Intoxicated Mathematics

Kyla Gorman Fall 2020 Mathematics Capstone

Modeling BAC

Blood Alcohol Content (BAC) refers to the concentration of alcohol within the blood, B(t). We also model alcohol content in the stomach, A(t). These depend on various factors, such as the amount of alcohol consumed, time between drinks, size of person, etc. The change in alcohol content over time can be modeled by the following system of differential equations:

$$A'(t) = -k_1 A(t) \quad (1)$$

$$B'(t) = k_1 A(t) - k_2 B(t) \quad (2)$$

k represents the proportionality constant of concentration of alcohol in the stomach and k2 represents the proportionality constant of concentration of alcohol in the blood. This system can be represented in the following vector-matrix system:

$$\begin{bmatrix} A'(t) \\ B'(t) \end{bmatrix} = \begin{bmatrix} -k_1 & 0 \\ k_1 & -k_2 \end{bmatrix} \cdot \begin{bmatrix} A(t) \\ B(t) \end{bmatrix}$$

Solving the Eigensystem Equation of the above vector-matrix system yields the following solution:

$$\begin{bmatrix} A(t) \\ B(t) \end{bmatrix} = c_1 \begin{bmatrix} \frac{k_1 - k_2}{k_1} \\ 1 \end{bmatrix} e^{-k_1 t} + c_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^{-k_2 t}$$

This is a linear combination of the two eigenvectors, and corresponds to the following solution set:

$$A(t) = c_1 e^{-k_1 t}$$

$$B(t) = c_1 e^{-k_1 t} + c_2 e^{-k_2 t}$$

The initial conditions are A(0) = A0, and B(0) = 0 when no alcohol is already in the body. We apply these to our solution set and solve for our integration constants which yields this solution:

$$A(t) = A_0 e^{-k_1 t}$$

$$B(t) = A_0\left(\frac{k_1}{k_2 - k_1}\right)(e^{-k_1 t} - e^{-k_2 t})$$

The initial conditions are A(0) = A0+A1, and B(0) = B0 when alcohol is already in the body where A1 is the concentration of alcohol already in the stomach at time zero, and B0 is the concentration of alcohol in the blood at time zero. We apply these to our solution set and solve for our integration constants which yields this solution:

$$A(t) = (A_1 + A_0)e^{-k_1 t}$$

$$B(t) = (A_1 + A_0)\left(\frac{k_1}{k_2 - k_1}\right)e^{-k_1 t} + \left(B_0 - (A_1 + A_0)\left(\frac{k_1}{k_2 - k_1}\right)\right)e^{-k_2 t}$$

Assumptions:

1. subject is 75kg
2. alcohol enters the stomach immediately after is consumed
3. alcohol is consumed all at once, not over a period of time (the more likely scenario)
4. subject is not drinking any water or eating any food

Figures & Tables

Figure 1.

The blood alcohol content quickly reaches its maximum of 0.0172% 19.8 minutes after the alcohol is consumed, and then exponentially decays at a slower rate. The proportionality constants for the stomach and blood are kept constant.

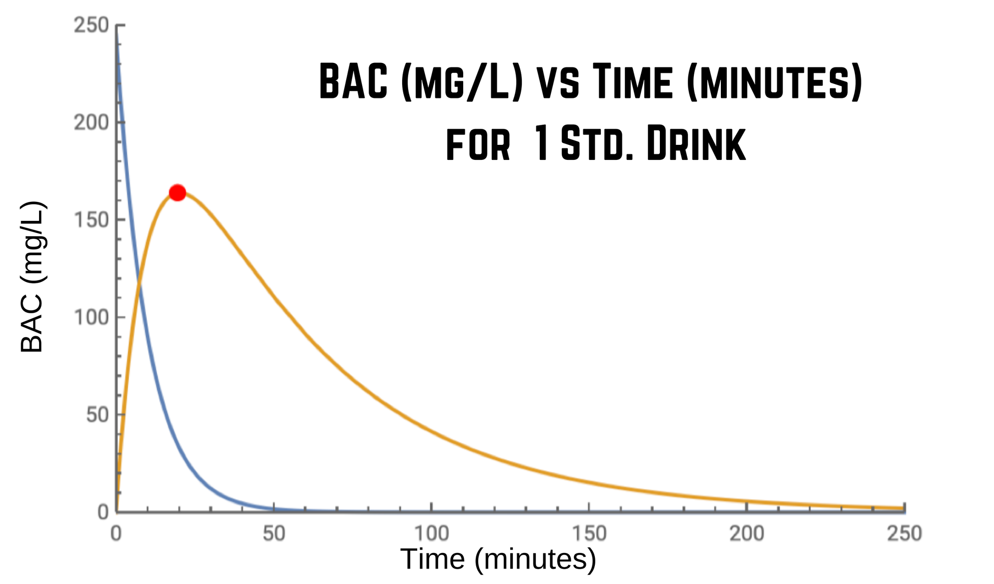


Figure 2.

