

Coriolis & Eötvös

The deflection everybody teaches — and the partner effect almost nobody does

The Earth turns once in a sidereal day, and we do all our navigating and all our forecasting from inside that turning frame. Anything that moves freely across the surface — an air mass, a current, a shell, a pendulum — travels in a straight line as far as space is concerned, but we watch it from a platform that has rotated out from under it. The apparent curve we see is the Coriolis effect. It is not a force acting on the object. It is the bookkeeping entry we owe for measuring motion against a rotating chart.

Everything in this handout comes out of one number and one angle.

$$\mathbf{f} = 2\Omega \sin \varphi$$

Ω is the Earth's rotation rate, 7.292×10^{-5} radians per second, so $2\Omega = 1.458 \times 10^{-4} \text{ s}^{-1}$. φ is your latitude. That product, $\mathbf{f}$, is the **Coriolis parameter**, and it sets the strength of the horizontal deflection you feel where you are standing. In the northern hemisphere the deflection is always to the **right** of the direction of motion; in the southern hemisphere, always to the left. Speed determines how hard the deflection bites (the acceleration is $\mathbf{f} \times \mathbf{V}$), but latitude alone determines the rate of turning.

The one thing students get wrong

Coriolis does not change a body's speed — only its direction. It always acts at right angles to the velocity, so it does no work. A 20-knot vessel that is deflected for six hours is still making 20 knots; it is simply no longer heading where it started.

Why latitude, and why the sine

Stand at the North Pole and you are spinning about your own local vertical, a full 360° in a day, so a straight-line path across the ice appears to curve at the maximum possible rate. Stand at the equator and your local vertical does not rotate about itself at all — it tumbles end over end instead. Nothing moving horizontally is deflected horizontally. Everywhere in between, the component of the Earth's rotation vector that points along your local vertical is $\Omega \sin \varphi$, and the sine is doing exactly that projection.

The consequence catches people out: the fall-off with latitude is not linear. At 45° — halfway from the equator to the pole in degrees — you still have about **71%** of the polar value, not half. You have to come all the way down to 30° before Coriolis is cut in half.

Strength by latitude — the working numbers

Latitude	$\sin \varphi$	$f = 2\Omega \sin \varphi$ horizontal (s^{-1})	% of max	$f^* = 2\Omega \cos \varphi$ vertical (s^{-1})	Inertial period $2\pi/f$	Foucault °/hr
0°	0.0000	0	0%	1.458×10^{-4}	—	0.00
30°	0.5000	0.729×10^{-4}	50%	1.263×10^{-4}	23.93 h	7.52
45°	0.7071	1.031×10^{-4}	71%	1.031×10^{-4}	16.92 h	10.64
60°	0.8660	1.263×10^{-4}	87%	0.729×10^{-4}	13.82 h	13.03
90°	1.0000	1.458×10^{-4}	100%	0.000×10^{-4}	11.97 h	15.04

