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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

Chapter 30:
Non-Maturity Deposits

In this chapter, we turn our attention to the liability side of the balance sheet of financial institutions, in particular to deposits of commercial banks. This is particularly appropriate in light of massive deposit run-offs in the credit crisis of 2007 to 2010, which we document later in this chapter and in Chapter 37 on liquidity risk. This focus on deposits is also critical to correct valuation of the Jarrow-Merton put option as the best comprehensive measure of integrated credit risk and interest rate risk in the commercial banking sector. Why? Because we need to value a put option that includes both assets and liabilities of the financial institution, since some of the liabilities may be providing a hedge of the assets. One of the advantages of the reduced form credit modeling technology of Chapter 16 is that it can handle the complex liability structure that is typical of large financial institutions, whereas the Merton model of Chapter 18 assumes a single zero-coupon bond as a liability. In fact, it is the multi-period cash flows of non-maturity deposits that explained much of the drama of the credit crisis.

The bulk of bank deposits and insurance liabilities are made up of 'securities' with explicit maturities, although they are often putable (back to the bank or insurance company) by the consumer who supplied the funds, whether they come in the form of a time deposit or life insurance policy. These liabilities can be analyzed with the techniques of Chapters 27 and 28, even if the consumer's right to put the security back to the financial institution is exercised irrationally. A substantial portion, however, of the liabilities of major banks world-wide consists of 'non-maturity deposits'. Similarly, the funds supplied to mutual fund managers of fixed income funds also have no specific maturities. This chapter is relevant to both types of liabilities, but we will refer to them as 'non-maturity deposits' within a banking industry context from here on. These deposits have no specific maturity, and individual depositors can freely add or subtract balances as they wish. Most importantly, there is no way to distinguish "new clients" from "old clients" because the same interest rates are paid to both, unlike a certificate of deposit where the rates are changing constantly for new clients. The interest rate on non-maturity deposits is usually, but not always, a function of open market interest rates. Similarly, the level of deposit balances in aggregate often moves in sympathy with open market interest rates. With the strong trend in mark-to-market based risk management in banking, all bankers are faced with the difficult task of calculating the mark-to-market 'value' of these non-maturity deposits. This chapter provides an introduction to that difficult topic.

In September 1993, the Federal Reserve Board [1993] published proposed regulations for interest rate risk and capital adequacy. In its regulations, the Federal Reserve Board noted '...the inherent difficulties in determining the appropriate treatment of non-maturity deposits'. In this chapter, we use the Heath Jarrow and Morton interest rate framework of Chapters 6 through 9 to measure the mark-to-market value of deposits. We do this in a way that is theoretically rigorous and yet easy to apply in practice. Most of the arguments which are applied here to the valuation of non-maturity deposits can be applied equally well to the valuation of charge card loans, where again the balance as well as the rate on the loan fluctuates in response to market rate movements. For more on both deposit and charge card valuation, see Jarrow and van Deventer [1996a, 1996b, 1998] and Janosi, Jarrow and Zullo [1998].

The 'Value' of the Deposit Franchise

When we measure the 'value' of deposits, what is it that we are trying to measure? If the bank had issued a bond to finance its assets, the 'value' of the bond is its present value—the present value of future payments that the bank must make on the security that it has issued. 'Value' is the present value of future costs.

In the case of non-maturity deposits, the 'value' of the deposit could also mean the premium that a third party would be willing to pay above the face value of the deposits to assume ownership of the bank's deposit 'franchise'.[1] In this case, 'value' means the net present value benefit of owning the deposit franchise—the value of cash provided by depositors less the present value cost of the deposit franchise.

In what follows, we use the word 'value' to refer to the present value of the cost of the deposit franchise, exactly in keeping with the analogy of the bank as bond issuer. The net present value benefit of having the deposit franchise can be calculated from the valuation formulas presented below by subtracting the 'values' we calculate from the face value of the deposit. For example, the present value cost of $1000 face value of savings deposits might be $650. The net present value benefit of the deposit franchise is $1000 - $650, or $350. All references to 'value' below refer to the present value of the cost of the franchise, $650.

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[1] The average premium paid by purchasers of failed bank deposit franchises was 2.32% in 1,225 auctions run through October 28 1994 by the Resolution Trust Corporation in the U.S. The highest premium paid was 25.33%. Most of the deposits auctioned (more than 70% on average) were time deposits, which would be expected to have very low premiums.

We also initially assume that the deposit franchise is guaranteed by a riskless third party, which is the case (in theory) in the U.S. and many other countries. This allows us to avoid dealing with the complexities of bankruptcy risk. We deal with that explicit rate later in the chapter.

Some bankers prefer to focus on the value of specific non-maturity deposits owned by current clients, i.e. valuing only deposit accounts on the books of the bank today and ignoring future business. This orientation has a logical basis, because we don't value the 'franchise' of any other business line on the bank balance sheet.

There is a good reason, however, for looking at the "franchise" for valuation. The pricing of non-maturity deposits is exactly the same for new customers as it is for old customers. The bank cannot control the flow of new demand deposit accounts or savings deposit accounts by changing pricing to attract new clients without immediately making the same price change on existing accounts. This is not the case for auto loans or consumer certificates of deposit or any other account on the balance sheet of the bank except for the charge card business. For this reason, we think the focus on the franchise is the right one, but we recognize that some will disagree.

Total Cash Flow of Non-Maturity Deposits

The first step in valuing non-maturity deposits is to isolate the total cash flows from the deposit franchise. Total costs consist of the following elements:

Total deposit cost cash flows in a given time period:

= interest paid on the deposit

+ non-interest expense of servicing the deposit

- non-interest revenue from the deposit franchise

- net increase in deposit balances

Many readers will be surprised to see the net increase in deposit balances subtracted from costs in the equation above. The net change in deposit balances on many deposit products is the single most important cash flow in the valuation process, and yet many analysts ignore this important determinant of value.