The blood alcohol content reaches a maximum 19.8 minutes after the first drink is consumed, and again about twenty minutes after the second drink is consumed (fifty minutes after consumption of first drink). The maximum blood alcohol content that is reached is 0.029% BAC. The proportionality constants for the stomach and blood are kept constan

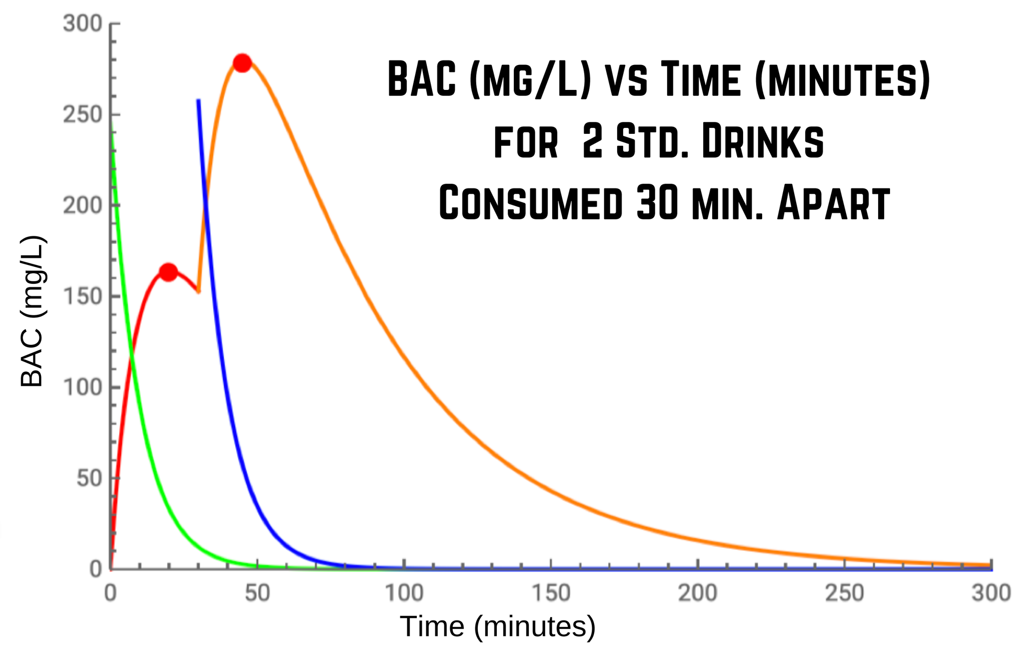


Figure 3.

Figure 3 shows the blood alcohol content over time for four different volumes of alcohol: 200mg/L, 300mg/L, 400mg/L, and 500 mg/L. The higher the concentration that is consumed, the higher the maximum that is reached. However, there is no difference in the time it takes to reach each maximum because the proportionality constants are kept constant.