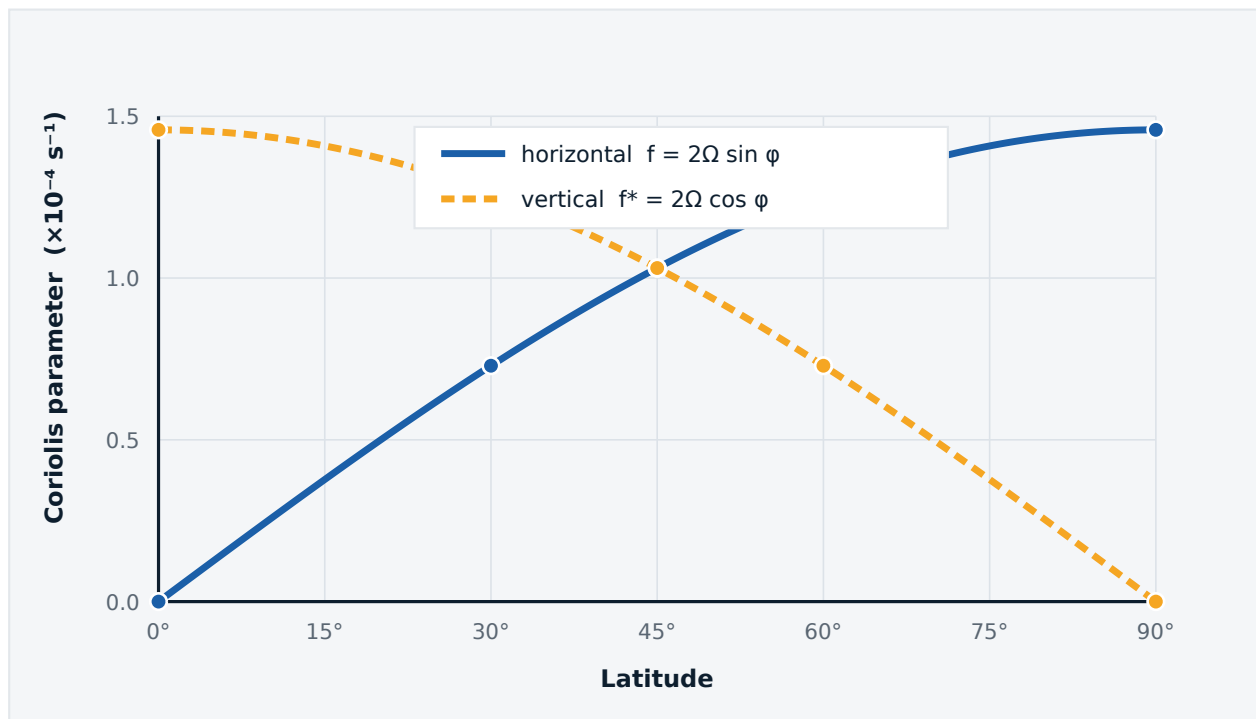


Figure 1 · Horizontal and vertical components of the Coriolis term. They are mirror images: where one is strongest, the other is zero.

Read the table across and the pattern is clean. The horizontal term climbs as the sine — 0 at the equator, half strength at 30°, 71% at 45°, 87% at 60°, full value at the pole. The vertical term does the exact opposite, because it goes as the cosine. Those two columns are the whole story of this handout: the first column builds weather, and the last one — the one nobody teaches — is Eötvös.

The inertial period, $2\pi/f$, is the most useful way to carry the scaling around in your head. It is 11.97 hours at the pole, 16.93 hours at 45°, 23.93 hours at 30°, and infinite at the equator. It is the time a freely moving body takes to turn a complete circle under Coriolis alone, and it is the same number that governs the Foucault pendulum, which rotates $15.04 \sin \varphi$ degrees per hour. In Fort Lauderdale at 26° N, a Foucault pendulum turns about 6.6° per hour — a bit over two and a half days for a full circuit.

What that does to something actually moving

Take 20 knots — a fast ship, a slow air parcel — and see what the deflection amounts to. The cross-track figures below assume no rudder, no wind, no current: pure Coriolis acting alone on a free body.

Latitude	Deflecting accel. $f \cdot V$ (m/s ²)	Cross-track drift first hour (nm, approx.)	Inertial-circle radius V/f (nm)
0°	0	0	∞
15°	0.388×10^{-3}	1	147
30°	0.750×10^{-3}	3	76
45°	1.061×10^{-3}	4	54
60°	1.299×10^{-3}	5	44
90°	1.500×10^{-3}	5	38

Two honest conclusions come out of that table. For a vessel under helm, Coriolis is irrelevant — a fraction of a degree of rudder swallows it, and the helmsman corrects it without ever knowing it was there. The old chestnut about long-range naval gunnery needing a Coriolis correction is true, but that is a case of a free body in flight for a minute or more. For an **air mass**, which is free, unsteered, and travels for days over thousands of miles, it is the dominant organizing force in the atmosphere. That difference in time-of-flight is the whole reason Coriolis is a meteorology topic and not a shiphandling topic.

Where the approximation quits

Within roughly 5° of the equator, f is so small that there is nothing left to balance the pressure gradient. The geostrophic approximation fails, and you get no tropical cyclogenesis in that band no matter how warm the water is — a disturbance there has no mechanism to spin up a closed circulation. That, not sea surface temperature, is the reason the belt within about 300 nm of the equator stays free of hurricanes.

Surface wind, and why it turns as you climb

Air starts moving because of a pressure gradient — it is pushed from high toward low, straight across the isobars. The moment it is moving, Coriolis begins deflecting it to the right. Left alone, that continues until the deflection has turned the flow far enough that Coriolis exactly opposes the pressure gradient force. At that point the two are in balance and the wind runs **parallel to the isobars**, low pressure on the left in the northern hemisphere. That is the **geostrophic wind**, and it is the single most counter-intuitive result in basic meteorology: air does not flow from high to low, it flows *around* them.

