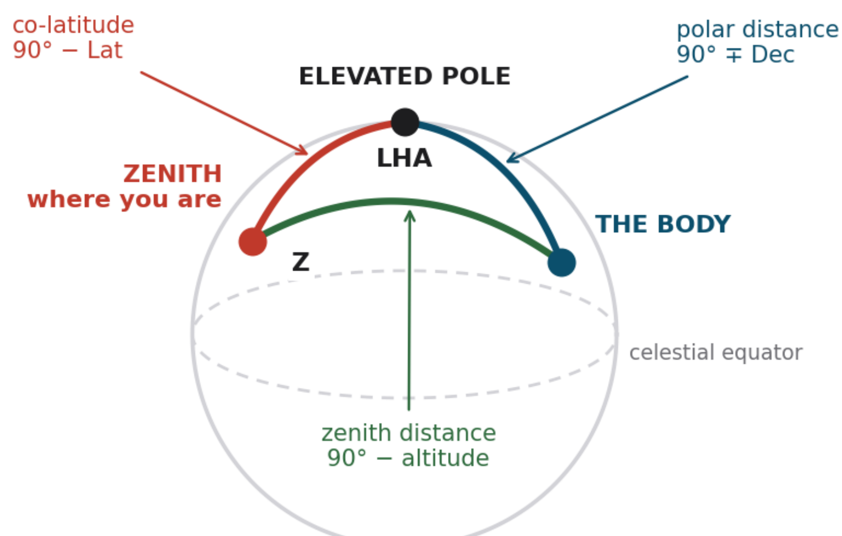
STUDY AID

Terrestrial Navigation

The earth, the compass, the clock
and the sailings

Capt. Georgia Hilton

THE NAVIGATIONAL TRIANGLE



Every celestial problem is this one triangle with a different corner unknown. Amplitude is the special case where the body sits ON the horizon: the zenith distance is exactly 90°, the triangle collapses, and all that is left is sin A = sin Dec / cos Lat.

This is the module people fail. Not because it is hard — almost none of it is — but because it is a dozen small conventions that all have a sign, and every sign has an opposite that looks equally reasonable at two in the morning in an examination room. Easterly or westerly. Add or subtract. Plus zone or minus zone. Toward the elevated pole or away from it. This guide is built around those signs, and every worked example in it is a real question from a Coast Guard examination or from your own demonstration set, worked longhand and checked against the published answer.

The five signs that decide whether you pass.

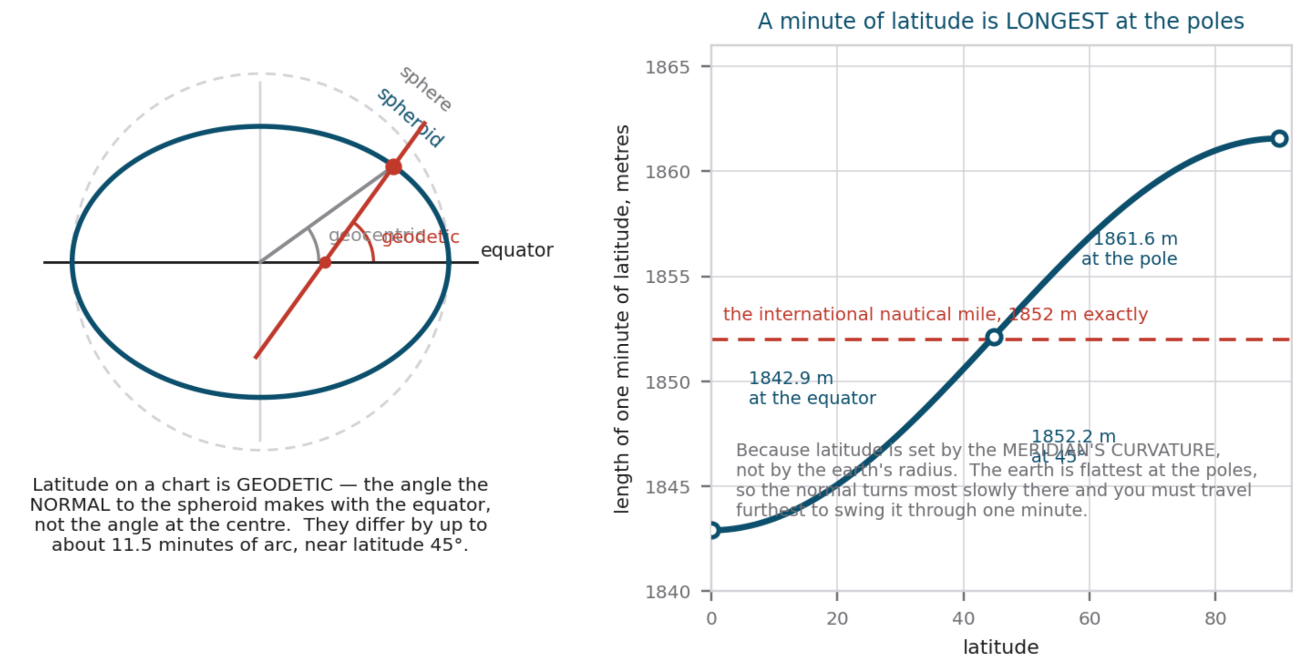
1. Correcting — compass to true — **ADD easterly, subtract westerly.** Uncorrecting is the reverse.
2. Zone description is **POSITIVE in west longitude**, negative in east. **UT = ZT + ZD.**
3. **LHA = GHA + east longitude**, and – west longitude.
4. A chronometer that is **FAST** reads ahead of the truth, so you **subtract** its error to get UT.
5. Amplitude is measured from **EAST or WEST**, never from north or south.

SECTION 1

The shape of the earth, and what a mile actually is

Two facts here are examinable and both are counter-intuitive.

The earth is an oblate spheroid — a sphere squashed at the poles. For navigation we use the **WGS 84** reference spheroid, which is what GPS and every modern chart are built on. Its two defining figures are a semi-major axis of **6 378 137 metres** and a flattening of **1/298.257223563**, which makes the semi-minor axis 6 356 752.3 metres. The difference between the two is about 21 kilometres — roughly a third of one per cent.



Left: latitude on a chart is the angle of the NORMAL, not the angle at the centre. Right: the consequence nobody expects.

Why a minute of latitude is longest at the poles

A nautical mile is one minute of latitude — except that it is not, quite, because a minute of latitude is not the same length everywhere. It runs from about **1 842.9 metres at the equator to 1 861.6 metres at the pole**. That is why the **international nautical mile was fixed at exactly 1 852 metres** in 1929, which is very close to the value at latitude 45°. In feet that is 6 076.1; the old United States value of 6 080.2 feet is obsolete and appears in exam questions only as a wrong answer.

The trap: students reason from the wrong radius.

The earth is *smaller* near the poles, so surely a minute of latitude is shorter there? No — that reasoning is correct for a minute of **longitude**, which does shrink with the cosine of the latitude and vanishes at the pole.

