THE ORB

BODY OF MATHEMATICAL EVIDENCE

Geometry • Field Composition • Invariant Reference • Objective Displacement

A technical foundation for independent mathematical examination

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Scope: mathematical correspondence and derived results, not a completed physical embodiment.

Abstract

This document reconciles the Orb investigation into a sequence that can be checked independently. It begins with the round three-sphere and the Hopf fibration; proceeds to the exact Hopfion solution of vacuum Maxwell theory and the energy changes produced by field superposition; examines finite-scale configurations in the Faddeev–Skyrme model; and derives, for a separately specified two-field interaction, the exact common-translation symmetry and the local relative-displacement stiffness tensor.

A significant external result is included: Guerra, Lamy, and Zemas (2026) prove that the Hopf map is the unique global minimizer, modulo rotations, in Hopf class one for a stated range of the Faddeev–Skyrme functional on the round S^3. In the coefficient convention used here, that range is $\kappa_4/\kappa_2 \geq 2$. This theorem applies to the compact spherical model and is not silently transferred to the Euclidean R^3 model or to the proposed coupled-field model.

The document distinguishes four evidentiary levels: (i) exact identities and derivations; (ii) established results from published mathematics and physics; (iii) consequences of an explicitly proposed model; and (iv) numerical observations that cannot yet be independently reproduced. Unsupported deformation results are excluded. The result is a reviewable mathematical foundation, with unresolved bridges and assumptions made visible.

Central proposition The objective embodiment may change its position while its internal reference relationship remains invariant.

How to read the status of a result

Status	Meaning in this document
Exact identity / derivation	Follows algebraically or variationally from the displayed definitions and assumptions.
Published theorem / exact solution	Established in cited literature for the precise model and hypotheses stated.
Model-dependent result	A rigorous consequence of a proposed functional or ansatz; the model itself is an assumption.
Provisional computation	A prior numerical observation for which the full executable record is not presently available.
Open correspondence	A proposed relationship whose mathematical map or physical realization has not been derived.

Executive finding

The investigation is ready for independent mathematical review as a foundation of correspondences. It is not ready to be presented as a completed physical theory. The strongest results are not the speculative numerical tables; they are the exact geometric invariance of the Hopf projection, the exact Maxwell Hopfion and its conserved momentum ratio, the Derrick scaling balance, the published global-minimality theorem on S^3, and the derived translation/displacement theorem for the proposed coupled-field energy.

At-a-glance: equation, application, result

Question	Governing equation	Application / result	Status
Invariant internal phase	$h(e^{i\theta}z_1,e^{i\theta}z_2)=h(z_1,z_2)$	Move along a Hopf fiber while the projected S^2 point stays fixed.	Exact identity
Field composition	$u_{tot}=u_A+u_B+E_A\cdot E_B+B_A\cdot B_B$	Relative phase and position alter the total energy density through cross terms.	Exact for Maxwell superposition
Finite field scale	$E(L)=aL+b/L$; $L^*=\sqrt{(b/a)}$	Two derivative orders balance along uniform dilation.	Exact one-parameter result
Compact spherical minimizer	$FS\rho(u)\geq FS\rho(h)$, $0<\rho\leq 1$	Hopf map minimizes the stated Hopf-charge-one functional on round S^3.	Published theorem
Common / relative motion	$E(X+a,\Delta)=E(X,\Delta)$; $K=(g_0/2)G+g_2Q$	Common translation preserves body-frame structure; relative shift has a local energy cost in the proposed coupling.	Exact conditional derivation
Objective centroid	$dR_E/dt=c^2P/U$	The center of energy follows total momentum in an isolated relativistic system.	Conservation law

The essential distinction has to be stated in three separate mathematical registers:

1. Hopf-fiber invariance: a common U(1) phase changes a point on the fiber while leaving its S^2 projection fixed.
2. Spatial translation invariance: translating an entire field configuration changes its external position while leaving its body-frame profile and translation-invariant energy unchanged.
3. Objective center-of-energy motion: for a relativistic isolated system, the energy centroid moves according to total momentum; it is not the same mathematical object as the internal fiber coordinate or the body-frame origin.

These are related by analogy and by the organizing question, but they are not one and the same transformation. The present mathematics establishes each relationship in its own setting; the map that would physically identify them remains an open part of the Orb theory.

1. The originating proposition and its mathematical translation

The original proposal is organized around one reference and a multiplicity of relational expressions. The reference is not intended as another ordinary component among the things being related. The objective expression is conceived through the composition, opposition, and variation of relational attributes. The embodiment may change its external position while its internal reference relationship remains invariant.

For mathematical examination, the proposition is separated into four testable questions:

1. Does a geometric construction contain a degree of freedom that changes while a projection remains invariant?
2. Can field composition alter an objective energy distribution through cross terms?
3. Can a nonlinear field energy support a finite-scale configuration, and under what exact conditions is a particular Hopf configuration minimizing?
4. Can a coupled configuration distinguish common translation from relative displacement, with different energetic consequences?

The philosophical language of reference and objective expression remains the organizing interpretation. The equations below describe the mathematical structures that correspond to parts of that interpretation; each equation is judged by its own definitions and consequences.

1.1 Three notions of “center” that must remain distinct

Name	Definition / role	What can change?
Internal fiber coordinate	The U(1) coordinate within a Hopf fiber.	It changes under common phase while the Hopf projection is fixed.
Body-frame reference	The fixed origin used to describe the translated internal profile $n_0(y)$, where $y=x-X$.	It is fixed in body coordinates; the full configuration can translate in external space.
Objective center of energy	$R_E = (1/U)\int x\, u(x,t)d^3x$.	It changes with the energy distribution and evolves according to total momentum.

The theoretical “unchanged reference” is most directly represented in the second row: the body-frame profile is unchanged under common translation. The center of energy, by contrast, moves in external coordinates when the configuration is translated. Confusing these statements would obscure the very distinction the investigation is trying to express.

2. The Orb geometry: S^3, Hopf coordinates, and projection

2.1 The three-sphere

Let $R>0$. The geometric Orb is represented first as a round three-sphere embedded in four-dimensional Euclidean space:

$$S^3(R) = \{(x,y,z,w) \in \mathbb{R}^4 : x^2 + y^2 + z^2 + w^2 = R^2\}.$$

Use Hopf coordinates $\eta\in[0,\pi/2]$ and $\xi,\chi\in\mathbb{R}/(2\pi\mathbb{Z})$:

$$x = R\cos\eta\cos\xi \qquad y = R\cos\eta\sin\xi$$
$$z = R\sin\eta\cos\chi \qquad w = R\sin\eta\sin\chi$$

The sphere constraint follows directly:

$$x^2+y^2+z^2+w^2 = R^2[\cos^2\eta(\cos^2\xi+\sin^2\xi)+\sin^2\eta(\cos^2\chi+\sin^2\chi)]$$
$$= R^2(\cos^2\eta+\sin^2\eta) = R^2.$$

Equivalently, with $\mathbb{R}^4\cong\mathbb{C}^2$, $z_1=R\cos\eta\, e^{i\xi}$ and $z_2=R\sin\eta\, e^{i\chi}$. For fixed η, the pair (ξ,χ) parameterizes a Clifford torus. The two coordinate circles obtained by varying ξ alone or χ alone are not individually Hopf fibers.

