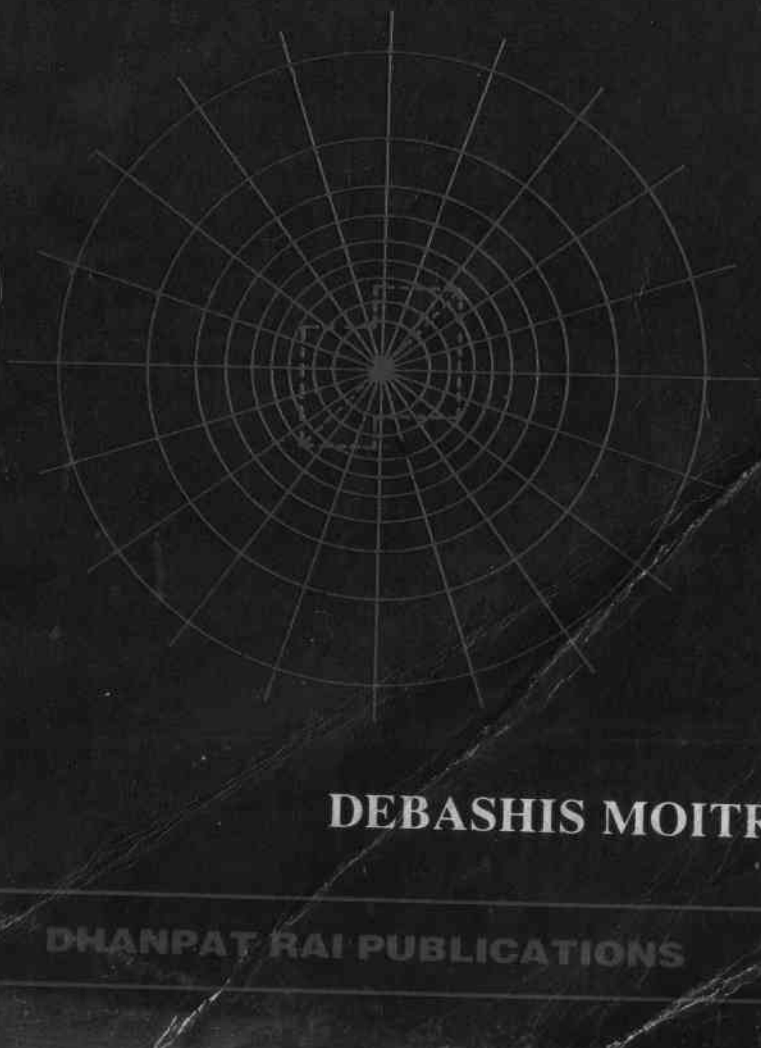
PROBLEMS IN SOIL MECHANICS AND FOUNDATION ENGINEERING



DEBASHIS MOITRA

DHANPAT RAI PUBLICATIONS

PROBLEMS
IN
SOIL MECHANICS
AND
FOUNDATION ENGINEERING

[For B.E.(Civil); M.E.(Civil); A.M.I.E.(India);
U.P.S.C. and other Competitive Examinations]

DEBASHIS MOITRA
Department of Civil Engineering
Bengal Engineering College – Deemed University
Howrah.

DHANPAT RAI PUBLICATIONS (P) LTD.
FIRST FLOOR, 67/4 MADRAS HOUSE
DARYAGANJ, NEW DELHI - 110 002
PHONES : 327 4073

OTHER USEFUL BOOKS

1. Advance Theory of Structures . . . . *N.C. Sinha*
2. Concrete Testing Manual . . . . *M.L. Gambhir*
3. Fundamentals of Limit Analysis of Structures . *V.K. Manicka Selvam*
4. Modern method of Structural Analysis . . . *V.K. Manicka Selvam*
5. Multistorey Building & Yield Line Analysis of Slabs . . . . *V.K. Manicka Selvam*
6. Energy Methods in Structural Mechanics . . . *V.K. Manicka Selvam*
7. Analysis of Structure in Earth Quake Region . *V.K. Manicka Selvam*
8. Dock and Harbour Engineering . . . . *S.P. Bindra*
9. Foundation Design Manual . . . . *N.V. Nayak*

First Edition 1993

Reprint : *1998*

Price : Rs. 60.00

Published by Ish Kapur for Dhanpat Rai Publications (P) Ltd.
Printed at : A.P. Offset, Naveen Shahdara, Delhi-110032.

Preface

This book is primarily intended for the undergraduate students of Civil Engineering. However, it will be helpful also to the diploma-level students, A.M.I.E. students, and, in some cases, even to the post-graduate students of Soil Mechanics and Foundation Engineering.

A thorough understanding of the basic principles of a subject like Soil Mechanics calls for the solution of a large number of numerical problems. In the present book a brief introduction to the contents of each chapter has been given, which is followed by a number of worked-out examples and quite a few practice problems. For a better understanding of the topics and students are required to solve all the problems by themselves. Effort has been made to explain the basic principles underlying the solution of the problems so that the students may develop the habit of having a logical insight into the numerical problems while solving them.

Comments and suggestions regarding the book, from the students as well as the teachers, will be highly appreciated.

Calcutta,
9, March 1993

DEBASHIS MOITRA

CONTENTS

1

WEIGHT-VOLUME RELATIONSHIPS

1.1 Introduction: Matter may exist in nature in three different states, viz., solid, liquid and gaseous. A soil mass in its natural state may consist of all three phases. The basic ingredient is the solid grains which form the soil skeleton, while the intermittent void spaces are filled up by either air, or water, or both. Thus, a soil mass in its natural state may be considered a three-phase system.

1.2 Soil Mass as a Three-phase System : In a soil mass in its natural state, the three phases, viz., solid, liquid and gas, are completely intermingled with one another. However, if one can determine the individual volumes of solid grains, liquid (i.e., water) and gas (i.e., air) present in a certain volume

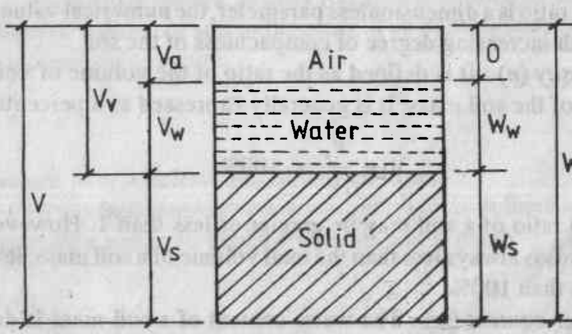


Fig. 1.1

of a soil, the entire soil mass can be represented by a schematic diagram, as shown in Fig. 1.1, where the volume of each constituent part is shown as a fraction of the total volume. The cross-sectional area of the soil mass is taken to be unity, so that, the volume of each constituent part is numerically equal to its height shown in the diagram. Again, the mass of each part may be obtained by multiplying its volume by the corresponding density.

The notations used in the diagram are defined below:

V = total volume of the soil mass

V_s = volume of solid particles in the soil
V_v = volume of voids in the soil
V_w = volume of water present in the voids
V_a = volume of air present in the voids
W = total mass of the soil
W_s = mass of the solid particles
W_w = mass of water present in the voids.

The mass of air present in the voids is negligible.

Thus, $V_v = V_a + V_w$

and, $V = V_s + V_v$

or, $V = V_s + V_a + V_w$

1.3 Basic Definitions : The fundamental physical properties which govern the engineering performance of a soil are defined below :

(i) *Void ratio* (e) : The void ratio of a soil is defined as the ratio of volume of voids to the volume of solids.

i.e., $$e = \frac{V_v}{V_s} \quad ...(1.1)$$

The void ratio is a dimensionless parameter, the numerical value of which decreases with increasing degree of compactness of the soil.

(ii) *Porosity* (n) : It is defined as the ratio of the volume of voids to the total volume of the soil mass. It is generally expressed as a percentage.

i.e., $$n = \frac{V_v}{V} \times 100\% \quad ...(1.2)$$

The void ratio of a soil may be greater or less than 1. However, as the volume of voids is always less than the total volume of a soil mass, its porosity is always less than 100%.

(iii) *Water content* (w) : The water content of a soil mass is defined as the ratio of the mass of water to the mass of solids. It is always expressed as a percentage.

i.e., $$w = \frac{W_w}{W_s} \times 100\% \quad ...(1.3)$$

(iv) *Degree of saturation* (s) : The degree of saturation of a soil mass is defined as the ratio of volume of water to the volume of voids. It is always expressed as a percentage.

i.e., $$s = \frac{V_w}{V_v} \times 100\% \quad ...(1.4)$$

The value of s may vary from 0% (for dry soils) to 100% (for fully saturated soils).

(v) *Specific gravity of solids* (G_s or G) : It is defined as the ratio of the mass of a given volume of solid grains to the mass of an equal volume of water, measured at the same temperature.

i.e., $$G = \frac{M_s}{M_w}$$

where, M_s = mass of any volume V of solid grains

M_w = mass of water of volume V.

If this volume V is arbitrarily taken as unity, then in the C.G.S. system M_s and M_w become numerically equal to the density of solid grains (γ_s) and density of water (γ_w) respectively. Thus,

$$G = \frac{\text{mass of unit volume of solids}}{\text{mass of unit volume of water}} = \frac{\gamma_s}{\gamma_w}$$

or, $$\gamma_s = G \cdot \gamma_w \quad ...(1.5)$$

(vi) *Mass specific gravity* (G_m) : It is defined as the ratio of the mass of a given volume of soil to the mass of an equal volume of water, measured at the same temperature.

i.e., $$G_m = \frac{M}{M_w} = \frac{\gamma}{\gamma_w} \quad ...(1.6)$$

where γ = unit weight of the soil mass.

(vii) *Bulk density or unit weight* (γ) : It is defined as the ratio of the total mass of a soil to its total volume. Its unit is gm/cc or t/m^3 or kN/m^3,

i.e., $$\gamma = \frac{W}{V} \quad ...(1.7)$$

(viii) *Unit weight of solids* (γ_s): It is defined as the mass of soil solids per unit volume of solids.

i.e., $$\gamma_s = \frac{W_s}{V_s} \quad ...(1.8)$$

(ix) *Dry density* (γ_d) : The dry density of a soil mass is defined as the mass of soil solids per unit of the total volume of the soil mass.

i.e., $$\gamma_d = \frac{W_s}{V} \qquad ...(1.9)$$

The difference between γ_s and γ_d should be clearly understood. The dry density of a fully or partly saturated soil is nothing but its bulk density in the dry state. The dry density of a soil depends on its degree of compactness, and hence, on its void ratio. But the unit weight of solids depends only on the properties of the minerals present in it and is independent of the state in which the soil exists.

(x) *Saturated unit weight* (γ_{sat}) : When a soil mass is fully saturated, its bulk density is termed as the saturated unit weight of the soil.

(xi) *Submerged density* (γ_{sub}) : The submerged density of a soil mass is defined as the submerged weight of the soil per unit of its total volume.

1.4 Functional Relationships : In order to assess the engineering performance and behaviour of a soil, it is required to evaluate the fundamental properties enumerated in Art. 1.3. While some of these properties (e.g., *w*, *G*, γ etc.) can be easily determined from laboratory tests, some others (e.g., *e*, *s*, γ_s etc.) cannot be evaluated directly. However, all of these properties are interdependent. Hence, if mathematical relationships between two or more such properties can be developed then the direct determination of a few of them will lead to the indirect determination of the others. Thus, the functional relationships have an important role to play in Soil Mechanics.

The most important relationships are established below :

(i) *Relation between e and n* :

By definition, $$e = \frac{V_v}{V_s}$$

But, $V = V_v + V_s$, or, $V_s = V - V_v$.

$$\therefore\ e = \frac{V_v}{V - V_v} = \frac{V_v/V}{(V - V_v)/V} = \frac{V_v/V}{1 - \frac{V_v}{V}} = \frac{n}{1 - n} \qquad \left[\because\ n = \frac{V_v}{V}\right]$$

$$\therefore\ e = \frac{n}{1 - n} \qquad ...(1.10)$$

Again, by definition, $$n = \frac{V_v}{V}$$

$$\text{or, } n = \frac{V_v}{V_s + V_v} = \frac{V_v/V_s}{(V_s + V_v)/V_s} = \frac{V_v/V_s}{1 + V_v/V_s} = \frac{e}{1 + e} \qquad \left[\because\ e = \frac{V_v}{V_s}\right]$$

$$\therefore\ n = \frac{e}{1 + e} \qquad ...(1.11)$$

Alternative proof: The same relationships may also be deduced considering the schematic diagram of a soil mass as shown in Fig. 1.2 (a) and (b).

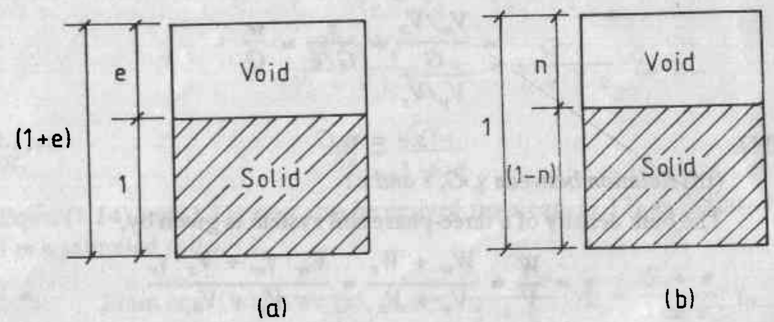


Fig. 1.2

We know that, $$e = \frac{V_v}{V_s}$$

$$\therefore\ V_v = e\,.\,V_s.$$

Let us consider a soil mass having unit volume of solids.

$\therefore\ V_s = 1$, or, $V_v = e \cdot 1 = e.$

$\therefore$ Total volume of the soil, $V = V_v + V_s = 1 + e$

Now, $$n = \frac{V_v}{V} = \frac{e}{1 + e}$$

Again, $$n = \frac{V_v}{V}, \quad \text{or} \quad V_v = n \cdot V$$

Considering a soil mass having a total volume $V = 1$,

$V_v = 1 \cdot n = n$, or, $V_s = V - V_v = 1 - n$.

$$\therefore\ e = \frac{V_v}{V_s} = \frac{n}{1 - n}$$

(ii) *Relation between e, G, w and s* :

With reference to Fig 1.1,

$$w = \frac{W_w}{W_s} = \frac{V_w \cdot \gamma_w}{V_s \cdot \gamma_s}$$

But, $G = \frac{\gamma_s}{\gamma_w}$, or, $\gamma_s = G \cdot \gamma_w$

$$\therefore \quad w = \frac{V_w \cdot \gamma_w}{V_s \cdot G\gamma_w} = \frac{V_w}{V_s \cdot G} = \frac{V_w/V_v}{(V_s/V_v) \cdot G}$$

$$= \frac{V_w/V_v}{\frac{G}{V_v/V_s}} = \frac{s}{G/e} = \frac{se}{G}$$

or, $$s.e = w.G \quad ...(1.12)$$

(iii) *Relation between γ, G, s and e* :

The bulk density of a three-phase soil system is given by,

$$\gamma = \frac{W}{V} = \frac{W_w + W_s}{V_v + V_s} = \frac{V_w \cdot \gamma_w + V_s \cdot \gamma_s}{V_v + V_s}$$

$$= \frac{V_w \cdot \gamma_w + V_s \cdot G\gamma_w}{V_v + V_s} = \frac{V_w + G \cdot V_s}{V_v + V_s} \cdot \gamma_w$$

Dividing the numerator and denominator by V_v, we get,

$$\gamma = \frac{V_w/V_v + G \cdot V_s/V_v}{1 + V_s/V_v} \cdot \gamma_w = \frac{s + G/e}{1 + 1/e} \cdot \gamma_w$$

$$= \frac{(s.e + G)/e}{(1 + e)/e} \gamma_w = \frac{G + se}{1 + e} \cdot \gamma_w$$

or, $$\gamma = \frac{G + se}{1 + e} \cdot \gamma_w \quad ...(1.13)$$

(iv) Expression for γ_{sat} :

By definition, $$\gamma_{sat} = \frac{W}{V} = \frac{W_w + W_s}{V_v + V_s} = \frac{V_w \cdot \gamma_w + V_s \cdot \gamma_s}{V_v + V_s}$$

For a saturated soil, $V_w = V_v$

$$\therefore \quad \gamma_{sat} = \frac{V_v \cdot \gamma_w + V_s \cdot G\gamma_w}{V_v + V_s} = \frac{V_v + G.V_s}{V_v + V_s} \cdot \gamma_w$$

$$= \frac{(V_v + GV_s)/V_v}{(V_v + V_s)/V_v} \cdot \gamma_w$$

$$= \frac{1 + G.(1/e)}{1 + 1/e} \cdot \gamma_w = \frac{(G + e)/e}{(1 + e)/e} \cdot \gamma_w = \frac{G + e}{1 + e} \cdot \gamma_w$$

or, $$\gamma_{sat} = \frac{G + e}{1 + e} \cdot \gamma_w \quad ...(1.14)$$

(v) *Expression for* γ_d :

By definition, $$\gamma_d = \frac{W_s}{V} = \frac{V_s \cdot \gamma_s}{V_v + V_s} = \frac{V_s \cdot G\gamma_w}{V_v + V_s}$$

$$= \frac{G.V_s/V_v}{(V_v + V_s)/V_v} \cdot \gamma_w = \frac{G/e}{1 + 1/e} \cdot \gamma_w$$

or, $$\gamma_d = \frac{G\gamma_w}{1 + e} \quad ...(1.15)$$

Eqns. (1.14) and (1.15) may also be derived from eqn. (1.13) as follows :

For a saturated soil $s = 1$,

∴ From eqn. (1.13) we get, $\gamma_{sat} = \frac{G + 1.e}{1 + e} \cdot \gamma_w = \frac{G + e}{1 + e} \cdot \gamma_w$

For a dry soil, $s = 0$

∴ From eqn. (1.13) we get, $\gamma_d = \frac{G + 0.e}{1 + e} \cdot \gamma_w = \frac{G\gamma_w}{1 + e}$

(vi) *Relation between* γ *and* γ_d :

We know that, $$\gamma = \frac{W}{V} = \frac{W_w + W_s}{V}$$

or, $$V = \frac{W_w + W_s}{\gamma} \quad ...(i)$$

Again, $$\gamma_d = \frac{W_s}{V}, \text{ or, } V = \frac{W_s}{\gamma_d} \quad ...(ii)$$

From (i) and (ii) we get,

$$\frac{W_w + W_s}{\gamma} = \frac{W_s}{\gamma_d}$$

or, $$\gamma = \frac{W_w + W_s}{W_s} \cdot \gamma_d = \left(1 + \frac{W_w}{W_s}\right) \cdot \gamma_d = (1 + w) \cdot \gamma_d$$

or, $$\gamma_d = \frac{\gamma}{1 + w} \quad ...(1.16)$$

(vii) *Relation between* γ_{sub} *and* γ_{sat} :

A soil is said to be submerged when it lies below the ground water table. Such a soil is fully saturated. Now, according to Archimedes' principle, when an object is submerged in a liquid, it undergoes an apparent reduction in mass, the amount of such reduction being equal to the mass of the liquid displaced by the object.

Consider a soil mass, having a volume V and mass W, which is fully submerged in water.

Volume of water displaced by the soil $= V$

$\therefore$ Mass of displaced water $= V \cdot \gamma_w$

$\therefore$ Apparent mass of the soil, $W' = W - V \cdot \gamma_w = V \cdot \gamma_{sat} - V \cdot \gamma_w$

$$= V(\gamma_{sat} - \gamma_w)$$

The apparent density or submerged density of the soil is given by,

$$\gamma_{sub} = \frac{W'}{V} = \frac{V(\gamma_{sat} - \gamma_w)}{V}$$

or, $$\gamma_{sub} = \gamma_{sat} - \gamma_w \quad ...(1.17)$$

EXAMPLES

Two different methods may be employed to solve the numerical problems in this chapter. They are :

Method I : *Solution using mathematical relationships* :

This process is somewhat mechanical, one has to memorise all the equations deduced in Art. 1.4 and should select the appropriate equation/s while solving a given problem. However, in most of the cases this method can yield the desired result fairly quickly.

Method II : *Solution from first principles* :

In this method the solution is obtained using only the basic definitions with reference to a three-phase diagram of the soil mass under consideration. This method always allows the student to have an insight into the problem. However, in some cases the solution becomes a little complicated and more time-consuming than method I.

After going through the worked out examples, quite a few of which illustrate the use of both of these methods, one should be able to realise as to which method of solution suits better to a particular type of problem. It may be pointed out that, the methods may also be used in conjunction with one another.

Problem 1.1. A soil sample has a unit weight of 1.9 gm/cc and a water content of 12%. If the specific gravity of solids be 2.65, determine the dry density, degree of saturation, void ratio and porosity of the soil.

Solution : From the consideration of degree of saturation, a soil sample may be :

(i) Completely dry ($s = 0$)

(ii) fully saturated ($s = 1$)

(iii) partially saturated ($0 < s < 1$)

Unless otherwise mentioned in the problem, a soil sample should always be taken to be partially saturated.

Method I : Given : $\boxed{\gamma, w, G}$ $\Rightarrow$ Required : $\boxed{\gamma_d, s, e, n}$

As e and n are mutually dependent on each other, effectively three unknown parameters have to be determined from the given data. Select the appropriate equations which may serve this purpose.

The value of γ_d can be determined from :

$$\gamma_d = \frac{\gamma}{1 + w}$$

Here, γ = unit weight of the soil = 1.9 gm/cc

w = water content = 12% = 0.12

$$\therefore \quad \gamma_d = \frac{1.9}{1 + 0.12} = 1.696 \text{ gm/cc}$$

In order to solve for the other two unknowns, viz., s and e, two equations are required. Evidently, the following equations will serve the purpose :

$$wG = se, \text{ or } se = (0.12)(2.65) = 0.318 \quad ...(i)$$

Again, $$\gamma = \frac{G + se}{1 + e} \cdot \gamma_w$$

or, $$1.9 = \left(\frac{2.65 + 0.318}{1 + e}\right)(1.0)$$

or, $$1 + e = 1.56, \text{ or, } e = 0.56$$

The expression of γ_d may also be used.

$$\gamma_d = \frac{G \cdot \gamma_w}{1 + e},$$

or, $$1.696 = \frac{(2.65)(1)}{1 + e},$$

or, $$1.696 + 1.696e = 2.65$$

or, $$e = \frac{0.954}{1.696} = 0.56$$

From (i), $s = \dfrac{0.318}{0.56} = 0.568 = 56.8\%$

$$n = \frac{e}{1+e} = \frac{0.56}{1+0.56} = 0.36 = 36\%$$

Answer. Dry density = 1.696 gm/cc, void ratio = 0.56

Degree of saturation = 56.8%, Porosity = 36%

Method II :

Let us consider a specimen of the given soil in which the mass of solid grains = 1 gm. The three-phase diagram of the soil is shown in Fig. 1.3.

