

THE GENERAL CIRCULATION OF THE ATMOSPHERE

Why a Rotating, Unequally Heated Sphere Builds Cells, Jets, and Waves — A Graduate Monograph with the Mathematics

Every forecast you will ever read is a snapshot of one machine at work. The high that gives you a fair-weather passage, the low you route around, the trade winds that carried sail for four centuries, the calms that stranded them, the band of squalls on the equator, the jet stream steering the depression that will find you on day three — these are not separate phenomena to memorize. They are the same rotating, unequally heated atmosphere seen from different latitudes, and once you understand the machine you stop memorizing weather and start anticipating it.

This brief is why that matters on a bridge at 0300. A mariner who knows only rules — wind along the isobars, back for a low, veer for a high — is a mariner who is surprised. A mariner who knows why the wind runs along the isobars knows the rule's limits: why a 1019 mb high off Florida is not “high means fair,” why a low can deepen without any ceiling while a high never can, why the storm two days ahead of the one on your chart is the one to watch, and why the tropics play by different rules entirely. The theory in these pages is not academic. It is the difference between reading the weather you have and forecasting the weather you are about to get.

The argument is a single chain. Rotation does not drive the circulation, but it constrains it, and the constraint changes with latitude. That change gives the air a restoring force, and therefore waves. In the latitudes where most of the world's shipping runs, those waves are the weather: they carry the heat the mean flow cannot, they build the jet that steers your storms, and they set the highs, lows, and trade belts you plan passages around. Read the eddies as the machine, not as noise, and the whole ocean's weather closes into one picture.

ORIENTATION

The spine, in one breath: differential heating loads a pole-to-equator temperature gradient → rotation forbids a simple overturning → the gradient is released instead by baroclinic waves → those waves flux heat and momentum → they build the jets and drive the mean cells → friction closes the budget.

On the bridge, it comes down to this: the pressure pattern on your chart is the *structure*; the small departures from it are the *weather* — the crossing winds, the vertical motion, the deepening low. Learn to see the pattern as the output of a turning, heated engine and the chart stops being a picture and becomes a forecast.

Contents — nine movements: (1) the rotating frame & Coriolis · (2) scale analysis & geostrophy · (3) vorticity & potential vorticity · (4) Rossby waves · (5) thermal wind & the two jets · (6) baroclinic instability · (7) the Hadley cell · (8) the Ferrel cell & the residual circulation · (9) energetics & variability.

1 · The Rotating Frame — Where Coriolis Comes From

Newton's laws hold in an inertial frame, and we refuse to use one: we stand on a spinning planet and write the equations as though our frame were at rest. That refusal is the entire origin of the Coriolis effect. It is not a force in the sense that pressure gradients and gravity are forces; it is the bookkeeping cost of doing physics in a frame that is itself accelerating. Transform the momentum equation into that rotating frame, turning at angular velocity Ω , and two extra terms appear. One of them is the deflecting force every forecaster already knows in plain terms — the apparent push that turns moving air to the right in the Northern Hemisphere and to the left in the Southern, so the wind circles a low instead of pouring straight into it and Buys Ballot's law holds at all:

$$\left(\frac{d\vec{v}}{dt}\right)_{\text{rot}} = \left(\frac{d\vec{v}}{dt}\right)_{\text{in}} - 2\vec{\Omega} \times \vec{v} - \vec{\Omega} \times (\vec{\Omega} \times \vec{r}) \quad (1.1)$$

The centrifugal term is steady, independent of the motion, and points outward from the axis — and we dispose of it entirely by folding it into gravity. This is genuinely elegant. The Earth is not a sphere; it is an oblate spheroid, flattened by about 21 km between equator and pole, and that shape is precisely the equilibrium a deformable rotating body settles into so that true gravitation plus centrifugal force is everywhere normal to the surface. The planet has already done the vector addition. What we call gravity is *effective* gravity, geopotential surfaces are spheroids rather than spheres, and the centrifugal term never appears explicitly again. Mean sea level — your chart datum — is a geopotential surface, and in a real sense a centrifugal artefact.

The Coriolis term cannot be absorbed, because it depends on the velocity itself. Being a cross product it is always perpendicular to the motion, and a force perpendicular to velocity does no work: it changes direction, never speed. That is the rigorous content of the rule you already teach — Coriolis turns, it does not drive.

For large-scale horizontal motion the component of Ω that counts is the locally vertical one, $\Omega \sin\phi$. Twice it is the Coriolis parameter:

$$f = 2\Omega \sin \phi \quad \beta \equiv \frac{df}{dy} = \frac{2\Omega \cos \phi}{a} \quad (1.2)$$

and the real payload is not f but its meridional gradient β . A constant f gives a fluid with no sense of latitude — displace a parcel and nothing objects. A *varying* f gives it a restoring tendency, and therefore waves. Nearly everything distinctive downstream — Rossby waves, westward phase propagation, the jets, the storm tracks — traces to $\beta \neq 0$. One honest caveat: we drop the horizontal component of Ω (the *traditional approximation*); it is justified for shallow, largely horizontal motion and fails for deep convection and in the tropics.

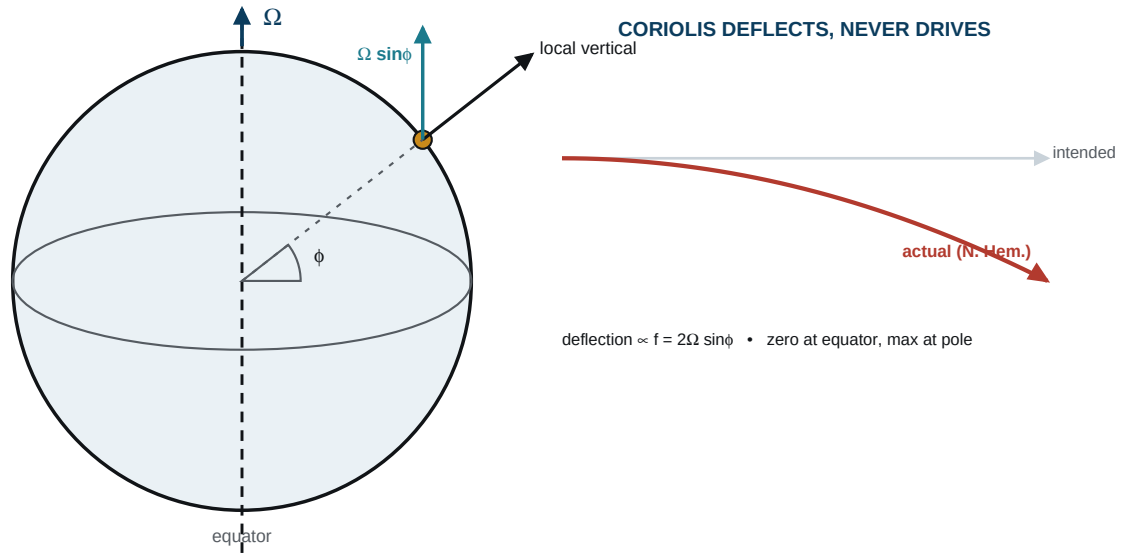


Fig. 1 — The locally-vertical component of Ω is $\Omega \sin \phi$, so $f = 2\Omega \sin \phi$ vanishes at the equator and maxes at the pole. Right: the deflection this produces is perpendicular to the motion — it turns the path, never speeds it.