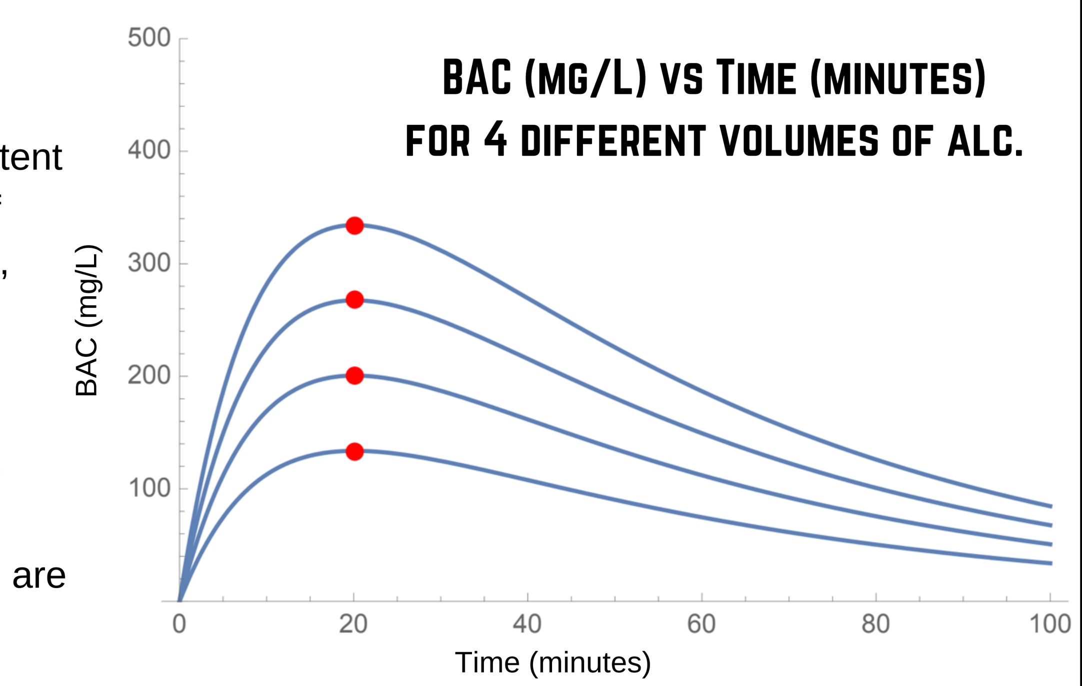
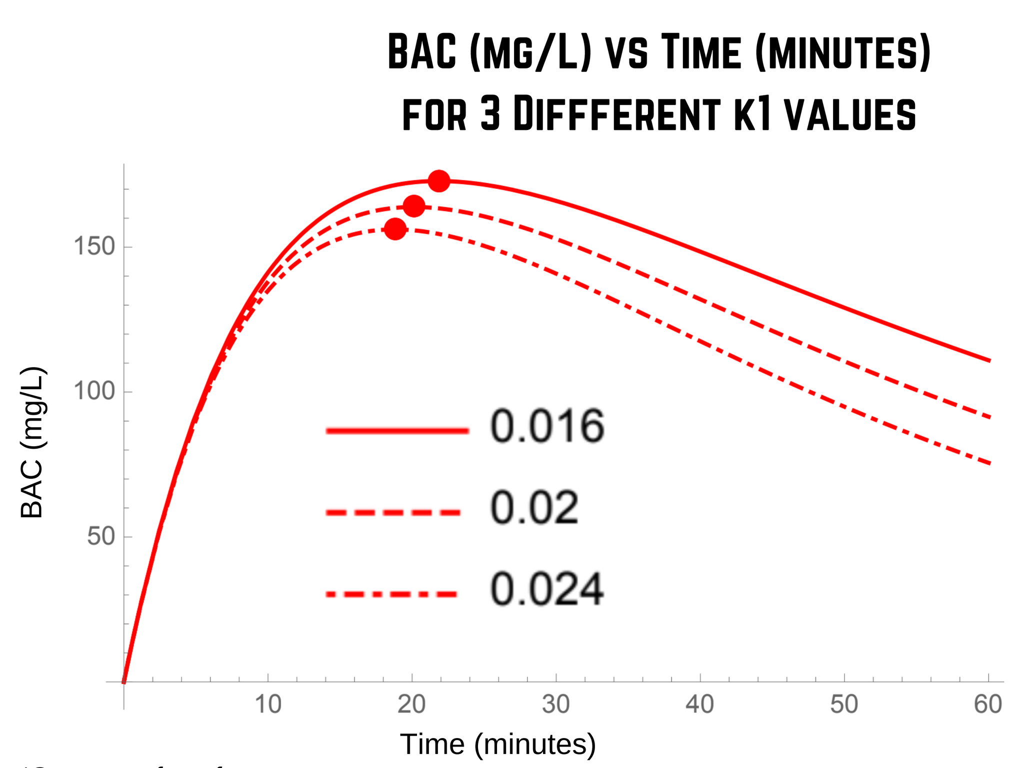


Figure 4.

0.016 is a 20% decrease from the original k2 = 0.02, and 0.024 is a 20% increase. Neither variance results in a significant change in max BAC. As the proportionality constant for the blood decreases, the time it takes to reach the maximum blood alcohol content increases. Likewise, the maximum blood alcohol content reached increases as well.



*See paper for references

Table 1.

% Change in k_1	k_1	Max B(t) (mg/L)	Time (minutes)	% Change In Max
-10%	0.0985	167.861	21.2305	2.60%
-5%	0.1040	170.248	20.5103	1.21%
0%	0.1095	172.337	19.8458	0%
5%	0.1149	174.162	19.2303	-1.06%
10%	0.1204	175.752	18.6584	-1.98%

A 10% in the proportionality constant in the blood does not result in a significant change in the maximum blood alcohol content reached. As the proportionality constant for the stomach decreases, the time it takes to reach the maximum blood alcohol content increases. However, unlike Figure 4, the maximum reached actually decreases ask1 decreases.

Conclusions

One weakness of this model is that each time a new drink is consumed, we must solve for a new set of initial conditions. Additionally, it is assumed that each drink is consumed immediately as opposed to over a period of time, which would be more accurate. This model also does not account for the consumption of food and/or water, which are also factors that affect blood alcohol content. Also, we do not account for gender and/or weight of the human, which are factors that also affect blood alcohol content.

Strengths of this model include the ease with which the eigenvectors and eigenvalues can be solved for using the eigensystem equation. The solving method for this problem is very efficient, although as mentioned above, its accuracy has yet to be tested. Another strength is that this model can be easily generalized to other biological processes. Finally, this model can be easily extended to apply to other organs within the body.

Further Work

There are various possibilities for further work. Firstly, we can model multiple drinks consumed over various periods of time. We could also figure out how to model a single drink consumed over a period of time, and possibly combine this model with the model of multiple drinks. Additionally, a model could be developed for how blood alcohol content changes over time while taking into account water and/or food consumption, as well of size or gender of human. Another opportunity for further work is modeling what happens to blood alcohol content when the half-life of alcohol is increased and/or decreased significantly and comparing these results to other known results. Lastly, a mathematical model could be developed for other organs in the body, such as concentration of alcohol in the brain, liver, etc.

Exponential decay functions can be used to describe a plethora of biological processes, so the mathematical model outlined in this paper might be applicable to other biological processes. It might be especially valuable to develop a mathematical model similar to this for substances metabolized by the human body that could be generalized to all such metabolic processes and would be somewhat accurate.

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