

Empirical bubble broadening and effects on decompression schedules

Abstract

The question of bubble broadening (Ostwald ripening) in the diver under compression-decompression is virtually unanswered and untractable. Effects in vivo have not been measured nor quantified to date and remain unlikely in the near future. We take up this question and suggest hypothetical impacts on diver staging using available data and recent experimental results in the laboratory. A well known and safe bubble model, RGBM, provides a framework to estimate hypothetical effects in mixed gas diving on open circuit (OC) and rebreather (RB) systems. These are estimates and are neither verified nor tested in divers. However the projections are conservative, increasing decompression time and shortening no decompression time limits (NDL), so that implementation in diver staging protocols, software, dive computers and dive tables is patently safe and of interest to modelers, table designers, training agencies, dive tenders, engineers, doctors, dive computer vendors and related professionals. Experiments impacting broadening are briefly detailed. Particular are the broadening studies in hydrocarbon and glycerol substrates. Features of bubble models affected by broadening are quantified within the RGBM framework. Comparative results are given with and without broadening. Broadening times can range from hours to days. Corresponding broadening estimates of decreases in NDLs and increases in decompression times range 2% to 8% for nominal (recreational) exposures and 10% to 18% for extreme diving and extended (technical) exposures. Overall effects are small to moderate within existing data and recreational to technical diving protocols but diver staging effects of broadening increase with depth and exposure time. Beyond 8 hrs broadening time scales effects are insignificant.

Keywords: bubble broadening, DCS risk, decompression models, staging procedures, laboratory experiments

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Abbreviations and acronyms

Standard (SI) and English units are employed. By convention, by usage or for ease, some nonstandard units are employed. Pressure and depth are both measured in feet-of-sea-water (*fsw*) and meters-of-seawater (*msw*) with $1atm = 33fsw = 10msw$ to good approximation. Also used for scale lengths of bubbles are *micron* with $1micron = 10^{-6}m$. Acronyms are employed herein. They are standard:

ANDI, association of nitrox diving instructors; BM, bubble phase model dividing the body into tissue compartments with halftimes that are coupled to inert gas diffusion across bubble film surfaces of exponential size distribution constrained in cumulative growth by a volume limit point; Bubble Broadening, noted laboratory effect that small bubbles increase and large bubbles decrease in number in liquid and solid systems due to concentration gradients that drive material from smaller bubbles to larger bubbles over time spans of hours to days; Bubble Regeneration, noted laboratory effect that pressurized distributions of bubbles in aqueous systems return to their original non-pressurized distributions in time spans of hours to days; CCR, closed circuit rebreather, a special RB system that allows the diver to fix the oxygen partial pressure in the breathing loop (setpoint); CMAS, Confederation Mondial des Activites Subaquatiques; Critical Radius, temporary bubble radius at equilibrium, that is, pressure inside the bubble just equals the sum of external ambient pressure and film surface tension; DB, data bank storing downloaded computer profiles in 5-10sec time-depth intervals; DCS, crippling malady resulting from bubble formation and tissue damage in divers breathing

compressed gases at depth and ascending too rapidly; decompression stop, necessary pause in a diver ascent strategy to eliminate dissolved gas and/or bubbles safely and is model based with stops usually made in $10fsw$ increments; deep stop, decompression stop made in the deep zone to control bubble growth; DAN, Divers alert network; diveware, diver staging software package usually based on USN, ZHL, VPM and RGBM algorithms mainly; diluents, any mixed gas combination used with pure oxygen in the breathing loop of RBs; diving algorithm, combination of a gas transport and/or bubble model with coupled diver ascent strategy; DOD, Department of Defense; DOE, Department of Energy; Doppler, a device for counting bubbles in flowing blood that bounces acoustical signals off bubbles and measures change in frequency; DSAT, Diving Science and Technology, a research arm of PADI; DSL, Diving Safety Laboratory, the European arm of DAN; EAHx, enriched air helium breathing mixture with oxygen fraction, x , above 21% often called helitrox; EANx, enriched air nitrox breathing mixture with oxygen fraction, x , above 21%; EOD, end of dive risk estimator computed after finishing dive and surfacing; ERDI, Emergency Response Diving International; FDF, Finnish Diving Federation; GF, gradient factor, multiplier of USN and ZHL critical gradients, G and H , that try to mimic BMs; GM, dissolved gas model dividing the body into tissue compartments with arbitrary half times for uptake and elimination of inert gases with tissue tensions constrained by limit points; GUE, Global Underwater Explorers; heliox, breathing gas mixture of helium and oxygen used in deep and decompression diving; IANTD, International Association of Nitrox and Technical Divers; ICD, isobaric counter diffusion, inert dissolved