§1 — KEEP THESE

Coriolis is a frame artefact, always $\perp$ to velocity $\rightarrow$ **it does no work.**

Centrifugal force is already inside “gravity” — the oblate Earth swallowed it.

$f = 2\Omega \sin \phi$: zero at equator, maximum at pole.

$\beta = df/dy$ is the prize — the gradient of rotation is what makes waves possible.

2 · Scale Analysis and Geostrophy — Structure vs. Evolution

Scale analysis is the most underrated tool in the subject. Assign every term in the horizontal momentum equation the magnitude it actually has for the motion of interest, compare, and discard what is small. For synoptic mid-latitude flow — length $L \approx 10^6$ m, speed $U \approx 10$ m s⁻¹, $f \approx 10^{-4}$ s⁻¹ — two terms stand an order of magnitude above the rest: the pressure-gradient force and Coriolis. What survives at leading order is geostrophic balance — the exact standoff between the pressure-gradient force and Coriolis, in which the wind ends up blowing along the isobars rather than across them:

$$fu_g = -\frac{1}{\rho} \frac{\partial p}{\partial y}, \quad fv_g = \frac{1}{\rho} \frac{\partial p}{\partial x} \quad (2.1)$$

This is why a weather chart is readable at all: wind runs along the isobars, not across them, and the spacing sets the speed. Everything you teach about Buys Ballot is this balance; the $\sim 20^\circ$ surface backing is friction climbing back into significance. The number that says when to trust it is the Rossby number — the ratio of the acceleration term to Coriolis:

$$Ro = \frac{U}{fL} \approx 0.1 \text{ (synoptic)} \quad (2.2)$$

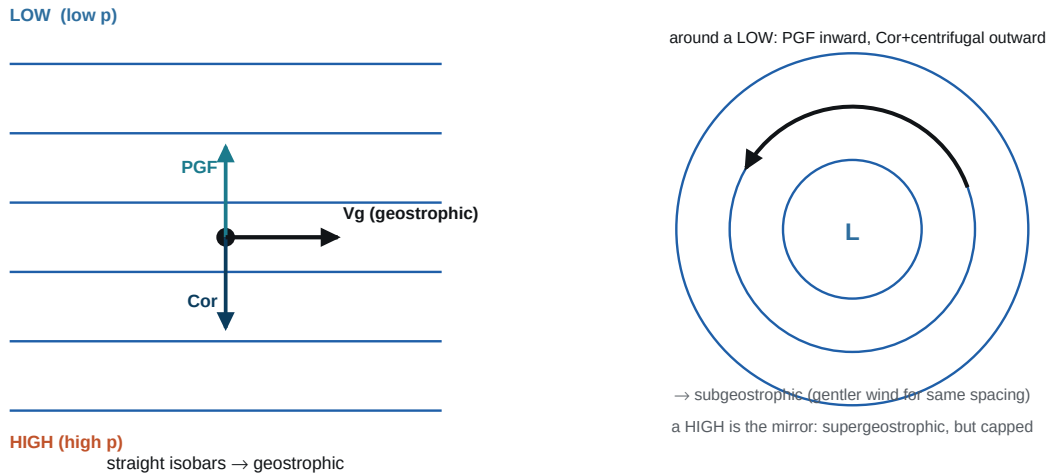


Fig. 2 — Left: along straight isobars, PGF balances Coriolis and the geostrophic wind blows parallel to them. Right: add curvature and you get gradient-wind balance — flow around a low is subgeostrophic, around a high supergeostrophic.

Watch the extremes, where the concept earns its keep. Shrink L to a tornado and $Ro \rightarrow$ thousands: rotation is irrelevant, pressure gradient balances centrifugal acceleration directly (cyclotrophic balance), and the vortex may spin either way — the bathtub myth dies in one line. Go equatorward and $f \rightarrow 0$, so Ro blows up again and geostrophy fails for the opposite reason. Tropical dynamics is genuinely different physics, not the same physics with smaller numbers — which is exactly why Florida at 1019 mb is not “high = fair.”

Now the hinge of the whole monograph. Geostrophic balance is *diagnostic*, not *prognostic* — there is no time derivative in (2.1). It tells you the wind now, given the pressure now; it cannot tell you what happens next. A perfectly geostrophic atmosphere is dead: flow parallel to isobars forever, no convergence, no vertical motion, no weather. Everything that *is* weather lives in the small residual scale analysis threw away — the ageostrophic wind — the small part of the wind that does *not* simply run along the isobars but cuts across them — typically $\sim 10\%$ of the total, and the 10% that does all the work. Ageostrophic flow crosses isobars; crossing isobars converges mass; convergence forces vertical motion; vertical motion makes cloud and deepens lows. The leading-order balance gives the *structure*; the small departure gives the *evolution*. Quasi-geostrophic theory is exactly the formalism for keeping geostrophy as the backbone while tracking that ageostrophic correction. On the bridge this is the barometer’s real lesson: the pressure pattern tells you the wind you *have*, but it is the *change* — the glass falling, the isobars packing in ahead — that tells you the wind you are about to *get*. The chart is the structure; the tendency is the forecast.

TWO REFINEMENTS WORTH CARRYING

Gradient wind: for the same spacing, a low is subgeostrophic and a high supergeostrophic — the curvature term adds to Coriolis in a low, opposes it in a high. That is why highs have gentle gradients and gentle winds. **The asymmetry that matters at sea:** the balance sets a hard ceiling on how tight a high can be, but *none* on a low. Lows can be arbitrarily intense; the strongest winds on Earth always belong to lows.

Regime	Ro	Balance	What dominates
Synoptic mid-latitude	~ 0.1	Geostrophic / QG	rotation — chart is readable
Gradient / curved flow	$\sim 0.1\text{--}0.3$	Gradient wind	+ centripetal term
Tornado, dust devil	$\gg 1$	Cyclotrophic	pressure vs. centrifugal; f irrelevant
Tropics ($f \rightarrow 0$)	$\gg 1$	(none simple)	convection & divergence, not isobars

§2 — KEEP THESE

Leading order = **structure** (geostrophy). Small residual = **evolution** (ageostrophic).

Ro = **U/fL** tells you which world you're in.

The ageostrophic ~10% crosses isobars → converges mass → makes the weather.

3 · Vorticity and Potential Vorticity — A Quantity the Atmosphere Conserves

Until now we have only *described* the flow. Vorticity gives us something the atmosphere *conserves*, and a conserved quantity is what lets us predict rather than diagnose. Vorticity is local spin — the curl of the velocity; for large-scale work we take its vertical component ζ , counterclockwise positive (cyclonic in the N. hemisphere). It has two physically distinct parts: relative vorticity ζ (the spin on your chart) and planetary vorticity f (every parcel at rest is already spinning with the planet). Their sum is absolute vorticity, the spin an observer in space measures — the quantity physics actually cares about.

The first payoff is deep. Because f varies with latitude, a parcel that merely moves north or south changes its planetary vorticity for free. If absolute vorticity is conserved, moving poleward ($f\uparrow$) forces $\zeta\downarrow$ (anticyclonic curvature); moving equatorward ($f\downarrow$) forces $\zeta\uparrow$ (cyclonic). That single sentence is the entire Rossby-wave mechanism, built out in §4.