In the valuation of any security, total cash flow is the basic building block of valuation. For most securities, the maturity date is fixed and cash flow from "principal" stems from a pre-determined payment schedule associated with that security. In the case of the non-maturity deposit, however, the security is a perpetual one and principal is never "returned." Instead, changes in deposit balances are simply another source of cash flow. A simple example illustrates the point.

Let us consider a deposit category with balances of $1000 where the sum of interest expense and processing costs is 2% of balances annually. Let's assume that this cash flow occurs at the end of every year. Let's also assume that the balances on this account grow at 2% a year, and that balances also increase in a one-time jump at the end of each year. What is the net cash flow associated with this account? On December 31 of each year, 2% of the deposit amount flows out in the form of interest expense and processing costs, and it flows back in on the same day in the form of increased deposit balances. The net cash flow from the deposit franchise is zero every year forever.

The present value cost of this deposit is zero, since there is never net cash inflow or outflow. Since the present value cost is zero, the 'net present value' or premium that would be paid to gain this deposit franchise is $1000, calculated as above: the deposit balance $1000 - the present value of deposit costs ($0) is $1000.

This example is deceptively simple: it is consistent with common sense, and yet it will lead us to some surprising conclusions about the value of non-maturity deposits. We will analyze the value of non-maturity deposits in what follows using realistic assumptions about random deposit rate and balance behavior. This is covered in more detail in Jarrow and van Deventer [1996a, 1996b, 1998] and Janosi, Jarrow and Zullo [1998], which provide explicit "closed form solutions" for a commonly used one factor term structure model. Step by step examples using a one factor term structure model can also be found in the first edition of this book. In this volume, however, consistent with our conclusions in Chapter 3, we believe that a one factor model is not accurate enough to meet the current 21st century best practice standard. In this chapter, we continue to use the 3 factor Heath Jarrow and Morton framework of Chapter 9.

As in Chapters 19 through 29, we will use the probability weighted discount factors from Chapter 9 to value non-maturity deposits. Our task in this section is to calculate the cash flows on each node of the bushy tree (or in each scenario of an HJM-based Monte Carlo simulation) that stem from the non-maturity deposit cash flows.

Specifying the Rate and Balance Movement Formulas

Step 1 in the valuation process is to specify how non-maturity deposit rates, non-interest income, and non-interest expense vary with interest rates and other macro-economic factors. Step 2 is to specify a similar formula for the variation in deposit balances. In this example, we ignore the credit risk of the institution. Later in this chapter, we explicitly incorporate the risk of the bank in light of actual experiences of "too big to fail" banks during the 2007 to 2010 credit crisis.

MODEL RISK ALERT: It is very important, for a bank that has never had credit problems previously, to realize that its own data is "too good" for fitting the rate and balance relationships. Accurate relationships can only be based on data from both "good" and "bad" banks; if we do not fit the relationships in this way, they cannot possibly capture the behavior of non-maturity deposits at a distressed bank in an accurate way. Let's assume we compile such a data set and we derive a relationship for the sum of deposit interest plus non-interest expense less non-interest revenue for the bank in question. Assume that the "effective deposit rate" is as follows:

Effective deposit rate = 0.10% + 0.2 (Lagged 1 period spot rate)

We are implicitly assuming that the effective deposit interest at time T=k is the effective rate at time T=k (which depends on the spot rate at time T=k-1). We base this calculation on the 1 period spot rates from Chapter 9:

Figure 1

		Spot Rate Process			
State	Row	0	1	2	3
S-1, S-1, S-1	1	0.3003%	2.1653%	4.6388%	6.1106%
S-1, S-1, S-2	2		0.6911%	1.4142%	5.5667%
S-1, S-1, S-3	3		1.0972%	2.6089%	5.1710%
S-1, S-1, S-4	4		1.3611%	4.9179%	8.0375%
S-1, S-2, S-1	5			2.2496%	3.1041%
S-1, S-2, S-2	6			0.5984%	2.5913%
S-1, S-2, S-3	7			0.8121%	2.0291%
S-1, S-2, S-4	8			1.2459%	4.0628%
S-1, S-3, S-1	9			1.9217%	5.2210%
S-1, S-3, S-2	10			1.4149%	1.9784%
S-1, S-3, S-3	11			0.8591%	3.1798%
S-1, S-3, S-4	12			2.8695%	5.5017%
S-1, S-4, S-1	13			2.2088%	5.4252%
S-1, S-4, S-2	14			1.7005%	4.8848%
S-1, S-4, S-3	15			1.1432%	4.4917%
S-1, S-4, S-4	16			3.1593%	7.3396%
S-2, S-1, S-1	17				4.1517%
S-2, S-1, S-2	18				0.9421%
S-2, S-1, S-3	19				2.1313%
S-2, S-1, S-4	20				4.4295%
S-2, S-2, S-1	21				2.2036%
S-2, S-2, S-2	22				0.5531%
S-2, S-2, S-3	23				0.7667%
S-2, S-2, S-4	24				1.2004%
S-2, S-3, S-1	25				2.6231%
S-2, S-3, S-2	26				-0.3734%
S-2, S-3, S-3	27				0.2593%
S-2, S-3, S-4	28				2.1679%
S-2, S-4, S-1	29				1.7764%
S-2, S-4, S-2	30				1.2703%
S-2, S-4, S-3	31				0.7153%
S-2, S-4, S-4	32				2.7228%
S-3, S-1, S-1	33				2.2804%
S-3, S-1, S-2	34				1.7718%
S-3, S-1, S-3	35				1.2141%
S-3, S-1, S-4	36				3.2315%
S-3, S-2, S-1	37				2.2804%
S-3, S-2, S-2	38				1.7718%
S-3, S-2, S-3	39				1.2141%
S-3, S-2, S-4	40				3.2315%
S-3, S-3, S-1	41				3.6354%
S-3, S-3, S-2	42				0.6093%
S-3, S-3, S-3	43				1.2483%
S-3, S-3, S-4	44				3.1757%
S-3, S-4, S-1	45				4.5070%
S-3, S-4, S-2	46				1.2864%
S-3, S-4, S-3	47				2.4797%
S-3, S-4, S-4	48				4.7858%
S-4, S-1, S-1	49				3.5013%
S-4, S-1, S-2	50				0.3117%
S-4, S-1, S-3	51				1.4934%
S-4, S-1, S-4	52				3.7773%
S-4, S-2, S-1	53				2.5831%
S-4, S-2, S-2	54				2.0730%
S-4, S-2, S-3	55				1.5136%
S-4, S-2, S-4	56				3.5370%
S-4, S-3, S-1	57				2.5831%
S-4, S-3, S-2	58				2.0730%
S-4, S-3, S-3	59				1.5136%
S-4, S-3, S-4	60				3.5370%
S-4, S-4, S-1	61				4.5061%
S-4, S-4, S-2	62				2.2686%
S-4, S-4, S-3	63				2.7252%
S-4, S-4, S-4	64				5.0230%

