Kamakura Corporation
2222 Kalakaua Avenue
Suite 1400
Honolulu, Hawaii 96815, USA
phone 1 808 791 9888
facsimile 1 808 791 9898
www.kamakuraco.com

Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 3: Risk Management Techniques for Credit Risk Analytics

Chapter 18
Legacy approaches to credit risk

In this chapter, we do for credit risk models what we did for interest rate risk models in Chapter 13. We go up into the risk management attic and dust off the tools that may have been best practice once but which are only described that way now by a few. We review legacy credit ratings and the Merton model using the tools of Chapter 16. We do this because the selection of credit risk tools is not a beauty contest. It's all about accuracy, period. Arguments to the contrary are usually about regulatory or corporate politics and the slow speed at which very large organizations respond to and absorb new technology. In that spirit, we explain why legacy credit ratings and the Merton default model, used almost exclusively for public firms, have been firmly out-performed by reduced form credit models. The dominance of the reduced form approach for retail borrowers and non-public firms has never been challenged by tools like legacy ratings and the Merton model, where the assumptions we outline below are even less appropriate than they are for public firms.

In this chapter, we return to using "MODEL RISK ALERTS" with respect to statements known to cause problems or known to be false but commonly stated by practitioners.

The Rise and Fall of Legacy Ratings

Almost every financial institution has used legacy credit ratings actively in both individual asset selection and in portfolio management at some point in its past. Ratings have a history measured in decades and they will probably be used actively many decades from now, despite the fatal flows we outline in this section. That being said, rating agencies and experts in internal ratings now face substantial pressure, both commercial and conceptual, from providers of quantitative default probabilities both inside and outside of the organization. In this section, we summarize some of the strengths and weaknesses of a traditional ratings approach for comparison with the quantitative approaches that increasingly dominant risk management and credit derivatives pricing. We present proof consistent with Hilscher and Wilson (2011) that ratings are consistently outperformed from an accuracy point of view by quantitative default models built using logistic regression and the reduced form modeling approach. We also present the even more serious problems with ratings summarized by Senator Carl Levin and the U.S. Senate in the wake of the 2007 to 2010 credit crisis.

Ratings: What They Do and Don't Do

For many, many years Moody's Investors Service and Standard & Poor's Corporation were essentially the only providers of credit risk assessment on large corporations and sovereign issuers of debt. This has changed dramatically, recently, with a plethora of credit rating agencies springing up around the world to challenge the older rating agencies. Competing rating agencies and vendors of default probabilities have arisen for a number of reasons, just some of which are noted here:

- The **"granularity"** of ratings offered by the major rating agencies is low. The number of ratings grades (AAA, AA, A, BBB, BB, B, CCC, CC and D for Standard & Poor's for example) is small, so companies with ratings are divided into only nine groups by major ratings groups and 21 groups if the pluses and minus are added to ratings from AA to CCC. Market participants need a much higher degree of precision than the rating alone allows, even giving credit for the pluses and minuses attached to each rating
- The **annual default rates by rating are very volatile** over time, so there is considerable uncertainty about what the default probability is that is associated with each rating.
- Ratings are an **ordinal measure of credit risk**, rather than a quantitative default probability
- **Ratings change very infrequently**, so market participants are getting a view of an issuer's credit that changes only every six to 12 months at best. General Electric's rating did not change between 1956 and 2009, a period of 53 years.
- Ratings take the long "through the cycle view" so **they show relatively little of the cyclicality** that many analysts believe is the key to credit risk management
- The **ratings universe is a very small percentage** of the total number of counterparties that a typical financial institution would lend to (retail clients, small business clients and corporate clients totaling in the millions at some large banks), so an institution relying on ratings as its primary credit risk management tool is forced by definition to take a piece meal approach to enterprise credit risk management. Each class of counterparties ends up having a different type of rating, some internal and some external.
- Rating agencies are normally paid by the structurer of a complex CDO or the issuer of the securities, so there is substantial **commercial pressure** on the agencies for a "good" rating. The report by Senator Levin's committee summarizes these problems in detail.
- Rating agencies have a substantial "non-rating" consulting business, so they are increasing being scrutinized as having **potential conflicts of interest** like the major accounting firms before they spun off their consulting arms in the last decade
- The **accuracy of legacy ratings has been consistently found to be much lower** than default probabilities developed using logistic regression and the reduced form approach.

Each of these issues could fill a book in and of itself. We deal with the most serious issues in this section.

"Through the Cycle" versus "Point in Time," A Distinction without a Difference

There has been a long debate in the risk management industry about whether legacy credit ratings are intended to be "point in time" ratings or whether they are intended to be "through the cycle" credit indicators. The reason that such a debate occurs is the vagueness of the maturity and default probabilities associated with a given rating for a specific company at any point in time. When viewed in the context of a modern quantitative default probability, the debate about "point in time" versus "through the cycle" is a distinction without a difference. This section explains the reason why.

Even after more than 100 years of experience with traditional credit ratings, practical credit portfolio managers still don't have clear answers to a number of critical questions:

- What is the default probability associated with a specific rating for a specific company over a given time horizon on a specific date?
- What is the default probability associated with the same issuer over different time horizons?
- How do the ratings and default probabilities associated with them change when macro-economic factors change?
- Put differently, how does the default probability associated with a given rating change as the company moves through various points in the business cycle?

Because the legacy rating agencies have been unable to articulate clear answers to these questions, two other questions are often asked and debated:

- Is a given credit rating a "point in time rating," an index of risk that prevails today but (by implication) not necessarily tomorrow?
- Is a given credit rating a "through the cycle rating" intended to measure the average level of credit risk for a given company over an unspecified but presumably long period of time that extends "though the cycle" both for the current business cycle and presumably many others?

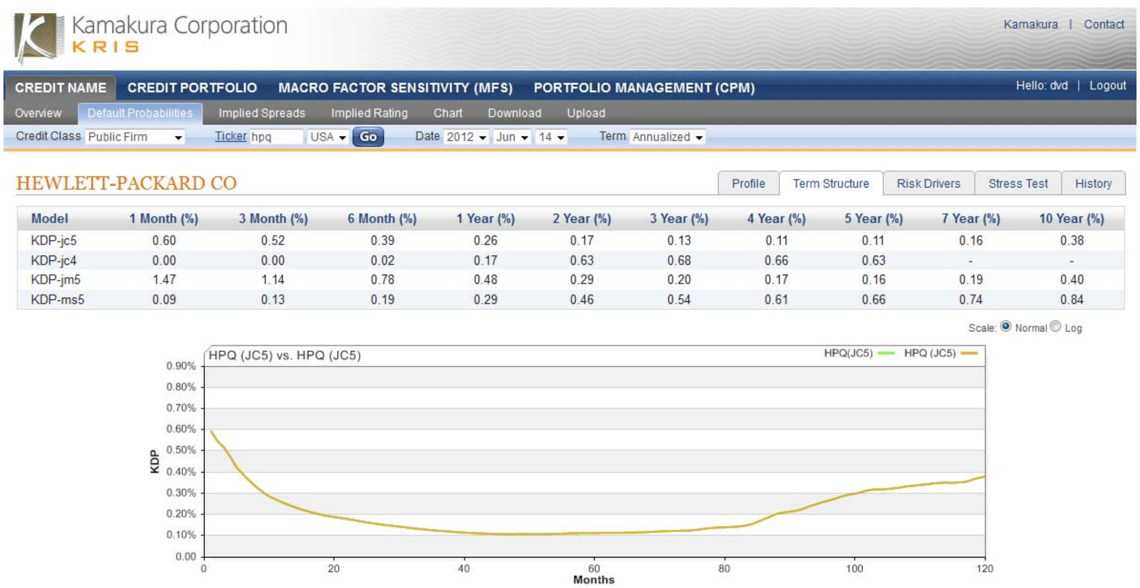
A precise answer to these questions only becomes more muddled if one poses it to senior people at the rating agencies. In February 2003 at a well-lubricated dinner in mid-town Manhattan, one of the authors posed this question to the head of corporate credit ratings at one of the two leading rating agencies: "What would your agency do with respect to a company's rating if the company was a long run risk level of BBB/Baa but faced 60 days of CCC/Caa risk?" His answer, consistent with the benign credit conditions of the time, was this: "We can't afford to damage our firm's reputation by letting a BBB/Baa rated company default, so we'd set the rating at CCC/Caa." This comment contrasts with more recent statements by other senior rating executives who argue that they have a special obligation to avoid 'type 1' error that overly harshly rates companies. The reason for this concern is that derivatives-related margin requirements, unfortunately, have been tied to ratings and the legacy rating agencies fear an error that would trigger a margin call that leads to bankruptcy. If the first quote is accurate in reflecting rating agency thinking, it means that a rating on any given date reflects the worse of (a) the short term risk of the company or (b) the long run risk of the company. From this view, the rating is neither "point in time" nor "through the cycle." A little reflection in a default modeling context makes the muddled semantics much more clear and transparent.

How does a modern quantitative default model differ from ratings? The differences are very great and very attractive from a practice use point of view:

- Each default probability has an explicit maturity
- Each default probability has an obvious meaning. A default probability of 4.25% for a three year maturity means what it says: there is a 4.25% (annualized) probability of default by this company over the three year period starting today. Cumulative default risk is calculated using the formulas in the appendix of Chapter 16.
- Each company has a full term structure of default probabilities at maturities from 1 month to 10 years, updated daily, in a widely used default probability service.