Latitude is defined by the **normal to the spheroid**, so a minute of latitude is the distance you must travel for that normal to swing through one minute. That depends on the **curvature of the meridian**. The earth is flattest at the poles, so the meridian is least curved there, the normal turns most slowly, and you must travel **furthest**. Flattest means longest.

The vocabulary, exactly as Bowditch has it

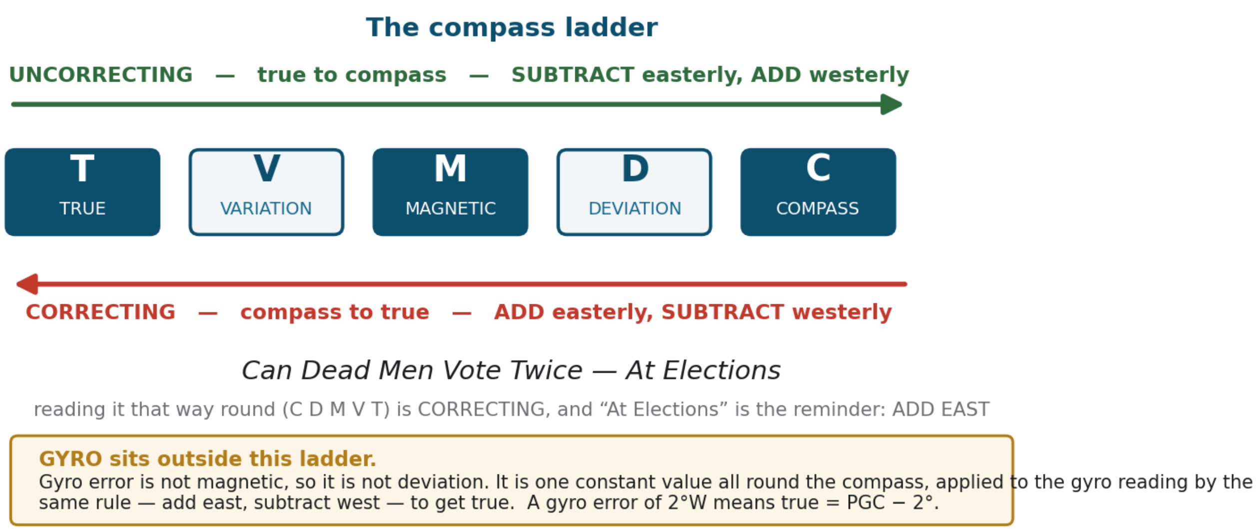
Term	Definition
Great circle	The intersection of the surface of a sphere and a plane through the centre of the sphere. The largest circle that can be drawn on the sphere
Small circle	The intersection of the surface of a sphere and a plane that does *not* pass through the centre. Every parallel of latitude except the equator is one
Rhumb line	A line that makes the same angle with every meridian it crosses, and appears as a straight line on a Mercator chart
Difference of latitude	The shorter arc of any meridian between the parallels of two places, in angular measure
Difference of longitude	The shorter arc of a parallel between the meridians of two places, in angular measure
Departure	The distance in nautical miles between two meridians at a given parallel. It decreases with the cosine of the latitude — d.long × cos Lat
Meridional parts	The length along a meridian on a Mercator chart, in units of the longitude scale, from the equator to that parallel. Taken from Bowditch Table 6 or Norie's
Geodetic latitude	The angle the normal to the spheroid makes with the plane of the equator. This is what your chart and your GPS give you
Geocentric latitude	The angle at the centre of the spheroid between the equator and a line to the point. Differs from geodetic by up to about 11.5 minutes of arc, near latitude 45°

SECTION 2

Direction: true, magnetic and compass

Three norths, two errors, and one ladder you climb in either direction.

Variation is the angle between the magnetic meridian and the true meridian at a place. It is a property of the earth, it is printed in the compass rose on every chart with its annual change, and it has nothing to do with your ship. **Deviation** is the deflection caused by your ship's own iron and electrics. It changes with the ship's head, which is why it comes from a table rather than a single number. **Compass error** is the algebraic sum of the two.



One ladder, climbed in either direction. Which way you are going decides the sign, and getting that backwards is the single most common error in the whole module.

Working it, both ways

Correcting — you have a compass reading and want the truth. **Add easterly, subtract westerly.**

Uncorrecting — you have a true course and want what to steer. **Subtract easterly, add westerly.**

Compass 127°, deviation 4°E, variation 9°W → Magnetic = 127 + 4 = 131° → True = 131 – 9 = **122°**

True 122° back the other way → Magnetic = 122 + 9 = 131° → Compass = 131 – 4 = **127°**. It closes, which is how you check yourself.

The range-light problem

A pair of range lights in line gives you a true bearing straight off the chart with no instrument error at all, so it is the classic way to find compass error at sea. From the Coast Guard's own Q138:

Q138 question 1

Ranges in line bear 315° per standard magnetic compass. The chart shows the range bearing is 312°T and variation is 6°W. What is the deviation?

True 312°, compass 315°, so **compass error = 312 – 315 = –3° = 3°W**.

Compass error = deviation + variation, so deviation = –3 – (–6) = **+3° = 3°E**.

Answer A. Note that the deviation is easterly even though the compass error is westerly — because the variation is larger and pulls the other way. If that surprises you, you have found the reason this question is on the paper.

