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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 28:
Irrational Exercise of Fixed Income Options

As we continue to progress toward the mark-to-market of the entire balance sheet of a financial institution in a way that integrates interest rate risk, market risk, liquidity risk and credit risk, we have to deal with the reality that the vast majority (by transaction count) of financial institutions investments are extensions of credit to individuals or small businesses whose credit worthiness is inseparable from the creditworthiness of the proprietor. Moreover, in Chapter 29, we deal with securitized pools of assets like this where the same problems exist.

One of the reasons that analysis of consumer-related financial products is so complex is because of the issue of the so-called 'irrationality' of consumers. Perhaps the most prominent example from a Wall Street perspective is the mortgage market, which is at the heart of the credit crisis of 2007 through 2010. Mortgages are often prepaid when current mortgage rates are higher than the borrower's mortgage rate, and many borrowers fail to prepay even when rates have fallen far below the rate on their loan. In the credit crisis, there was usually due to the fact that the house price had fallen so low that it was worth less than the principal on the loan. Besides these common examples of "irrationality," partial prepayments are common, something that is impossible in a traditional Black-Scholes context or in the context of the fixed income options analysis we explored in Chapter 21. In both of those cases, options exercise is “all or nothing” so many observers classify partial prepayments by consumers as “irrational.”

As a working assumption, the hypothesis that borrowers are not very intelligent runs contrary to the assumptions behind all developments in modern financial theory over the 40 years since the Black-Scholes options formula was first published. The perception of ‘irrationality’ is simply a short hand description for the fact that lenders and academic researchers do not have enough data to ‘see through’ the individual loan or loan pool data to understand why the individual borrower’s behavior is more rational than it appears at first glance. The authors believe even more strongly that the underlying assumption of rationality is the right one since one of them has failed to refinance his own mortgage loan in the past, even though it must seem irrational from the bank's perspective. The bank was simply unaware that the first edition of this book was more important than a few yen earned from refinancing a Japanese mortgage in the midst of writing the book. Once the book was done, the ‘irrational’ author suddenly regained his rationality from the bank’s perspective.

The need to deal with hard-to-explain consumer behavior is essential for accurate risk management in the insurance, investment management, and banking industries. Insurance companies must deal with the reality that some traditional whole life insurance policy holders will cancel their policies in return for the surrender value when interest rates rise, ‘putting’ the policy back to the insurance company. Investment managers who suffer poor performance know that some customers of the firm will ‘put’ their ownership in a mutual fund back to the investment management company. Bankers who make home equity loans know that some of these loans will be ‘called’ (as in Chapter 27) or prepaid by the borrower. Bankers also know that some time deposit customers will ‘put’ the deposit back to the bank and willingly pay an early withdrawal penalty when rates rise in order to earn more on their funds. Without dealing with the correct degree of irrational consumer behavior behind these products, institutions can neither price properly nor hedge their risk properly. This chapter deals explicitly with a method for doing so.

In the last section of the chapter, we explore the strong synergies between our approach to ‘irrational’ behavior and the credit-adjusted valuation and simulation we have been discussing in earlier chapters.

Analysis of Irrationality: Criteria for a Powerful Explanation

The irrationality problem is so all-pervasive that we deal with the problem in general terms in this chapter. We deal with alternative approaches to irrationality in Chapter 29. This approach has two virtues. Firstly, it will help us deal with the problem from a fresh perspective, unburdened by conventional wisdom. Secondly, armed with a general solution, we can compare this general solution to the conventional wisdom and judge the relative strengths of the two approaches without prejudice. In Chapter 29, armed with the tools of this chapter, we will analyze all the traditional practices of mortgage-related products in light of both this chapter and the truths revealed (or confirmed) by the 2007 to 2010 credit crisis. In this chapter, however, we need to establish principles that will solve problems in insurance, investment management, and in banking. They have to apply equally well to call options, put options, and securities that are callable but have rate caps and floors as well. A few basic principles come to mind:

a. ***A general approach to irrational behavior should take advantage to as great an extent possible of the advances in finance in the last 40 years.***

All derivative pricing theory is based on the premise of no-arbitrage and rational behavior. Behavioral finance has revealed that, in many cases, "rational behavior" leads to exceptions that turn out to be rational after all, if only the academic researchers had made more realistic assumptions in the first place. If all consumers were totally irrational, we would not need to price the options embedded in retail finance products because the exercise of those options would be truly random and uncorrelated with economic events. Portfolio theory would allow us to argue that we can diversify away this random behavior and ignore the option all together. The reality, as we shall see in Chapter 29, however, is that consumer behavior is highly correlated with economic variables and behavior is not totally irrational. It is this mixture of rational and irrational behavior (which in fact is largely due to rational acts based on unobservable information), that makes the problem both important and difficult.

