	23				0.7667%
S-2, S-2, S-4	24				1.2004%
S-2, S-3, S-1	25				2.6231%
S-2, S-3, S-2	26				-0.3734%
S-2, S-3, S-3	27				0.2593%
S-2, S-3, S-4	28				2.1679%
S-2, S-4, S-1	29				1.7764%
S-2, S-4, S-2	30				1.2703%
S-2, S-4, S-3	31				0.7153%
S-2, S-4, S-4	32				2.7228%
S-3, S-1, S-1	33				2.2804%
S-3, S-1, S-2	34				1.7718%
S-3, S-1, S-3	35				1.2141%
S-3, S-1, S-4	36				3.2315%
S-3, S-2, S-1	37				2.2804%
S-3, S-2, S-2	38				1.7718%
S-3, S-2, S-3	39				1.2141%
S-3, S-2, S-4	40				3.2315%
S-3, S-3, S-1	41				3.6354%
S-3, S-3, S-2	42				0.6093%
S-3, S-3, S-3	43				1.2483%
S-3, S-3, S-4	44				3.1757%
S-3, S-4, S-1	45				4.5070%
S-3, S-4, S-2	46				1.2864%
S-3, S-4, S-3	47				2.4797%
S-3, S-4, S-4	48				4.7858%
S-4, S-1, S-1	49				3.5013%
S-4, S-1, S-2	50				0.3117%
S-4, S-1, S-3	51				1.4934%
S-4, S-1, S-4	52				3.7773%
S-4, S-2, S-1	53				2.5831%
S-4, S-2, S-2	54				2.0730%
S-4, S-2, S-3	55				1.5136%
S-4, S-2, S-4	56				3.5370%
S-4, S-3, S-1	57				2.5831%
S-4, S-3, S-2	58				2.0730%
S-4, S-3, S-3	59				1.5136%
S-4, S-3, S-4	60				3.5370%
S-4, S-4, S-1	61				4.5061%
S-4, S-4, S-2	62				2.2686%
S-4, S-4, S-3	63				2.7252%
S-4, S-4, S-4	64				5.0230%

Let's assume we want to value a loan with $1000 in principal that matures at time T=4. Let us also assume that the interest rate on the loan equals the spot rate of interest times principal plus $50, paid in arrears. The reader can confirm that the cash flow table is consistent with Figure 8 and that the present value of the loan is 1,193.6932:

Figure 8

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	53.0032	71.6527	96.3884	1,111.1062
2		53.0032	71.6527	96.3884	1,105.6672
3		53.0032	71.6527	96.3884	1,101.7103
4		53.0032	71.6527	96.3884	1,130.3747
5			56.9105	64.1419	1,081.0407
6			56.9105	64.1419	1,075.9132
7			56.9105	64.1419	1,070.2913
8			56.9105	64.1419	1,090.6283
9			60.9723	76.0890	1,102.2100
10			60.9723	76.0890	1,069.7841
11			60.9723	76.0890	1,081.7977
12			60.9723	76.0890	1,105.0166
13			63.6107	99.1794	1,104.2520
14			63.6107	99.1794	1,098.8481
15			63.6107	99.1794	1,094.9168
16			63.6107	99.1794	1,123.3960
17				72.4959	1,091.5175
18				72.4959	1,059.4211
19				72.4959	1,071.3126
20				72.4959	1,094.2955
21				55.9835	1,072.0359
22				55.9835	1,055.5310
23				55.9835	1,057.6673
24				55.9835	1,062.0037
25				58.1208	1,076.2311
26				58.1208	1,046.2660
27				58.1208	1,052.5932
28				58.1208	1,071.6787
29				62.4592	1,067.7642
30				62.4592	1,062.7027
31				62.4592	1,057.1531
32				62.4592	1,077.2283
33				69.2173	1,072.8044
34				69.2173	1,067.7178
35				69.2173	1,062.1408
36				69.2173	1,082.3153
37				64.1486	1,072.8044
38				64.1486	1,067.7178
39				64.1486	1,062.1408
40				64.1486	1,082.3153
41				58.5911	1,086.3541
42				58.5911	1,056.0934
43				58.5911	1,062.4830
44				58.5911	1,081.7567
45				78.6949	1,095.0704
46				78.6949	1,062.8645
47				78.6949	1,074.7966
48				78.6949	1,097.8579
49				72.0883	1,085.0128
50				72.0883	1,053.1169
51				72.0883	1,064.9341
52				72.0883	1,087.7735
53				67.0053	1,075.8313
54				67.0053	1,070.7297
55				67.0053	1,065.1361
56				67.0053	1,085.3704
57				61.4322	1,075.8313
58				61.4322	1,070.7297
59				61.4322	1,065.1361
60				61.4322	1,085.3704
61				81.5926	1,095.0608
62				81.5926	1,072.6860
63				81.5926	1,077.2519
64				81.5926	1,100.2298
Risk Neutral Value =					**1,193.6932**