Now, $w = \dfrac{W_w}{W_s} = \dfrac{W_w}{1} = W_w$

or, $W_w = 0.12$ gm

$\therefore$ Total mass of the sample, $W = W_w + W_d = 1.12$ gm

Volume of solids, $V_s = \dfrac{W_s}{\gamma_s} = \dfrac{W_s}{G\gamma_w} = \dfrac{1}{(2.65)(1)} = 0.377$ cc

Volume of water, $V_w = \dfrac{W_w}{\gamma_w} = \dfrac{0.12}{1} = 0.12$ cc

Total volume of soil, $V = \dfrac{W}{\gamma} = \dfrac{1.12}{1.9} = 0.589$ cc

$\therefore$ Volume of air, $V_a = V - (V_s + V_w) = 0.092$ cc

$\therefore$ Volume of voids, $V_v = V_a + V_w = 0.12 + 0.092 = 0.212$ cc

Degree of saturation, $s = \dfrac{V_w}{V_v} = \dfrac{0.12}{0.212} \times 100\% = 56.6\%$

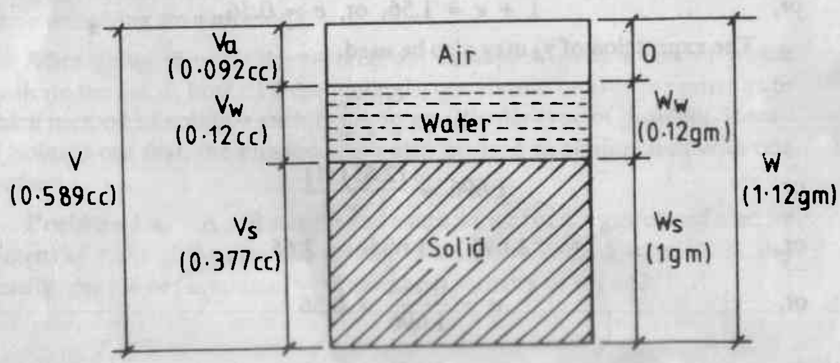


Fig. 1.3.

Void ratio, $e = \dfrac{V_v}{V_s} = \dfrac{0.212}{0.377} = 0.56$

Porosity, $n = \dfrac{V_v}{V} = \dfrac{0.212}{0.589} \times 100\% = 36\%$

Dry density, $\gamma_d = \dfrac{W_s}{V} = \dfrac{1}{0.589} = 1.697$ gm/cc

Problem 1.2. An undisturbed specimen of soil has a volume of 300 cc and weighs 498 gm. After drying in oven at 105°C for 24 hours, its weight reduced to 456 gm. Determine the void ratio, porosity, degree of saturation and water content. Assume $G = 2.70$.

Solution : Method I :

Given : $\boxed{V, W, W_d, G}$ $\Rightarrow$ Required : $\boxed{e, n, s, w}$

After drying in oven, the water present in the soil evaporates and the soil becomes completely dry.

Now, weight of the moist sample, $W = 498$ gm

And, weight of the dry sample, $W_d = 456$ gm.

$\therefore$ Weight of water evaporated, $W_w = W - W_d = 498 - 456 = 42$ gm.

Water content, $w = \dfrac{W_w}{W_d} = \dfrac{42}{456} = 0.0921 = 9.21\%$

Dry density, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

But $\gamma_d = \dfrac{W_d}{V} = \dfrac{456}{300} = 1.52$ gm/cc

$$\therefore \quad \frac{G\gamma_w}{1+e} = 1.52$$

or, $1.52(1+e) = (2.7)(1)$

or, $1.52e + 1.52 = 2.7$

or, $e = 0.78$

$\therefore$ Void ratio = 0.78

Again, porosity $n = \dfrac{e}{1+e} = \dfrac{0.78}{1+0.78} = 0.438 = 43.8\%$

From eqn. (1.12), $wG = se$, or, $s = \dfrac{wG}{e}$

or, $$s = \frac{(0.0921)(2.7)}{0.78} = 0.319 = 31.9\%$$

Method II : With reference to the three-phase diagram shown in Fig. 1.4,

Weight of water, $W_w = 498 - 456 = 42$ gm

Volume of water, $$V_w = \frac{W_w}{\gamma_w} = 42 \text{ cc}$$

Volume of solids, $$V_s = \frac{W_s}{\gamma_s} = \frac{W_s}{G\gamma_w}$$

$$= \frac{456}{(2.7)(1)} = 168.89 \text{ cc}$$

Total volume, $V = 300$ cc

Volume of voids, $V_v = V - V_s$

or, $V_v = 300 - 168.89 = 131.11$ cc

$$\therefore \quad e = \frac{V_v}{V_s} = \frac{131.11}{168.89} = 0.78$$

$$n = \frac{V_v}{V} = \frac{131.11}{300} = 0.437 = 43.7\%$$

$$s = \frac{V_w}{V_v} = \frac{42}{131.11} = 0.32 = 32\%$$

$$w = \frac{W_w}{W_s} = \frac{42}{456} = 0.0921 = 9.21\%$$

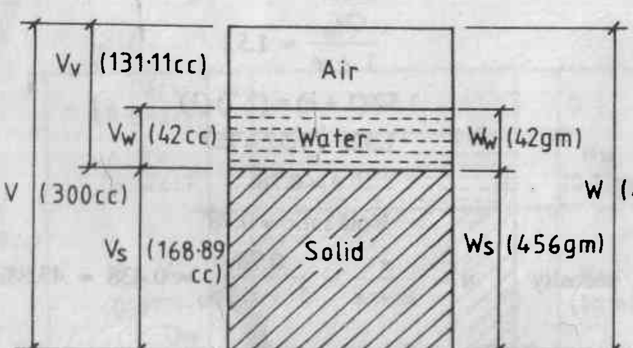


Fig. 1.4

Problem 1.3. A saturated soil sample, weighing 178 gm, has a volume of 96 cc. If the specific gravity of soil solids be 2.67, determine the void ratio, water content and unit weight of the soil.

Solution : Given : W, V, G, s ⇒ Required : e, w, γ

Unit weight of the soil,

$$\gamma_{sat} = \frac{W}{V} = \frac{178}{96} = 1.854 \text{ gm/cc}$$

But, $$\gamma_{sat} = \frac{G + e}{1 + e}\cdot\gamma_w$$

$$\therefore \quad \left(\frac{2.67 + 1 \times e}{1 + e}\right)(1.0) = 1.854$$

or, $1.854 + 1.854e = 2.67 + e$

or, $0.854e = 0.816$

or, $e = 0.955$

Again, $$w = \frac{s.e}{G} = \frac{(1)(0.955)}{2.67} = 0.358 = 35.8\%$$

Problem 1.4. A fully saturated soil sample has a volume of 28 cc. The sample was dried in oven and the weight of the dry soil pat was found to be 48.86 gm. Determine the void ratio, moisture content, saturated density and dry density of the soil mass. Given $G = 2.68$.

Solution : Given : V, W_s, G, s ⇒ Required : $e, w, \gamma_{sat}, \gamma_d$

A schematic representation of the given soil is shown in Fig. 1.5.

Here, total volume $V = 28$ cc

Volume of dry soil, $$V_s = \frac{48.86}{2.68} \text{ cc} = 18.23 \text{ cc}$$

Assuming that there was no change in void ratio during oven-drying, volume of water evaporated, $V_w = V - V_s = (28 - 18.23)$ cc $= 9.77$ cc

$$\therefore \quad \text{Void ratio,} \quad e = \frac{V_v}{V_s} = \frac{V_w}{V_s} \qquad [\because V_v = V_w]$$

$$= \frac{9.77}{18.23} = 0.536$$

Weight of water, $W_w = V_w \cdot \gamma_w = (9.77)(1.0)$

$= 9.77$ gm

Moisture content, $w = \dfrac{W_w}{W_s} = \dfrac{9.77}{48.86} = 0.2 = 20\%$

Total weight of the soil,

$W = W_w + W_s = (9.77 + 48.86)$ gm $= 58.63$ gm

Saturated density, $\gamma_{sat} = \dfrac{W}{V} = \dfrac{58.63}{28} = 2.09$ gm/cc

Dry density, $\gamma_d = \dfrac{W_s}{V} = \dfrac{48.86}{28} = 1.745$ gm/cc

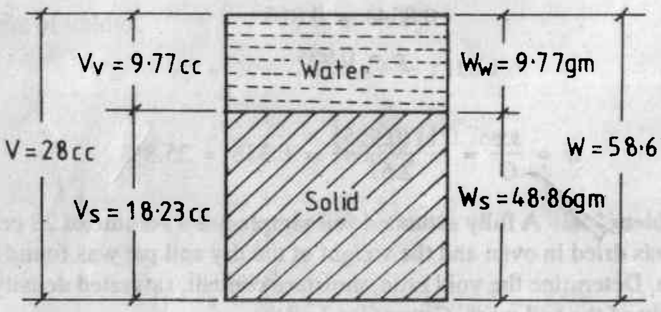


Fig. 1.5

Problem 1.5. An undisturbed sample of saturated clay has a volume of 16.5 cc and weighs 35.1 gm. On oven-drying, the weight of the sample reduces to 29.5 gm. Determine the void ratio, moisture content, dry density and the specific gravity of solids.

Solution : Method I :

Given : V, W, W_d, s ⇒ Required : e, w, γ_d, G

Weight of the saturated sample, $W = 35.1$ gm

Weight of the dry sample, $W_d = 29.5$ gm

∴ Weight of water evaporated, $W_w = W - W_d = (35.1 - 29.5)$ gm

$= 5.6$ gm

Now, $\gamma_{sat} = \dfrac{W}{V} = \dfrac{35.1}{16.5} = 2.127$ gm/cc

$\gamma_d = \dfrac{W_d}{V} = \dfrac{29.5}{16.5} = 1.788$ gm/cc

But, $\gamma_{sat} = \dfrac{G+e}{1+e} \cdot \gamma_w$

∴ $2.127 = \dfrac{G+e}{1+e} \cdot 1$

or, $2.127 + 2.127e = G + e$

or, $G = 1.127e + 2.127$ ...(i)

Again, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

∴ $1.788 = \dfrac{G \cdot 1}{1+e}$

or, $G = 1.788e + 1.788$ ...(ii)

From (i) and (ii) we get,

$1.788e + 1.788 = 1.127e + 2.127$

or, $0.661e = 0.339$

or, $e = 0.51$

From (i) we get, $G = (1.27)(0.51) + 2.127 = 2.7$

Now, $wG = se$

or, $w = \dfrac{se}{G} = \dfrac{(1)(0.51)}{2.7} = 0.189 = 18.9\%$

Method II : A three-phase diagram of the given soil is shown in Fig. 1.6.

Here, wet weight of the sample, $W = 35.1$ gm

Dry weight of the sample, $W_d = 29.5$ gm

Weight of water, $W_w = W - W_d = (35.1 - 29.5)$ gm $= 5.6$ gm

Volume of water $V_w = V_v = 5.6$ cc

Total volume $V = 16.5$ cc

∴ Volume of solids, $V_s = V - V_v = (16.5 - 5.6)$ cc $= 10.9$ cc

Void ratio, $e = \dfrac{V_v}{V_s} = \dfrac{5.6}{10.9} = 0.51$

Moisture content, $w = \dfrac{W_w}{W_s} = \dfrac{5.6}{29.5} = 0.189 \approx 18.9\%$

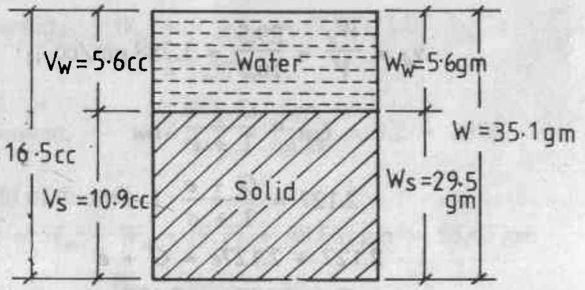


Fig. 1.6

Dry density, $\gamma_d = \dfrac{W_s}{V} = \dfrac{29.5}{16.5} = 1.79$ gm/cc

Unit weight of solids, $\gamma_s = \dfrac{W_s}{V_s} = \dfrac{29.5}{10.9} = 2.70$ gm/cc

Specific gravity of solids, $G = \dfrac{\gamma_s}{\gamma_w} = \dfrac{2.70}{1.0} = 2.70$

Problem 1.6 The initial void ratio of an inorganic clay is found to be 0.65, while the specific gravity of solids is 2.68. Determine the dry density and saturated density of the soil. Also determine its bulk density and moisture content, if the soil is 50% saturated.

Solution : Given : e, G, s ⇒ Required : $\gamma_d, \gamma_{sat}, \gamma, w$

Saturated density of the soil, $\gamma_{sat} = \dfrac{G+e}{1+e} \cdot \gamma_w$

$$= \left(\frac{2.68 + 0.65}{1 + 0.65}\right)(1) = 2.02 \text{ gm/cc}$$

Dry density, $\gamma_d = \dfrac{G\gamma_w}{1+e} = \dfrac{(2.68)(1)}{1+0.65} = 1.62$ gm/cc

When the soil is 50% saturated, its bulk density

$$\gamma = \frac{G + se}{1+e} \cdot \gamma_w = \frac{2.68 + (0.5)(0.65)}{1 + 0.65}(1) = 1.82 \text{ gm/cc}$$

Moisture content at 50% saturation,

$$w = \frac{se}{G} = \frac{(0.5)(0.65)}{2.68} = 0.12 = 12\%$$

Problem 1.7. The volume and weight of a partially saturated clay sample are 185 cc and 362 gm respectively. After drying in an oven at 105°C for 24 hours, its weight reduced to 326 gm. If the natural void ratio of the soil was 0.54, determine the moisture content, dry density, bulk density, degree of saturation and specific gravity of solids.

Solution : Given : W, V, W_s, e ⇒ Required : $\gamma, \gamma_d, w, s, G$

Total volume $V = 185$ cc

Total weight, $W = 362$ gm

Dry weight, $W_d = 326$ gm

∴ Bulk density, $\gamma = \dfrac{W}{V} = \dfrac{362}{185} = 1.96$ gm/cc

Dry density, $\gamma_d = \dfrac{W_d}{V} = \dfrac{326}{185} = 1.76$ gm/cc

Weight of water evaporated, $W_w = W - W_s$

$= (362 - 326)$ gm $= 36$ gm

Moisture content, $w = \dfrac{W_w}{W_s} = \dfrac{36}{326} = 0.11 = 11\%$

Now, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

∴ $1.76 = \dfrac{G \cdot 1}{1 + 0.54}$

or, $G = (1.76)(1.54) = 2.71$

Again, $wG = se$,

or, $s = \dfrac{wG}{e} = \dfrac{(0.11)(2.71)}{0.54} = 0.55 = 55\%$

Problem 1.8. A sample of silty clay has a void ratio of 0.8. The soil is allowed to absorb water and its saturated density was found to be 1.92 gm/cc. Determine the water content of the saturated sample.

Solution : Method I :

It is assumed that the void ratio of the soil did not change due to absorption of water.

The saturated density is given by,

$$\gamma_{sat} = \frac{G+e}{1+e} \cdot \gamma_w$$

∴ $\dfrac{G + 0.8}{1 + 0.8}(1) = 1.92$

or, $G = (1.92)(1.8) - 0.8 = 2.656$

Now, using the relation $wG = se$, we, get,

$$w = \frac{se}{G} = \frac{(1)(0.8)}{2.656} = 0.30$$

∴ Required water content = 30%

Method II : Fig. 1.7 shows the three-phase diagram of the given soil.

Let the weight of solids be unity. Let w be the moisture content of the saturated soil,

$$\text{Now, } w = \frac{W_w}{W_s}, \quad \text{or,} \quad W_w = w \cdot W_s = w \cdot 1 = w \text{ gm}$$

∴ Volume of water, $V_w = w$ cc

Now, void ratio $e = 0.8$

$$\therefore \quad \frac{V_v}{V_s} = 0.8$$

$$\text{or,} \quad V_s = \frac{V_v}{0.8} = \frac{V_w}{0.8} = \frac{w}{0.8} = 1.25w \text{ cc}$$

Total volume of the soil,

$$V = V_s + V_w$$

$$= 1.25w + w = 2.25w \text{ cc}$$

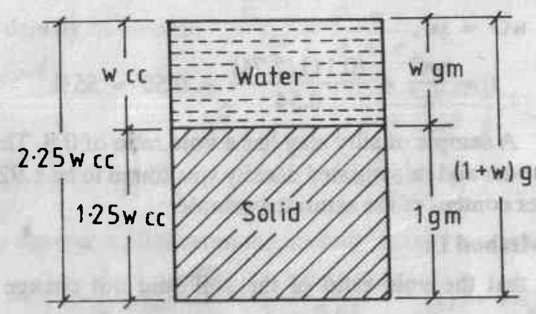


Fig. 1.7

Total weight of the soil, $W = W_w + W_d = (1 + w)$ gm

$$\therefore \quad \gamma_{sat} = \frac{W}{V} = \frac{1+w}{2.25w}$$

But, $\gamma_{sat} = 1.92$ gm/cc

$$\therefore \quad \frac{1+w}{2.25w} = 1.92$$

$$\text{or,} \quad 4.32w = 1 + w$$

$$\text{or,} \quad w = 0.30 = 30\%$$

Note : Try to solve the problem assuming the volume of solids to be unity.

Problem 1.9. The bulk density and dry density of a partially saturated soil are 1.95 gm/cc and 1.80 gm/cc respectively. The specific gravity of solids is 2.68. Determine the void ratio, moisture content and degree of saturation of the soil.

Solution :

We have, $\gamma_d = \dfrac{\gamma}{1+w}$

Here, $\gamma_d = 1.80$ gm/cc, $\gamma = 1.95$ gm/cc

$$\therefore \quad 1.80 = \frac{1.95}{1+w}$$

$$\text{or,} \quad 1 + w = 1.95/1.80 = 1.0833$$

$$\text{or,} \quad w = 0.0833 = 8.33\%$$

Again, we have, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

$$\therefore \quad 1.80 = \frac{(2.68)(1)}{1+e}$$

$$\text{or,} \quad 1 + e = \frac{2.68}{1.80} = 1.49$$

$$\text{or,} \quad e = 0.49$$

Now, $wG = se$

$$\text{or,} \quad s = \frac{wG}{e} = \frac{(0.0833)(2.68)}{0.49} = 0.456 = 45.6\%$$

Problem 1.10. The density of a partially saturated soil was found to be 1.88 gm/cc. If the moisture content and void ratio of the soil be 24.8% and 0.76 respectively, determine the specific gravity of solids, and the degree of saturation.

Solution :

We have $\gamma = \dfrac{G + se}{1+e} \cdot \gamma_w$ ...(i)

and, $wG = se$ ...(ii)

Substituting for *se* in eqn (i), we get,

$$\gamma = \frac{G + wG}{1 + e} \cdot \gamma_w$$

or, $$\gamma = \frac{G(1 + w)}{1 + e} \cdot \gamma_w$$

∴ $$1.88 = \frac{G(1 + 0.248)}{1 + 0.76}(1)$$

or, $$G = \frac{(1.88)(1.76)}{1.248} = 2.65$$

Problem 1.11. A given soil mass has a moisture content of 10.5% and a void ratio of 0.67. The specific gravity of soil solids is 2.68. It is required to construct three cylindrical test specimens of diameter 3.75 cm and height 7.5 cm from this soil mass. Each specimen should have a moisture content of 15% and a dry density of 1.6 gm/cc. Determine :

(i) the quantity of the given soil to be used for this purpose
(ii) quantity of water to be mixed with it.

Solution : (i) Volume of each specimen $= \pi r^2 h$

$$= \pi (3.75/2)^2 \; (7.5) \text{ cc}$$

$$= 82.83 \text{ cc}$$

Total volume of three specimens, $V = (3)(82.83) = 248.49$ cc

Weight of dry soil required, $W_d = V \times \gamma_d$ $\left[\because \gamma_d = \frac{W_d}{V}\right]$

$$= (248.49)(1.6)$$

$$= 397.58 \text{ gm}$$

Moisture content of finished specimens, $w = 15\%$

But, $w = \frac{W_w}{W_d}$, or, $W_w = w \times W_d$

∴ Weight of water in the specimens, $W_w = (0.15)(397.58)$

$$= 59.64 \text{ gm}$$

Now, dry density of the given soil mass,

$$\gamma_d = \frac{G\gamma_w}{1 + e} = \frac{(2.68)(1)}{1 + 0.67} = 1.605 \text{ gm/cc}$$

i.e., 1.605 gm of dry soil is obtained from 1 cc of moist soil

∴ 397.58 gm of dry soil is obtained from $\frac{397.58}{1.605}$ cc

$$= 247.71 \text{ cc of moist soil}$$

∴ Volume of moist soil to be used = 247.71 cc.

Now, bulk density $\gamma = \gamma_d (1 + w)$

$$= (1.605)(1 + 0.105) = 1.773 \text{ gm/cc}$$

∴ Total weight of moist soil required $= \gamma \times V$

$$= (1.773)(247.71) \text{ gm} = 439.19 \text{ gm}$$

(ii) Weight of water present in this soil

$$= (439.19 - 397.58) \text{ gm} = 41.61 \text{ gm}$$

Weight of water finally required = 59.64 gm

∴ Weight of water to be added = (59.64 − 41.61) gm

= 18.03 gm

Volume of water to be added = 18.03 cc

Ans : 439.19 gm of given soil is to be taken and 18.03 cc of water is to be added to it.

EXERCISE 1

1.1. A soil sample has a porosity of 35%. The soil is 75% saturated and the specific gravity of solids is 2.68. Determine its void ratio, dry density, bulk density and moisture content.

[Ans : $e = 0.54$, $\gamma_d = 1.74$ gm/cc, $\gamma = 2.0$ gm/cc, $w = 15\%$]

1.2. The mass specific gravity of a soil is 1.95, while the specific gravity of soil solids is 2.7. If the moisture content of the soil be 22%, determine the following :

(i) Void ratio (ii) porosity (iii) degree of saturation (iv) dry density (v) saturated density.

[Ans : (i) 0.69 (ii) 41% (iii) 86% (iv) 1.597 gm/cc (v) 2.00 gm/cc]

1.3. The saturated and dry densities of a soil are 1.93 gm/cc and 1.47 gm/cc respectively. Determine the porosity and the specific gravity of the solid grains. [Ans : n = 45.9%, G = 2.72]

1.4. A partially saturated soil sample has a natural moisture content of 17% and a bulk density of 2.05 gm/cc. If the specific gravity of soil solids be 2.66, determine the void ratio, degree of saturation and dry density of the soil. What will be the bulk density of the soil if it is :

(i) Fully saturated

(ii) 60% saturated ?

[Ans : Part 1 : $e = 0.52$, $s = 87\%$, $\gamma_d = 1.75$ gm/cc Part 2 : (i) 2.09 gm/cc (ii) 1.95 gm/cc]

1.5. An undisturbed soil sample has a volume of 50 cc and weighs 96.5 gm. On oven-drying, the weight reduces to 83.2 gm. Determine the water content, void ratio and degree of saturation of the soil. Given, $G = 2.65$.