gases (helium, nitrogen) moving in opposite directions in tissue and blood; IDF, Irish Diving Federation; LSW theory, Lifschitz-Slyasov-Wagner Ostwald bubble ripening theory and model; M-values, set of limiting tensions for dissolved gas buildup in tissue compartments at depth; mirroring, the gas switching strategy on OC ascents of reducing the helium fraction and increasing the oxygen fraction in the same amount thereby keeping nitrogen constant; mixed gases, combination of oxygen, nitrogen and helium gas mixtures breathed underwater; NAUI, National Association of Underwater Instructors; NDL, no decompression limit, maximum allowable time at given depth permitting direct ascent to the surface; NEDU, Naval Experimental Diving Unit, diver testing arm of the USN in Panama City; nitrox, breathing gas mixture of nitrogen and oxygen used in recreational diving; OC, open circuit, underwater breathing system using mixed gases from a tank exhausted upon exhalation; Ostwald ripening, large bubble growth at the expense of small bubbles in liquid and solid systems; OT, oxtox, pulmonary and/or central nervous system oxygen toxicity resulting from over exposure to oxygen at depth or high pressure; PADI, Professional Association of Diving Instructors; PDE, Project Dive Exploration, a computer dive profile collection project at DAN; phase volume, surfacing limit point for bubble growth under decompression; RB, rebreather, underwater breathing system using mixed gases from a cannister that are re-circulated after carbon dioxide is scrubbed with oxygen from another cannister injected into the breathing loop; recreational diving, air and nitrox nonstop diving; RGBM algorithm, an American bubble staging model correlated with DCS computer outcomes by Wienke; RN, Royal Navy; SDI, Scuba Diving International; shallow stop, decompression stop made in the shallow zone to eliminate dissolved gas; SI, surface interval, time between dives; SSI, Scuba Schools International; TDI, Technical Diving International; technical diving, mixed gas (nitrogen, helium, oxygen), OC and RB, deep and decompression diving; TMX x/y, trimix with oxygen fraction, x, helium fraction, y, and the rest nitrogen; trimix, breathing gas mixture of helium, nitrogen and oxygen used in deep and decompression diving; USAF, United States Air Force; USCG, United States Coast Guard; USN, United States Navy; USN algorithm, an American dissolved gas staging model developed by Workman of the US Navy; UTC, United Technologies Center, an Israeli company marketing a message sending-receiving underwater computer system (UDI) using sonar, GPS and underwater communications with range about 2miles; VPM algorithm, an American bubble staging model based on gels by Yount; Z-values, another set of limiting tensions extended to altitude and similar to M-values; ZHL algorithm, a Swiss dissolved gas staging model developed and tested at altitude by Buhlmann

Introduction

Bubbles, birth, growth, evolution, destruction and elimination in the body of human divers are central issues in safe diver staging protocols from exposures at depth. Today, despite incredible technological advances in medical and physiological science, we really know very little about bubbles *in vivo* and their complex behaviour under pressure and environmental changes. Measuring bubbles and their properties *in vivo* by invasive means often destroys or changes what is being measured. Measuring with non-invasive techniques is very limited. Doppler scoring of moving body bubbles is only able to count numbers. Experiments using materials with properties similar to blood and tissue can be useful as a starting point for simulating bubble

behaviour but of course blood and tissue are metabolic and perfused adding additional complexity and unknowns to coupled modelling and simulation. In this vein, therefore, we investigate bubble data and experiments in the laboratory to make some hypothetical estimates of possible impacts of bubble broadening on diver staging regimens. We emphasize these are speculative based on experiments in the laboratory not in the field or in divers. The fact that tissue and blood are both perfused and metabolic always complicates biophysical modelling. A simple rheological assignment of laboratory variables and physical constants doesn't always extrapolate to divers.

