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Advanced Financial Risk Management:

Tools and Techniques for Integrated Credit Risk, Market Risk, Liquidity Risk and Interest Rate Risk Management

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Part 4: Risk Management Applications, Instrument by Instrument

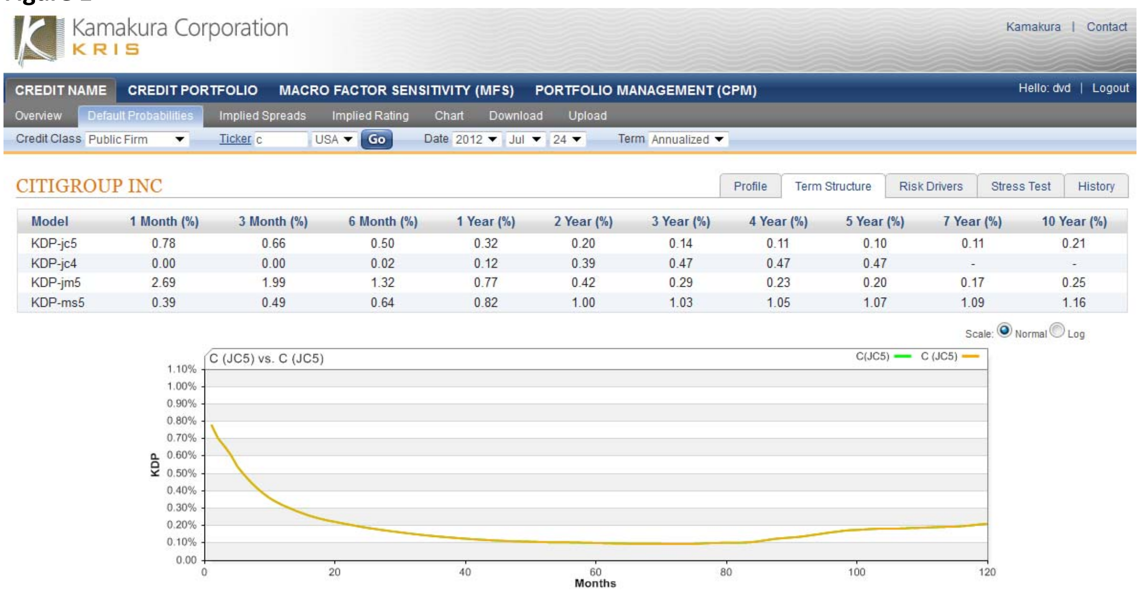
Chapter 34:
Modeling common stock and convertible bonds on a default-adjusted basis

A very large proportion of the world's financial institutions are substantial direct or indirect owners of common stock and convertible bonds. Financial institutions, via their own pension funds, often have a large exposure to equities that is overlooked in risk analysis. In addition, the direct investment in equity-related securities is large and linked in a complex way with other assets and liabilities on the balance sheet.

Many sophisticated investors have major concerns with the inconsistency in their management of fixed income and equity portfolios. Credit risk is typically given lots of attention on the fixed income side and ignored on the equity side. Risk analysis of a position in IBM bonds and IBM common stock is typically done in a way that recognizes the losses of default on the bonds and ignores the simultaneous impact of default on the common stock of IBM. This inconsistency should be very troubling to management, boards of directors, and regulators of the financial institutions involved. The inconsistency stems from the largely independent development of financial theory for fixed income securities and for equities. In the latter cash, a fundamental premise of the capital asset pricing model and arbitrage pricing theory is that the returns on common stocks is normally distributed. MODEL RISK ALERT: This assumption is false. Figure 1, provided by Kamakura Corporation, shows that there is a 0.32% probability that the stock price of Citigroup is zero at the end of one year. This is the best estimate, on an annualized basis, that Citigroup will be bankrupt over that time horizon, using the reduced form models of Chapter 16.

