IMPROVING FRACTION UNDERSTANING THROUGH DEEP TEACHING

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Think back to the times you have taught students the meaning of a verb. Chances are you explained a verb is a 'doing' word and presented them with sentences such as 'The family swam at the beach'. You may then have asked your students to identify the word 'swam' as a verb. As we know, a verb is needed in every sentence.

You may then ask students to start writing descriptive sentences, such as 'The tiger has black spots'. How confident are you that your students could identify the verb in this example?

In this sentence, the tiger isn't doing anything. In reality, the definition of a verb is much more than just a 'doing word'. An action verb is only one category of verbs. Too often, we narrow definitions for our students to make them easier to understand and 'cherry-pick' examples to ensure that they fit the definition we provide.

However, as we discover in mathematics, this can hurt far more than it helps.

Cast your mind to the learning of place value where students are taught that the tens place tells you the number of tens in a number. For 2 digit-numbers this works perfectly. 67 has 6 tens. 83 has 8 tens. All is right in the world. But what about 112? Yes, there is a 1 in the tens place. But that is not how many tens are in the number. It actually has 11 tens.

Renaming also rears its head when we look at using algorithms. As a teacher you may have introduced multi-digit addition without renaming first (63 + 24). Once students can do this, they might be moved onto addition with renaming (67 + 28). But what does this teach the student? They learn that what they have been shown previously needs to change according to the numbers. No wonder students see maths as a collection of separate ideas, rather than an interconnected series of relationships.

Fractions, a concept widely considered to be one of the most difficult for students to learn, is often 'simplified' to make it more understandable. Looking at Figure 1, you would be confident that students could correctly identify the fraction as one quarter. But what about Figure 2?

Anecdotally from my work in classrooms, I would suggest that less than 10% of students in Years 3-5 (and only slightly more Year 6 students) could successfully identify the fraction in Figure 2 as one quarter

Figure 1.

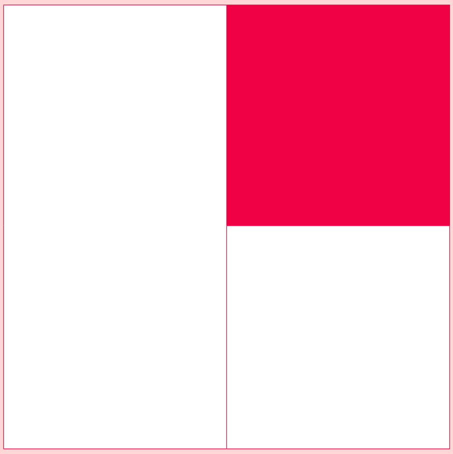

Figure 2.

But why is this the case? A common misconception relates to how the denominator is named. Often I hear students define the numerator as the number of pieces needed, and the denominator as the number of pieces altogether. It is this definition that severely limits the understanding of fractions. Not only does it make dealing with diagrams such as Figure 2 difficult, it also makes improper fractions very difficult to grasp. Think of seven fifths.

If you understand this as seven parts needed and five parts altogether, how could this be represented? Rather, we should advocate for the definition,

'In the fraction $\frac{a}{b}$, b is the name or size of the parts (e.g., fifths have this name because 5 equal parts can fill a whole), and a is the number of parts of that name or size' (Clarke, Roache & Mitchell, 2008, p. 375).

A fantastic activity that allows teachers to uncover what their students understanding of naming fractions is shown in Figures 3-5. Students are given a strip of paper and are invited to fold them equally into two parts. They then name the fraction they have made (halves – Figure 3). They will then fold one of the halves in half again, shade the first part they made and name it (fourths, see Figure 4).

Figure 3.

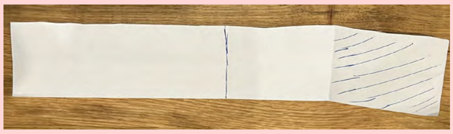

Figure 4.

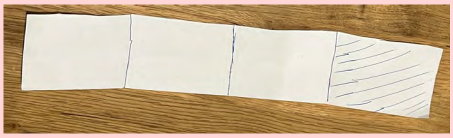

Figure 5.

Many students will identify this part as one-third, demonstrating a common misconception related to naming fractions based on the number of parts. What it also shows is the lack of connection between the names of fractions and how they can be partitioned, Siemon (2003) states this is the missing link in fractional understanding. A narrow view of a fourth is that it is four equal pieces, but a deeper understanding would allow students to recognise that a fourth is half of a half, which is demonstrated in this activity. The final part of the activity asks students to fold the other half and then name the shaded part (Figure 5).