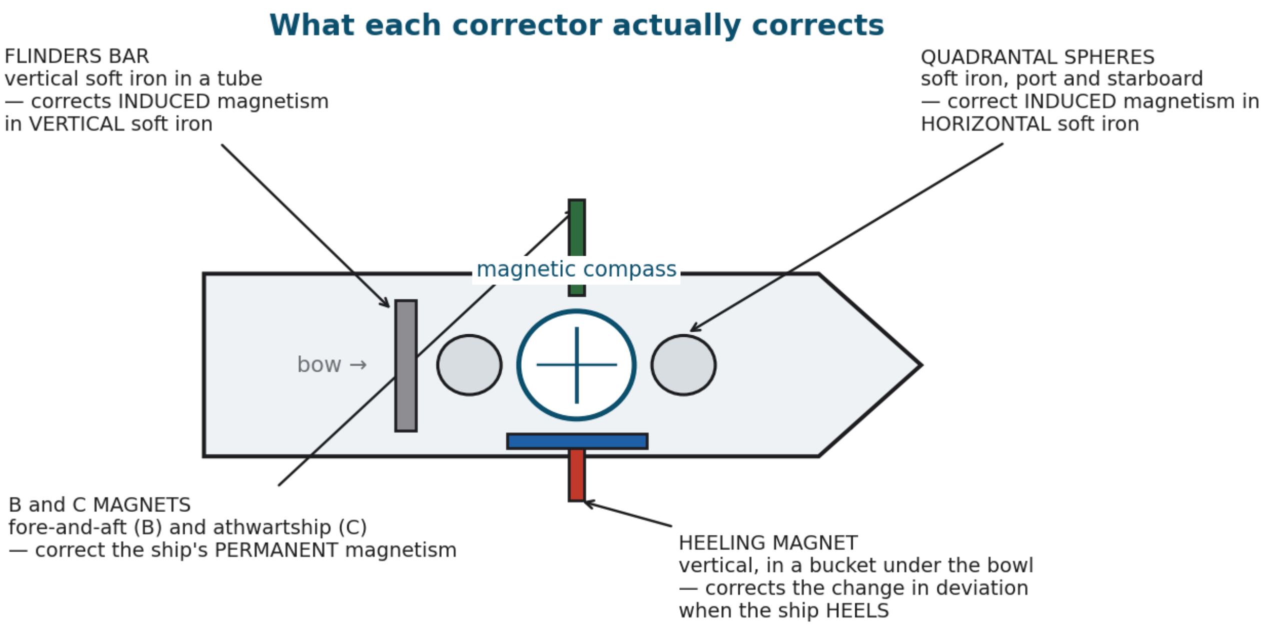
SECTION 3

The ship's own magnetism

What each corrector corrects, which is the only part of this that gets examined.

A ship acquires **permanent** magnetism while she is built — hammering and vibration in the earth's field, sitting on one heading on the ways for months. She acquires **induced** magnetism the moment she is placed in the earth's field, and that part changes as she changes heading and latitude. The compass adjuster's job is to cancel each kind with the right corrector.

Dip, or magnetic inclination, is the angle the earth's field makes with the horizontal. It is 0° at the magnetic equator and 90° at the magnetic poles — the needle stands on end. That is why a magnetic compass becomes useless in high latitudes, and why the vertical component that the Flinders bar deals with grows as you go north.



Learn which corrector answers which kind of magnetism. That is the question, every time.

Corrector	What it is	What it corrects
Flinders bar	A vertical soft iron bar in a tube on the forward or after side of the binnacle	Induced magnetism in the ship's **vertical soft iron**. It is the one that matters when you change latitude
Quadrantal spheres	Two soft iron spheres, port and starboard of the bowl	Induced magnetism in the ship's **horizontal soft iron**
Heeling magnet	A vertical magnet in a bucket directly beneath the compass	The change in deviation when the ship **heels**. Adjusted with the ship upright, checked when she lists
B magnets	Fore-and-aft permanent magnets	The fore-and-aft component of the ship's **permanent** field
C magnets	Athwartship permanent magnets	The athwartship component of the ship's **permanent** field

SECTION 4

The gyrocompass

Two properties, one clever trick, and five errors with names.

A gyroscope has two useful properties. **Gyroscopic inertia**: spun fast, it tends to keep its spin axis pointing in the same direction in space, indefinitely. **Precession**: apply a force to tilt that axis and it moves not in the direction of the force but at right angles to it. Everything a gyrocompass does comes out of those two.

Left alone, a free gyro keeps pointing at the same star while the earth turns under it — which is no use for finding north. The trick is to make it **bottom-heavy**, usually with mercury ballistics. As the earth's rotation appears to tilt the spin axis, gravity applies a torque; because of precession the axis then swings horizontally, toward the meridian. Instead of tracing an ellipse it traces a spiral, and with damping it settles pointing north. It is **north-seeking**, not north-pointing — it hunts its way there, which is why it takes hours to settle from cold.

Error	When	Which way
Speed error	Always, whenever the ship has a north-south component of speed. The gyro is chasing a meridian that is moving under it	**Westerly on northerly courses, easterly on southerly courses.** Larger at higher speed and higher latitude
Tangent latitude error	A property only of gyros with mercury ballistics	**Easterly in north latitude, westerly in south latitude**
Ballistic deflection	During a marked change in the north-south component of speed	A temporary offset while the ballistic responds
Ballistic damping error	A temporary oscillation introduced by a change of course or speed	Dies away as the damping works
Quadrantal error	From a misplaced centre of gravity, or the apparent vertical shifting as the ship rolls	Varies with the ship's head

Gyro error is not deviation, and it does not belong on the compass ladder.

Gyro error has nothing to do with the earth's magnetic field, so it is **one constant value all the way round the compass**. An error of 1°E applies to every bearing equally. Deviation does not — it changes with the ship's head, which is why it needs a table.

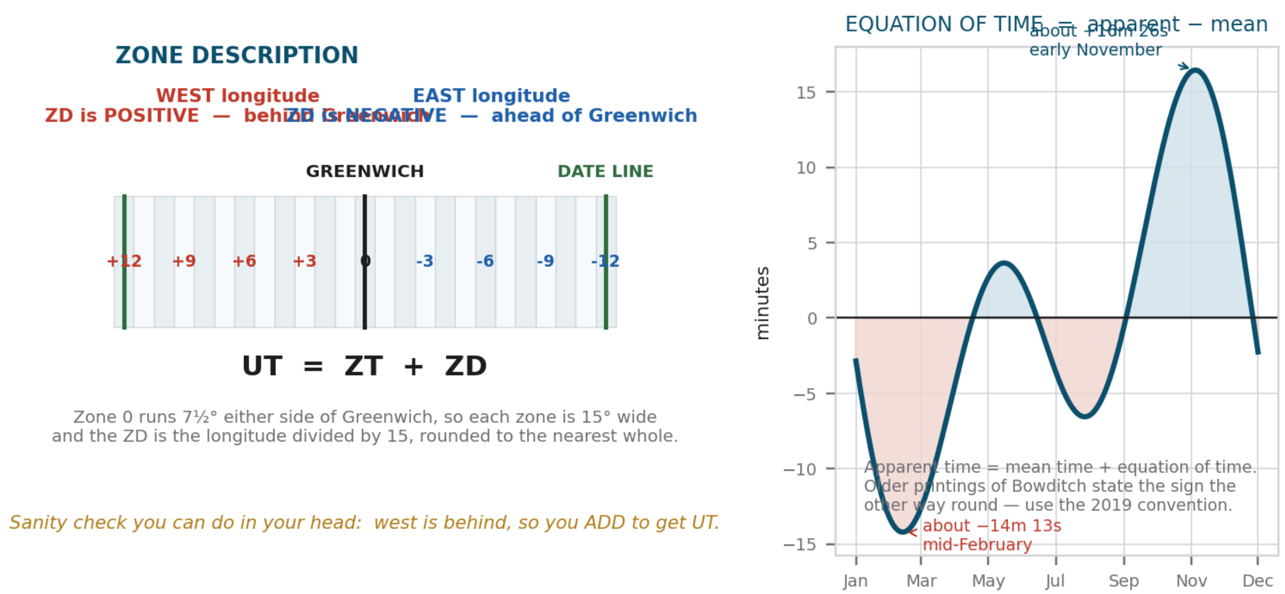
Apply it by the same rule though: **add east, subtract west** to go from gyro reading to true. Gyro error 2°W means true = PGC – 2°.