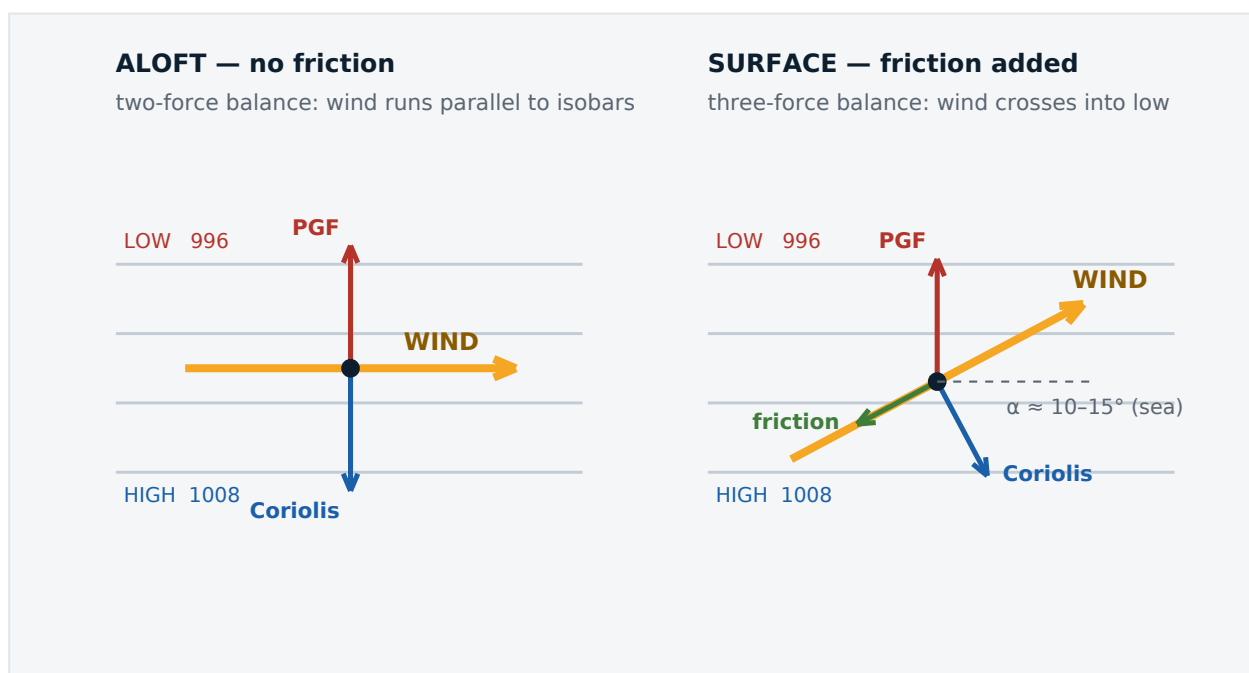


Figure 2 · Two-force balance aloft versus three-force balance at the surface. Friction shortens the wind vector, which weakens Coriolis, which lets the pressure gradient win and pull the flow across the isobars into the low.

At the surface the balance has a third member: friction. Drag against the sea or the land acts directly opposite the wind, so it slows the air down. And because the Coriolis force is proportional to speed, slowing the air **weakens Coriolis**. The pressure gradient force does not care — it is set by the isobar spacing and is unchanged. So the balance tips, and the wind ends up blowing at an angle *across* the isobars, toward the low.

The crossing angle, in practice

Over open water the surface is relatively smooth and the crossing angle is small — about **10° to 15°**, sometimes up to 20° in light winds or a stable layer. Over land it is much larger, commonly **25° to 45°**, because the surface is rougher. Speed follows the same logic: surface wind over water runs roughly two-thirds of the geostrophic value, over land closer to one-third to one-half. This is exactly why the wind on a surface analysis does not lie along the isobars, and why Buys Ballot's law needs a qualifier at the surface — back to the wind in the northern hemisphere and low pressure is on your left and *slightly behind* you, not dead abeam.

Climbing out of the friction layer

Friction is a surface effect, and it is transmitted upward only by turbulent mixing. As you climb, that mixing weakens, drag falls away, the wind speeds up, and Coriolis strengthens with it. The crossing angle shrinks and the flow swings clockwise (in the northern hemisphere) until, at the top of the friction layer, it is running essentially parallel to the isobars. The tips of the wind vectors, plotted level by level, trace the **Ekman spiral**.