Using the effective deposit rate formula above, we calculate the effective deposit rate at each node of the bushy tree. The rates we derive are shown in Figure 2:

Figure 2

Effective Deposit Rate Paid

State	Row	0	1	2	3	4
S-1, S-1, S-1	1		0.1601%	0.5331%	1.0278%	1.3221%
S-1, S-1, S-2	2		0.1601%	0.5331%	1.0278%	1.2133%
S-1, S-1, S-3	3		0.1601%	0.5331%	1.0278%	1.1342%
S-1, S-1, S-4	4		0.1601%	0.5331%	1.0278%	1.7075%
S-1, S-2, S-1	5			0.2382%	0.3828%	0.7208%
S-1, S-2, S-2	6			0.2382%	0.3828%	0.6183%
S-1, S-2, S-3	7			0.2382%	0.3828%	0.5058%
S-1, S-2, S-4	8			0.2382%	0.3828%	0.9126%
S-1, S-3, S-1	9			0.3194%	0.6218%	1.1442%
S-1, S-3, S-2	10			0.3194%	0.6218%	0.4957%
S-1, S-3, S-3	11			0.3194%	0.6218%	0.7360%
S-1, S-3, S-4	12			0.3194%	0.6218%	1.2003%
S-1, S-4, S-1	13			0.3722%	1.0836%	1.1850%
S-1, S-4, S-2	14			0.3722%	1.0836%	1.0770%
S-1, S-4, S-3	15			0.3722%	1.0836%	0.9983%
S-1, S-4, S-4	16			0.3722%	1.0836%	1.5679%
S-2, S-1, S-1	17				0.5499%	0.9303%
S-2, S-1, S-2	18				0.5499%	0.2884%
S-2, S-1, S-3	19				0.5499%	0.5263%
S-2, S-1, S-4	20				0.5499%	0.9859%
S-2, S-2, S-1	21				0.2197%	0.5407%
S-2, S-2, S-2	22				0.2197%	0.2106%
S-2, S-2, S-3	23				0.2197%	0.2533%
S-2, S-2, S-4	24				0.2197%	0.3401%
S-2, S-3, S-1	25				0.2624%	0.6246%
S-2, S-3, S-2	26				0.2624%	0.0253%
S-2, S-3, S-3	27				0.2624%	0.1519%
S-2, S-3, S-4	28				0.2624%	0.5336%
S-2, S-4, S-1	29				0.3492%	0.4553%
S-2, S-4, S-2	30				0.3492%	0.3541%
S-2, S-4, S-3	31				0.3492%	0.2431%
S-2, S-4, S-4	32				0.3492%	0.6446%
S-3, S-1, S-1	33				0.4843%	0.5561%
S-3, S-1, S-2	34				0.4843%	0.4544%
S-3, S-1, S-3	35				0.4843%	0.3428%
S-3, S-1, S-4	36				0.4843%	0.7463%
S-3, S-2, S-1	37				0.3830%	0.5561%
S-3, S-2, S-2	38				0.3830%	0.4544%
S-3, S-2, S-3	39				0.3830%	0.3428%
S-3, S-2, S-4	40				0.3830%	0.7463%
S-3, S-3, S-1	41				0.2718%	0.8271%
S-3, S-3, S-2	42				0.2718%	0.2219%
S-3, S-3, S-3	43				0.2718%	0.3497%
S-3, S-3, S-4	44				0.2718%	0.7351%
S-3, S-4, S-1	45				0.6739%	1.0014%
S-3, S-4, S-2	46				0.6739%	0.3573%
S-3, S-4, S-3	47				0.6739%	0.5959%
S-3, S-4, S-4	48				0.6739%	1.0572%
S-4, S-1, S-1	49				0.5418%	0.8003%
S-4, S-1, S-2	50				0.5418%	0.1623%
S-4, S-1, S-3	51				0.5418%	0.3987%
S-4, S-1, S-4	52				0.5418%	0.8555%
S-4, S-2, S-1	53				0.4401%	0.6166%
S-4, S-2, S-2	54				0.4401%	0.5146%
S-4, S-2, S-3	55				0.4401%	0.4027%
S-4, S-2, S-4	56				0.4401%	0.8074%
S-4, S-3, S-1	57				0.3286%	0.6166%
S-4, S-3, S-2	58				0.3286%	0.5146%
S-4, S-3, S-3	59				0.3286%	0.4027%
S-4, S-3, S-4	60				0.3286%	0.8074%
S-4, S-4, S-1	61				0.7319%	1.0012%
S-4, S-4, S-2	62				0.7319%	0.5537%
S-4, S-4, S-3	63				0.7319%	0.6450%
S-4, S-4, S-4	64				0.7319%	1.1046%

