2026 Illinois Middle School Math Olympiad

May 16, 2026

Set A

1. In concave quadrilateral $ABCD$, $\angle A = \angle B = \angle D = 45^\circ$. Prove that $AC = BD$.

2. The function $f(x)$ has domain and range $[0, 1]$. It also satisfies

$$f(0) = 0, \quad f(1) = 1, \quad f(\frac{x+y}{2}) = \frac{f(x) + f(y)}{2}$$

for all x, y such that $x, y \in [0, 1]$.

Prove that $f(x) = x$ for all rational values of x in $[0, 1]$.

3. Prove that every integer $n \geq 2$ has a proper divisor d ($d < n$) such that $n + d$ has at least as many divisors as n has. For example, if $n = 50$, one may take $d = 10$, giving $n + d = 60$, which has more divisors than 50.

4. An infinite sequence $\{a_n\}$ is defined by

$$a_1 = c + 1, \quad a_{n+1} = c + \frac{1}{a_n}$$

for all positive integers n, where c is a constant and $0 < c < 1$.

Prove that every term of the sequence is greater than 1.

5. A pile of 100 chips is placed on a table. Two players A and B take turns removing chips, with A going first. On each turn:

   - At least one chip must be removed.
   - On the first move, A may remove at most 99 chips.
   - On each subsequent move, the number of chips removed is at most twice the number removed on the previous move.

   The player who removes the last chip wins.

   Determine whether player A has a winning strategy. (A winning strategy for a player is a complete plan of action that ensures victory from a given position against every possible sequence of legal moves by the opponent.)

Set B

6. For positive integers a, b, c, d, suppose

$$a + b = cd, \quad ab = c + d.$$

Find all possible values of $a + b + c + d$.

7. Let $ABCD$ be a trapezoid with $AD \parallel BC$ and $AD < BC$. Suppose there exists a point P on segment BC such that triangles APB, APD, and DPC all have equal perimeters.

Prove that $BC = 2AD$.

8. Let n be a fixed positive integer. Suppose a, b, c are positive integers satisfying

$$\frac{1}{n} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}.$$

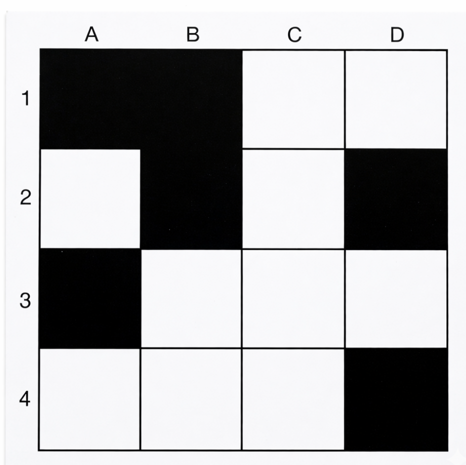
Determine, in terms of n, the largest possible value among a, b, c.

(Hint: When $n = 2$, the value is 42.)

9. Let $\triangle ABC$ be an acute triangle with $AB < BC < AC$. Points D and E lie on AC such that $BD = DC$ and $AE = EB$. Suppose the circumcircles of $\triangle ADB$ and $\triangle BCE$ intersect again at a point X.

Prove that $AX = XC$.

10. Let k be a positive integer. Consider a $2k \times 2k$ grid in which exactly $3k$ unit squares are colored black. Prove that there exist k rows and k columns such that every black square lies in at least one of these rows or columns.



Example shown for $k = 2$ with 6 black unit squares in a 4×4 grid. Rows 1 and 3, and columns B and D can be picked to include all 6 black unit squares.