Further Remarks About Example 4.5
in CRC Book "Water Environment Modeling"

By Clark C.K. Liu

Example 4.5 in the Chapter 4 of a CRC book "Water Environment Modeling" was prepared to illustrate the derivation and application of a simple detention pond model. The example consists of two parts. The first part presents the CSTR detention pond flow modeling and the second part presents the corresponding detention pond water quality modeling.

In the first part of Example 4.5, the outflow hydrograph from a detention pond was calculated on an Excel spread sheet by using an inflow hydrograph and pond hydraulic property as modeling input (Viessman et al, 2009). The calculated results are shown in Table 4.2, from columns 1 to 7.

In the second part of Example 4.5. the outflow pollutant concentration vs time curve was calculated by using the known inflow water quality conditions and the calculated outflow hydrograph as modeling input. The calculated results are shown in Table 4.2, from columns 8 to 14. The outflow pollutant concentration vs time curve is also called as the outflow pollutograph. The subject of the simple detention pond water quality modeling was not covered by Viessman et al (2009).

It is noted that Table 4.2 in Example 4.5 shows modeling results of both flow and water quality modeling; the information provided by Table 4.1 in Example 4.5 has been included completely by Table 4.2, and thus Table 4.1 should be removed from Example 4.5.

On Sept 21, 2021, we informed CRC of a missed citation in Example 4.5 and made a request to revise the example. However, CRC Press told us "*…book went to press Sept. 13, and no further corrections can be made at this point.*"

We are sorry for not identifying and citing properly the source of inflow hydrograph data used by Example 4.5. We will make the corrections in the future editions of this book.

A revised Example 4.5 is presented in the following pages.

4.3 Formulation and Application of Simple Water Environment Models

4.3.1 Modeling a detention pond system as a CSTR with time-variable flow

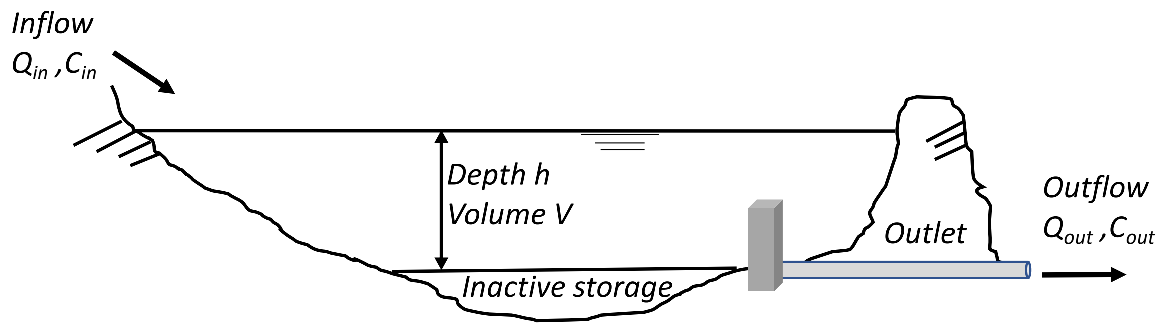


Figure 4.11 Flow and transport simulation of a storm detention pond

As one of the most popular non-point source pollution control measures, a storm runoff detention pond provides temporal storage of storm runoff produced over a watershed to manage its flow and water quality.

By taking the concentration of the transporting chemical $C = M/V$ in a detention pond, the transport equation or Eqn. (4-4) is modified to get,

$$\frac{dM}{dt} = Q_{in}C_{in} - Q_{out}\frac{M}{V} - kM \qquad (4.29)$$

In applied hydrology, storm runoff produced over a watershed is estimated and expressed by flood hydrograph analysis (see Section 5.2). Because the CSTR model of a detention pond uses flood hydrograph as its inflow, both Q_{in} and Q_{out} in Eqn. (4-29) are time-variable functions. Generally, Eqn. (4-29) contains two unknown variables of mass *M* and flow Q_{out}; therefore, an independent flow equation must be solved together. Based on mass conservation principle, the flow equation is formulated as,

$$\frac{dV}{dt} = Q_{in}(t) - Q_{out}(t) \qquad (4.30)$$

As shown in Figure 4-11, volume increment *dV* can be expressed in terms of water depth increment d*h* or *dV*=*A*(*h*) *dh*. Note that *A*(*h*) does not change with time and d*h* can be used to replace *dV* as an independent variable in Eqn. (4-30) to get,

$$A(h)\frac{dh}{dt} = Q_{in}(t) - Q_{out}(t) \qquad (4.31)$$

Example 4.5 Modeling a detention pond system as a CSTR with time-variable flow **(Revised Sept 2021)**