Absolute vorticity is conserved only for horizontal, non-divergent flow. Real columns stretch and squash, and stretching changes spin (the skater's arms). So we use potential vorticity. In its barotropic form,

$$q = \frac{\zeta + f}{H} = \text{const following the motion} \quad (3.1)$$

Stretch a column ($H\uparrow$) and its absolute vorticity must rise — it spins up cyclonically; squash it and it spins down. You can watch this over topography: westerlies climbing the Rockies are compressed ($\zeta\downarrow$, turning anticyclonically, swinging equatorward), then stretched in the lee ($\zeta\uparrow$, cyclonic), overshooting and setting up a standing wave train — which is why there is a climatological trough over eastern North America.

The full weapon is Rossby–Ertel PV, using potential-temperature surfaces (isentropes) as the layer, with stability playing the role depth played:

$$q = \frac{(\vec{\zeta} + f) \cdot \nabla \theta}{\rho} = -g(\zeta_\theta + f) \frac{\partial \theta}{\partial p} \quad (3.2)$$

Two properties make it extraordinary. First, it is **materially conserved** for frictionless, adiabatic flow, so PV is a tracer — a filament of high-PV stratospheric air drawn down into the troposphere can be followed like dye. Second, it is **invertible**: given the PV field plus a balance condition and boundaries, you can recover the entire balanced state — winds, temperatures, heights, all of it. PV is not one diagnostic among many; it contains the whole flow, which is why modern dynamics is written in its language. Practically, the tropopause is essentially a PV surface: a fold that brings high-PV air down over a low-level baroclinic zone is the classic bomb-cyclogenesis setup — the vertical coupling of those two anomalies is the engine. You already read this on a satellite loop without naming it: the sharp dry slot curling into a deepening low is high-PV stratospheric air folding down, the tracer made visible. When it drops over a surface front the two lock together and the low bombs — watching that intrusion is watching potential vorticity do its work.

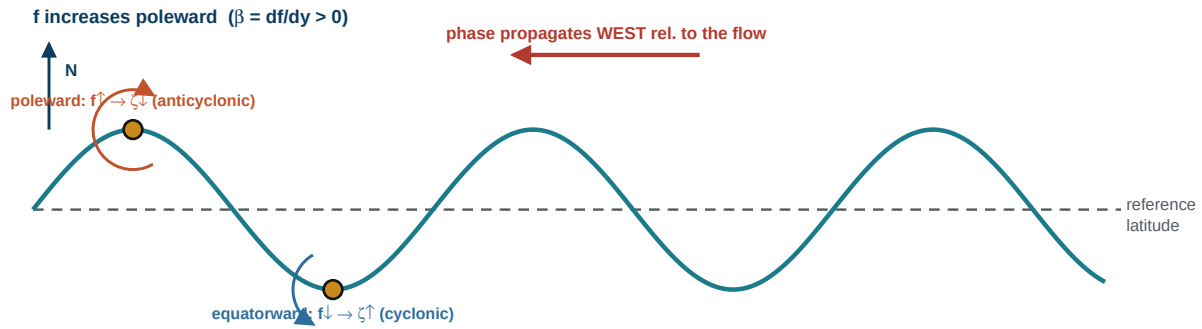


Fig. 3 — The β mechanism. Displaced parcels conserving absolute vorticity acquire anticyclonic spin when pushed poleward ($f\uparrow$) and cyclonic when pushed equatorward ($f\downarrow$); the induced flow shifts the pattern westward. This is the restoring force behind every Rossby wave.

§3 — KEEP THESE

Absolute vorticity = $\zeta + f$. Move poleward, $f\uparrow \rightarrow \zeta\downarrow$; move equatorward, $f\downarrow \rightarrow \zeta\uparrow$.

PV is conserved (adiabatic, frictionless) $\rightarrow$ it is a tracer you can follow.

PV is invertible $\rightarrow$ one scalar field contains the entire balanced flow.

Tropopause = a PV surface; a PV fold over a baroclinic zone = rapid cyclogenesis.

4 · Rossby Waves — The Heart of the Circulation

Take a line of parcels conserving absolute vorticity and displace one poleward: $f\uparrow$ forces $\zeta\downarrow$ (anticyclonic), which pushes its neighbours so as to propagate the disturbance sideways — and always so the pattern moves *westward* relative to the air. β is the restoring force; the wave is the response. For a barotropic atmosphere with mean westerly U , the dispersion relation and phase speed are

$$\omega = Uk - \frac{\beta k}{k^2 + l^2} \quad \Rightarrow \quad c = \frac{\omega}{k} = U - \frac{\beta}{k^2 + l^2} \quad (4.1)$$

Read what it says. The intrinsic phase speed — relative to the air — is $-\beta/(k^2 + l^2)$: *always* westward; the minus sign is β at work. But the wave sits in a westerly current carrying it east, so the observed speed is the sum, and which wins depends on wavelength. Short waves (large k) have a tiny westward intrinsic speed — the flow wins, and they run east at nearly the steering wind (your daily troughs). Long waves (small k) have a large westward speed that nearly cancels the advection — they crawl, quasi-stationary. At the *stationary wavenumber* the two exactly balance and the wave stands still over a continent; push past it and a very long wave *retrogrades*, moving west across the map.

$$K_s = \sqrt{\frac{\beta}{U}} \quad (\text{stationary}) \quad L_d = \frac{NH}{f} \quad (\text{Rossby radius}) \quad (4.2)$$

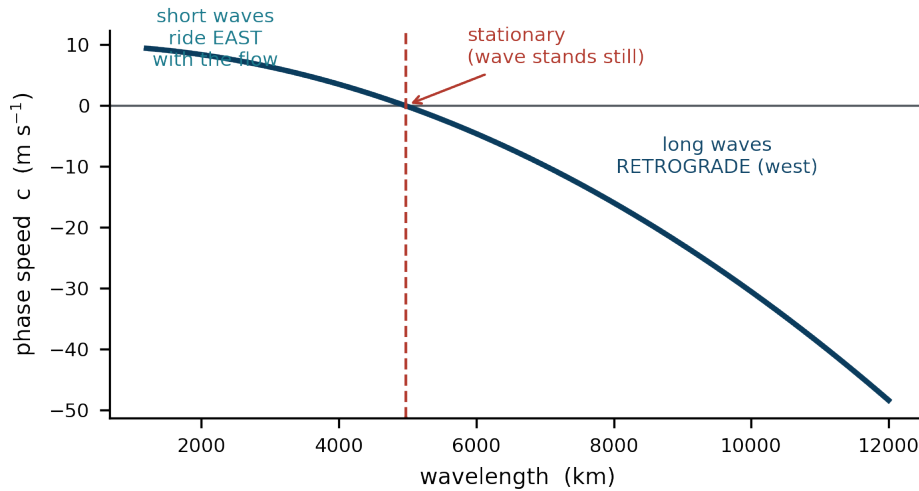


Fig. 4 — Phase speed vs. wavelength from (4.1). Short waves ride east with the flow; a single stationary wavelength stands still; long waves retrograde westward. Earth's Ω , radius and winds put the stationary scale at wavenumbers $\sim 3\text{--}6$ — which is why so few long waves circle each hemisphere.

Why only three to six long waves? Two reasons converge: geometrically the wave must fit an integer number of times around a latitude circle, and dynamically the stationary wavenumber falls in that low range for realistic U and β . The long waves set the stage; shorter waves run through them eastward as the actors.