What does "point in time" mean in this context? We illustrate the issues with the default probabilities for Hewlett Packard on June 14, 2012 as displayed by Kamakura Risk Information Services in Figure 1.

Figure 1



Model	1 Month (%)	3 Month (%)	6 Month (%)	1 Year (%)	2 Year (%)	3 Year (%)	4 Year (%)	5 Year (%)	7 Year (%)	10 Year (%)
KDP-jc5	0.60	0.52	0.39	0.26	0.17	0.13	0.11	0.11	0.16	0.38
KDP-jc4	0.00	0.00	0.02	0.17	0.63	0.68	0.66	0.63	-	-
KDP-jm5	1.47	1.14	0.78	0.48	0.29	0.20	0.17	0.16	0.19	0.40
KDP-ms5	0.09	0.13	0.19	0.29	0.46	0.54	0.61	0.66	0.74	0.84

All of the default probabilities for Hewlett Packard on June 14, 2012 are default probabilities at that are "point in time" for HP. Default probabilities at a different point in time, say June 15, will change if the inputs to the default models are different on June 15 than they were on June 14. What does "through the cycle" mean with respect to the default probabilities for HP on June 14? "Through the cycle" implies the longest default probability available on the term structure of default probabilities because this maturity does the best job of extending through as much of the business cycle as possible. For KRIS version 5, the longest default probability available is the 10 year default probability. If the default probability for Hewlett Packard on June 14 is 0.38% at 10 years, it means that the "through the cycle" default probability for Hewlett Packard prevailing on June 14, 2012 is a 0.38% (annualized) default rate over the 10 years ending June 14, 2022. What could be clearer than that?

To summarize, all of the default probabilities prevailing for Hewlett Packard on June 14, 2012 are the "point in time" default probabilities for HP at all maturities from 1 month to 10 years. The "through the cycle" default probability for HP on June 14 is the 10 year default probability, because this is the longest maturity available. The 10 year default probability is also obviously a point in time default probability because it prevails on June 14, the "point in time" we care about. On June 15, all of the point in time default probabilities for HP will be updated, including the 10 year default probability, which has a dual role as the "through the cycle" default probability.

There is no uncertainty about these concepts: all default probabilities that exist today at all maturities are "point in time" default probabilities for HP, and the longest maturity default probability is also the "through the cycle" default probability.

How can these default probabilities be mapped to ratings, and what rating would be "point in time" and what rating would be "through the cycle"? An experienced user of quantitative default probabilities would ask in return "Why would you want to go from an explicit credit risk assessment with a known maturity and 10,000 grades (from 0 basis points to 10,000 basis points) to a vague credit assessment with no known maturity and only 20 grades?" A common answer is that management is used to ratings and ratings have to be produced, even if they're much less accurate and much less useful than the default probabilities themselves. MODEL RISK ALERT: Most of the tens of billions of losses incurred by investors in collateralized debt obligations (which we discuss in Chapter xx) during the 2007 to 2010 credit crisis were explained after the fact by the statement "But it was rated AAA when we bought it."

Stress Testing, Legacy Ratings, and Transition Matrices

In the wake of the 2007 to 2010 credit crisis, financial institutions regulators have implemented a wide variety of "stress tests": which require financial institutions to calculate the market value of assets, liabilities, and capital in different economic scenarios. Many of the regulations also require projections of net income under the accounting standard relevant for that institution. When ratings, instead of default probabilities, are at the heart of the credit risk process, institutions are literally unable to do accurate calculations required by regulators because the rating agencies themselves are unable to articulate the quantitative links between macro-economic factors, legacy ratings, and probabilities of default. We spend a lot of time on these links in later chapters. These stress tests contrast heavily with the much-criticized reliance on ratings in the Basel II Capital Accords. The Dodd-Frank legislation in the United States in 2010 has hastened the inevitable demise of the rating agencies by requiring U.S. government agencies to remove any rules or requirements that demand the use of legacy ratings.

Transition Matrices: Analyzing the Random Changes in Ratings from One Level to Another

It goes without saying that the opaqueness of the link between ratings, macro-factors, and default risk makes "transition matrices" an idea useful in concept but useless in practice. The transition matrix is the probability that a firm moves from ratings class J to ratings class K over a specific period of time. Most users of the transition matrix concept make the assumption (MODEL RISK ALERT) that the transition probabilities are constant. Chapter 16 illustrates clearly that this assumption is false in its graph of the number of bankruptcies in the United States from 1990 to 2008. Default risk of all firms changes in a highly correlated fashion. The very nature of the phrase "business cycle" conveys the meaning that most default probabilities rise when times are bad and fall when times are good.

This means that transition probabilities to lower ratings should rise in bad times and fall in good times. Since the very nature of the ratings process is non-transparent, there is no valid basis for calculating these transition probabilities that matches the reduced form approach in accuracy.

Moral Hazard in "Self-Assessment" of Ratings Accuracy by Legacy Rating Agencies

As we noted in the introduction to this chapter, the choice of credit models is not a beauty contest. It is all about accuracy and nothing else (except when corporate and regulatory politics interferes with the facts). One of the challenges facing an analyst is the moral hazard of accuracy "self-assessments" by the legacy rating agencies, a topic to which we now turn.

On March 9, 2009, one of the authors and our colleague Prof. Robert Jarrow wrote a blog entitled "The Rating Chernobyl," predicting that rating agency errors during the 2007 to 2010 credit crisis were so serious that even their own self assessments would show once and for all that legacy ratings are a hopelessly flawed measure of default risk. That 2009 prediction hasn't come true for a very simple reason: the rating agencies have left their mistakes out of their self-assessments. This section illustrates the dangers of relying on rating agency default rates in credit risk management, because the numbers are simply not credible.

A copy of the original blog "The Ratings Chernobyl" was redistributed on www.riskcenter.com and by the Global Association of Risk Professionals. For a copy of that blog, please use this link:

http://www.kamakuraco.com/Blog/tabid/231/EntryId/11/The-Ratings-Chernobyl.aspx

We predicted that the ratings errors during the credit crisis were so egregious that even the rating agencies' own annual self-assessments of ratings performance would show that an analyst of credit risk should not place any serious reliance on legacy ratings. For an example of such a recent self-assessment, please see this example from Standard & Poor's released in 2011:

http://www.standardandpoors.com/ratings/articles/en/us/?assetID=1245302234237

The other two major rating agencies release similar self-assessments. In this section, in the interests of brevity, we use the S&P self-assessment. An analysis of the other two rating agencies would yield similar results.

In the wake of the financial crisis, rating agencies were forced to make available their ratings for public firms for the years 2007-2009. We use those disclosed ratings from Standard & Poor's in what follows. Like the rating agency self-assessments, we analyze firms' subsequent default rates based on their legacy ratings on January 1 of each year. If we use all of the data released by Standard & Poor's, we have 6,664 observations for the three year period, using the January 1 rating in each case for each firm:

Analysis of 2007-2009 Default Rates by Rating Grade
Source: Kamakura Corporation and Standard & Poor's

		Number of Observations			
Ratings Rank	Rating	2007	2008	2009	Total Observations
1	AAA	17	15	12	44
2	AA+	8	8	8	24
3	AA	35	65	48	148
4	AA-	101	67	72	240
5	A+	120	113	104	337
6	A	193	172	172	537
7	A-	238	202	202	642
8	BBB+	272	253	236	761
9	BBB	306	255	273	834
10	BBB-	207	213	225	645
11	BB+	149	132	121	402
12	BB	186	147	138	471
13	BB-	201	196	183	580
14	B+	161	153	138	452
15	B	92	104	101	297
16	B-	52	44	61	157
17	CCC+	18	15	19	52
18	CCC	10	6	11	27
19	CCC-	1	1	2	4
20	CC	2	3	5	10
	Grand Total	2,369	2,164	2,131	6,664

How many firms failed during this three year period among firms with legacy ratings? To answer that question, we first use the Kamakura Risk Information Services data base and then compare with results from Standard & Poor's. The KRIS definition of failure includes the following cases:

- A D or uncured SD rating from a legacy rating agency
- An ISDA event of default
- A delisting for obvious financial reasons, such as failure to file financial statements, failure to pay exchange fees, or failure to maintain the minimum required stock price level

The KRIS data base very carefully distinguishes failures from rescues. Bear Stearns, for example, failed. After the failure, Bear Stearns was rescued by J.P. Morgan in return for massive U.S. government assistance that is documented in this blog:

