The Effects of Frost on Soil-Water Potential ($φ)$

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INTRODUCTION

Michigan is subject to drastic variations in climate and weather due to the presence of the Great Lakes and their capacity to act as a heat source and heat sink. Combining the lake effects with winds, fronts, and greater climatological events such as La Niña creates a recipe for intense weather events and frequent fluctuations. Though these fluctuations have always been common in the region at the turn of seasons, changes to global climate and their related effects are extending meteorological irregularities into the bulk of the seasons. This produces unusual and problematic effects – that are increasing in severity as their contributing factors are prolonged – predominantly on plant growth and agricultural development, the growth and development of various micro/macro organisms, and soil hydraulic properties.

Though the weather in the Great Lakes Region is a significant case, no matter where one lives seasonal variability of any intensity will have an undeniable ecological effect. As such, the purpose of this analysis is to examine the general impact of seasonally frozen soils and frost-conditions on soil-water potential through its deviations in soil permeability, hydraulic conductivity, and infiltration, and its subsequent propensity for increased runoff generation.

Research has consistently concluded that both current and recently frozen soils inhibit the infiltration of snowmelt and rainfall, which thereby results in earlier and higher-volume springtime runoff (Yang and Niu, 2006). The repeated thawing and refreezing of frozen soils - as generated by the aforementioned fluctuations, for example - has a particular influence according to a report by Stähli, M. (2005) referenced by Iwata et al. (2010). Frozen soils can occupy 55% to 60% of land surface in the Northern Hemisphere during the winter season (Zhang et al., 1999) and can freeze anywhere between zero to a recorded maximum depth of 2.4 meters in the United States (Selezneva et al., 2008). The depth of frost-penetration and type of frost formed (e.g., concrete vs. porous concrete frost) can vary based upon the presence and type of vegetation, soil saturation, and the length of time soil is exposed to freezing temperatures. The volume of runoff generated by frozen soil conditions is contingent on the volume of water produced through precipitation and snowmelt at the time of thaw, and the infiltration capacity of the frozen ground (which, in turn, is formulated from soil-water content, porosity, and structure).

Broadly, the behaviour of soil is dependent on a variety of hydro-physical, geological, thermal, and mechanical properties. In order to thoroughly and effectively analyze the associations between freezing conditions, soil-water potential, and resulting runoff in relation to the former, relative concepts of these fundamental properties must first be addressed.

CONCEPTS

Foremost, the infiltration and transfer of water throughout the soil matrix is based upon the overall soil-water potential, or its potential energy, represented by the Greek letter $φ.$ $φ$ is defined as the ‘work’ of water as it moves between present and reference states. This property in and of itself is comprised of multiple components, of which the sum determines overall soil-water potential and the direction of water flow:

$$φ=φ\_{0}+φ\_{π}+φ\_{p}+φ\_{g}+φ\_{v}+φ\_{m}$$

in which:

$$φ\_{0}=reference correction$$

$$φ\_{π}=soil-water osmotic potential$$

$$φ\_{p}=pressure component$$

$$φ\_{g}=gravitational potential$$

$$φ\_{v}=potential due to humidity$$

$$φ\_{m}=matrix-effect potential [e.g. surface tension, cohesion, etc.]$$

Even minimal alterations in surface and subsurface conditions, including climatological changes such as barometric pressure and temperature, or a shift in osmotic potential due to the introduction of solutes to the water supply, can drastically alter soil-water potential.

It is equally important to note, in relation to frost, that soil-water potential is a function of temperature, as represented by the Clausius-Clapeyron equation:

$$φ=φ\_{π}+φ\_{m}=L\_{f}\left(\frac{T-T\_{frz}}{T}\right)$$

in which:

$$φ=total water potential$$

$$φ\_{π}=soil-water osmotic potential$$

$$φ\_{m}=soil matric potential$$

$$T=absolute temperature$$

$$T\_{frz}=freezing point of bulk water \left[typically, 0°C\right]$$

$$L\_{f}=latent heat of fusion$$

The latent heat of fusion (also referred to as the enthalpy of fusion) is the amount of heat energy required to change the state of a solid of given mass to a liquid, or vice versa, without causing shifts the mass’ temperature. For example, when ice begins to melt it initially remains at$ 0°C$, and therefore the liquid water formed is also at $ 0°C$. In order to maintain this, ice must melt with a latent heat of fusion of 334 kJ/kg.