We take up bubble broadening in a model framework after experiments are briefly recounted. There are many popular models and frameworks to estimate impacts of bubble broadening, USN,¹ ZHL,² VPM³ and RGBM⁴ specifically, either directly with bubble mechanics or indirectly with dissolved gas limiters,⁵ and we will employ the RGBM. The RGBM is detailed and features of the model impacted by broadening are quantified. Comparative results for real diving schedules are given in each case. Cases include OC and RB mixed gas diving in decompression and no decompression scenarios. Comparative results across these cases will be seen to be small to moderate for nominal exposures. Nonetheless, it is still important to spell out the techniques used to address bubble broadening in general. Specific model constructs addressing bubble broadening are underscored. The starting points for model alterations are the permissible supersaturations which vary across all models, dissolved gas (GM) or bubble (BM), but are most easily handled in bubble models like the RGBM. Turns out the USN, ZHL, VPM and RGBM are safe and sane models^{6,7} across technical and recreational diving and considerations of bubble regeneration and broadening effects will be reported for USN, ZHL and VPM algorithms in the future. Staging comparisons are given only within the RGBM framework here. This represents a first time study of bubble broadening effects, hypothetical or otherwise, on decompression staging.

RGBM framework

Nitrogen tissue compartments in the Wienke RGBM range,

$$\tau_{N_2} = (2, 5, 10, 20, 40, 80, 120, 160, 200, 240, 300) \text{min} \quad (1)$$

With helium compartments,

$$\tau_{He} = \frac{\tau_{N_2}}{3} \quad (2)$$

And dissolved gas tensions, p , for buildup and elimination in time, t , follow the well known tissue equations for nitrogen and helium independently,

$$p = p_a + (p_i - p_a) \exp(-\lambda t) \quad (3)$$

For p_i initial tensions and p_a ambient pressures for each species. The constant, λ , takes the usual form,

$$\lambda = \frac{0.693}{\tau} \quad (4)$$

And helium, τ_{He} , and nitrogen, τ_{N_2} , halftimes related by Graham's law for bulk diffusion,

$$\tau_{He} = \frac{\tau_{N_2}}{3} \quad (5)$$

The total gas tension, Π , is the sum of helium, p_{He} , and nitrogen, p_{N_2} , components,

$$\Pi = p_{He} + p_{N_2} \quad (6)$$

In each tissue compartments. Ambient partial pressures, p_a , are simple functions of gas fractions, f , in the

breathing mixture and depth, d , at sea level, that is, in f_{sw} or m_{sw} ,

$$p_a = f(d + 33) \quad (7)$$

For f_{He} and f_{N_2} . Thus in the tissue equations, nitrogen and helium exhibit different gas exchange properties. The bubble dynamical protocol in the RGBM model amounts to staging on the minimal seed number averaged, free-dissolved gradient, Y , across all tissue compartments for P permissible ambient pressure, Π total inert gas tissue tension, n excited bubble radial distribution (exponential), γ bubble surface tension and r bubble radius,

$$Y \int_{\varepsilon}^{\infty} ndr = (\Pi - P) \int_{\varepsilon}^{\infty} ndr \leq \int_{\varepsilon}^{\infty} \left[\frac{2\gamma}{r} \right] ndr \quad (8)$$

So that,

$$Y = (\Pi - P) \leq \beta \exp(\beta \varepsilon) \int_{\varepsilon}^{\infty} \exp(-\beta r) \left[\frac{2\gamma}{r} \right] dr \quad (9)$$

For ε the excitation radius⁸ at P , that is, for nitrogen,

$$\varepsilon_{N_2} = 0.007655 + 0.016543 \left[\frac{T}{P} \right]^{1/3} + 0.041602 \left[\frac{T}{P} \right]^{2/3} \quad (10)$$

And helium,

$$\varepsilon_{He} = 0.003114 + 0.015731 \left[\frac{T}{P} \right]^{1/3} + 0.025893 \left[\frac{T}{P} \right]^{2/3} \quad (11)$$

For T measured in absolute temperature, $^{\circ}K$, and P in usual diving pressure metric, f_{sw} . Time spent at each stop is iteratively calculated so that the total separated phase, ϕ , is maintained at, or below, its limit point, ϕ . This requires some computing power but is attainable in diver wrist computers presently marketed. The limit point to phase separation, ϕ , is near 600micron^3 and the distribution scaling length, \hat{a} , is close to 0.60micron^{-1} for both nitrogen and helium. Both excitation radii, ε , and surface tension, γ , are functions of ambient pressure and temperature and not constant. The equation-of-state (EOS) assigned to the bubble surface renders the surface tension below lipid estimates, on the order of 20dyne/cm and excitation radii are below 1micron .