We need, then, an approach that allows us to scale the degree of irrationality to fit the particular product, company, and market at hand. We need an approach where consumer action is partially rational so we can use the first 27 chapters of this book.

b. ***History is not necessarily a good guide to the future, so we want to be able to derive the level of irrationality implied by observable market prices as a supplement to the analysis of historical behavior.***

Wall Street firms have spent millions of dollars examining historical data for clues to consumer behavior as reflected in historical prepayment activity. As we shall see in Chapter 29, this work has been at best partially satisfactory. For financial institutions which need to make an immediate decision about the proper hedge for a security whose market price and local rate sensitivity (i.e. the value change for small changes in interest rates) is observable, the historical approach can be too slow, too inaccurate and too expensive. The parallels in the debate about the use of historical volatility versus implied volatility in the Black-Scholes options model are very strong. We want an approach that gives us the 'implied' level of irrationality. Most bankers who price retail banking products which contain embedded options come to the conclusion that most products are unprofitable if the options risk is fully hedged. Bankers implicitly recognize that consumers are partially rational and that a partial hedge is necessary. How can we use market data to calculate exactly how much rationality is embedded in a given product?

c. ***Any model of irrationality must be able to explain observable phenomenon in pools of consumer related securities: path dependent prepayment behavior, decreased interest rate sensitivity over time ('burnout'), and a lower propensity to exercise embedded options as the financial product nears its expiration.***

d. ***The model must allow for very rapid calculation time and maximum use of analytical solutions for security valuation.***

One approach that meets these criteria is the transactions cost approach, which we explain in the next section.

The Transactions Cost Approach

Over the last 20 years, the rational approach to irrational behavior has steadily gained prominence among both academics (see McConnell and Singh [1994] and Stanton [1995] for early examples) and practitioners as the best method to model consumer behavior. The rational, or transactions cost approach, holds that consumers are rational but exercise their options subject to transactions costs. These transaction costs can be both explicit financial costs (the 'points' from refinancing a mortgage or the penalty for early withdrawal of bank deposits) and more subtle costs that reflect the fact that consumers are maximizing a utility function with more arguments in it than the present value of rational action in one specific aspect of their lives. For example, someone with a life insurance policy that rationally should be canceled and re-initiated at current market pricing may have a serious disease that would cause the applicant to be rejected at the health check on the new policy. The disease is a transactions cost that at least partially blocks rational action. J. Thurston Howell III, the wealthy castaway in the old American comedy "Gilligan's Island," may fail to refinance the mortgage on his ski chalet in Zermatt because his yacht has run aground on some deserted desert island without a Bloomberg screen to keep him abreast of the benefits of refinancing. In the authors' case, the principal amount of our bank time deposits is so small (since other investments are so superior in risk-adjusted yield), that the benefits of early withdrawal of our time deposits, while positive, are smaller than the costs of the time it takes to go to the bank and negotiate the transaction. The cost of our time is a transactions cost. Finally, in the 2007 to 2010 credit crisis, the impact of home prices on both the desire and the ability to prepay a mortgage is a hugely important factor in whether the mortgage repays or not. The "underwater" amount by which the loan balance exceeds the current home price value can be thought of as a large transaction cost as well.

The objective of the transactions cost approach is to firmly divorce rational from irrational behavior. Accordingly, the transactions cost function can be any time dependent function that is not dependent on the level of interest rates. Determinants of the level of transactions costs could be any of the following:

- time to maturity on the security;
- the level of the coupon on the security;
- the fixed dollar cost of exercising the option embedded in the security;
- the dollar opportunity cost of the time it takes to exercise the embedded option;
- the value of the collateral on the loan, relative to the loan balance
- the month of the year, recognizing that the transactions cost of refinancing a home in Alaska in January is higher than the cost in August;
- the date of origination of the security;
- the length of time the security has been outstanding without the embedded option being exercised.