2.2 Hopf projection and exact fiber invariance

Normalize to the unit three-sphere by dividing z_1,z_2 by R. A standard Hopf map to the unit two-sphere is:

$$h(z_1,z_2) = (\, 2\,\mathrm{Re}(z_1\,\bar{z}_2),\, 2\,\mathrm{Im}(z_1\,\bar{z}_2),\, |z_1|^2 - |z_2|^2\,),$$
$$\text{with } |z_1|^2 + |z_2|^2 = 1.$$

Its image has unit norm, since

$$|h|^2 = 4|z_1|^2|z_2|^2 + (|z_1|^2-|z_2|^2)^2$$
$$= (|z_1|^2+|z_2|^2)^2 = 1.$$

In Hopf coordinates,

$$h(\eta,\xi,\chi) = (\sin(2\eta)\cos(\xi-\chi),\ \sin(2\eta)\sin(\xi-\chi),\ \cos(2\eta)).$$

The projected point depends on η and the phase difference ξ−χ. Under the common phase action

$$(z_1,z_2) \mapsto (e^{i\theta}z_1, e^{i\theta}z_2),$$

the product $z_1\bar{z}_2$ and both squared moduli are unchanged. Therefore

$$h(e^{i\theta}z_1, e^{i\theta}z_2) = h(z_1,z_2).$$

Exact result A full U(1) phase degree of freedom exists along each Hopf fiber, while the S^2 projection remains invariant.

This is a precise geometric example of variation within a configuration that leaves a projected reference unchanged. It is not a spatial translation in $\mathbb{R}^3$; it is a fiber action on S^3.

2.3 Common and opposed phase motion

The distinction between common and relative phase rates is especially relevant to the earlier counter-rotation investigations. Write

$$q = (\xi+\chi)/2 \qquad \text{and} \qquad \delta = (\xi-\chi)/2.$$

The Hopf base depends on δ, while q parameterizes the common fiber phase (up to the standard periodic identifications). Therefore:

$$\dot{\xi} = \dot{\chi} = \omega \quad \Rightarrow \quad \dot{\delta} = 0,\ \dot{q} = \omega \quad \text{(fiber motion; base fixed).}$$
$$\dot{\xi} = -\dot{\chi} = \omega \quad \Rightarrow \quad \dot{\delta} = \omega,\ \dot{q} = 0 \quad \text{(base motion; fiber coordinate fixed).}$$

Thus equal-sense phase motion traverses a Hopf fiber; opposed phase motion changes the projected base point. These are exact consequences of the coordinate definitions. A physical rotor or field system would still need its own dynamical law to realize these phase motions.

2.4 Stereographic projection and inverse

Project from $N=(0,0,0,R)$ onto $w=0$. The stereographic map is

$$\sigma(x,y,z,w) = X = [R/(R-w)]\,(x,y,z), \qquad w \neq R.$$

In Hopf coordinates, define $D=1-\sin\eta\sin\chi$. Then

$$\begin{aligned} X &= (R\cos\eta\cos\xi)/D, \\ Y &= (R\cos\eta\sin\xi)/D, \\ Z &= (R\sin\eta\cos\chi)/D. \end{aligned}$$

The pole occurs at $\eta=\pi/2$, $\chi=\pi/2 \pmod{2\pi}$, where $D=0$. Approaching the pole produces unbounded Euclidean coordinates. This is a coordinate singularity of the stereographic chart, not by itself a physical divergence or acceleration.

For an external point $X=(X,Y,Z)$, let $\rho^2=X^2+Y^2+Z^2$. The inverse is

$$\begin{aligned} x &= 2R^2X/(R^2+\rho^2), \quad y = 2R^2Y/(R^2+\rho^2), \\ z &= 2R^2Z/(R^2+\rho^2), \quad w = R(\rho^2-R^2)/(\rho^2+R^2). \end{aligned}$$

Direct substitution gives $x^2+y^2+z^2+w^2=R^2$ and $\sigma(\sigma^{-1}(X))=X$. These formulas give a complete coordinate lift from ordinary three-space to S^3, away from the projection pole. They do not prescribe a physical mechanism for moving an embodiment.

2.5 Optional irrational phase winding

The earlier Orb construction considered the golden ratio $\varphi=(1+\sqrt{5})/2$ as a slope relating the two torus phases. The mathematically well-defined object is a one-parameter linear flow on the torus:

$$\gamma(s) = (\xi_0 + \varphi s \bmod 2\pi,\ \chi_0 + s \bmod 2\pi).$$

Because φ is irrational, there is no nonzero period T satisfying both $\varphi T=2\pi m$ and $T=2\pi n$ for integers m,n. The orbit is dense in the two-torus. This is a standard irrational linear flow. It is not a consequence that the Hopf map selects φ; any irrational slope gives a dense orbit.

The shorthand equation $\xi-\varphi\chi=\text{constant}$ is not a globally single-valued equation on the angle torus when φ is irrational, because χ itself is defined modulo 2π. The lifted one-parameter flow above avoids that ambiguity.

For fixed $\eta<\pi/2$, stereographic projection is continuous on the corresponding Clifford torus, so the projected orbit is dense in the image of that torus. It is not dense in all of $\mathbb{R}^3$. The phase offset changes a parameterized locus; it does not establish a physical location-selection mechanism.