This further demonstrates a view of the number of parts rather than their size, as most students will now name the fraction as one-fourth. They see that there are four pieces without identifying the fact that the size of the piece shaded didn't change with the second fold.

Below are suggestions of possible changes within classroom practice to address this common issue.

USE MULTIPLE REPRESENTATIONS

Students need to see and explore multiple representations of the same fraction. This should include, but not be limited to, using different shapes. This means students don't view fractions as just something you 'do' with circles or rectangles. Students also need to see a mixture of discrete and continuous representations. We need to broaden the representations we show, which includes having different factions within the same whole (Figure 2).

An example of an activity is called *Guess My Fraction*. The beauty of this activity is that it can be extended into equivalent fractions, fractions on a number line and adding fractions. It can also be simplified to show fractions as common fractions.

In the original version, each student needs paper/whiteboard, a pen, and two dice (six-sided or ten-sided). Student A rolls the two dice (without showing their partner) and draws the fraction. In this game, students attempt to not draw all parts when drawing their fraction (see Figure 6). Once drawn, Student B will attempt to name the fraction their partner rolled $\left(\frac{2}{5}\right)$.

If they are correct, they get a point and swap roles. If not, Student A receives a point, and they swap roles. Play continues until one player has ten points.

Extensions

For each of the following variations, the game is played the same way. All that changes is the diagrams that the students are drawing.

Number lines

Instead of drawing diagrams, students mark the fraction that they have rolled on the number line (Figure 7).

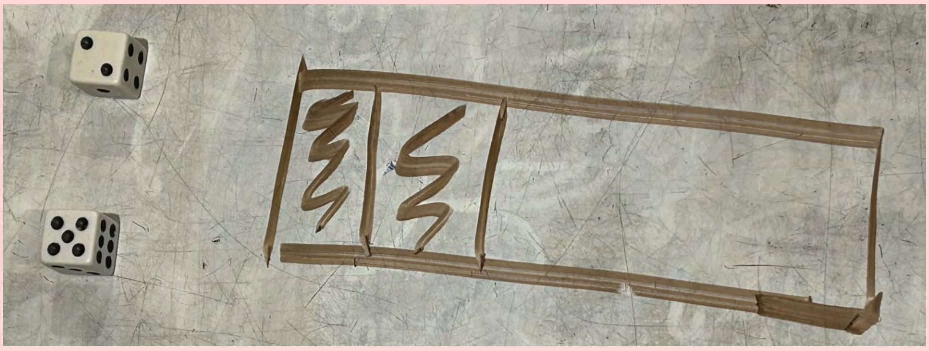

Figure 6.

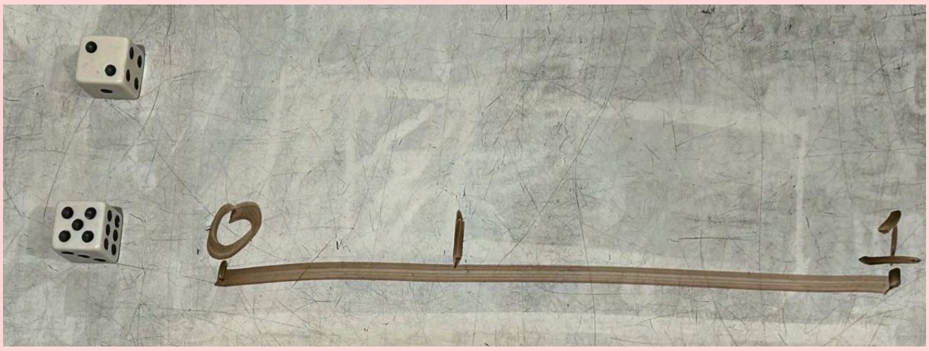

Figure 7.

Equivalent fractions

Students draw a representation of an equivalent fraction (Figure 8). Students can be given more guesses (such as three or five) to guess the original fraction, as there are several possibilities.

Figure 8.

CONCLUSION

In order to ensure our students develop the necessary skills and understandings in fractions, it is important we use multiple representations to explore common misconceptions. A deep understanding of fractions is crucial to accessing further critical mathematics and will lay strong foundations for our students as they progress through primary school.

REFERENCES

Clarke, D., Roche, A., & Mitchell, A. (2008). 10 Practical Tips for Making Fractions Come Alive and Make Sense. *Mathematics Teaching In The Middle School*, *13*(7), 375.

Siemon, D. (2003). Partitioning: the missing link in building fraction knowledge and confidence. *Australian Mathematics Teacher*, *59*(3), 22–24.