[Ans : $w = 16\%$, $e = 0.59$, $s = 72\%$]

1.6. The bulk density and dry density of a soil are 1.95 gm/cc and 1.58 gm/cc respectively. Assuming $G_s = 2.68$, determine the porosity, water content and degree of saturation of the soil.

[Ans : $n = 41\%$, $w = 23\%$, $s = 89.2\%$]

1.7. A cylindrical sample of saturated clay, 7.6 cm high and 3.8 cm in diameter, weighs 149.6 gm. The sample was dried in an oven at 105°C for 24 hours, and its weight reduced by 16.9 gm. Determine the dry density, void ratio, moisture content and specific gravity of solids.

[Ans : $\gamma_d = 1.54$ gm/cc, $e = 0.74$, $w = 12.7\%$, $G = 2.68$]

1.8. The moisture content and bulk density of a partially saturated silt sample were 18% and 19.6 kN/m^3 respectively. The sample was kept in an oven at 105° C for 15 minutes, resulting in a partial evaporation of the pore water. The bulk density of the sample reduced to 18.3 kN/m^3. Assuming the void ratio to remain unchanged, determine the final water content of the sample. What would have been its bulk density if the sample was kept in the oven for 24 hours ? [Ans : 10%, 16.6 kN/m^3]

1.9. An embankment was constructed with a clayey soil at a moisture content of 12%. Just after construction, the degree of saturation of the soil was found to be 55%. The soil absorbed water during the monsoon and its degree of saturation increased to 90%. Determine the water content of the soil at this stage. What will be the degree of saturation if the moisture content reduces to 5% in the dry season ? Given, $G = 2.68$. [Ans : 19.6%, 22.9%]

1.10. The natural moisture content of a soil mass is 11%, while its void ratio is 0.63. Assuming the void ratio to remain unchanged, determine the quantity of water to be added to 1 m^3 of this soil in order to double its moisture content. Given, specific gravity of solids = 2.72. [Ans : 183.3 kg]

1.11. The in-situ density of a soil mass is to be determined by the core-cutter method. The height and diameter of the core are 13 cm and 10 cm respectively. The core, when full of soil, weighs 3155 gm, while the self-weight of the empty core is 1250 gm. The natural moisture content and the specific gravity of solids are 12% and 2.66 respectively. Determine the bulk density, dry density and void ratio of the soil.

[Ans : $\gamma = 1.87$ gm/cc, $\gamma_d = 1.67$ gm/cc, $e = 0.59$]

1.12. In problem 1.11, what will be the water content and bulk density of the soil if, without undergoing any change in the void ratio, the soil becomes :

(i) Fully saturated

(ii) 80% saturated [Ans : (i) 22% ; 2.04 gm/cc, (ii) 17.7%, 1.97 gm/cc]

1.13. A 4 m high embankment, with a top width of 5 m and side slopes of 1 : 1, has to be constructed by compacting soil from a nearby borrow pit. The unit weight and natural moisture content of the soil are 1.8 t/m^3 and 8% respectively. Determine the volume of earth to be excavated from the borrow pit and the quantity of water to be added to it for every km of finished embankment, if the required dry density and moisture content of the embankment soil be 1.82 gm/cc and 18% respectively. Given, $G = 2.70$.

[Ans : Vol. of excavation = 39304 m^3 ; Vol. of water = 6552 m^3]

2

INDEX PROPERTIES AND SOIL CLASSIFICATION

2.1 Introduction: Various physical and engineering properties with the help of which a soil can be properly identified and classified are called the index properties. Such properties can be broadly divided into the following two categories:

(a) *Soil grain properties*: These are the properties pertaining to individual solid grains and remain unaffected by the state in which a particular soil exists in nature. The most important soil grain properties are the specific gravity and the particle size distribution.

(b) *Soil aggregate properties*: These properties control the behaviour of the soil in actual field. The most important aggragate properties are:

(i) for cohesionless soils: the relative density

(ii) for cohesive soils: the consistency, which depends on the moisture content and which can be measured by either the Atterberg limits or the unconfined compressive strength.

2.2 Specific Gravity: The specific gravity of a soil can be determined by a pycnometer (i.e., a specific gravity bottle of 500 ml capacity). Fig. 2.1 gives a schematic representation of the process. Let,

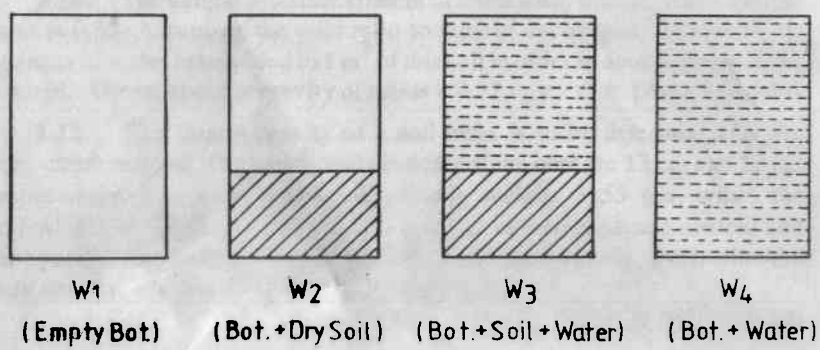


Fig. 2.1

W_1 = empty weight of pycnometer.

W_2 = weight of pycnometer and dry soil.

W_3 = weight of pycnometer, soil and water.

W_4 = weight of pycnometer filled with water.

Now, weight of soil solids = $W_2 - W_1$

and, weight of an equal volume of water = $(W_4 - W_1) - (W_3 - W_2)$

$$\therefore \quad G = \frac{W_4 - W_1 - W_3 + W_2}{W_2 - W_1} \qquad \text{...(2.1)}$$

2.3 Particle Size Distribution: This is determined in the laboratory by the mechanical analysis, which consists of:

(a) *Dry mechanical analysis or sieve analysis*: In this method the sample is sieved through a set of sieves of gradually diminishing opening sizes. The percent finer corresponding to each sieve size is determined and the results are plotted on a semilog graph paper to obtain the particle size distribution curve. However, this method is applicable only to the coarser fractions of soils and not to the silt and clay frictions as sieves having open sizes less than 0.075 mm are practically impossible to manufacture.

(b) *Wet mechanical analysis or hydrometer analysis*: The percentage of finer fractions (i.e., silt and clay) in a soil can be analysed indirectly using a hydrometer. The method is based on Stokes' law which states that the terminal velocity of a falling sphere in a liquid is given by

$$v = \frac{\gamma_s - \gamma_w}{18\mu} D^2 \qquad \text{...(2.2)}$$

where, γ_s and γ_w are the unit weights of the sphere and the liquid respectively

D = diameter of the sphere

μ = absolute viscosity of the liquid

Fig. 2.2 shows the sketch of a hydrometer. After immersing the hydrometer in the measuring cylinder containing the soil-water suspension, readings are taken at $\frac{1}{2}$, 1, 2, 4, 8, 15, 30, 60, 120, and 1440 minutes. Let r_1 be the reading of hydrometer at time t. The particle size and the corresponding value of percent finer are obtained from the following equations:

$$D = \sqrt{\frac{1800 \cdot \mu}{\gamma_s - \gamma_w}} \times \sqrt{\frac{Z_r}{t}} \qquad \text{...(2.3)}$$

and, $$N = \frac{\gamma_s}{\gamma_s - \gamma_w} \cdot \frac{V}{W_s} \cdot \gamma_w \; (r_1 + C_m - r_w) \times 100\% \qquad \text{...(2.4)}$$

where, D = particle size in mm

γ_s = unit weight of soil solids = $G_s \cdot \gamma_w$

γ_w = unit weight of distilled water at the room temperature

t = time interval in sec

r_1 = reading of hydrometer in suspension at time t

μ = viscosity of water at room temperature in gm–sec/cm^2

Z_r = distance from the surface of suspension to the centre of gravity of hydrometer bulb at time t, which can be determined from :

$$Z_r = H_1 + \frac{1}{2}\left(h - \frac{V_h}{A}\right) \text{ cm} \qquad \text{...(2.5)}$$

where, V_h = volume of hydrometer in cc

A = area of cross-section of measuring cylinder in cm^2

H_1 = distance between the surface of suspension and the neck of bulb, in cm

h = length of the bulb in cm

The distance H_1 may be measured by a scale. However, a better proposition is to determine H_1 from the following equation:

$$H_1 = \frac{(r_d + 1) - r_1}{r_d} \times L \qquad \text{...(2.6)}$$

where, r_d = difference between the maximum and minimum calibration marks on the stem of hydrometer

L = length of calibration (≈ length of stem)

In eqn. (2.4),

N = percent finer.

V = Volume of suspension in cc

W_s = weight of dry soil taken in gm

r_w = reading of hydrometer in distilled water at room temperature

C_m = meniscus correction

If W_s be the weight of dry soil passing through the 75 μ sieve during sieve analysis, which is subsequently used for hydrometer analysis, and if W_t be the total weight of sample taken for combined dry and wet mechanical analysis, then the percent finer, N', of the particle size D mm, with respect to the total quantity of sample, is given by,

$$N' = N \times \frac{W_s}{W} \qquad \text{...(2.7)}$$

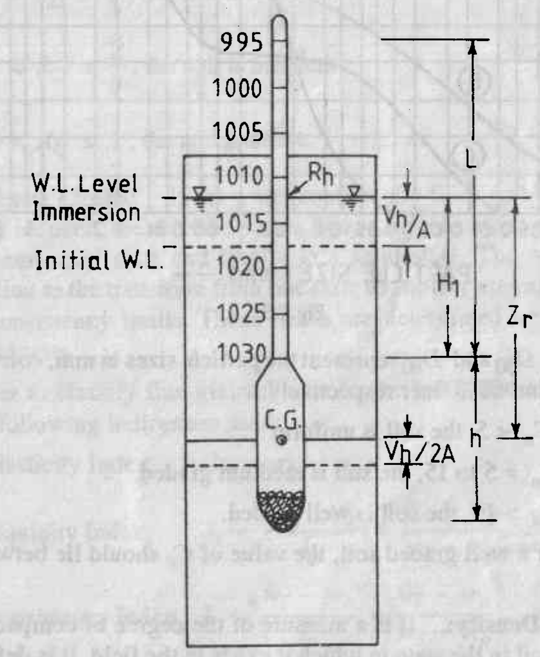


Fig. 2.2

2.3.1 *Particle Size Distribution Curve*: Fig. 2.3 shows typical particle size distribution curves for various types of soils. Curves A, B and C represent a uniform soil, a well graded soil and a gap graded soil respectively.

With reference to the particle size distribution curve of a given soil, the following two factors are helpful for defining the gradation of the soil:

(i) Uniformity Co-efficient:

$$C_u = \frac{D_{60}}{D_{10}} \qquad \text{...(2.8)}$$

(ii) Co-efficient of Curvature:

$$C_c = \frac{(D_{30})^2}{D_{10} \times D_{60}} \qquad \text{...(2.9)}$$

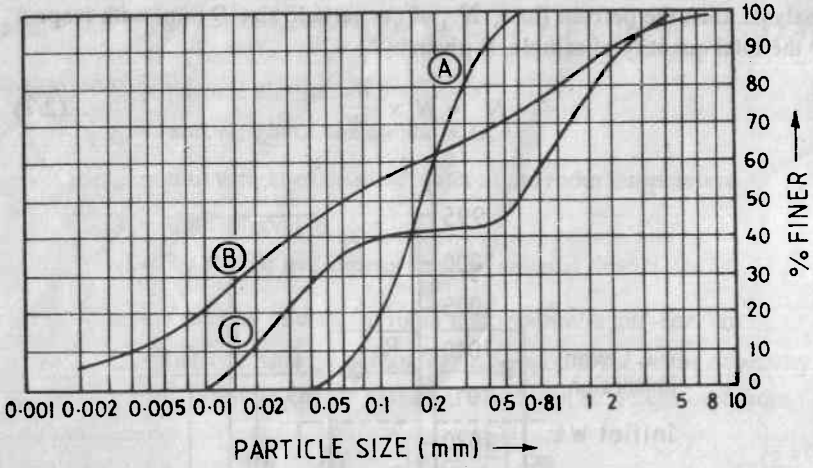


Fig. 2.3

where, D_{10}, D_{30} and D_{60} represent the particle sizes in mm, corresponding to 10%, 30% and 60% finer respectively.

When $C_u < 5$, the soil is uniform

$C_u = 5$ to 15, the soil is medium graded.

$C_u > 15$, the soil is well graded.

Again, for a well graded soil, the value of C_c should lie between 1 and 3.

2.4. Relative Density: It is a measure of the degree of compactness of a cohesionless soil in the state in which it exists in the field. It is defined as,

$$R_D = \frac{e_{max} - e}{e_{max} - e_{min}} \qquad ...(2.10)$$

where, e_{max} = void ratio of the soil in its loosest state

e_{min} = void ratio at the densest state

e = natural void ratio in the field.

The relative density of a soil may also be determined from:

$$R_D = \frac{\gamma_{dmax}}{\gamma_d} \cdot \frac{\gamma_d - \gamma_{dmin}}{\gamma_{dmax} - \gamma_{dmin}} \qquad ...(2.11)$$

where, γ_{dmax} = maximum dry density of the soil

γ_{dmin} = minimum dry density of the soil

γ_d = in-situ dry density of the soil.

On the basis of the relative density, coarse-grained soils are classified as loose, medium or dense as follows:

If $0 \le R_D \le \frac{1}{3}$, the soil is loose

$\frac{1}{3} < R_D \le \frac{2}{3}$, the soil is medium

$\frac{2}{3} < R_D \le 1$, the soil is dense.

2.5. Atterberg Limits: If the water content of a thick soil-water mixture is gradually reduced, the mixture passes from a liquid state to a plastic state, then to a semi-solid state and finally to a solid state. The water contents corresponding to the transition from one state to another are called Atterberg limits or consistency limits. These limits are determined by arbitrary but standardised tests.

In order to classify fine-grained soils on the basis of their consistency limits, the following indices are used:

(i) Plasticity Index, $I_p = w_l - w_p$ ...(2.12)

(ii) Liquidity Index, $I_l = \frac{w_n - w_p}{I_p} = \frac{w_n - w_p}{w_l - w_p}$ ...(2.13)

(iii) Consistency Index, $I_c = \frac{w_l - w_n}{I_p} = \frac{w_l - w_n}{w_l - w_p}$ ...(2.14)

where, w_l, w_p and w_n stand for the liquid limit, plastic limit and the natural water content of the soil.

(iv) Flow Index (I_f): It is defined as the slope of the w vs. $\log_{10} N$ curve obtained from the liquid limit test.

i.e., $$I_f = \frac{w_1 - w_2}{\log_{10} N_2/N_1} \qquad ...(2.15)$$

where, N_1 and N_2 are the number of blows corresponding to the water contents w_1 and w_2.

(v) Toughness Index, $I_t = \frac{I_p}{I_f}$ ...(2.16)

(vi) Activity Number, $A = \frac{\text{Plasticity Index}}{\text{Percent finer then 0.002 mm}}$ ...(2.17)

Soils can be classified according to various indices, as follows:

(a) *Classification according to the plasticity index*:

Plasticity Index	*Degree of Plasticity*	*Type of Soil*
0	Non - plastic	Sand
< 7	Low plastic	Silt
7 - 17	Medium plastic	Silty clay or clayey silt
> 17	Highly plastic	Clay

(b) *Classification according to the liquidity index*: A soil for which the liquidity index is –ve (i.e., $w_n < w_P$) is in either semi-solid or solid state. The soil is very stiff if $I_l = 0$ (i.e., $w_n = w_P$) and very soft if $I_l = 1$ (i.e. $w_n = w_l$) Soils having $I_l > 1$ are in the liquid state. For most soils, however, I_l lies between 0 and 1. Accordingly, the soils are classified as follows:

I_l	*Consistency*
0.0 - 0.25	Stiff
0.25 - 0.50	Medium to soft
0.50 - 0.75	Soft
0.75 - 1.00	Very soft

(c) *Classification according to the activity number*: The activity number of a soil represents the tendency of a soil to swell or shrink due to absorption or evaporation of water. The classification is as follows:

Activity Number	*Type of Soil*
< 0.75	Inactive
0.75 - 1.25	Normal
> 1.25	Active

2.5.1 *Determination of Shrinkage Limit*: The shrinkage limit of a soil is defined as the water content below which a reduction in the water content does not result in a decrease in the total volume of the soil. This is the minimum water content at which a soil can still be saturated.

In order to determine the shrinkage limit, a sample of soil having a high moisture content is filled up in a mould of known volume. The mould containing the sample is then kept in the oven at 105°C for 24 hours. After taking it out from the oven, the weight of the dry soil pat is taken and its volume is measured by the mercury displacement method.

Fig. 2.4(a) and 2.4(c) represent the schematic diagrams of the initial and final states of the sample while Fig. 2.4(b) represents that corresponding to

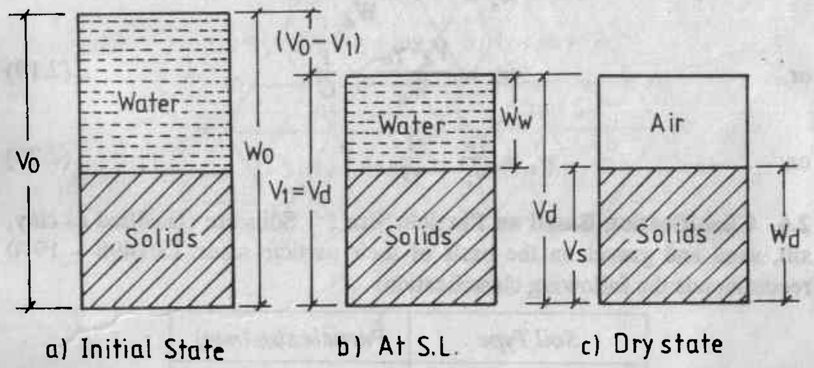


Fig. 2.4

the shrinkage limit. With reference to these figures, the shrinkage limit can be determined by the following two methods:

Method I: *When G is unknown*:

Let V_0 and V_d be the initial and final volumes of the sample and W_0 and W_d be its corresponding weights. By definition, the volume of the soil at shrinkage limit is equal to its final volume. Let W_w be the weight of water at this stage. The shrinkage limit is then given by,

$$w_s = \frac{W_w}{W_d}$$

At the initial stage, weight of water = $W_0 - W_d$.

Weight of water evaporated upto shrinkage limit = $(V_0 - V_d)\gamma_w$

$$\therefore \quad W_w = (W_0 - W_d) - (V_0 - V_d)\gamma_w$$

$$\therefore \quad W_s = \frac{(W_0 - W_d) - (V_0 - V_d)\gamma_w}{W_d} \quad ...(2.18)$$

Method II: *When G is known*:

Let V_s = volume of solids

$$\therefore \quad V_s = \frac{W_d}{\gamma_s} = \frac{W_d}{G\gamma_w}$$

But, $$W_w = (V_d - V_s)\,\gamma_w = \left(V_d - \frac{W_d}{G\gamma_w}\right)\gamma_w$$

$$= V_d \cdot \gamma_w - \frac{W_d}{G}$$

$$\therefore \quad w_s = \frac{V_d \cdot \gamma_w - W_d/G}{W_d}$$

or, $$w_s = \frac{V_d \cdot \gamma_w}{W_d} - \frac{1}{G} \qquad ...(2.19)$$

or, $$w_s = \frac{\gamma_w}{\gamma_s} - \frac{1}{G} \qquad ...(2.20)$$

2.6. Classification Based on Particle Size : Soils are classified as clay, silt, sand and gravel on the basis of their particle sizes. IS:1498 – 1970 recommends the following classification:

Soil Type	*Particle size (mm)*
Clay	< 0.002
Silt	0.002 to 0.075
Sand :	
(i) Fine sand	0.075 to 0.425
(ii) Medium sand	0.425 to 2.0
(iii) Coarse sand	2.0 to 4.75
Gravel	4.75 to 80

***2.6.1. Textural Classification System*:** Any soil, in its natural state, consists of particles of various sizes. On the basis of the percentages of particle sizes, and following certain definite principles, broad classification of such mixed soil is possible.

Fig. 2.5 shows the triangular classification chart of the Mississippi River Commission, USA. It essentially consists of an equilateral triangle ABC. The percentages of sand, silt and clay (ranging from 0% to 100%) are plotted along the sides AB, BC and CA respectively. The area of the triangle is divided into a number of segments and each segment is given a name. In order to find out the group to which a given soil belongs, three lines are required to be drawn from the appropriate points on the three sides along the directions shown by the arrows. These three lines intersect at a single point. The nomenclature of the soil is then determined according to the name of the segment in which the intersection point lies.

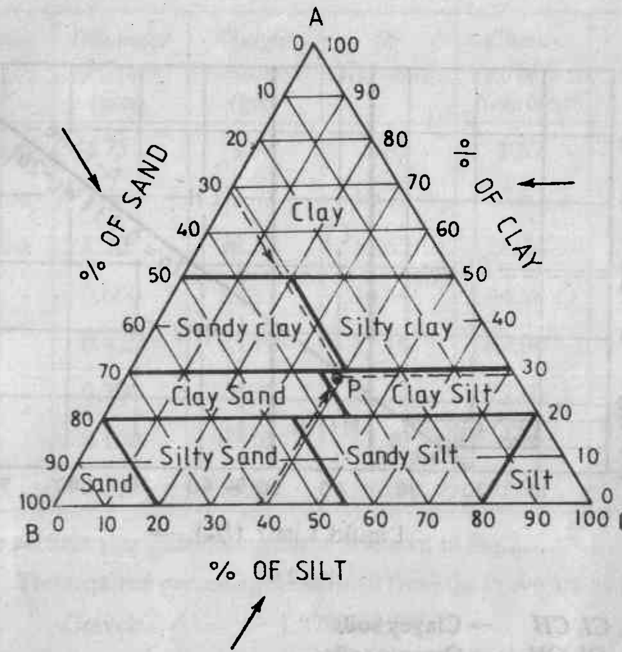


Fig. 2.5

2.7. Plasticity Chart: This chart is useful for identifying and classifying fine-grained soils. In this chart the ordinate and abscissa represent the values of plasticity index and liquid limit respectively. A straight line called A-line, represented by the equation $I_p = 0.73\,(w_L - 20)$, is drawn and the area under the chart is divided into a number of segments. On the chart any fine-grained soil can be represented by a single point if its consistency limits are known. The segment in which this point lies determines the name of the soil.