Note that the value of $50 received at times 1, 2, 3, and 4 was valued at $193.6932 in Figure 6, so we expect that the present value of a risk free loan that pays the 1 period spot rate plus Note that the value of $50 received at times 1, 2, 3, and 4 was valued at $193.6932 in Figure 6, so we expect that the present value of a risk free loan that pays the 1 period spot rate plus zero premium is exactly $1000.0000. We insert those cash flows into the case flow table, multiply by the probability-weighted discount factors, and indeed get exactly $1,000. This should come as no surprise because on each branch of the bushy tree, we are discounting $1000 (1 + spot rate) by (1 + spot rate), consistent with no arbitrage valuation in Chapter 9.

Figure 9

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	3.0032	21.6527	46.3884	1,061.1062
2		3.0032	21.6527	46.3884	1,055.6672
3		3.0032	21.6527	46.3884	1,051.7103
4		3.0032	21.6527	46.3884	1,080.3747
5			6.9105	14.1419	1,031.0407
6			6.9105	14.1419	1,025.9132
7			6.9105	14.1419	1,020.2913
8			6.9105	14.1419	1,040.6283
9			10.9723	26.0890	1,052.2100
10			10.9723	26.0890	1,019.7841
11			10.9723	26.0890	1,031.7977
12			10.9723	26.0890	1,055.0166
13			13.6107	49.1794	1,054.2520
14			13.6107	49.1794	1,048.8481
15			13.6107	49.1794	1,044.9168
16			13.6107	49.1794	1,073.3960
17				22.4959	1,041.5175
18				22.4959	1,009.4211
19				22.4959	1,021.3126
20				22.4959	1,044.2955
21				5.9835	1,022.0359
22				5.9835	1,005.5310
23				5.9835	1,007.6673
24				5.9835	1,012.0037
25				8.1208	1,026.2311
26				8.1208	996.2660
27				8.1208	1,002.5932
28				8.1208	1,021.6787
29				12.4592	1,017.7642
30				12.4592	1,012.7027
31				12.4592	1,007.1531
32				12.4592	1,027.2283
33				19.2173	1,022.8044
34				19.2173	1,017.7178
35				19.2173	1,012.1408
36				19.2173	1,032.3153
37				14.1486	1,022.8044
38				14.1486	1,017.7178
39				14.1486	1,012.1408
40				14.1486	1,032.3153
41				8.5911	1,036.3541
42				8.5911	1,006.0934
43				8.5911	1,012.4830
44				8.5911	1,031.7567
45				28.6949	1,045.0704
46				28.6949	1,012.8645
47				28.6949	1,024.7966
48				28.6949	1,047.8579
49				22.0883	1,035.0128
50				22.0883	1,003.1169
51				22.0883	1,014.9341
52				22.0883	1,037.7735
53				17.0053	1,025.8313
54				17.0053	1,020.7297
55				17.0053	1,015.1361
56				17.0053	1,035.3704
57				11.4322	1,025.8313
58				11.4322	1,020.7297
59				11.4322	1,015.1361
60				11.4322	1,035.3704
61				31.5926	1,045.0608
62				31.5926	1,022.6860
63				31.5926	1,027.2519
64				31.5926	1,050.2298
Risk Neutral Value =					**1,000.0000**

This result is a practical application of this formula from Chapter 4 in the context of a 3 factor Heath Jarrow and Morton bushy tree:

$$Value\ of\ Floating\ Rate\ Bond$$
$$= P(t_1)\left[\left(1+\frac{index}{m}\right)principal\right] + spread\left[\sum_{i=1}^{n} P(t_i)\right]$$

This seems simple enough—what about the credit risky case?