Now the concept that separates those who have had this course from those who have not: **phase** velocity vs. **group** velocity. The crest speed is c ; the speed at which *energy* travels is the group velocity, obtained by differentiating (4.1):

$$c_{gx} = \frac{\partial \omega}{\partial k} = U + \frac{\beta(k^2 - l^2)}{(k^2 + l^2)^2} \quad (4.3)$$

For Rossby waves the two genuinely differ and are often opposite: even when individual crests move west, the energy of a packet moves *east* relative to the flow. The operational name is **downstream development** — energy radiates ahead and builds a *new* trough downstream, which builds another, so weather propagates faster and farther than any single trough. A forecaster tracking only features is caught out every time; this is the difference between a good medium-range forecast and a blown one.

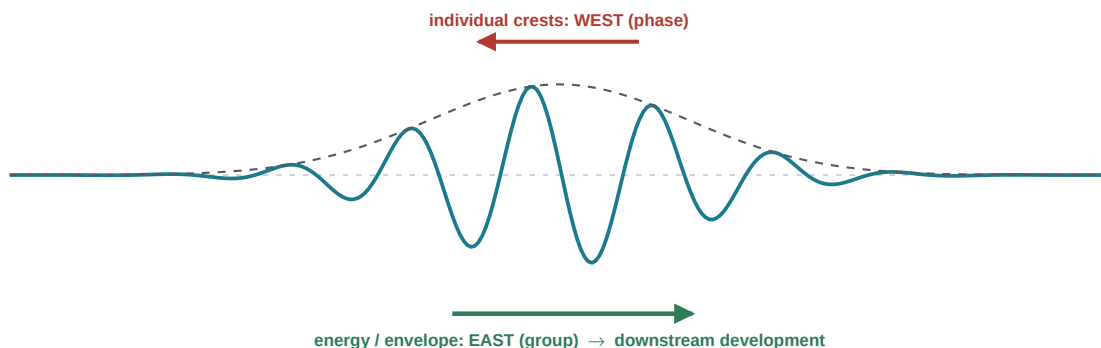


Fig. 5 — A wave packet: the carrier crests move west (phase) while the envelope — the energy — moves east (group). Downstream development is this picture in the atmosphere.

Two closers. The **Rossby radius of deformation** $L_d = NH/f$ is the scale at which rotation and stratification trade off; it is a few thousand km in the troposphere and essentially sets the size of synoptic systems and the preferred

scale of baroclinic growth (§6). And Rossby waves can propagate **vertically**, but only under the Charney–Drazin condition — the mean westerly must be positive but below a critical value. In winter the stratospheric winds sit in that window, planetary waves climb, deposit momentum, and can trigger a sudden stratospheric warming; in summer the winds are easterly and the waves are trapped. That is why winter and summer stratospheres are utterly different places, and why the stratosphere now buys weeks of extended-range skill. These are the features you plan weeks around, not hours. The long waves are the semi-permanent lows and ridges — the Icelandic and Aleutian lows, the ridge that hands you a week of fair weather — and when one locks in place (a block) the weather it carries parks over your route and will not move. Downstream development is why the gale that finds you on day three is not on today’s chart yet.

§4 — KEEP THESE

$c = U - \beta / (k^2 + l^2)$: intrinsic propagation is *always westward*.

Short waves → east with the flow; long waves → stationary or retrograde. Only ~3–6 fit a hemisphere.

Phase ≠ group. Crests go west; energy goes east → **downstream development**.

Charney–Drazin: waves climb into the stratosphere only through a westerly window → winter SSWs.

5 · Thermal Wind and the Two Jets

Combine geostrophic and hydrostatic balance and you get thermal wind — not a statement about wind, but about how wind *changes with height*, tied directly to the horizontal temperature gradient:

$$\frac{\partial u_g}{\partial z} = - \frac{g}{fT} \frac{\partial T}{\partial y} \quad (5.1)$$

Wherever there is a horizontal temperature contrast, the wind must change with altitude. The atmosphere is warm at the equator and cold at the pole through the whole troposphere, so (5.1) says the westerly must strengthen with height all the way up — integrate the shear over ~10 km and you arrive at jet-stream speeds. The jet is not a surface feature pushed aloft; it is the integrated signature of the pole-to-equator temperature difference. And because the gradient concentrates into narrow baroclinic zones and fronts, the shear — and the wind maximum aloft — concentrates above them. That is the polar-front jet, sitting over the strongest low-level temperature contrast, collocated with the storm track that shares the same gradient.

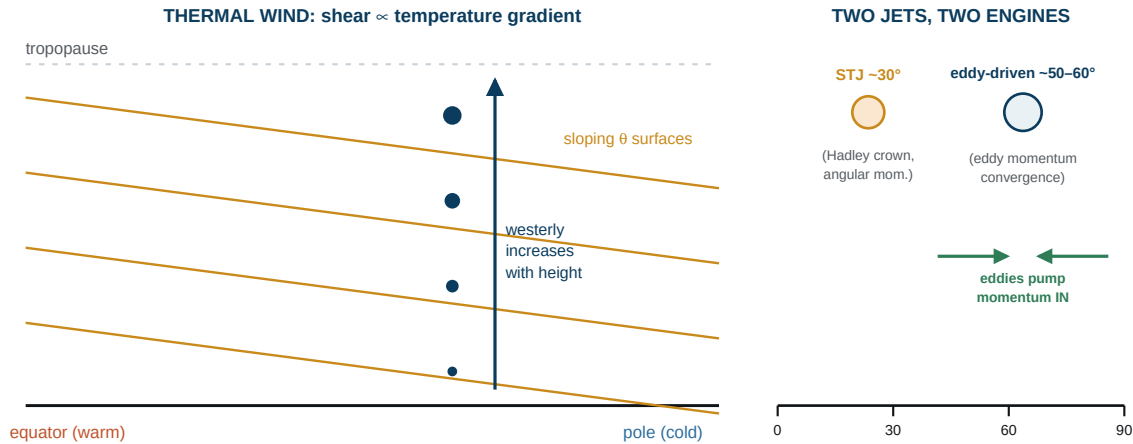


Fig. 6 — Left: sloping θ surfaces mean the westerly grows with height (thermal wind). Right: two jets from two mechanisms — the subtropical jet at $\sim 30^\circ$ (Hadley's angular-momentum crown) and the eddy-driven jet at $\sim 50\text{--}60^\circ$ (built by eddy momentum-flux convergence).

There are two jets, with completely different parents — a distinction that is constantly blurred and is management-level material. The **subtropical jet** comes from angular momentum: air moving poleward aloft in the Hadley cell conserves angular momentum, so as it nears the axis it spins up into a strong westerly, crowning the cell near 30° (derived in §7). It is smooth, zonal, thermally driven. The **polar-front (eddy-driven) jet** is not driven by the mean overturning at all — it is built, maintained, and located by the eddies. When Rossby waves propagate away from their mid-latitude source and break elsewhere, they carry *westerly* momentum back *into* the source latitude: the eddies flux momentum **up-gradient**, sharpening the wind maximum instead of smearing it. The rigorous accounting is the Eliassen–Palm flux — its divergence measures what the waves do to the mean flow:

$$\frac{\partial \bar{u}}{\partial t} = \frac{1}{\rho_0 a \cos \varphi} \nabla \cdot \vec{F} + \dots \quad \vec{F} = (F_\varphi, F_z) \quad (5.2)$$

Where the EP flux converges, the waves deposit westerly momentum and accelerate the mean flow; where it diverges, they remove it. That same equatorward wave propagation aloft is what drives the Ferrel cell backward in §8.

