Kamakura Corporation

2222 Kalakaua Avenue
Suite 1400
Honolulu, Hawaii 96815, USA
phone 1 808 791 9888
facsimile 1 808 791 9898
www.kamakuraco.com

Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

April 2012

Donald R. van Deventer
Kenji Imai
Mark Mesler

Part 4: Risk Management Applications, Instrument by Instrument

Chapter 25:

Interest Rate Swaps and Swaptions

Interest rate swaps have become the most successful over-the-counter derivative security in the world, and for this reason it is particularly important that we include their credit-adjusted valuation in this book. We emphasize, in the wake of the scandal that enveloped Barclays for Libor manipulation, that nothing in this chapter is specific to Libor. As in Chapter 24, we assume that the short term reference rate and other point on the yield curve from which swap pricing is derived are determined in a perfectly competitive transparent market. Libor does not fit that description and it should be rejected as a pricing index by sophisticated (and honest) market participants.

Even when swap counterparties offer transactions through an AAA-rated special purposes vehicle, credit-adjusted valuation is critical because the default probabilities of major financial institutions are highly correlated, as the credit crisis chronology in the Introduction documents. This means that a guarantee by Bank A of another investment bank's special purpose vehicle has much less value than a guarantee by another entity with a much lower correlation in default probabilities and events of default.

Valuation and simulation of interest rate swaps and swaptions is also critical to meet the accounting requirements for hedges under the constantly evolving hedge accounting requirements of Generally Accepted Accounting Principles in the United States and International Financial Reporting Standards around the world. In this chapter, we will continue to employ the 3 factor Heath Jarrow and Morton framework of Chapter 9 for valuation analysis.

Interest Rate Swap Basics

The most common plain vanilla interest rate swap between two counterparties calls for counterparty A to make fixed-rate payments at even intervals (normally semi-annual) to counterparty B and for counterparty B to make floating-rate payments, typically at three-month or six-month intervals, to counterparty A. Seen from the perspective of counterparty A, the swap has the same net cash flow as if counterparty A issued a fixed-rate bond and purchased a floating-rate bond (except in the case of default, where the principal amount is relevant in the bond case but not in the swap case). From the perspective of counterparty B, the swap cash flows are identical to the case where counterparty B purchases a fixed-rate bond with the proceeds of a floating-rate bond issue (again, except in the case of default).

Why are interest rate swaps so popular? The primary reasons include the vast liquidity of the swap markets in the major currencies, the low issuance costs compared to traditional bonds even under a medium-term note program or shelf registration, the ease of reversing a position with an offsetting transaction, and the (historical) lack of daily mark-to-market margin requirements for swap market participants of good quality. In the wake of the credit crisis of 2007 to 2010, however, nearly all counterparties now face daily margin requirements, but at least margins are settled on a net basis rather than on a transaction by transaction basis. While the cash flows would be the same in either case, the operational efficiency of margining on net exposure is overwhelming. Operationally, a swap has historically been much less work than a similar position in interest rate futures with the daily mark-to-market requirements. In the way of the pervasive troubles of the largest swap counterparties, however, the move toward exchange-settled transactions may well reverse this trend in the years ahead.

Many market participants value swaps as if both the fixed and the floating side involved principal payments at maturity. Since the amounts net to zero, this is a good approximation when both parties have zero credit risk. As the credit risk of each party diverges from zero, this becomes a more dangerous assumption, so we won't use it in this chapter.

Valuing the Interest Rate Swaps

To illustrate the valuation of interest rate swaps in the general HJM valuation framework, we will use the assumptions of the 3 factor HJM example in Chapter 9. We assume that the short term floating rate side of the interest rate swap pays at one year intervals at the 1 year spot rate. We know from Figure 22 in Chapter 9 that the value of a bond paying 3% interest at annual intervals has a present value of 104.7071. This is the value of the fixed side of a swap when the annual interest rate is 3% and we include an exchange of principal. We know from Figure 9 in Chapter 19 that the present value of a floating rate transaction with $100 principal amount that matures at time T=4 is par: 100.0000. The value of this "swap" to the party who receives the fixed rate side and pays the floating rate side is 104.7071-100.0000 = 4.7071. If this were not true, our no arbitrage assumptions about the market place would be proven false.