The RGBM staging model ranges across recreational to technical, deep to shallow, nonstop to near saturation, sea level to altitude, air to mixed gas and OC to RB diving. It has been correlated and validated over the past 25 yrs or so by safe utilization within tables, meters and software and by formal data correlations with the LANL DB profile

set of computer downloads.^{5,6,14} Safe commercial implementations span computer vendors, software purveyors, table designers, Training Agencies, exploration and underwater scientific projects to name a few.

Bubble properties and behaviour

Most questions of seed distributions, lifetimes, persistence and origins in the body are unanswered today. And while we have yet to measure microbubble distributions and lifetimes in the body, we can gain some insight from laboratory measurements and statistical mechanics. Microbubble distributions have been studied extensively. Our companion biophysics work⁸ details some interesting studies about microbubbles and properties in general and follows in abbreviated form. Microbubbles typically exhibit size distributions that decrease exponentially in radius, r . Holography measurements of cavitation nuclei in water tunnels suggest,

$$N = N_0 \exp(-\beta r) \quad (12)$$

With,

$$N_0 = 1.017 \times 10^{12} m^{-3} \quad (13)$$

$$\beta = 0.0512 \text{micron}^{-1} \quad (14)$$

Experiments in gels also display exponential dependences in cavitation radii,

$$N = N_0 \exp(-r / \alpha) \quad (15)$$

With,

$$N_0 = 662.5 ml^{-1} \quad (16)$$

$$\alpha = 0.0237 \text{micron} \quad (17)$$

Both MRI and Doppler laser measurements of water and ice droplets in the atmosphere underline exponential decrease in number density as droplet diameter increases. Ice and water droplets in clouds typically range, $2 \text{micron} \leq r \leq 100 \text{micron}$. Dust and pollutants are also exponentially distributed, potentially serving as heterogeneous nucleation sites. It might be a surprise if micronuclei in the body were not exponentially distributed in number density versus size.

The lifetimes of cavitation voids are not known, nor measured, in the body. The radial growth equations provide a framework for estimation using nominal blood and tissue constants. Consider first the mass transfer equation,

$$\frac{\partial r}{\partial t} = \frac{DS}{r} \left[\Pi - P - \frac{2\gamma}{r} \right] \quad (18)$$

With all quantities as before, that is, r bubble radius, D diffusivity, S solubility, γ surface tension, P ambient pressure and Π total gas tension. The time to collapse, τ , can be obtained by integrating over time and radius, taking initial bubble radius, r_i ,

$$\tau = \int_0^{\tau} dt = \int_{r_i}^0 \left[\frac{r}{DS} \right] \left[\frac{1}{\Pi - P - 2\gamma/r} \right] dr = \left[\frac{\Delta p r_i (4\gamma + \Delta p r_i) - 8\gamma^2 \ln(1 - \Delta p r_i / 2\gamma)}{2DS\Delta p^3} \right] \quad (19)$$

With,

$$\Delta p = P - \Pi \quad (20)$$

If surface tension is suppressed, we get,

$$\tau = \frac{r_i^2}{2DS\Delta p} \quad (21)$$

In both cases, small tension gradients, Δp and small transport coefficients, DS , lead to long collapse times and vice-versa. Large bubbles take a longer time to dissolve than small bubbles. Taking nominal transport coefficient for nitrogen, $DS = 56.9 \times 10^{-6} \text{micron}^2/\text{sec}$ fsw , and initial bubble radius, $r_i = 10.0 \text{micron}$, for $\Delta p = 3.0 fsw$, and $\gamma = 40 \text{dynes/cm}$, we find,

$$\tau = 0.25 \text{sec} \quad (22)$$

In the Rayleigh-Plesset picture,⁸ the radial growth equation takes the form, neglecting viscosity,

$$\left[\frac{\partial r}{\partial t} \right]^2 = \frac{2(\Pi - P)}{3\rho} \left[\frac{r_i^3}{r^3} - 1 \right] + \frac{2\gamma}{\rho r} \left[\frac{r_i^2}{r^2} - 1 \right] \quad (23)$$

So that the collapse time by diffusion only is,

$$\tau = \int_0^\tau dt = \left[\frac{3\rho}{2(\Pi - P)} \right]^{1/2} \int_{r_i}^0 \left[\frac{r_i^3}{r^3} - 1 \right]^{-1/2} dr = r_i \frac{\Gamma(5/6)}{\Gamma(1/3)} \left[\frac{3\rho}{2\Delta P} \right]^{1/2} \quad (24)$$