Problem

A detention pond receives inflow from watershed runoff (Figure 4-11). The runoff contains a contaminant at a concentration C_{in} = 100 g/m^3. The contaminant undergoes a first-order decay in the pond with a reaction coefficient of k = 0.1 d^{-1}. Initially, the contaminant concentration in the pond is zero. The surface area and water depth of the pond is related by an empirical formula $A(h) = 400h^{0.7}$. Water flows out of the detention pond through a culvert with diameter d_c=8-inch. Outflow rate Q_{0ut} can be determine by an orifice formula $Q_0 = C_d A_0 \sqrt{2gh}$ in which the orifice coefficient C_d has a value of 0.9, $A_0 = C_d \, \pi d_c^{\,2}/4$, or $C_0 A_0$ = 0.029. The hydraulic properties and the inflow hydrograph of a hypothetical detention pond as given by Viessman et al., (2009) are used by this example.

Assume the pond can be simulated by a simple CSTR model. Calculate the outflow hydrograph $Q_{out}(t)$ and the outflow contaminant concentration over time curve or pollutograph $C_{out}(t)$.

Solution

Insert the orifice equation into Eqn. (4-31) to get,

$$\frac{dh}{dt} = \frac{Q_i(t) - C_0 A_0 \sqrt{2gh}}{A(h)} = \frac{Q_i(t) - 0.029\sqrt{19.6h}}{400h^{0.7}} \qquad (4.32)$$

By Euler approximation, $\frac{dh}{dt}$ can be expressed as,

$$\frac{dh}{dt} \approx \frac{h(t+\Delta t) - h(t)}{\Delta t} \qquad (4.33)$$

Combining Equs. (4-32) and (4-33),

$$h(t+\Delta t) = h(t) + \Delta t\left(\frac{Q_{in}(t) - C_o A_o \sqrt{2gh}}{A(h)}\right) \qquad (4.34)$$

Eqn.(4.34) indicates that the water depth of the detention pond at any time step *h(t)* can be used to calculate the water depth at the next time step *h(t+Δt)* if the rate of inflow $Q_{in}(t)$ and other hydraulic properties of the pond are known.

Again, by Euler approximation, $\frac{dM}{dt}$ is expressed as,

$$\frac{dM}{dt} \approx \frac{M(t+\Delta t) - M(t)}{\Delta t} \qquad (4.35)$$

Combining Eqn. (4-35) and Eqn. (4-29) to get,

$$M(t+\Delta t) = M(t) - \Delta t\left[Q_{in}C_{in} - Q_{out}C(t) - kM(t)\right] \quad (4.36)$$

Eqn.(4.36) indicates that the mass flux out of the pond *M(t)* and the pollutant concentration *C(t)* at a time step *t* can be used together to calculate the mass flux $M(t+\Delta t)$ out of the pond at the next time step; if the rate of inflow $Q_{in}(t)$, the rate of outflow $Q_{out}(t)$, and reaction coefficient *k* are known. The pollutant concentration at the next time step is then calculated simply by $C(t+\Delta t) = M(t+\Delta t)/V(t+\Delta t)$ *where V is* the water volume in the pond.

Successive values of water depth *h*(t), the rate of outflow $Q_o(t)$ and the outflow pollutant concentration *C(t)* can be calculated on an Excel spread sheet as shown in Table 4.2. In Table 4.2, hydraulic parameters are shown in Columns 1 to 7 and water quality parameters are shown in columns 8 to 14.

Each cell in Table 4.2 is denoted by cell (*a.b*), where *a* is the number of column and *b* is the number of row. Thus, the initial inflow rate $Q_i(0) = 0$ is denoted by cell (2,1), the initial water depth $h(0) = 0.5$ is denoted by cell (3,1), the initial inflow pollutant concentration $C_{in}(0) = 0$ is denoted by cell (9,1), and the initial outflow pollutant concentration $C(0) = 0$ is denoted by cell (12,1).

The initial outflow rate can be calculated by orifice equation $Q_0(0) = 0.029\sqrt{2(9.8)(0.5)} = 0.09141\ m^3/sec$ and is denoted by cell (4,1). The initial water surface area $A\text{-}surf(0) = 400\ (0.5^{0.7}) = 246.2289\ m^2$ and is denoted by cell (5,1).