In order to calculate the dollars of effective interest paid, we need to model deposit balances. With all of the caveats above still in mind, we derive a deposit balance equation that has the following terms:

Deposit balance = 1.005 (Lagged deposit balance) - 200 (Lagged 1 period spot rate)

This gives the deposit balances for each node of the bushy tree in Figure 3, assuming our time 0 initial balance is $1,000:

Figure 3

Deposit Balance

State	Row	0	1	2	3	4
S-1, S-1, S-1	1	1000	1004.3994	1005.0908	1000.8386	993.62154
S-1, S-1, S-2	2		1004.3994	1005.0908	1000.8386	994.70935
S-1, S-1, S-3	3		1004.3994	1005.0908	1000.8386	995.50071
S-1, S-1, S-4	4		1004.3994	1005.0908	1000.8386	989.76785
S-1, S-2, S-1	5			1008.0393	1007.2879	1006.1162
S-1, S-2, S-2	6			1008.0393	1007.2879	1007.1417
S-1, S-2, S-3	7			1008.0393	1007.2879	1008.2661
S-1, S-2, S-4	8			1008.0393	1007.2879	1004.1987
S-1, S-3, S-1	9			1007.2269	1004.8985	999.48095
S-1, S-3, S-2	10			1007.2269	1004.8985	1005.9661
S-1, S-3, S-3	11			1007.2269	1004.8985	1003.5634
S-1, S-3, S-4	12			1007.2269	1004.8985	998.91964
S-1, S-4, S-1	13			1006.6992	1000.2804	994.43139
S-1, S-4, S-2	14			1006.6992	1000.2804	995.51217
S-1, S-4, S-3	15			1006.6992	1000.2804	996.29843
S-1, S-4, S-4	16			1006.6992	1000.2804	990.60259
S-2, S-1, S-1	17				1008.5803	1005.3197
S-2, S-1, S-2	18				1008.5803	1011.739
S-2, S-1, S-3	19				1008.5803	1009.3607
S-2, S-1, S-4	20				1008.5803	1004.7641
S-2, S-2, S-1	21				1011.8827	1012.535
S-2, S-2, S-2	22				1011.8827	1015.836
S-2, S-2, S-3	23				1011.8827	1015.4087
S-2, S-2, S-4	24				1011.8827	1014.5414
S-2, S-3, S-1	25				1011.4553	1011.2663
S-2, S-3, S-2	26				1011.4553	1017.2594
S-2, S-3, S-3	27				1011.4553	1015.9939
S-2, S-3, S-4	28				1011.4553	1012.1768
S-2, S-4, S-1	29				1010.5876	1012.0877
S-2, S-4, S-2	30				1010.5876	1013.1
S-2, S-4, S-3	31				1010.5876	1014.2099
S-2, S-4, S-4	32				1010.5876	1010.1949
S-3, S-1, S-1	33				1008.4196	1008.9008
S-3, S-1, S-2	34				1008.4196	1009.9181
S-3, S-1, S-3	35				1008.4196	1011.0335
S-3, S-1, S-4	36				1008.4196	1006.9986
S-3, S-2, S-1	37				1009.4333	1009.9196
S-3, S-2, S-2	38				1009.4333	1010.9369
S-3, S-2, S-3	39				1009.4333	1012.0523
S-3, S-2, S-4	40				1009.4333	1008.0174
S-3, S-3, S-1	41				1010.5448	1008.3267
S-3, S-3, S-2	42				1010.5448	1014.3789
S-3, S-3, S-3	43				1010.5448	1013.1009
S-3, S-3, S-4	44				1010.5448	1009.2462
S-3, S-4, S-1	45				1006.524	1002.5426
S-3, S-4, S-2	46				1006.524	1008.9838
S-3, S-4, S-3	47				1006.524	1006.5973
S-3, S-4, S-4	48				1006.524	1001.9851
S-4, S-1, S-1	49				1007.3151	1005.3491
S-4, S-1, S-2	50				1007.3151	1011.7283
S-4, S-1, S-3	51				1007.3151	1009.3648
S-4, S-1, S-4	52				1007.3151	1004.7969
S-4, S-2, S-1	53				1008.3317	1008.2071
S-4, S-2, S-2	54				1008.3317	1009.2274
S-4, S-2, S-3	55				1008.3317	1010.3461
S-4, S-2, S-4	56				1008.3317	1006.2992
S-4, S-3, S-1	57				1009.4463	1009.3273
S-4, S-3, S-2	58				1009.4463	1010.3476
S-4, S-3, S-3	59				1009.4463	1011.4663
S-4, S-3, S-4	60				1009.4463	1007.4194
S-4, S-4, S-1	61				1005.4142	1001.4291
S-4, S-4, S-2	62				1005.4142	1005.9041
S-4, S-4, S-3	63				1005.4142	1004.9909
S-4, S-4, S-4	64				1005.4142	1000.3953

The effective dollar amount of deposit interest (which includes non-interest expense less non-interest income on deposits) is the product of the rates in Figure 2 and the balances in Figure 3 for each node of the three. This dollar cash outflow is shown in Figure 4 as a positive number, just like interest on a bond:

Figure 4

Dollars of Effective Deposit Interest Paid						
State	Row	0	1	2	3	4
S-1, S-1, S-1	1	0	1.6077	5.3577	10.286	13.137
S-1, S-1, S-2	2		1.6077	5.3577	10.286	12.069
S-1, S-1, S-3	3		1.6077	5.3577	10.286	11.291
S-1, S-1, S-4	4		1.6077	5.3577	10.286	16.9
S-1, S-2, S-1	5			2.4013	3.8563	7.2522
S-1, S-2, S-2	6			2.4013	3.8563	6.2268
S-1, S-2, S-3	7			2.4013	3.8563	5.1001
S-1, S-2, S-4	8			2.4013	3.8563	9.164
S-1, S-3, S-1	9			3.2175	6.2483	11.436
S-1, S-3, S-2	10			3.2175	6.2483	4.9864
S-1, S-3, S-3	11			3.2175	6.2483	7.3858
S-1, S-3, S-4	12			3.2175	6.2483	11.99
S-1, S-4, S-1	13			3.7471	10.839	11.784
S-1, S-4, S-2	14			3.7471	10.839	10.721
S-1, S-4, S-3	15			3.7471	10.839	9.9464
S-1, S-4, S-4	16			3.7471	10.839	15.532
S-2, S-1, S-1	17				5.5464	9.353
S-2, S-1, S-2	18				5.5464	2.9181
S-2, S-1, S-3	19				5.5464	5.3118
S-2, S-1, S-4	20				5.5464	9.9061
S-2, S-2, S-1	21				2.2228	5.475
S-2, S-2, S-2	22				2.2228	2.1396
S-2, S-2, S-3	23				2.2228	2.5725
S-2, S-2, S-4	24				2.2228	3.4502
S-2, S-3, S-1	25				2.6542	6.3166
S-2, S-3, S-2	26				2.6542	0.2576
S-2, S-3, S-3	27				2.6542	1.5429
S-2, S-3, S-4	28				2.6542	5.4007
S-2, S-4, S-1	29				3.5288	4.6079
S-2, S-4, S-2	30				3.5288	3.5869
S-2, S-4, S-3	31				3.5288	2.4652
S-2, S-4, S-4	32				3.5288	6.5114
S-3, S-1, S-1	33				4.8842	5.6104
S-3, S-1, S-2	34				4.8842	4.5886
S-3, S-1, S-3	35				4.8842	3.466
S-3, S-1, S-4	36				4.8842	7.5153
S-3, S-2, S-1	37				3.8659	5.616
S-3, S-2, S-2	38				3.8659	4.5933
S-3, S-2, S-3	39				3.8659	3.4695
S-3, S-2, S-4	40				3.8659	7.5229
S-3, S-3, S-1	41				2.7469	8.3397
S-3, S-3, S-2	42				2.7469	2.2506
S-3, S-3, S-3	43				2.7469	3.5424
S-3, S-3, S-4	44				2.7469	7.4193
S-3, S-4, S-1	45				6.7829	10.04
S-3, S-4, S-2	46				6.7829	3.605
S-3, S-4, S-3	47				6.7829	5.9986
S-3, S-4, S-4	48				6.7829	10.593
S-4, S-1, S-1	49				5.4573	8.0454
S-4, S-1, S-2	50				5.4573	1.6424
S-4, S-1, S-3	51				5.4573	4.0242
S-4, S-1, S-4	52				5.4573	8.5957
S-4, S-2, S-1	53				4.4377	6.2169
S-4, S-2, S-2	54				4.4377	5.1934
S-4, S-2, S-3	55				4.4377	4.0689
S-4, S-2, S-4	56				4.4377	8.1249
S-4, S-3, S-1	57				3.3175	6.2238
S-4, S-3, S-2	58				3.3175	5.1992
S-4, S-3, S-3	59				3.3175	4.0734
S-4, S-3, S-4	60				3.3175	8.134
S-4, S-4, S-1	61				7.3581	10.026
S-4, S-4, S-2	62				7.3581	5.5699
S-4, S-4, S-3	63				7.3581	6.4826
S-4, S-4, S-4	64				7.3581	11.05

The last step in the cash flow calculation is to calculate the change in deposit balances. An increase in deposit balances is a NEGATIVE number, because that represents "cash in." A decrease in deposit balances is shown as a positive number, because we are trying the value the cost of the deposit franchise and a reduction in balances is a reduction in principal, like a principal payment on a bond. Figure 5 shows the net cash outflow on the deposit franchise at each node of the tree, with the additional assumption that we terminate our valuation as of time T=4 by having all deposits “mature” or run-off.