2.6 Projection sensitivity Jacobian

If the optional phase flow is written locally as $\xi=\varphi\chi+\lambda_p$, with phase offset λ_p, define $q=\varphi\chi+\lambda_p$ and $D=1-\sin\eta\sin\chi$. The projected map is

$$X = R\cos\eta\cos q / D, \quad Y = R\cos\eta\sin q / D,$$
$$Z = R\sin\eta\cos\chi / D.$$

Its Jacobian $J=\partial(X,Y,Z)/\partial(\eta,\chi,\lambda_p)$ has columns:

$$\partial_\eta X = R(\sin\chi-\sin\eta)\cos q/D^2,$$
$$\partial_\eta Y = R(\sin\chi-\sin\eta)\sin q/D^2,$$
$$\partial_\eta Z = R\cos\eta\cos\chi/D^2;$$

$$\partial_\chi X = R\cos\eta\sin\eta\cos\chi\cos q/D^2 - \varphi R\cos\eta\sin q/D,$$
$$\partial_\chi Y = R\cos\eta\sin\eta\cos\chi\sin q/D^2 + \varphi R\cos\eta\cos q/D,$$
$$\partial_\chi Z = R\sin\eta(\sin\eta-\sin\chi)/D^2;$$

$$\partial_{\lambda_p} X = -R\cos\eta\sin q/D,$$
$$\partial_{\lambda_p} Y = R\cos\eta\cos q/D,$$
$$\partial_{\lambda_p} Z = 0.$$

For small parameter changes, $\delta X\approx J(\delta\eta,\delta\chi,\delta\lambda_p)^T$. This is an exact first-order coordinate-sensitivity formula. It quantifies how parameter changes alter a stereographic image. It is not a force law, a control law, or a proof of physical relocation.

3. Electromagnetic field composition

3.1 Maxwell equations in complex form

Use rationalized natural units $c=\varepsilon_0=\mu_0=1$. Define the Riemann–Silberstein vector

$$\mathcal{R} = E + iB.$$

Vacuum Maxwell equations become

$$\nabla\cdot\mathcal{R} = 0, \qquad \nabla\times\mathcal{R} = i\,\partial_t\mathcal{R}.$$

Bateman's construction sets

$$\mathcal{R} = \nabla\alpha \times \nabla\beta,$$

where α and β are complex scalar functions satisfying the Bateman compatibility condition

$$\nabla\alpha \times \nabla\beta = i[(\partial_t\alpha)\nabla\beta - (\partial_t\beta)\nabla\alpha].$$

The divergence equation is automatic because the divergence of a cross product of two gradients vanishes. The compatibility condition supplies the time-dependent Maxwell equation. It also gives nullness: the Bateman condition writes $\mathcal{R}$ as i times a linear combination of ∇α and ∇β, while $\mathcal{R}=\nabla\alpha\times\nabla\beta$ is orthogonal to both. Hence $\mathcal{R}\cdot\mathcal{R}=0$, and therefore

$$E\cdot B = 0, \qquad |E|^2 = |B|^2.$$

3.2 Exact Hopfion solution

In dimensionless coordinates with characteristic length a and time scale a/c set to one, define $r^2=x^2+y^2+z^2$ and

$$\begin{aligned} A &= \tfrac{1}{2}(r^2 - t^2 + 1),\\ \alpha &= (A-1+iz)/(A+it),\\ \beta &= (x-iy)/(A+it). \end{aligned}$$

Then $\mathcal{R}=\nabla\alpha\times\nabla\beta$ is the standard finite-energy null electromagnetic Hopfion. At each time slice, the Bateman variables provide a map with the Hopf topology; its electric and magnetic field lines have linked structure. The solution is exact for vacuum Maxwell theory [1]. Multiplying the field by a constant changes its energy normalization but not the dimensionless momentum-to-energy ratio.

This example matters because nontrivial topology does not require nonlinear Maxwell equations. A linear field equation can carry a knotted configuration. However, Maxwell linearity also means that superposition alone does not create a nonlinear binding force or a static soliton.

3.3 Composition and cross terms

For two vacuum solutions, linearity allows

$$\mathcal{R}_{tot} = \mathcal{R}_A + \mathcal{R}_B, \qquad E_{tot} = E_A+E_B, \qquad B_{tot} = B_A+B_B.$$

In the same normalized units, the energy density is

$$u = \tfrac{1}{2}(|E|^2+|B|^2).$$

Consequently,

$$u_{tot} = u_A + u_B + E_A\cdot E_B + B_A\cdot B_B.$$

The cross terms depend on relative phase, position, and field orientation. The total distribution is therefore not generally the sum of two independent energy distributions. The momentum density also contains cross terms, since

$$\begin{aligned} E_{tot}\times B_{tot} = {} & E_A\times B_A + E_B\times B_B\\ & + E_A\times B_B + E_B\times B_A. \end{aligned}$$

This is the exact mathematical basis for saying that field composition changes the objective distribution. It does not by itself imply attraction, binding, or a preferred stationary shape.

3.4 Hopfion momentum and objective translation

The exact Hopfion solution above has, in c=1 units, conserved momentum

$$P = (0, 0, -U/2),$$

where U is its total energy. Restoring SI dimensions gives $P_z=-U/(2c)$. The center-of-energy law therefore yields

$$dR_E/dt = c^2P/U = -(c/2)\, e_z.$$

Thus this exact knotted electromagnetic field has an objective energy centroid that translates at speed c/2 in the negative z direction, while its linked field-line topology is preserved. This is a concrete example of a changing objective location with persistent internal topological organization. It is not a static, self-confined orb.

For two reflected configurations, total momentum cancellation follows only when the complete composed stress-energy distribution has the required reflection symmetry. The cross momentum terms must be included. Cancellation cannot be inferred merely by adding the individual momenta when the fields overlap.

4. Nonlinear field embodiment: Faddeev–Skyrme

4.1 Field and energy definition

Let $n:\mathbb{R}^3\to S^2$ be a unit-vector field with $n(x)\to n_\infty$ at spatial infinity. Define

$$F_{ij} = n \cdot (\partial_i n \times \partial_j n), \qquad i,j\in\{1,2,3\}.$$

The static Faddeev–Skyrme energy in the convention used in this document is

$$E_{FS}[n] = \int_{\mathbb{R}^3} \left[(\kappa_2/2)|\nabla n|^2 + (\kappa_4/4) \textstyle\sum_{i<j} F_{ij}^2 \right] d^3x, \qquad \kappa_2>0,\ \kappa_4>0.$$

The boundary condition compactifies the domain topologically: $\mathbb{R}^3\cup\{\infty\}\cong S^3$. The map can therefore be regarded as $n:S^3\to S^2$. Its homotopy class is characterized by the Hopf invariant $Q_H\in\mathbb{Z}$. With ω_2 the standard unit-sphere area form and $dA=n^*\omega_2$, one normalization is

$$Q_H = (1/16\pi^2) \int_{S^3} A\wedge dA, \qquad dA = n^*\omega_2.$$

The Hopf invariant is unchanged by a smooth homotopy that preserves the boundary conditions. This is a topological restriction on admissible deformations, not an energy-minimization theorem by itself.