There are an endless number of factors that can go into the transactions cost function. In short, however, as of current time t, all of these factors are non-random functions of time. We call this transactions cost function X and make it, without loss of generality, a function of the remaining time to maturity on the security:

$$\text{transactions cost} = X(t,T) = X(\tau)$$

We can model any degree of irrationality using this function. If the consumer is a retired Salomon Brothers partner living in Greenwich with a Bloomberg screen in his living room and a healthy bank account, $X = 0$. The consumer is totally rational. If the consumer is working hard on a book on risk management, *X* is infinity, the consumer is totally irrational, and no embedded options will be exercised. Any level of irrationality in between these two extremes can be modeled in the same way. We illustrate the approach for European options in the next section.

Irrational Exercise of European Options

In Chapter 21, we analyzed the value of a European option on a zero-coupon bond using both the 3 factor Heath Jarrow and Morton framework and Jamshidian's valuation formula from the 1 factor Vasicek term structure model. How would the analysis of a European option in Chapter 21 differ from the results presented in that chapter if there were barriers to options exercise due to transactions costs? If the option at issue is a European option, the time-dependent nature of the transactions cost *X* is irrelevant with regard to a single European option. Only the level of *X* that will prevail at the exercise time T_1 matters, so we drop the time-dependent notation. The only difference between the rationally exercised option on a zero coupon bond P in Chapter 21 and the irrationally exercised option is easy to state. The "irrational" holder of a fixed income option exercises it only when the short rate of interest on date T_1 reaches a critical level s^* (in the Vasicek context) or state s_t in the HJM context such that:

$$P(s^*, T_1, T_2) = K + X$$

When *X* is zero, the analysis produces the same answer as the assumption that the option is exercised rationally. If *X* is infinity, the option will never be exercised, and the option is worthless from the perspective of the holder. We illustrate the power of this formulation with a specific example.

The Irrational Exercise of American Options

In Chapter 27, we reviewed the fundamentals of the pricing of American fixed income options. An American call option was valued on the premise of rational behavior throughout the life of the call, such that the holder of the call always acted in such a way as to maximize the value of the call. In the case of a callable security, the holder of the call always acts in such a way as to minimize the value of the security. Working backwards, one period from maturity, the holder of the call is assumed to prepay if there is even a one cent advantage of prepayment at par when compared to the present value of leaving the security outstanding at least one more period.

In the case of an irrationally exercised American call option, at every point (working backwards from maturity) the holder of an option to call their bond or loan with principal amount B will exercise that call option only if:

$$\textit{Value if Prepayment Option Unexercised} > B(\tau) + X(\tau)$$

That is, exercise will take place at that point in time and at that state of interest rates only if the present value of the security, if the call option is left unexercised, exceeds the principal amount *B* by more than the transactions cost *X* prevailing at that instant in time. The fact that *X* varies over time will result in a very rich array of realistic behavior in securities modeled in this way.

A Worked Example Using An Amortizing Loan with Rational and Irrational Prepayment Behavior

In this example, we modify the American options analysis of Chapter 27 using the 3 factor Heath Jarrow and Morton valuation framework from Chapter 9. In Chapter 27, we analyzed a "bullet" bond with all principal ($100) due at the maturity of the bond at time T=4. In this chapter, we assume instead that the loan has a more common structure for consumer credits, an amortizing structure with a level payment that ultimately retires all principal at maturity at T=4. If the loan has a principal amount at time 0 and a 2.00% coupon rate, the reader can verify that the loan will have annual payments of $26.2624, divided into interest and principal payments due at each annual time period (we use annual, not monthly, periods for expositional convenience) as shown in Figure 1:

Figure 1

Scheduled Amortization

Coupon Rate:	2.0000%
Time 0 Book Value	100
Periods to Maturity	4
Periodic Payment Amount	26.2624

Period Number	Initial Principal	Amount Paid	Interest	Principal	Ending Principal
0	100.0000				100.0000
1	100.0000	26.2624	2.0000	24.2624	75.7376
2	75.7376	26.2624	1.5148	24.7476	50.9900
3	50.9900	26.2624	1.0198	25.2426	25.7474
4	25.7474	26.2624	0.5149	25.7474	0.0000

If there are infinitely high transactions costs, the loan will never be prepaid and we will have cash flows of $26.2624 at times T=1, 2, 3, and 4 and in every state or at each node on the HJM bushy tree. This is shown in Figure 2, where the valuation comes from dropping these cash flows into the cash flow table that is then multiplied by the same probability-weighted discount factors that we have employed since Chapter 9. Figure 2 shows a risk neutral value of 101.7369.