This example assumed that the principal amounts were exchanged at maturity. If we do away with that assumption, we know that the value of $3 received annually will be equal to the value of the bond (104.7071) less the present value of $100 received in 4 years. Using our standard zero coupon bonds as input, we know the present value of a dollar received in 4 years is 0.9308550992. This means the value of $3 received annually is 104.7071 – 93.0855 = 11.6216. We can confirm this by simply valuing the fixed coupons using the zero coupon bond prices we are given for maturity at times 1, 2, 3, and 4 or, as shown in Figure 1, we can insert $3 in the cash flow table at every node on the bushy tree to confirm the value of the fixed payments is indeed 11.6216:

Figure 1

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	3.0000	3.0000	3.0000	3.0000
2		3.0000	3.0000	3.0000	3.0000
3		3.0000	3.0000	3.0000	3.0000
4		3.0000	3.0000	3.0000	3.0000
5			3.0000	3.0000	3.0000
6			3.0000	3.0000	3.0000
7			3.0000	3.0000	3.0000
8			3.0000	3.0000	3.0000
9			3.0000	3.0000	3.0000
10			3.0000	3.0000	3.0000
11			3.0000	3.0000	3.0000
12			3.0000	3.0000	3.0000
13			3.0000	3.0000	3.0000
14			3.0000	3.0000	3.0000
15			3.0000	3.0000	3.0000
16			3.0000	3.0000	3.0000
17				3.0000	3.0000
18				3.0000	3.0000
19				3.0000	3.0000
20				3.0000	3.0000
21				3.0000	3.0000
22				3.0000	3.0000
23				3.0000	3.0000
24				3.0000	3.0000
25				3.0000	3.0000
26				3.0000	3.0000
27				3.0000	3.0000
28				3.0000	3.0000
29				3.0000	3.0000
30				3.0000	3.0000
31				3.0000	3.0000
32				3.0000	3.0000
33				3.0000	3.0000
34				3.0000	3.0000
35				3.0000	3.0000
36				3.0000	3.0000
37				3.0000	3.0000
38				3.0000	3.0000
39				3.0000	3.0000
40				3.0000	3.0000
41				3.0000	3.0000
42				3.0000	3.0000
43				3.0000	3.0000
44				3.0000	3.0000
45				3.0000	3.0000
46				3.0000	3.0000
47				3.0000	3.0000
48				3.0000	3.0000
49				3.0000	3.0000
50				3.0000	3.0000
51				3.0000	3.0000
52				3.0000	3.0000
53				3.0000	3.0000
54				3.0000	3.0000
55				3.0000	3.0000
56				3.0000	3.0000
57				3.0000	3.0000
58				3.0000	3.0000
59				3.0000	3.0000
60				3.0000	3.0000
61				3.0000	3.0000
62				3.0000	3.0000
63				3.0000	3.0000
64				3.0000	3.0000
Risk Neutral Value =					11.62159

Similarly, the floating rate payments, in the absence of arbitrage, should total 100.0000 – 93.0855 = 6.9145. We can confirm this by inserting the floating rate payments (but no principal payments) in the cash flow table and confirming the 6.9145 value in Figure 2:

Figure 2

		Time Period				
State	Row	0	1	2	3	4
S-1, S-1, S-1	1		0.3003	2.1653	4.6388	6.1106
S-1, S-1, S-2	2		0.3003	2.1653	4.6388	5.5667
S-1, S-1, S-3	3		0.3003	2.1653	4.6388	5.1710
S-1, S-1, S-4	4		0.3003	2.1653	4.6388	8.0375
S-1, S-2, S-1	5			0.6911	1.4142	3.1041
S-1, S-2, S-2	6			0.6911	1.4142	2.5913
S-1, S-2, S-3	7			0.6911	1.4142	2.0291
S-1, S-2, S-4	8			0.6911	1.4142	4.0628
S-1, S-3, S-1	9			1.0972	2.6089	5.2210
S-1, S-3, S-2	10			1.0972	2.6089	1.9784
S-1, S-3, S-3	11			1.0972	2.6089	3.1798
S-1, S-3, S-4	12			1.0972	2.6089	5.5017
S-1, S-4, S-1	13			1.3611	4.9179	5.4252
S-1, S-4, S-2	14			1.3611	4.9179	4.8848
S-1, S-4, S-3	15			1.3611	4.9179	4.4917
S-1, S-4, S-4	16			1.3611	4.9179	7.3396
S-2, S-1, S-1	17				2.2496	4.1517
S-2, S-1, S-2	18				2.2496	0.9421
S-2, S-1, S-3	19				2.2496	2.1313
S-2, S-1, S-4	20				2.2496	4.4295
S-2, S-2, S-1	21				0.5984	2.2036
S-2, S-2, S-2	22				0.5984	0.5531
S-2, S-2, S-3	23				0.5984	0.7667
S-2, S-2, S-4	24				0.5984	1.2004
S-2, S-3, S-1	25				0.8121	2.6231
S-2, S-3, S-2	26				0.8121	-0.3734
S-2, S-3, S-3	27				0.8121	0.2593
S-2, S-3, S-4	28				0.8121	2.1679
S-2, S-4, S-1	29				1.2459	1.7764
S-2, S-4, S-2	30				1.2459	1.2703
S-2, S-4, S-3	31				1.2459	0.7153
S-2, S-4, S-4	32				1.2459	2.7228
S-3, S-1, S-1	33				1.9217	2.2804
S-3, S-1, S-2	34				1.9217	1.7718
S-3, S-1, S-3	35				1.9217	1.2141
S-3, S-1, S-4	36				1.9217	3.2315
S-3, S-2, S-1	37				1.4149	2.2804
S-3, S-2, S-2	38				1.4149	1.7718
S-3, S-2, S-3	39				1.4149	1.2141
S-3, S-2, S-4	40				1.4149	3.2315
S-3, S-3, S-1	41				0.8591	3.6354
S-3, S-3, S-2	42				0.8591	0.6093
S-3, S-3, S-3	43				0.8591	1.2483
S-3, S-3, S-4	44				0.8591	3.1757
S-3, S-4, S-1	45				2.8695	4.5070
S-3, S-4, S-2	46				2.8695	1.2864
S-3, S-4, S-3	47				2.8695	2.4797
S-3, S-4, S-4	48				2.8695	4.7858
S-4, S-1, S-1	49				2.2088	3.5013
S-4, S-1, S-2	50				2.2088	0.3117
S-4, S-1, S-3	51				2.2088	1.4934
S-4, S-1, S-4	52				2.2088	3.7773
S-4, S-2, S-1	53				1.7005	2.5831
S-4, S-2, S-2	54				1.7005	2.0730
S-4, S-2, S-3	55				1.7005	1.5136
S-4, S-2, S-4	56				1.7005	3.5370
S-4, S-3, S-1	57				1.1432	2.5831
S-4, S-3, S-2	58				1.1432	2.0730
S-4, S-3, S-3	59				1.1432	1.5136
S-4, S-3, S-4	60				1.1432	3.5370
S-4, S-4, S-1	61				3.1593	4.5061
S-4, S-4, S-2	62				3.1593	2.2686
S-4, S-4, S-3	63				3.1593	2.7252
S-4, S-4, S-4	64				3.1593	5.0230
	Risk Neutral Value =					6.91449

Alternatively, we can calculate the net cash flow received by the counterparty who receives the fixed rate payment. MODEL RISK ALERT: This netting is only appropriate if the two swap counterparties are riskless. If we calculate the fixed payment less the floating payment at every node, we get the cash flows in Figure 3. The value of the swap should be the value of the fixed payments less the value of the floating payments, 11.6216 – 6.9145 = 4.7071 and indeed it is:

Figure 3

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	2.6997	0.8347	-1.6388	-3.1106
2		2.6997	0.8347	-1.6388	-2.5667
3		2.6997	0.8347	-1.6388	-2.1710
4		2.6997	0.8347	-1.6388	-5.0375
5			2.3089	1.5858	-0.1041
6			2.3089	1.5858	0.4087
7			2.3089	1.5858	0.9709
8			2.3089	1.5858	-1.0628
9			1.9028	0.3911	-2.2210
10			1.9028	0.3911	1.0216
11			1.9028	0.3911	-0.1798
12			1.9028	0.3911	-2.5017
13			1.6389	-1.9179	-2.4252
14			1.6389	-1.9179	-1.8848
15			1.6389	-1.9179	-1.4917
16			1.6389	-1.9179	-4.3396
17				0.7504	-1.1517
18				0.7504	2.0579
19				0.7504	0.8687
20				0.7504	-1.4295
21				2.4016	0.7964
22				2.4016	2.4469
23				2.4016	2.2333
24				2.4016	1.7996
25				2.1879	0.3769
26				2.1879	3.3734
27				2.1879	2.7407
28				2.1879	0.8321
29				1.7541	1.2236
30				1.7541	1.7297
31				1.7541	2.2847
32				1.7541	0.2772
33				1.0783	0.7196
34				1.0783	1.2282
35				1.0783	1.7859
36				1.0783	-0.2315
37				1.5851	0.7196
38				1.5851	1.2282
39				1.5851	1.7859
40				1.5851	-0.2315
41				2.1409	-0.6354
42				2.1409	2.3907
43				2.1409	1.7517
44				2.1409	-0.1757
45				0.1305	-1.5070
46				0.1305	1.7136
47				0.1305	0.5203
48				0.1305	-1.7858
49				0.7912	-0.5013
50				0.7912	2.6883
51				0.7912	1.5066
52				0.7912	-0.7773
53				1.2995	0.4169
54				1.2995	0.9270
55				1.2995	1.4864
56				1.2995	-0.5370
57				1.8568	0.4169
58				1.8568	0.9270
59				1.8568	1.4864
60				1.8568	-0.5370
61				-0.1593	-1.5061
62				-0.1593	0.7314
63				-0.1593	0.2748
64				-0.1593	-2.0230
Risk Neutral Value =					4.7071

We now ask the question “what level of interest rate should prevail on the fixed side of the swap” for new swaps?

The Observable Fixed Rate in the Swap Market

We know from the prior section and Chapter 19 that the floating side of a new swap should have a market value of par value, 100.0000, if the principal exchange at maturity is included. We also know, from no arbitrage principles, that for a new swap the fixed side should also have a net present value of 100.0000 so that the net value to the receiver of the fixed rate is 100.0000 – 100.0000 = 0. If this were not the case, there would be arbitrage opportunities. For what fixed rate will market value exactly equal par value? We know the answer from Chapter 4:

$$\textit{Dollar Coupon Amount of Fixed Coupon Bond} = \frac{\textit{Value} - P(t_n)\textit{Principal}}{\sum_{i=1}^{n} P(t_i)}$$

When “Value” = 100, using the zero coupon bond prices for time 0 given in Chapter 9, the “par coupon” level is 1.78490814. We can verify that a 4 year bond with annual payments at this level and principal of 100 has a net present value of 100.0000 either using the original zero coupon bond values and formulas from Chapter 4 or using the bushy tree from Chapter 9. If we do the latter, we get the cash flows in Figure 4 and confirm a value of 100.0000:

Figure 4

	Maturity of Cash Flow Received				
	Current Time				
Row	0	1	2	3	4
1	0.0000	1.7849	1.7849	1.7849	101.7849
2		1.7849	1.7849	1.7849	101.7849
3		1.7849	1.7849	1.7849	101.7849
4		1.7849	1.7849	1.7849	101.7849
5			1.7849	1.7849	101.7849
6			1.7849	1.7849	101.7849
7			1.7849	1.7849	101.7849
8			1.7849	1.7849	101.7849
9			1.7849	1.7849	101.7849
10			1.7849	1.7849	101.7849
11			1.7849	1.7849	101.7849
12			1.7849	1.7849	101.7849
13			1.7849	1.7849	101.7849
14			1.7849	1.7849	101.7849
15			1.7849	1.7849	101.7849
16			1.7849	1.7849	101.7849
17				1.7849	101.7849
18				1.7849	101.7849
19				1.7849	101.7849
20				1.7849	101.7849
21				1.7849	101.7849
22				1.7849	101.7849
23				1.7849	101.7849
24				1.7849	101.7849
25				1.7849	101.7849
26				1.7849	101.7849
27				1.7849	101.7849
28				1.7849	101.7849
29				1.7849	101.7849
30				1.7849	101.7849
31				1.7849	101.7849
32				1.7849	101.7849
33				1.7849	101.7849
34				1.7849	101.7849
35				1.7849	101.7849
36				1.7849	101.7849
37				1.7849	101.7849
38				1.7849	101.7849
39				1.7849	101.7849
40				1.7849	101.7849
41				1.7849	101.7849
42				1.7849	101.7849
43				1.7849	101.7849
44				1.7849	101.7849
45				1.7849	101.7849
46				1.7849	101.7849
47				1.7849	101.7849
48				1.7849	101.7849
49				1.7849	101.7849
50				1.7849	101.7849
51				1.7849	101.7849
52				1.7849	101.7849
53				1.7849	101.7849
54				1.7849	101.7849
55				1.7849	101.7849
56				1.7849	101.7849
57				1.7849	101.7849
58				1.7849	101.7849
59				1.7849	101.7849
60				1.7849	101.7849
61				1.7849	101.7849
62				1.7849	101.7849
63				1.7849	101.7849
64				1.7849	101.7849
Risk Neutral Value =					100.0000

Similarly, we can do away with the exchange of principal and value the fixed payments minus the floating rate payments. When we do so, we confirm in Figure 5 that the "at market" fixed rate swap rate produces a mark to market value of the swap equal to par value of 100.0000:

Figure 5

	Current Time	Maturity of Cash Flow Received			
Row	0	1	2	3	4
1	0.0000	1.4846	-0.3804	-2.8539	-4.3257
2		1.4846	-0.3804	-2.8539	-3.7818
3		1.4846	-0.3804	-2.8539	-3.3861
4		1.4846	-0.3804	-2.8539	-6.2526
5			1.0939	0.3707	-1.3192
6			1.0939	0.3707	-0.8064
7			1.0939	0.3707	-0.2442
8			1.0939	0.3707	-2.2779
9			0.6877	-0.8240	-3.4361
10			0.6877	-0.8240	-0.1935
11			0.6877	-0.8240	-1.3949
12			0.6877	-0.8240	-3.7167
13			0.4238	-3.1330	-3.6403
14			0.4238	-3.1330	-3.0999
15			0.4238	-3.1330	-2.7068
16			0.4238	-3.1330	-5.5547
17				-0.4647	-2.3668
18				-0.4647	0.8428
19				-0.4647	-0.3464
20				-0.4647	-2.6446
21				1.1866	-0.4187
22				1.1866	1.2318
23				1.1866	1.0182
24				1.1866	0.5845
25				0.9728	-0.8382
26				0.9728	2.1583
27				0.9728	1.5256
28				0.9728	-0.3830
29				0.5390	0.0085
30				0.5390	0.5146
31				0.5390	1.0696
32				0.5390	-0.9379
33				-0.1368	-0.4955
34				-0.1368	0.0131
35				-0.1368	0.5708
36				-0.1368	-1.4466
37				0.3700	-0.4955
38				0.3700	0.0131
39				0.3700	0.5708
40				0.3700	-1.4466
41				0.9258	-1.8505
42				0.9258	1.1756
43				0.9258	0.5366
44				0.9258	-1.3908
45				-1.0846	-2.7221
46				-1.0846	0.4985
47				-1.0846	-0.6948
48				-1.0846	-3.0009
49				-0.4239	-1.7164
50				-0.4239	1.4732
51				-0.4239	0.2915
52				-0.4239	-1.9924
53				0.0844	-0.7982
54				0.0844	-0.2881
55				0.0844	0.2713
56				0.0844	-1.7521
57				0.6417	-0.7982
58				0.6417	-0.2881
59				0.6417	0.2713
60				0.6417	-1.7521
61				-1.3744	-2.7212
62				-1.3744	-0.4837
63				-1.3744	-0.9403
64				-1.3744	-3.2381
Risk Neutral Value =					**0.0000**