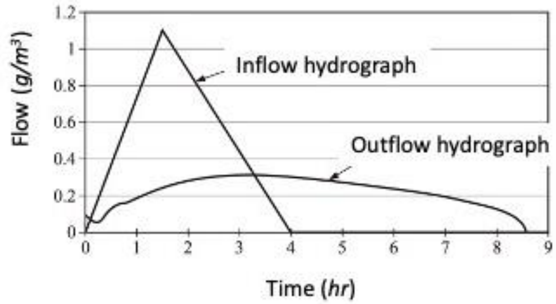
Eqn. (4-34) is used to calculate the water depth at $t = 0.25$ *hours* to get $h(0.25) = 0.165882\ m$ which is denoted by cell (3,2). The orifice equation is then used to calculate the rate of outflow at $t = 0.25$ *hours* to get $Q_0(0.25) = 0.052652\ m^3/sec$ and the results is denoted by cell (4,2).

Eqn. (4-36) is used to calculate the mass flux at $t = 0.25$ *hours* to get $M(0.25) = 0$ which is denoted by cell (11,2). The pollutant concentration in the outflow can then be readily calculate as $C_0(t) = M(t)/V$, where V is the volume of water in the pond. Note that, by taking the pond as a CSTR model, the pollutant concentration in the outflow equals the pollutant concentration inside the pond. For example, at time t = 0.5 *hour*, C (0.5) can be calculated to be M(0.5)/V(0.5) = (16200 *g*)/[(1.173547 *m*)(447.415 $m^{2)}$] = 30.85 g/m^3 and is denoted by cell (12,3).

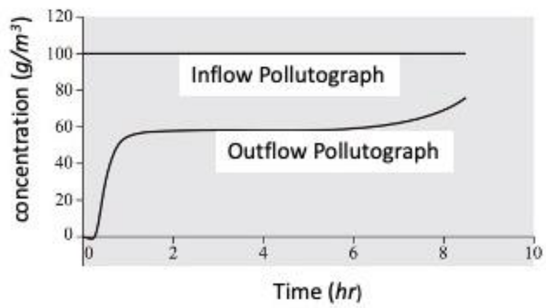
Table 4-2 is completed by repeating the procedures for the remaining rows. Figure 4-13 (a) shows calculated inflow and outflow hydrographs and Figure 4-13 (a) shows calculated inflow and outflow pollutographs.