Figure 2

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	26.2624	26.2624	26.2624	26.2624
2		26.2624	26.2624	26.2624	26.2624
3		26.2624	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			26.2624	26.2624	26.2624
6			26.2624	26.2624	26.2624
7			26.2624	26.2624	26.2624
8			26.2624	26.2624	26.2624
9			26.2624	26.2624	26.2624
10			26.2624	26.2624	26.2624
11			26.2624	26.2624	26.2624
12			26.2624	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			26.2624	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				26.2624	26.2624
18				26.2624	26.2624
19				26.2624	26.2624
20				26.2624	26.2624
21				26.2624	26.2624
22				26.2624	26.2624
23				26.2624	26.2624
24				26.2624	26.2624
25				26.2624	26.2624
26				26.2624	26.2624
27				26.2624	26.2624
28				26.2624	26.2624
29				26.2624	26.2624
30				26.2624	26.2624
31				26.2624	26.2624
32				26.2624	26.2624
33				26.2624	26.2624
34				26.2624	26.2624
35				26.2624	26.2624
36				26.2624	26.2624
37				26.2624	26.2624
38				26.2624	26.2624
39				26.2624	26.2624
40				26.2624	26.2624
41				26.2624	26.2624
42				26.2624	26.2624
43				26.2624	26.2624
44				26.2624	26.2624
45				26.2624	26.2624
46				26.2624	26.2624
47				26.2624	26.2624
48				26.2624	26.2624
49				26.2624	26.2624
50				26.2624	26.2624
51				26.2624	26.2624
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				26.2624	26.2624
56				26.2624	26.2624
57				26.2624	26.2624
58				26.2624	26.2624
59				26.2624	26.2624
60				26.2624	26.2624
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624
Risk Neutral Value =					101.7369

In a no arbitrage world, this value should equal the product of our annual cash flow $26.2624 and the sum of the zero coupon bond prices for maturity at times 1, 2, 3, and 4 used as inputs to the HJM bushy tree: 0.9970057865, 0.9841101497, 0.9618922376, and 0.9308550992. The reader can verify that value is indeed 101.7369, consistent with Chapter 9 and the formulas of Chapter 4. How do the cash flows vary if the borrower under this loan is completely rational with zero transactions costs? We use the same analysis we employed in Chapter 27 to answer this question. We start at the last options exercise point, time T=3, and analyze the optimal options exercise policy. We prepay or call the loan whenever the risk-neutral value of the payment $26.2624 due at time T=4 is more than the principal that remains on the loan ($25.7474 as shown in Figure 1) after making the payment of $26.2624 due at time T=3. Figure 3 gives the rational prepayment behavior for each of the 64 nodes on the HJM bushy tree at time T=3:

Figure 3

Annual Payment					**26.2624**
Period End Principal					**25.7474**
		Maturity of Cash Flow Received			
	Current Time				
Row Number	**0**	**1**	**2**	**3**	**Action?**
1	0.0000	0.0000	0.0000	24.7500	No Call
2		0.0000	0.0000	24.8775	No Call
3		0.0000	0.0000	24.9711	No Call
4		0.0000	0.0000	24.3086	No Call
5			0.0000	25.4717	No Call
6			0.0000	25.5990	No Call
7			0.0000	25.7401	No Call
8			0.0000	25.2370	No Call
9			0.0000	24.9593	No Call
10			0.0000	25.7529	Call
11			0.0000	25.4530	No Call
12			0.0000	24.8929	No Call
13			0.0000	24.9109	No Call
14			0.0000	25.0393	No Call
15			0.0000	25.1335	No Call
16			0.0000	24.4666	No Call
17				25.2155	No Call
18				26.0173	Call
19				25.7143	No Call
20				25.1484	No Call
21				25.6961	No Call
22				26.1179	Call
23				26.0625	Call
24				25.9509	Call
25				25.5911	No Call
26				26.3608	Call
27				26.1944	Call
28				25.7051	No Call
29				25.8040	Call
30				25.9330	Call
31				26.0759	Call
32				25.5663	No Call
33				25.6768	No Call
34				25.8052	Call
35				25.9474	Call
36				25.4403	No Call
37				25.6768	No Call
38				25.8052	Call
39				25.9474	Call
40				25.4403	No Call
41				25.3411	No Call
42				26.1033	Call
43				25.9386	Call
44				25.4540	No Call
45				25.1298	No Call
46				25.9288	Call
47				25.6269	No Call
48				25.0629	No Call
49				25.3740	No Call
50				26.1808	Call
51				25.8759	Call
52				25.3065	No Call
53				25.6011	No Call
54				25.7290	No Call
55				25.8708	Call
56				25.3652	No Call
57				25.6011	No Call
58				25.7290	No Call
59				25.8708	Call
60				25.3652	No Call
61				25.1300	No Call
62				25.6798	No Call
63				25.5657	No Call
64				25.0063	No Call

