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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 31:
Foreign Exchange Markets

In this chapter, we continue to progress toward our integrated measure of interest rate risk, market risk, liquidity risk and credit risk using the Jarrow-Merton put option concept from Chapter 1. A high percentage of smaller financial institutions have the luxury of financial assets and liabilities that are almost all denominated in a single currency. In fact, due in part to the advent of the Euro, the percentage of financial institutions in this situation is rising. Nonetheless, for most of the largest financial institutions foreign currency risk is a critical element of total risk. As van Deventer and Imai [2003] showed with respect to the Korean Development Bank and many other issuers, credit risk and foreign exchange rates of the home country can move very strongly together.

The Heath Jarrow and Morton term structure model approach that we have taken in Chapters 6 through 9 again provides a sound foundation with which to approach foreign currency denominated securities and derivatives. This chapter is heavily based on the HJM approach to FX options valuation of Amin and Jarrow [1991, "Pricing foreign currency options under stochastic interest rates"]. The variety of foreign exchange (FX) related securities is great, but we will limit ourselves to an introductory view of the HJM approach's implications for FX analysis. In this chapter, we concentrate solely on foreign exchange forward contracts and European options on the spot foreign exchange rate. Foreign exchange futures contracts and options on FX futures involve a combination of the techniques in this chapter and those in Chapters 22 and 23, but we leave such analysis as advanced exercises for interested readers. Jarrow and Turnbull [1996, chapter 11] is an excellent reference on this topic.

The realism of the HJM term structure model framework is essential for modeling of foreign currency derivatives in order to minimize the model error inherent in the legacy versions of FX options formulas, which assumed that interest rates in both countries are constant.

Setting the Stage: Assumptions for the Domestic and Foreign Economies

Initially, we adopt the assumptions of Amin and Jarrow [1991]. Later in the chapter we discuss how the 3 factor HJM model from Chapter 9 can be used in a way consistent with the Amin and Jarrow [1991] approach.

Amin and Jarrow base their model, without loss of generality, on four independent standard Brownian motions, which are commonly used in the HJM framework and in the legacy term structure models reviewed in Chapter 13. Amin and Jarrow assume that interest rates in both countries 1 and 2 are driven by a two factor interest rate model like the model of Chapter 8. They assume that interest rates are driven in country 1 by risk factors 1 and 2. They assume that interest rates in country 2 are driven by risk factors 2 and 3. The presence of risk factor 2 as a driver of interest rates in both countries means that we can model correlated interest rate movements in this framework, whether that correlation is positive or negative. In both country 1 and 2, HJM no arbitrage conditions are imposed and the resulting random evolution of interest rates in each country is similar to the modeling in Chapter 8. The authors note that “the analysis is easily generalized to a finite number of independent Brownian motions with any subset common across the two term structures” (Amin and Jarrow, 1991, page 313). This is consistent with our findings in Chapter 3 that 5 to 10 risk factors drive interest rate movements.

The link between the two countries is the spot exchange rate. Amin and Jarrow assume that all three of the risk factors introduced so far are allowed to impact the spot exchange rate, allowing for correlation with the interest rate risk factors in both countries. To these three factors, they add a fourth independent risk factor which drives the spot exchange rate but not interest rates in either country. Amin and Jarrow note that, since there are 4 sources of risks in this economy, in general it will require four unique assets to have total risk. With the addition of three assumptions (concerning complete markets, existence of a unique asset-specific martingale measure, and independence of the market prices of risk for the four risk factors), the authors derive no arbitrage conditions, showing that the result is consistent with the HJM conditions imposed on one country as in Chapters 6 through 9.

This framework allows the valuation of any European or American style contingent claim in this two country economy. The authors illustrate valuation techniques in the special case where the volatilities of the risk factors are non-stochastic, deterministic functions of time (like our interest rate assumptions of Chapter 6 for the one factor HJM interest rate model). They go on to derive closed form solutions for options on the spot exchange rate, the pricing of forwards and futures, and the pricing of options on forwards and futures.

Foreign Exchange Forwards

The pricing of foreign exchange forwards can be set by an arbitrage argument that doesn't rely on any of the assumptions above. Let's assume it's time *t* and the observable forward price of country 2 currency at time *T* is a function of the spot rate *S* and interest rates in the two countries is *H(t,T,S)*. Let's assume we want to invest, from the perspective of our position in country 1, in country 2 currency for *T* years and maximize our currency 2 holdings at time *T*. We define $P_1 = P_1(t,T)$ as the *T* maturity country 1 zero-coupon bond and $P_2 = P_2(t,T)$ as the country 2 *T* maturity zero-coupon bond. Assume that country 1 is the U.S. and country 2 is Japan. *S* and *H* both represent the dollar cost of one yen. There are two investment strategies:

Strategy 1

1. Borrow P_1 dollars, which will become $1 at time *T*.
2. Sell $1 forward at current time *t* for yen deliverable at time *T*, which will generate yen proceeds of *1/H*

Strategy 2

1. Borrow P_1 dollars.
2. At current time *t*, convert the dollars to P_1/S yen at the spot exchange rate *S*.
3. Invest *P1/S* yen in a zero-coupon yen bond maturing at time *T*, resulting in time *T* yen proceeds of $P_1/(SP_2)$.

