Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 4: Risk Management Applications, Instrument by Instrument

Chapter 23

European Options on Forward and Futures Contracts

This chapter continues our analysis of standard instruments found on the balance sheet of large financial institutions as we continue with our objective of establishing a unified measure of interest rate risk, market risk, liquidity risk and credit risk. In order to value the Jarrow-Merton put option proposed in Chapter 1 as the best measure of total risk, we need a methodology for valuation and for simulation of cash flows and values at many future dates. As in Chapter 22, we need this capability for European options on forward and futures contracts for three reasons:

- To understand correct hedging amounts from a shareholder value-added perspective, as discussed in the early chapters of this book;
- To meet the requirements of constantly evolving accounting standards for hedging
- To meet the requirements of regulatory capital calculations like Basel II, Basel III and Solvency II.

This chapter combines the valuation formulas for futures and forwards from Chapter 22 and the options approach from Chapter 21. As in Chapters 21 and 21, we use 3 factor Heath Jarrow and Morton framework to value options on forwards and futures.

We remind the reader that Chapter 3 illustrated the need for an interest rate model with 5 to 10 driving risk factors. We use the 3 factor HJM model from Chapter 9 for expositional purposes. In the first edition of this book, we showed in detail how the 1 factor Vasicek model can be used to derive closed form valuation formulas for this important class of instruments. While this approach provides many useful insights, we don't believe that a one factor model is sufficiently accurate to emphasize in this volume. Similarly, the $450 million settlement announced by Barclays PLC on June 27, 2012 for manipulation of the Libor market calls into question the pervasive use of the "Libor market model" for money market instruments. A more general approach like the HJM approach provides the most reliable and accurate framework for analysis.

Valuing Options on Forwards and Futures: Notations and Useful Formulas

Valuing options on forward and futures contracts can be done in using one of two methods. First, we can employ the HJM framework using a bushy tree or Monte Carlo simulation, properly parameterized, to obtain accurate values and hedges using a term structure model with a realistic number of risk factors (5 to 10). We use the Chapter 9 HJM 3 factor example for exposition purposes in this chapter. A second approach, which the authors admire but now judge insufficiently accurate, is to take a simple term structure model with 1 or 2 factors and derive analytical valuation and hedging formulas. For example, we can take advantage of the Jamshidian general solution for security pricing under the Vasicek model from Chapter 21 to obtain analytical solutions.

Market participants have historically preferred analytical solutions over numerical solutions because of a desire for total control over the calculation on the part of the trader/analyst. In the modern era, with 64-bit operating systems and powerful enterprise risk systems, this relentless urge by Felix Salmon's "F9 Model Monkeys" is not in the best interest of most financial services firms. For that reason we emphasize the HJM numerical solutions in this chapter.

Figure 1 summarizes the current list of interest rate futures and options contracts on interest rates available at www.cmegroup.com:

Figure 1

Interest Rates Futures & Options Products - CME Group

Block Trades

STIR (CME)

Eurodollar	FUT | OPT
Euribor	FUT
Mid-Curve Options	OPT
1-month Eurodollar	FUT | OPT
Euroyen TIBOR	FUT | OPT
3-Month OIS Futures	FUT | OPT
Eurodollar Calendar Spread Options	OPT

STIR (CBOT)

30-Day Federal Funds	FUT | OPT

Cleared OTC

Interest Rate Swaps

Sovereign Yield Spreads (CME)

US-UK 10-Year Sovys Futures	FUT
US-DE 10-Year Sovys Futures	FUT
US-FR 10-Year Sovys Futures	FUT
US-IT 10-Year Sovys Futures	FUT
US-Nd 10-Year Sovys Futures	FUT
UK-DE 10-Year Sovys Futures	FUT
UK-FR 10-Year Sovys Futures	FUT
UK-IT 10-Year Sovys Futures	FUT
UK-ND 10-Year Sovys Futures	FUT
DE-FR 10-Year Sovys Futures	FUT
DE-IT 10-Year Sovys Futures	FUT
DE-ND 10-Year Sovys Futures	FUT

U.S. Treasury Futures and Options (CBOT)

Ultra T-Bond	FUT | OPT
U.S. Treasury Bond	FUT | OPT
10-Year U.S. Treasury Note	FUT | OPT
5-Year U.S. Treasury Note	FUT | OPT
3-Year U.S. Treasury Note	FUT
2-Year U.S. Treasury Note	FUT | OPT
OTR 2-Year U.S. Treasury Note	FUT
OTR 5-Year U.S. Treasury Note	FUT
OTR 10-Year U.S. Treasury Note	FUT