The cash flows that result from this call strategy at time T=3 (ignoring our ability to call or prepay the loan at times T=1 and 2) are shown in Figure 4:

Figure 4

	Maturity of Cash Flow Received				
	Current Time				
Row Number	0	1	2	3	4
1		26.2624	26.2624	26.2624	26.2624
2		26.2624	26.2624	26.2624	26.2624
3		26.2624	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			26.2624	26.2624	26.2624
6			26.2624	26.2624	26.2624
7			26.2624	26.2624	26.2624
8			26.2624	26.2624	26.2624
9			26.2624	26.2624	26.2624
10			26.2624	52.0098	0.0000
11			26.2624	26.2624	26.2624
12			26.2624	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			26.2624	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				26.2624	26.2624
18				52.0098	0.0000
19				26.2624	26.2624
20				26.2624	26.2624
21				26.2624	26.2624
22				52.0098	0.0000
23				52.0098	0.0000
24				52.0098	0.0000
25				26.2624	26.2624
26				52.0098	0.0000
27				52.0098	0.0000
28				26.2624	26.2624
29				52.0098	0.0000
30				52.0098	0.0000
31				52.0098	0.0000
32				26.2624	26.2624
33				26.2624	26.2624
34				52.0098	0.0000
35				52.0098	0.0000
36				26.2624	26.2624
37				26.2624	26.2624
38				52.0098	0.0000
39				52.0098	0.0000
40				26.2624	26.2624
41				26.2624	26.2624
42				52.0098	0.0000
43				52.0098	0.0000
44				26.2624	26.2624
45				26.2624	26.2624
46				52.0098	0.0000
47				26.2624	26.2624
48				26.2624	26.2624
49				26.2624	26.2624
50				52.0098	0.0000
51				52.0098	0.0000
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				52.0098	0.0000
56				26.2624	26.2624
57				26.2624	26.2624
58				26.2624	26.2624
59				52.0098	0.0000
60				26.2624	26.2624
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624

Following the procedures of Chapter 27, we then step back to time T=2 and again re-examine the optimal call or prepayment strategy. When we do that, we may decide to prepay the loan on a branch of the bushy tree for which prepaying at time T=3 was also rational. In this case, once the prepayment is made at time T=2, the cash flows at time T=3 and 4 on this branch of the bushy tree are set to zero. We set the strategy for time T=2 and then step back to time T=1 and repeat the process. When we do that, we get the cash flows and valuation shown in Figure 5 for zero transactions costs and a 100% rational borrower:

Figure 5

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	26.2624	26.2624	26.2624	26.2624
2		102.0000	26.2624	26.2624	26.2624
3		102.0000	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			0.0000	26.2624	26.2624
6			0.0000	26.2624	26.2624
7			0.0000	26.2624	26.2624
8			0.0000	26.2624	26.2624
9			0.0000	26.2624	26.2624
10			0.0000	52.0098	0.0000
11			0.0000	26.2624	26.2624
12			0.0000	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			77.2524	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				0.0000	0.0000
18				0.0000	0.0000
19				0.0000	0.0000
20				0.0000	0.0000
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				0.0000	0.0000
34				0.0000	0.0000
35				0.0000	0.0000
36				0.0000	0.0000
37				0.0000	0.0000
38				0.0000	0.0000
39				0.0000	0.0000
40				0.0000	0.0000
41				0.0000	0.0000
42				0.0000	0.0000
43				0.0000	0.0000
44				0.0000	0.0000
45				0.0000	0.0000
46				0.0000	0.0000
47				0.0000	0.0000
48				0.0000	0.0000
49				26.2624	26.2624
50				52.0098	0.0000
51				52.0098	0.0000
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				52.0098	0.0000
56				26.2624	26.2624
57				0.0000	0.0000
58				0.0000	0.0000
59				0.0000	0.0000
60				0.0000	0.0000
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624
Risk Neutral Value =					101.4234