To avoid arbitrage, the results of these two strategies must be equal, so the forward rate *H* must equal:

$$H(t,T,S) = \frac{S\,P_2}{P_1}$$

This well-known expression makes it clear that the forward rate depends on all of the risk factors that determine domestic interest rates, foreign interest rates, and the spot foreign exchange rate. With this as background, we turn to numerical valuation of foreign currency derivatives in the HJM framework.

Numerical Methods for Valuation of Foreign Currency Derivatives

Jarrow and Turnbull [1996, Chapter 11] discuss the construction of a lattice to model movements of the spot foreign exchange rate in the special case where (a) the spot foreign exchange rate is lognormally distributed and (b) interest rates are constant. In the Amin and Jarrow example, 4 risk factors drive interest rates in both countries (2 factors each, with one factor affecting both countries) and the spot exchange rate. In Chapter 9, we showed how to construct a bushy tree for interest rates in one country when there are three random factors. Using these insights, how should we value foreign currency derivatives? The elegant mathematics in Amin and Jarrow, even using a restricted number of interest rate factors, is complex. At the same time, the assumptions are too severe to be realistic. This leaves numerical methods as the only realistic alternative for valuation. As in Chapters 6 through 9, the two principal tools to use are Monte Carlo simulation and bushy trees.

To do Monte Carlo simulation with the Amin and Jarrow assumptions, we would simulate the movements of the 4 risk factors at M points in time and over N scenarios. At each point in time for each scenario, the Amin and Jarrow/Heath Jarrow and Morton no arbitrage restrictions give us the complete term structure of interest rates (as we derived in Chapter 8) from the 2 factors affecting each country plus a no-arbitrage simulated value of the spot rate. From the expression for H above, we would also have a complete term structure of forward foreign exchange rates. For most of the securities we have studied in Chapters 19 through 30, we can determine the case flows in the base currency at each point in time. We discount at the correct HJM discount factor (using the value of the "money market fund" equal to the compounded investment in the spot rate from time 0 onward) to get valuation.

What about the 3 factor HJM example of Chapter 9? How would we use that in a foreign exchange context? Let's assume that there are three factors driving interest rates in country 1, three other (non-overlapping) factors driving interest rates in country 2, and one unique factor driving the spot rate. In this special case, it is easy to describe the augmented bushy tree that would result. As in Chapter 9, at time step 1, we would have four nodes on the tree for country 1 interest rates with a complete term structure of interest rates at each node. We would have four independent nodes at time step 1 for country 2 as well. For the value of the spot rate, the random variation of the spot rate is driven by only one factor and the drift term is constrained by the no-arbitrage conditions in Amin and Jarrow. We can model the spot rate movements with an up shift and a down shift, properly constrained by no arbitrage restrictions. Therefore, at time step 1 we will have 2 values for the spot rate, 4 country 1 term structures, and 4 country 2 term structures. From the expression for forward rates H given above, this means that there are 2 x 4 x 4 =32 term structures of forward foreign exchange rates at time step 1.

What about time step 2? Let's assume that the volatility assumptions for the spot rate are such that a bushy tree results (i.e. the lattice does not recombine). At time step 2, then, we will have 4 values for the spot rate, 16 nodes or term structures for country 1, and 16 term structures for country 2. This means that at time step 2 there will be 4 x 16 x 16 = 1,024 term structures for forward foreign exchange rates. At time step 3, there will be 8 x 64 x 64 = 32,768 term structures for forward rates. While this means that a lot of numbers will get thrown around, this is what computers are for: they do what they are told. In the world of modern 64-bit operating systems, it is a routine task.

What happens when some of 3 factors in country 1 and 3 factors in country 2 are shared? And what if some of these factors affect the spot rate? This will affect the construction of the bushy tree (just as it impacts the results of a Monte Carlo simulation) but the result is still completely consistent with the no arbitrage restrictions of Amin and Jarrow/Heath Jarrow and Morton.

Before closing the chapter, we finish with a brief review of the results from a one factor term structure model.

Legacy Approaches to Foreign Exchange Options Valuation

Hilliard, Madura, and Tucker [1991] ('HMT') assume that bond prices in country 1 and country 2 follow the one factor Vasicek stochastic process and that the spot rate follows a lognormal process with the form given in the previous section. MODEL RISK ALERT: We include the HMT results here for expository purposes but the one factor Vasicek model is not accurate enough to meet best practice standards. Prior to Amin and Jarrow [1991], the most popular model of foreign exchange options had been the Garman-Kohlhagen [1983] model, a variation on the Black-Scholes model which assumes that the interest rates in country 1 and country 2 are constant at levels r_1 and r_2. Garman and Kohlhagen show that the value of a European call option maturing at time *T* from the perspective of time *t* to purchase one unit of country 2's currency at a strike price of *K* is:

$$V(t,T,S,r_1,r_2) = Se^{-r_2\tau} N(h) - K e^{-r_1\tau} N(h - \sigma_S\sqrt{\tau})$$

where

$$\tau = T - t$$

and

$$h = \frac{\ln\left(\frac{S}{K}\right) + \tau\left[r_1 - r_2 + \frac{\sigma_S^2}{2}\right]}{\sigma_S\sqrt{\tau}}$$