Fig. 2.6 shows a plasticity chart. The meaning of the symbols used in the chart are as follows:

M : Silty soils.
C : Clayey soils.
O : Organic soils.
L : Low plasticity
I : Medium or intermediate plasticity
H : High plasticity

Main groups of fine-grained soils are

ML, MI, MH → Silty soils

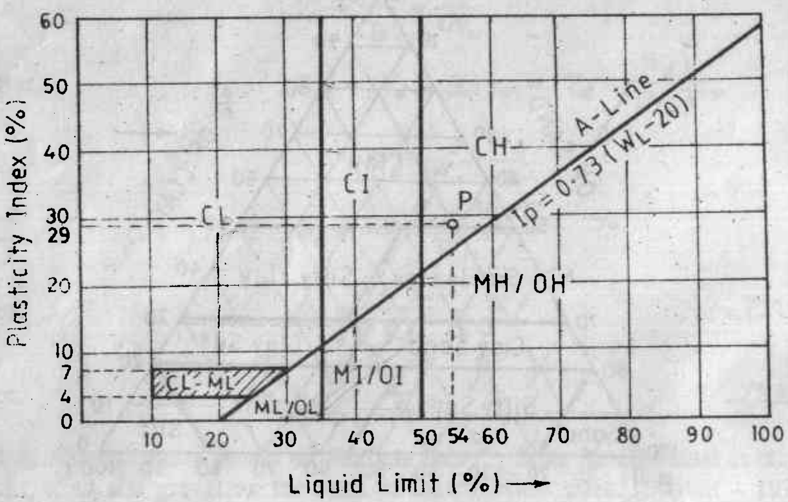


Fig. 2.6

CL, CI, CH → Clayey soils
OL, OI, OH → Organic soils.

EXAMPLES

Problem 2.1. The results of a sieve analysis performed on a dry soil sample weighing 500 gm are given below:

I. S. Sieve	4.75 mm	2.40 mm	1.20 mm	600 μ	425 μ	300 μ	150 μ	75 μ
Wt. of soil retained (gm)	9.36	53.75	78.10	83.22	85.79	76.82	67.02	33.88

(i) Plot the particle size distribution curve of the soil.

(ii) Find out the percentage of gravel, coarse sand, medium sand, fine sand and silt present in the soil.

(iii) Determine the uniformity co-efficient and the co-efficient of curvature. Hence comment on the type of soil.

Solution: (i) The computations necessary for plotting the particle size distribution curve are shown below:

I.S. Sieve	*Diameter of Grains (mm)*	*Weight Retained (gm)*	*% Retained*	*Cumulative % Retained*	*% Finer*
4.75 mm	4.75	9.36	1.87	1.87	98.13
2.40 mm	2.40	53.75	10.75	12.62	87.38
1.20 mm	1.20	78.10	15.62	28.24	71.76
600 μ	0.600	83.22	16.64	44.88	55.12
425 μ	0.425	85.79	17.16	62.04	37.96
300 μ	0.300	76.82	15.36	77.40	22.60
150 μ	0.150	67.02	13.40	90.80	9.20
75 μ	0.075	33.88	6.78	97.58	2.42

The particle size distribution curve is shown in Fig.2.7.

(ii) The required percentages obtained from the curve are as follows:

Gravel:	1.87% ≈ 1.9%	
Coarse sand :	98.1% – 92%	= 6.1%
Medium sand :	92% – 38%	= 54%
Fine sand :	38% – 2.4%	= 35.6%
Silt :	2.42% ≈ 2.4%	

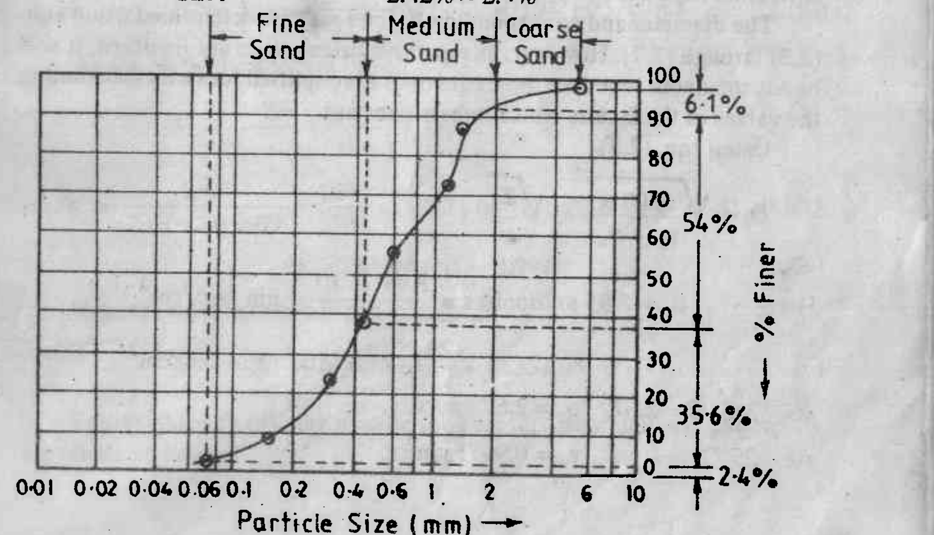


Fig. 2.7

Problem 2.2. 500 gm of dry soil sample was used in a sieve analysis. 178.85 gm of soil passed through the 75 μ sieve and was collected in the steel pan, out of which 50 gm was taken and a 1 litre suspension was made by adding distilled water and dispersing agent to it in a measuring cylinder having a diameter of 6.15 cm. The volume of the hydrometer was 50 cc, the height of bulb 15.5 cm and the length of calibration on its stem 9.7 cm. The minimum and maximum marks on its stem were 990 and 1040 respectively. A hydrometer test was then performed at the room temperature of 25°C and the following readings were recorded:

Elapsed time (min)	$\frac{1}{2}$	1	2	4	8	15	30	60
Hydrometer reading	1024	1023	1020	1017	1013	1010	1006	1001

When the hydrometer was immersed in distilled water containing the same quantity of dispersing agent as that present in the suspension, the reading was found to be 999.5. At 25°C, the unit weight of water is 0.9971 gm/cc and its viscosity is 8.95 millipoises. The specific gravity of soil solids is 2.67. The meniscus correction may be taken as 0.5.

Find out the diameter of particles settled corresponding to each hydrometer reading and the respective % finer values. Neglect volumetric expansion due to temperature change.

Solution: The temperature correction and the dispersing agent correction need not be applied here.

The diameter and corresponding % finer may be determined using eqn. (2.3) through (2.7). However, as repetitive calculations are involved, it will be advantageous to reduce these equations to simplified forms by substituting the values of the factors which remain constant.

Using eqn. (2.3)

$$D = \sqrt{\frac{1800\,\mu}{\gamma_s - \gamma_w}} \times \sqrt{\frac{Z_r}{t}}$$

Here $\mu = 8.95 \text{ millipoises} = \frac{8.95 \times 10^{-3}}{981} \text{ gm-sec/cm}^2$

$$= 9.123 \times 10^{-6} \text{ gm-sec/cm}^2$$

As $G = 2.67,\quad \gamma_s = 2.67$ gm/cc.

At 25°C, $\gamma_w = 0.9971$ gm/cc.

$$\therefore \quad D = \sqrt{\frac{(1800)(9.123 \times 10^{-6})}{2.67 - 0.9971}} \times \sqrt{\frac{Z_r}{t}}$$

$$\text{or,} \quad D = 0.0991\sqrt{\frac{Z_r}{t}} \qquad \text{...(i)}$$

Using eqn. (2.5),

$$Z_r = H_1 + \frac{1}{2}\left(h - \frac{V_h}{A}\right)$$

$$\text{Here,} \quad A = \frac{\pi}{4}(6.15)^2 = 29.706 \text{ cm}^2$$

$$\therefore \quad Z_r = H_1 + \frac{1}{2}(15.5 - 50/29.706)$$

$$\text{or,} \quad Z_r = H_1 + 6.908 \qquad \text{...(ii)}$$

Using eqn. (2.6),

$$H_1 = \frac{(r_d + 1) - r_1}{r_d} \times L$$

$$\text{Here,} \quad r_d = 1.040 - 0.99 = 0.05,$$

$$L = 9.7 \text{ cm},$$

$$\therefore \quad H_1 = \left(\frac{1 + 0.05 - r_1}{0.05}\right)(9.7) = 194(1.05 - r_1) \qquad \text{...(iii)}$$

Again, % finer on 50 gm of soil

$$N = \frac{\gamma_s}{\gamma_s - \gamma_w} \cdot \frac{V}{Ws_1} \cdot \gamma_c (r_1 + C_m - r_w) \times 100$$

$$\text{or,} \quad N = \frac{2.67}{2.67 - 0.9971} \times \frac{1000}{50} \times 0.9971\,(r_1 + 0.0005 - 0.9995) \times 100$$

$$\text{or,} \quad N = 3182.8\,(r_1 - 0.999) \qquad \text{...(iv)}$$

% finer on 500 gm of soil taken initially

$$N' = N \times \frac{178.85}{500} = 0.3577\,N \qquad \text{...(v)}$$

Eqn. (i) through (v) may now be used for the computations. The results are tabulated below.

Time (sec)	Hydro-meter reading r_1	$H_1 = 195 \times (1.05 - r_1)$ (cm)	$Z_r = H_1 + 6.908$ (cm)	$D = 0.0991\sqrt{\frac{Z_r}{t}}$ (mm)	$N = 3182.8 \times (\gamma_1 - 0.999)$ (%)	$N' = 0.3577 \times N$ (%)
30	1.024	5.044	11.952	0.0625	79.57	28.46
60	1.023	5.238	12.146	0.0446	76.39	27.32
120	1.020	5.820	12.728	0.0323	66.84	23.91
240	1.017	6.402	13.310	0.0233	57.29	20.49
480	1.013	7.178	14.086	0.1697	44.56	15.94
900	1.010	7.760	14.668	0.0126	35.01	12.52
1800	1.006	8.536	15.444	0.0092	22.28	7.97
3600	1.001	9.506	16.414	0.0067	6.37	2.28

Problem 2.3. Distilled water was added to 60 gm of dry soil to prepare a suspension of 1 litre. What will be the reading of a hydrometer in the suspension at $t = 0$ sec, if the hydrometer could be immersed at that time? Assume, density of water = 1 gm/cc and specific gravity of solids = 2.70.

Solution: At $t = 0$ sec, the solid grains have not started to settle. The suspension, therefore, is homogeneous, having constant density at any point in it.

As $G = 2.70$, $\gamma_s = 2.70$ gm/cc.

Total volume of solids in the suspension

$$= \frac{60}{2.70} = 22.22 \text{ cc.}$$

∴ Volume of solids in unit volume of suspension,

$$V_s = \frac{22.22}{1000} = 0.0222 \text{ cc.}$$

Volume of water in unit volume of suspension,

$$V_w = 1 - 0.0222 = 0.9778 \text{ cc.}$$

Weight of solids in unit volume of suspension,

$$W_s = (0.0222)(2.70) = 0.0599 \text{ gm.}$$

Weight of water in unit volume of suspension,

$$W_w = (0.9778)(1) = 0.9778 \text{ gm.}$$

∴ Total weight of unit volume of suspension,

$$W = W_s + W_w = 0.0599 + 0.9778 = 1.0377 \text{ gm.}$$

∴ Density of the suspension = 1.0377 gm/cc ≈ 1.038 gm/cc.

Therefore, reading of the hydrometer = 1038.

Problem 2.4. A sample of dry soil ($G_s = 2.68$) weighing 125 gm is uniformly dispersed in water to form a 1 litre suspension at a temperature of 28°C.

(i) Determine the unit weight of the suspension immediately after its preparation.

(ii) 10 cc of the suspension was removed from a depth of 20 cm beneath the top surface after the suspension was allowed to settle for 2.5 min. The dry weight of the sample in the suspension drawn was found to be 0.398 gm. Determine a single point on the particle size distribution curve corresponding to this observation. Given, at 28°C, viscosity of water = 8.36 millipoises and unit weight of water = 0.9963 gm/cc

Solution: (i) Volume of solids in the suspension = $\frac{125}{2.68} = 46.64$ cc.

Considering unit weight of suspension,

Volume of solids present $= \frac{46.64}{1000} = 0.0466$ cc

Volume of water present = 1 − 0.0466 = 0.9534 cc

Weight of 0.466 cc of solids = (0.0466)(2.68) = 0.1249 gm

Weight of 0.9534 cc of water at 28°C = (0.9534) (0.9963) = 0.9499 gm.

∴ Total weight of 1 cc of suspension

$$= 0.1249 + 0.9499 = 1.0748 \text{ gm.}$$

Therefore, unit weight of suspension = 1.0748 gm/cc.

(ii) We have, from Stokes' law,

$$v = \frac{\gamma_s - \gamma_w}{18\,\mu} \cdot D^2$$

or,

$$D = \sqrt{\frac{18\,\mu}{\gamma_s - \gamma_w}} \times \sqrt{v}$$

Let D be the diameter of the particles settled to a depth of 20 cm at t = 2.5 min. with a uniform velocity v.

$$\therefore \quad v = \frac{Z}{t} = \frac{20}{(2.5)(60)} = 0.133 \text{ cm/sec}$$

$$\mu = 8.36 \text{ millipoises} = \frac{8.36 \times 10^{-3}}{981} = 8.522 \times 10^{-6} \text{ gm-sec/cm}^2$$

$$\gamma_s = 2.68 \text{ gm/cc}, \ \gamma_w = 0.9963 \text{ gm/cc}$$

$$\therefore \quad D = \sqrt{\frac{(18)\,(8.522 \times 10^{-6})}{2.68 - 0.9963}} \times \sqrt{0.133} \text{ cm}$$

$$= 3.48 \times 10^{-3} \text{ cm} = 0.035 \text{ mm}$$

Again, at time $t = 0$, weight of solids present in 1 cc of suspension = 0.1249 gm.

∴ Weight of solids present in 10 cc of suspension = 1.249 gm.

At time $t = 2.5$ min., weight of solids present in 10 cc of suspension = 0.398 gm

$$\therefore \quad \% \text{ finer} = \frac{0.398}{1.249} \times 100 = 31.86\%$$

Hence the co-ordinates of the required point on the particle size distribution curve are:

$$D = 0.035 \text{ mm}$$

$$N = 31.86\%$$

Problem 2.5. The results of a liquid limit test are given below:

No. of blows	48	38	29	20	14
Water content (%)	32.1	35.9	40.7	46.1	52.8

(a) Determine the liquid limit of the soil.

(b) If the plastic limit of the soil be 23%, find out the plasticity index, flow index and toughness index. Hence comment on the nature of the soil.

Solution: (a) From the given data, a curve between the water content and the number of blows is plotted on a semi-log graph paper. Fig. 2.8 shows this w vs. $\log_{10} N$ curve. The water content corresponding to 25 blows, as obtained from the curve, is 43%. Hence the liquid limit of the soil is 43%.

(b) Plasticity index, $I_p = w_l - w_p = 43\% - 23\% = 20\%$

$$\text{Flow index,} \quad I_f = \frac{52.8 - 32.1}{\log_{10} 48/14} = = 38.68\%$$

$$\text{Toughness index,} \quad I_t = \frac{I_p}{I_f} = \frac{20}{38.68} = 0.52.$$

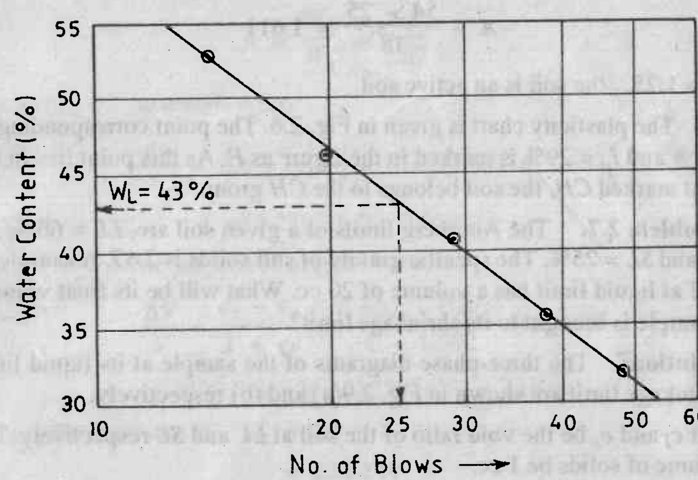


Fig. 2.8

As the plasticity index is greater than 17%, the soil is highly plastic in nature.

As the toughness index is less than 1, the soil is friable at liquid limit.

Problem 2.6. Laboratory tests on a soil sample yielded the following results:

Liquid limit = 54%

Plastic limit = 25%

Natural moisture content = 29%

% finer than 0.002 mm = 18%

(a) Determine the liquidity index of the soil and comment on its consistency.

(b) Find out the activity number and comment on the nature of the soil.

(c) Classify the soil with the help of a plasticity chart.

Solution: (a) Liquidity index, $I_l = \dfrac{w_n - w_p}{w_l - w_p}$

$$= \frac{29 - 25}{54 - 25} = 0.138$$

As $0 < I_l < 0.25$, the soil is in the plastic state and is stiff.

(b) Activity number, $A = \dfrac{I_p}{\% < 0.002 \text{ mm}}$

or, $$A = \frac{54 - 25}{18} = 1.611$$

As $A > 1.25$, the soil is an active soil.

(c) The plasticity chart is given in Fig. 2.6. The point corresponding to $w_l = 54\%$ and $I_p = 29\%$ is marked in the figure as P. As this point lies in the segment marked CH, the soil belongs to the CH group.

Problem 2.7. The Atterberg limits of a given soil are, $LL = 60\%$, $PL = 45\%$ and $SL = 25\%$. The specific gravity of soil solids is 2.67. A sample of this soil at liquid limit has a volume of 20 cc. What will be its final volume if the sample is brought to its shrinkage limit?

Solution: The three-phase diagrams of the sample at its liquid limit and shrinkage limit are shown in Fig. 2.9(a) and (b) respectively.

Let e_l and e_s be the void ratio of the soil at LL and SL respectively. Let the volume of solids be 1 cc.

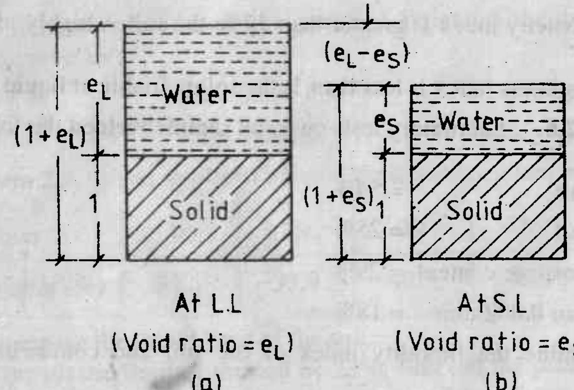


Fig. 2.9

We have, $$e = \frac{V_v}{V_s}\text{ , or, } V_v = e \cdot V_s$$

At liquid limit, $V_v = e_l \cdot 1 = e_l$ cc

$\therefore$ Volume of water present $= e_l$ cc

Weight of this water $= e_l \times 1 = e_l$ gm

Weight of solids $= V_s \cdot G\gamma_w = (1)(2.67) = 2.67$ gm

$$\therefore \quad w = \frac{W_w}{W_s} = \frac{e_l}{2.67}$$

But, at LL, $w = 60\% = 0.6$.

$$\therefore \quad \frac{e_l}{2.67} = 0.6\text{ , or, } e_l = (0.6)(2.67) = 1.602$$

Similarly, at SL, $e_s = (0.25)(2.67) = 0.668$

Change in volume per unit of original volume,

$$\frac{\Delta V}{V} = \frac{e_l - e_s}{1 + e_l} = \frac{1.602 - 0.668}{1 + 1.602} = 0.359$$

$$\therefore \quad \Delta V = 0.359\, V = (0.359)(20) = 7.18 \text{ cc}$$

Hence, final volume at $SL = 20 - 7.18$

$= 12.82$ cc

Problem 2.8. The consistency limits of a soil sample are:
$LL = 52\%, PL = 35\%, SL = 17\%$

If a specimen of this soil shrinks from a volume 10 cc at liquid limit to 6.1 cc at plastic limit, determine the specific gravity of solids.

Solution: Let e_l and e_s be the void ratio corresponding to the liquid limit and plastic limit.

Let volume of solids be 1 cc.

$\therefore$ At liquid limit, volume of water $= e_l$ cc

Weight of water $= e_l$ gm

Weight of solids $= V_s \cdot G\gamma_w = 1 \cdot G \cdot 1 = G$ cc

$$\therefore \quad w = \frac{W_w}{w_l} = e_l$$

But at liquid limit, $w = 52\% = 0.52$

$$\therefore \quad \frac{e_l}{G} = 0.52\text{ , or, } e_l = 0.52\,G$$

Similarly we obtain, $e_s = 0.17\,G$

Now, change in volume per unit of original volume,

$$\frac{\Delta V}{V} = \frac{e_l - e_s}{1 + e_l} = \frac{0.52\,G - 0.17\,G}{1 + 0.52\,G} = \frac{0.35\,G}{1 + 0.52\,G}$$

But, $$\frac{\Delta V}{V} = \frac{10 - 6.1}{10} = 0.39$$

$$\therefore \quad \frac{0.35\,G}{1 + 0.52\,G} = 0.39$$

$$\text{or.} \quad G = 2.65$$

Problem 2.9: An oven dried pat of clay weighs 26.20 gm and displaces 190 gm of mercury when fully immersed in it. If the specific gravity of solids be 2.7, determine the shrinkage limit of the soil.

Solution: (i) Solution from first principles:

Fig. 2.10 shows the schematic diagram of the dry soil pat.

Volume of the dry pat $= \frac{W_{Hg}}{\gamma_{Hg}} = \frac{190}{13.6} = 13.97$ cc

Unit weight of solids, $\gamma_s = G\gamma_w = (2.7)(1.0) = 2.7$ gm/cc

Volume of solids, $V_s = 26.20/2.7 = 9.7$ cc

$\therefore$ Volume of voids, $V_v = 13.97 - 9.7 = 4.27$ cc

When the soil is at shrinkage limit, this volume of 4.27 cc will be just filled up with water.

Weight of this water = 4.27 gm

Moisture content at that stage,

$$w = \frac{W_w}{W_s} = \frac{4.27}{26.20} = 0.163 = 16.30\%$$

$\therefore$ Shrinkage limit = 16.3%

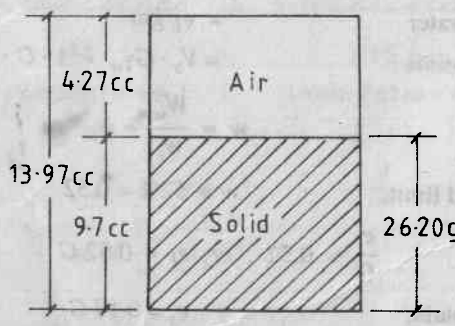


Fig. 2.10

(ii) *Solution using eqn. (2.19):*

The shrinkage limit is given by,

$$w_s = \frac{V_d \cdot \gamma_w}{W_d} - \frac{1}{G}$$

Here, V_d = volume of dry soil pat = 13.97 cc.

W_d = weight of dry soil pat = 26.20 gm

$G = 2.7$.

$$\therefore \quad w_s = \frac{(13.97)(1)}{26.20} - \frac{1}{2.7} = 0.163$$

Hence, Shrinkage limit = 16.3%

Problem 2.10. A sample of coarse sand was found to have void ratios of 0.87 and 0.52 in its loosest and densest states respectively. The in-situ density and water content of the sand were 1.95 gm/cc and 23%. Determine the degree of saturation and relative density of the sand in the field. Given, $G = 2.66$.