For completeness, the stationary field equation can be stated without hiding the unit-length constraint. Let $\eta(x)$ be any smooth compactly supported tangent variation, $n\cdot\eta=0$, and define $a_i=n\cdot(\eta\times\partial_i n)$. The first variation of the pulled-back area form is $\delta F_{ij}=\partial_i a_j-\partial_j a_i$. Thus a critical point satisfies the following weak Euler–Lagrange identity for every such η:

$$\begin{aligned} 0 &= -\kappa_2 \int \Delta n\cdot\eta \, d^3x \\ &\quad + (\kappa_4/2) \textstyle\sum_{i<j} \int F_{ij}(\partial_i a_j-\partial_j a_i) d^3x, \\ a_i &= n\cdot(\eta\times\partial_i n), \qquad n\cdot\eta=0. \end{aligned}$$

This is the constrained first-variation equation for the displayed static functional. It is the appropriate field equation for stationary profiles in this paper. A time-dependent equation would require a kinetic term and a full action; it is not supplied by the static energy alone.

4.2 Derrick scaling and a preferred scale

For $n_L(x)=n(x/L)$, the quadratic and quartic contributions scale differently:

$$E_2(L) = L\, E_2(1), \qquad E_4(L) = L^{-1}\, E_4(1).$$

Writing $a=E_2(1)>0$ and $b=E_4(1)>0$,

$$E(L) = aL + b/L.$$

Differentiating,

$$dE/dL = a - b/L^2 = 0 \;\Rightarrow\; L^* = \sqrt{(b/a)}.$$

The second derivative at L^* is $2b/L^{*3}>0$. At the stationary scale, the two contributions balance:

$$E_2(L^*) = E_4(L^*) = \sqrt{(ab)}, \qquad E(L^*) = 2\sqrt{(ab)}.$$

This proves a strict minimum along the one-parameter uniform-dilation family. It is a necessary scale-balance mechanism for a finite-size configuration. It does not prove that the profile is stationary or stable against all shape variations.

4.3 The 2026 global-minimality theorem on the round S^3

A result published in Communications in Mathematical Physics on 9 July 2026 supplies a stronger statement for a precise compact model. On the unit round S^3, define

$$FS\rho(u) = \int_{S^3} |du|^2 + (1/4\rho^2) \int_{S^3} |du \wedge du|^2,$$
$$u: S^3 \to S^2, \qquad 0<\rho\le 1.$$

Since $|u^*\omega_2|^2=(1/4)|du\wedge du|^2$ in the authors' convention, this is equivalently

$$FS\rho(u) = \int_{S^3} |du|^2 + \rho^{-2} \int_{S^3} |u^*\omega_2|^2.$$

Guerra, Lamy, and Zemas [3] prove: if $Q_H(u)=1$ and $0<\rho\le 1$, then

$$FS\rho(u) \ge FS\rho(h),$$

with equality if and only if $u=h\circ R$ for some $R\in SO(4)$, where h is the Hopf map. Thus, in that coupling regime, the Hopf map is the unique global minimizer in its Hopf class, modulo domain rotations.

To translate this theorem to the coefficients used in this document, multiply E_{FS} by $2/\kappa_2$. Since the coefficient of $\int|u^*\omega_2|^2$ then becomes $\kappa_4/(2\kappa_2)$, identify

$$\rho^{-2} = \kappa_4/(2\kappa_2).$$

The theorem's range $\rho\le 1$ therefore corresponds to

$$\kappa_4/\kappa_2 \ge 2.$$

Important result For the round S^3 Faddeev–Skyrme model with $Q_H=1$ and $\kappa_4/\kappa_2\ge 2$, the fully spherical Hopf map is globally energy-minimizing, modulo SO(4) rotations. This is a theorem from the cited paper, not a numerical inference.

This theorem establishes unrestricted shape minimization only for the stated compact round-S^3 model and coupling range. It does not transfer to flat $\mathbb{R}^3$ or to the separate two-field functional: the Faddeev–Skyrme energy is not conformally invariant, and stereographic projection does not preserve its energy.

4.4 Explicit Hopf-map energy on unit S^3

For the standard Hopf map with round unit-sphere metrics, its two nonzero horizontal singular values are 2. Hence

$$|dh|^2 = 8, \qquad |h^*\omega_2|^2 = 16.$$

Since $Vol(S^3)=2\pi^2$,

$$\int_{S^3}|dh|^2 = 16\pi^2,$$
$$\int_{S^3}|h^*\omega_2|^2 = 32\pi^2,$$
$$FS\rho(h) = 16\pi^2 + 32\pi^2/\rho^2.$$

In the coefficient convention above,

$$E_{FS}[h] = 8\pi^2(\kappa_2+\kappa_4).$$

5. Coupled fields: common translation and relative displacement

The next model is a proposed mathematical extension. It is designed to put two different changes into one variational framework: common translation of an entire configuration and relative displacement between its constituent fields. The form of the interaction is a modeling choice; it is not derived uniquely from Maxwell theory or from Hopf topology.

5.1 Definition of the two-field model

Let $n_A, n_B : \mathbb{R}^3 \to S^2$, with the same constant limit at infinity. Define

$$F^A_{ij} = n_A \cdot (\partial_i n_A \times \partial_j n_A),$$
$$F^B_{ij} = n_B \cdot (\partial_i n_B \times \partial_j n_B).$$

The proposed energy is

$$E[n_A, n_B] = E_{FS}[n_A] + E_{FS}[n_B] + E_{int},$$

with

$$E_{int} = (g_0/2) \int (1 - n_A \cdot n_B) d^3x + (g_2/2) \int \Sigma_{i<j} (F^A_{ij} - F^B_{ij})^2 \, d^3x,$$
$$g_0, g_2 \geq 0.$$

The first interaction measures pointwise disagreement of the unit-vector fields. The second measures disagreement of their pulled-back area two-forms. Both terms are nonnegative and both vanish when the fields coincide.

5.2 Common and relative coordinates

Take two translated copies of a common localized profile n_H:

$$n_A(x) = n_H(x - X_A), \qquad n_B(x) = n_H(x - X_B).$$

Define the midpoint and separation:

$$X = (X_A + X_B)/2, \qquad \Delta = X_A - X_B.$$

Equivalently, $X_A = X + \Delta/2$ and $X_B = X - \Delta/2$. The variable X changes the external position of the whole pair; Δ changes their internal separation.

5.3 Exact common-translation invariance

Apply the same vector a to both centers:

$$X_A \mapsto X_A + a, \qquad X_B \mapsto X_B + a.$$

Then Δ is unchanged and $X \mapsto X + a$. Because every term in the energy is an integral over all $\mathbb{R}^3$ with no externally fixed spatial reference, the change of variables $y = x - a$ gives

$$E(X + a, \Delta) = E(X, \Delta), \qquad \partial E / \partial X = 0.$$

Exact result The common-position coordinate is a symmetry direction of the proposed coupled functional. The energy depends on the relative configuration, not on the absolute common position.