Two practical closers. **Jet streaks** — wind maxima embedded in the jet — force four-quadrant transverse ageostrophic circulations: rising motion under the right-entrance and left-exit quadrants, sinking under the others. Surface cyclones preferentially deepen beneath the rising quadrants, where upper divergence evacuates mass. And seasonally the jets strengthen and shift equatorward in winter, when the temperature gradient is steepest — thermal wind again, directly. For the mariner the lesson is where to expect trouble: the jet sits above the sharpest temperature contrast, which is the polar front, which is the storm track. Cross it on a North Atlantic passage and you are crossing the assembly line where the depressions are built — the jet aloft is the surveyor's mark for the worst of the surface weather below.

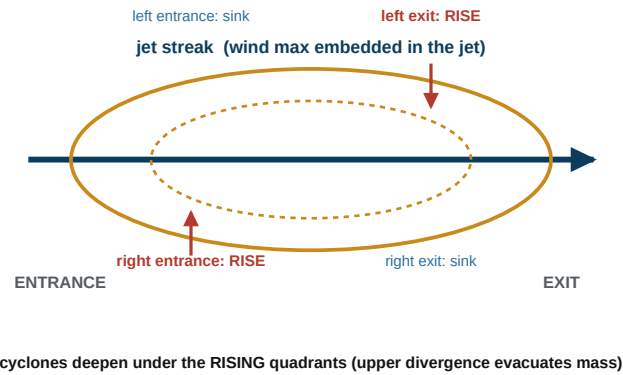


Fig. 7 — The four-quadrant circulation of a jet streak. Cyclogenesis is favoured under the right-entrance and left-exit (rising) quadrants — an everyday forecasting tool that falls straight out of the ageostrophic response.

	Subtropical jet	Eddy-driven (polar-front) jet
Origin	angular-momentum conservation	eddy momentum-flux convergence
Latitude	~30° (Hadley terminus)	~50–60° (baroclinic zone)
Character	smooth, zonal, steady	meanders, pulses with storms
Driven by	the mean Hadley overturning	the eddies themselves (up-gradient)
Tied to	subtropical dry zones	the storm track & annular modes

§5 — KEEP THESE

Thermal wind: vertical shear $\propto$ horizontal temperature gradient $\rightarrow$ the westerlies build with height.
Subtropical jet = angular momentum (Hadley crown, ~30°). **Eddy-driven jet = eddy momentum flux** (~50–60°).
Eddies flux momentum **up-gradient** — they build the jet, not smear it (EP-flux convergence).
Jet-streak rising quadrants (right-entrance, left-exit) = favoured cyclogenesis.

6 · Baroclinic Instability — The Engine Room

The balanced state of §5 is quiet; nothing in it makes a storm. Baroclinic instability answers why the atmosphere will not sit in that state. The pole-to-equator gradient is *loaded* — cold dense air poleward, warm light air equatorward, the centre of mass higher than it needs to be, a reservoir of available potential energy. Let the cold air slide under and the warm ride over, and the centre of mass drops and that energy becomes motion. The whole mid-latitude machine is that conversion, running continuously.

But it is locked: stratification resists lifting warm air over cold, and rotation resists through the balance constraints. The lock opens only if parcels move along paths *shallower* than the sloping isentropes — warm poleward-and-up, cold equatorward-and-down, within the wedge. That is the signature motion of every extratropical cyclone: the warm conveyor rising poleward, the cold air sinking behind. The storm is the release, organized.

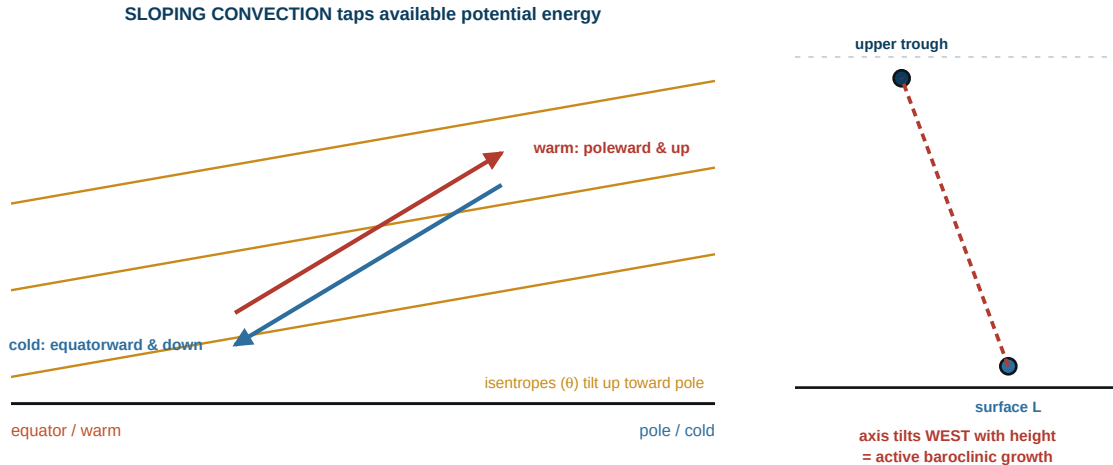


Fig. 8 — Left: sloping convection within the isentropic wedge taps available potential energy — warm poleward and up, cold equatorward and down. Right: the growing wave tilts *westward with height*, the fingerprint of active baroclinic conversion; when it becomes vertically stacked, growth stops and the system occludes.

Two idealized models bracket the problem. The **Eady model** — uniform shear between two rigid boundaries, no β — gives a most-unstable wavelength of a few thousand km (the synoptic scale, near L_d) and a growth rate that doubles the disturbance in about a day, both matching observed cyclogenesis:

$$\sigma_{max} \approx 0.31 \frac{f}{N} \left| \frac{\partial u}{\partial z} \right| \quad (\text{Eady growth rate}) \quad (6.1)$$

Its solution is two edge waves, one on each boundary, phase-locked and amplifying, tilting westward with height — the rigorous version of “upper support sits upstream of the surface low.” The **Charney model** restores β and drops the lid, tying the vertical scale to stratification and β and connecting growth to the mean-state PV. The general necessary condition (Charney–Stern) is that the PV gradient must *change sign* somewhere — a boundary temperature gradient acting like a PV anomaly, the instability being the interaction of boundary and interior PV. Everything ties back to PV.

The instability does not grow forever. It amplifies, saturates as it rearranges the mean state, fronts sharpen, the occlusion forms — and then the wave breaks. *How* it breaks sets what it does to the jet: anticyclonic breaking sheds wave activity equatorward and nudges the jet poleward; cyclonic breaking wraps into a cutoff and shifts the jet equatorward. These two life cycles are how the storm track pushes the eddy-driven jet around, and their balance sets the mean jet latitude — the individual storm is one realization; the climatological jet is the statistical residue of thousands.

Here is the closure that makes the system self-consistent. In releasing its energy the eddy transports heat poleward and so *reduces* the very gradient that fed it — a negative feedback. Left alone it would erase the gradient and shut off; solar heating continuously rebuilds it. The mid-latitudes therefore sit slightly supercritical — always a little more unstable than neutral, the eddies never quite catching the heating — which is why there is always weather at 45° and rarely any at the equator or pole. The eddies are not disturbances on the circulation; in mid-latitudes they *are* it. In plain terms this is the extratropical cyclone itself — the wave that forms on a front, deepens in a day, and throws the gale across your track. The practical tell is the westward tilt: while the upper trough still lies upstream of the surface low, the system is still drawing energy and still deepening — do not relax until it stacks vertically and occludes.