When interest rates change, the volatility used in the option calculation must change. Interest rates in both countries and their instantaneous correlation with the spot rate have an impact on the variance in the price of forward foreign exchange for delivery at time *T*. HMT show that the value of the call option under the stochastic interest rate case is:

$$\begin{aligned} V(t,T,S,r_1,r_2,K) &= P_1(r_1,t,T)\left[H(t,T,S,r_1,r_2)N(h^*) - KN(h^* - v_h)\right] \\ &= S\,P_2(r_2,t,T)N(h^*) - P_1(r_1,t,T)KN(h^* - v_h) \end{aligned}$$

where

$$h^* = \frac{\ln\left(\frac{H(t,T,S,r_1,r_2)}{K}\right) + \frac{1}{2}v_h^2}{v_h}$$

and v_h is the variance of the price of the forward foreign exchange rate *H* at time *T* from the perspective of time *t*.

Using our standard notation for each country for the volatility of interest rates in each country as of time *T* from the perspective of time *t:*

$$v_i^2(t,T) = \frac{\sigma_i^2}{2\alpha_i}\left[1 - e^{-2\alpha_i\tau}\right]$$

and

$$F_i = \frac{1}{\alpha_i}\left[1 - e^{-\alpha_i\tau}\right]$$

allows us to write v_h as in HMT as follows:

$$v_h^2 = \sigma_s^2\tau + M_2 + M_5 + 2(\rho_{s1}\sigma_s M_1 - \rho_{s2}\sigma_s M_4 - \rho_{12} M_3)$$

where HMT use the notation:

$$M_1 = \frac{\sigma_1}{\alpha_1}\left[\tau - F_1\right]$$

$$M_2 = \frac{\sigma_1^2}{\alpha_1^2}\left[\tau - 2F_1\right] + \frac{v_1^2}{\alpha_1^2}$$

$$M_3 = \frac{\sigma_1\sigma_2}{\alpha_1\alpha_2}\left[\tau - F_1 - F_2 + \frac{1}{\alpha_1 + \alpha_2}\left(1 - e^{-(\alpha_1+\alpha_2)\tau}\right)\right]$$

$$M_4 = \frac{\sigma_2}{\alpha_2}\left[\tau - F_2\right]$$

$$M_5 = \frac{\sigma_2^2}{\alpha_2^2}\left[\tau - 2F_2\right] + \frac{v_2^2}{\alpha_2^2}$$

Implications of a Term Structure Model-Based FX Options Formula

In an environment where the correlation between risk exposures, as popularized by the value-at-risk approach we discuss in later chapters, is a very high priority, the HJM term structure model approach to FX option valuation provides an immense improvement over the constant interest rate-based Garman-Kohlhagen approach:

- o Correlations between country 1 interest rates, country 2 interest rates and the spot rate are explicitly recognized.
- o The option valuation can be stressed tested for changes in any risk factor and any parameter in the Amin and Jarrow/HJM framework.

- The Amin and Jarrow formulation explicitly recognizes that the volatility of the spot rate at time *T*, the option exercise date, is directly affected by the volatility of all risk factors driving interest rates in the two countries. The more traditional approach ignores this source of volatility.

- Interest rate-related hedges stemming from FX options and forward positions can be done using the general HJM approach. The number of hedging instruments has to equal the number of risk factors for a “perfect hedge” (which comes about only when the model is true). We discuss this in more detail in later chapters, in a way fully consistent with all other fixed income and interest rate derivative hedging. Hedges using maturities other than the maturity of the FX option can be calculated and put into place. Like its relative the Black-Scholes model, the Garman-Kohlhagen model's fixed interest rate assumption makes this impossible without some additional (and contradictory) assumptions.

All of these benefits are powerful ones, and they come with an ease of use far superior to complex but inaccurate closed form solutions.

The Impact of Credit Risk on Foreign Exchange Risk Formulas

The HMT FX option formula above, like the Garman-Kohlhagen model, implicitly assumes that our options counterparty is riskless. As JPMorgan discovered in 1998 in a $500 million FX dispute with SK Securities of Korea, it is much more likely that the default probability of the counterparty rises sharply as the amount of money owed on the FX position increases.[1] For this reason it is very important for FX rates to be explicitly modeled as drivers of default probabilities so that that the default probabilities of all international counterparties will vary properly as exchange rates move.

A failure to incorporate this critical link with credit risk will result in a substantial understatement of total risk and of the correlation between the default probabilities in broad classes of borrowers.

We will return to this point in Chapters 36 through 41 frequently as we value the Jarrow-Merton put option as a comprehensive measure of total risk—interest rate risk, market risk, liquidity risk, credit risk and foreign exchange risk.

[1] See van Deventer and Imai [2003] for a discussion of this incident.