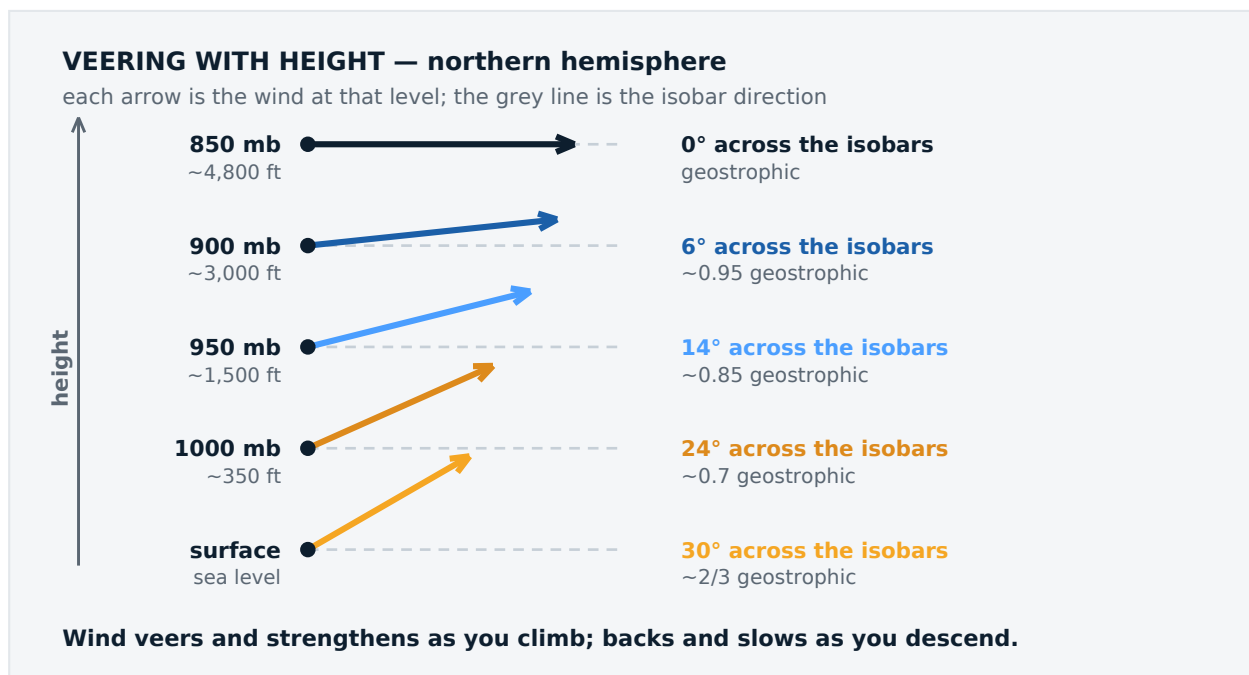


Figure 3 · Wind veers and strengthens with height through the friction layer. The 1000 mb surface sits only about 350 ft up in a standard atmosphere — still firmly inside the friction layer, not a geostrophic level.

Worth being precise here, because the chart levels invite a mistake: the **1000 mb surface is only about 350 feet above sea level** in a standard atmosphere. It is not an upper-air level in any meaningful sense — it is still deep inside the friction layer, and a 1000 mb wind still crosses the isobars at nearly the surface angle. The real transition to geostrophic flow happens at the **gradient level**, roughly 1,500 to 3,000 feet over water and up to 3,000–5,000 feet over rough terrain. By **850 mb** — about 4,800 feet — the geostrophic approximation is a good working fit over open ocean, and by 700 mb and above it is very good indeed.

Veering versus backing — the practical shorthand

Northern hemisphere: wind **veers** (turns clockwise) with increasing height, and **backs** as you descend, so the surface wind is backed relative to the wind aloft. Compare the direction the low cloud is moving with the wind on deck: the difference is the friction-layer veer, and its sense tells you whether warm or cold advection is going on overhead. Southern hemisphere: reverse both.

One last refinement: real isobars are curved, and around a low or a high the centripetal term joins the balance. That gives the **gradient wind** — sub-geostrophic around a low, super-geostrophic around a high — which is why a tight low can have isobars packed hard and still not blow quite as hard as a straight-line calculation predicts.

The partner effect: Eötvös

Here is the part that gets left out. When we say “there is no Coriolis at the equator,” what we mean is that the *horizontal* component vanishes. The Coriolis term is a three-dimensional vector, and it has a vertical component too:

$$f^* = 2\Omega \cos \varphi$$

That component is **maximum at the equator and zero at the poles** — the exact opposite of the horizontal term. It acts on east-west motion, and it pushes vertically. Move east and you are adding your own speed to the Earth's eastward rotation, so you are travelling a little faster around the axis, which means a little more centrifugal effect, which means you weigh a little less. Move west and you subtract, and you weigh a little more. This is the **Eötvös effect**, named for Loránd Eötvös, who worked it out around 1900 after noticing systematic discrepancies in gravity measurements taken aboard moving ships.