If the borrower is not allowed to prepay at time 0, the loan has a value of 101.4234. If the borrower is allowed to prepay at time zero, the borrower (if he is totally rational) will prepay so that the value = minimum (par value, risk-neutral value if not prepaid) = minimum (100, 101.4234) = 100.

Now, let's allow our client to be rational but subject to transactions costs. If prepayment is allowed at time zero, we know that the loan will be prepaid and have a time zero value of 100 for any transactions cost that is 101.4234 - 100 = 1.4234 or less. Let's take a more interesting case. Let's assume that the loan was made 1 second earlier and that time T=1 is the borrower's first opportunity to prepay. Let's assume we can see the loan trading in the secondary market at a net present value (price plus accrued interest, consistent with our comments in Chapter 4) of 101.56. What constant transactions cost (relevant to exercise at times T= 1, 2, and 3) is implied by this market price? We use iteration with respect to the transaction cost in common spreadsheet software to find the answer. Note that the typical solver function isn't very helpful in this case because our valuation answer is in some sense a step function, rather than a smooth continuous function, due to the size of the time steps and the small number of branches on the bushy tree relative to what would be used in practice. We find that a transactions cost of $0.92 produces a value of 101.56 and generates the cash flows (and implied prepayment strategies) shown in Figure 6:

Figure 6

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	26.2624	26.2624	26.2624	26.2624
2		102.0000	26.2624	26.2624	26.2624
3		26.2624	26.2624	26.2624	26.2624
4		26.2624	26.2624	26.2624	26.2624
5			0.0000	26.2624	26.2624
6			0.0000	26.2624	26.2624
7			0.0000	26.2624	26.2624
8			0.0000	26.2624	26.2624
9			26.2624	26.2624	26.2624
10			26.2624	26.2624	26.2624
11			26.2624	26.2624	26.2624
12			26.2624	26.2624	26.2624
13			26.2624	26.2624	26.2624
14			26.2624	26.2624	26.2624
15			26.2624	26.2624	26.2624
16			26.2624	26.2624	26.2624
17				0.0000	0.0000
18				0.0000	0.0000
19				0.0000	0.0000
20				0.0000	0.0000
21				0.0000	0.0000
22				0.0000	0.0000
23				0.0000	0.0000
24				0.0000	0.0000
25				0.0000	0.0000
26				0.0000	0.0000
27				0.0000	0.0000
28				0.0000	0.0000
29				0.0000	0.0000
30				0.0000	0.0000
31				0.0000	0.0000
32				0.0000	0.0000
33				26.2624	26.2624
34				26.2624	26.2624
35				26.2624	26.2624
36				26.2624	26.2624
37				26.2624	26.2624
38				26.2624	26.2624
39				26.2624	26.2624
40				26.2624	26.2624
41				26.2624	26.2624
42				26.2624	26.2624
43				26.2624	26.2624
44				26.2624	26.2624
45				26.2624	26.2624
46				26.2624	26.2624
47				26.2624	26.2624
48				26.2624	26.2624
49				26.2624	26.2624
50				26.2624	26.2624
51				26.2624	26.2624
52				26.2624	26.2624
53				26.2624	26.2624
54				26.2624	26.2624
55				26.2624	26.2624
56				26.2624	26.2624
57				26.2624	26.2624
58				26.2624	26.2624
59				26.2624	26.2624
60				26.2624	26.2624
61				26.2624	26.2624
62				26.2624	26.2624
63				26.2624	26.2624
64				26.2624	26.2624
Risk Neutral Value =					101.5589

We now turn to the implications of the transactions cost approach to modeling "irrational" behavior.