SECTION 5

Time

Where the sign errors live.

Apparent solar time is what a sundial reads — the real sun, which does not keep good time because the earth's orbit is elliptical and its axis is tilted. **Mean** solar time replaces it with a fictitious mean sun that moves uniformly, and that is the time our clocks keep. **Sidereal** time uses the first point of Aries instead of the sun; a sidereal day is about **23h 56m 04s** of ordinary time, which is why a given star crosses your meridian about four minutes earlier each night.



Left: the zones, and the sign that decides half the marks. Right: the difference between the sun you see and the clock on the bulkhead.

The equation of time

The equation of time is **apparent time minus mean time**. It reaches about –**14m 13s in mid-February** and about **+16m 26s in early November**, and it is zero four times a year. Apparent time = mean time + equation of time.

Bowditch contradicts itself on this one, between editions.

The 2019 edition defines the sign as above, and agrees with the US Naval Observatory. Some older printings state the rule the other way round and are self-contradictory when you test them against November.

Use the 2019 convention. If a question seems to demand the opposite, it is testing the old wording — work it both ways and pick the answer that is offered.

Zone description

Each zone is 15° of longitude wide, and zone zero straddles Greenwich, running 7½° either side. The zone description is the longitude divided by 15 and rounded to the nearest whole number, **positive in west longitude and negative in east**.

The rule, and the check that saves you

UT = ZT + ZD and going the other way, ZT = UT – ZD.

West longitude is **behind** Greenwich — the sun gets there later — so you **add** to get UT. If you can remember that one sentence you never need the rule.

Longitude 115°24.0′E → 115.4 / 15 = 7.69 → zone 8, east, so **ZD =** –**8**. Zone time 1516 gives UT = 1516 – 8 = **0716**.

Arc to time: 360° = 24h · 15° = 1h · 1° = 4m · 15′ = 1m · 1′ = 4s

Crossing the **date line**: going **west** across it the date becomes one day **later**; going **east** across it, one day **earlier**. The line is the anti-meridian of Greenwich, bent around inhabited places.

SECTION 6

The chronometer

A clock nobody is allowed to reset, and the arithmetic that follows from that.

A marine chronometer is never adjusted. It is allowed to be wrong, and its wrongness is measured and recorded instead, because a known error is more useful than a clock somebody has been fiddling with.

- **Chronometer error (CE)** — how far it differs from UT, recorded as **fast** or **slow**, to the nearest half second.
- **Chronometer rate** — the change in that error in 24 hours, recorded as **gaining** or **losing**, to the nearest tenth of a second per day. A steady rate is worth more than a small error.
- **Comparing watch** — a watch you carry to the wing, compare against the chronometer, and read at the instant of the sight. Its own offset is the **watch error**.
- **Applying it — fast means it reads ahead of the truth, so subtract. Slow means it reads behind, so add.**

The radio time tick, and what you are actually listening to

WWV (Fort Collins, Colorado) broadcasts on 2.5, 5, 10, 15 and 20 MHz; **WWVH** (Kekaha, Kauai) on 2.5, 5, 10 and 15 MHz.

A tick every second — **except the 29th and the 59th**, which are silent. That silence is how you know the minute is coming.

The minute is marked by a long **0.8-second** tone; the hour by an 0.8-second tone at 1500 Hz. WWV has a man's voice, WWVH a woman's, and WWVH speaks first.

The exam question describes a **nine-second** silence, which is the older time-signal convention where the beats of the 51st to 59th seconds were omitted and the long dash began exactly on the minute. Either way, what you need from it is the same: **the long dash IS the minute.**

Worked: Q141 question 1

Taking a time tick on the 2000 signal from WWVH. You hear ticks, a nine-second silence, then a long 1.3-second dash. At the beginning of the long dash the comparing watch reads 08h 00m 49s. When compared to the chronometer, the watch reads 08h 01m 33s and the chronometer reads 08h 00m 56s. What is the chronometer error?

Step	Working	Result
The long dash **is** 2000 UT, which a twelve-hour chronometer shows as 08h 00m 00s	watch 08h 00m 49s at that instant	watch is **49s fast**
Take the watch reading at the comparison and remove its error	08h 01m 33s – 49s	true UT **08h 00m 44s**
Compare the chronometer against that	08h 00m 56s – 08h 00m 44s	**CE = 12s FAST**

Answer B. The whole question is one idea: the watch is a middleman, so you have to take the watch's own error out before you can say anything about the chronometer. Skip that step and you land on one of the other offered answers — 0m 56s is what you get by comparing the chronometer against the tick and ignoring the watch altogether, and 0m 44s is simply the UT at the moment of comparison.

SECTION 7

The celestial sphere

Only the part a terrestrial course needs — but that part has to be exact.

Imagine the sky as a sphere with the earth at its centre. Project the earth's equator onto it and you have the **celestial equator**; project the poles and you have the celestial poles. Every body then has a position on that sphere given by two coordinates that behave exactly like latitude and longitude.

Term	What it is
Declination (Dec)	Angular distance north or south of the celestial equator, 0° to 90°. The celestial equivalent of latitude. If the sun's declination is N20°, the sun is overhead at latitude 20°N
Greenwich hour angle (GHA)	Angular distance **west** of the Greenwich celestial meridian, measured westward through 360°
Local hour angle (LHA)	The same thing measured from **your** meridian. **LHA = GHA + east longitude, or GHA** – **west longitude**, adding or subtracting 360° as needed
Sidereal hour angle (SHA)	Angular distance west of the first point of Aries. Used for stars, whose SHA barely changes
Altitude (h or Hc)	Angular distance above the horizon, 0° at the horizon to 90° at the zenith
Zenith distance (z)	90° – altitude
True azimuth (Zn)	Bearing of the body, measured **east from north through 360°** — an ordinary three-figure bearing
Azimuth angle (Z)	Measured either way along the horizon through 180°, starting from north in north latitude and from south in south latitude. This is what the sight reduction tables give you, and it has to be converted