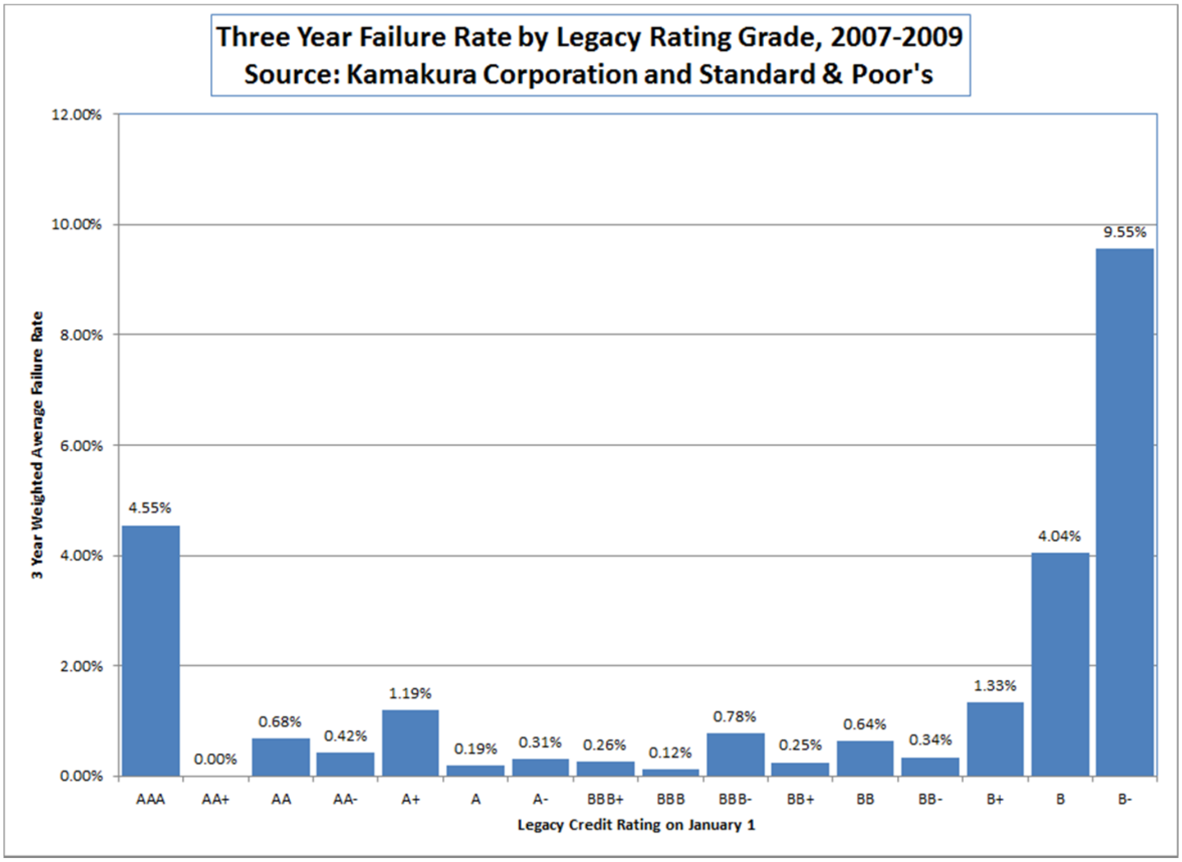
van Deventer, Donald R. "Case Studies in Liquidity Risk: JPMorgan Chase, Bear Stearns and Washington Mutual," Kamakura blog, www.kamakuraco.com, July 8, 2011.

The Kamakura failure count includes all firms which would have failed without government assistance. In many of the government rescues, common shareholders and preferred shareholders would be amused to hear that the rescued firm "did not default" just because the senior debt holders were bailed out. As the Kamakura "Case Studies in Liquidity Risk" series makes clear, the government's definition of "too big to fail" changes within 2 or 3 days of the failure of Lehman Brothers, so the prediction of a probability of rescue is a much more difficult task than the prediction of failure. Using the KRIS data base of failures in the 2007-2009, we have 86 failed firms during this period. This count of failed firms is being revised upward by Kamakura in KRIS version 6 in light of recent government documents:

Analysis of 2007-2009 Default Rates by Rating Grade
Source: Kamakura Corporation and Standard & Poor's

		Number of Defaults			
Ratings Rank	Rating	2007	2008	2009	Total Defaults
1	AAA	0	2	0	2
2	AA+	0	0	0	0
3	AA	0	1	0	1
4	AA-	0	1	0	1
5	A+	1	3	0	4
6	A	0	1	0	1
7	A-	0	1	1	2
8	BBB+	0	0	2	2
9	BBB	0	1	0	1
10	BBB-	0	2	3	5
11	BB+	0	1	0	1
12	BB	1	1	1	3
13	BB-	0	2	0	2
14	B+	0	4	2	6
15	B	0	2	10	12
16	B-	1	3	11	15
17	CCC+	1	1	8	10
18	CCC	2	3	7	12
19	CCC-	0	0	1	1
20	CC	1	1	3	5
	Grand Total	7	30	49	86

We can calculate the failure rate in each year and the weighted average 3 year failure rate by simply dividing the number of failures by the number of observations. We do that and display the results in graphical form from AAA to B- rating:



The results are consistent with Jarrow and van Deventer's "The Ratings Chernobyl." The 3 year weighted average failure rate for AAA rated firms was 4.55%, much higher than any other failure rate except B- ratings in the B to AAA range. By contract, BBB firms failed at only a 0.12% rate and BB+ firms failed at only a 0.25% rate. In tabular form the results can be summarized as follows:

Analysis of 2007-2009 Default Rates by Rating Grade
Source: Kamakura Corporation and Standard & Poor's

Default Rate by Year					
Ratings Rank	Rating	2007	2008	2009	3 Year Default Rate
1	AAA	0.00%	13.33%	0.00%	4.55%
2	AA+	0.00%	0.00%	0.00%	0.00%
3	AA	0.00%	1.54%	0.00%	0.68%
4	AA-	0.00%	1.49%	0.00%	0.42%
5	A+	0.83%	2.65%	0.00%	1.19%
6	A	0.00%	0.58%	0.00%	0.19%
7	A-	0.00%	0.50%	0.50%	0.31%
8	BBB+	0.00%	0.00%	0.85%	0.26%
9	BBB	0.00%	0.39%	0.00%	0.12%
10	BBB-	0.00%	0.94%	1.33%	0.78%
11	BB+	0.00%	0.76%	0.00%	0.25%
12	BB	0.54%	0.68%	0.72%	0.64%
13	BB-	0.00%	1.02%	0.00%	0.34%
14	B+	0.00%	2.61%	1.45%	1.33%
15	B	0.00%	1.92%	9.90%	4.04%
16	B-	1.92%	6.82%	18.03%	9.55%
17	CCC+	5.56%	6.67%	42.11%	19.23%
18	CCC	20.00%	50.00%	63.64%	44.44%
19	CCC-	0.00%	0.00%	50.00%	25.00%
20	CC	50.00%	33.33%	60.00%	50.00%
	Grand Total	0.30%	1.39%	2.30%	1.29%

By any statistical test over the B to AAA range, legacy ratings were nearly worthless in distinguishing strong firms from weak firms. How does the Standard & Poor's self-assessment present these results? It doesn't. Here is what it shows in Table 24:

Table 24

Global Corporate Average Cumulative Default Rates (1981-2010)

(%)	–Time horizon (years)–														
Rating	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
AAA	0.00	0.03	0.14	0.26	0.38	0.50	0.56	0.66	0.72	0.79	0.83	0.87	0.91	1.00	1.09
	(0.00)	(0.20)	(0.39)	(0.47)	(0.59)	(0.69)	(0.75)	(0.82)	(0.83)	(0.83)	(0.83)	(0.84)	(0.84)	(0.91)	(0.99)
AA	0.02	0.07	0.15	0.26	0.37	0.49	0.58	0.67	0.74	0.82	0.90	0.97	1.04	1.10	1.15
	(0.08)	(0.12)	(0.19)	(0.28)	(0.36)	(0.47)	(0.56)	(0.63)	(0.68)	(0.72)	(0.72)	(0.73)	(0.70)	(0.70)	(0.71)
A	0.08	0.19	0.33	0.50	0.68	0.89	1.15	1.37	1.60	1.84	2.05	2.23	2.40	2.55	2.77
	(0.12)	(0.21)	(0.27)	(0.36)	(0.44)	(0.48)	(0.54)	(0.59)	(0.67)	(0.77)	(0.86)	(0.91)	(0.88)	(0.87)	(0.83)
BBB	0.25	0.70	1.19	1.80	2.43	3.05	3.59	4.14	4.68	5.22	5.78	6.24	6.72	7.21	7.71
	(0.27)	(0.60)	(0.88)	(1.09)	(1.32)	(1.48)	(1.59)	(1.61)	(1.64)	(1.57)	(1.40)	(1.33)	(1.19)	(1.04)	(1.00)
BB	0.95	2.83	5.03	7.14	9.04	10.87	12.48	13.97	15.35	16.54	17.52	18.39	19.14	19.78	20.52
	(1.05)	(2.32)	(3.39)	(4.08)	(4.64)	(4.87)	(4.78)	(4.61)	(4.43)	(4.24)	(4.43)	(4.50)	(4.56)	(4.51)	(4.63)
B	4.70	10.40	15.22	18.98	21.76	23.99	25.82	27.32	28.64	29.94	31.09	32.02	32.89	33.70	34.54
	(3.31)	(5.69)	(6.93)	(7.70)	(8.10)	(7.87)	(7.33)	(6.84)	(6.27)	(5.97)	(5.58)	(5.35)	(4.55)	(3.91)	(3.89)
CCC/C	27.39	36.79	42.12	45.21	47.64	48.72	49.72	50.61	51.88	52.88	53.71	54.64	55.67	56.55	56.55
	(12.69)	(13.97)	(13.61)	(14.09)	(14.05)	(12.98)	(12.70)	(12.10)	(11.65)	(10.47)	(10.75)	(11.42)	(12.06)	(10.38)	(9.61)
Investment grade	0.13	0.34	0.59	0.89	1.21	1.53	1.83	2.12	2.39	2.68	2.94	3.16	3.37	3.59	3.83
	(0.12)	(0.28)	(0.41)	(0.52)	(0.61)	(0.66)	(0.69)	(0.72)	(0.79)	(0.84)	(0.86)	(0.85)	(0.78)	(0.68)	(0.65)
Speculative grade	4.36	8.53	12.17	15.13	17.48	19.45	21.13	22.59	23.93	25.16	26.21	27.10	27.93	28.66	29.40
	(2.80)	(4.55)	(5.61)	(6.18)	(6.41)	(6.12)	(5.61)	(4.98)	(4.34)	(4.07)	(3.94)	(3.95)	(3.72)	(3.53)	(3.57)
All rated	1.61	3.19	4.60	5.80	6.79	7.64	8.38	9.02	9.62	10.18	10.67	11.08	11.47	11.82	12.20
	(1.06)	(1.85)	(2.40)	(2.73)	(2.88)	(2.82)	(2.68)	(2.54)	(2.43)	(2.39)	(2.28)	(2.15)	(1.91)	(1.90)	(2.08)

Numbers in parentheses are standard deviations. Sources: Standard & Poor's Global Fixed Income Research and Standard & Poor's CreditPro®.