With,

$$\Gamma(5/6) = 1.128 \quad (25)$$

$$\Gamma(1/3) = 2.679 \quad (26)$$

Suppressing the diffusion term in the growth equation, there similarly obtains,

$$\tau = \int_0^\tau dt = \left[\frac{\rho}{2\gamma} \right]^{1/2} \int_{r_i}^0 r^{1/2} \left[\frac{r_i^2}{r^2} - 1 \right]^{-1/2} dr = r_i \frac{\Gamma(-3/4)}{\Gamma(-1/4)} \left[\frac{\pi \rho r_i}{4\gamma} \right]^{1/2} \quad (27)$$

With,

$$\Gamma(-3/4) = -4.834 \quad (28)$$

$$\Gamma(-1/4) = -4.062 \quad (29)$$

Collapse time in the Rayleigh-Plesset picture is linear in initial bubble radius, r_i and inversely proportional to the square root of the tension gradient, Δp , or the surface tension, γ . Taking all quantities as previously, with density, $\rho = 1.15 \text{g/cm}^3$, we find with surface tension suppressed,

$$\tau = 2.91 \times 10^{-3} \text{sec} \quad (30)$$

and, for the diffusion term suppressed with only the surface tension term contributing,

$$\tau = 2.52 \times 10^{-6} \text{sec} \quad (31)$$

Dissolution times above range,

$$10^{-6} \text{sec} \leq \tau \leq 10^{-1} \text{sec} \quad (32)$$

In the Yount model of persistent nuclei, within the permeable gas transfer region, seed nuclei lifetimes, τ , range,

$$10^{-6} \text{sec} \leq \tau \leq 10^{-2} \text{sec} \quad (33)$$

The collapse rate increases with both γ and Δp and inversely with r_i . Small bubbles collapse more rapidly than large bubbles, with large bubble collapse driven most by outgassing diffusion gradients and small bubble collapse driven most by constrictive surface tension. Between these extrema, both diffusion and surface tension play a role. In any media, if stabilizing material attaches to micronuclei, the effective surface tension can be reduced considerably and bubble collapse arrested temporarily, that is, as $\gamma \rightarrow 0$ as a limit point. For small bubbles, this seems more plausible than for large bubbles because smaller amounts of material need adhere. For large bubbles, bubble collapse is not aided by surface tension as much as for small bubbles, with outgassing gradients taking longer to dissolve large bubbles than small ones. In both cases, collapse times are likely to lengthen over the short times estimated above. Additionally, external influences on the bubble, like crevices and surface discontinuities, may prevent bubble growth or collapse. All this adds to bubble complexities faced by modelers. The question of regeneration is equally complex and follows.

Bubble broadening and diving implications

Bubble broadening is a phenomena observed by Ostwald⁹ whereby small bubbles diminish in size and large bubbles grow over time spans of hours to days. Concentration gradients (diffusion) drive the transport of material across bubble interfaces with small bubbles at higher concentrations than large bubbles because of their increased curvature and surface tension pressure. An everyday example is the recrystallization of water within ice cream which gives old ice cream a gritty, crunchy texture. Larger ice crystals grow at the expense of smaller ones within the ice cream creating a coarser surface texture. A systematic theory of bubble broadening developed by Lifshitz, Slyozov and Wagner¹⁰ (LSW) suggests that in supersaturated and solid solutions the distribution mean bubble radius, r_m , evolves in time as,

$$r_m^3 = r_0^3 + Kt \quad (34)$$

with r_0 the unbroadened (initial) mean radius and K the transport coefficient a function of temperature, bubble surface tension, diffusivity, gas solubility and gas molar volume. For a wide range of experiments¹¹ the relationship holds with the transport coefficient, K , varying across materials of course. Two of interest include the Kabalnov and Del Cima studies.