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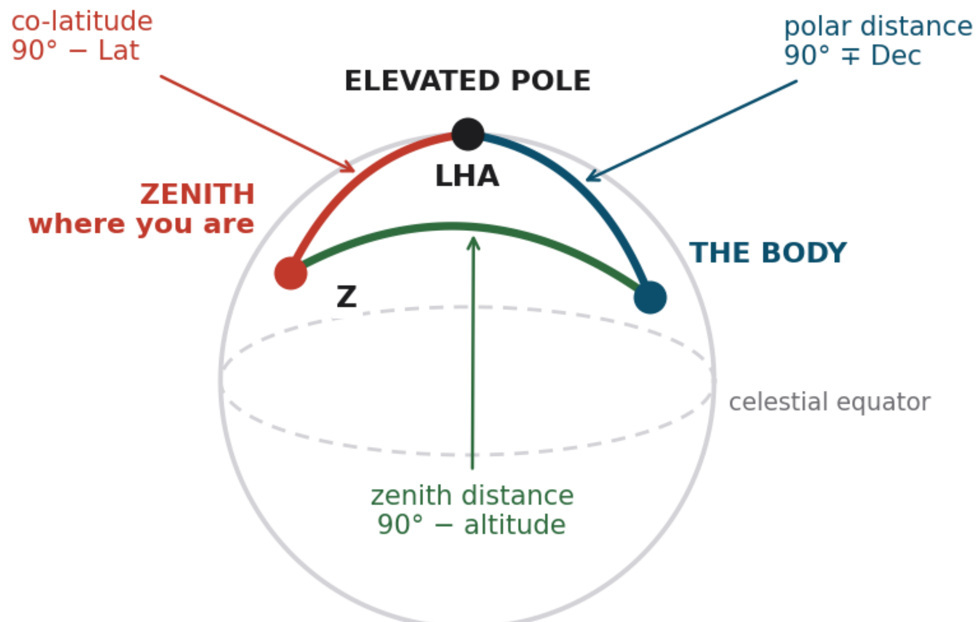


Every celestial problem is this one triangle with a different corner unknown. Amplitude is the special case where the body sits ON the horizon: the zenith distance is exactly 90°, the triangle collapses, and all that is left is sin A = sin Dec / cos Lat.

Learn this once and most of celestial navigation stops being a collection of unrelated formulas.

> **Azimuth angle Z to true azimuth Zn — the rule printed in Pub. 229**
>
> **North latitude:** LHA greater than 180° → Zn = Z · LHA less than 180° → Zn = 360° – Z
>
> **South latitude:** LHA greater than 180° → Zn = 180° – Z · LHA less than 180° → Zn = 180° + Z

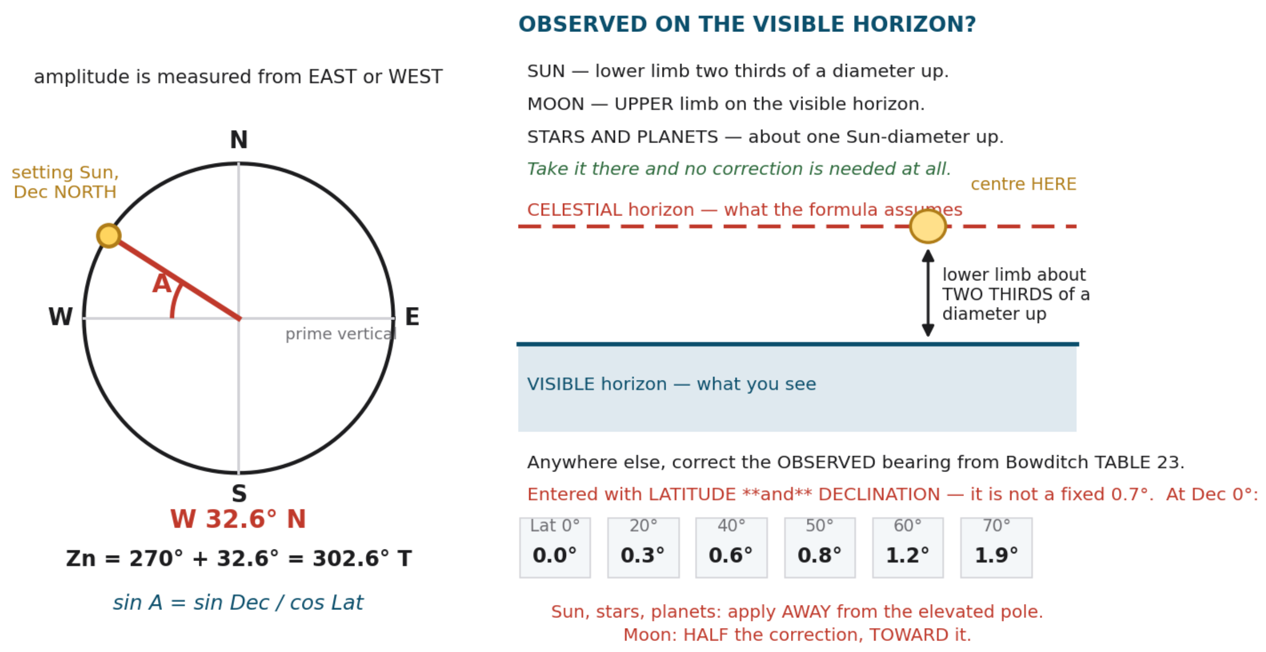
SECTION 8

Compass error by amplitude

The easiest compass check at sea, and the one with the most traps in it.

When a body is **on the horizon**, the navigational triangle collapses: the zenith distance is exactly 90° and the whole problem reduces to one line of trigonometry. That is amplitude, and it is the reason it is the quickest way to get gyro error at sea.

> **sin A = sin Dec / cos Lat** (the same thing as sin Dec × sec Lat)
>
> Name it **E if the body is rising, W if it is setting**, and then **N or S to match the name of the declination**.
>
> Then convert to a true bearing by measuring that angle from east or from west — **never from north or south**.
>
> Or take it straight out of **Bowditch Table 22** (Table 27 in some editions), entering with declination and latitude.



Left: the naming, and how it becomes a bearing. Right: the correction that separates a pass from a fail.

The horizon you observed it on

The formula assumes the body's centre is on the **celestial** horizon. What you actually see is the **visible** horizon, and they are not the same. There are two ways to deal with that, and the exam uses both.

- **Take the sight at the right moment and no correction is needed.** For the **Sun**, when the lower limb is about **two thirds of a diameter** above the visible horizon — roughly 20 minutes of arc. For **stars and planets**, about **one Sun-diameter** up, roughly 30 minutes. For the **Moon**, when its **upper limb** is on the visible horizon.
- **Or observe it on the visible horizon and correct.** The correction comes from **Bowditch Table 23** (Table 28 in some editions) and it is applied to the **observed bearing**.

Two things everyone gets wrong here.

The correction is not a fixed 0.7°. Table 23 is entered with **latitude AND declination**. At declination 0° it runs about 0.0° at the equator, 0.6° at 40°, 1.2° at 60° and 3.0° at 77° — but at a high declination, where the body barely rises at all, it climbs past 10°. The figure –0.7° that appears in the general formula is something else entirely — it is the assumed *altitude* of the body's centre when it is seen on the visible horizon, not a bearing correction.

The direction depends on the body. For the Sun, stars and planets, apply it **away from the elevated pole**. For the Moon, apply **half** of it, **toward** the elevated pole. The Moon is the odd one out in both respects.

Worked: the correction actually changing the answer

This is one of your own demonstration problems, and it is a good one precisely because leaving the correction out gives you a clean, plausible, wrong number.