The steady-state, one-dimensional flux of water through soil can be represented by Darcy’s equation:

$$q\_{i}=-K\left(\frac{∂(φ\_{m}+φ\_{g})}{∂\_{z}}\right)$$

in which:

$$K=unsaturated conductivity$$

$$φ\_{m}=soil matric potential$$

$$φ\_{g}=gravitational potential$$

$$z=depth within soil$$

$$\left(\frac{∂(φ\_{m}+φ\_{g})}{∂\_{z}}\right)=gradient in soil-water potential$$

Darcy’s equation represents the rate at which water moves throughout saturated soils and the overall distribution of water potential within a one-dimensional soil column, specifically. However, when the soil is unsaturated or nonhomogeneous the equation can be modified to represent the unsaturated hydraulic conductivity function, $K\left(θ\right)$ or $ K(φ\_{m})$, and its relationship to either soil matric potential ($φ\_{m})$ or volumetric water content ($θ)$:

$q\_{i}=-K(θ)\left(\frac{∂(φ\_{m}+φ\_{g})}{∂\_{z}}\right)$ or $q\_{i}=-K(φ\_{m})\left(\frac{∂(φ\_{m}+φ\_{g})}{∂\_{z}}\right)$

Lastly, there is the water transient mass balance equation:

$$\frac{∂θ\_{1}}{∂t}+\frac{p\_{i}}{p\_{1}}\frac{∂θ\_{i}}{∂t}=\frac{∂q\_{1}}{∂z}+U$$

in which:

$$θ\_{1}=volumetic water content$$

$$p\_{1}=density of water$$

$$θ\_{i}=volumetric ice content$$

$$p\_{i}=density of ice$$

and, overall:

$$\frac{∂θ\_{1}}{∂t}=rate of change \left(time\right)of liquid water content$$

$$\frac{p\_{i}}{p\_{1}}\frac{∂θ\_{i}}{∂t}=rate of change \left(time\right)of ice content$$

$$\frac{∂q\_{1}}{∂z}=water flux gradient$$

$$U=water source/sink term$$

This equation is inclusive of the effects of freezing and thawing on soil condition and summarizes that the net flux of liquid water, in a layer of soil, must be equal to the combined change in volumetric ice and water content.

PRIOR STUDIES

A report by Gray and Granger (1969) studied the necessity of infiltration augmentation in seasonally frozen soils to decrease snowmelt runoff and peak discharge. In the original study, the effect of different types of frost, as well as the number and orientation of connected macropores, on the absorption of meltwater was observed. They stated: given that “the presence of ice in a soil reduces both its effective porosity and permeability, leads one intuitively to expect an inverse relationship between infiltration and frozen soil moisture.” Several years of study were conducted using a physically-based model, reliant on empirical calculations and the knowledge of basic physical relationships, considering this an efficient addition to some of the extensively-developed operational models recognized and utilized by many U.S. governmental departments. The results garnered from the application of their model showed that, in relating runoff volume “equal to the areally-weighted snowcover water equivalent minus the areally-weighted infiltration” to the measured streamflow from snowmelt, and using data from 1974 and 1975, was a ratio of 1.20 and 1.16, respectively (Gray and Granger, 1969). In further analysis, it was “suggested that significant increases in soil water augmentation by snowmelt infiltration to uncracked, seasonally-frozen soils cannot be realized without altering their structure, by some mechanical or other means, so as to increase the number of macropores which will allow more meltwater to percolate deep into the root zone” (Gray, Granger, and Nicholaichuk, 1985).