Table 4.2 Computation of outflow hydrograph and pollutogragh in Example 4.5

(1) Time (hr)	(2) Q_{in} (cms)	(3) $h(t)$ (m)	(4) Q_{out} (cms)	(5) A-surf m^2	(6) dh/dt m/sec	(7) $h(t+dt)$ (m)	(8) k hr^{-1}	(9) C_{in} g/m^3	(10) Flux-1 kg/hr	(11) $M(t)$ g	(12) $C(t)$ g/m^3	(13) Flux-0 kg/hr	(14) dM/dt g/hr
0	0	0.5	0.09141	246.2289	−0.000371	0.166	0.1	0	0	0	0	0	0
0.25	0.18	0.165882	0.052652	113.742	0.00112	1.174	0.1	100	64.8	0	0	0	64800
0.5	0.36	1.173547	0.140043	447.415	0.000492	1.616	0.1	100	129.6	16200	30.85	15.55	112425
0.75	0.54	1.616003	0.164336	559.718	0.000671	2.220	0.1	100	194.4	44306	48.98	28.98	160990
1	0.72	2.220053	0.192616	699.060	0.000754	2.899	0.1	100	259.2	84554	54.48	37.78	212966
1.25	0.9	2.89903	0.220109	842.630	0.000807	3.625	0.1	100	324	137795	56.41	44.7	265523
1.5	1.1	3.625211	0.246137	985.356	0.000867	4.405	0.1	100	396	204176	57.16	50.65	324935
1.75	0.99	4.405108	0.271325	1129.353	0.000636	4.978	0.1	100	356.4	285410	57.37	56.04	271822
2	0.88	4.977833	0.288424	1230.235	0.000481	5.411	0.1	100	316.8	353365	57.7	59.91	221549
2.25	0.77	5.410611	0.3007	1304.165	0.00036	5.734	0.1	100	277.2	408753	57.93	62.71	173617
2.5	0.66	5.734473	0.309569	1358.330	0.000258	5.967	0.1	100	237.6	452157	58.05	64.69	127692
2.75	0.55	5.966661	0.315774	1396.600	0.000168	6.118	0.1	100	198	484080	58.09	66.04	83554
3	0.44	6.117901	0.319743	1421.238	8.46E-05	6.194	0.1	100	158.4	504969	58.08	66.85	41050
3.25	0.33	6.193754	0.321727	1433.599	5.77E-06	6.199	0.1	100	118.8	515231	58.03	67.21	70.478
3.5	0.22	6.198947	0.321862	1434.441	−7.1E-05	6.135	0.1	100	79.2	515249	57.95	67.14	−39466
3.75	0.11	6.135037	0.320199	1424.072	−0.000148	6.002	0.1	100	39.6	505382	57.85	66.68	−77618
4	0	6.002193	0.316713	1402.416	−0.000226	5.799	0.1	100	0	485978	57.73	65.83	−114424
4.25	0	5.798943	0.311304	1369.002	−0.000227	5.594	0.1	100	0	457372	57.61	64.57	−110303
4.5	0	5.594287	0.305762	1335.000	−0.000229	5.388	0.1	100	0	429796	57.55	63.35	−106326
4.75	0	5.388156	0.300076	1300.373	−0.000231	5.180	0.1	100	0	403214	57.55	62.17	−102489
5	0	5.18047	0.294236	1265.081	−0.000233	4.971	0.1	100	0	377592	57.61	61.03	−98788
5.25	0	4.971146	0.28823	1229.078	−0.000235	4.760	0.1	100	0	352895	57.76	59.93	−95221
5.5	0	4.760088	0.282045	1192.314	−0.000237	4.547	0.1	100	0	329090	57.98	58.87	−91784
5.75	0	4.547191	0.275666	1154.729	−0.000239	4.332	0.1	100	0	306144	58.3	57.86	−88476
6	0	4.332336	0.269074	1116.260	−0.000241	4.115	0.1	100	0	284025	58.73	56.89	−85294
6.25	0	4.115391	0.262251	1076.832	−0.000244	3.896	0.1	100	0	262702	59.28	55.97	−82236
6.5	0	3.896206	0.255171	1036.357	−0.000246	3.675	0.1	100	0	242143	59.97	55.09	−79302
6.75	0	3.674609	0.247809	994.736	−0.000249	3.450	0.1	100	0	222317	60.82	54.26	−78491
7	0	3.4504	0.24013	951.850	−0.000252	3.223	0.1	100	0	203195	61.87	53.48	−73803
7.25	0	3.223352	0.232094	907.560	−0.000256	2.993	0.1	100	0	184744	63.15	52.77	−71240
7.5	0	2.99319	0.223655	861.696	−0.00026	2.760	0.1	100	0	166934	64.72	52.11	−68805
7.75	0	2.759594	0.21475	814.051	−0.000264	2.522	0.1	100	0	149732	66.65	51.53	−66503
8	0	2.52217	0.205304	764.367	−0.000269	2.280	0.1	100	0	133107	69.04	51.03	−64341
8.25	0	2.280436	0.195218	712.316	−0.000274	2.034	0.1	100	0	117022	72.04	50.63	−62331
8.5	0	2.033781	0.184358	657.465	−0.00028	1.781	0.1	100	0	101439	75.86	50.35	−60493



(a) Hydrograph (adopted from Viessman et al, 2009)



(b) Pollutograph

Figure 4.13 Calculated outflow hydrograph $Q_{out}(t)$ and pollutograph $C_{out}(t)$

Comments

1. Detention ponds have been widely used as management measures for flood mitigation and non-point source pollution control. This example illustrates the derivation and testing of a simple CSTR detention pond model.
2. The outflow hydrograph $Q_{out}(t)$ and the outflow pollutograph $C_{out}(t)$ are given in Table 4.2 and Figure 4.13. These Modeling results show that a detention pond reduces the flood peak and the contaminant load during a particular heavy rainstorm.
3. After the model of a retention pond is developed, it can be used to simulate outflow hydrographs and pollutographs under varying inflow sequences and pond hydraulic condition. The results can then be used to identify the most cost-effective pond design to achieve specific flow and water quality objectives.

The following reference will be added to the reference list of Chapter 4 of the CRC book "Water Environment Modeling":

Viessman Jr., W., Hammer, M.J., Perez, E.M., and Chadik, P.A. (2009). Water Supply and Pollution Control, 8th ed., Pearson Prentice Hall, Upper Saddle River, NJ.