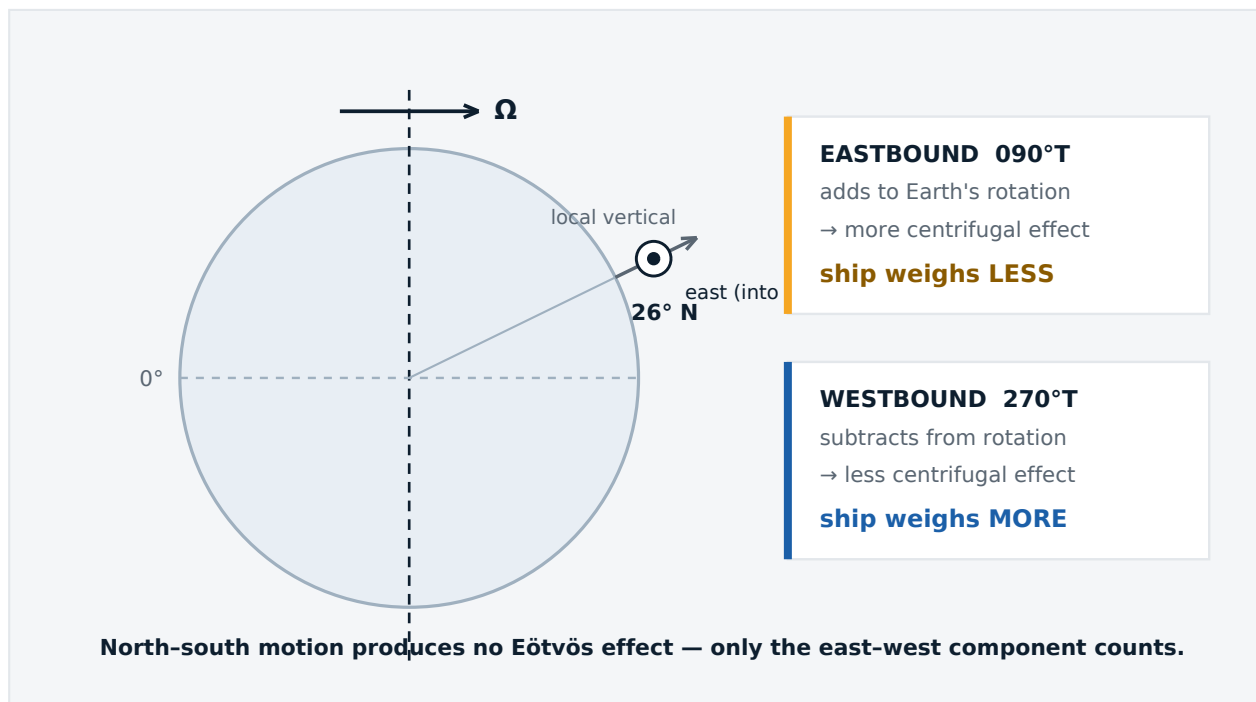


Figure 4 · The Eötvös effect. Only the east-west component of velocity matters; pure north-south motion produces none.

The standard marine form, with speed in knots and heading true, gives the correction in milligals:

$$E = 7.503 \cdot V \cdot \cos \varphi \cdot \sin \alpha + 0.004154 \cdot V^2$$

The first term is the rotation-rate term and dominates; the second is the small self-centrifugal term from the vessel's own curved path over the globe, and it is independent of heading.

Worked example — a ship off Fort Lauderdale

Take a 50,000-tonne vessel making **20 knots due east** at **26° N**, roughly the latitude of the Port Everglades sea buoy.

Step 1 — the acceleration.

$$2\Omega = 1.458 \times 10^{-4} \text{ s}^{-1} \cdot V = 20 \text{ kt} = 10.29 \text{ m/s} \cdot \cos 26^\circ = 0.8988$$

$$a = 2\Omega \cdot V \cdot \cos \varphi = 1.458 \times 10^{-4} \times 10.29 \times 0.8988 = \mathbf{1.348 \times 10^{-3} \text{ m/s}^2}$$

In survey units that is about **135 mGal**, and the knots formula agrees: $7.503 \times 20 \times 0.8988 = 134.9$, plus 1.7 for the V^2 term = **136.6 mGal**.