Note that the first year default rate for AAA rated companies is zero. What happened to FNMA and FHLMC? And what about the 2007-2009 failure rate of 1.19% for A+ weighted companies? Why is the A rated first year default rate so small in light of that? The reason is simple. A large number of important failed companies have been omitted from the self-assessment, avoiding a very embarrassing self-assessment.

Table 15

Investment-Grade Defaults In The Five-Year 2006 Static Pool

Company	Country	Industry	Default date	Next-to-last rating	Date of next-to-last rating	First rating	Date of first rating	Year of default
Aiful Corp.	Japan	Financial institutions	9/24/2009	CC	9/18/2009	BBB	10/6/2003	2009
Ambac Assurance Corp.	U.S.	Insurance	11/18/2009	CC	7/28/2009	AAA	12/31/1980	2009
Ambac Financial Group, Inc.	U.S.	Insurance	11/2/2010	CC	6/9/2008	AA+	7/30/1991	2010
BluePoint Re Limited	Bermuda	Insurance	8/14/2008	A	6/9/2008	AA	10/25/2004	2008
Caesars Entertainment Corp.	U.S.	Leisure time/media	12/24/2008	CC	11/18/2008	A	12/31/1980	2008
CIT Group, Inc.	U.S.	Financial institutions	8/17/2009	CC	7/16/2009	AA	12/31/1980	2009
Clear Channel Communications Inc.	U.S.	Leisure time/media	12/23/2008	CC	12/5/2008	BBB-	9/26/1997	2008
Colonial BancGroup Inc.	U.S.	Financial institutions	8/17/2009	CC	7/30/2009	BBB-	1/17/1997	2009
Colonial Bank	U.S.	Financial institutions	8/17/2009	CCC-	7/30/2009	BBB	1/21/1997	2009
Commonwealth Land Title Insurance Co.	U.S.	Insurance	12/4/2008	BB-	11/24/2008	A-	6/25/1997	2008
Controladora Comercial Mexicana, S. A. B. de C. V.	Mexico	Consumer/service sector	10/9/2008	CC	10/8/2008	BB+	3/31/1998	2008
Downey Financial Corp.	U.S.	Financial institutions	11/24/2008	CCC-	11/21/2008	BBB-	6/7/1999	2008
Downey S&L Assn	U.S.	Financial institutions	11/24/2008	CCC	11/21/2008	A+	12/31/1980	2008
Energy Future Holdings Corp.	U.S.	Energy and natural resources	11/16/2009	CC	10/5/2009	BBB	10/3/1997	2009
FGIC Corp.	U.S.	Insurance	8/3/2010	CC	9/24/2008	AA	1/5/2004	2010
General Growth Properties, Inc.	U.S.	Real estate	3/17/2009	CC	12/24/2008	BBB-	6/2/1998	2009
IndyMac Bank, FSB	U.S.	Financial institutions	7/14/2008	B-	7/9/2008	BBB-	9/4/1998	2008
LandAmerica Financial Group Inc.	U.S.	Insurance	11/26/2008	B-	11/24/2008	BBB-	11/19/2004	2008
Lehman Brothers Holdings Inc.	U.S.	Financial institutions	9/16/2008	A	6/2/2008	AA-	1/1/1985	2008
Lehman Brothers Inc.	U.S.	Financial institutions	9/23/2008	BB-	9/15/2008	AA	10/5/1984	2008
Mashantucket Western Pequot Tribe	U.S.	Leisure time/media	11/16/2009	CCC	8/26/2009	BBB-	9/16/1999	2009
McClatchy Co. (The)	U.S.	Leisure time/media	6/29/2009	CC	5/22/2009	BBB-	2/8/2000	2009
Residential Capital, LLC	U.S.	Financial institutions	6/4/2008	CC	5/2/2008	BBB-	6/9/2005	2008
Sabre Holdings Corp.	U.S.	Transportation	6/16/2009	B	3/30/2009	A-	2/7/2000	2009
Scottish Annuity & Life Insurance Co. (Cayman) Ltd.	Cayman Islands	Insurance	1/30/2009	CC	1/9/2009	A-	12/18/2001	2009
Takefuji Corp.	Japan	Financial institutions	12/15/2009	CC	11/17/2009	A-	2/10/1999	2009
Technicolor S.A.	France	Consumer/service sector	5/7/2009	CC	1/29/2009	BBB+	7/24/2002	2009
Tribune Co.	U.S.	Leisure time/media	12/9/2008	CCC	11/11/2008	AA	3/1/1983	2008
Washington Mutual Bank	U.S.	Financial institutions	9/26/2008	BBB-	9/15/2008	B+	1/24/1989	2008
Washington Mutual, Inc.	U.S.	Financial institutions	9/26/2008	CCC	9/15/2008	BBB	7/17/1995	2008
YRC Worldwide Inc.	U.S.	Transportation	1/4/2010	CC	11/2/2009	BBB-	11/19/2003	2010

Which firms did Kamakura have listed as failures in the ratings grades from A to AAA?

Kamakura included these 11 firms:

AAA	FANNIE MAE
AAA	FEDERAL HOME LOAN MORTG CORP
AA	AMERICAN INTERNATIONAL GROUP
AA-	WACHOVIA CORP
A+	NORTHERN ROCK PLC
A+	MERRILL LYNCH & CO INC
A+	LEHMAN BROTHERS HOLDINGS INC
A+	AGEAS SA/NV
A	BEAR STEARNS COMPANIES INC
A-	WASHINGTON MUTUAL INC
A-	ANGLO IRISH BANK CORP

Remember also that American International Group was rated AAA in January 2005. Which firms were omitted from the Standard & Poor's list? Nine of the 11 failures tallied by Kamakura were omitted by S&P from their self-assessment.

FANNIE MAE (FNMA)

FEDERAL HOME LOAN MORTG CORP

AMERICAN INTERNATIONAL GROUP

WACHOVIA CORP

NORTHERN ROCK PLC

MERRILL LYNCH & CO INC

AGEAS SA/NV

BEAR STEARNS COMPANIES INC

ANGLO IRISH BANK CORP

For those who wish to debate whether or not FNMA and FHLMC were in fact failures, here are the credit default swap auction protocols triggered by their "credit event" and announced by ISDA:

http://www.dechert.com/library/FS_FRE_10-08_ISDA_Credit_Default_Swap_Auction_Protocols.pdf

When Standard & Poor's was asked by a regional bank in the United States why FNMA and FHLMC had been omitted from the corporate ratings self-assessment, the response from Standard & Poor's did not discuss the nuances of a rescue of a failed firm. Instead, the e-mail reply stated only that FNMA and FHLMC were no longer exchanged listed privately owned corporations and that they were therefore no longer an object of this study. This justification, obviously, could be used to exclude every firm that went bankrupt from the legacy ratings accuracy self-assessment.

A complete third-party audit of the entire rating agency default history is mandatory before historical rating agency default rates can relied upon with confidence. Kamakura Risk Information Services maintains an independent and active data base of corporate failures. For more information see info@kamakuraco.com.