In body coordinates $y = x - X$, the internal profile is unchanged under common translation. If $n_X(x) = n_H(x - X)$, then $n_{X+a}(x + a) = n_X(x)$. This identity is the precise body-frame statement that the internal reference relationship remains the same while the objective configuration is displaced.

5.4 Derivation of the relative-displacement stiffness K

Set $y = x - X$ and define $n_-(y) = n_H(y - \Delta/2)$, $n_+(y) = n_H(y + \Delta/2)$. Since $|n_\pm| = 1$,

$$1-n_-\cdot n_+ = \tfrac{1}{2}|n_- - n_+|^2.$$

For a sufficiently smooth profile, symmetric Taylor expansion gives

$$n_- - n_+ = -\Delta_i \partial_i n_H + O(|\Delta|^3).$$

Therefore the first interaction contributes

$$(g_0/2)\int(1-n_-\cdot n_+)d^3y = (g_0/4)\,\Delta_i\Delta_j \int \partial_i n_H\cdot\partial_j n_H\, d^3y + O(|\Delta|^4).$$

Now define the dual vector of the pulled-back two-form:

$$B_H = (F^H_{yz}, F^H_{zx}, F^H_{xy}).$$

The dual-vector difference has the symmetric expansion

$$B_- - B_+ = -\Delta_i \partial_i B_H + O(|\Delta|^3).$$

Since $\Sigma_{i<j}(F^A_{ij}-F^B_{ij})^2=|B_A-B_B|^2$, the second interaction contributes

$$(g_2/2)\int|B_- - B_+|^2 d^3y = (g_2/2)\,\Delta_i\Delta_j \int \partial_i B_H\cdot\partial_j B_H\, d^3y + O(|\Delta|^4).$$

Define the two symmetric matrices

$$G_{ij} = \int \partial_i n_H\cdot\partial_j n_H\, d^3x,$$
$$Q_{ij} = \int \partial_i B_H\cdot\partial_j B_H\, d^3x.$$

Combining the two contributions yields

$$E_{int}(\Delta) = \tfrac{1}{2}\,\Delta_i\, K_{ij}\,\Delta_j + O(|\Delta|^4),$$
$$K_{ij} = (g_0/2)G_{ij} + g_2 Q_{ij}.$$

Derived local result K is the Hessian of the interaction energy in the relative-translation coordinates at coincident copies. It is not an assumed restoring matrix.

5.5 Positivity and the meaning of “restoring”

For any vector v,

$$v_i G_{ij} v_j = \int |v_i \partial_i n_H|^2 d^3x \geq 0,$$
$$v_i Q_{ij} v_j = \int |v_i \partial_i B_H|^2 d^3x \geq 0.$$

Hence G and Q are positive semidefinite, and for $g_0, g_2 \geq 0$,

$$K \geq 0.$$

If $g_0>0$ and n_H is a smooth nonconstant localized field approaching a single vacuum value at infinity, G is positive definite: a null direction v would imply $\partial_v n_H=0$ everywhere, so n_H would be constant along every line parallel to v; the boundary condition at infinity would then force the entire field to be constant, contrary to nonzero Hopf charge. Thus in that stated class K is positive definite when $g_0>0$.

A positive K means that small relative translations raise the interaction energy quadratically. The negative energy gradient points back toward $\Delta=0$. Calling this a dynamical restoring force requires a time-dependent action and kinetic terms; the present construction is a static energy functional.

Because the energy is symmetric under exchanging A and B, $E_{int}(\Delta)=E_{int}(-\Delta)$. With sufficient smoothness, odd powers vanish and the next Taylor term is fourth order. The result is local: it does not determine the entire interaction potential at large separation.

6. Objective center of energy and translation

6.1 Definition and conservation law

For a localized energy density u(x,t), total energy U and center of energy are

$$U = \int u(x,t)d^3x,$$
$$R_E = (1/U)\int x\, u(x,t)d^3x.$$

For electromagnetism in SI units, the local energy balance is $\partial_t u+\nabla\cdot S=0$. If the boundary flux vanishes,

$$d/dt \int x\, u\, d^3x = -\int x\, \nabla\cdot S\, d^3x = \int S\, d^3x.$$

Using total momentum $P=(1/c^2)\int S\, d^3x$ and conserved U gives

$$dR_E/dt = c^2P/U.$$

This is the center-of-energy theorem for an isolated electromagnetic system with the stated boundary behavior. For a general relativistic field-plus-matter system, the same relation follows from conservation of the total symmetric stress-energy tensor, with total momentum replacing the electromagnetic Poynting integral. It applies to the Maxwell Hopfion and other such systems when the required integrals and boundary conditions are well-defined.

6.2 How common translation changes the centroid

If a configuration is translated by a, so $u_a(x)=u(x-a)$, then

$$U_a = U, \qquad R_E[a] = R_E[0] + a.$$

The internal energy distribution in body coordinates is unchanged; its objective centroid shifts by the same external displacement. This gives a precise mathematical formulation of the two aspects that the theoretical proposition distinguishes.

6.3 Do not identify the three transformations

Transformation	What it changes	What it preserves
Hopf U(1) phase	Location along a fiber in S^3	The S^2 Hopf projection
Common spatial translation	External location X in $\mathbb{R}^3$	Body-frame profile, relative separation, translation-invariant energy
Relativistic time evolution	The objective energy/momentum distribution	The conservation laws; topology for the exact null Hopfion

The first transformation is internal geometry. The second is a symmetry of the proposed spatial field functional. The third is dynamics governed by a time-dependent field theory. The current investigation has found meaningful correspondences among them, but has not derived a single physical mechanism that identifies all three.

7. The original six relational coordinates

The later Light Speed Fields discussions corrected an earlier visual interpretation: the sixfold construction is not to be treated as six fixed spatial points or as two literal physical triangles. It is better recorded as two triples of relational phase or reference coordinates:

$$\Theta_A = (\theta_{A1}, \theta_{A2}, \theta_{A3}),$$
$$\Theta_B = (\theta_{B1}, \theta_{B2}, \theta_{B3}).$$

$$\theta_{Ak}(t) = \Omega_A\, t + 2\pi k/3 + \theta_{A0},$$
$$\theta_{Bk}(t) = -\Omega_B\, t + 2\pi k/3 + \theta_{B0}, \qquad k = 0,1,2.$$

These equations encode two threefold phase samplings with opposite signed rates. Their relative phase evolves at rate $\Omega_A+\Omega_B$. For equal rates $\Omega_A=\Omega_B=\Omega$, the relative rate is 2Ω. The 120-degree spacing is a parameter choice for each triple; the six entries are relational coordinates, not six imposed spatial points.

$$\Delta_{\text{phase}}(t) = \theta_{A0} - \theta_{B0} + (\Omega_A + \Omega_B)t.$$

This kinematic description records the original counter-rotating proposal in a precise form. It does not yet derive the six phases from a single field action, nor show that a material system realizes them. The two triples remain the proposed relational input whose coupling to field variables is an open modeling step.