In the Kabalnov¹² fluorocarbon experiments, the LSW transport coefficient was determined,

$$K_f = 4.2 \times 10^3 \mu\text{m hr}^{-1} \quad (35)$$

for the emulsion. Rheological scaling suggests the extrapolation to body blood and tissue, K_b ,

$$K_f = \frac{K_b}{5.7} \quad (36)$$

with,

$$r_0 = 16.56 \text{ micron} \quad (37)$$

In the Del Cima¹³ glycerol-water studies, K_g deviated from the LSW value,

$$K = 5.1 \times 10^3 \mu\text{m}^3 \text{ hr}^{-1} \quad (38)$$

According to,

$$K_g = 2.0 \times 10^7 \text{ m}^{1/0.1956} \text{ sec}^{-1} \quad (39)$$

with,

$$r_0 = 18.42 \text{ micron} \quad (40)$$

and the fitted bubble number, nb , and distribution mean radius, r_m , in time t , given by,

$$nb = \frac{12019.0 \times 2.0205^3}{2.0205^3 \times 9.9865 t^{0.6132}} \quad (41)$$

$$r_m = \left[16.977^3 + 14203.0 t^{0.67637} \right]^{1/3} \quad (42)$$

for t in hr and r_m in micron (10^{-6}m). Glycerol suspensions again are not tissue and blood and the transport coefficient in glycerol is empirically $1/7.8$ the value in blood. To use the fitted expression from glycerol we then take for tissue and blood,

$$K_g = \frac{K_b}{7.8} \quad (43)$$

as an approximation. Other representations in different materials with rheological scaling will also be investigated in the future and the Kabalnov study is sufficient here.

In BMs, the excitation radii, ε , are central to the staging regimens. A simple approach to broadening is to require the integrals of the initial and broadened distributions from excitation radius to ∞ to be equal. This obviously just scales the distributions to the excitation radii while conserving growing bubble numbers. The distributions are assumed to be exponential. The process is straight forward as follows.

Normalizing the initial distribution of bubbles to the unbroadened mean radius, r_0 , and the final distribution of bubbles to the broadened mean radius, r_m , we have,

$$\alpha \int_0^{\infty} \exp(-\alpha r) r dr = \frac{1}{\alpha} = r_0 \quad (44)$$

and,

$$\beta \int_0^{\infty} \exp(-\beta r) r dr = \frac{1}{\beta} = r_m \quad (45)$$

To find the broadened critical radius, ε_m , in terms of the unbroadened radius, ε_0 , we set the normalized integrals from critical radii to ∞ equal, thereby conserving growing bubble numbers,

$$\alpha \int_{\varepsilon_0}^{\infty} \exp(-\alpha r) dr = \beta \int_{\varepsilon_m}^{\infty} \exp(-\beta r) dr \quad (46)$$

which yields in lowest order,

$$\exp(-\alpha \varepsilon_0) = \exp(-\beta \varepsilon_m) \quad (47)$$

so that,

$$\alpha \varepsilon_0 = \beta \varepsilon_m \quad (48)$$

and the new critical radii, ε_m , in the VPM and RGBM obtain by simple scaling of the initial critical radii, ε_0 , by the ratio of broadened, r_m , to unbroadened, r_0 , distribution mean radii,

$$\varepsilon_m = \frac{r_m}{r_0} \varepsilon_0 \quad (49)$$

A higher order approximation scheme equates the integrals of growing bubbles over radius, r , that is,

$$\alpha = \int_0^{\infty} \exp(-\alpha r) r dr = \beta \int_{\varepsilon_m}^{\infty} \exp(-\beta r) r dr \quad (50)$$

yielding a transcendental relationship,

$$\exp(-\alpha \varepsilon_0) \frac{(1 + \alpha \varepsilon_0)}{\alpha} = \exp(-\beta \varepsilon_m) \frac{(1 + \beta \varepsilon_m)}{\beta} \quad (51)$$

which is solvable numerically. The correction to the low order approximation is small and neglected herein. Finally, as Doppler counting peaks in an hour or so suggesting bubble washout and since LSW bubble broadening increases linearly in time, the RGBM (and VPM) broadened critical radius, ε_m , is modulated with a relaxation time, ω , such that over dive time, t , the new modulated critical radius, $\varepsilon(t)$, is given by,

$$\varepsilon(t) = \varepsilon_0 + \left[1 + \exp(-\omega t) \right] \left[\varepsilon_m - \varepsilon_0 \right] \quad (52)$$

with relaxation halftime roughly 4 hrs,

$$\omega = \frac{1}{240} \text{ min}^{-1} \quad (53)$$

Modifications described hold for the VPM with gel parameters while the RGBM relies on fits in lipid and aqueous materials [14]. Both employ the similarity relationship,

$$P_0 + \frac{2\gamma}{\varepsilon_0} = P + \frac{2\gamma}{\varepsilon(t)} \quad (54)$$

with P_0 and ε_0 surface values and P and $\varepsilon(t)$ values at depth. This relationship imparts bubble crushing at depth thereby yielding larger excitation radii, $\varepsilon(t)$, than surfacing excitation radii, ε_0 . The critical radius, $\varepsilon(t)$, is then modulated over relaxation scales, ω , as described. The above relationship is also a staging criteria limiting ambient pressure, P , as a function of excitation radii, $\varepsilon(t)$. Effects on three decompression dives on trimix and air with switches on the way up follow, as well as effects on air NDLs.