The other maturities in Figure 1 show the bankruptcy probabilities, all on an annualized basis, at different time horizons:

Figure 1



Model	1 Month (%)	3 Month (%)	6 Month (%)	1 Year (%)	2 Year (%)	3 Year (%)	4 Year (%)	5 Year (%)	7 Year (%)	10 Year (%)
KDP-jc5	0.78	0.66	0.50	0.32	0.20	0.14	0.11	0.10	0.11	0.21
KDP-jc4	0.00	0.00	0.02	0.12	0.39	0.47	0.47	0.47	-	-
KDP-jm5	2.69	1.99	1.32	0.77	0.42	0.29	0.23	0.20	0.17	0.25
KDP-ms5	0.39	0.49	0.64	0.82	1.00	1.03	1.05	1.07	1.09	1.16

The existence of bankruptcy is inconsistent with the assumption of normally distributed returns. While bankruptcy risk was explicitly noted in the early models of Merton ["Theory of Rational Options Pricing," 1973], it has been largely ignored in practice. This has massive implications for measured risk. For example, if one calculated the mean monthly returns and volatility of monthly returns for Lehman Brothers and Bear Stearns from 1990 to 2007, they would have implied that the probability of a -100% return in one month (due to bankruptcy) was 0.000000%. The failure of both firms in 2008 shows that the explicit recognition of bankruptcy in the modeling of common stock and convertible bonds is essential for realism.

Credit risk on the equity side has another more subtle form as well, a topic we introduced in Chapter 2. Many fund managers are judged on their performance versus an equity benchmark, such as the S&P 500 index in the U.S. If a company is a 'member' of the list of 500 companies making up the index, it has to be of fairly good credit quality. If the company's default probability rises to a certain level, the odds that it is removed from the S&P500 index become very high. If the company is to be dropped, the announcement is normally made after the close of business. The stock will drop dramatically at the opening the next day, leaving the fund manager with a large loss versus the index because the weakened company is no longer in the index, but the fund manager still has the position from the previous day. This credit risk-related loss can be substantial and has also been almost completely overlooked by many financial institutions. Campbell, Hilscher and Szilagyi [2011], in a paper given the Markowitz award for the best paper in the Journal of Investment Management, show that high default risk equity portfolios persistently underperform other equity portfolios after making all of the standard industry adjustments for risk-adjusted return calculations.

With this as background, we move on to a discussion of how to incorporate common stock and convertible bonds on the list of instruments that we can value as we progress toward our objective—valuing the Jarrow-Merton put option on company assets and liabilities as the comprehensive, integrated measure of interest rate risk, market risk, liquidity risk, foreign exchange risk and credit risk.

Modeling Equities: The Traditional Fund Management Approach

For more than 40 years, it has been the fund management industry standard to model the return on a common stock as a linear combination of the return on various other factors. This approach is justified by assuming that stock prices are lognormally distributed and therefore their returns are normally distributed. A linear combination of normally distributed inputs produces the desired normal distribution. This approach dates to the original formulation of the capital asset pricing model, where the original single factor was the 'return on the market'. In this model, the return on the common stock of company *J* is the sum of the risk-free rate and its beta times the [return on the market less the risk-free rate} plus a random error term. For decades, there has been an enormous amount of research on correctly measuring the correlation between the returns on the common stock of company *J* and the return on the market. The beta is the measure of this correlation of returns.

Fund managers quickly realized that there were multiple factors driving common stock returns, some of which were macro-economic factors (like those in the reduced form credit models of Chapter 16) and some of which were company specific. The company specific factors included things like company size, its price earnings ratio, its industry sector, and so on. Many of these factors too are incorporated in the reduced form default probability models of Chapter 16. It is fairly common for a fund manager to be using a multi-factor model of common stock returns to manage 'tracking error', the amount of potential deviation between the return on the fund manager's portfolio and the return on the index.

What is wrong with this approach?

It totally ignores both the direct and indirect impact of default. Clearly, when a company has a 1% probability of bankruptcy, a normal distribution can't be an accurate portrayal of the potential distribution of returns on the common stock because there is a 1% probability that the return will be minus 100%![1] Moreover, in the indirect risk category, there is a large drop in the common stock price that would occur if the company is dropped from the S&P index for credit risk-related or other reasons.

The great strength of this approach, however, is that it already incorporates many of the macro factors and company-specific factors that are related to default risk—they are just not used properly for default adjusted tracking error management. We turn now to general assumptions about common stock price movements in the derivatives world.

Modeling Equities: The Derivatives Approach

In the derivatives world, key assumptions about the movement in equity prices also employ the lognormal assumption about stock price movements that results in normally distributed returns on

[1] We note that, even in the event of bankruptcy, in many cases common stock prices do not drop to zero in large part because of the common violation of absolute priorities of debt holders over equity holders. This is another of the many challenges facing analysts using the Merton model of risky debt from Chapter 18.

common stock. As we noted in our term structure analysis of Chapters 3 through 10 and in Chapter 18 on the Merton model of risky debt, the analytical benefits of this assumption can be so powerful that it is worth the cost of glossing over some real world realities.