Step 2 — what it means on the scale.

As a fraction of gravity, $1.348 \times 10^{-3} \div 9.807 = \mathbf{0.0137\%}$, about 137 parts per million. Our 50,000-tonne ship therefore appears to lose roughly **6.9 tonnes** of weight while running east at 20 knots. Turn her onto 270° and she gains the same 6.9 tonnes — an **east-to-west swing of about 13.8 tonnes** on the same hull, same displacement, same cargo.

Latitude	Heading	Speed	Eötvös accel. (mGal)	Apparent weight change 50,000 t ship
26°	090° T	20 kt	+136.5	−7.0 t
26°	270° T	20 kt	-133.2	+6.8 t
26°	000° T	20 kt	+1.7	−0.1 t
45°	090° T	20 kt	+107.8	−5.5 t
60°	090° T	20 kt	+76.7	−3.9 t
0°	090° T	20 kt	+151.7	−7.7 t
26°	090° T	8 kt	+54.2	−2.8 t

Table · Eötvös for a 50,000-tonne vessel. Negative weight change = lighter. The 000° T row shows the heading-independent V^2 term only — due north, the main Eötvös term is zero.

So does it change your draft?

No — and this is the honest answer to give the class. 6.9 tonnes on a ship with a TPC of 50 tonnes per centimetre is about 1.4 mm of immersion, far below anything you could read on a draft mark or feel in the handling. The effect is real, perfectly measurable with the right instrument, and operationally invisible to the watch officer.

Where it actually matters

It matters anywhere a gravity measurement is taken from a moving platform. Marine and airborne gravity surveys — hydrographic work, offshore resource exploration, geoid and datum determination — must apply the **Eötvös correction** to every reading, and at survey speeds it is one of the largest corrections in the reduction, far bigger than the anomalies being hunted. Get the ship's speed and heading a little wrong and you have invented a geological feature. It is also a live term in inertial navigation.

Quick reference card

Item	Value / rule
Horizontal Coriolis parameter	$f = 2\Omega \sin \phi$ · max at poles, zero at equator
Vertical (Eötvös) parameter	$f^* = 2\Omega \cos \phi$ · max at equator, zero at poles
2Ω	$1.458 \times 10^{-4} \text{ s}^{-1}$ ($\Omega = 7.292 \times 10^{-5} \text{ rad/s}$)
Deflection sense	right of motion in N hemisphere, left in S
Deflecting acceleration	$f \times V$ · always perpendicular to motion, does no work
Inertial period	$11.967 / \sin \phi$ hours
Foucault rotation rate	$15.04 \sin \phi$ degrees per hour
Strength at 45°	71% of polar value — not 50%
Strength at 30°	50% of polar value
Geostrophic wind	PGF balanced by Coriolis; flow parallel to isobars
Surface crossing angle	$10\text{--}15^\circ$ over water · $25\text{--}45^\circ$ over land, toward the low
Surface wind speed	$\sim 2/3$ geostrophic over water · $\sim 1/3\text{--}1/2$ over land
Gradient level	$\sim 1,500\text{--}3,000$ ft over water; 1000 mb is NOT above it
Veering	N hemisphere wind veers with height, backs with descent
Eötvös correction (mGal)	$E = 7.503 V \cos \phi \sin \alpha + 0.004154 V^2$ (V in knots)
Eötvös sense	eastbound lighter · westbound heavier · N-S no effect
No tropical cyclogenesis	within $\sim 5^\circ$ of the equator — f too small

Five questions to close the block

1. A vessel is steaming due north at 18 knots at 40° N. Which way does Coriolis deflect her, and does the Eötvös effect change her apparent weight? (Right; no — Eötvös acts only on the east-west component.)
2. The surface wind is 220° at 25 knots over open water. Roughly what is the direction of the isobars, and where is the low? (Isobars about $230\text{--}235^\circ$; low on the left of the wind and slightly behind — roughly northwest.)
3. Why does a hurricane not form at 2° N even over 30°C water? (f is essentially zero; nothing can balance the pressure gradient, so no closed rotating circulation can organize.)
4. Low cloud is moving from 260° while the deck wind is from 240° . Warm or cold advection? (Wind veers with height — warm advection.)
5. Two identical ships at 26° N, one on 090° and one on 270° , both 20 knots. Which is heavier and by how much? (The westbound one, by about 13.8 tonnes apparent — about 1.4 mm of draft. Unmeasurable aboard, unmissable on a gravimeter.)