Set A

2025 Illinois Middle School Math Olympiad - May 17th, 2025

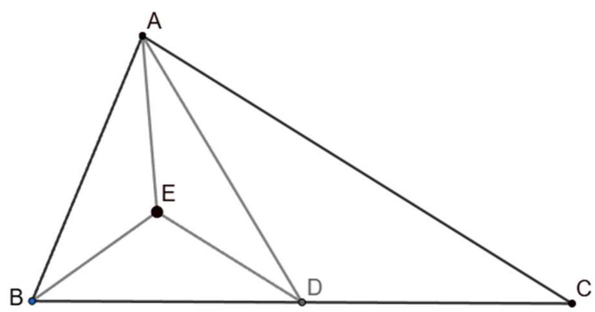
1. Find all integer solutions to the 3-variable equation:

   $$\mathbf{2025^x - 2025^y = Z^2.}$$

   Justify your answer.

2. A(x) and B(x) are two quadratic functions about x with real coefficients only. If A(A(x)) = B(B(x)) for all real values of x, prove that A(x) and B(x) are identical functions.

3. In △ABC, point D is the midpoint of BC, and point E is inside triangle ABD. It is given that △ABE is similar to △ACD. Prove DE is parallel to AC.



4. John and Joe play the following game: John picks a positive real number r, and then Joe must determine a positive integer n such that n multiplies r, rounded to the nearest tenth, is an integer. Once Joe chooses n, subject to this constraint, John gets n points. What is the greatest number of points that John can guarantee himself? Justify your answer.

5. Inside a tour bus, there are several tourists. An interesting property holds: For every group of 10 tourists chosen from the bus, there is always exactly one person who is a mutual friend of all 10. What is the maximum number of tourists that could be on the bus? Justify your answer. Notice friendship is mutual (if A is a friend of B, then B is a friend of A), and no one is their own friend.

Set B

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6. From the positive integers 1 through 2025, what is the greatest number of integers that can be selected such that the sum of any three distinct chosen numbers is divisible by 33? Justify your answer.

7. a, b, c, d are positive integers such that $ab = cd$. Prove that the sum $a+b+c+d$ cannot be a prime number.

8. Let point I be the incenter of triangle ABC. AI and BI meet the circumcircle of triangle ABC again at points D and E, respectively. Suppose DE meets BC and AC at points F and G, respectively. Prove that quadrilateral IGCF is a rhombus.

9. n is a positive integer with $n > 3$, and let $a_0, a_1, a_2, \ldots, a_n$ be a strictly increasing sequence of positive integers such that: $a_n \leq 2n - 3$. Prove that there exist five distinct indices p, q, r, s, t in $\{0, 1, 2, \ldots, n\}$ such that:

$$a_p + a_q = a_r + a_s = a_t$$

10. Let $\triangle ABC$ be a right triangle with $\angle BAC = 90^0$. Point D lies on side BC, and point E is the midpoint of segment AD. Suppose $\angle BED = \angle CED$.

    Prove $\angle BDA = 2\angle BAD$.