18 August, sunrise observed with the Sun's centre on the VISIBLE horizon, latitude 52°N, declination N13°00′, observed bearing 065° pgc

sin A = sin 13° / cos 52° = 0.2250 / 0.6157 = 0.3654 → **A = 21.4°**

Rising, declination north → **E 21.4° N** → true bearing = 090° – 21.4° = **068.6° T**

Table 23 at latitude 52°, declination 13° gives **1.0°**. The Sun, so apply it to the observed bearing **away from the elevated pole** — the pole is north, the body is in the east, so the bearing increases: 065.0 + 1.0 = **066.0° pgc**

Gyro error = 068.6 – 066.0 = **2.6° E**

Skip the correction and you get 3.6°E — a whole degree out, and on a multiple-choice paper that is a different letter.

SECTION 9

Compass error by azimuth

The same triangle, with the body anywhere in the sky.

Amplitude only works at rising and setting. An azimuth works at any time the body is visible, which is most of the day, so it is what you use for a routine check. You need the body's declination and its local hour angle, both from the almanac, and your latitude.

The three ways to get the azimuth

By table. Pub. 229 or Pub. 249, entered with latitude, declination and LHA; read Z and convert to Zn with the rule on the previous page. Pub. 229 comes in six volumes of 15° of latitude each; Pub. 249 is the air-navigation set and is quicker but coarser.

By ABC tables (Norie's, Burton's). A is entered with LHA and latitude and is named *opposite* to the latitude, **except** when LHA lies between 90° and 270°. B is entered with LHA and declination and is *always* named the same as the declination. Same names add, different names subtract, to give C; then the azimuth comes from C and the latitude.

By formula. tan Z = sin LHA / (cos Lat × tan Dec – sin Lat × cos LHA), resolving the quadrant carefully — or use a two-argument arctangent and let it do the quadrant for you.

The A-table rule is stated backwards on a lot of teaching websites.

Norie's own wording is: named **opposite** to the latitude, **EXCEPT** when the hour angle is between 90° and 270°. At least one widely used site prints it as ‘opposite *when* the hour angle is between 90° and 270°’, which is the reverse and will fail you. Norie's is the authority; check any rule you were taught from the web against the book.

Worked: Q141 question 9

On 17 April your 1516 zone time DR position is 27°24.0′N, 115°24.0′E. You observe the Sun bearing 247° psc. The chronometer reads 07h 16m 26s, chronometer error 00m 32s slow. Variation is 4.5°E. What is the deviation?

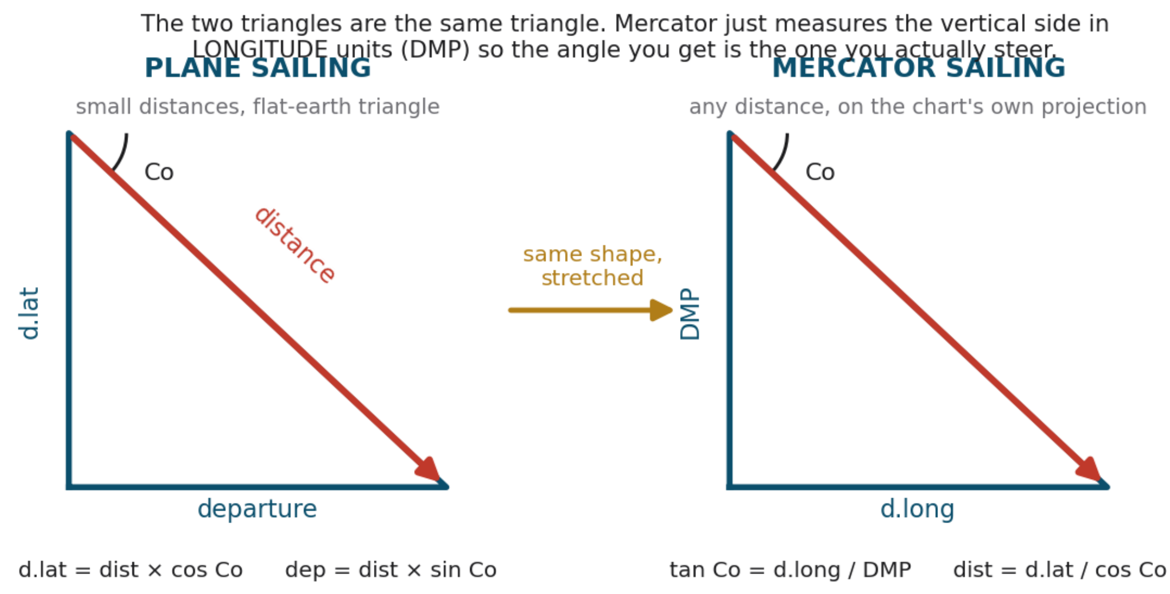
Step	Working
1. Get UT	The chronometer is **slow**, so **add** the error: 07h 16m 26s + 32s = **07h 16m 58s UT**. Check it: 115°24′E is zone –8, so UT = 1516 – 8 = 0716. It agrees, which tells you the chronometer is on the right hour
2. Almanac	For 17 April at 07h 16m 58s UT the Sun's **GHA is about 289.3°** and its **declination about N10.4°**
3. LHA	East longitude **adds**: LHA = 289.3 + 115.4 = 404.7 – 360 = **44.7°**
4. Azimuth	By table or formula, **Zn = 256.9° T**
5. Compass error	256.9 – 247.0 = **9.9° E** (true is greater than compass, so the error is easterly)
6. Deviation	Compass error = deviation + variation, so deviation = 9.9 – 4.5 = **5.4° E**

Answer B. Six steps and six chances to drop a sign. The two that actually catch people are step 1 — a *slow* chronometer is added, not subtracted — and step 3, where east longitude adds to GHA. Get either backwards and nothing downstream can save you.

SECTION 10

The sailings

Plane for short legs, Mercator for long ones, and one table that decides the answer.



The same triangle twice. Mercator measures the vertical side in longitude units so the angle it gives you is the one you steer.

Sailing	Use it for	The working
Plane	Short distances, where the earth's curvature does not matter	d.lat = dist × cos Co · dep = dist × sin Co
Parallel	Running due east or west along a parallel	dep = d.long × cos Lat
Mid-latitude	Middling distances, as a shortcut	dep = d.long × cos(mid-lat)
Mercator	Any distance, and the one the exam wants	tan Co = d.long / DMP · dist = d.lat / cos Co
Great circle	Long ocean passages	Covered in the companion guide, *Passage Planning & Weather Routing*

Worked: Q141 question 4, and the trap inside it

A vessel steams 1106 miles on course 249°T from 13°30.0′N, 144°30.3′E. What is the position of arrival by Mercator sailing?

d.lat = 1106 × cos 249° = –396.4′ = 6°36.4′ S → **Lat 2 = 06°53.6′N**

Course 249°T is a course **angle** of **S 69° W**, and the d.long will be **westerly**.