Comparing the Accuracy of Ratings and Reduced Form Default Probabilities

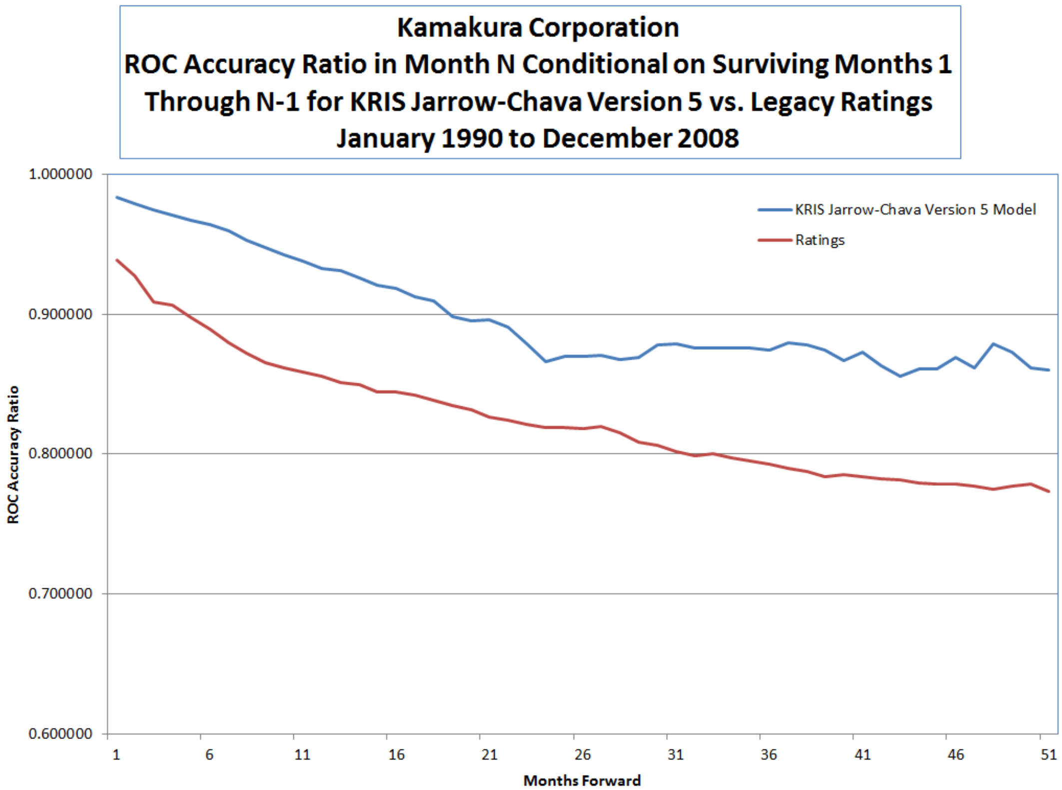
In a recent paper, Prof. Jens Hilscher (Brandeis University and senior research fellow at Kamakura Corporation) and Mungo Wilson (Oxford University) compared the accuracy of legacy credit ratings with modern default probabilities ("Credit Risk and Credit Ratings," Brandeis University working paper, 2011). The authors concluded the following:

> "This paper investigates the information in corporate credit ratings. We examine the extent to which firms' credit ratings measure raw probability of default as opposed to systematic risk of default, a firm's tendency to default in bad times. We find that credit ratings are dominated as predictors of corporate failure by a simple model based on publicly available financial information (`failure score'), indicating that ratings are poor measures of raw default probability."

We encourage the serious reader to review the full paper to understand the details of this and related conclusions.

The Hilscher and Wilson results are consistent with the Kamakura Risk Information Services Technical Guides (versions 2, 3, 4, and 5) which have been released in sequence beginning in 2002. Version 5 of the Technical Guide (by Robert A. Jarrow, Sean P. Klein, Mark Mesler, and Donald R. van Deventer, March 2011) includes ROC accuracy ratio comparisons for legacy ratings and the version 5 Jarrow-Chava default probabilities discussed in Chapter 16. The testing universe was much smaller than the 1.76 million observations and 2,046 company failures in the full KRIS version 5 sample. The rated universe included only 285,000 observations and 276 company failures. The results in Figure 2 show that the ROC accuracy ratio for the version 5 Jarrow-Chava model is 7-10 percentage points more accurate than legacy ratings at every single time horizon studied. Note that the accuracy reported for month N is the accuracy of predicting failure in month N of those companies who survived the period from period 1 through period N-1:

Figure 2



Problems with Legacy Ratings in the 2007 to 2010 Credit Crisis

One of the reasons for the moral hazard in accuracy self-assessment by the rating agencies is the conflicting pressures on management of the agencies themselves. Management is put in the situation where the revenue growth "demanded" by shareholders of the firm in the short run was in conflict with the need for accuracy in ratings to preserve the "ratings franchise of the firm," in the words of our rating agency friend mentioned above. This conflict was examined in detail by the U.S. Senate in the wake of the credit crisis in a very important research report led by Senator Carl Levin:

Figure 3

United States Senate
PERMANENT SUBCOMMITTEE ON INVESTIGATIONS
Committee on Homeland Security and Governmental Affairs

Carl Levin, Chairman
Tom Coburn, Ranking Minority Member

WALL STREET AND THE FINANCIAL CRISIS: Anatomy of a Financial Collapse

MAJORITY AND MINORITY STAFF REPORT

PERMANENT SUBCOMMITTEE ON INVESTIGATIONS

UNITED STATES SENATE

April 13, 2011

We list the conclusions of the Levin Report with respect to the rating agencies in this excerpt, which beings on page 245:

A. Subcommittee Investigation and Findings of Fact

For more than one year, the Subcommittee conducted an in-depth investigation of the role of credit rating agencies in the financial crisis, using as case histories Moody's and S&P. The

Subcommittee subpoenaed and reviewed hundreds of thousands of documents from both companies including reports, analyses, memoranda, correspondence, and email, as well as documents from a number of financial institutions that obtained ratings for RMBS and CDO securities. The Subcommittee also collected and reviewed documents from the SEC and reports produced by academics and government agencies on credit rating issues. In addition, the Subcommittee conducted nearly two dozen interviews with current and former Moody's and S&P executives, managers, and analysts, and consulted with credit rating experts from the SEC, Federal Reserve, academia, and industry. On April 23, 2010, the Subcommittee held a hearing and released 100 hearing exhibits. In connection with the hearing, the Subcommittee released a joint memorandum from Chairman Levin and Ranking Member Coburn summarizing the investigation into the credit rating agencies and the problems with the credit ratings assigned to RMBS and CDO securities.

The memorandum contained joint findings regarding the role of the credit rating agencies in the Moody's and S&P case histories, which this Report reaffirms. The findings of fact are as follows.

1. **Inaccurate Rating Models.** From 2004 to 2007, Moody's and S&P used credit rating models with data that was inadequate to predict how high risk residential mortgages, such as subprime, interest only, and option adjustable rate mortgages, would perform.

2. Competitive Pressures. Competitive pressures, including the drive for market share and need to accommodate investment bankers bringing in business, affected the credit ratings issued by Moody's and S&P.

3. Failure to Re-evaluate. By 2006, Moody's and S&P knew their ratings of RMBS and CDOs were inaccurate, revised their rating models to produce more accurate ratings, but then failed to use the revised model to re-evaluate existing RMBS and CDO securities, delaying thousands of rating downgrades and allowing those securities to carry inflated ratings that could mislead investors.

4. Failure to Factor in Fraud, Laxity, or Housing Bubble. From 2004 to 2007, Moody's and S&P knew of increased credit risks due to mortgage fraud, lax underwriting standards, and unsustainable housing price appreciation, but failed adequately to incorporate those factors into their credit rating models.

5. Inadequate Resources. Despite record profits from 2004 to 2007, Moody's and S&P failed to assign sufficient resources to adequately rate new products and test the accuracy of existing ratings.

6. Mass Downgrades Shocked Market. Mass downgrades by Moody's and S&P, including downgrades of hundreds of subprime RMBS over a few days in July 2007, downgrades by Moody's of CDOs in October 2007, and actions taken (including downgrading and placing securities on credit watch with negative implications) by S&P on over 6,300 RMBS and 1,900 CDOs on one day in January 2008, shocked the financial markets, helped cause the collapse of the subprime secondary market, triggered sales of assets that had lost investment grade status, and damaged holdings of financial firms worldwide, contributing to the financial crisis.

7. Failed Ratings. Moody's and S&P each rated more than 10,000 RMBS securities from 2006 to 2007, downgraded a substantial number within a year, and, by 2010, had downgraded many AAA ratings to junk status.

8. Statutory Bar. The SEC is barred by statute from conducting needed oversight into the substance, procedures, and methodologies of the credit rating models.

9**. Legal Pressure for AAA Ratings.** Legal requirements that some regulated entities, such as banks, broker-dealers, insurance companies, pension funds, and others, hold assets with AAA or investment grade credit ratings, created pressure on credit rating agencies to issue inflated ratings making assets eligible for purchase by those entities.

These "findings of fact" summarize briefly the reasons why legacy ratings have been called into question generally, even though many of the specific comments above refer to ratings of mortgage-backed securities and collateralized debt obligations, which we discuss later in this chapter.

The Jarrow-Merton Put Option and Legacy Ratings

We end this section by noting the big picture objectives of Chapters 1 and 2. We want to measure market risk, interest rate risk, liquidity risk and credit risk on an integrated basis and we want to be able to do something about it if we are uncomfortable with the level of risk we have. We want to be able to compute the value of the Jarrow-Merton put option (say in one year at the par value of the liabilities of the financial institution) on the value of company assets that is the best coherent estimate of the risk the institution has taken on.

In simpler language, we want to know "what's the hedge" if our risk is too large. The use of credit ratings leaves us short of this goal line. The link between ratings and pricing, valuation and hedging is very tenuous in most financial institutions. We now turn to another approach that once seemed promising but which has been exposed as inadequate in the light of the 2007 to 2010 credit crisis: the Merton model of risky debt.

The Merton Model of Risky Debt

In Geneva in December 2002, Robert Merton was speaking to a large conference audience of 400 risk managers when a question arose from the back of the room. "Professor Merton," the questioner started, "what else do you recommend to financial institutions using commercial versions of your credit model?" After a long pause, Professor Merton said, "Well, the first thing you have to remember is that the model is 28 years old." This is both the strength and the weakness of the Merton model. As it ages it is better understood. At the same time, more extensive credit model testing like that in Chapter 16 have made it apparent that the model simply has not performed well from an accuracy point of view. This view is shared by Professor Merton himself, who was once of a panel of four Nobel Prize winners who awarded the Harry Markowitz Award for best paper of 2011 to Campbell, Szilagyi and Hilscher (2011) for their paper in the Journal of Investment Management which concluded the following about their reduced form credit model: "Our model is… between 49% and 94% more accurate than [a Merton distance to default model]."