Figure 5

Change in Deposit Balance (Net Cash OUT Flow)						
State	**Row**	**0**	**1**	**2**	**3**	**4**
S-1, S-1, S-1	1	0	-4.399	-0.691	4.2522	993.62
S-1, S-1, S-2	2		-4.399	-0.691	4.2522	994.71
S-1, S-1, S-3	3		-4.399	-0.691	4.2522	995.5
S-1, S-1, S-4	4		-4.399	-0.691	4.2522	989.77
S-1, S-2, S-1	5			-3.64	-2.197	1006.1
S-1, S-2, S-2	6			-3.64	-2.197	1007.1
S-1, S-2, S-3	7			-3.64	-2.197	1008.3
S-1, S-2, S-4	8			-3.64	-2.197	1004.2
S-1, S-3, S-1	9			-2.828	0.1924	999.48
S-1, S-3, S-2	10			-2.828	0.1924	1006
S-1, S-3, S-3	11			-2.828	0.1924	1003.6
S-1, S-3, S-4	12			-2.828	0.1924	998.92
S-1, S-4, S-1	13			-2.3	4.8104	994.43
S-1, S-4, S-2	14			-2.3	4.8104	995.51
S-1, S-4, S-3	15			-2.3	4.8104	996.3
S-1, S-4, S-4	16			-2.3	4.8104	990.6
S-2, S-1, S-1	17				-0.541	1005.3
S-2, S-1, S-2	18				-0.541	1011.7
S-2, S-1, S-3	19				-0.541	1009.4
S-2, S-1, S-4	20				-0.541	1004.8
S-2, S-2, S-1	21				-3.843	1012.5
S-2, S-2, S-2	22				-3.843	1015.8
S-2, S-2, S-3	23				-3.843	1015.4
S-2, S-2, S-4	24				-3.843	1014.5
S-2, S-3, S-1	25				-3.416	1011.3
S-2, S-3, S-2	26				-3.416	1017.3
S-2, S-3, S-3	27				-3.416	1016
S-2, S-3, S-4	28				-3.416	1012.2
S-2, S-4, S-1	29				-2.548	1012.1
S-2, S-4, S-2	30				-2.548	1013.1
S-2, S-4, S-3	31				-2.548	1014.2
S-2, S-4, S-4	32				-2.548	1010.2
S-3, S-1, S-1	33				-1.193	1008.9
S-3, S-1, S-2	34				-1.193	1009.9
S-3, S-1, S-3	35				-1.193	1011
S-3, S-1, S-4	36				-1.193	1007
S-3, S-2, S-1	37				-2.206	1009.9
S-3, S-2, S-2	38				-2.206	1010.9
S-3, S-2, S-3	39				-2.206	1012.1
S-3, S-2, S-4	40				-2.206	1008
S-3, S-3, S-1	41				-3.318	1008.3
S-3, S-3, S-2	42				-3.318	1014.4
S-3, S-3, S-3	43				-3.318	1013.1
S-3, S-3, S-4	44				-3.318	1009.2
S-3, S-4, S-1	45				0.7029	1002.5
S-3, S-4, S-2	46				0.7029	1009
S-3, S-4, S-3	47				0.7029	1006.6
S-3, S-4, S-4	48				0.7029	1002
S-4, S-1, S-1	49				-0.616	1005.3
S-4, S-1, S-2	50				-0.616	1011.7
S-4, S-1, S-3	51				-0.616	1009.4
S-4, S-1, S-4	52				-0.616	1004.8
S-4, S-2, S-1	53				-1.632	1008.2
S-4, S-2, S-2	54				-1.632	1009.2
S-4, S-2, S-3	55				-1.632	1010.3
S-4, S-2, S-4	56				-1.632	1006.3
S-4, S-3, S-1	57				-2.747	1009.3
S-4, S-3, S-2	58				-2.747	1010.3
S-4, S-3, S-3	59				-2.747	1011.5
S-4, S-3, S-4	60				-2.747	1007.4
S-4, S-4, S-1	61				1.285	1001.4
S-4, S-4, S-2	62				1.285	1005.9
S-4, S-4, S-3	63				1.285	1005
S-4, S-4, S-4	64				1.285	1000.4

The total cash outflow on the deposit franchise is the sum of the effective interest paid in Figure 4 and the net cash outflow on the deposit franchise shown in Figure 5. We add these numbers at each node of the bushy tree to get the cash flow cost of the deposit franchise at each node of the bushy tree. We drop those numbers into the cash flow table in Figure 6 and derive the risk-neutral value (cost) of this non-maturity deposit franchise with an initial balance of $1,000. That value is $946.7281:

Figure 6

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1		-2.7917	4.6662	14.5385	1,006.7585
2		-2.7917	4.6662	14.5385	1,006.7786
3		-2.7917	4.6662	14.5385	1,006.7918
4		-2.7917	4.6662	14.5385	1,006.6681
5			-1.2386	1.6592	1,013.3684
6			-1.2386	1.6592	1,013.3685
7			-1.2386	1.6592	1,013.3661
8			-1.2386	1.6592	1,013.3626
9			0.3900	6.4406	1,010.9170
10			0.3900	6.4406	1,010.9525
11			0.3900	6.4406	1,010.9492
12			0.3900	6.4406	1,010.9100
13			1.4472	15.6493	1,006.2158
14			1.4472	15.6493	1,006.2335
15			1.4472	15.6493	1,006.2448
16			1.4472	15.6493	1,006.1344
17				5.0053	1,014.6727
18				5.0053	1,014.6570
19				5.0053	1,014.6724
20				5.0053	1,014.6701
21				-1.6207	1,018.0099
22				-1.6207	1,017.9755
23				-1.6207	1,017.9812
24				-1.6207	1,017.9916
25				-0.7618	1,017.5829
26				-0.7618	1,017.5169
27				-0.7618	1,017.5369
28				-0.7618	1,017.5775
29				0.9805	1,016.6956
30				0.9805	1,016.6869
31				0.9805	1,016.6751
32				0.9805	1,016.7063
33				3.6916	1,014.5112
34				3.6916	1,014.5067
35				3.6916	1,014.4995
36				3.6916	1,014.5139
37				1.6594	1,015.5356
38				1.6594	1,015.5302
39				1.6594	1,015.5218
40				1.6594	1,015.5403
41				-0.5710	1,016.6664
42				-0.5710	1,016.6294
43				-0.5710	1,016.6433
44				-0.5710	1,016.6655
45				7.4858	1,012.5821
46				7.4858	1,012.5888
47				7.4858	1,012.5960
48				7.4858	1,012.5777
49				4.8415	1,013.3944
50				4.8415	1,013.3707
51				4.8415	1,013.3890
52				4.8415	1,013.3927
53				2.8053	1,014.4239
54				2.8053	1,014.4208
55				2.8053	1,014.4150
56				2.8053	1,014.4242
57				0.5704	1,015.5510
58				0.5704	1,015.5468
59				0.5704	1,015.5397
60				0.5704	1,015.5534
61				8.6432	1,011.4556
62				8.6432	1,011.4740
63				8.6432	1,011.4735
64				8.6432	1,011.4456
Risk Neutral Value =					946.7281