Table 1 contrasts reductions in air NDLs for broadening on time scale of bottom time. These estimates also fall within the USN 1%-5% DCS incidence Tables just mentioned in passing.^{1,14}

Table 1 Air NDLs and Broadening

ω^{-1} depth f sw	0 time min	240 min time min
30	8.7	8.2
120	10.3	9.5
110	12.6	11.6
100	15.2	14.2
90	19.4	17.7
80	26.1	24.1
70	36.7	33.3
60	54.7	48.5
50	99.2	91.9
40	206.6	196.2

Reductions in NDLs are small in the deep zone and nominal elsewhere.

Table 2 tabulates decompression times for a 16/14 trimix dive down to 300fsw for 30min with a stop for 2min at 160fsw and switch to 20/10 trimix and another switch at 80fsw to EAN50 using the RGBM and broadening time scale bottom time.

Table 2 Decompression Schedules For 16/14 Trimix Dive and Broadening

ω^{-1} depth f sw	0 time min	240 min time min
300	30	30
210	0.5	0.5
200	1	1.5
190	1.5	1.5
180	1.5	1.5
170	1.5	1.5
160	2	2
150	2	2
140	2.5	2.5
130	2.5	3
120	4	4
100	4.5	4.5
90	6.5	7

Table continued..

ω^{-1} depth f sw	0 time min	240 min time min
80	3	3.5
70	4.5	5
60	6	6
50	7.5	8
40	10.5	11
30	13	14
20	20	20.5
10	29.5	32
	173.5	182

Differences are nominal and small increasing slightly in the shallow zone for this OC decompression dive. This dive is in the LANL DB.

Table 3 tabulates decompression times for an air dive down to 240fsw for 15min with switches to EAN50 at 80fsw and EAN20 at 20fsw. This is a fairly comfortable technical dive compared to the previous trimix dive. The VPM is used here with broadening time scale decompression plus bottom time.

Table 3 Decompression Schedules for Short Air Dive and Broadening

ω^{-1} depth f sw	0 time min	240 min time min
240	15	15
140	1	1
130	1	1
120	1	1
110	1.5	2
100	2	2
90	2	2
80	2	2
70	1.5	1.5
60	1.5	1.5
50	7.5	8
40	2.5	2.5
30	4	4
20	5	5.5
10	8.5	8.5
	62.5	64

Differences here are in the noise. As dive times and depth increase broadening effects increase obviously. Effects show up more strongly for very extreme diving and diving on the envelope but certainly outside recreational and technical regimes. Beyond 4-8hr broadening time scales effects are small But if we extend the bottom time to 60min effects are notable as seen for the same dive in Table 4.

The increase in overall decompression time in both cases reminds that decompression debt accrues rapidly with increasing depth and exposure time. Broadening effects in this case increase decompression time by roughly 10%.

Table 4 Decompression Schedules For Long Air Dive and Broadening

ω^{-1} depth f sw	0 time min	240 min time min
240	60	60
160	0	0.5
150	1.5	2
140	3	2.5
130	3	3.5
120	4.5	5
110	5	5
100	5.5	6
90	8	8.5
80	8	8.5
70	12	13
60	13	14
50	18.5	20
40	21	24
30	32	34
20	43	48
10	63.5	71
	313.5	338.5

Table 5 tabulates broadening for a heliair (21/79 heliox or EAH21) CCR dive to 420 fsw for 15min with setpoint 1.1atm and broadening time scale bottom time. This is not an easy dive by any means, technical or otherwise. If broadening time scales include decompression time, effects are larger in dive times on the order of 15%. This one is in the LANL DB too and was performed off Dry Tortugas.

Table 5 Decompression Schedules For 420 fsw CCR Dive And Broadening.

ω^{-1} depth f sw	0 time min	240 min time min
420	15	15
340	0	0
330	0.5	0.5
320	0.5	0.5
310	0.5	0.5
300	0.5	0.5
290	1	1
280	1	1
270	1	1
260	1	1
250	1.5	1.5
240	1.5	1.5
230	1.5	1.5
220	1.5	1.5

Table continued..