Many authors have examined this topic and reached the same conclusions that we reached in Chapter 16 about the superior performance of reduced form models versus the Merton model. The

Chapter 16 results were originally reported in Kamakura Risk Information Services Technical Guides 2, 3, 4 and 5 released between 2002 and 2010. Most of the following papers have been in circulation in banking and academic circles for more than a decade:

Bharath, S. and Shumway, T. (2008). "Forecasting default with the Merton distance to default model," Review of Financial Studies 21, 1339–1369.

Campbell J.Y., Hilscher, J., and Szilagyi, J. (2008). "In search of distress risk," Journal of Finance 63, 2899–2939.

Chava, S. and Jarrow, R.A. (2004). "Bankruptcy prediction with industry effects," Review of Finance 8, 537–569.

Hilscher, J. and Wilson, M. (2011). "Credit ratings and credit risk," unpublished paper, Brandeis University and Oxford University.

Shumway, T. (2001). "Forecasting bankruptcy more accurately: A simple hazard model," Journal of Business 74, 101–124.

Shumway and Bharath (2008) conducted an extensive test of the Merton approach. They tested two hypotheses. Hypothesis 1 was that the Merton model is a "sufficient statistic" for the probability of default, i.e. a variable so powerful that in a logistic regression like the formula in the previous section no other explanatory variables add explanatory power. Hypothesis 2 was the hypothesis that the Merton model adds explanatory power even if common reduced form model explanatory variables are present. They specifically tested modifications of the Merton structure partially disclosed by commercial vendors of the Merton model. The Shumway and Bharath (2008) conclusions, based on all publicly traded firms in the United States (except financial firms) using quarterly data from 1980 to 2003 are as follows:

- "We conclude that the …Merton model does not produce a sufficient statistic for the probability of default…,"
- "Models 6 and 7 include a number of other covariates: the firm's returns over the past year, the log of the firm's debt, the inverse of the firm's equity volatility, and the firm's ratio of net income to total assets. Each of these predictors is statistically significant, making our rejection of hypothesis one quite robust. Interestingly, with all of these predictors included in the hazard model, the …Merton probability is no longer statistically significant, implying that we can reject hypothesis two,"
- "Looking at CDS implied default probability regressions and bond yield spread regressions, the …Merton probability does not appear to be a significant predictor of either quantity when our naïve probability, agency ratings and other explanatory variables are accounted for."

The Campbell, Hilscher, and Szilagyi (2008) paper was done independently on a separate monthly data set and reaches similar conclusions, all consistent with the results in Chapter 16.

Any rational analyst who has read these papers and replicated their tests on a large data set (more than 500,000 monthly observations is recommended) will reach the same conclusions and would issue a MODEL RISK ALERT, because the Merton model alone is simply not an accurate tool for credit risk assessment.

Why then include the Merton model in this chapter? First of all, this is a chapter on credit risk management antiques, and the Merton model is an antiqued that held the affections of one of the authors of this book for more than a decade. That author was ultimately convinced by this comment by John Maynard Keynes (quoted by Malabre) that he should turn to other models "When the facts change, I change my mind. What do you do, sir?"

That being said, the intuition of the Merton model is attractive and sometimes useful in thinking about credit risk issues, even though the default probabilities derived from the model are much less accurate than the reduced form approach. For that reason, we summarize the attributes of the model and its derivation here.

The Intuition of the Merton Model

The intuition of the Merton model is dramatically appealing, particularly to one of the authors, a former treasurer at a major bank holding company who used the model's insights in thinking about financial strategy. Merton [1974] was a pioneer in options research at MIT at the time that Black and Scholes [1973] had just published their now well-known options model. One of the reasons for the enduring popularity of the Merton credit model is the even greater popularity of the Black-Scholes model. The Black-Scholes model is simple enough that it can be modeled in popular spreadsheet software. Because of this simplicity, Hull [1993] and Jarrow and Turnbull [1996] note with chagrin that traders in interest rate caps and floors (a MODEL RISK ALERT that we issue in later chapters) frequently use the related Black commodity options model, even though it implicitly assumes interest rates are constant. In the same way, extensive use of the Merton model has persisted even though it is now clear that other modeling techniques for credit risk are much more effective.

Merton's great insight was to recognize that the equity in a company can be considered an option on the assets of the firm. More formally, he postulated that the assumptions which Black and Scholes applied in their stock options model can be applied to the assets of corporations. Merton [1974] assumes that the assets of corporations are completely liquid and traded in frictionless markets. MODEL RISK ALERT: Cash is liquid, but most other assets are not, invalidating many of the model's conclusions. Merton assumed that interest rates are (MODEL RISK ALERT) constant, and that the firm has only (MODEL RISK ALERT) one zero coupon debt issue outstanding. He assumed, like Black and Scholes, a (MODEL RISK ALERT) one period world where management choices were made only at (MODEL RISK ALERT) time zero and the consequences of those decisions would become apparent at the end of the single period.

Given these assumptions, what happens at the end of the single period at time T? If the value of company assets V(T) is greater than the amount of debt that has to be paid off, K, then the excess value V(T) – K goes to equity holders in exactly the same way as if the company were liquidated in the perfectly liquid markets Merton assumes. If the value of company assets is less than K, then equity holders get nothing. More important, from a Basel II and III perspective, debt holders have a loss given default of K – V(T). MODEL RISK ALERT: Loss given default is built into the Merton model, an insight many market participants have overlooked.

The payoffs on the common stock of the firm are identical to that of a European call option at current time t with these characteristics:

- The option has maturity T-t, the same maturity as the debt. The original maturity of the option was T-0.
- The option has a "strike price" equal to the amount of debt K
- The option's value is determined by the current value of company assets V(t) and their volatility σ

We label this call option, the value of the company's equity, C(V(t),t,T,K). Then Merton invokes an argument of Nobel prize winners Modigliani and Miller[1958]. Merton assumes that the value of company assets equals the value of its equity plus the value of its debt. That means we can write the value of the company's debt

$$D(V(t),t,T,K) = V(t) - C(V(t),t,T,K)$$

That, in essence, is the heart of the Merton model intuition. Alternative "versions" of the Merton model are relatively minor variations around this theme. Many "versions" in fact are just alternative methods of estimating the Merton parameters.

In the next few sections, we summarize some of the variations on the Merton model. After that, we turn to practical use of the Merton model.

The Basic Merton Model

In order to be precise about the Merton model and related default probabilities, we need to be more precise about the assumptions used by Black, Scholes and Merton[1].

r	Interest rates are constant at a continuously compounded rate r
σ	The value of company assets has a constant instantaneous variance of the return on company assets, $\sigma(V(t),t)^2=\sigma$
K	K is the amount of zero coupon debt payable at maturity, time T

Black, Scholes and Merton assume that the value of company assets follows a lognormal distribution and that there are no dividends on these assets and no other cash flows from them during the period. Formally, we can write the evolution of the value of company assets over time as

[1] For readers who would like a more detailed explanation of options concepts, the authors recommend Ingersoll[1987], Hull[1993] and Jarrow and Turnbull [1996].

$$\frac{dV(t)}{V(t)} = \alpha dt + \sigma dW(t)$$

W(t) is normally distributed with mean zero and variance t—it is the "random number generator" which produces shocks to the value of company assets. The magnitude of these shocks is scaled by the volatility of company assets, σ. The term α is the drift in the value of company assets. We discuss its value later in this chapter, but we don't need to spend time on that now. From this formula, we know that V(t), the value of company assets, is lognormally distributed. Taking advantage of these facts, the value of company assets at a future point in time can be written like this:

$$V(t) = V(0)e^{[(\alpha - \frac{1}{2}\sigma^2)t + \sigma W(t)]}$$

We can use this equation to get the value of company assets and its distribution at maturity T. W(t), as mentioned above, is normally distributed with mean zero and variance t (standard deviation of √t). This allows us to get the well-known "distance to default" in the Merton model. We can do this by first rewriting the formula for the value of company assets to get the instantaneous return on company assets. The return on company assets is normally distributed because W is normally distributed:

$$y(t) = \frac{1}{t}\left[(\alpha - \frac{1}{2}\sigma^2)t + \sigma W(t)\right]$$

The distance from default is the number of standard deviations that this return is (when t=T, the date of maturity) from the instantaneous return just needed to "break even" versus the value of debt that must be paid, K. We can write this breakeven return as y(K,T) or

$$y(K,T) = \frac{1}{T}\ln\left[\frac{K}{V(0)}\right]$$

The standard deviation of the return on company assets y(t) when t=T is σ /√T so we can write the distance to default z(T) as

$$z(T) = \frac{y(T) - y(K,T)}{\frac{\sigma}{\sqrt{T}}}$$

Later in this chapter, we discuss how this distance to default or the Merton default probabilities themselves can be mapped to actual historical default rates. In order to use the Merton default probabilities, we need to evaluate α. We turn to that in the next section.