Implied Irrationality and Hedging

When irrationality is modeled in this way, the level of irrationality can be implied from observable market prices just like implied volatility in the Black-Scholes model, achieving one of our primary objectives for a model of irrationality. Almost no observable security reflects the behavior of one consumer, so the best replication of the market price movement of a given security reflects a combination of irrational behavior by many consumers. Using standard HJM procedures, we know value of the loan as time changes from time 0 to one of the four shift states at time T=1. From these value changes, we can solve for the hedging position that generates zero risk, i.e. a return equal to the return on the money market fund (an investment for one period at the risk free rate).

This powerful insight of the HJM framework gives us an alternative to historical research on prepayment behavior--we can imply the degree of rationality of prepayment behavior from observable prices. We have the tools for efficient hedging of securities which contain irrationally exercised embedded options, and we can achieve very fast calculation times with much greater accuracy than we could using the legacy prepayment tools discussed in the next chapter, historical prepayment functions or prepayment tables. We close this chapter by exploring the links between our credit risk analysis of Chapters 16 and 17 and prepayment analysis.

Credit Risk and Irrational Prepayment Behavior

One of the realities of consumer lending is that consumers appear irrational in the three ways discussed in the example above:

- They sometimes fail to prepay when they 'should';
- They sometimes prepay when they 'shouldn't';
- They often partially prepay instead of fully prepaying as they 'should'.

In each of these cases, the word 'should' stems from an analysis of the consumer's behavior that is based only on what the lender knows—the terms of the loan and not much else. Many of these allegedly irrational acts stem from the complex interaction of the long-term creditworthiness and financial well-being of the borrower and the loan itself. We can give rational explanations for each of these types of irrational behavior.

- They sometimes fail to prepay when they 'should':

  - Case A: The price of the home may be less than the principal amount of the loan and the consumer may not have enough liquid financial assets to bridge this gap. This is extremely common in the wake of the 2007 to 2010 credit crisis and home price collapse
  - Case B: The consumer may be in financial distress and unable to qualify for a new loan.
  - Case C: The consumer may be on a one year sabbatical at the University of Tahiti.
  - Case D: The loan may be so close to maturity that the benefits of refinancing are not worth the costs of going through the process.
  - Case E: The consumer's personal wealth may now be so great that they have no intention of borrowing any more money and prepayment may simply be delayed until the consumer has enough liquid assets to prepay with cash.

There are many other similar examples we could give to further illustrate this point.

- They sometimes prepay when they 'shouldn't':

  Even in this circumstance, there are many explanations that are quite rational for this superficially irrational response, many of which again are due to the credit worthiness and net worth of the consumer:

  - Case A: The borrower may have sold the house (due to the famous trio of death, divorce, or relocation).
  - Case B: The borrower may have just received very large payments from structuring CDOs for Goldman Sachs in the credit crisis; prepaying the mortgage is in fact a better risk-adjusted investment than U.S. Treasuries since the credit risk of the now very wealthy consumer is the same as the Treasury: zero. The mortgage is prepaid because it has a higher yield than Treasuries and the lender is not reflecting the borrower's credit risk correctly.
  - Case C: The borrower may have been a problem borrower in the past and prepays the loan simply because the borrower can no longer tolerate a relationship with a financial institution that has pursued overly aggressive tactics to get the borrower current on the loan; this is a common refrain from consumers these days.

- They often partially prepay instead of fully prepaying as they 'should':

  This phenomenon is inconsistent with the 'all or nothing' implications of fixed income options explained in Chapters 21 and 27, but it is very common practice. It has a lot in common with the partial draw-downs of commercial lines of credit and with revolving charge card balances that we discuss in later chapters.

  - Case A: The consumer's credit quality has improved considerably since the mortgage was granted and the true 'market mortgage rate' for the consumer is now less than the rate on the loan, but the bank doesn't know this. The consumer signals this by prepaying in whatever amounts fit his total financial plan.
  - Case B: Market interest rates are now lower but the consumer's total financial plan suggests that occasional partial prepayments are better than refinancing the loan. There are a number of reasons why this may be the case. Most revolve around the fact that partial prepayments are less costly (with 'cost' broadly defined) to the consumer than one lump sum prepayment.

We explore the implications of these links between credit risk, market risk, liquidity risk, and interest rate risk and prepayment behavior for the Jarrow-Merton put option as a measure of integrated risk in detail using U.S. mortgage and consumer loan data in the next chapter.