From Bowditch Table 6: MP(13°30.0′N) = 812.2, MP(06°53.6′N) = 411.9 → **DMP = 400.3** — enter the table with the *unrounded* arrival latitude you just computed, not with 06°54′

d.long = DMP × tan 69° = 400.3 × 2.6051 = 1042.9′ W = 17°22.9′ W

Long 2 = 144°30.3′E – 17°22.9′ = **127°07.4′E** → **answer C**

The trap: meridional parts from the tables are SPHEROIDAL.

If you compute meridional parts on a sphere — which is what every simple formula and most calculators give you — you get DMP 403.0 instead of 400.3, and the answer comes out **127°00.6′E**.

That is nearly seven miles out, and on this paper it is a *different multiple-choice answer*. Bowditch Table 6 and Norie's are computed on the spheroid. **Use the tables, and do not mix a tabulated value with a computed one inside the same problem.**

SECTION 11

Four more from the paper

Speed, distance, time and the propeller — quick marks if you know the form.

ETA across time zones

2190 miles at 15.0 knots, with a 16.0 hour layover. Depart 0654 (ZD +6) on 27 November. ETA at ZD +4?

> Steaming time = 2190 / 15.0 = **146.0 h**; plus 16.0 h layover = **162.0 h** = 6 days 18 hours.
>
> **Work in UT and convert once at the end.** Depart 0654 + 6 = **1254 UT 27 November**.
>
> 1254 UT + 6d 18h = **0654 UT 4 December**.
>
> ZD +4, so local = UT – 4 = **0254 on 4 December** — answer C.

Propeller slip

Engine distance is what the propeller would have driven the ship if the water were solid: pitch × revolutions. **Slip** is the percentage the ship falls short of it. Positive slip is normal; negative slip means the ship went further than the propeller turned for, which happens with a following current.

> **Engine distance** = pitch (ft) × RPM × 60 × hours / 6076.1
>
> **Slip %** = (engine distance – observed distance) / engine distance × 100
>
> *Q141 q6:* 21.4 ft pitch, 76 RPM, 24 hours → 21.4 × 76 × 1440 / 6076.1 = **385.4 NM**. Observed 378 → (385.4 – 378) / 385.4 = **+1.9%**
>
> *Q141 q10, backwards:* need 18.7 kt with 3% slip → engine speed = 18.7 / 0.97 = 19.28 kt → RPM = 19.28 × 6076.1 / (24.25 × 60) = **80.5, so 81 RPM**

Deviation from a swing table

Gyro error 2°W, variation 8°W. Find the deviation on a true heading of 236°, from a table of PSC against PGC.

> Work **down** the ladder from true to compass, one step at a time.
>
> True 236°, gyro error 2°W → uncorrecting, add westerly: **PGC = 238°**
>
> Enter the table at PGC 238°. Between 234→239.5 and 264→269.0, interpolate: **PSC = 243.4°**
>
> Magnetic = true – variation = 236 – (–8) = **244°**
>
> **Deviation = magnetic – compass = 244 – 243.4 = 0.6°E** → nearest offered answer **0.5°E**
>
> *The gyro column is only there to get you to a compass heading. Once you have PSC, the gyro has done its job and drops out.*

SECTION 12

The four equations, one line at a time

The same equation written out again at every step, with one more thing filled in. Nothing is rearranged and nothing is skipped.

Every worked example earlier in this guide is correct, but it moves fast. This section does the four that matter slowly. The rule throughout: **the equation never changes shape.** It is written out again on every line exactly as Bowditch gives it, and the only thing that happens is that one more number replaces one more symbol. If you can follow a recipe, you can follow this.

1. AMPLITUDE — sunrise, 18 August, latitude 52°N, observed 065° pgc

WHAT YOU DO NOW	THE EQUATION, WITH WHAT YOU KNOW SO FAR
THE EQUATION straight from Bowditch — do not rearrange it	**sin A = sin Dec / cos Lat**
Step 1. Go to the Nautical Almanac for the date and the time, and take out the declination. Dec = N 13° 00.0′	sin A = sin **13°00.0′** / cos Lat
Step 2. Put in your own latitude. It is given in the question. Lat = 52° N	sin A = sin 13°00.0′ / cos **52°**
Step 3. Work out the top of the fraction.	sin A = **0.2250** / cos 52°
Step 4. Work out the bottom of the fraction.	sin A = 0.2250 / **0.6157**
Step 5. Divide.	sin A = **0.3654**
Step 6. Take the inverse sine. This is the amplitude.	**A = 21.4°**
Step 7. Name it. **E** because the Sun is rising. **N** because the declination is north.	**A = E 21.4° N**
Step 8. Turn it into a true bearing. East is 090°; the amplitude is measured *from* east, and N means toward north, so it comes off.	Zn = 090° – 21.4° = **068.6° T**
Step 9. The Sun's centre was on the *visible* horizon, so correct the OBSERVED bearing. Bowditch Table 23 at Lat 52°, Dec 13° gives 1.0°, applied away from the elevated pole.	observed = 065.0° + **1.0°** = **066.0° pgc**
Step 10. Gyro error is what the compass should have read minus what it did read.	GE = 068.6° – 066.0° = **2.6° EAST**

Leave step 9 out and you get 3.6°E. It is a clean, plausible, wrong answer, and it is on the paper as a distractor.

2. AZIMUTH — 17 April, DR 27°24.0′N 115°24.0′E, Sun bearing 247° psc

WHAT YOU DO NOW	THE EQUATION, WITH WHAT YOU KNOW SO FAR
THE EQUATION straight from Bowditch — do not rearrange it	**tan Z = sin LHA / (cos Lat × tan Dec – sin Lat × cos LHA)**
Step 1. Get the time right first. The chronometer is **slow**, so **add** the error. 07h 16m 26s + 32s = 07h 16m 58s UT	time settled — the equation is untouched so far
Step 2. Almanac, for that UT. Take out GHA and declination. GHA 289.3° · Dec N 10.4°	tan Z = sin LHA / (cos Lat × tan **10.4°** – sin Lat × cos LHA)
Step 3. Turn GHA into LHA. The longitude is **EAST**, so it **ADDS**. 289.3 + 115.4 = 404.7 – 360 = 44.7°	tan Z = sin **44.7°** / (cos Lat × tan 10.4° – sin Lat × cos **44.7°**)
Step 4. Put in your latitude. Lat = 27.4° N	tan Z = sin 44.7° / (cos **27.4°** × tan 10.4° – sin **27.4°** × cos 44.7°)
Step 5. Evaluate the top.	tan Z = **0.7034** / (cos 27.4° × tan 10.4° – sin 27.4° × cos 44.7°)
Step 6. Evaluate the first product in the bracket.	tan Z = 0.7034 / (**0.1630** – sin 27.4° × cos 44.7°)
Step 7. Evaluate the second product in the bracket.	tan Z = 0.7034 / (0.1630 – **0.3271**)
Step 8. Do the subtraction inside the bracket. Note it goes **negative** — that is what puts the body in the western half.	tan Z = 0.7034 / **(–0.1641)**
Step 9. Divide.	tan Z = **–4.286**
Step 10. Inverse tangent, then resolve the quadrant. LHA is less than 180°, so the body is **west** of the meridian, and the denominator was negative.	**Zn = 256.9° T**
Step 11. Compass error is true minus compass.	CE = 256.9° – 247.0° = **9.9° EAST**
Step 12. Compass error is deviation *plus* variation, so take the variation out. Variation 4.5°E	Dev = 9.9° – 4.5° = **5.4° EAST**

Two steps do all the damage: step 1, where a SLOW chronometer is ADDED, and step 3, where EAST longitude ADDS to GHA. Get either backwards and every line after it is wrong.