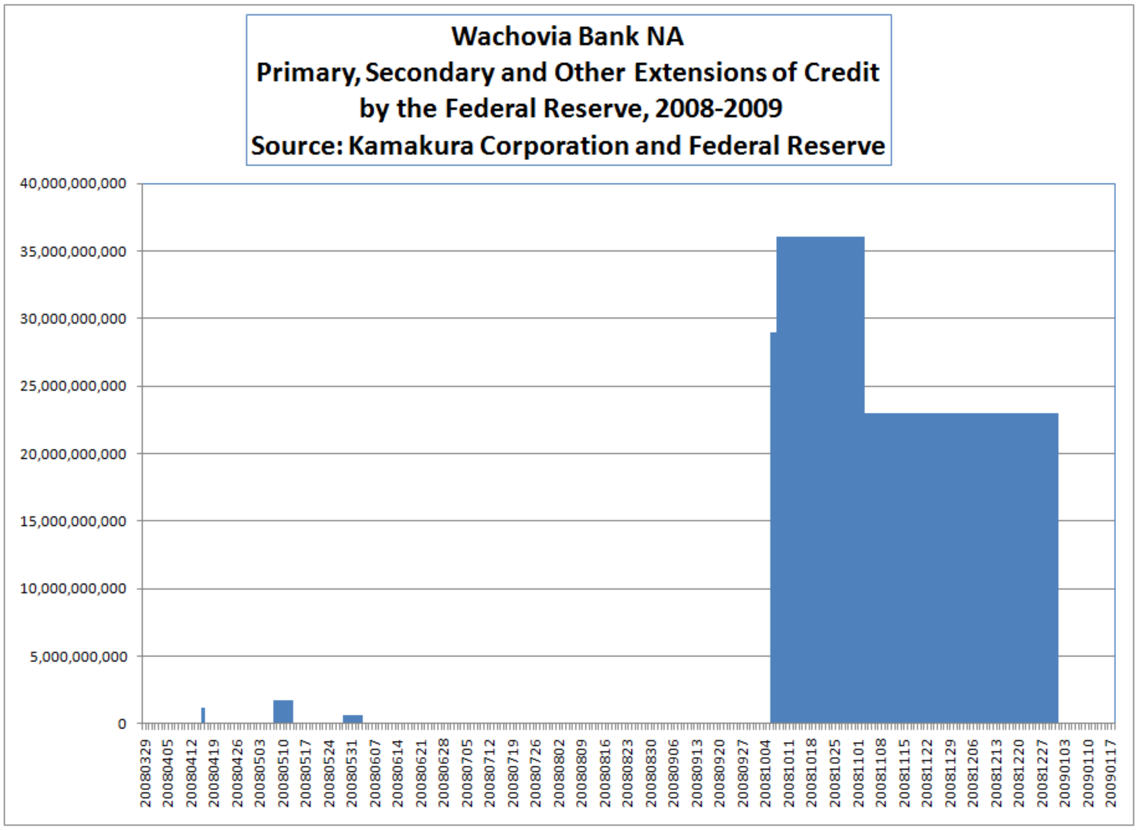
In a rational world, a bank would be willing to pay up to $1,000 - 946.7281 = 53.2719 to buy this deposit franchise, a premium of 5.32719% above par value of the deposits.

This example shows that there is nothing mysterious or different about the valuation (and by extension, hedging) of non-maturity deposits. Using the standard HJM methodology like our example of Chapter 9, we populate the bushy tree with the appropriate cash flows and valuation follows directly. The same process gives us information for market risk ("value at risk") and liquidity risk ("cash flow at risk") from the non-maturity deposit franchise. This is the power of the HJM framework in action. Results are both more accurate and simpler to derive than the purely mathematical approaches we cited earlier in the chapter, which rely on one factor term structure models.

The Impact of Bank Credit Risk on Deposit Rates and Balances

We learned one more time in the credit crisis of 2007 to 2010 that deposit insurance does not mean deposits will not run off if the bank has credit problems. Northern Rock PLC was seized by the Bank of England early in 2008 even after the Bank of England declared its full support for Northern Rock late in 2007. Similarly, as we discuss in detail in Chapter 37, the U.S. Federal Reserve lent more than $1 trillion to both U.S. and international financial institutions in the 2008-2009 period, even though almost all of them were beneficiaries of U.S. government deposit insurance. Figure 7, provided by Kamakura Corporation, shows that Wachovia Bank was forced to borrow $29 billion on October 6, 2008 from the Federal Reserve and that it increased borrowings to $36 billion on October 8 because its liquid asset holdings were insufficient to fully pay off deposits withdrawn by depositors of all types:

Figure 7



How do we take the impact of the bank's own risk into account in the analysis above? We make that modification in this section. In a sense, non-maturity deposits have an embedded put option or credit default swap that is triggered when a bank's risk hits a high enough level. In our example above, we are assuming that all deposits run off at time T=4. If we incorporate credit risk, it will affect deposit rates and balances at times 1, 2 or 3. One sample modification is to say the following: the deposit balance will follow the equation above except when the bank's one period funding cost (the spot rate, not the deposit rate) is 4.00% or higher. In this circumstance, it is assumed that 70% of the bank's deposits run-off immediately. This happens at 8 of the 64 nodes at time T=3, as seen in Figure 8:

Figure 8

Maturity of Cash Flow Received

	Current Time				
Row	**0**	**1**	**2**	**3**	**4**
1		-2.7917	4.6662	706.6626	294.6586
2		-2.7917	4.6662	706.6626	295.4432
3		-2.7917	4.6662	706.6626	296.0126
4		-2.7917	4.6662	706.6626	291.8598
5			-1.2386	1.6592	1,013.3684
6			-1.2386	1.6592	1,013.3685
7			-1.2386	1.6592	1,013.3661
8			-1.2386	1.6592	1,013.3626
9			0.3900	6.4406	1,010.9170
10			0.3900	6.4406	1,010.9525
11			0.3900	6.4406	1,010.9492
12			0.3900	6.4406	1,010.9100
13			1.4472	706.8309	295.6470
14			1.4472	706.8309	296.4236
15			1.4472	706.8309	296.9871
16			1.4472	706.8309	292.8769
17				5.0053	1,014.6727
18				5.0053	1,014.6570
19				5.0053	1,014.6724
20				5.0053	1,014.6701
21				-1.6207	1,018.0099
22				-1.6207	1,017.9755
23				-1.6207	1,017.9812
24				-1.6207	1,017.9916
25				-0.7618	1,017.5829
26				-0.7618	1,017.5169
27				-0.7618	1,017.5369
28				-0.7618	1,017.5775
29				0.9805	1,016.6956
30				0.9805	1,016.6869
31				0.9805	1,016.6751
32				0.9805	1,016.7063
33				3.6916	1,014.5112
34				3.6916	1,014.5067
35				3.6916	1,014.4995
36				3.6916	1,014.5139
37				1.6594	1,015.5356
38				1.6594	1,015.5302
39				1.6594	1,015.5218
40				1.6594	1,015.5403
41				-0.5710	1,016.6664
42				-0.5710	1,016.6294
43				-0.5710	1,016.6433
44				-0.5710	1,016.6655
45				7.4858	1,012.5821
46				7.4858	1,012.5888
47				7.4858	1,012.5960
48				7.4858	1,012.5777
49				4.8415	1,013.3944
50				4.8415	1,013.3707
51				4.8415	1,013.3890
52				4.8415	1,013.3927
53				2.8053	1,014.4239
54				2.8053	1,014.4208
55				2.8053	1,014.4150
56				2.8053	1,014.4242
57				0.5704	1,015.5510
58				0.5704	1,015.5468
59				0.5704	1,015.5397
60				0.5704	1,015.5534
61				8.6432	1,011.4556
62				8.6432	1,011.4740
63				8.6432	1,011.4735
64				8.6432	1,011.4456
Risk Neutral Value =					**948.2673**

The risk neutral cost of the non-maturity deposit franchise rises slightly to $948.2673, because the effective maturity of the non-maturity deposits is shortened. While the impact on value is relatively small in this example, the impact on "cash flow at risk" at time T=3 is very very large. In Chapters 36 through 41, we will analyze the impact of deposit outflows on the bankruptcy risk of the organization in much more detail.

Case Study: German Three-Month Notice Savings Deposits[2]

In this section, we use the Kamakura Risk Manager enterprise wide risk management system to value and analyze the interest rate sensitivity of three-month notice savings deposits in Germany. Instead of the 3 factor HJM model we have illustrated above, we use a 1 factor model consistent with the papers by Janosi, Jarrow and van Deventer above. This data set is of interest because it is perhaps the longest deposit history available. There are 469 monthly observations on the deposit balance data running from December 1959 to December 1998. There are 361 monthly observations on the deposit rate running from June 1967 to February 1999. Note that these figures are national deposit figures, rather than the numbers for a particularly bank. As such, they dramatically understate the risk of deposit outflows. With this caveat, we proceed.

The valuation yield curve derived in Kamakura Risk Manager was based on April 28 1999 interest rates in Germany, including one-month, three-month, six-month, nine-month and 12 month money market rates and two-year, three-year, five-year, seven-year, 10-year, 15-year, 20-year and 30-year Deutsche Mark interest rate swap rates. The maximum smoothness forward rate smoothing technique of Chapter 5 was selected for yield curve smoothing.

The speed of mean reversion α was set at 0.01 and the interest rate volatility σ in the Extended Vasicek model was set at 0.008.

Kamakura Risk Manager was used to fit the deposit interest rate regression of the Jarrow-van Deventer [1998] model to the deposit rate history. The regression explains deposit rate variation as a function of the lagged deposit rate, a constant, and short-term interest rate in Germany. The resulting relationship explains 98.76% in monthly variation of the three-month notice savings deposit rate, and the relationship realistically says, all other things being equal, that a 100 basis point move in short-term open market rates in Germany leads to a two basis point move in the same month in the deposit rate. Of course, the long-term equilibrium level of the deposit rate is much higher than this because of the impact of the lagged deposit rate.

The deposit balance relationship was also fitted in Kamakura Risk Manager using linear regression. The relationship explains 99.97% of the variation in deposit balances and shows quite clearly that (all other things being equal) a rise in money market rates in Germany lowers savings deposit balances—disintermediation takes place as has been found in many countries.

This excellent fit comes in spite of the impact of the unification with East Germany during the life of the data set, which caused a jump in deposit balances.

[2] The authors would like to thank Robert Fiedler, formerly of Deutsche Bank, and Karin Gradischnig of Reuters for helpful comments on this section.

Using representative assumptions on long-term deposit balance levels, three-month notice deposits in Germany were found to have a present value cost of about 67% of their nominal amount and a negative duration of 2.29 years. This means that, all other things being equal, when interest rates rise, the present value cost of the deposits actually declines because of the reduction in balances that comes about as noted above. This is identical to the experience of the U.S. savings and loan industry during its high interest rate crisis in the late 1970s and 1980s.

Negative duration was also found for parallel shifts in the German yield curve, much the same as found by Chase Manhattan Bank in its own analysis of the effective interest rate sensitivity of non-maturity deposits in the U.S.[3]

The Regulators' View

The regulatory agencies around the world clearly have a strong interest in valuation of non-maturity deposits. James O'Brien [2000] of the Board of Governors of the Federal Reserve takes a Monte Carlo simulation approach to the valuation of non-maturity deposits with a strong similarity to the Jarrow-van Deventer model. The National Credit Union Administration of the United States in December 2001 commissioned a study of non-maturity deposit evaluation techniques by National Economic Research Associates, with David Ellis and James Jordan [2001] as lead authors. We urge readers with a strong interest in this topic to review both papers in detail.

Conclusions

The default probability of the financial institution offering non-maturity deposits to consumers is a critical determinant of non-maturity deposit balances and rates. This in turn has a big impact on the valuation and risk of both the firm as a whole and non-maturity deposits in particular. We revisit these issues in great detail in Chapters 36 through 41.

[3] Private conversation with Dr. Robert Selvaggio, formerly of Chase Manhattan Bank.