These two triples represent opposed or complementary reference descriptions within the theoretical construction. In the Light Speed Fields investigation, each triple was treated as three relational field contributions in its own reference, not as a physical triangle. The six variables are therefore retained as six relational coordinates. They are not yet derived as six independent dynamical coordinates of the Maxwell or Faddeev–Skyrme models presented above. To do that, an explicit map from these six relational variables to field variables, an action, or a constraint system would be required.

Where a threefold phase sample is used, 120-degree spacing can be a convenient coordinate representation. It should not be elevated into a claim that the underlying field literally consists of two rigid triangles. The mathematical object under examination is the relation among the phase variables and their common and relative combinations.

The Hopf phase decomposition supplies one exact two-angle example of such a distinction: common phase changes the fiber coordinate while relative phase changes the base. A full six-variable model would require additional structure beyond the standard Hopf map.

7.1 Original symbolic sequence (interpretive register)

The wider Orb conception also uses a numbered sequence. It is included here to preserve the original language, not as a set of mathematical identities or additional physical dimensions:

Number	Project-specific correspondence
0	Unseen reference / ground.
1	One point; first expression; center.
2	Polarity; one and other.
3	Relation; one happening.
4	Spatial embodiment.
5	Senses and inward questions.
6	Two complementary triples; sixfold relational expression.

Number	Project-specific correspondence
7	Reference together with sixfold expression.
8	Waking/sleeping; inner/outer confluence.
9	Motion and cycle.
10	Full cycle / return.
11	Presence or observer recognized within the cycle.
12	Clock's completed circle.
13	Recognition beyond the clock's visible cycle.

The labels 0–13 are project-specific symbolic roles. They are not universal mathematical definitions, and the field equations in this paper do not derive the sequence. Their inclusion records the conceptual path that motivated the mathematical questions without treating the labels as proof.

8. Experimental and computational record

The experimental sequence was valuable in finding candidate structures and identifying where earlier interpretations needed correction. For an independent mathematical offering, however, the record must distinguish reproducible results from exploratory observations.

8.1 Results that can be stated now

Finding	Basis	Status
Hopf projection is invariant under common phase.	Direct algebra from the Hopf map.	Exact identity.
Stereographic map and inverse lift.	Direct substitution using the displayed formulas.	Exact coordinate result.
Irrational linear flow is dense on the phase torus.	Kronecker theorem for a two-frequency flow with irrational ratio.	Exact dynamical-systems result.
Bateman Hopfion solves vacuum Maxwell equations and is null.	Bateman condition and cited exact solution.	Published exact solution.
Hopfion momentum ratio gives centroid speed c/2.	Published conserved charges plus center-of-energy law.	Exact for the specified normalized solution.
Two-derivative and four-derivative FS terms balance at finite scale.	Derrick scaling under uniform dilation.	Exact one-parameter variational result.
Hopf map globally minimizes the specified compact S^3 FS functional for $Q_H=1$ and $\kappa_4/\kappa_2 \geq 2$.	Guerra–Lamy–Zemas theorem, translated into this normalization.	Published global theorem.

Finding	Basis	Status
Common translation leaves the proposed two-field energy unchanged.	Change of variables in the full-space functional.	Exact model-dependent theorem.
Relative-displacement Hessian equals $(g_0/2)G+g_2Q$.	Taylor expansion of the stated coupling.	Exact model-dependent derivation.
K is positive semidefinite; positive definite under the stated nonconstant localized-field and $g_0>0$ condition.	Integral-of-squares argument.	Exact model-dependent result.

8.2 Earlier Maxwell-grid experiments (exploratory)

The earlier numerical sessions also evaluated a single Hopfion, reflected pairs, and a threefold composition on finite Cartesian grids. These calculations were exploratory finite-domain integrations; the old chat record does not contain the executable code, complete quadrature specification, or convergence data needed to certify their reported values. The single-Hopfion momentum ratio is independently confirmed by the exact published charge calculation in Section 3.4. The pair and triad observations remain provisional.

Configuration	Recorded observation	Status
Single Hopfion; grid $[-10,10]^3$, 101^3; c=1	$U\approx19.739$ ($\approx2\pi^2$), $P_z\approx-9.86966$ ($\approx-\pi^2$); centroid slope $z/t\approx-0.5$.	Finite-grid result; ratio agrees with exact solution [1].
Reflected pair	Opposing momenta nearly cancel; energy centroid remains near the symmetry plane while the spatial RMS width grows.	Exploratory; cross terms and grid/domain convergence require rerun.
Threefold composition	$U\approx44.412$, $P_z\approx-22.207$; reported centroid slope $z/t\approx-0.5$.	Exploratory; normalization and convergence record incomplete.
Relative shifts, timing offsets, amplitude imbalance	Change the interference pattern and the computed energy distribution/centroid.	Qualitative finite-grid observation; no verified quantitative table retained.

8.3 Provisional radial-profile computation

A prior numerical run used the degree-one hedgehog lift

$$U(x)=\cos f(r)\, I + i \sin f(r)\, \hat{x}\cdot\sigma,$$
$$n_H = U\dagger\, \sigma_3\, U,$$

with endpoint conditions $f(0)=\pi$ and $f(\infty)=0$, initially using $f(r)=2\arctan(1/r)$. The profile $f(r)$ was then varied on a radial grid while the angular structure of the ansatz was held fixed.

The retained chat record reports one run with $N_r=300$, $N_\theta=20$, $N_\varphi=20$, $r_{max}=20$, $r_{min}=0.005$, $\kappa_2=\kappa_4=1$. It reports:

Reported quantity	Reported value	Record status
Total E	200.4632643	Provisional numerical observation
E_2	102.9921770	Provisional numerical observation

Reported quantity	Reported value	Record status
E_4	97.4710873	Provisional numerical observation
Degree integral	−1.0001104	Numerical check of $\lvert\text{degree}\rvert \approx 1$
G	diag(70.05223, 70.05223, 65.87989)	Provisional integral tensor
Q	diag(1135.56219, 1135.56219, 486.98653)	Provisional integral tensor
Uniform scale minimizer	$s \approx 0.972827$	Derived from reported E_2, E_4

The uniform scaling relation gives $s^* = \sqrt{(E_4/E_2)} \approx 0.972827$ and the rescaled energy $2\sqrt{(E_2 E_4)} \approx 200.3872$. This is internally consistent with the scaling law. The reported G and Q tensors have axial form $\text{diag}(G_\perp, G_\perp, G_\parallel)$ and $\text{diag}(Q_\perp, Q_\perp, Q_\parallel)$, reflecting the axial orientation built into $n_H = U^\dagger \sigma_3 U$. They are not evidence of unrestricted three-dimensional symmetry breaking.