3. COMPASS ERROR — range lights in line, bearing 315° psc, chart says 312°T, variation 6°W

WHAT YOU DO NOW	THE EQUATION, WITH WHAT YOU KNOW SO FAR
THE EQUATION straight from Bowditch — do not rearrange it	**Compass Error = Deviation + Variation**
Step 1. First get the compass error on its own, from the two bearings you have. It is **true minus compass**.	CE = **312°** – **315°**
Step 2. Do the subtraction. A **negative** answer means **WEST**; a positive one means east.	CE = **–3°** = **3° WEST**
Step 3. Now go back to the Bowditch relation and put that in. Westerly is negative throughout.	**–3°** = Deviation + Variation
Step 4. Put in the variation. It is 6°**W**, so it goes in as –6°.	–3° = Deviation + **(–6°)**
Step 5. Rearrange for the one thing you do not know.	Deviation = –3° – **(–6°)**
Step 6. Subtracting a negative adds.	Deviation = **+3°**
Step 7. Positive means east.	**Deviation = 3° EAST**

The compass error is WESTERLY and the deviation is EASTERLY at the same time. That is not a mistake — the variation is larger and pulls the other way. This is exactly why the question is on the paper.

4. MERCATOR SAILING — 1106 miles on 249°T from 13°30.0′N 144°30.3′E

WHAT YOU DO NOW	THE EQUATION, WITH WHAT YOU KNOW SO FAR
THE EQUATION straight from Bowditch — do not rearrange it	**d.lat = distance × cos Co tan Co = d.long / DMP**
Step 1. Start with the first equation and put the distance in. distance = 1106 miles	d.lat = **1106** × cos Co
Step 2. Put the course in. Co = 249° T	d.lat = 1106 × cos **249°**
Step 3. Evaluate. It comes out negative, which simply means **southerly**.	d.lat = **–396.4′** = 6°36.4′ S
Step 4. Apply it to the latitude you left from. This is your arrival latitude.	Lat 2 = 13°30.0′N – 6°36.4′ = **06°53.6′ N**
Step 5. Now the second equation. Go to **Bowditch Table 6** for the meridional parts of BOTH latitudes. Enter with the unrounded arrival latitude. MP 13°30.0′ = 812.2 · MP 06°53.6′ = 411.9	tan Co = d.long / (**812.2 – 411.9**)
Step 6. Subtract to get the difference of meridional parts. Same names, so subtract.	tan Co = d.long / **400.3**
Step 7. Put the course angle in. 249°T is **S 69° W**, so the angle is 69° and the d.long will be **west**.	tan **69°** = d.long / 400.3
Step 8. Rearrange for d.long.	d.long = 400.3 × tan 69°
Step 9. Evaluate the tangent.	d.long = 400.3 × **2.6051**
Step 10. Multiply.	d.long = **1042.9′ W** = 17°22.9′ W
Step 11. Apply it to the longitude you left from. West takes away from an east longitude.	Long 2 = 144°30.3′E – 17°22.9′ = **127°07.4′ E**

Step 5 is the one that decides it. Meridional parts from the TABLES are computed on the spheroid. Compute them on a sphere instead and you get DMP 403.0, d.long 1049.7′ and a final longitude of 127°00.6′E — which is a different answer on the multiple-choice paper.

SECTION 13

The tear-out card

Everything with a sign in it, on one sheet.

THE COMPASS LADDER

Can Dead Men Vote Twice — At Elections C → D → M → V → T is **CORRECTING**, and **ADD EAST**.

Correcting (compass → true): **add easterly, subtract westerly**. Uncorrecting: the reverse.

Compass error = deviation + variation. Gyro error is separate, constant all round the compass, and applied by the same add-east rule.

TIME

UT = ZT + ZD ZD is **POSITIVE WEST**, negative east. West is behind Greenwich, so you add.

Arc to time: 360°=24h · 15°=1h · 1°=4m · 15′=1m · 1′=4s

Chronometer **FAST** → subtract. **SLOW** → add.

Equation of time = apparent – mean. Apparent = mean + EoT.

Date line: **westward, the date advances**; eastward it goes back.

Sidereal day = 23h 56m 04s.

CELESTIAL

LHA = GHA + E long / GHA – **W long** (±360°)

Zenith distance = 90° – altitude.

Navigational triangle: co-latitude (90–Lat), polar distance (90■Dec), zenith distance (90–alt).

Z to Zn — N lat: LHA>180 → Zn=Z; LHA<180 → Zn=360–Z. S lat: LHA>180 → Zn=180–Z; LHA<180 → Zn=180+Z.

AMPLITUDE

sin A = sin Dec / cos Lat — or Bowditch Table 22 (27 in some editions).

Named **E rising / W setting**, then **N or S to match the declination**. Measured from EAST or WEST.

On the celestial horizon — no correction — when: **Sun**, lower limb 2/3 of a diameter up (about 20′); **stars and planets**, one Sun-diameter up (about 30′); **Moon**, UPPER limb on the visible horizon.

Otherwise correct the OBSERVED bearing from **Table 23** (28 in some editions). It is **latitude-dependent**: about 0.0° at the equator, 0.6° at 40°, 1.2° at 60°.

Sun, stars, planets: AWAY from the elevated pole. Moon: HALF, TOWARD it.

SAILINGS AND THE EARTH

Plane: d.lat = dist × cos Co · dep = dist × sin Co

Parallel: dep = d.long × cos Lat

Mercator: tan Co = d.long / DMP · dist = d.lat / cos Co — and take DMP from the TABLES, not a formula.

Nautical mile = **1852 m exactly = 6076.1 ft**. A minute of latitude runs 1842.9 m at the equator to 1861.6 m at the pole — **longest at the poles**.

Engine distance = pitch × RPM × 60 × hours / 6076.1 · slip% = (engine – observed) / engine × 100

IF YOU ONLY REMEMBER ONE THING

Work every problem in UT, all the way through, and convert to local time once, at the very end.

Terrestrial Navigation · Capt. Georgia Hilton · captaingeorgia.com · free to use for study