Making use of the derivation so far, Merton shows that the value of company equity C(V(0),0,T,K) is equal to the value of a Black-Scholes call option:

$$C[V(0),0,T,K] = V(0)N(d_1) - KP(0,T)N(d_2)$$

N() is the cumulative normal distribution function that can be found in any popular spreadsheet package. The variable d_1 = ln[V(0)/KP(0,T)], and $d_2 = d_1$ - σ√(T-0). The expression P(0,T) is the zero coupon bond price of $1 paid at time T, discounting at the constant risk-free interest rate r. Using the Modigliani and Miller argument that the value of company assets must equal the sum of the value of this call option the value of the debt D gives us

$$V(0) = C[V(0),0,T,K] + D[V(0),0,T,K]$$

Rearranging this gives the Merton formula for risky debt. The second expression takes advantage of the symmetrical nature of the normal distribution.

$$D[V(0),0,T,K] = V(0) - V(0)N(d_1) + KP(0,T)N(d_2) = V(0)N(-d_1) + KP0,T)N(d_2)$$

As we have stated throughout this book, our objective is to come up with an integrated approach to market risk, interest rate risk, liquidity risk and credit risk that provides pricing, hedging, and valuation. Ultimately, we want to be able to value the Jarrow-Merton put option on the assets of our company or business unit that provides the best measure of risky. Therefore it is very important to note that all of the Black-Scholes hedging concepts can be applied to the Merton model of risky debt, in theory, but these hedges have been proven to be wildly inaccurate in practice.

The **delta** of risky debt shows the change in the value of risky debt that results from a change in the value of company assets:

$$Delta = \frac{\partial D}{\partial V} = 1 - N(d_1) > 0$$

The value of risky debt rises when the value of company assets V rises. The gamma of risky debt is the second derivative of the Merton risky debt formula with respect to the value of company assets:

$$Gamma = \frac{\partial_2 D}{\partial V^2} = \frac{-f(d_1)}{V\sigma\sqrt{T}} < 0$$

Similarly, we can derive derivatives for changes in time, volatility, and the level of interest rates, in a fashion parallel to that of the Black-Scholes model.

$$Theta = \frac{\partial D}{\partial t} = \frac{Vf(d_1)\sigma}{2\sqrt{T}} + rKP(0,T)N(d_2) > 0$$

$$Vega = \frac{\partial D}{\partial \sigma} = -V\sqrt{T}\, f(d_1) < 0$$

$$Rho = \frac{\partial D}{\partial r} = -TKP(0,t)N(d_2) \le 0$$

Note that we can hedge our position in risky debt by taking the appropriate positions in the assets of the company, just like a trader of stock options can hedge his position in options with a replicating position in the underlying common stock. Similarly, in the Merton model, both the value of the debt and the value of the common stock are functions of the value of company assets. This means that a position in the debt of the company can be hedged with the appropriate "short" position in the common stock of the company. MODEL RISK ALERT: Note that when the value of common stock approaches zero this hedge will break down in practice, for good sound theoretical reasons. The value of debt can keep declining as the value of company assets declines, but the stock price can't go below zero.

Finally, note that we can do pricing, hedging and valuation in the Merton model without any discussion whatsoever of default probabilities and loss given default. They are totally unnecessary, since the valuation formulas take into account all possible values of company assets, the probability of default at each asset level, and the loss given default at each asset level. This is obvious from a theoretical point of view but (MODEL RISK ALERT) it is often forgotten by practitioners who follow this process:

1. Estimate the Merton model of risky debt
2. Derive the Merton default probability
3. Apply this default probability to calculate the value of a debt instrument

If our objective is valuation and hedging, and if the Merton model is true (it isn't), steps 2 and 3 are unnecessary given the sound theoretical structure that Merton has laid out. Before turning to the subject of default probabilities, we emphasize this point by discussing valuation of a bond with many payments.

Valuing Multi-payment Bonds with the Merton Model of Risky Debt

How do we apply the single period Merton model to value a bond with many payments, say a ten year bond with semiannual payments? To do the valuation, we need 20 zero coupon bond prices to apply the present value techniques of Chapter 4. Remember, if we have a smoothed yield curve using the techniques in Chapter 5, we don't need the Merton model for valuation because we can pull the 20 zero coupon bond prices we need directly from the yield curve with need to resort to the Merton theory. When we say "use the Merton model" for valuation, what we are really saying is this—how do we fit the parameters of the model to coupon-paying bonds with observable prices? We will return to this issue below when we discuss parameter fitting. Let's say we have a three year bond with an observable price, a five year bond with an observable price and a ten year bond with an observable price. How can we fit the Merton model to this data? If all the bonds have the same payment dates (say June 15 and December 15) then we can guess the value of debt to be paid off K, the value of company assets V(0), and the volatility of company assets σ. We are immediately confronted with a conflict with model assumptions because we will see from the smoothed risk-free yield curve that the value of the continuously compounded zero yield r will not be constant; there will be 20 different values, one for each payment date.

At this point users of the Merton model get to the quick and dirty nature of application that is consistent with using the constant interest rate Black model to value interest rate caps! Given the 20 values of the risk free rate, in our example we have three pricing equations for the bonds and three variables to solve for, K, V(0), and σ. At the same time, theory says we should also be fitting the model to the stock price, but we don't know which of the 20 maturities we are using is most relevant to the stock price.

Let's squint our eyes, hold our nose, and get this done. As a practical matter, the authors would fit the model to the shortest maturity observable debt issue, the longest observable debt issue, and stock price. For any pricing error that arises, we can "extend" the model just like we extended the Vasicek term structure model to get the Hull and White model. We can use the techniques of Chapter 14 to find the "best fitting" Merton parameters just like we did for term structure model parameters.

When we do this, what should we "extend" to fit the observable term structure for say Hewlett Packard using the Merton model. Users can extend K, the value of debt to be paid off, to be different at each maturity. Another option is to extend the volatility of company assets σ. A final option is to allow for drift in the value of company assets. All of these are options. Van Deventer and Imai [2003] show using data from First Interstate Bancorp new issue spreads that the pricing error can be large when using the Merton model without some form of extension.

Estimating the Probability of Default in the Merton Model

As shown above, we don't need the Merton model default probabilities for valuation, pricing and hedging in the Merton framework. The model implicitly considers all possible default probabilities and the associated loss given default for each possible ending value of company assets V(T). As an index of risk, however, it was once hoped that Merton default probabilities would be an accurate measure of default risk. As we saw in Chapter 16, this hope did not materialize.

One of the curious features of the Black-Scholes options model in the Merton context is that the drift term in the return on company assets α doesn't affect the valuation formulas just like the drift in the return on a common stock doesn't affect its option pricing. We do need to know α when it comes to the probability of default on an empirical or statistical basis (as opposed to the risk neutral default probabilities that are implicitly embedded in the valuation formulas).

Jarrow and van Deventer [1998] derive the empirical default probabilities using Merton's intertemporal capital asset pricing model. That model recognizes that the value of company assets of all companies are not independent. They will be priced in a market that recognizes the benefits of diversification, a market that knows lending to a suntan lotion company and an umbrella maker diversifies away weather risk. These benefits of correlation in the returns on company assets affect the expected return on the assets. Jarrow and van Deventer note that, when interest rates are constant, the expected return on the assets of company i in Merton's [1973] equilibrium asset pricing model must be

$$\alpha_i = r + \frac{\sigma_i \rho_{iM}}{\sigma_M}(\alpha_M - r)$$ where

r is the continuously compounded risk free rate, the subscript M refers to the "market" portfolio or equivalently, the portfolio consisting of all assets of all companies in the economy, and ρ_{iM} denotes the correlation between the return on firm i's asset value and the return on the market portfolio of all companies' assets.

Given this formula for α, the probability of default in the Merton model becomes

$$\text{Probability(Default)} = \Pr obability(V_i(T) < K) = N\left(\frac{\ln(K / V_i(0)) - \mu_i T}{\sigma_i \sqrt{T}}\right)$$

The variable μ is defined as

$$\mu_i = -\frac{1}{2}\sigma_i^2 + (1 - b_i)r + b_i a_M$$

where

$b_i = \dfrac{\sigma_i \rho_{iM}}{\sigma_M}$ is the ith firm's beta, which measures the degree to which the return on the assets of the ith company are correlated with the return on the assets of all companies.

When interest rates are not constant, α takes on a more complex form given by Merton.

Implying the Value of Company Assets and Their Return Volatility σ

The market value of company assets in the Merton model is not observable, nor is its volatility. If the asset volatility is known, we can solve implicitly for the value of company assets V(0) by solving the equation relating the total value of the company's equity (stock price S times number of shares outstanding M) to the Black-Scholes call option formula:

$$MS = C[V(0), 0, T, K] = V(0)N(d_1) - KP(0,T)N(d_2)$$

The volatility of the return on company assets is not observable, but we can observe the stock price history of the firm and calculate the volatility of the return on the stock σ_c in typical Black-Scholes fashion. MODEL RISK ALERT: For highly distressed companies, the stock price may only be observable intermittently, it can remain near zero for an extended period of time, and it can bounce randomly between bid and offered levels. All of these facts lead to inaccurate volatility assessment and inaccurate default probabilities. MODEL RISK ALERT: Let's assume we are true believers in the Merton model and assume all of these real world complications don't exist. In Appendix A, we use Ito's lemma in the same way we did earlier in the book, when we introduced term structure models of interest rate movements. This allows us to link the observable volatility of equity returns σ_c to the volatility of the returns on company assets σ:

$$\frac{\sigma_C C}{C_V V} = \sigma$$

Together with the call option formula above, we have two equations in two unknowns V(0) and σ. We solve for both of them and create a time series of each. From these time series on the value of company assets for all companies, we can calculate the betas necessary to insert in the default probability formula above. The expected return on the assets of all companies is usually estimated as a multiple of the risk free rate in such a way that model performance (as we discussed in Chapter 16) is optimized.