Solution: We have,

$$\gamma = \frac{G + se}{1 + e} \cdot \gamma_w = \frac{G + wG}{1 + e} \cdot \gamma_w = \frac{G(1 + w)}{1 + e} \cdot \gamma_w$$

According to the given field conditions

$$\gamma = 1.95 \text{ gm/cc}, w = 0.23, G = 2.66$$

$$\therefore \quad 1.95 = \frac{2.66\,(1 + 0.23)}{1 + e} (1)$$

$$\text{or,} \quad e = 0.678.$$

$$\therefore \text{ Degree of saturation, } s = \frac{wG}{e} = \frac{(0.23)(2.66)}{0.678} = 0.902 = 90.2\%$$

Again, using eqn. 2.10.

$$R_D = \frac{e_{max} - e}{e_{max} - e_{min}}$$

Here, $e_{max} = 0.87$, $e_{min} = 0.52$, $e = 0.678$.

$$\therefore \quad R_D = \frac{0.87 - 0.678}{0.87 - 0.52} = 0.55$$

As $\frac{1}{3} < R_D < \frac{2}{3}$, the soil is a medium sand.

Problem 2.11. The composition of a given soil is as follows:

Sand = 32%, Silt = 39%, Clay = 29%.

Draw a triangular classification chart and classify the soil.

Solution: The triangular classification chart is given in Fig. 2.5. In order to classify the soil, proceed as follows:

(i) On the side *AB* of the chart, which represents the percentage of sand, choose the point corresponding to 32%. Draw a straight line from that point in the direction of the arrow (i.e., parallel to the side AC representing the percentage of clay).

(ii) Similarly on the side BC, locate the point corresponding to 39% and draw another straight line making it parallel to BA. These two lines intersect each other at P.

(iii) If now a third line is drawn from the appropriate point (29%) on the clay side, making it parallel to AB, it will pass through P.

The point P then represents the given soil in the triangular classification chart. The point lies in the sector marked 'clay silt'. Hence the given soil is classified as a clay silt.

EXERCISE 2

2.1 The following data were obtained from a specific gravity test performed in the laboratory:

Weight of empty pycnometer = 201.25 gm

Weight of pycnometer and dry soil = 298.76 gm

Weight of pycnometer, soil and water = 758.92 gm

Weight of pycnometer full of water = 698.15 gm

Determine the specific gravity of the soil. [Ans. 2.654]

2.2 The results of a sieve analysis are given below:

I.S. Sieve	4.75 mm	2.40 mm	1.20 mm	600 μ	425 μ	300 μ	212 μ	150 μ	75 μ
Wt. of Soil Retained (gm)	32.34	41.60	47.29	58.14	71.23	74.99	46.24	58.14	38.17

The total weight of dry soil taken was 500 gm.

(a) Plot the particle size distribution curve.

(b) Determine the percentage of gravel, coarse sand, medium sand, fine sand and fine fractions in the soil.

(c) Determine the co-efficient of curvature and the uniformity co-efficient.

(d) Comment on the type of soil.

2.3 A combined mechanical analysis was carried out on a dry soil sample weighing 500 gm. The following are the results:

(a) Sieve analysis:

I.S. Sieve	4.75 mm	2.40 mm	1.20 mm	600 μ	425 μ	300 μ	150 μ	75 μ
Wt. of Soil Retained (gm)	6.85	50.45	67.10	72.31	52.51	51.27	84.01	58.15

(b) Hydrometer analysis:

Time (min)	$\frac{1}{2}$	1	2	4	8	15	30	60	120
Hydrometer Reading	1024	1023	1021	1019	1016	1013	1008	1005	1001

During the hydrometer test, 50 gm of soil retained on the steel pan was mixed with distilled water and dispersing agent to form a suspension of 1200 cc in a measuring cylinder having a diameter of 6.2 cm. The hydrometer had a volume of 50 cc. The length of its bulb and the calibration on the stem were 16 cm and 10 cm respectively. The range of calibrations was from 995 to 1035. When immersed in distilled water containing dispersing agent, the hydrometer read 998.5. Meniscus correction may be taken as 0.4. The specific gravity of solids was 2.69. The viscosity and unit weight of water at the room temperature of 28°C were respectively 8.36 millipoise and 0.9963 gm/cc.

Plot the particle size distribution curve and determine the percentage of gravel, sand, silt and clay.

2.4 Draw a rough sketch of the particle size distribution curve of a sand sample having the following properties:

Effective size (D_{10}) = 0.17 mm

Uniformity co-efficient = 5.5

Co-efficient of curvature = 1.2.

2.5 100 gm of dry soil was mixed with water at 4°C to form a 1000 cc suspension. If G = 2.72, determine the initial unit weight of the suspension. To what depth with the particles having effective diameter of 0.05 mm settle after 5 minutes? What will be the time required by a 5 micron particle to settle through 10 cm? The viscosity of water at 4°C may be assumed as 0.85×10^{-3} poise.

2.6 The results of a liquid limit test are given below:

No. of blows	11	15	23	30	46	53
Water content (%)	53.9	50.6	48.1	46.0	43.3	41.0

Draw the flow curve and determine the liquid limit and flow index of the soil.

[Ans: 47%, 18.9%]

2.7 The Atterberg limits of a given soil are:

$$LL = 68\%, PL = 37\%, SL = 22\%$$

If the natural moisture content of this soil at the site be 42%, then determine:

(i) Plasticity index (ii) Consistancy index (iii) Liquidity index.

Comment on the nature of the soil on the basis of these indices.

[Ans. (i) 31% (ii) 0.839 (iii) 0.161]

2.8 A single liquid limit test was performed with Casagrande's liquid limit device on a soil sample with known Atterberg limits. The number of blows required to close the groove was recorded as 53. The corresponding moisture content of the sample was found to be 28%. If the liquid limit and plastic limit of the soil be 74% and 41% respectively, determine its toughness index. [Ans. 0.23]

2.9 The weight and volume of a fully saturated soil sample were 55.4 gm and 29.2 cc respectively. After drying in an oven for 24 hours, its weight and volume reduced to 39.8 gm and 21.1 cc respectively. Find out the shrinkage limit of the soil. [Ans.18.8%]

2.10 If the dry density and unit weight of solids of a soil be 1.68 gm/cc and 2.65 gm/cc respectively, determine its shrinkage limit. [Ans. 21.8%]

2.11 A cylindrical soil sample of 7.5 cm height and 3.75 cm diameter has been prepared at the shrinkage limit. If the sample is now allowed to absorb water so that its water content reaches the liquid limit, what will be its volume? Given $LL = 62\%, PL = 34\%, SL = 21\%, G = 2.68$.

[Ans. 48.67cc]

2.12 A cylindrical mould of 10 cm internal diameter and 11.7 cm height weighs 1894 gm. The mould was filled up with dry soil, first at its loosest state and then at the densest state, and was found to weigh 3273 gm and 3538 gm respectively. If the natural soil existing at the field be submerged below the ground water table and has a water content of 23%, determine the relative density of the soil and comment on its state of compactness. Given, $G = 2.65$.

[Ans. 54.75%]

2.13 The Atterberg limits of a given soil are as follows:

$$LL = 41\%, PL = 29\%, SL = 18\%.$$

Draw a plasticity chart and classify the soil.

2.14 Draw a triangular classification chart and classify the soil having the following composition:

Sand = 43%. Silt = 31%, Clay = 26%.

3

CAPILLARITY AND PERMEABILITY

3.1 Capillarity: The interconnected pore spaces in a soil mass may be assumed to form innumerable capillary tubes. At any given site, the natural ground water table normally exists at a certain depth below the ground level. Due to surface tension, water gradually rises from this level through the capillary tubes. This causes the soil above the ground water table to be partially or even fully saturated.

In Fig. 3.1, h_c represents the maximum height of capillary rise of water in a capillary tube of diameter d. The upper meniscus of water is concave upwards and makes an angle α with the vertical (if the tube is perfectly clean and wet, $\alpha = 0$). The surface tension, T_s, also acts in this direction. The vertical component of T_s is responsible for balancing the self-weight of the water column.

Now, volume of capillary water $= \dfrac{\pi d^2}{4} . h_c$

Weight of capillary water $= \dfrac{\pi d^2}{4} . h_c . \gamma_w$

Again, vertical component of the surface tension force

$$= T_s . \pi d . \cos\alpha$$

$$\therefore \quad \frac{\pi d^2}{4} . h_c . \gamma_w = T_s . \pi d . \cos\alpha$$

or
$$h_c = \frac{4\, T_s \cos\alpha}{d . \gamma_w} \qquad ...(3.1)$$

At 4°C, $T_s = 75.6$ dynes/cm $= 75.6 \times 10^{-8}$ kN/cm

and, $\gamma_w = 1$ gm/cc $= 9.807$ kN/m^3 $= 9.807 \times 10^{-6}$ kN/cm^3

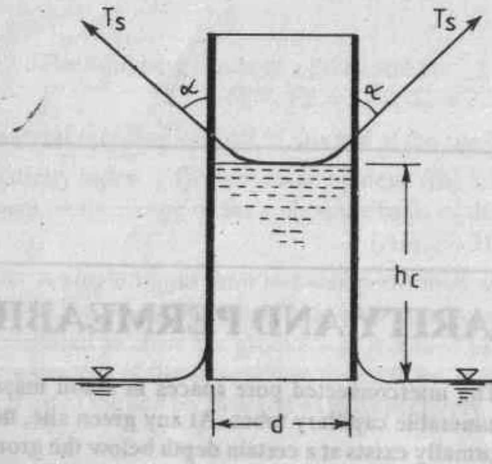


Fig. 3.1

Assuming the tube to be perfectly clean and wet, $\cos \alpha = \cos 0° = 1$

$$\therefore \quad h_c = \frac{(4)(75.6 \times 10^{-8})}{(9.807 \times 10^{-6}).d}(1)$$

or, $$h_c = \frac{0.3084}{d} \text{ cm} \quad ...(3.2)$$

The value of h_c may also be determined from:

$$h_c = \frac{C}{e\,d_{10}} \quad ...(3.3)$$

where, e = void ratio

d_{10} = particle size corresponding to 10% finer

C = empirical constant, the value of which depends on the shape and surface impurities of the grains and lies between 0.1 and 0.5 cm^2.

3.2 Pressure Due to Capillary Water : The capillary water rises against gravity and is held by the surface tension. Therefore, the capillary water exerts a tensile force on the soil. However, the free water exerts a pressure due to its own self weight, which is always compressive.

The distribution of vertical pressure in a soil saturated upto a height h_c due to capillary water is shown in Fig. 3.2.

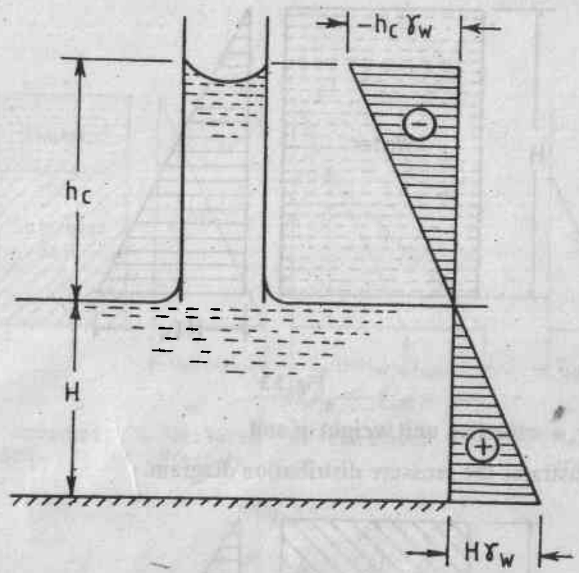


Fig. 3.2

3.3 Total, Effective and Neutral Stresses : When an external load is applied on a saturated soil mass, the pressure is immediately transferred to the pore water. At this point, the soil skeleton does not share any load. But with passage of time, the pore water gradually escapes due to the pore water pressure induced and a part of the external stress is transferred to the solid grains. The total stress σ is therefore divided into the following components:

(i) Effective stress or intergranular pressure, σ'

(ii) Pore water pressure or neutral stress, u.

or, $$\sigma = \sigma' + u \quad ...(3.4)$$

3.4 Distribution of Vertical Stress in Various Soil-water Systems

(i) *Free water* : In free water, the hydrostatic pressure distribution is linear. At any depth z below the water level, the vertical pressure is given by,

$$u = z\gamma_w \quad ...(3.5)$$

The pressure distribution diagram is shown in Fig. 3.3.

(ii) *Dry soil* : In a dry soil mass, the distribution of vertical stress is similar to a hydrostatic pressure distribution. At any depth z, the pressure is given by,

$$\sigma' = \gamma z$$

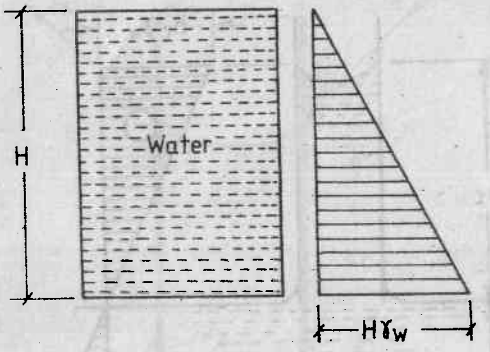


Fig. 3.3

where, γ = effective unit weight of soil

Fig. 3.4 illustrates the pressure distribution diagram.

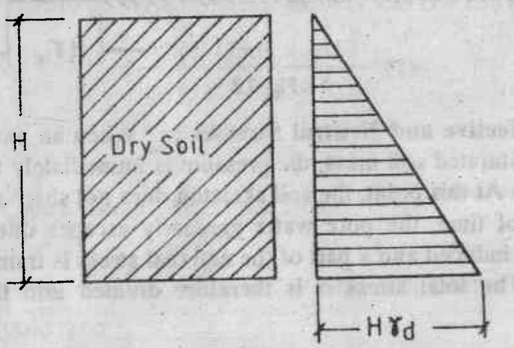


Fig. 3.4

(iii) *Submerged soil* : Fig. 3.5 shows a soil mass submerged in water with free water standing upto a height H_w. If H be the height of the soil, the total pressure at the bottom of it is given by,

$$\sigma = \gamma_{sub} \cdot H + \gamma_w \, (H_w + H)$$

or, $$\sigma = (\gamma_{sub} + \gamma_w) \, H + \gamma_w H_w$$

or, $$\sigma = \gamma_{sat} \cdot H + \gamma_w H_w \qquad ...(3.6)$$

Pore water pressure, $$u = \gamma_w \, (H + H_w) \qquad ...(3.7)$$

$\therefore$ Effective stress $$\sigma' = \sigma - u$$

$$= \gamma_{sat} \cdot H + \gamma_w H_w - \gamma_w \, (H + H_w)$$

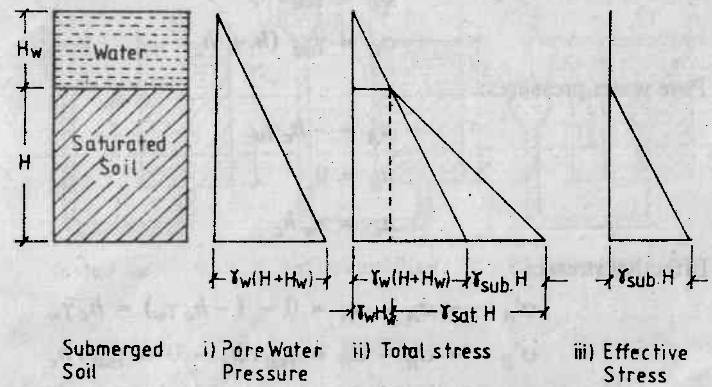


Fig. 3.5

$$= H \, (\gamma_{sat} - \gamma_w)$$

or, $$\sigma' = \gamma_{sub} \cdot H \qquad ...(3.8)$$

(iv) *Saturated soil with capillary water* : In Fig. 3.6, the soil mass is saturated upto a height h_c above the water level, due to capillary rise of water. The total stresses, pore water pressures and the effective stresses at various levels are worked out below:

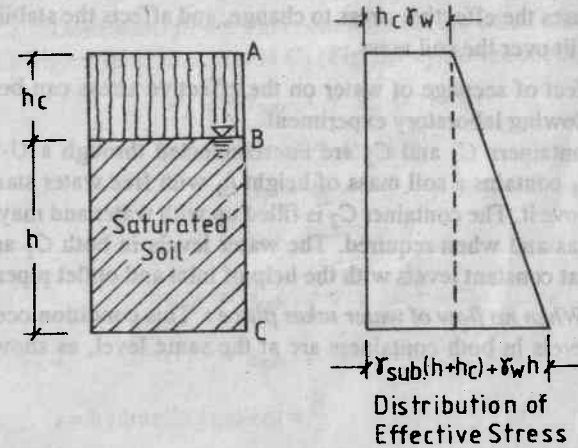


Fig. 3.6

(a) Total stresses:

$$\sigma_A = 0$$

$$\sigma_B = \gamma_{sat} \cdot h_c$$

$$\sigma_C = \gamma_{sat}\ (h + h_c)$$

(b) Pore water pressures:

$$u_A = -h_c \gamma_w$$

$$u_B = 0$$

$$u_C = \gamma_w h_c$$

(c) Effective stresses :

$$\sigma'_A = \sigma_A - u_A = 0 - (-h_c \gamma_w) = h_c \gamma_w$$

$$\sigma'_B = \sigma_B - u_B = \gamma_{sat} \cdot h_c - 0 = \gamma_{sat} \cdot h_c$$

$$\sigma'_C = \sigma_C - u_C = \gamma_{sat}(h + h_c) - \gamma_w h_c$$

$$= \gamma_{sat} \cdot h + (\gamma_{sat} - \gamma_w) h_c$$

or, $$\sigma'_C = \gamma_{sat} \cdot h + \gamma_{sub} \cdot h_c \quad ...(3.9)$$

$$= \gamma_{sub} \cdot h + \gamma_w \cdot h + \gamma_{sub} \cdot h_c$$

or, $$\sigma'_C = \gamma_{sub}(h + h_c) + \gamma_w h \quad ...(3.10)$$

3.5 Pore Pressure in Seepage Water : The shear strength of a soil is governed by the effective stress. When no flow of water takes place through a soil, the effective stress at a given point remains constant. However, seepage of water causes the effective stress to change, and affects the stability of any structure built over the soil mass.

The effect of seepage of water on the effective stress can be analysed with the following laboratory experiment.

Two containers C_1 and C_2 are interconnected through a U-tube. The container C_1 contains a soil mass of height h_1 with free water standing to a height h_2 above it. The container C_2 is filled up with water and may be raised or lowered as and when required. The water levels in both C_1 and C_2 are maintained at constant levels with the help of inlet and outlet pipes.

Case 1 : When no flow of water takes place : This condition occurs when the water levels in both containers are at the same level, as shown in Fig. 3.7(a).

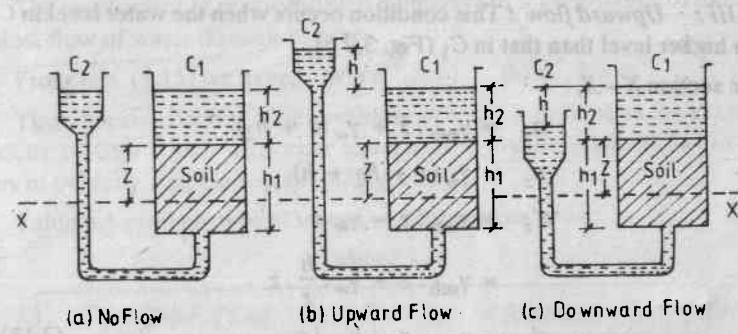


Fig. 3.7

At any depth z below the top of the soil mass (i.e., sec. X – X)

$$\sigma_z = \gamma_{sub} \cdot z + \gamma_w (z + h_2)$$

and , $$u_z = \gamma_w (z + h_2)$$

$\therefore$ $$\sigma'_z = \sigma_z - u_z$$

$$= \gamma_{sub} \cdot z + \gamma_w (z + h_2) - \gamma_w (z + h_2)$$

or, $$\sigma'_z = \gamma_{sub} \cdot z \quad ...(3.11)$$

Thus , at any depth z, the effective stress depends only on the submerged density of the soil.

Case II : Downward flow : This condition occurs when the water level in C_1 is at a higher level than that in C_2 (Fig. 3.7 c). At the section $X-X$,

$$\sigma_z = \gamma_{sub} \cdot z + \gamma_w (z + h_2)$$

and $$u_z = \gamma_w (z + h_2 - h)$$

$\therefore$ $$\sigma'_z = \sigma_z - u_z$$

$$= \gamma_{sub} \cdot z + \gamma_w \cdot h$$

$$= \gamma_{sub} \cdot z + \gamma_w \cdot \frac{h}{z} \cdot z$$

or, $$\sigma'_z = \gamma_{sub} \cdot z + \gamma_w\, i z \quad ...(3.12)$$

where, i = hydraulic gradient = $\frac{h}{z}$

A comparison between equations (3.11) and (3.12) clearly shows that a downward flow causes the effective stress to increase.

Case III : Upward flow : This condition occurs when the water level in C_2 is at a higher level than that in C_1 (Fig. 3.7 b).

At the section $X - X$,

$$\sigma_z = \gamma_{sub} \cdot z + \gamma_w (z + h_2)$$

$$u_z = \gamma_w (z + h_2 + h)$$

$$\therefore \quad \sigma'_z = \gamma_{sub} \cdot z - \gamma_w h$$

$$= \gamma_{sub} \cdot z - \gamma_w \cdot \frac{h}{z} \cdot z$$

or,

$$\sigma'_z = \gamma_{sub} \cdot z - \gamma_w \, i \, z \quad \text{...(3.13)}$$

Thus an upward flow of water causes the effective stress to decrease.

3.6 Quicksand Condition : Eqn. (3.13) suggests that the reduction in effective stress at any depth z due to upward flow of water depends on the existing hydraulic gradient, i. If at any site, the hydraulic gradient reaches a certain critical value (i.e., $i = i_c$), the seepage pressure may become equal to the pressure due to the self-weight of the soil. In such cases, the effective stress will be zero. In other words, the solid grains will not carry any load any more, and the entire load is transmitted to the pore water. The entire soil mass will then behave as if it were a liquid, and any external load placed on the soil will settle immediately. At this stage the soil loses its shear strength and does not have any bearing power. Such a condition is known as the quicksand condition. The corresponding hydraulic gradient is called the critical hydraulic gradient.