ω^{-1} depth f sw	0 time min	240 min time min
210	1.5	2
200	2	2
190	2	2
180	2	2
170	2.5	3
160	3	3
150	3	3
140	4	4
130	5	5
120	5	5
110	5.5	5.5
100	6.5	7.5
90	7	7.5
80	7	9
70	10	10.5
60	10	11.5
50	13	14.5
40	15	17.5
30	18	21.5
20	24	26.5
10	30	35.5
	219.5	228.5

Beyond 8hr broadening time scales, ω^{-1} , hypothetical effects are small. This also is not a nominal dive and effects of broadening are more pronounced here.

Summary

We have presented a hypothetical study of possible impacts of bubble broadening on diver staging using empirical laboratory data and tests. A framework was constructed and detailed for assessing effects within well established and safe diving models, namely VPM and RGBM. The effects impacting NDIs and decompression staging times are relatively small for nominal recreational and technical diving but increase in magnitude as exposure times and depth increase. Broadening effects relax over time scales of 4hrs. Non of the broadening effects have been seen nor measured in actual divers and likely will not be in the near future. However, we have given a computational framework to investigate such effects and their impacts on diving and dive operations. Hopefully that is of interest to diving organizations and a useful research and operational tool in the future.

Biosketches

Bruce Wienke is a Program Manager in the Weapons Technology/Simulation Office at LANL. He received a BS in physics and mathematics (Northern Michigan), MS in nuclear physics (Marquette)

and a PhD in particle physics (Northwestern). He has authored 350+ articles in peer reviewed journals, media outlets, trade magazines, workshop proceedings and has published 12 books on diving science, biophysics and decompression theory. He heads up the C&C Dive Team vested with worldwide underwater search, assessment and disablement of nuclear, chemical and biological WMDs. He is a Fellow of the APS, Technical Committee Member of the ANS, Member of the UHMS and serves as a Consultant to the EPA, DHS, ADA, US Military and Dive Industry. Bruce is an Editor/Reviewer for CBM, PR, TTSP, NSE, JQSRT and CEO of Southwest Enterprises Consulting. He is the developer of the Reduced Gradient Bubble Model (RGBM) implemented in decompression meters, tables and dive software worldwide. He has dived all over the world on OC and RB systems in military, scientific, exploration, testing and training activities. Bruce is a NAUI Tec/Rec Instructor Trainer and Course Director. Interests include USSA Masters ski racing, USTA Seniors tennis, golf and windsurfing. Bruce is a Certified Ski Instructor (PSIA) and Racing Coach (USSCA). He has won Masters National Titles in SL, GS, SG and DH and Quarterbacked the Northern Michigan Wildcats to a NCAA II Title in the Hickory Bowl. Tim O'Leary heads up NAUI Technical Diving Operations having developed and co-authored training manuals, support material, tech dive tables, monographs and related media along with Tech Course Standards. He is a practicing commercial diver and CEO of American Diving & Marine Salvage on the Texas Gulf Coast.

Tim received a BS in zoology (Texas A&M) and a DMT and CHT from Jo Ellen Smith Medical Center at the Baromedical Research Institute. He was a Commercial Diving and Hyperbaric Chamber Instructor at the Ocean Corporation. Tim is a member of the UHMS, SNAME and NADMT. He is an Admiral in the Texas Navy, a USCG 100 Ton Vessel Master and a Consultant to Texas Parks & Wildlife, Canadian Corporation, Rimkus Group and Offshore Oil Industry. His diving experience is global on OC and RB systems in commercial, exploration, training and testing activities. He is a NAUI Tec/Rec Instructor Trainer, Course Director and Workshop Director. Other interests include skiing, deep wreck diving, and dive travel. Tim and NAUI Dive Team are credited with the discovery and exploration of the USS Perry in approximately 250 fsw off Anguar and diving it for a week on RBs.

Ostwaldo Del Cima is a Professor in the Department of Physics at the University of Vicoso with interests in quantum field theory, computational physics, bubble dynamics and decompression theory. He has initiated studies of bubble dynamics and Ostwald broadening in glycerol-water substrates with an eye on implications for diver staging in recreational and technical activities. He has dived all over South America and is a NAUI Diving Instructor. He is the Technical Director of NAUI Brazil.

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Conflict of interest

Authors have no conflicts of interest in publishing this paper.

Animal disclaimer

Humans and animals were not used for testing in this paper.

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