Typically, in the derivatives world, the stock price itself is used as a random factor without modeling the *N* macro-economic and company specific factors driving movements in the stock price. This is due in large part because the primary focus of derivatives analysts, dating from the original options model of Black and Scholes [1973], is on 'synthetic replication' of the derivative with an appropriate position in the common stock. Jarrow and Turnbull [1996] discuss the importance of this argument in detail.

It is argued that because movements in the price of the derivative can be hedged by appropriate positions in the common stock, we don't need to know the drivers of returns on the common stock.

For most of the last 40 years, equity derivatives valuation formulas have largely assumed away both the potential default of the counterparty on an option on common stock and the potential default of the issuer of the common stock underlying the derivative. Lately, the issue of these two potential defaults has getting a lot more attention in the literature and the real world for two reasons. First, the credit crisis of 2007 to 2010 has made it obvious that default risk is a massive risk management issue. Secondly, because of the insights triggered by the Jarrow-Turnbull [1995] reduced form credit model and research by many other talented researchers, it is now well understood how to deal with credit risk issues explicitly.

Modeling Equities: A Credit-Risk Adjusted Approach

The Merton [1974] model of risky debt, which we discussed in Chapter 18, is not just a model of debt prices. It is implicitly a single-factor model of equity prices where the value of a company's common stock is a function of the value of company assets. Similarly, the Shimko, Tejima and van Deventer model [1993] implies that the value of a company's common stock is a function of a one factor Vasicek interest rate term structure and the value of company assets, which in turn can have any arbitrary correlation with interest rates. As we discussed in Chapters 16, 17 and 18, tests of the consistency of these models with relative movements in common stock prices and credit spreads on bonds have proved to be disappointing. Consequently, the Merton insights have not been used by equity market participants to the extent they have been used in the debt markets.

Jarrow and Turnbull [1995] present a framework for modeling equity securities on a full credit-adjusted basis. Using a simplified assumption of a constant default intensity, they assume that stock prices are log-normally distributed and driven by random interest rates and a company specific risk factor which can have any correlation with interest rates, provide that the company does not go bankrupt. They assume that the stock price drops to zero in the event of bankruptcy. Jarrow and Turnbull show that, under these assumptions, the risk neutral drift term in the common stock's return formula has to increase by the instantaneous probability of default. This has a number of important implications for modeling not only the common stock but also options on a credit-risky company's common stock and convertible bonds, which we discuss below.

Using the Jarrow-Turnbull framework for modeling common stock returns, we can directly address all of the concerns expressed in the first sections of this chapter while preserving (in modified form) the multi-factor approach to explaining lognormally distributed returns.

Jarrow [1999] takes this approach still further and models common stock in a framework with a random default intensity driven by interest rates and macro factors as discussed in Chapter 16. A common stock pays dividends until bankruptcy, when the value of common stock goes to zero. Amin and Jarrow [1992] provide the general framework for the valuation of risky assets and derivatives on them in an economy where the risk free term structure is driven by n random factors (like we analyzed in Chapters 6 through 9) and d other random factors. Our analysis of foreign exchange risk (Chapter 31) and the value of collateral (Chapter 32) take advantage of these insights. Amin and Jarrow show that the excess return on a risky asset over the instantaneous spot rate of interest is a function of these d risk factors, the volatility δ of the risky asset's return with respect to each risk factor, and the market price of risk n associated with each risk factor:

$$\mu(t,x) - r(t) = -\sum_{i=1}^{d} \delta_i(t,x) n_i(t)$$

If this constraint were not imposed on the risky asset's expected return, one could achieve excess risk-adjusted returns by buying the risky asset and hedging each of the risk factors. If one estimates the excess return over the risk-free rate using one year returns on the common stock of Citigroup, the joint importance of random interest rates and other macro factors is extremely obvious. For expositional purposes, we use 244 over-lapping annual intervals from January 1991 to April 2011 and fit a linear relationship between the excess return variable and the change in 1 year U.S. Treasury yields. The excess return is calculated by subtracting the 1 year U.S. Treasury yield at the beginning of the one year interval from the 1 year return on Citigroup common stock. We ignore dividends and the many econometric problems with this specification to emphasize more important aspects of the results, which are summarized in Figure 2:

Figure 2

Regression Statistics	
Multiple R	0.28256864
R Square	0.07984504
Adjusted R Square	0.07604275
Standard Error	0.34035526
Observations	244

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	1	2.432581	2.432581	20.99918	7.36384E-06
Residual	242	28.03369	0.115842		
Total	243	30.46627			