From eqn. (3.13) we get,

$$0 = \gamma_{sub} \cdot z - \gamma_w \cdot i_c \cdot z$$

or,

$$i_c = \frac{\gamma_{sub}}{\gamma_w} = \frac{(G-1)}{1+e} \cdot \gamma_w / \gamma_w$$

or,

$$i_c = \frac{G-1}{1+e} \quad \text{...(3.14)}$$

3.7. Darcy's Law: This law states that, the velocity of flow of water through a soil mass is proportional to the hydraulic gradient.

i.e., $v \propto i$

or, $v = k\,i$ ...(3.15)

where, k = constant of proportionality, termed as the co-efficient of permeability of soil.

The co-efficient of permeability is a measure of the resistance of the soil against flow of water through its pores.

From eqn. (3.15) we have, when $i = 1$, then $k = v$.

Thus, the co-efficient of permeability of a soil is defined as the average velocity of flow which will occur under unit hydraulic gradient. It has the units of velocity, i.e., cm/sec, or, m/day, etc.

Table 3.1 presents typical values of k for various soils :

Table 3.1

Type of Soil	*k (cm/sec)*
Gravel	1 to 10^2
Coarse and medium sand	10^{-3} to 1
Fine sand, loose silt	10^{-5} to 10^{-3}
Dense silt, clayey silt	10^{-6} to 10^{-5}
Silty clay, clay	10^{-9} to 10^{-6}

Eqn. (3.15) may also be written as

$$q = k\,i\,A \quad \text{...(3.16)}$$

where, q = unit discharge, i.e., the quantity of water flowing through a cross-sectional area A in unit time.

3.8. Allen Hazen's Formula : Allen Hazen found experimentally that for loose filter sands,

$$k = C \cdot D_{10}^2 \quad \text{...(3.17)}$$

where, k = co-efficient of permeability in cm/sec

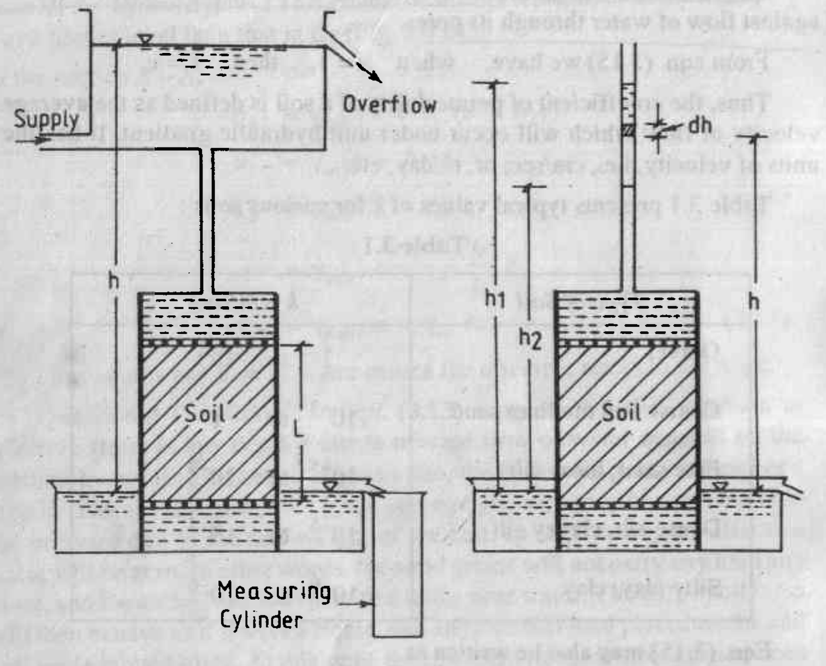
C = a constant, being approximately equal to 100 $cm^{-1}\ sec^{-1}$

D_{10} = Particle size corresponding to 10% finer, in cm.

3.9. Laboratory Determination of k : The co-efficient of permeability of a soil can be determined in the laboratory using permeameters, which are of the following two types:

(a) Constant head permeameter

(b) Falling head permeameter



(a) Constant Head Test (b) Falling Head Test

Fig. 3.8

The test arrangements for these two types of permeameters are shown in Fig. 3.8 (a) and (b) respectively.

3.9.1. ***Constant head permeameter*:** In this type of permeameters, arrangements are made to keep the water levels at the top and bottom of the soil sample constant. Water flowing through the soil from top to bottom is collected in a graduated glass cylinder and its volume is measured.

Let, Q = quantity of discharge in time t

L = length of the sample

h = difference in head of water at top and bottom.

Now, discharge per unit time, $q = \dfrac{Q}{t}$

We have from Darcy's law, $q = k\,i\,A$

Here, $i = \dfrac{h}{L}$

$\therefore \quad \dfrac{Q}{t} = k \cdot \dfrac{h}{L} \cdot A$

or, $k = \dfrac{Q \cdot L}{h\,A\,t}$ ...(3.18)

3.9.2. ***Falling head permeameter*:** In this case, a stand-pipe containing water is attached to the top of the soil mass. As water percolates through the soil from top to the bottom, the water level in the standpipe gradually falls down. Instead of measuring the discharge quantity, the fall of water level in the stand-pipe over a certain time interval t is measured.

Let, L = length of the soil sample

A = cross-sectional area of the sample

a = cross-sectional area of the stand-pipe

h_1 = head of water causing flow at time t_1

h_2 = head of water causing flow at time t_2

Let, in any small interval of time dt, the change in head is given by $-dh$ (the negative sign indicates that the head decreases).

Hence, the quantity of water flowing in time $dt = -dh \cdot a$

And, the discharge per unit time, $q = -\dfrac{dh}{dt} \cdot a$

But, we have from Darcy's law, $q = k\,i\,A$

$\therefore \quad k\,i\,A = -\dfrac{dh}{dt} \cdot a$

$\therefore \quad k \cdot \dfrac{h}{L} \cdot A = -\dfrac{dh}{dt} \cdot a$

or, $\dfrac{Ak}{aL}\,dt = -\dfrac{dh}{h}$ ...(3.19)

Integrating between proper limits, we get,

$$\frac{Ak}{aL}\int_{t_1}^{t_2} dt = -\int_{h_1}^{h_2}\frac{dh}{h}$$

or, $\dfrac{Ak}{aL}(t_2 - t_1) = -\log_e \dfrac{h_2}{h_1}$

or, $$k = \frac{aL}{At} \cdot \log_e \frac{h_1}{h_2} \qquad ...(3.20)$$

where, $t = t_2 - t_1$

The constant head permeameter is suitable for coarse-grained soils while the falling head permeameter is suitable for fine-grained ones.

3.10. Field Determination of k : In the field, the co-efficient of permeability of a stratified or heterogeneous deposit can be determined by either pumping-out tests or pumping-in tests. The pumping-out tests for unconfined as well as confined aquifers are described below:

(a) *Unconfined aquifer* : Fig. 3.9 illustrates a test well fully penetrating an unconfined aquifer. As water is pumped out from the well, water percolates from all sides into it. When the discharge q equals the rate of percolation, the water level in the well becomes steady.

Consider a point P on the drawdown curve at a radial distance r from the centre of the well. The hydraulic gradient at this point is given by,

$$i = \frac{dy}{dx}$$

Again, if h be the head of water at P then the rate of radial flow of water through a cylinder of radius r and height h is given by,

$$q = k\,i\,A = k \cdot \frac{dy}{dx} \cdot 2\pi x y$$

or, $$\frac{dx}{x} = \frac{2\pi k}{q} \cdot y\,dy$$

Integrating between proper limits,

$$\int_{r_1}^{r_2} \frac{dx}{x} = \frac{2\pi k}{q} \cdot \int_{h_1}^{h_2} y\,dy$$

where, r_1 and r_2 represent the radial distances of two observation wells
and h_1 and h_2 represent the height of water levels in them.

$\therefore$ $$\log_e \frac{r_2}{r_1} = \frac{2\pi k}{q} \cdot \frac{(h_2^2 - h_1^2)}{2}$$

or, $$k = \frac{q \log_e (r_2/r_1)}{\pi\,(h_2^2 - h_1^2)} \qquad ...(3.21)$$

Alternatively, when observation wells are not used,

$$k = \frac{q \cdot \log_e (R/a)}{\pi\,(H^2 - h^2)} \qquad ...(3.22)$$

where, a = radius of test well
R = radius of influence

The value of R may be determined from

$$R = 3000\, s\sqrt{k} \text{ m}$$

where, s = drawdown in the test well, m
k = co-efficient of permeability, m/sec.

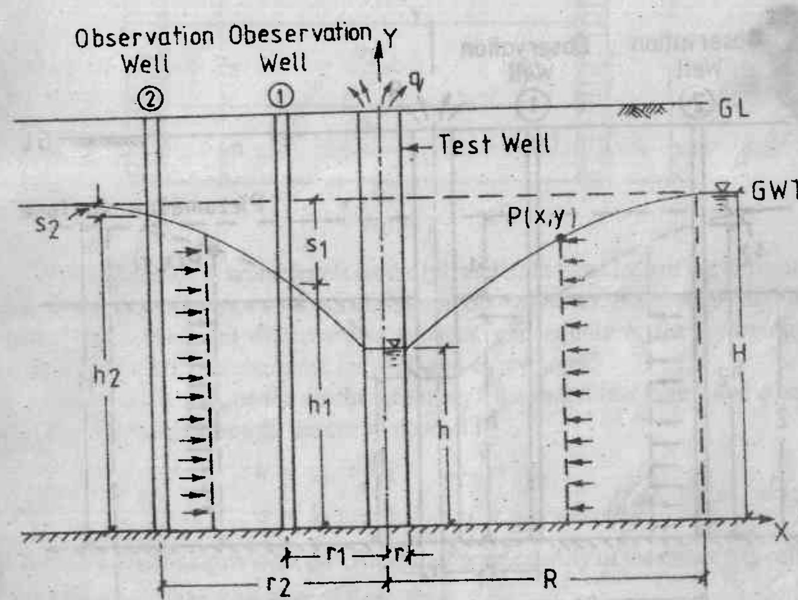


Fig. 3.9

(b) Confined aquifer : Fig. 3.10 illustrates a test well fully penetrating into a confined aquifer of thickness z.

From Darcy's law, $q = k\,i\,A$

or, $$q = k \cdot \frac{dy}{dx} \cdot 2\pi x z$$

or, $$\frac{dx}{x} = \frac{2\pi k z}{q}\,dy$$

Integrating, we get,

$$\int_{r_1}^{r_2} \frac{dx}{x} = \frac{2\pi k z}{q} \int_{h_1}^{h_2} dy$$

or, $$\log_e (r_2/r_1) = \frac{2\pi k z}{q}(h_2 - h_1)$$

or, $$k = \frac{q \cdot \log_e (r_2/r_1)}{2\pi k z (h_2 - h_1)} \quad ...(3.23)$$

Alternatively, $$k = \frac{q \log_e R/a}{2\pi k z (h_2 - h_1)} \quad ...(3.24)$$

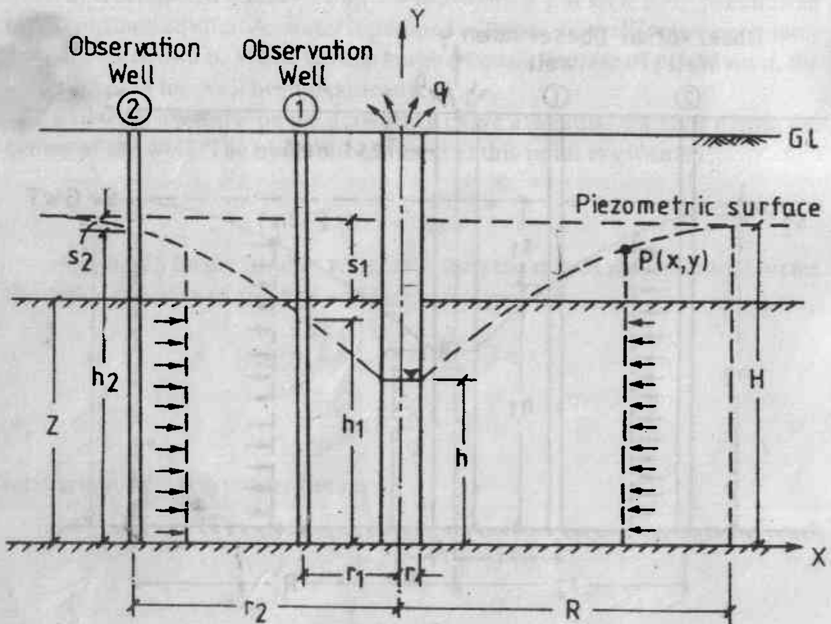


Fig. 3.10

3.11 Permeability of Stratified Deposits : Natural soil deposits generally are not homogeneous, but consist of a number of layers. The thickness and the co-efficient of permeability of the layers may vary to a large extent. In such cases, it is required to compute the equivalent co-efficient of permeability of the entire soil deposit.

3.11.1. ***Equivalent permeability parallel to the bedding planes*****:** Fig. 3.11 shows a stratified soil deposit consisting of n layers. Let $z_1, z_2, \ldots\ldots, z_n$ be the thickness of the layers while $k_1, k_2, \ldots\ldots, k_n$ be their co-efficients of permeability.

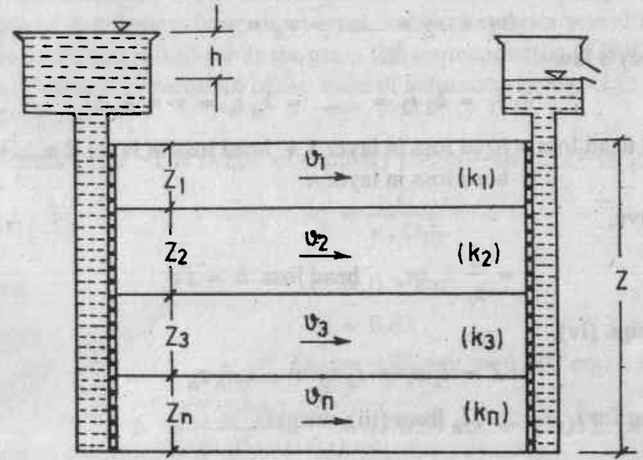


Fig. 3.11

The difference in water levels on the left and right hand side of the deposit is h. This head difference causes a horizontal flow of water. Since at any depth below G.L. the head difference is constant and equals h, the hydraulic gradient $i\,(= h/L)$ is the same for each and every layer.

Let $q_1, q_2, \ldots\ldots, q_n$ be the discharge through the individual layers and q be the total discharge through the entire deposit.

$\therefore$ $$q = q_1 + q_2 + \ldots\ldots + q_n$$

or $$q = k_1 i z_1 + k_2 i z_2 + \ldots\ldots + k_n i z_n \quad ...(i)$$

Again, if k_h be the equivalent co-efficient of permeability of the entire deposit of thickness z in the direction of flow, then

$$q = k_h iz \quad ...(ii)$$

From (i) and (ii) we get,

$$k_h i z = k_1 i z_1 + k_2 i z_2 + \ldots\ldots\ldots + k_n i z_n$$

or, $$k_h = \frac{k_1 z_1 + k_2 z_2 + \ldots\ldots\ldots + k_n z_n}{z_1 + z_2 + \ldots\ldots + z_n} = \frac{\sum_{i=1}^{n} k_i z_i}{\sum_{i=1}^{n} z_i} \quad ...(3.25)$$

3.11.2. *Equivalent permeability perpendicular to the bedding planes* : For flow in vertical direction (Fig. 3.12), the discharge velocities in each layer must be the same.

$$\therefore \quad v_1 = v_2 = \ldots\ldots = v_n = v$$

Using Darcy's law

$$k_1 i_1 = k_2 i_2 = \ldots\ldots = k_n i_n = v = k_v i \quad \ldots\text{(iii)}$$

Now, total head loss = head loss in layer 1 + head loss in layer 2 + ...+ head loss in layer n ...(iv)

But, we have,

$$i = \frac{h}{z,} \quad \text{or,} \quad \text{head loss } h = i z$$

$\therefore$ From eqn. (iv),

$$i Z = i_1 z_1 + i_2 z_2 + \ldots\ldots i_n z_n$$

Substituting for $i_1, i_2, \ldots, i_n$ from (iii), we get,

$$\frac{v}{k_v} . Z = \frac{v}{k_1} . z_1 + \frac{v}{k_2} . z_2 + \ldots\ldots\ldots + \frac{v}{k_n} . z_n$$

or,

$$\frac{Z}{k_v} = \frac{z_1}{k_1} + \frac{z_2}{k_2} + \ldots\ldots + \frac{z_n}{k_n}$$

or,

$$k_v = \frac{Z}{\frac{z_1}{k_1} + \frac{z_2}{k_2} + \ldots\ldots + \frac{z_n}{k_n}} = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} \frac{z_i}{k_i}} \quad \ldots(3.26)$$

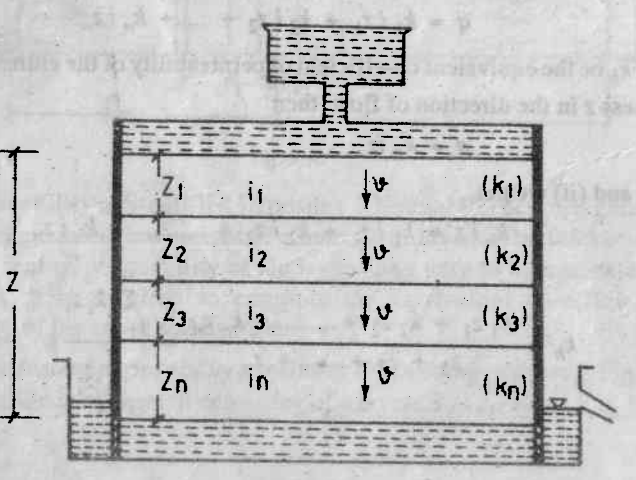


Fig. 3.12

EXAMPLES

Problem 3.1 The natural ground water table at a site is located at a depth of 2 m below the ground level. Laboratory tests reveal that the void ratio of the soil is 0.85 while the grain size corresponding to 10% finer is 0.05 mm. Determine the depth of the zone of saturation below G.L. Assume, $C = 0.3\ \text{cm}^2$.

Solution. The height of capillary rise of water is given by,

$$h_c = \frac{C}{e \,.\, D_{10}}$$

Here,

$$C = 0.3\ \text{cm}^2$$
$$e = 0.85$$
$$D_{10} = 0.05\ \text{mm} = 0.005\ \text{cm}.$$

$$\therefore \quad h_c = \frac{0.3}{(0.85)\,(0.005)} = 70.59\ \text{cm} = 0.706\ \text{m}.$$

Hence, the depth of saturation below G.L.

$$= 2.0 - 0.706 = 1.294\ \text{m}.$$

Problem 3.2. A capillary glass tube of 0.1 mm internal diameter is immersed vertically in a beaker full of water. Assuming the tube to be perfectly clean and wet, determine the height of capillary rise of water in the tube when the room temperature is 20°C. Given, at 20°C, unit weight of water = 0.9980 gm/cc and surface tension = 72.8 dynes/cm.

Solution : When a capillary tube is perfectly clean and wet, the upper meniscus of water in the tube is tangential (i.e., $\alpha = 0^\circ$). The height of capillary rise is then given by,

$$h_c = \frac{4\,T_s}{\gamma_w\, d\, g}$$

Here, $T_s = 72.8\ \text{dynes/cm}, \ \gamma_w = 0.9980\ \text{gm/cc},$

$d = 0.1\ \text{mm} = 0.01\ \text{cm}, \ g = 981\ \text{cm/sec}^2.$

$$\therefore \quad h_c = \frac{(4)\,(72.8)}{(0.998)\,(0.01)\,(981)}\ \text{cm} = 29.74\ \text{cm}.$$

Problem 3.3. The void ratio of a given soil *A* is twice that of another soil *B*, while the effective size of particles of soil *A* is one-third that of soil *B*. The height of capillary rise of water in soil *A* on a certain day is found to be 40 cm. Determine the corresponding height of capillary rise in soil *B*.

Solution: We have, $h_c = \dfrac{C}{e \cdot D_{10}}$

Let h_A and h_B be the heights of capillary rise in soil A and B respectively. Also, let e_A and e_B be the respective void ratios and D_A and D_B be the respective effective sizes.

From the question,

$$\frac{e_B}{e_A} = \frac{1}{2} \quad \text{and} \quad \frac{D_B}{D_A} = 3.$$

Now, $$\frac{h_A}{h_B} = \frac{C}{e_A \cdot D_A} \times \frac{e_B \cdot D_B}{C} = \frac{e_B}{e_A} \cdot \frac{D_B}{D_A} = (1/2)(3) = 1.5$$

$$\therefore \quad h_B = \frac{h_A}{1.5} = \frac{40}{1.5} = 26.67 \text{ cm.}$$

Problem 3.4. At a site the subsoil consists of a 8 m thick layer of dry sand (G = 2.65, e = 0.85, D_{10} = 0.14 mm) which is underlain by a 6 m thick clay layer (G = 2.75, w = 22%) below which there exists a thick layer of hardpan. The water table is located at a depth of 6 m below the ground level. Plot the distribution of total, neutral and effective stresses.

Solution : The soil profile is presented in Fig. 3.13 (a).

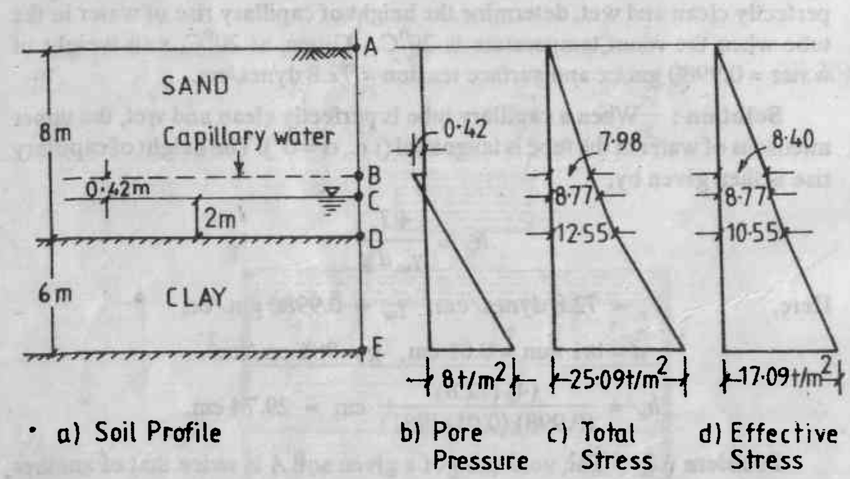


Fig. 3.13

Height of capillary rise in the sand layer,

$$h_c = \frac{C}{e \cdot D_{10}} = \frac{0.5}{(0.85)(0.014)} \quad (\text{assuming } C = 0.5 \text{ cm}^2)$$

$$= 42.0 \text{ cm} = 0.42 \text{ m.}$$

Hence the sand will be saturated upto 0.42 m above the water table. The remaining portion of the sand above this level will be dry.

For the sand layer,

$$\gamma_{sat} = \frac{G + e}{1 + e} \cdot \gamma_w = \frac{2.65 + 0.85}{1 + 0.85}(1.0) = 1.89 \text{ t/m}^3$$

$$\gamma_d = \frac{G\,\gamma_w}{1 + e} = \frac{(2.65)(1.0)}{1 + 0.85} = 1.43 \text{ t/m}^3$$

As the clay layer is submerged below water, it is saturated.