U.S. Treasury Futures and Options (CME)

13-week T-bill	FUT

Intercommodity Spreads (CBOT)

Treasury and Swap Spreads	FUT

Swap Futures and Options (CBOT)

5-Year Interest Rate Swap	FUT | OPT
7-Year Interest Rate Swap	FUT | OPT
10-Year Interest Rate Swap	FUT | OPT
30-Year Interest Rate Swap	FUT | OPT

Interest Rate Indexes (CME)

Barclays U.S. Aggregate Bond Index Futures	FUT
Eurozone HICP Futures	FUT

Because of the wide array of interest rate options currently available, this chapter will emphasize options on stylized types of instruments. Space does not permit a detailed analysis of each contract, and, because contract specifications are rapidly evolving, such a product-specific focus would soon be rendered obsolete. We start with options on forward contracts on zero coupon bonds, as in Chapter 22.

European Options on Forward Contracts on Zero-coupon Bonds

In this section, we return to our example from Chapter 22 of a three year forward contract on a zero coupon bond maturing in year 4. The forward contract had an exercise price of 0.97. The cash flows on such a contract, using the HJM 3 factor model of Chapter 9, were shown in Figures 2 and 3 of Chapter 22. We reproduce the cash flow table for the forward contract as Figure 2, along with the forward contract's value of -0.0022:

Figure 2

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.0000	0.0000	-0.0276	0.0000
2		0.0000	0.0000	-0.0227	0.0000
3		0.0000	0.0000	-0.0192	0.0000
4		0.0000	0.0000	-0.0444	0.0000
5			0.0000	-0.0001	0.0000
6			0.0000	0.0047	0.0000
7			0.0000	0.0101	0.0000
8			0.0000	-0.0090	0.0000
9			0.0000	-0.0196	0.0000
10			0.0000	0.0106	0.0000
11			0.0000	-0.0008	0.0000
12			0.0000	-0.0221	0.0000
13			0.0000	-0.0215	0.0000
14			0.0000	-0.0166	0.0000
15			0.0000	-0.0130	0.0000
16			0.0000	-0.0384	0.0000
17				-0.0099	0.0000
18				0.0207	0.0000
19				0.0091	0.0000
20				-0.0124	0.0000
21				0.0084	0.0000
22				0.0245	0.0000
23				0.0224	0.0000
24				0.0181	0.0000
25				0.0044	0.0000
26				0.0337	0.0000
27				0.0274	0.0000
28				0.0088	0.0000
29				0.0125	0.0000
30				0.0175	0.0000
31				0.0229	0.0000
32				0.0035	0.0000
33				0.0077	0.0000
34				0.0126	0.0000
35				0.0180	0.0000
36				-0.0013	0.0000
37				0.0077	0.0000
38				0.0126	0.0000
39				0.0180	0.0000
40				-0.0013	0.0000
41				-0.0051	0.0000
42				0.0239	0.0000
43				0.0177	0.0000
44				-0.0008	0.0000
45				-0.0131	0.0000
46				0.0173	0.0000
47				0.0058	0.0000
48				-0.0157	0.0000
49				-0.0038	0.0000
50				0.0269	0.0000
51				0.0153	0.0000
52				-0.0064	0.0000
53				0.0048	0.0000
54				0.0097	0.0000
55				0.0151	0.0000
56				-0.0042	0.0000
57				0.0048	0.0000
58				0.0097	0.0000
59				0.0151	0.0000
60				-0.0042	0.0000
61				-0.0131	0.0000
62				0.0078	0.0000
63				0.0035	0.0000
64				-0.0178	0.0000
Risk Neutral Value =					-0.0022

Now consider the simplest possible call option on this forward contract. Let the option be a European call option with an exercise period of three years, exactly the maturity of the forward contract. Let the exercise price of the option on the forward contract be zero. In this case, since the forward contract will converge to the zero coupon bond price minus 0.97 at maturity, the call option will be exercised only when there are gains at the maturity of the futures contract. To value the call option, we modify the cash flow table above so that it registers only values equal to Maximum (0, cash flow). When we make this modification, we see the cash flows for the three year European call option on this forward contract in Figure 3:

Figure 3

Gain or Loss on 3 Year Forward Contract on Zero Coupon Bond Maturing in Year 4					
State Name	State Number	Zero Coupon	Forward Price	Forward Cash Flow	Call option Cash Flow
Time 3 S-1, S-1, S-1	21	0.942413	0.97	-0.0276	0.0000
Time 3 S-1, S-1, S-2	22	0.947268	0.97	-0.0227	0.0000
Time 3 S-1, S-1, S-3	23	0.950832	0.97	-0.0192	0.0000
Time 3 S-1, S-1, S-4	24	0.925605	0.97	-0.0444	0.0000
Time 3 S-1, S-2, S-1	25	0.969894	0.97	-0.0001	0.0000
Time 3 S-1, S-2, S-2	26	0.974741	0.97	0.0047	0.0047
Time 3 S-1, S-2, S-3	27	0.980112	0.97	0.0101	0.0101
Time 3 S-1, S-2, S-4	28	0.960958	0.97	-0.0090	0.0000
Time 3 S-1, S-3, S-1	29	0.950381	0.97	-0.0196	0.0000
Time 3 S-1, S-3, S-2	30	0.9806	0.97	0.0106	0.0106
Time 3 S-1, S-3, S-3	31	0.969182	0.97	-0.0008	0.0000
Time 3 S-1, S-3, S-4	32	0.947852	0.97	-0.0221	0.0000
Time 3 S-1, S-4, S-1	33	0.94854	0.97	-0.0215	0.0000
Time 3 S-1, S-4, S-2	34	0.953427	0.97	-0.0166	0.0000
Time 3 S-1, S-4, S-3	35	0.957014	0.97	-0.0130	0.0000
Time 3 S-1, S-4, S-4	36	0.931623	0.97	-0.0384	0.0000
Time 3 S-2, S-1, S-1	37	0.960138	0.97	-0.0099	0.0000
Time 3 S-2, S-1, S-2	38	0.990667	0.97	0.0207	0.0207
Time 3 S-2, S-1, S-3	39	0.979132	0.97	0.0091	0.0091
Time 3 S-2, S-1, S-4	40	0.957583	0.97	-0.0124	0.0000
Time 3 S-2, S-2, S-1	41	0.978439	0.97	0.0084	0.0084
Time 3 S-2, S-2, S-2	42	0.994499	0.97	0.0245	0.0245
Time 3 S-2, S-2, S-3	43	0.992391	0.97	0.0224	0.0224
Time 3 S-2, S-2, S-4	44	0.988139	0.97	0.0181	0.0181
Time 3 S-2, S-3, S-1	45	0.974439	0.97	0.0044	0.0044
Time 3 S-2, S-3, S-2	46	1.003748	0.97	0.0337	0.0337
Time 3 S-2, S-3, S-3	47	0.997413	0.97	0.0274	0.0274
Time 3 S-2, S-3, S-4	48	0.978781	0.97	0.0088	0.0088
Time 3 S-2, S-4, S-1	49	0.982546	0.97	0.0125	0.0125
Time 3 S-2, S-4, S-2	50	0.987457	0.97	0.0175	0.0175
Time 3 S-2, S-4, S-3	51	0.992898	0.97	0.0229	0.0229
Time 3 S-2, S-4, S-4	52	0.973493	0.97	0.0035	0.0035
Time 3 S-3, S-1, S-1	53	0.977704	0.97	0.0077	0.0077
Time 3 S-3, S-1, S-2	54	0.982591	0.97	0.0126	0.0126
Time 3 S-3, S-1, S-3	55	0.988005	0.97	0.0180	0.0180
Time 3 S-3, S-1, S-4	56	0.968696	0.97	-0.0013	0.0000
Time 3 S-3, S-2, S-1	57	0.977704	0.97	0.0077	0.0077
Time 3 S-3, S-2, S-2	58	0.982591	0.97	0.0126	0.0126
Time 3 S-3, S-2, S-3	59	0.988005	0.97	0.0180	0.0180
Time 3 S-3, S-2, S-4	60	0.968696	0.97	-0.0013	0.0000
Time 3 S-3, S-3, S-1	61	0.964921	0.97	-0.0051	0.0000
Time 3 S-3, S-3, S-2	62	0.993944	0.97	0.0239	0.0239
Time 3 S-3, S-3, S-3	63	0.987671	0.97	0.0177	0.0177
Time 3 S-3, S-3, S-4	64	0.969221	0.97	-0.0008	0.0000
Time 3 S-3, S-4, S-1	65	0.956873	0.97	-0.0131	0.0000
Time 3 S-3, S-4, S-2	66	0.987299	0.97	0.0173	0.0173
Time 3 S-3, S-4, S-3	67	0.975803	0.97	0.0058	0.0058
Time 3 S-3, S-4, S-4	68	0.954328	0.97	-0.0157	0.0000
Time 3 S-4, S-1, S-1	69	0.966172	0.97	-0.0038	0.0000
Time 3 S-4, S-1, S-2	70	0.996893	0.97	0.0269	0.0269
Time 3 S-4, S-1, S-3	71	0.985286	0.97	0.0153	0.0153
Time 3 S-4, S-1, S-4	72	0.963601	0.97	-0.0064	0.0000
Time 3 S-4, S-2, S-1	73	0.974819	0.97	0.0048	0.0048
Time 3 S-4, S-2, S-2	74	0.979691	0.97	0.0097	0.0097
Time 3 S-4, S-2, S-3	75	0.98509	0.97	0.0151	0.0151
Time 3 S-4, S-2, S-4	76	0.965838	0.97	-0.0042	0.0000
Time 3 S-4, S-3, S-1	77	0.974819	0.97	0.0048	0.0048
Time 3 S-4, S-3, S-2	78	0.979691	0.97	0.0097	0.0097
Time 3 S-4, S-3, S-3	79	0.98509	0.97	0.0151	0.0151
Time 3 S-4, S-3, S-4	80	0.965838	0.97	-0.0042	0.0000
Time 3 S-4, S-4, S-1	81	0.956882	0.97	-0.0131	0.0000
Time 3 S-4, S-4, S-2	82	0.977817	0.97	0.0078	0.0078
Time 3 S-4, S-4, S-3	83	0.973471	0.97	0.0035	0.0035
Time 3 S-4, S-4, S-4	84	0.952173	0.97	-0.0178	0.0000