An Introduction to Swaptions

A swaption is an agreement between two counterparties which allows counterparty A to enter into an interest rate swap with a pre-agreed fixed-rate payment *C* at the option of counterparty A. Swaptions come in two forms. The 'constant maturity' form of swaption prescribes that the interest rate swap has a set maturity, say five years, regardless of when the swaption was exercised. For example, if the exercise period on the swaption is two years, the underlying swap would end up being a five- year swap regardless of whether the swaption was entered into after three months, 12 months or two years. The 'fixed maturity date' swaption prescribes a maturity date for the swap at the signing date of the contract, and therefore the effective maturity of the swap will shorten if exercise is delayed. Consider a three-year swaption of the fixed maturity type on a swap with an original maturity of 10 years. If the swaption is exercised after two years, the swap that is entered into will have an eight-year life.

Both forms are common. If the swaption is a European swaption, the two methods are equivalent.

Valuation of European Swaptions

The valuation of European swaptions is a direct application of European bond options that we covered in Chapter 21. In that chapter, we showed how to use the HJM 3 factor model from Chapter 9 to value such options. The valuation in a swap context is nearly identical. If one has the lucky (and extremely unlikely) circumstance where a 1 factor term structure model accurately describes interest rates, Jamshidian's formula for options on coupon-bearing bonds could be employed. MODEL RISK ALERT: A one factor model will probably never be accurate enough for practical use. As shown previously in this chapter, the market value of a new swap must be such that the fixed-rate on the swap would cause a coupon-bearing bond with the same payment frequency and maturity date[1] to trade at par. This means we can ignore the floating-rate side of the swap for purposes of valuation.[2] Therefore a swaption that allows the holder to receive the fixed-rate side of the swap is a call option to buy the equivalent underlying bond. A swaption which allows the holder to pay the fixed-rate side of the swap is a put option on the equivalent underlying bond. Having made this translation, the formula is identical to that of Chapter 21's options on coupon-bearing bonds.

Valuation of American Swaptions

Because of the intervening cash flows on swaps, there is the possibility of early exercise on an American swaption and therefore the European swaption valuation approach is only a rough approximation to the value of an American swaption. Correct valuation requires the numerical methods we analyze in Chapter 27.

Defaultable Interest Rate Swaps and Swaptions

---

[1] And for purposes of this chapter, credit risk.

[2] We could not ignore the floating side if the floating side involved a non-zero spread from the pricing index.

As discussed in the introduction of this chapter, even the existence of AAA-rated special purpose vehicles does not insulate a financial market participant from the risk of default by a counterparty on an interest rate swap or swaption. Many firms found this out the hard way when Lehman Brothers filed for bankruptcy in September 2008. The 2007 to 2010 credit crises provides an almost uncountable number of cases where interest rates and other macro factors (most notably home prices) drove correlated default risk among counterparties.

Therefore we need to take the same approach as that discussed in Chapters 19 through 24, using Monte Carlo simulation of interest rate-driven (and other macro-factor driven) default probabilities to measure the valuation and future distribution of values and cash flows on a credit-adjusted basis. What this means for swap valuation is that different yield curves and different HJM bushy trees should be used to value the different legs of a swap transaction, even if the reference curve used is the same for both halves of the swap. It is the difference in the value of a promise to pay by Company A and Company B that causes this slight complication to be necessary.

We now turn to more exotic swap and option structures.