We have, $wG = se$,

$$\therefore \quad e = \frac{wG}{s} = \frac{(0.22)(2.75)}{1} = 0.605$$

$$\therefore \quad \gamma_{sat} = \frac{2.75 + 0.605}{1 + 0.605}(1) = 2.09 \text{ t/m}^3.$$

At A (z = 0), the total, neutral and effective stresses are all equal to zero.

At B (z = 5.58 m), total stress, $\sigma = (1.43)(5.58) = 7.98$ t/m²

neutral stress, $u = -h_c \cdot \gamma_w$

$= -(0.42)(1) = -0.42$ t/m²

effective stress, $\sigma' = \sigma - u$

$= 7.98 - (-0.42)$

$= 8.40$ t/m².

At C (z = 6.0 m), $\sigma = (1.43)(5.58) + (1.89)(0.42) = 8.77$ t/m²

$u = 0$

$\therefore \quad \sigma' = \sigma - u = \sigma = 8.77$ t/m²

At D (z = 8.0 m), $\sigma = (1.43)(5.58) + (1.89)(2.42) = 12.55$ t/m²

$u = (2.0)(1.0) = 2.0$ t/m²

$\sigma' = 12.55 - 2.0 = 10.55$ t/m²

At E (z = 14.0 m),

$\sigma = (1.43)(5.58) + (1.89)(2.42) + (2.09)(6.0)$

$= 25.09$ t/m²

$u = (2.0 + 6.0)(1.0) = 8.0 \text{ t/m}^2$

$\sigma' = 25.09 - 8.0 = 17.09 \text{ t/m}^2$

The distribution of total, neutral and effective stresses are shown in Fig. 3.13 (b), (c) and (d) respectively.

Problem 3.5. For the soil profile shown in Fig. 3.14, determine the total stress, pore water pressure and intergranular pressure at a depth of 15 m below the ground level.

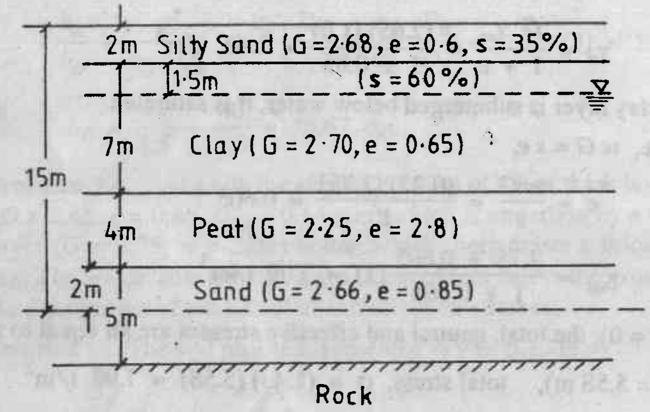


Fig. 3.14

Solution:

Bulk density of silty sand $(s = 35\%)$

$$= \frac{2.68 + (0.35)(0.60)}{1 + 0.60}(1.0) = 1.81 \text{ t/m}^3$$

Bulk density of clay above G.W.T. $(s = 60\%)$

$$= \frac{2.70 + (0.60)(0.65)}{1 + 0.65}(1.0) = 1.87 \text{ t/m}^3$$

Saturated density of clay below G.W.T.

$$= \frac{2.70 + 0.65}{1 + 0.65}(1) = 2.03 \text{ t/m}^3$$

Saturated density of peat $= \dfrac{2.25 + 2.8}{1 + 2.8}(1) = 1.33 \text{ t/m}^3$

Saturated density of sand $= \dfrac{2.66 + 0.85}{1 + 0.85}(1) = 1.90 \text{ t/m}^3$

At a depth of 15 m below G.L.:

$$\text{total stress } \sigma = (1.81)(2) + (1.87)(1.5) + (2.03)(5.5) + (1.33)(4) + (1.90)(2) = 26.71 \text{ t/m}^2$$

$$\text{pore water pressure} = (15 - 2 - 1.5)(1.0) = 11.5 \text{ t/m}^2$$

$$\therefore \quad \text{effective stress} = (26.71 - 11.5) = 15.21 \text{ t/m}^2$$

Problem 3.6. The void ratio of a sand sample at the loosest and densest possible states are found to be 0.55 and 0.98 respectively. If the specific gravity of soil solids be 2.67, determine the corresponding values of the critical hydraulic gradient.

Solution: The critical hydraulic gradient is given by,

$$i_c = \frac{G - 1}{1 + e} \cdot \gamma_\omega$$

At the densest state, $i_c = \dfrac{2.67 - 1}{1 + 0.55}(1) = 1.08$

At the loosest state, $i_c = \dfrac{2.67 - 1}{1 + 0.98}(1) = 0.84$

Problem 3.7. It is required to excavate a long trench in a sand deposit upto a depth of 3.5 m below G.L. The sides of the trench should be vertical and are to be supported by steel sheet piles driven upto 1.5 m below the bottom of the trench. The ground water table is at 1 m below G.L. In order to have a dry working area, water accumulated in the trench will be continuously pumped out. If the sand has a void ratio of 0.72 and the specific gravity of solids be 2.66, check whether a quick sand condition is likely to occur. If so, what remedial measures would you suggest?

Solution. Fig. 3.15 illustrates the given site conditions. It is evident that there will be an upward flow of water through the soil mass MNDB. The differential head which causes this flow is,

$$h = 2.5 \text{ m}$$

Again, thickness of the soil mass through which this flow occurs is, $L = MB = ND = 1.5$ m.

$$\therefore \quad \text{Hydraulic gradient,} \quad i = \frac{h}{L} = \frac{2.5}{1.5} = 1.67$$

$$\text{Critical hydraulic gradient,} \quad i_c = \frac{G - 1}{1 + e} = \frac{2.66 - 1}{1 + 0.72} = 0.965$$

$$\therefore \quad i > i_c$$

Hence, quick sand condition will occur.

The following remedial measures can be recommended :

(i) The depth of embedment of sheet piles below the bottom of the trench should be increased. This will increase the thickness of soil layer through which water percolates, and hence will reduce the hydraulic gradient.

Let x be the required depth of sheet piles below the bottom of the trench, which gives a factor of safety of 1.5 against quick sand condition.

$$\therefore \quad i = \frac{h}{L} = \frac{2.5}{x} \qquad \text{...(i)}$$

Now, $$F.S. = \frac{i_c}{i} = 1.5,$$

or $$i = \frac{i_c}{1.5} = \frac{0.965}{1.5} = 0.643 \qquad \text{...(ii)}$$

From (i) and (ii) we have,

$$\frac{2.5}{x} = 0.643, \quad \text{or,} \quad x = 3.89 \text{ m}$$

(ii) Alternatively, water table at the site may be lowered by any suitable dewatering method. This will reduce the differential head and hence the hydraulic gradient will be reduced.

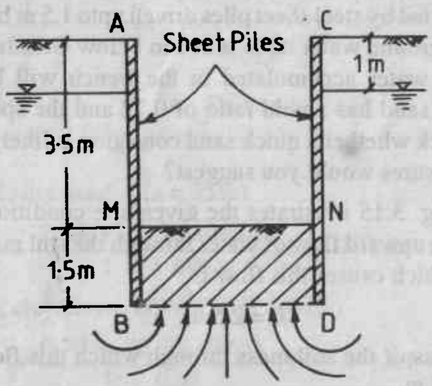


Fig. 3.15

Problem 3.8. In the experimental set-up shown in Fig. 3.16, if the area of cross-section of the soil sample be 0.28 m^2, and the quantity of water flowing through it be 0.03 cc/sec, determine the co-efficient of permeability in m/day.

Solution: From Darcy's law, $q = k\,i\,A$, or $k = \dfrac{q}{iA}$

where, i = hydraulic gradient = $\dfrac{h}{L}$

Here, h = differential head of water causing flow

$= 1.6 - 1.0 = 0.6$ m

L = length of soil mass through which flow takes place

$= 2.0$ m.

$$\therefore \quad i = \frac{0.6}{2.0} = 0.3$$

Again, we have, $q = 0.03$ cc/sec

and, $A = 0.28 \text{ m}^2 = 0.28 \times 10^4 \text{ cm}^2 = 2800 \text{ cm}^2$

$$\therefore \quad k = \frac{0.03}{(0.3)(2800)} \text{ cm/sec}$$

$$= \frac{(0.03)(60)(60)(24)}{(0.3)(2800)(100)} \text{ m/day}$$

$$= 0.0308 \text{ m/day.}$$

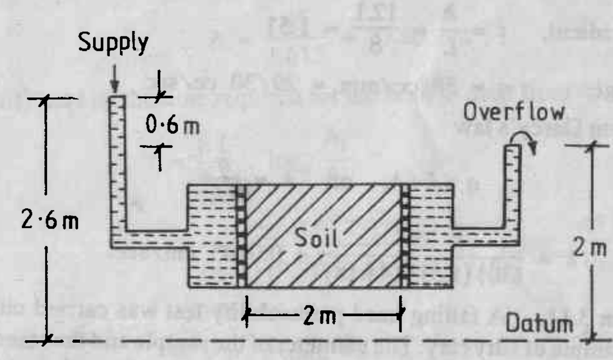


Fig. 3.16

Problem 3.9. A sample of coarse sand is tested in a constant head permeameter. The sample is 20 cm high and has a diameter of 8 cm. Water flows through the soil under a constant head of 1 m for 15 minutes. The mass of discharged water was found to be 1.2 kg. Determine the coefficient of permeability of the soil.

Solution: We have, for a constant head permeability test,

$$k = \frac{QL}{hAt}$$

Now, mass of discharged water = 1.2 kg = 1200 gm.

∴ Volume of discharged water, Q = 1200 cc.

Time of flow, t = 15 min. = (15) (60) = 900 sec.

Head of water, h = 1 m = 100 cm.

Area of cross-section of sample, $A = \frac{\pi}{4} \times 8^2 = 50.26 \text{ cm}^2$

Length of flow path, L = 20 cm.

$$\therefore \quad k = \frac{(1200)(20)}{(100)(50.26)(900)} = 0.0053 \text{ cm/sec.}$$

Problem 3.10. A cylindrical mould of diameter 7.5 cm contains a 15 cm long sample of fine sand. When water flows through the soil under constant head at a rate of 58 cc/min., the loss of head between two points 8 cm apart is found to be 12.1 cm. Determine the co-efficient of permeability of the soil.

Solution: Area of cross-section of the sample,

$$A = (\pi/4)(7.5)^2 = 44.18 \text{ cm}^2$$

Hydraulic gradient, $i = \frac{h}{L} = \frac{12.1}{8} = 1.51$

Unit discharge, $q = 58 \text{ cc/min} = 29/30 \text{ cc/sec}$

We have, from Darcy's law

$$q = k\,i\,A, \quad \text{or,} \quad k = \frac{q}{i\,A}$$

$$\therefore \quad k = \frac{29}{(30)(1.51)(44.18)} = 0.0145 \text{ cm/sec.}$$

Problem 3.11. A falling head permeability test was carried out on a 15 cm long sample of silty clay. The diameter of the sample and the stand-pipe were 9.8 cm and 0.75 cm respectively. The water level in the stand-pipe was observed to fall from 60 cm to 45 cm in 12 minutes. Determine :

(i) the co-efficient of permeability of the soil in m/day

(ii) height of water level in the stand-pipe after another 20 minutes.

(iii) time required for the water level to drop to 10 cm.

Solution: (i) For a falling head permeability test, we have,

$$k = \frac{aL}{At} \cdot \log_e \frac{h_1}{h_2}$$

Here, $a = (\pi/4)(0.75)^2 \text{ cm}^2$

$A = (\pi/4)(9.8)^2 \text{ cm}^2$

$L = 15$ cm, $t = 12$ min $= (12) \times (60) = 720$ sec.

$h_1 = 60$ cm, $h_2 = 45$ cm

$$\therefore \quad k = \frac{(\pi/4)(0.75)^2(15)}{(\pi/4)(9.8)^2(720)} \cdot \log_e \frac{60}{45}$$

$$= 3.51 \times 10^{-5} \text{ cm/sec}$$

$$= \frac{(3.51 \times 10^{-5})(86400)}{100} \text{ m/day} = 0.03 \text{ m/day.}$$

(ii) Let h be the head at the end of another 20 minutes.

$$\therefore \quad 3.51 \times 10^{-5} = \frac{(\pi/4)(0.75)^2(15)}{(\pi/4)(9.8)^2(20)(60)} \cdot \log_e \frac{45}{h}$$

or, $\log_e \frac{45}{h} = 0.479$

or, $\frac{45}{h} = e^{0.479} = 1.615$

or, $h = \frac{45}{1.615} = 27.86$ cm

(iii) Let t be the time required for the head to drop from 45 cm to 10 cm.

Now, $t = \frac{aL}{Ak} \cdot \log_e \frac{h_1}{h_2}$

$$= \frac{(\pi/4)(0.75)^2(15)}{(\pi/4)(9.8)^2(3.51 \times 10^{-5})} \cdot \log_e \frac{45}{10}$$

$= 3764.65$ sec.

$= 1$ hr. 2 min. 45 sec.

Problem 3.12. A well is fully penetrated into a 16 m thick layer of sand which is underlain by a rock layer. Water is pumped out of the well at a constant rate of 450000 litres/hour. The water level in two observation wells situated at 15 m and 30 m from the test well are found to be at 3.7 m and 2.6 m respectively below the ground level. Determine the co-efficient of permeability of the soil.

Solution: For an unconfined aquifer, the co-efficient of permeability is given by:

$$k = \frac{Q \cdot \log_e (r_2/r_1)}{\pi (h_2^2 - h_1^2)}$$

Here, $Q = 450000$ litre/hour

$$= \frac{(450000)(1000)}{(60)(60)} \text{ cc/sec} = 125000 \text{ cc/sec.}$$

$r_1 = 15 \text{ m} = 1500 \text{ cm}$

$r_2 = 30 \text{ m} = 3000 \text{ cm}$

$h_1 = (16 - 3.7) \text{ m} = 12.3 \text{ m} = 1230 \text{ cm}$

$h_2 = (16 - 2.6) \text{ m} = 13.4 \text{ m} = 1340 \text{ cm}$

$$\therefore \quad k = \frac{(125000)[\log_e (3000/1500)]}{(3.14)[(1340)^2 - (1230)^2]} = 0.098 \text{ cm/sec}$$

Problem 3.13. A pumping-out test was carried out in the field in order to determine the average co-efficient of permeability of a 18 m thick sand layer. The ground water table was located at a depth of 2.2 m below the ground level. A steady state was reached when the discharge from the well was 21.5 lit/sec. At this stage, the drawdown in the test well was 2.54 m, while the drawdowns in two observation wells situated at 8 m and 20 m from the test well were found to be 1.76 m and 1.27 m respectively. Determine:

(i) co-efficient of permeability of the sand layer in m/day.
(ii) radius of influence of the test well
(iii) effective size of the sand.

Solution: (i) We have, $k = \dfrac{Q \cdot \log_e (r_2/r_1)}{\pi (h_2^2 - h_1^2)}$

Here, $Q = 21.5$ lit/sec

$$= \frac{(21.5)(1000)(86400)}{10^6} \text{ m}^3/\text{day} = 1857.6 \text{ m}^3/\text{day}$$

$r_1 = 8 \text{ m}, \quad r_2 = 20 \text{ m}.$

Height of the water table above the base of the well,

$$H = (18 - 2.2) \text{ m} = 15.8 \text{ m}$$

Drawdown in the observation wells, $s_1 = 1.76 \text{ m}, \quad s_2 = 1.27 \text{ m}$

$\therefore$ Height of water in the observation wells,

$h_1 = H - s_1 = (15.8 - 1.76) \text{ m} = 14.04 \text{ m}$

$h_2 = H - s_2 = (15.8 - 1.27) \text{ m} = 14.53 \text{ m}$

$$\therefore \quad k = \frac{(1857.6)[\log_e (20/8)]}{\pi[(14.53)^2 - (14.04)^2]} = 38.70 \text{ m/day}$$

(ii) The radius of influence is given by,

$$R = 3000\, s \sqrt{k}$$

Here, $k = 38.70$ m/day

$$= \frac{38.70}{86400} \text{ m/sec} = 4.48 \times 10^{-4} \text{ m/sec}$$

$$\therefore \quad R = (3000)(2.54)\sqrt{4.48 \times 10^{-4}} \text{ m} = 161.29 \text{ m}$$

(iii) The effective size can be determined from Allen Hazen's formula :

$$k = C \cdot D_{10}^2$$

or, $D_{10} = \sqrt{k/C}$

Assuming $C = 100 \text{ cm}^{-1} \text{ sec}^{-1}$

$$D_{10} = \sqrt{\frac{4.48 \times 10^{-4}}{100}} = 2.12 \times 10^{-3} \text{ cm}$$

$$= 0.0212 \text{ mm}$$

Problem 3.14. The subsoil at a site consists of a fine sand layer lying in between a clay layer at top and a silt layer at bottom. The co-efficient of permeability of the sand is 100 times that of clay and 20 times that of silt, while the thickness of the sand layer is one-tenth that of clay and one third that of silt. Find out the equivalent co-efficient of permeability of the deposit in directions parallel and perpendicular to the bedding planes, in terms of the co-efficient of permeability of the clay layer.

Solution: Let k be the co-efficient of permeability of the clay layer.

$\therefore$ co-efficient of permeability of sand $= 100\,k$

and, co-efficient of permeability of silt $= \dfrac{100\,k}{20} = 5k$

Again, let z be the thickness of the sand layer.

$\therefore$ Thickness of clay layer $= 10\,z$

and, thickness of silt layer $= 3\,z.$

Equivalent co-efficient of permeability parallel to the bedding planes,

$$k_h = \frac{k_1 z_1 + k_2 z_2 + k_3 z_3}{z_1 + z_2 + z_3} = \frac{(k)(10\,z) + (100\,k)(z) + (5\,k)(3\,z)}{10\,z + z + 3\,z}$$

$$= \frac{10 + 100 + 15}{10 + 1 + 3} \cdot k = \frac{125}{14} k = 8.93\, k$$

Equivalent co-efficient of permeability perpendicular to the bedding planes,

$$k_v = \frac{z_1 + z_2 + z_3}{\dfrac{z_1}{k_1} + \dfrac{z_2}{k_2} + \dfrac{z_3}{k_3}} = \frac{10\,z + z + 3\,z}{\dfrac{10\,z}{k} + \dfrac{z}{100\,k} + \dfrac{3\,z}{5\,k}}$$

$$= \frac{10 + 1 + 3}{10 + \dfrac{1}{100} + \dfrac{3}{5}} k = \frac{14\,k}{\dfrac{1000 + 1 + 60}{100}} = \frac{1400}{1061} k = 1.319\, k$$

Problem 3.15. Fig. 3.17 shows a soil profile at a given site. Determine:

(i) Average co-efficient of permeability of the deposit.

(ii) Equivalent co-efficient of permeability of the deposit in the horizontal and vertical directions.

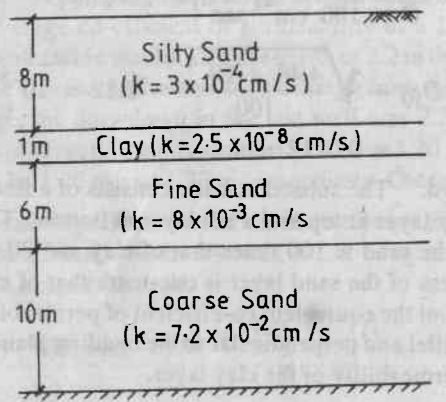


Fig. 3.17

Solution : (i) Average co-efficient of permeability of the deposit,

$$h_{av} = \frac{k_1 + k_2 + k_3 + k_4}{4}$$

$$= \frac{3 \times 10^{-4} + 2.5 \times 10^{-8} + 8 \times 10^{-3} + 7.2 \times 10^{-2}}{4}$$

$$= 2 \times 10^{-2} \text{ cm/sec} = 0.02 \text{ cm/sec.}$$

(ii) Equivalent co-efficient of permeability in the horizontal direction,

$$k_h = \frac{\sum_{i=1}^{4} k_i\, z_i}{\sum_{i=1}^{4} z_i}$$

$$= \frac{(8)(3 \times 10^{-4}) + (1)(2.5 \times 10^{-8}) + (6)(8 \times 10^{-3}) + (10)(7.2 \times 10^{-2})}{(8 + 1 + 6 + 10)}$$

$$= 0.0308 \text{ cm/sec}$$

Equivalent co-efficient of permeability in the vertical direction.

$$k_v = \frac{\sum_{i=1}^{4} z_i}{\sum_{i=1}^{4} \dfrac{z_i}{k_i}}$$

$$= \frac{8 + 1 + 6 + 10}{\dfrac{8}{3 \times 10^{-4}} + \dfrac{1}{2.5 \times 10^{-8}} + \dfrac{6}{8 \times 10^{-3}} + \dfrac{10}{7.2 \times 10^{-2}}}$$

$$= 6.24 \times 10^{-7} \text{ cm/sec}$$

EXERCISE 3

3.1 Determine the height of capillary rise of water above the ground water table in a homogeneous bed of sand having an effective size of 0.12 mm. The moisture content of the soil below the ground water table was found to be 25%. Take, $G = 2.67$ and $C = 0.5\ \text{cm}^2$. [Ans : 62.5 cm]

3.2 A perfectly clean and wet capillary tube of 0.1 mm radius is immersed in a container full of water. The room temperature is 30°C and the water level in the tube is found to rise to a height of 14.54 cm. If the unit weight of water at 30°C be 0.996 gm/cc, determine the surface tension at 30°C. [Ans : 71.03 dynes/cm]

3.3 A dry capillary tube of 0.3 mm diameter was immersed in distilled water at 4°C. The upper meniscus of the water column in the tube was found to be inclined at 30°C to the vertical. Find out the height of the water column. Given, at 4°C, unit weight of water = 1 gm/cc and surface tension of water = 75.6 dynes/cm. [Ans : 8.9 cm]

3.4 The subsoil at a site consists of a 2 m thick layer of clay which is underlain by a deep sand layer. The natural ground water table is at 3 m below

G.L. The unit weight of clay is 1.8 t/m^3, while that of sand above and below water table are 1.75 t/m^3 and 1.92 t/m^3 respectively. Find out the total and effective stresses at a depth of 5 m below the ground level.