Valuing Bonds with Credit Risk

Is the process of valuing bonds with credit risk any different from valuing bonds with credit risk? As we saw in Chapter 17, we need to do more work to fit credit risky bonds to observable market data and we need to be able to separate out the credit loss-related portion of the credit spread from the liquidity portion as we discussed then. What if we do that? Having done that and having calculated the value of a risky zero coupon bond with maturity t, P(t), can we use the formulas for value given above?

$$Present\ value = \sum_{i=1}^{n} P(t_i)C(t_i)$$

Robert Jarrow [2004] provides the answer. The necessary and sufficient conditions for using the present value formula directly, even when the issuer can default, can be summarized as follows:

- The present value formula can be used correctly if the recovery rate δ is constant and the same for all bonds with the same seniority, the assumption used by Jarrow and Turnbull [1995]
- The present value formula can be used if the recovery rate is random and depends only on time and seniority.
- If the default process is a Cox process (like the reduced form models in Chapter 16), then a sufficient condition to use the present value formula is for recovery to be a fraction of the bond's price an instant before default

In reduced form models, under these assumptions then, the present value formula can be used in a straightforward way. In the Merton model and its variations, things are not so easy, and that is why it has been relegated to our review of legacy credit models, models past their prime, in Chapter 18. Use of the Merton model for enterprise risk management is not accurate or practical and will not be covered in the rest of this book except in passing when reviewing the problems of using the copula method of portfolio simulation for valuation of collateralized debt obligations (see Chapter 20).

Under the Jarrow [2004] conditions, the valuation of the structures above (a "bullet" bond, an amortizing loan, and a floating rate loan) is exactly the same for credit risky bonds as it is for risk-free bonds. Only the values in the bushy tree would be different because the zero coupon bond inputs are different for a credit risky borrower.
What happens when the Jarrow conditions are not met? We deal with that special case in our chapter on American fixed income options, Chapter 27.

Simulating the Future Values of Bonds with Credit Risk

Simulating the value of risky bonds involves a straightforward number of steps using the reduced form models. For the reduced form model, the steps are as follows:

1. Simulate the risk-free term structure as given above.
2. Choose a formula and the risk-drivers for the liquidity component of credit spread as discussed in Chapter 17
3. Choose a formula and the risk-drivers for the default intensity process in the Jarrow model as discussed in Chapter 16
4. Simulate the random values of the drivers of liquidity risk and the default intensity for K risk factors and M time periods over N scenarios, consistent with the risk free term structure
5. Calculate the default intensity and the liquidity component at each of the M time steps and N scenarios. Note that these will be changing randomly over time because interest rates and other factors are each of them. This is essential to capturing cyclicality in bond prices and defaults
6. Apply the Jarrow model as we have done in Chapters 16 and 17 to get the zero coupon bond prices for each maturity relevant to the bond
7. Calculate the bond's value at each of the M time steps and N scenarios, conditional on the company not defaulting in that period or in a prior date.

This is only slightly more complicated than the risk-free bond simulation. It relies on the identical interest rate scenarios and simulation process. Note that if the simulation is done for many companies with common risk factors driving default, we will see correlated default behavior that we have discussed through out this book.

Valuing the Jarrow –Merton Put Option

If we have a portfolio of straight bonds, either fixed rate or floating rate, on the asset side of our financial institution, we are almost ready to value the Jarrow-Merton put option on the assets of the firm. We will defer that until we reach Chapter 21, but the goal is near at hand. We move first to the discussion of credit default swaps and collateralized debt obligations because they help set a more general portfolio context for the problem.