The United States Geological Survey (USGS) conducted a related model, prepared by Douglas Emerson (1994), using an adaptation of Fourier’s law of heat conduction to improve upon their preexisting Precipitation-Runoff Modeling System (PRMS). This adaptation relates the heat and water transfer of seasonally frozen soils and the freeze-thaw cycle to soil field capacity and infiltration rates as they are reliant upon soil conditions and snowmelt, and is comprised of numerous equations expanding upon the properties of thermodynamics, soil, frost, and thaw.

Firstly, the depth of frost must be known. An equation developed in 1949 by the U.S. Army Corps of Engineers calculates the depth of frost penetration as a function of time exposed to freezing temperatures:

$$X\_{f}=\left(\frac{86,400K\_{f}I\_{f}}{L+C\left(T\_{a}+\frac{I\_{f}}{2t}\right)}\right)$$

in which:

$$X\_{f}=depth of frost$$

$K\_{f}=thermal conductivity of frozen soil$

$$I\_{f}=frost index$$

$$L=latent heat$$

$$C=volumetric heat capacity$$

$$T\_{a}=mean annual temperature of given soil layer$$

$$t=duration of freezing period, in days$$

From this point, thawing can then be calculated:

$$X\_{t}=\left(\frac{86,400K\_{u}I\_{t}}{L+CI\_{t}}\right)^{0.5}$$

in which:

$$X\_{t}=thaw depth$$

$$K\_{u}=thermal conductivity of nonfrozen soil$$

$$I\_{t}=thaw index$$

$$L=latent heat$$

$$C=volumetric heat capacity$$

Remember that, due to the properties of latent heat of fusion, in order for thawing conditions to be met the soil only needs to reach $0°C$ and a temperature gradient be maintained.

That being said, in running the model presented by the USGS, they were also required to define composite thermal conductivity based upon a ratio of a substance’s average temperature gradient to that of the main heat-conducting substance (water, in this instance):

$$K\_{n}=\frac{K\_{s}V\_{s}G\_{s}+K\_{w}V\_{w}G\_{w}+K\_{a}V\_{a}G\_{a}}{V\_{s}G\_{s}+V\_{w}G\_{w}+V\_{a}G\_{a}}$$

in which, for the nth layer of soil:

$$K\_{n}=composite thermal conductivity$$

$$K\_{s}=thermal conductivity of soil$$

$$K\_{w}=thermal conductivity of water$$

$$K\_{a}=thermal conductivity of air$$

$$V\_{s}=volumetric fraction of soil$$

$$V\_{w}=volumetric fraction of water$$

$$V\_{a}=volumetric fraction of air$$

$$G\_{s}=average temperature gradient of soil, with respect to water$$

$$G\_{w}=average temperature gradient of water, with respect to water (1:1)$$

$$G\_{a}=average temperature gradient of air, with respect to water$$

Volumetric heat capacity – involved in the previous calculations for depth of both frost and thaw – can also be determined in relation to a particular nth layer of soil:

$$C\_{n}=V\_{s}C\_{s}+V\_{w}C\_{w}+V\_{a}C\_{a}$$

in which:

$$C\_{n}=volumetric heat capacity$$

$$V\_{s}=volumetric fraction of soil$$

$$V\_{w}=volumetric fraction of water$$

$$V\_{a}=volumetric fraction of air$$

$$C\_{s}=volumetric heat capacity of soil$$

$$C\_{w}=volumetric heat capacityof water$$

$$C\_{a}=volumetric heat capacityof air$$

In application, this equation can be written to represent the volumetric heat capacity of the nth layer in respect to liquid water (where$ C\_{s}$ is 0.46 and $C\_{w}$ is 1.00):

$$C\_{n}=\left(1-P\_{n}\right)0.46+V\_{w}(1.00)$$

in which:

$$C\_{n}=volumetric heat capacity$$

$$P\_{n}=porosity of the n^{th} layer $$

$$V\_{w}=volumetric fraction of water$$

or in respect to ice (where$ C\_{s}$ is 0.46 and is 0.45 for ice):