§6 — KEEP THESE

The pole-to-equator gradient is stored energy; **sloping convection** along the isentropic wedge releases it. **Eady**: ~1-day doubling, ~few-thousand-km scale — real cyclogenesis. **Westward tilt with height = active growth**.

Charney–Stern: instability needs the PV gradient to change sign somewhere.

Eddies flux heat poleward → erode their own gradient (negative feedback); heating rebuilds it → permanent storm track.

7 · The Hadley Cell — What Sets Its Edge

Drop to the tropics, where the eddies quiet and a genuine mean overturning takes charge. The sun heats most strongly near the latitude of maximum surface heating; deep convection there builds the Intertropical Convergence Zone (ITCZ). Air rises narrow and wet, spreads poleward aloft, cools and dries, sinks in a broad gentle descent near 30°, and returns as the trade winds. The sinking branch is where the world's great subtropical deserts and highs sit — the Sahara, the Kalahari, the Australian interior, the horse latitudes at sea. You can teach the entire distribution of subtropical desert from one descending branch.

The intellectual core is angular momentum. A parcel rising at the equator and moving poleward aloft feels little friction, so it conserves angular momentum about the axis:

$$M = \Omega a^2 \cos^2 \varphi + u a \cos \varphi = \text{const} \implies u(\varphi) = \Omega a \frac{\sin^2 \varphi}{\cos \varphi} \quad (7.1)$$

As the parcel moves poleward its distance from the axis shrinks, so its west-to-east velocity must climb — and fast. Starting from rest at the equator, (7.1) gives an implied westerly of order 130 m s^{-1} by 30° and hundreds of m s^{-1} by 45° — physically impossible, and that impossibility is the point. Long before mid-latitudes the spun-up westerly becomes baroclinically unstable and breaks into eddies; the cell *must* terminate. Where it terminates, that accumulated westerly *is* the subtropical jet. The jet is not bolted onto the Hadley cell — it is the cell's poleward edge. So Hadley's ~30° extent is set not by where the heating stops but by where angular-momentum-conserving flow goes unstable.

poleward aloft: radius shrinks → u speeds up

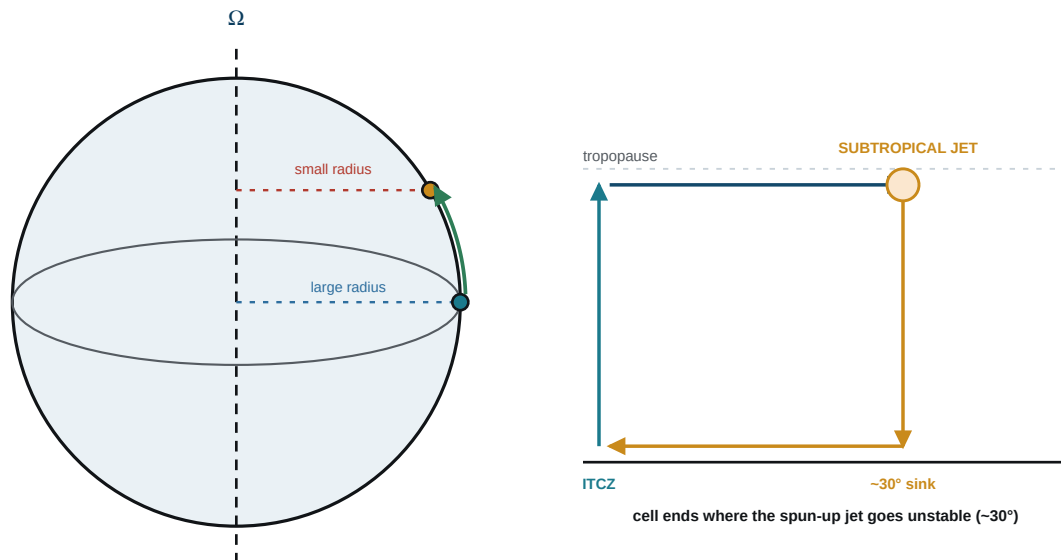


Fig. 9 — Left: poleward-moving air aloft nears the rotation axis, so conserving M forces u to increase. Right: the overturning loop — rising at the ITCZ, poleward aloft, sinking near 30°, trade-wind return — crowned by the subtropical jet where the spun-up flow goes unstable.

The landmark theory is **Held–Hou**: an axisymmetric cell that conserves angular momentum aloft and comes to thermal-wind balance, predicting the cell’s width, the jet strength, and the poleward heat transport as functions of Ω , g , the atmosphere’s depth, and the heating. Its structure is right — a faster-rotating planet has a narrower cell, which is why fast-spinning Jupiter shows many narrow bands rather than one cell per hemisphere. But show students its limits, because they are instructive: it has no eddies (which do most of the real work), so the real subtropical jet is weaker than its angular-momentum limit; it predicts a threshold-then-abrupt onset the real seasonal cycle does not follow; and it misses the strong seasonal asymmetry.

That asymmetry matters. Except briefly at the equinoxes the cell is not symmetric: with the sun over the summer hemisphere, the rising branch crosses into it and one giant winter cell dominates, far stronger than the annual mean suggests. That cross-equatorial flow is the dynamical heart of the **monsoon** — a seasonally migrating, longitudinally confined Hadley circulation with its ascending branch parked over the heated continent. And a contemporary thread your students will meet: the Hadley cells are **widening**, pushing the subtropical dry zones and highs poleward — shifting the storm track off the subtropical margins (the Mediterranean, southern Australia, the American Southwest). It is the general circulation moving under a changing climate, traceable to the same angular-momentum and stability arguments. This is the wind climatology under every long ocean passage: the steady trades to carry you, the horse latitudes and their calms to avoid, and the doldrums — the ITCZ — to punch through as narrowly as you can. The cell you can never see on any single chart set those belts centuries before there were satellites to confirm them.

§7 — KEEP THESE

Hadley = the one truly thermally-**direct** cell: rise at ITCZ, sink at $\sim 30^\circ$, trades return.

Its sinking branch makes the subtropical deserts and highs.

Angular momentum sets the edge: poleward flow spins up until it goes unstable near 30° — and that spun-up flow is the **subtropical jet**.

Held–Hou predicts width $\propto$ rotation; the real cell is eddy-limited, seasonally cross-equatorial (monsoon), and widening.

8 · The Ferrel Cell and the Residual Circulation

The zonally-averaged meridional circulation shows three cells per hemisphere. Hadley is direct (warm up, cold down). The polar cell is weak and broadly direct. Between them the **Ferrel cell runs backward**: in the zonal mean it rises near 60° (cool) and sinks near 30° (warm) — cold air up, warm air down, a thermally *indirect* circulation that *consumes* potential energy. It appears to run uphill, which looks impossible. It is not: it is driven by the eddies.

The resolution is the intellectual summit of the subject: in mid-latitudes the zonal-mean cell is a *residual* left after the eddies have done the real work. Take momentum first — the cleaner driver. The eddies converge westerly momentum into the jet (§5). A sustained zonal force in a rotating fluid must be balanced by a meridional flow across it (a Coriolis torque), so the eddy momentum convergence aloft drives a poleward branch aloft, which by continuity demands rising at 60° , sinking at 30° , and an equatorward return below — exactly the sense of the Ferrel cell. It runs backward because it serves the *momentum* budget, not the heat budget. It is the mechanical residual of the eddy stresses — driven by the storms embedded within it, not by the cells on either side. Every “three meshing gears” cartoon inverts cause and effect.