We drop these cash flows into the cash flow table and multiply times the corresponding probability-weighted discount factors from Chapter 9. When we do this, we see that the value of the 3 year call option on this forward contract is 0.0048 in Figure 4:

Figure 4

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.0000	0.0000	0.0000	0.0000
2		0.0000	0.0000	0.0000	0.0000
3		0.0000	0.0000	0.0000	0.0000
4		0.0000	0.0000	0.0000	0.0000
5			0.0000	0.0000	0.0000
6			0.0000	0.0047	0.0000
7			0.0000	0.0101	0.0000
8			0.0000	0.0000	0.0000
9			0.0000	0.0000	0.0000
10			0.0000	0.0106	0.0000
11			0.0000	0.0000	0.0000
12			0.0000	0.0000	0.0000
13			0.0000	0.0000	0.0000
14			0.0000	0.0000	0.0000
15			0.0000	0.0000	0.0000
16			0.0000	0.0000	0.0000
17				0.0000	0.0000
18				0.0207	0.0000
19				0.0091	0.0000
20				0.0000	0.0000
21				0.0084	0.0000
22				0.0245	0.0000
23				0.0224	0.0000
24				0.0181	0.0000
25				0.0044	0.0000
26				0.0337	0.0000
27				0.0274	0.0000
28				0.0088	0.0000
29				0.0125	0.0000
30				0.0175	0.0000
31				0.0229	0.0000
32				0.0035	0.0000
33				0.0077	0.0000
34				0.0126	0.0000
35				0.0180	0.0000
36				0.0000	0.0000
37				0.0077	0.0000
38				0.0126	0.0000
39				0.0180	0.0000
40				0.0000	0.0000
41				0.0000	0.0000
42				0.0239	0.0000
43				0.0177	0.0000
44				0.0000	0.0000
45				0.0000	0.0000
46				0.0173	0.0000
47				0.0058	0.0000
48				0.0000	0.0000
49				0.0000	0.0000
50				0.0269	0.0000
51				0.0153	0.0000
52				0.0000	0.0000
53				0.0048	0.0000
54				0.0097	0.0000
55				0.0151	0.0000
56				0.0000	0.0000
57				0.0048	0.0000
58				0.0097	0.0000
59				0.0151	0.0000
60				0.0000	0.0000
61				0.0000	0.0000
62				0.0078	0.0000
63				0.0035	0.0000
64				0.0000	0.0000
Risk Neutral Value =					0.0048

What if the option contract were a put option with the same three year term? Then the cash flows that would be relevant would be Minimum (0, cash flow). If we calculate these cash flows, drop them in the cash flow table, and multiply times the relevant probability weighted discount factors, we see in Figure 5 that the value of this European put option is -0.0070:

Figure 5

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.0000	0.0000	-0.0276	0.0000
2		0.0000	0.0000	-0.0227	0.0000
3		0.0000	0.0000	-0.0192	0.0000
4		0.0000	0.0000	-0.0444	0.0000
5			0.0000	-0.0001	0.0000
6			0.0000	0.0000	0.0000
7			0.0000	0.0000	0.0000
8			0.0000	-0.0090	0.0000
9			0.0000	-0.0196	0.0000
10			0.0000	0.0000	0.0000
11			0.0000	-0.0008	0.0000
12			0.0000	-0.0221	0.0000
13			0.0000	-0.0215	0.0000
14			0.0000	-0.0166	0.0000
15			0.0000	-0.0130	0.0000
16			0.0000	-0.0384	0.0000
17				-0.0099	0.0000
18				0.0000	0.0000
19				0.0000	0.0000
20				-0.0124	0.0000
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				0.0000	0.0000
34				0.0000	0.0000
35				0.0000	0.0000
36				-0.0013	0.0000
37				0.0000	0.0000
38				0.0000	0.0000
39				0.0000	0.0000
40				-0.0013	0.0000
41				-0.0051	0.0000
42				0.0000	0.0000
43				0.0000	0.0000
44				-0.0008	0.0000
45				-0.0131	0.0000
46				0.0000	0.0000
47				0.0000	0.0000
48				-0.0157	0.0000
49				-0.0038	0.0000
50				0.0000	0.0000
51				0.0000	0.0000
52				-0.0064	0.0000
53				0.0000	0.0000
54				0.0000	0.0000
55				0.0000	0.0000
56				-0.0042	0.0000
57				0.0000	0.0000
58				0.0000	0.0000
59				0.0000	0.0000
60				-0.0042	0.0000
61				-0.0131	0.0000
62				0.0000	0.0000
63				0.0000	0.0000
64				-0.0178	0.0000
Risk Neutral Value =					-0.0070

We now check to make sure that these valuations are consistent with no arbitrage. A forward contract with a 3 year maturity and a strike price of 0.97 has a value of

-0.0022. We can replicate this forward contract by (a) buying a 3 year call option on the forward contract at a strike price of zero and (b) selling a 3 year put option on the forward contract at a strike price of zero. The cash flow from this replication strategy would be -0.0048 + 0.0070 = 0.0022, exactly the amount of cash a rational investor would need to receive before the investor would take on a forward contract with a current value of -0.0022.

European Option on Forward Rate Agreements

We now examine the valuation of a European call option on a forward rate agreement with the same terms and cash flows as in Figures 4 and 5 in Chapter 22. Recall that the FRA from Chapter 22 paid $25 per basis point for every basis point over the exercise price of 4.00%. Time zero value of this FRA was -15.4927, as shown in Figure 5 of Chapter 22. What would be the value of a European call option to buy this FRA contract with a strike price of zero and an exercise period of 3 years? The call option would earn the holder the FRA cash flows such that the cash flows were Maximum (0, FRA Cash Flow), i.e. only the positive cash flows from the ownership of the FRA. These cash flows are shown in Figure 6:

Figure 6

FRA at 4% on One Year Instrument					
State Name	State Number	Spot Rate at	Exercise Spot	FRA Cash Time T = 4	Call Option Cash Flow
Time 3 S-1, S-1, S-1	21	6.11%	4.00%	52.77	52.77
Time 3 S-1, S-1, S-2	22	5.57%	4.00%	39.17	39.17
Time 3 S-1, S-1, S-3	23	5.17%	4.00%	29.28	29.28
Time 3 S-1, S-1, S-4	24	8.04%	4.00%	100.94	100.94
Time 3 S-1, S-2, S-1	25	3.10%	4.00%	-22.40	0.00
Time 3 S-1, S-2, S-2	26	2.59%	4.00%	-35.22	0.00
Time 3 S-1, S-2, S-3	27	2.03%	4.00%	-49.27	0.00
Time 3 S-1, S-2, S-4	28	4.06%	4.00%	1.57	1.57
Time 3 S-1, S-3, S-1	29	5.22%	4.00%	30.53	30.53
Time 3 S-1, S-3, S-2	30	1.98%	4.00%	-50.54	0.00
Time 3 S-1, S-3, S-3	31	3.18%	4.00%	-20.51	0.00
Time 3 S-1, S-3, S-4	32	5.50%	4.00%	37.54	37.54
Time 3 S-1, S-4, S-1	33	5.43%	4.00%	35.63	35.63
Time 3 S-1, S-4, S-2	34	4.88%	4.00%	22.12	22.12
Time 3 S-1, S-4, S-3	35	4.49%	4.00%	12.29	12.29
Time 3 S-1, S-4, S-4	36	7.34%	4.00%	83.49	83.49
Time 3 S-2, S-1, S-1	37	4.15%	4.00%	3.79	3.79
Time 3 S-2, S-1, S-2	38	0.94%	4.00%	-76.45	0.00
Time 3 S-2, S-1, S-3	39	2.13%	4.00%	-46.72	0.00
Time 3 S-2, S-1, S-4	40	4.43%	4.00%	10.74	10.74
Time 3 S-2, S-2, S-1	41	2.20%	4.00%	-44.91	0.00
Time 3 S-2, S-2, S-2	42	0.55%	4.00%	-86.17	0.00
Time 3 S-2, S-2, S-3	43	0.77%	4.00%	-80.83	0.00
Time 3 S-2, S-2, S-4	44	1.20%	4.00%	-69.99	0.00
Time 3 S-2, S-3, S-1	45	2.62%	4.00%	-34.42	0.00
Time 3 S-2, S-3, S-2	46	-0.37%	4.00%	-109.33	0.00
Time 3 S-2, S-3, S-3	47	0.26%	4.00%	-93.52	0.00
Time 3 S-2, S-3, S-4	48	2.17%	4.00%	-45.80	0.00
Time 3 S-2, S-4, S-1	49	1.78%	4.00%	-55.59	0.00
Time 3 S-2, S-4, S-2	50	1.27%	4.00%	-68.24	0.00
Time 3 S-2, S-4, S-3	51	0.72%	4.00%	-82.12	0.00
Time 3 S-2, S-4, S-4	52	2.72%	4.00%	-31.93	0.00
Time 3 S-3, S-1, S-1	53	2.28%	4.00%	-42.99	0.00
Time 3 S-3, S-1, S-2	54	1.77%	4.00%	-55.71	0.00
Time 3 S-3, S-1, S-3	55	1.21%	4.00%	-69.65	0.00
Time 3 S-3, S-1, S-4	56	3.23%	4.00%	-19.21	0.00
Time 3 S-3, S-2, S-1	57	2.28%	4.00%	-42.99	0.00
Time 3 S-3, S-2, S-2	58	1.77%	4.00%	-55.71	0.00
Time 3 S-3, S-2, S-3	59	1.21%	4.00%	-69.65	0.00
Time 3 S-3, S-2, S-4	60	3.23%	4.00%	-19.21	0.00
Time 3 S-3, S-3, S-1	61	3.64%	4.00%	-9.11	0.00
Time 3 S-3, S-3, S-2	62	0.61%	4.00%	-84.77	0.00
Time 3 S-3, S-3, S-3	63	1.25%	4.00%	-68.79	0.00
Time 3 S-3, S-3, S-4	64	3.18%	4.00%	-20.61	0.00
Time 3 S-3, S-4, S-1	65	4.51%	4.00%	12.68	12.68
Time 3 S-3, S-4, S-2	66	1.29%	4.00%	-67.84	0.00
Time 3 S-3, S-4, S-3	67	2.48%	4.00%	-38.01	0.00
Time 3 S-3, S-4, S-4	68	4.79%	4.00%	19.64	19.64
Time 3 S-4, S-1, S-1	69	3.50%	4.00%	-12.47	0.00
Time 3 S-4, S-1, S-2	70	0.31%	4.00%	-92.21	0.00
Time 3 S-4, S-1, S-3	71	1.49%	4.00%	-62.66	0.00
Time 3 S-4, S-1, S-4	72	3.78%	4.00%	-5.57	0.00
Time 3 S-4, S-2, S-1	73	2.58%	4.00%	-35.42	0.00
Time 3 S-4, S-2, S-2	74	2.07%	4.00%	-48.18	0.00
Time 3 S-4, S-2, S-3	75	1.51%	4.00%	-62.16	0.00
Time 3 S-4, S-2, S-4	76	3.54%	4.00%	-11.57	0.00
Time 3 S-4, S-3, S-1	77	2.58%	4.00%	-35.42	0.00
Time 3 S-4, S-3, S-2	78	2.07%	4.00%	-48.18	0.00
Time 3 S-4, S-3, S-3	79	1.51%	4.00%	-62.16	0.00
Time 3 S-4, S-3, S-4	80	3.54%	4.00%	-11.57	0.00
Time 3 S-4, S-4, S-1	81	4.51%	4.00%	12.65	12.65
Time 3 S-4, S-4, S-2	82	2.27%	4.00%	-43.29	0.00
Time 3 S-4, S-4, S-3	83	2.73%	4.00%	-31.87	0.00
Time 3 S-4, S-4, S-4	84	5.02%	4.00%	25.57	25.57