$$C\_{n}=\left(1-P\_{n}\right)0.46+V\_{w}(0.45)$$

In a report by Riche and Schneebeli (2013), it was concluded that the most effective and reliable method of determining the thermal conductivity of snow is through direct numerical simulations (SIM), as opposed to the more time-consuming Heat Flux Plate (HFP) or Single Needle Probe (NP) models. In the model run by the USGS, the equation used to calculate snow’s thermal conductivity is based upon density – though it should be noted that this is not the only contributing factor:

$$K\_{sn}=0.0068D\_{s}^{2}$$

in which:

$$K\_{sn}=thermal conductivity of snow$$

$$D\_{s}=density of snow$$

Branching off of thermal conductivity, the thermal resistance can also be calculated. This is defined as:

$$R\_{n}=\frac{X\_{n}}{K\_{n}}$$

in which, for the nth layer of soil:

$$R\_{n}=thermal resistance$$

$$X\_{n}=thickness$$

$$K\_{n}=thermal conductivity$$

The total thermal resistance, represented below as $R\_{t}$, throughout all n-layers of affected soil can be calculated via a summation of the above equation:

$$R\_{t}=\frac{X\_{1}}{K\_{1}}+\frac{X\_{2}}{K\_{2}}+\cdots +\frac{X\_{n}}{K\_{n}}$$

while the total effective thermal conductivity, as represented in the same equation, can be calculated by restructuring the formula as:

$$K\_{t}=\frac{X\_{1}+X\_{2}+\cdots +X\_{n}}{R\_{t}}$$

However, if snow has formed a layer above the soil then the heat capacity and latent heat for the affected layers are to be accounted for through a series of alternate equations:

$$C\_{t}=\frac{\sum\_{}^{}C\_{n}X\_{n}}{\sum\_{}^{}X\_{n}}$$

and

$$L\_{t}=\frac{\sum\_{}^{}L\_{n}X\_{n}}{\sum\_{}^{}X\_{n}}$$

in which, for the nth layer of soil:

$$C\_{t}=total volumetric heat capacity (mulitple layers)$$

$$L\_{t}=total latent heat (mulitple layers)$$

$$L\_{n}=latent heat$$

The next step in modeling precipitation-runoff as a result of frozen soils is to determine the type of frost present. Given that the frost type classification implies a significant imposition on water movement throughout the soil, the field capacity, infiltration rate, or both can be altered in the following equations. Antecedent to that, for a general comparison of effect it should be observed that when the first layer of soil is at field capacity, 90% of water is anticipated to infiltrate; if both the first and second layer are at field capacity, 80% infiltration is expected, and so forth. The percentage not infiltrated is assumed to run off.

 The ratio of soil-water content to field capacity in the last layer to hold water and the next lowest is written as:

$$D\_{n}=\frac{M\_{n}}{M\_{fn}}$$

and

$$D\_{n+1}=\frac{M\_{n+1}}{M\_{f(n+1)}}$$

in which, for the nth layer of soil:

$$D\_{n}=ratio of soil water$$

$$M\_{n}=soil-water content$$

$$M\_{fn}=field capacity$$

$$D\_{n+1}=ratio of soil water for the n^{th} layer+1 $$

$$M\_{n+1}=soil-water content for the n^{th} layer+1 $$

$$M\_{f\left(n+1\right)}=field capacity for the n^{th} layer+1 $$

At the end of the United States Geological Survey’s period of study and development upon the preceding equations and their use in the PRMS, it was concluded that, with data entered, the simulations ran successfully, and that the model performs “very well”, therefore accurately correlating frost-related parameters to those of runoff.

CONCLUSION

In consideration of the above, the relationship between freezing conditions, soil-water potential, and resulting runoff is empirically significant and quantifiable. As such, scientific studies can be conducted to determine the exact volume of runoff that will be generated based upon certain seasonal climatological and geographic criteria. Relatedly, infiltration augmentation and stormwater best management practices can be studied for both their efficacy in infiltrating snowmelt runoff in freeze-thaw cycles, and their dependency on existing soil-conditions to perform.

It should be noted, however, that freeze-thaw cycles and frost conditions are extremely sensitive and can be quite difficult to capture reliably and as such, models should be performed in a carefully controlled, simulated environment whenever possible.

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