For heat the same near-cancellation occurs: the eddies carry a large poleward heat flux, and the mean Ferrel cell sets up partly to oppose it; the small difference is the net transport the climate needs. The formalism that removes the paradox is the **Transformed Eulerian Mean** — the ordinary zonal average is the wrong average

because it splits a transport the eddies and mean flow do together and largely cancel. Fold the eddy heat flux into the mean and you get the **residual circulation**:

$$\bar{v}^* = \bar{v} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \left(\rho_0 \overline{v'\theta'} \right), \quad \bar{w}^* = \bar{w} + \frac{1}{a \cos \varphi} \frac{\partial}{\partial \varphi} \left(\frac{\cos \varphi \overline{v'\theta'}}{\partial \theta / \partial z} \right) \quad (8.1)$$

In the residual frame the spurious indirect Ferrel cell largely disappears: what remains is a single, sensible, nearly direct overturning from equator to pole — what air parcels, and the ozone and moisture they carry, actually follow. Stratospheric transport is described this way for exactly this reason: the **Brewer–Dobson circulation** — the slow poleward drift of ozone — is a residual circulation driven by Rossby waves propagating up (Charney–Drazin, §4), breaking, and depositing momentum, precisely the Ferrel mechanism one layer up. **Wave breaking drives mean meridional circulation** — that single sentence is most of the general circulation of the middle atmosphere. The polar cell, finally, is the weak, broadly direct run: cold air sinking at the pole, partly radiatively and partly eddy-driven at its equatorward edge. The plain-chart version: this is the belt of prevailing westerlies and the endless procession of lows riding them eastward across the mid-latitudes — the reason an eastbound North Atlantic or Pacific crossing runs with the weather at its back and a westbound one fights it the whole way.

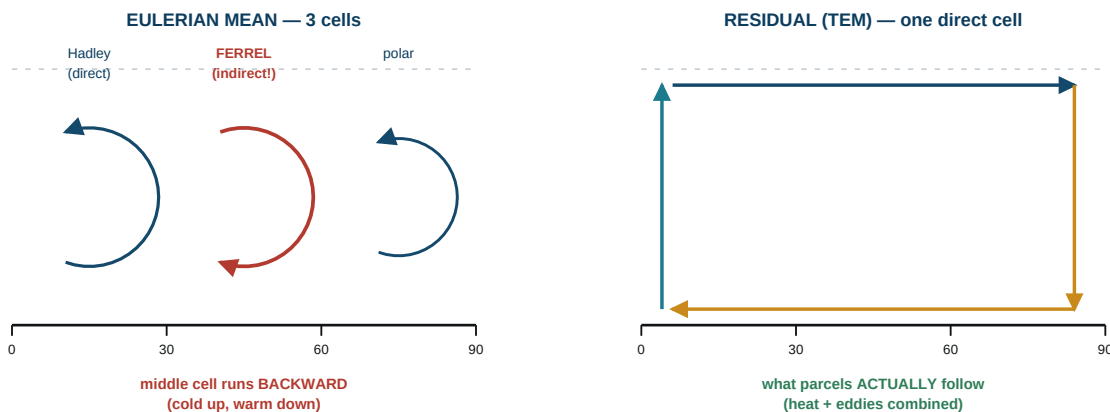


Fig. 10 — Left: the Eulerian mean shows a backward (indirect) Ferrel cell between two direct cells. Right: in the Transformed Eulerian Mean, the eddy heat flux folds in and the residual circulation is a single direct cell — what parcels actually follow.

§8 — KEEP THESE

The Ferrel cell is thermally **indirect** — it runs backward because it balances **eddy momentum**, not heat. It is **eddy-driven**, a residual of the storm stresses — not a gear turned by Hadley and polar cells. The **residual (TEM) circulation** removes the paradox: one direct cell, what parcels truly follow. Same physics aloft = the **Brewer–Dobson** circulation. Wave breaking drives mean circulation.

9 · Energetics and Variability — Closing the Loop

Account for the energy and the parts prove they fit. Edward Lorenz — the same Lorenz of chaos, and no accident — gave the scheme: four reservoirs and a cycle between them. Energy enters as *zonal available potential energy* P_Z , the pole-to-equator temperature gradient (“available” because only the part stored in the horizontal gradient can be tapped by rearrangement). Solar heating continuously generates it. The cycle then runs:

$$\text{sun} \rightarrow P_Z \rightarrow P_E \rightarrow K_E \rightarrow K_Z \rightarrow \text{friction} \quad (9.1)$$

P_Z converts to *eddy* APE as baroclinic eddies grow on the gradient; P_E to *eddy kinetic* energy as warm air rises and cold sinks within the storm (the baroclinic conversion — the main pathway of the extratropical atmosphere); K_E to *zonal kinetic* energy as the eddies feed the jet; and K_Z is dissipated by friction. The certifying fact: the dominant path runs *through the eddies* — not straight from the zonal gradient to the zonal jet. The working fluid of the mid-latitude engine is the eddy field. Note too the sign of the last step — eddies giving kinetic energy *to* the mean flow, the opposite of ordinary turbulence, where the mean feeds the eddies. Here turbulence organizes the mean flow. That is why the circulation is so stable in structure while so chaotic in detail — Lorenz’s other discovery, the same physics.

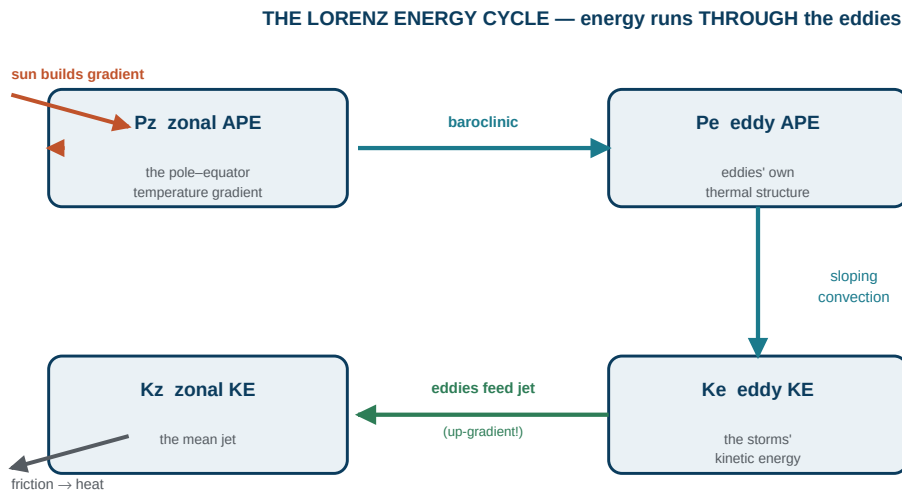


Fig. 11 — The Lorenz energy cycle. The sun builds P_Z ; energy flows through the eddies ($P_E \rightarrow K_E$) before reaching the mean jet (K_Z); friction closes the budget. The eddies are the drive train, not noise on it.