	Coefficients	*Standard Error*	*t Stat*	*P-value*	*Lower 95%*	*Upper 95%*	*Lower 95.0%*	*Upper 95.0%*
Intercept	-0.0106092	0.022582	-0.46982	0.63891	-0.055090964	0.033873	-0.05509	0.033873
Change in 1 Year US Treasury Yields	6.83197348	1.490888	4.582486	7.36E-06	3.895199806	9.768747	3.8952	9.768747

First, the change in the one year interest rate is statistically significant with a coefficient that is large in magnitude. It is not a surprise to find that interest rates are an important driver of the common stock returns of Citigroup. Secondly, interest rates explain only 7.6% of the total variation in Citigroup common stock returns. Other macro-economic factors make a large contribution to returns as does the non-zero default probability of the firm. This default probability, in turn, depends on an over-lapping set of macro factors like those we discussed in Chapters 19 and 20. The annual standard deviation of these missing variables is an extremely large 0.34.

Jarrow [1999] shows how to explicitly incorporate the possibility that the underlying risky asset could fall to zero value with a default intensity λ, consistent with the Jarrow and Turnbull [1995] reduced form model and our exposition in Chapter 16.

A full exploitation of the virtues of the Jarrow/Amin and Jarrow [1992] approach has been embedded in state of the art risk management systems that we describe in Chapters 36 through 41. In this chapter, we restrict our detailed comments on equity derivatives on the common stock of a credit risky company to the Jarrow-Turnbull [1995] and closely related approaches.

Options on the Common Stock of a Company Which Can Go Bankrupt

Jarrow and Turnbull [1995] use their model for the common stock of a company which can go bankrupt, to value put and call options on that credit-risky common stock.

They reach a very powerful and yet intuitive conclusion. The zero-coupon bond prices in the call options formula become the zero-coupon bond prices extracted from the yield curve of the risky company (see Chapters 5, 16, 17 and 19 for how to do this) instead of from the risk-free yield curve. These risky zero-coupon bond prices directly reflect the default intensity of the issuer, as we discussed in Chapters 16, 17 and 19.

Using the Jarrow-Turnbull [1995] formula for options on the common stock of a firm which can go bankrupt, we can explain much of the volatility smile issues with the constant interest rate/no default Black-Scholes options model. As many analysts have noted, equity options prices seem to be based on a probability distribution which reflects much greater downside risk than a simple lognormal distribution on stock price would imply. This is due to the risk of default (among other things) and the Jarrow-Turnbull model will improve the volatility smile problems because it deals with this increased downside risk directly. Moreover, the Jarrow-Turnbull model can be used to imply the probability of default from equity options on a company which can go bankrupt.

Convertible Bonds of a Company Which Can Go Bankrupt

The same approach proposed by Jarrow-Turnbull [1995] can be applied to convertible bonds, which contain a complex set of options controlled by both the issuer and by the holder of the bonds. After many years of trying to model convertible bonds as a combination of the straight bond and an option on the common stock, hedge funds and other analysts have employed both structural and reduced form modeling approaches. Three recent papers by Ayache, Forsyth, and Vetzal [2003], Andersen and Buffum [2003], and Bermudez and Webber [2003] show great promise in tackling one of the most complex securities on the balance sheet of financial institutions. Ultimately, one should use a bushy tree like the 3 factor Heath Jarrow and Morton example, coupled with a bushy of stock prices much like the collateral bushy tree of home prices that we employed in Chapter 32 on the impact of collateral.

Convertible bonds normally contain a call option granted to the issuer to call the bonds at a preset schedule of call prices. Separately, the holder of the convertible bonds has an option to convert the bonds normally to a preset number of common shares. This option is maintained for a pre-specified number of days even if the bonds are called by the issuer to allow time for conversion. Both of these options are significantly affected by the probability of default, which is why initial approaches which ignored default led to large losses at hedge funds investing in convertible bonds.

Many of the recent papers on convertible bond valuation combine reduced form default intensities with the finite difference method of securities valuation that we discussed in earlier chapters. Andersen and Buffum [2003] note that 'While it is in principle possible to build convertible bond models using the structural approach…the reduced-form approach is, by far, the most natural for trading applications and shall be the sole focus of this paper.'[2] At this stage it is safe to say that tremendous progress is being made on the valuation of convertible securities on a full credit-adjusted basis and they can be included in valuation of the Jarrow-Merton put option, our integrated measure of credit risk and interest rate risk.

[2] Andersen and Buffum, footnote 1.