These values are not included as validated numerical results because the complete executable source, exact quadrature implementation, saved optimized profile, and full convergence logs are not presently attached to the recoverable record. They may be retained as a provisional lab notebook entry, but must be independently rerun before publication as numerical evidence.

8.4 Deformation results withdrawn

The later axisymmetric-relaxation tables and P_2/P_4 angular-deformation numbers in the conversation were not supported by a verified computational run. They are withdrawn and are not evidence for either spherical stability or asymmetric preference. Earlier reported Hessian-sector eigenvalues and finite-displacement tables that lack a verifiable derivation are also excluded.

Computational discipline No numerical table should be promoted to a finding until the field definitions, units, domain, quadrature, optimizer, initial data, stopping criterion, code, and convergence record are available.

9. What the investigation establishes—and what it does not

9.1 Established mathematical relationships

- The Hopf fibration contains an exact internal U(1) fiber symmetry that leaves the S^2 projection unchanged.
- The Orb's S^3 coordinates and stereographic projection are explicit, invertible coordinate constructions away from the projection pole.
- A null Maxwell Hopfion is an exact finite-energy field solution with nontrivial linked topology and a known conserved momentum ratio.
- Field superposition changes energy and momentum densities through exact cross terms.
- The Faddeev–Skyrme energy balances its quadratic and quartic derivative contributions under dilation, creating a preferred scale along that variation.
- For the round S^3 compact model, the Hopf map is a proven global minimizer in Hopf class one when $\kappa_4/\kappa_2 \geq 2$.
- For the explicitly proposed two-field coupling, common translation is an exact symmetry and relative displacement has a derived positive-semidefinite local stiffness tensor.
- The objective center of energy moves according to conserved momentum in an isolated relativistic theory.

9.2 Theoretical correspondence that remains open

- The subjective reference is the organizing primitive of the philosophical proposal. Current field variables do not operationalize it as a measurable physical coordinate.
- The two triples of relational coordinates have not yet been mapped into a unique field ansatz or dynamical action.
- The coupling coefficients g_0 and g_2 and the particular interaction energy are proposed choices. Their derivation from the underlying Orb geometry or from a known physical interaction remains open.
- The common translation coordinate is a symmetry parameter, not an actuator or a dynamical mechanism. A physical law connecting internal state to chosen external displacement is not yet derived.
- The complete coupled-field minimum and its full fluctuation spectrum have not been proved.
- The static Faddeev–Skyrme functional does not by itself supply relativistic time evolution. A time-dependent action and stress-energy tensor would be required for a fully dynamical embodiment.
- No material implementation or experimental measurement has been established by the mathematical work described here.

9.3 Additional symbolic correspondences

π is intrinsic to circular and spherical geometry and appears in Hopf-map normalization, sphere volume, and topological integrals. φ is an optional irrational slope that generates a dense linear flow on a phase torus; it is not selected by the Hopf map. Other irrational slopes have the same density property. Further number correspondences (including clock cycles, 432/440, inverse fine-structure constant, or integer patterns) are not derived by the field models in this document and should be kept in a separate symbolic register unless a dimensionally valid derivation is supplied.

10. Is the work ready to offer for review?

Yes—with a precise description of what is being offered. The work is ready as a mathematical foundation and a set of derived correspondences for independent examination. It is not ready to be offered as a completed mathematical proof of every theoretical principle or as a validated physical theory of the Orb.

I do not think another broad round of exploratory experiments is a prerequisite for submitting this foundation to mathematicians. The exact geometric identities, published Maxwell Hopfion solution, Derrick scaling identity, 2026 compact-S^3 global-minimality theorem, and analytic coupled-field translation/displacement derivation are sufficient to make the mathematical foundation reviewable. The prior exploratory computations are supporting leads, not prerequisites for review, and must not be represented as certified numerical evidence until their computational record is reconstructed.

There are two outstanding qualifications for the package: first, the provisional radial numerical record must be reconstructed from code before being presented as verified computation; second, the six-coordinate theoretical construction must be connected to a definite field ansatz if it is to become a single unified dynamical model. Those are transparent next stages, not reasons to withhold the exact mathematical foundation.

10.1 Recommended form of the offering

1. Main paper: the exact geometry, field equations, variational derivations, compact S^3 theorem, coupled-field result, center-of-energy result, and evidence ledger contained here.
2. Computational supplement: source code and reproducibility logs for the radial-profile experiment, followed only by any numerical values that survive independent rerun.
3. Interpretive note: the original proposition and the two-triple relational picture, clearly separated from the formal results.

The offering should invite mathematicians to verify the displayed calculations, assess the model choices, and investigate the open bridges. It should not instruct them what conclusion to reach.

11. Consolidated statement

The mathematical correspondence A geometric configuration can contain an invariant internal relation while admitting transformations that change its representation or external position. In the Hopf fibration, common phase changes preserve the projected base point. In a translation-invariant field functional, common spatial displacement preserves the body-frame profile and energy. In a coupled-field model, relative displacement can cost energy even though common translation does not. In a relativistic field, the objective center of energy moves according to total momentum.

These are distinct mathematical results, joined by a common organizing relation: what changes and what remains invariant depend on which transformation is being considered. That is the strongest defensible mathematical statement at the center of this investigation.

The offering is therefore not the claim that the entire Orb has been built or that every interpretation has been proved. It is the presentation of an exact geometric foundation, established field-theoretic examples, a published variational theorem, and a derived coupled-field result—together with the explicit limits of each. The next reader is free to reproduce, challenge, extend, or reject any part of it on mathematical grounds.

The mathematical invitation is deliberately narrow: verify the identities and derivations as written; assess the assumptions of the proposed coupled-field functional; and determine which open correspondences merit further work. The philosophical proposition is offered as the question that organized the investigation, not as a substitute for the mathematics.