The assembled system never sits still — it swings through preferred modes. The **annular modes** (Northern/Southern; the Arctic Oscillation regionally, close kin to the NAO) are the eddy-driven jet wobbling north and south, or strengthening and weakening — a mass see-saw between the polar cap and mid-latitudes. Positive phase: a strong, poleward, tight jet, cold air bottled over the pole (mild wet northern-European winters). Negative phase: a weak, buckled, equatorward jet, cold air spilling south (hard winters, cold outbreaks). It is exactly the §6 life cycles playing out statistically, with a genuine feedback — the jet organizes the eddies that keep driving it — so anomalies persist for weeks. The **NAO** is the Atlantic piece, the single strongest control on European winter.

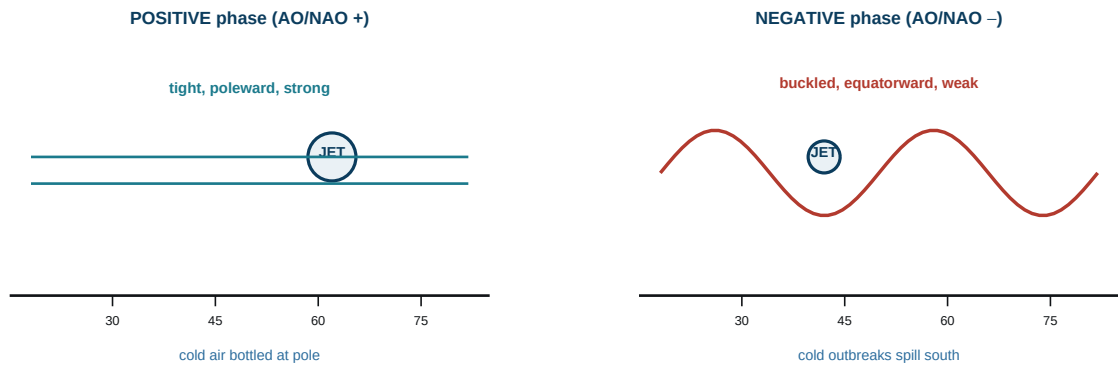


Fig. 12 — The annular mode. Positive: tight, poleward, strong jet, cold air contained. Negative: buckled, equatorward, weak jet, cold outbreaks spill south. The same baroclinic life cycles of §6, viewed statistically.

Coupling down from above: planetary waves can shatter the stratospheric polar vortex (a sudden stratospheric warming), and the signal propagates *downward* — weeks later the surface annular mode tends negative and cold outbreaks follow, now a real source of extended-range skill. The tropical modes complete the picture: **ENSO** reorganizes tropical convection and the Walker circulation and teleconnects to mid-latitudes by launching Rossby wave trains (§4) that bend the jet continents away; the **MJO** is the 30–60-day intraseasonal envelope; the **QBO** is the stratospheric equatorial wind reversal driven by vertically propagating waves depositing momentum — the Ferrel/Brewer–Dobson physics once more. For passage planning on the seasonal scale, this is where it pays off: a negative-NAO winter drives the storm track and the cold south; an El Niño reshuffles the tropics and the teleconnected jet worldwide. Knowing which mode the ocean is in is knowing what kind of season you are sailing into.

Mode	Timescale	What it is	Why it matters
AO / NAM	weeks–months	eddy-driven jet north–south	cold-air containment vs. outbreaks
NAO	weeks–seasons	Atlantic jet / storm track shift	dominant control on European winter
SSW	days, weeks lag	polar-vortex breakdown	downward coupling → surface cold
ENSO	1–7 years	coupled tropical Pacific	global teleconnections via wave trains
MJO	30–60 days	eastward tropical convection	intraseasonal tropics & beyond
QBO	~28 months	stratospheric wind reversal	wave-driven; modulates SSWs, tropics

Step back and the whole subject is one machine. Rotation constrains, and the *gradient* of rotation gives waves. Waves propagate, break, and deposit momentum somewhere other than where they were born, driving mean circulations — the Ferrel cell, Brewer–Dobson, the QBO. Waves flux momentum up-gradient to build jets and carry the heat the mean flow cannot. Energy runs from the sun-built gradient through the eddy field into the jets before friction takes it. Coriolis, Rossby, the jets, Hadley, Ferrel, polar are not six topics but six faces of one system: a thin fluid on a rotating sphere, heated unequally, that cannot overturn simply because rotation will not let it — and so does its heat transport by growing waves. Every large-scale feature we have named is those waves, the flow that makes them, or the flow they leave behind.

§9 — KEEP THESE

Energy path: **sun** → **P_Z** → **P_E** → **K_E** → **K_Z** → **friction** — it runs *through the eddies*.

Eddies give KE **to** the mean flow (negative viscosity) — turbulence organizes the jet.

Annular modes / NAO = the eddy-driven jet wobbling; SSWs couple downward to surface cold.

One law throughout: **waves deposit momentum where they break, and that drives the mean circulation.**

R · Symbols, Constants, and the Governing Balances

Symbol	Meaning	Typical value / relation
Ω	Earth's rotation rate	$7.29 \times 10^{-5} \text{ s}^{-1}$
a	Earth radius	$6.37 \times 10^6 \text{ m}$
f = $2\Omega \sin\phi$	Coriolis parameter	$\sim 10^{-4} \text{ s}^{-1}$ at 45°
$\beta = df/dy$	planetary vorticity gradient	$\sim 1.6 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ at 45°
Ro = U/fL	Rossby number	~ 0.1 synoptic; $>>1$ tornado/tropics
N	buoyancy (Brunt–Väisälä) freq.	$\sim 10^{-2} \text{ s}^{-1}$ troposphere
L_d = NH/f	Rossby radius of deformation	$\sim 1000\text{--}2000 \text{ km}$ mid-latitude
$\zeta + f$	absolute vorticity	conserved (horizontal, non-divergent)
q	potential vorticity	conserved, adiabatic frictionless; invertible
K_s = $\sqrt{(\beta/U)}$	stationary wavenumber	selects the quasi-stationary long waves

The three cells	Sense	Driven by	Signature at the surface
Hadley (0–30°)	direct	heating (thermal)	ITCZ rains; trade winds; subtropical deserts & highs
Ferrel (30–60°)	indirect	eddies (momentum)	the westerlies; the storm track
Polar (60–90°)	weak, direct	radiation + eddies	polar easterlies; weak

SAY THESE UNTIL THEY ARE AUTOMATIC

Why does Coriolis do no work?	It is always perpendicular to velocity — it turns, never speeds.
What matters more, f or β?	β . The gradient of rotation is what makes Rossby waves possible.
Geostrophy: what can it not do?	Predict. It is diagnostic — the ageostrophic 10% makes the weather.
Which way do Rossby waves go?	Intrinsically westward; observed motion = U minus that → short east, long retrograde.
Phase vs. group velocity?	Crests west, energy east → downstream development.
Where does the subtropical jet come from?	Angular-momentum conservation — the Hadley cell's poleward crown at $\sim 30^\circ$.
Where does the polar-front jet come from?	Eddy momentum-flux convergence — the eddies build it, up-gradient.
What unlocks baroclinic energy?	Sloping convection along the isentropic wedge; growth tilts west with height.
Why does the Ferrel cell run backward?	It balances eddy momentum, not heat — it is eddy-driven, a residual.
One law for the middle atmosphere?	Waves deposit momentum where they break, and that drives the mean circulation.

Written from the watch, where the theory has to survive contact with the sea. If you can say the ten lines above without hesitating, you understand the general circulation — not as six topics, but as one turning, heated, wave-making machine.

Fair winds, and mind the gradient of rotation.

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