The value of the call option on the FRA is the value of these cash flows. We drop them into the cash flow table and multiply times the probability-weighted discount factors to get a call option value of 9.0716:

Figure 7

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	0.0000	0.0000	0.0000	52.7655
2		0.0000	0.0000	0.0000	39.1679
3		0.0000	0.0000	0.0000	29.2759
4		0.0000	0.0000	0.0000	100.9367
5			0.0000	0.0000	0.0000
6			0.0000	0.0000	0.0000
7			0.0000	0.0000	0.0000
8			0.0000	0.0000	1.5707
9			0.0000	0.0000	30.5250
10			0.0000	0.0000	0.0000
11			0.0000	0.0000	0.0000
12			0.0000	0.0000	37.5414
13			0.0000	0.0000	35.6299
14			0.0000	0.0000	22.1202
15			0.0000	0.0000	12.2920
16			0.0000	0.0000	83.4900
17				0.0000	3.7937
18				0.0000	0.0000
19				0.0000	0.0000
20				0.0000	10.7387
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				0.0000	0.0000
34				0.0000	0.0000
35				0.0000	0.0000
36				0.0000	0.0000
37				0.0000	0.0000
38				0.0000	0.0000
39				0.0000	0.0000
40				0.0000	0.0000
41				0.0000	0.0000
42				0.0000	0.0000
43				0.0000	0.0000
44				0.0000	0.0000
45				0.0000	12.6759
46				0.0000	0.0000
47				0.0000	0.0000
48				0.0000	19.6447
49				0.0000	0.0000
50				0.0000	0.0000
51				0.0000	0.0000
52				0.0000	0.0000
53				0.0000	0.0000
54				0.0000	0.0000
55				0.0000	0.0000
56				0.0000	0.0000
57				0.0000	0.0000
58				0.0000	0.0000
59				0.0000	0.0000
60				0.0000	0.0000
61				0.0000	12.6520
62				0.0000	0.0000
63				0.0000	0.0000
64				0.0000	25.5745
Risk Neutral Value =					9.0716

If instead we valued a put option on the FRA, the cash flows would be Minimum (0, FRA Cash Flow). The value of the put option is shown in Figure 8 as -24.5643:

Figure 8

	Maturity of Cash Flow Received				
	Current Time				
Row	**0**	**1**	**2**	**3**	**4**
1	0.0000	0.0000	0.0000	0.0000	0.0000
2		0.0000	0.0000	0.0000	0.0000
3		0.0000	0.0000	0.0000	0.0000
4		0.0000	0.0000	0.0000	0.0000
5			0.0000	0.0000	-22.3981
6			0.0000	0.0000	-35.2169
7			0.0000	0.0000	-49.2719
8			0.0000	0.0000	0.0000
9			0.0000	0.0000	0.0000
10			0.0000	0.0000	-50.5397
11			0.0000	0.0000	-20.5056
12			0.0000	0.0000	0.0000
13			0.0000	0.0000	0.0000
14			0.0000	0.0000	0.0000
15			0.0000	0.0000	0.0000
16			0.0000	0.0000	0.0000
17				0.0000	0.0000
18				0.0000	-76.4473
19				0.0000	-46.7184
20				0.0000	0.0000
21				0.0000	-44.9103
22				0.0000	-86.1725
23				0.0000	-80.8318
24				0.0000	-69.9907
25				0.0000	-34.4222
26				0.0000	-109.3349
27				0.0000	-93.5170
28				0.0000	-45.8032
29				0.0000	-55.5895
30				0.0000	-68.2432
31				0.0000	-82.1172
32				0.0000	-31.9293
33				0.0000	-42.9890
34				0.0000	-55.7054
35				0.0000	-69.6481
36				0.0000	-19.2117
37				0.0000	-42.9890
38				0.0000	-55.7054
39				0.0000	-69.6481
40				0.0000	-19.2117
41				0.0000	-9.1149
42				0.0000	-84.7666
43				0.0000	-68.7926
44				0.0000	-20.6082
45				0.0000	0.0000
46				0.0000	-67.8388
47				0.0000	-38.0085
48				0.0000	0.0000
49				0.0000	-12.4680
50				0.0000	-92.2078
51				0.0000	-62.6646
52				0.0000	-5.5663
53				0.0000	-35.4218
54				0.0000	-48.1758
55				0.0000	-62.1597
56				0.0000	-11.5740
57				0.0000	-35.4218
58				0.0000	-48.1758
59				0.0000	-62.1597
60				0.0000	-11.5740
61				0.0000	0.0000
62				0.0000	-43.2850
63				0.0000	-31.8703
64				0.0000	0.0000
Risk Neutral Value =					**-24.5643**

We can again confirm the absence of arbitrage by noting that the total of the cash flows from a replication strategy (buy the call option and sell the put option, -9.0716 + 24.5643) equals the cash flow that an investor would receive for taking on an existing FRA position (15.4927) currently valued at a loss of 15.4927.