Appendix A. Verification derivations

A.1 Hopf map norm and phase invariance

$$\text{Let } a=|z_1|^2,\ b=|z_2|^2,\ a+b=1.$$
$$|h|^2 = 4ab+(a-b)^2 = (a+b)^2 = 1.$$

$$h(e^{i\theta}z_1,e^{i\theta}z_2) = h(z_1,z_2),$$

because $(e^{i\theta}z_1)\overline{(e^{i\theta}z_2)}=z_1\bar{z}_2$ and $|e^{i\theta}z_j|^2=|z_j|^2$.

A.2 Stereographic inverse check

$$x^2+y^2+z^2 = 4R^4\rho^2/(R^2+\rho^2)^2,$$
$$w^2 = R^2(\rho^2-R^2)^2/(R^2+\rho^2)^2.$$

$$x^2+y^2+z^2+w^2 = R^2[4R^2\rho^2+(\rho^2-R^2)^2]/(R^2+\rho^2)^2$$
$$= R^2(R^2+\rho^2)^2/(R^2+\rho^2)^2 = R^2.$$

Substituting the inverse into $R(x,y,z)/(R-w)$ returns (X,Y,Z), verifying $\sigma\circ\sigma^{-1}=\mathrm{Id}$.

A.3 Relative-displacement Hessian: compact proof

$$n_{\pm}(x)=n(x\mp\Delta/2),\quad |n_{\pm}|=1,$$
$$1-n_-\cdot n_+ = \tfrac{1}{2}|n_- - n_+|^2.$$

$$n_- - n_+ = -\Delta_i\partial_i n + O(|\Delta|^3).$$

$$E_0 = (g_0/2)\int(1-n_-\cdot n_+)d^3x$$
$$= (g_0/4)\Delta_i\Delta_j G_{ij} + O(|\Delta|^4).$$

$$B_{-}-B_{+} = -\Delta_i\partial_i B + O(|\Delta|^3),$$
$$E_2 = (g_2/2)\int |B_{-}-B_{+}|^2 d^3x = (g_2/2)\Delta_i\Delta_j Q_{ij} + O(|\Delta|^4).$$
$$E_{int} = \tfrac{1}{2}\Delta_i[(g_0/2)G_{ij}+g_2Q_{ij}]\Delta_j + O(|\Delta|^4).$$

This gives $K_{ij}=(g_0/2)G_{ij}+g_2Q_{ij}$. The even remainder follows because exchanging A and B sends Δ to −Δ but leaves E_int unchanged.

A.4 Center-of-energy identity

$$\partial_t u + \nabla\cdot S = 0,$$
$$d/dt \int x\, u\, d^3x = -\int x\, \nabla\cdot S\, d^3x = \int S\, d^3x,$$
$$P = c^{-2}\int S\, d^3x, \qquad U=\text{constant},$$
$$dR_E/dt = c^2 P/U.$$

Appendix B. Reproduction checklist for the numerical record

A numerical result can be upgraded from provisional to reproducible only when the following are archived:

- Exact ansatz, including the map U(x), the convention for Pauli matrices, and the induced n_H(x).
- Exact definitions and normalizations of κ_2, κ_4, F_{ij}, G_{ij}, and Q_{ij}.
- Radial and angular domains, endpoint regularity, grid nodes, quadrature weights, and the treatment of r=0 and r→∞.
- Optimization algorithm, line-search settings, tolerances, gradient norms, initialization, and termination reason.
- Every optimized profile f(r), not just scalar energy outputs.
- Independent checks of the Hopf charge, the virial identity $E_2=E_4$ after scale optimization, and tensor symmetry.
- Grid refinement and domain enlargement studies with tabulated relative changes.
- Direct finite-separation calculations E_int(Δ) compared against $\tfrac{1}{2}\Delta^T K\Delta$ for a sequence of decreasing |Δ|.
- Executable source code, package versions, fixed random seeds if applicable, and an output file containing all reported values.

The finite-separation comparison is the direct numerical check of the analytic Hessian: for a unit vector v,

$$2\,E_{int}(\varepsilon v)/\varepsilon^2 \to v^T K v \quad \text{as } \varepsilon\to 0.$$

For a smooth symmetric construction, the leading truncation error should be $O(\varepsilon^2)$ in this ratio. A logarithmic or tabulated convergence test in ε, plus numerical integration refinement, can verify the expansion independently.

Appendix C. Number, proportion, and clock correspondences

Several numerical relationships appeared in the wider theoretical development. The following separates exact arithmetic or standard constants from proposed interpretation:

Item	Mathematical fact	Status in this investigation
π	Circumference/diameter; sphere and Hopf integrals contain π.	Intrinsic to circular/spherical geometry.
φ=(1+√5)/2	Irrational number; a linear flow with slope φ on a two-torus is dense.	Optional phase slope; not selected by the Hopf map.
10/12=5/6; 12/10=6/5	Exact rational identities.	Clock-cycle comparisons; no field equation derived from them.
432/440=54/55	Exact ratio of two reference frequencies.	Tuning comparison; not a field constant in the present model.

Item	Mathematical fact	Status in this investigation
$\alpha^{-1} \approx 137.036$	Inverse electromagnetic fine-structure constant, dimensionless.	No derivation from the Orb field equations is supplied here.
$444 = 37 \times 12$	Exact integer factorization.	Numerical correspondence only; no variational or geometric theorem follows.
$2 - c \approx 0.137$	Dimensionally invalid if c denotes a speed.	Excluded unless c is explicitly nondimensionalized and a derivation is supplied.

The symbolic role of π and φ can remain part of the interpretive register. Their appearance in a formula is not sufficient to establish a physical law; each proposed relation requires a defined map, units, and a derivation. The same standard applies to clock and integer correspondences.

References

[1] C. Hoyos, N. Sircar, and J. Sonnenschein, “New knotted solutions of Maxwell’s equations,” Journal of Physics A: Mathematical and Theoretical 48 (2015), 255204. DOI: 10.1088/1751-8113/48/25/255204. Sections 2.1–2.2 give the Bateman condition, exact Hopfion variables, null-field property, and conserved energy-momentum charges. The paper’s normalization conventions should be retained when reproducing absolute energy values.

[2] B. Bode, “Stable Knots and Links in Electromagnetic Fields,” Communications in Mathematical Physics 387 (2021), 1757–1770. DOI: 10.1007/s00220-021-04219-3. This source establishes persistent knotted field-line configurations for exact vacuum Maxwell solutions using Bateman’s construction.

[3] A. Guerra, X. Lamy, and K. Zemas, “Global Minimality of the Hopf Map in the Faddeev–Skyrme Model with Large Coupling Constant,” Communications in Mathematical Physics 407 (2026), article 171. DOI: 10.1007/s00220-026-05702-5. Published 9 July 2026. This source proves the global-minimum theorem on the round S^3 stated in Section 4.3.