Mapping the Theoretical Merton Default Probabilities to Actual Defaults

Falkenstein and Boral[2000] and Lando ["Credit Risk Modeling: Theory and Applications," 2004], in his excellent book, note the theoretical values of Merton default probabilities produced in this manner typically result in large bunches of companies with very low default probabilities and large bunches of companies with very high default probabilities. In both cases, the Merton default probabilities are generally much higher than the actual levels of default. The best way to resolve this problem is to use the theoretical Merton model as an explanatory variable in the reduced form framework, using logistic regression. This is one of the models ("KDP-ms") offered by Kamakura Risk Information Services. As long as this route is taken, however, as Chapter 16 makes clear, it is better to simply abandon the Merton default probability completely because it adds little or no incremental explanatory power to a well-specified reduced form model.

As we note in Chapter 16, this mapping of Merton default probabilities does not change the ROC accuracy ratio, the standard statistical measure of model performance for 0/1 events like bankruptcy. The mapping process does improve the Falkenstein and Boral test we employed in Chapter 16.

The Merton Model When Interest Rates are Random

As mentioned in the introduction to this chapter, one of the earliest concerns expressed about the accuracy of the Merton model was its reliance on the assumption that interest rates are constant, exactly as in the Black-Scholes model. Shimko, Tejima and van Deventer [1993] solved this problem by applying Merton's option model with random interest rates to the risky debt problem. It turns out that the random interest rates version of Black-Scholes can be used with many of the single factor term structure models which we reviewed in Chapter 13, which is one of the reasons for their importance in this volume.

The Merton Model with Early Default

Another variation on the Merton problem is focused on another objection to the basic structure of the Merton model: default never occurs as a surprise. It either occurs at the end of the period or it doesn't, but it never occurs prior to the end of the period. Analysts have adopted the "down and out" option in order to trigger early default in the Merton model. In this form of the model, equity is a "down and out" call option on the assets of the firm. Equity becomes worthless if the value of company assets falls to a certain barrier level, often the amount of risky debt that must be repaid at the end of the period K. Adopting this formulation, the total value of company equity again equals an option on the assets of the firm, but this option is a down-and-out option. Lando [2004] discusses the variations on this approach extensively. Unfortunately, these variations have little or no ability to improve the accuracy of the default probabilities implied by the model by the standards of Chapter 16.

Loss Given Default in the Merton Model

As mentioned earlier in this chapter, analysts are divided between those who seek to use the Merton model for valuation, pricing and hedging, and those who use it to estimate default probabilities and then in turn try to use the default probabilities and a separate measure of loss given default for pricing, valuation and hedging outside the Merton model structure. The former group is truer to the model, because the Merton model incorporates all potential probabilities of default and loss given default in its valuation formulas. For this reason, there is no need to specify default probabilities and loss given default for valuation the model—they are endogenous, just like the Black-Scholes model considers all potential rates of return on common stock in the valuation of an option on the common stock.

Loss given default is the expectation (with respect to the value of company assets V) for the amount of loss K-V(T) which will be incurred given that the event of default has occurred. The derivation of loss given default in the model is left as an exercise for the reader.

Copulas and Correlation Between the Events of Default of Two Companies

The Merton model is normally fitted to market data in a univariate manner, one company at a time. As a result, there is no structure in the risky debt model itself which describes how the events of default of two companies would be correlated. Because this correlation is not derived from the model's structure, many analysts *assume* a correlation structure[2]. This contrasts sharply with the implementation of the reduced form models in Chapter 16, where the correlation between the events of default of all companies is jointly derived when the model is fitted.

Back to the Merton case. Let's assume that we need to estimate the correlation that causes two companies to have defaults and default probabilities that are not independent. If we are true to the

[2] Private conversation with Robert Jarrow, February 4, 2004.

model we could make the argument that the returns on the value of company assets for both companies has correlation σ:

$$\frac{dV_a}{V_a} = \mu_a dt + \sigma_a dZ_a$$

$$\frac{dV_b}{V_b} = \mu_b dt + \sigma_b dZ_b$$

$$(dZ_a dZ_b) = \rho dt$$

The correlation of the returns on the assets of both companies can be estimated using the formulas above from the historical implied series on the value of company assets. This is the proper specification for correlation in the "copula" method of triggering correlated events of default. We return to this issue in Chapter 20, and the results of our investigation in that chapter are not for the squeamish.
There are a number of other ways of doing the same thing. If the analyst has an observable probability that both companies will default between time t and T, then the value of the correlation coefficient ρ can be implied from the Merton model default probability formulas and the joint multi-variate normal distribution of the returns on company assets for the two firms. There is no reason to assume that the pairwise correlation between all pairs of companies is the same—in fact, this is a serious error in analyzing collateralized debt obligation values, as we discuss in Chapter 20.

There are two more elegant alternatives that the authors prefer. First choice is the reduced form models, which we examine in much greater detail in the following chapters, based on the introduction in Chapter 16. A second choice, which stays true to the Merton theory, is to specify the macro-economic factors driving the returns on company assets and to allow them to drive the correlations in the events of default. This approach is much less accurate.

Problems with the Merton Model: Summing Up

Van Deventer and Imai [2003] extensively summarize the strengths and concerns with each of the models they review, including the Merton and Shimko, Tejima, van Deventer random interest rates version of the Merton model. In their view, the key factors that contribute to the model performance issues in the Merton model can be summarized as follows:

- The model implicitly assumes a highly volatile capital structure by positing that the dollar amount of debt stays constant while the value of company assets varies randomly
- The model (in traditional form) assumes interest rates are constant, but they are not
- The model assumes that company assets are traded in a perfectly liquid market so the Black-Scholes replication arguments can apply. In reality, most corporate assets are very illiquid, which makes the application of the Black-Scholes model inappropriate in both theory and practice
- The model does not allow for early default—default only occurs at the end of the period

- The model's functional form is very rigid and does not allow the flexibility to incorporate other explanatory variables in default probability prediction.

Finally from a portfolio management point of view, one prefers a model where the correlation in defaults can be derived, not assumed. As we have emphasized throughout this volume, we are seeking pricing, valuation and hedging at both the transaction level and the portfolio level. The Merton model leaves wondering "how do I hedge." Even more serious, we showed in Chapter 16 that many naïve models of default have a greater accuracy in the ordinal ranking of companies by riskiness than the Merton model. This result alone has led a very large number of financial institutions and regulators to abandon their experimentation with the Merton idea. It is simply too much to hope that the value of debt and equity of a firm are driven by only one random factor, the value of company assets. We proved in Chapter 16 that the model simply doesn't work, consistent with the results of the many authors listed above.

Appendix

In the Merton model of risky debt, the volatility of returns on company assets (not the volatility of the value of assets) σ is a critical parameter but it is not observable.

We can observe the historical volatility of returns on the company's common stock (note, this is NOT the same as the volatility of the stock price or standard deviation of the stock price). By linking this observable historical volatility of common stock returns by formula to the historical volatility of company asset returns, we can derive the value of the volatility of company asset returns σ.

Assumptions

The value of company assets V follows the same stochastic process as specified in Black Scholes:

$$\frac{dV}{V} = \alpha dt + \sigma dZ$$

We will use Ito's lemma to expand the formula for the value of equity, the Black-Scholes call option C[V(0),0,T,K], and then link its volatility to the volatility of company assets σ.

Using Ito's Lemma to Expand Changes in the Value of Company Equity

Merton's assumptions lead to the conclusion that the value of company assets is a call option on the assets of the firm C[V(0),0,T,K]. Using Ito's lemma as we did in the chapter on term structure models, we can write the stochastic formula for changes in stock price as a function of V:

$$dC = C_V dV + \frac{1}{2} C_{VV} (dV)^2 dt + C_t dt$$

Substituting for dV gives

$$dC = C_V (V\alpha dt + V\sigma dZ) + \frac{1}{2} C_{VV} (dV)^2 dt + C_t dt$$

We can then rearrange, separating the drift from the stochastic terms:

$$dC = \left[\; C_V Vrdt + \frac{1}{2} C_{VV} (dV)^2 dt + C_t dt \;\right] + C_V V\sigma dZ$$

The equation above is for the change in stock price C. The volatility of stock price (not return on the stock price) is $C_V V\sigma$. We can get the expression for the volatility of the return on the stock price by dividing both sides by stock price C:

$$\frac{dC}{C} = \frac{\left[\; C_V Vrdt + \frac{1}{2} C_{VV} (dV)^2 dt + C_t dt \;\right]}{C} + \frac{C_V V\sigma}{C} dZ$$

Therefore, the market convention calculation for the volatility of the return on common stock σ_C (which is observable given the observable stock price history) is

$$\sigma_C = \frac{C_V V\sigma}{C}$$

We solve this equation for σ to express the volatility of the return on the assets of the firm as a function of the volatility of the return of the firm's common stock:

$$\frac{\sigma_C C}{C_V V} = \sigma$$

This formula can be used for σ in the Merton model of risky debt. MODEL RISK ALERT: While this approach is fine in theory, it does not work well in practice. Volatilities estimated in this way are distorted by a number of common phenomenon: stock prices near constant as they near zero, intermittent trading, trading at bid and offered prices, and so on.