European Options on a Eurodollar Futures-type Forward Contract

In Figure 6 in Chapter 22, we dealt with a forward contract which pays like the standard money market futures contracts, i.e. at the expiration of the futures contract or forward contract instead of on the interest payment date of the underlying instrument. Options on this type of forward contract are exactly the same as the analysis in Figures 7 and 8 except for the timing of the cash flows. These valuations are left as an exercise for the interested reader.

European Options on Futures on Coupon-Bearing Bonds

In Chapter 22, we argued that the valuation of futures contracts on coupon-bearing bonds is best accomplished using numerical methods like the HJM 3 factor bushy tree or related Monte Carlo simulation. There were three reasons for this conclusion: first, as we saw in Chapter 3, 5-10 risk factors drive interest rates and there is, in general, no “closed form” solution for futures in this case; second, daily mark to market cash flows compound the complexity of the futures contract beyond the complexity of the cash flows on the underlying instrument; third, the “cheapest to deliver” option embedded in most bond futures contracts requires numerical methods for an accurate valuation. We saw in Chapter 21 that the Jamshidian bond option formula makes use of the critical short rate level that triggers the “moneyness” of the bond option. A similar technique can be used in a one factor term structure model to identify the transition points at which the bond which is cheapest to deliver changes from bond J to bond J+1. In a term structure model where many factors drive interest rates, an option on a bond future that has an embedded “cheapest to deliver” option is a compound option. In general, this kind of compound option is best analyzed using the HJM framework rather than via closed form approximations.

European Options on Money Market Futures Contracts

For similar reasons, the HJM framework is the best approach to value and hedge positions that involve money market futures contracts as well. This challenging task has been further complicated by the Libor manipulation scandal in two dimensions. First, the manipulation of Libor that Barclays admitted on June 27, 2012 “pollutes” any historical data set that one would use for Libor-based interest rate modeling. Second, until there is a dramatic reform of setting short term interest rates for pricing caps, floors, and swaps, the potential for future manipulation remains. To ignore this in modeling options on futures would be naïve at best and suicidal at worst. The best modeling strategy will be dependent on further revelations that emerge from on-going investigations into Libor manipulation.

Defaultable Options on Forward and Futures Contracts

As we noted in Chapter 22, Jarrow and Turnbull [1995] show that, in special circumstances, the adjustment made to the price of a defaultable instrument is a simple ratio times the value of the same instrument if it was not defaultable. Can we analyze the possibility of a default on the options in this chapter in the same way?

As we noted in Chapter 22, the answer is no. Since the macro factors impact options prices, forward prices, futures prices and the default probabilities of the counterparties dealing in the forwards and futures, the neat separation of the credit-adjusted value of futures and forwards that Jarrow and Turnbull found for simpler instruments will not generally apply. This doesn't mean that this approach can't be used as a first approximation, but it will significantly understate risk. As J.P. Morgan found out in 1998 in a well-known incident with S.K. Securities of Korea, the default probability of the counterparty can be dangerous correlated with the amount of money they owe you.[1] This is because the same macro factor drives both the legally required payoff on the security at hand and it drives the default probability of the counterparty. A true credit-adjusted valuation requires the kind of 'scenario specific' default probabilities that are embedded in the rich flexibility of the reduced form models discussed in detail in Chapter 16.

[1] See van Deventer and Imai [2003] for a detailed discussion of this incident.

How do we simulate future values and cash flows on options on futures and forwards in a true default-adjusted fashion? Again, a careful Monte Carlo simulation like that specified in Chapters 19 and 20 is essential to recognizing the slim, but real possibility of the default of the futures Exchanges in times of crisis. Similarly, the default of our counterparty on an over-the-counter option on futures and forwards could be analyzed analytically or numerically. As suggested in the previous chapter, rather than doing this mathematical derivation, a carefully structured Monte Carlo simulation can capture the same effects. This is the principal topic in Chapters 36 to 41.