[Ans : 9.19 t/m^2, 7.19 t/m^2]

3.5 Plot the distribution of total, neutral and effective stress for the soil profile shown in Fig. 3.18.

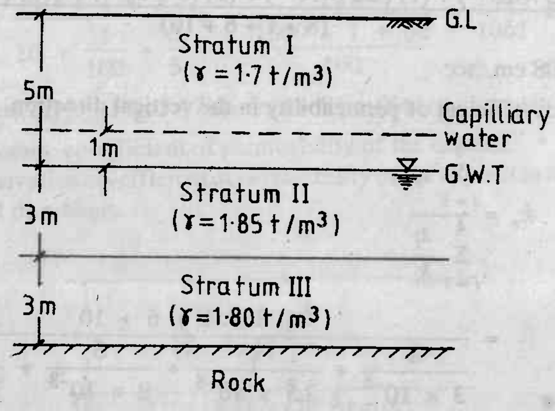


Fig. 3.18

3.6 A sand sample is 50% saturated and has a bulk density of 1.75 t/m^3. The specific gravity of solids is 2.65. Determine the critical hydraulic gradient. [Ans : 0.96]

3.7 How will the critical hydraulic gradient of the soil in Prob. 3.6 change, if the soil is compacted to increase its bulk density by 10%, without any change in its water content? [Ans : Increases by 13.8%]

3.8 At a site the subsoil consists of a deep layer of medium sand. It is required to excavate a trench upto 3 m below the ground level. The water table lies at depth of 1.5 below G.L.. In order to have a dry working area, sheet piles are driven along the sides of the trench upto a depth of 5 m below G.L. and water accumulated in the trench is pumped out as the excavation progresses. Determine the factor of safety against the occurrence of quick sand condition. Given, $e = 0.8$, $G = 2.7$. [Ans : 1.24]

3.9 The void ratio of a soil is 0.76, while its co-efficient of permeability is 1.2×10^{-4} cm/sec. If, keeping all other factors constant, the soil is compacted so as to reduce the void ratio to 0.60, what will be the co-efficient of permeability of the soil? [Hints : $k \propto e^3/1 + e$]

[Ans : 6.5×10^{-5}cm/sec]

3.10 In a constant head permeability test, water is allowed to pass through a cylindrical soil sample, 15 cm high and 10 cm in diameter, under a constant head of 1 m. The water flowing out of the sample is collected in a glass cylinder of 1200 cc capacity. It is observed that the cylinder just starts to overflow after 1 hr. 13 min. and 51 sec. Find out the co-efficient of permeability. [Ans : 5.17×10^{-4} cm/sec]

3.11 A specimen of a coarse-grained soil was subjected to a constant head permeability test. The sample was compacted in a cylindrical mould having a height of 9.5 cm and an internal volume of 987 cc. Under a constant head of 50 cm, 756.6 cc of water passed through the soil in 10 minutes . Determine the co-efficient of permeability and the effective size of the soil.

[Ans : 0.012 cm/sec , 0.11 mm]

3.12 How many litres of water will flow through a cylindrical soil sample of 8 cm diameter and 12 cm height in a day under a constant head of 65 cm, if the co-efficient of permeability of the soil be 0.01 mm/sec ?

[Ans : 23.5 litres]

3.13 In a falling head permeability test, the water level in the stand-pipe dropped from 40 cm to 20 cm in 1 hour. The diameter of the sample and the stand-pipe were 8 cm and 0.5 cm respectively, while the height of the sample was 9.5 cm. Find out the co-efficient of permeability of the soil.

[Ans : 7.15×10^{-6} cm/sec]

3.14 A falling head test was performed on a soil specimen having a diameter of 10 cm and a height of 12 cm. The stand-pipe had a diameter of 1.2 cm and the water level in it dropped from 55 cm to 41 cm in 2 hours. Determine the time required for the water level in the stand-pipe to come down to 20 cm. Also determine the height of water level in the stand-pipe after a period of 24 hours from the beginning of the test.

[Ans : 6 hours and 48.5 minutes; 3.53 cm]

3.15 A pumping-out test was carried out in an 18 m thick layer of pervious soil which is underlain by an impermeable shale. The water table was located at 1 m below the ground level. A steady state was reached when the discharge from the well was 9 cu.m/min. The corresponding water levels in two observation wells situated at 4 m and 8 m from the pumping well were found to be 2 m and 0.5 m respectively below the initial ground water table. Compute the co-efficient of permeability of the deposit. [Ans : 0.07 cm/sec]

3.16 In order to compute the co-efficient of permeability of a non-homogeneous deposit, a pumping out test was conducted by fully penetrating a well of 20 cm diameter into a 50 m thick unconfined aquifer. When the drawdown in the pumping well reached 4.2 m a steady discharge of 300 m^3/hr was obtained from it. The drawdown in an observation well at a distance of 30 m from the pumping well was found to be 1.1 m. If the initial ground water table was at 1.5 m below G.L., compute :

(i) the field co-efficient of permeability of the soil

(ii) the radius of influence. [Ans : (i) 5.3×10^{-2} cm/sec (ii) 290 m]

3.17 A pumping well of 20 cm diameter penetrates fully into a confined aquifer of 25 m thickness. A steady discharge of 26.5 lit/sec is obtained from the well under a drawdown of 3.2 m. Assuming a radius of influence of 300 m, find out the co-efficient of permeability of the soil in m/day.

[Ans : 33.31 m/day]

3.18 A pumping well of 25 cm diameter was fully penetrated into a 20 m thick bed of sand which lies between two clay layers of negligible permeability. Laboratory tests revealed that the sand had a co-efficient of permeability of 0.03 cm/sec. A steady state was reached when the drawdown in the test well was 4.3 m and the corresponding discharge was 12 litres/sec.

Estimate the drawdown in an observation well sunk at a distance of 20 m from the pumping well. [Ans : 1.51 m]

3.19 A stratified soil deposit consists of four layers. The thickness of the second, third and fourth layers are equal to half, one-third and one-fourth, respectively, the thickness of the top layer, while their co-efficients of permeability are respectively twice, thrice and four times that of the top layer. Find out :

(i) average co-efficient of permeability of the deposit.

(ii) equivalent co-efficient of permeability of the deposit

(a) parallel to (b) perpendicular to, the bedding planes.

[Ans (i) 2.5 k (ii) (a) 1.92 k (b) 1.46 k, k being the co-efficient of permeability of the first layer]

4

SEEPAGE AND FLOWNETS

4.1 Introduction : When a water-retaining structure (e.g., an earth or rockfill dam, a concrete dam or weir, sheet-pile cut-off wall etc.) is constructed to maintain a differential head of water, seepage through the structure itself and/or the foundation soil takes place. The quantity of water which flows from the upstream to the downstream side, termed as the seepage loss, is of paramount importance in designing such a structure. Moreover, the percolating water exerts a pressure on the soil, which is called the seepage pressure. In impermeable structures (viz., a concrete dam) the seepage of water results in a vertical uplift pressure on the base of the dam. When the seepage water reaches the downstream side, soil particles may be lifted up resulting in a 'piping' failure. The stability of the side slopes of an earth dam may be substantially reduced due to seepage of water.

All of these problems can be analysed graphically by constructing flow-nets.

4.2. Equation of Continuity: Laplace's equation of continuity, as applicable to two-dimensional flow problems, is given below:

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0 \quad ...(4.1)$$

Where, k_x and k_y are the co-efficients of permeability in the x and y directions respectively.

For an isotropic soil, $k_x = k_y$. Therefore,

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad ..(4.2)$$

Eqn. (4.2) is satisfied by the potential function $\phi(x, y)$ and the stream function $\psi(x, y)$. The properties of these functions are as follows:

$$\left.\begin{aligned} v_x &= \frac{\partial \phi}{\partial x}, \quad v_y = \frac{\partial \phi}{\partial y} \\ \text{and,} \qquad v_x &= \frac{\partial \psi}{\partial y}, \quad v_y = -\frac{\partial \psi}{\partial x} \end{aligned}\right\} \quad ...(4.3)$$

The potential function ϕ can be represented by a family of curves, each having a particular constant value of ϕ. These curves are called the equipotential lines. Similarly, the stream function ψ may be represented by a number of curves, known as the stream lines or the flow lines. A stream line represents the path along which a water particle flows. An equipotential line is a curve at any point of which the piezometric head is constant. It can be proved that the product of the gradients of the ϕ function and the ψ function equals –1. Thus, an equipotential line should always intersect a stream line orthogonally.

The combination of stream lines and flow lines in the proper flow domain is called a flownet.

4.3. Properties of a Flownet: A flownet has the following properties:

1. All flow lines and equipotential lines are smooth curves.
2. A flow line and an equipotential line should intersect each other orthogonally.
3. No two flow lines can intersect each other.
4. No two equipotential lines can intersect each other.

4.4. Construction of a Flownet : In order to construct a flownet, the boundary conditions, i.e., the location of the two extreme flow lines and the two extreme equipotential lines, have to be identified first. For example, Fig. 4.1 shows a flownet for a sheet-pile wall. Here the boundary conditions are :

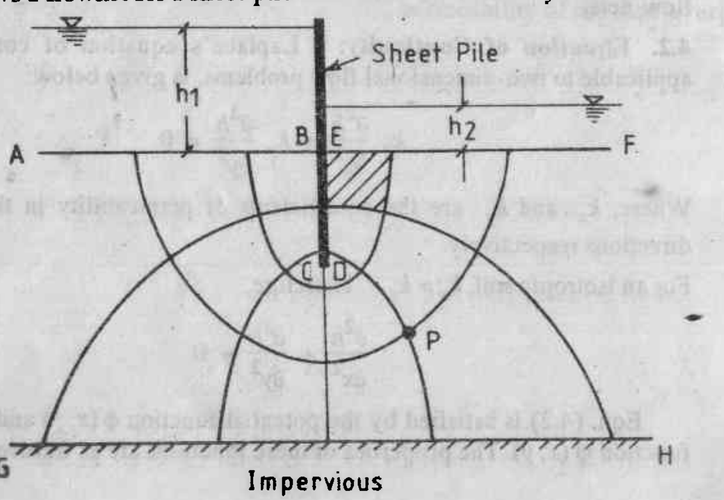


Fig. 4.1

1. AB is the equipotential line having the maximum piezometric head ($h = h_1$).
2. EF is the equipotential line having the minimum piezometric head ($h = h_2$).
3. $BCDE$ (i.e., the surface of the sheet pile) is the shortest flow line.
4. GH (i.e., the imperious boundary) is the longest flow line.

Once the boundary conditions are identified, the flownet can be drawn by trial and error. The process is tedious and each line has to be drawn, erased and redrawn a number of times.

4.5. Uses of a Flownet: A flownet enables one to determine the following:

(i) *Quantity of seepage* : Fig. 4.2 shows a portion of a flownet. Let Δq_1 and Δq_2 be the quantity of seepage in unit time through two consecutive flow channels. Let b_1 and l_1 be the width and length respectively of the flow element $ABCD$, and Δh be the head drop between two consecutive equipotential lines.

From Darcy's law we have,

$$q = k\,i\,A$$

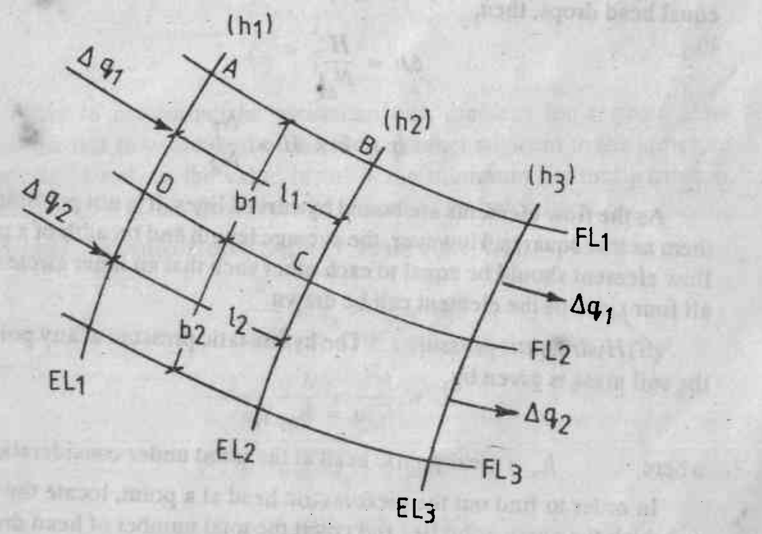


Fig. 4.2

Considering unit thickness of the soil mass, cross-sectional area of the element $ABCD = b_1 \times 1 = b_1$.

Hydraulic gradient, $i = \frac{\Delta h}{l_1}$

$$\therefore \quad \Delta q_1 = k \cdot \frac{\Delta h}{l_1} \cdot b_1 = k \times \Delta h \times \frac{b_1}{l_1}$$

Similarly, $\Delta q_2 = k \times \Delta h \times \frac{b_2}{l_2}$

The discharge quantity through all flow channels will be equal if,

$$\frac{b_1}{l_1} = \frac{b_2}{l_2} = \dots\dots\dots = \frac{b_n}{l_n}$$

However, if the elements are made orthogonally squared (i.e., $b = l$), then, $\Delta q_1 = \Delta q_2 = \dots\dots\dots = \Delta q_n = k \times \Delta h$

If N_f be the number of flow channels present in the flownet then the total quantity of seepage is given by,

$$q = k \times N_f \times \Delta h$$

Again, if H be the initial difference of head and N_d be the number of equal head drops, then,

$$\Delta h = \frac{H}{N_d}$$

$$\therefore \quad q = k \times H \times \frac{N_f}{N_d} \qquad \dots(4.4)$$

As the flow elements are bound by curved lines, it is not possible to draw them as true squares. However, the average length and breadth of a particular flow element should be equal to each other such that an inner circle touching all four sides of the element can be drawn.

(ii) *Hydrostatic pressure* : The hydrostatic pressure at any point within the soil mass is given by,

$$u = h_w \gamma_w$$

where, h_w = piezometric head at the point under consideration.

In order to find out the piezometric head at a point, locate the flow line on which the given point lies and count the total number of head drops in the flownet as well as the number of head drops occurred upto the given point. The piezometric head at the given point is then obtained from,

$$h_w = h_1 - n\Delta h$$

or, $$h_w = h_1 - \frac{n(h_1 - h_2)}{N_d} \qquad \dots(4.5)$$

For example, in Fig. 4.1, the piezometric head at P is,

$$h_w = h_1 - \frac{4(h_1 - h_2)}{6} = h_1 - 0.67(h_1 - h_2)$$

The uplift pressure at any point below the base of a concrete dam is given by,

$$p_u = h_w \gamma_w \qquad \dots(4.6)$$

(iii) *Exit gradient* : When the percolating water comes out of the soil at the downstream end, it applies a seepage pressure on the soil which is given by

$$p_s = i_e \cdot \gamma_w \qquad \dots(4.7)$$

where, i_e is the exit gradient = $\frac{\Delta h}{a}$

and a is the average dimension of the last element of a flow channel.

Piping may occur if the exit gradient becomes greater than the critical hydraulic gradient. The factor of safety against piping is given by,

$$F_s = \frac{i_e}{i_c} \qquad \dots(4.8)$$

In order to determine the maximum exit gradient for a given flow problem, the last flow element of the flow channel adjacent to the structure is to be considered, as the value of 'a' is the minimum for that particular element. In Fig. 4.1, this flow element is marked by hatch lines.

4.6. Flownet in Anisotropic Soils : From eqn. (4.1),

$$k_x \frac{\partial^2 h}{\partial x^2} + k_y \frac{\partial^2 h}{\partial y^2} = 0.$$

or, $$\frac{1}{k_y/k_x} \frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0.$$

Let, $$\sqrt{k_y/k_x}\; x = x' \qquad \dots(4.9)$$

$$\therefore \quad \frac{1}{k_y/k_x} \cdot \frac{\partial^2 h}{\partial x^2} = \frac{\partial^2 h}{\partial x'^2}$$

or, $$\frac{\partial^2 h}{\partial x'^2} + \frac{\partial^2 h}{\partial y^2} = 0 \qquad \dots(4.10)$$

In order to draw the flownet for anisotropic soils, a transformed section has to be drawn first by multiplying all horizontal dimensions by $\sqrt{k_y/k_x}$, but keeping the vertical dimensions unaltered. An orthogonally squared flownet is then drawn as usual for the transformed section. The structure, alongwith the flownet, is then retransformed by multiplying all horizontal dimensions by $\sqrt{k_x/k_y}$. The final flownet will consist of rectangular elements.

4.7. Multiple Permeability Conditions : When the flow lines pass from one soil to another having a different permeability, they deviate from the interface of the two soils and this deviation is similar to the refraction of light rays. This is illustrated in Fig. 4.3.

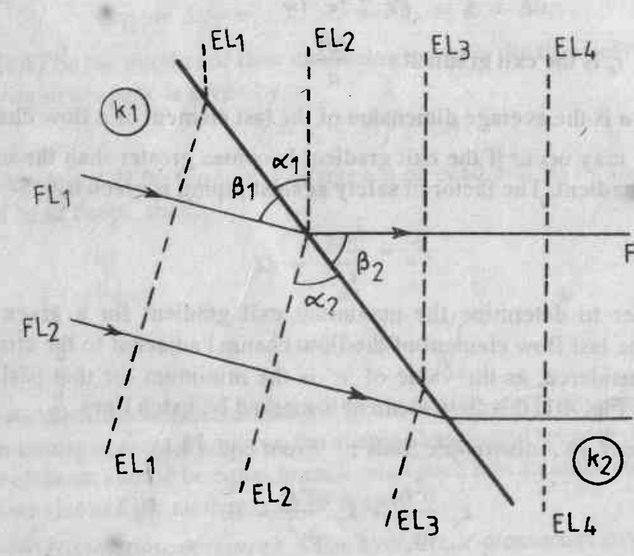


Fig. 4.3

The portion of the flow-net lying in layer 1 is first drawn in the usual manner with square flow elements. When the flow lines as well as equipotential lines enter layer 2, they undergo deviations according to the following equation:

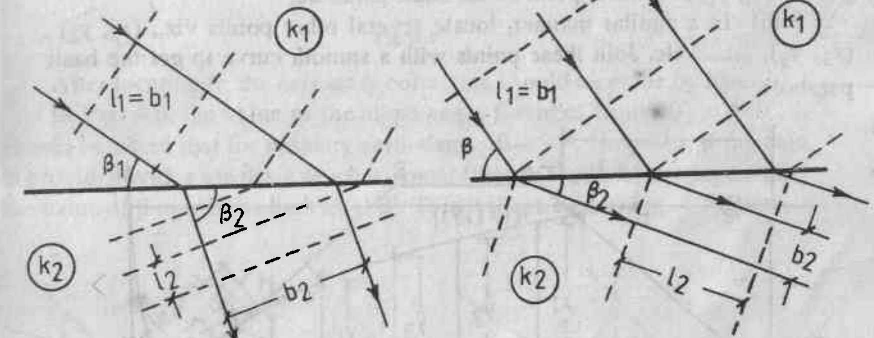
$$\frac{k_1}{k_2} = \frac{\tan\ \beta_1}{\tan\ \beta_2} \qquad ...(4.11)$$

Consequently, the flow elements in layer 2 are not squares any more, but become rectangles, and their width-to-height ratios are given by,

$$\frac{b}{l} = \frac{k_1}{k_2}.$$

If $k_1 > k_2$, the flow channels will get broadened, while if $k_1 < k_2$, they will get shortened. In other words, when a flow channel carrying a certain discharge enters a less permeable soil, it requires a greater area to carry the same discharge. However, when it enters a more permeable soil, a smaller area is sufficient.

These conditions are illustrated in Fig. 4.4 (a) and (b) respectively.



a) When $k_1 > k_2$

b) When $k_1 < k_2$

Fig. 4.4

4.8. Unconfined Flow : Phreatic Line: When an impermeable structure (e.g., a sheet pile or a concrete weir) retains water, all the boundary conditions are known. Such a flow is known as the confined flow or pressure flow. However, when the structure itself is pervious (e.g., an earth dam) the upper boundary or the uppermost flow line is unknown. Such a flow is termed as an unconfined flow or a gravity flow, and this upper boundary is called the phreatic line.

In order to obtain the phreatic line, the basic parabola has to be drawn first and then the necessary corrections at the entry and exit points have to be made.

4.8.1. *Construction of the Basic Parabola* : In Fig. 4.5, *ABCD* is the cross-section of an earth dam. In order to draw the basic parabola, proceed as follows:

(i) Measure the horizontal projection *L* of the wetted portion, *ED*, of the upstream face.

(ii) Locate the point P such that $EP = 0.3\,L$. The point P is the first point of the basic parabola.

(iii) With P as centre and PC radius draw an arc to intersect the extended water surface at F.

(iv) From F draw $FG \perp DC$. The line FG is the directrix of the basic parabola, while C is the focus.

(v) Locate the mid-point Q of CG.

(vi) Let G be the origin, GF the Y-axis and GD the X-axis.

(vii) Choose any point H on CD, such that $GH = x_1$. With C as centre and x_1 radius, draw an arc to intersect the vertical line through H at R. The point $R(x_1, y_1)$ is another point on the basic parabola.

(viii) In a similar manner, locate several other points viz., (x_2, y_2), (x_3, y_3), ..., etc. Join these points with a smooth curve to get the basic parabola.

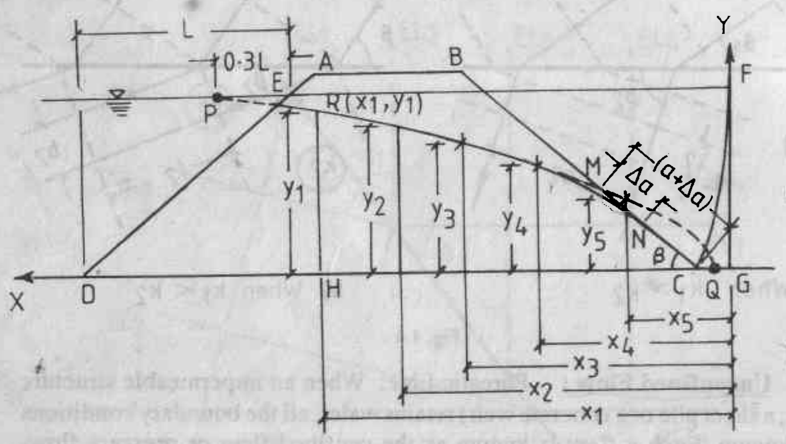


Fig. 4.5

4.8.2 *Corrections at Entry and Exit Points*: ED is an equipotential line and the phreatic line is a flow line. These two should meet each other at right angles. This necessitates the correction at the entry point, which should be drawn by hand.

The phreatic line should meet the down stream face BC tangentially. This necessitates the correction at the exit point. The basic parabola intersects BC at M. But the phreatic line should meet BC at N. Let $CN = a$ and $NM = \Delta a$, The magnitude of $\Delta a/(a + \Delta a)$ depends on the slope angle, β of the downstream face. Its value may be obtained from Fig. 4.6.

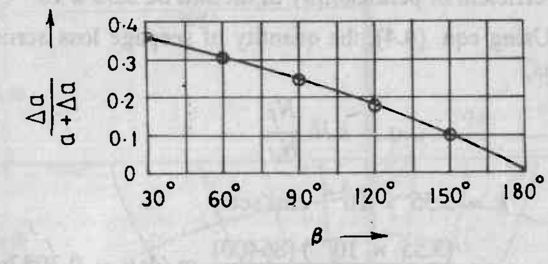


Fig. 4.6

After locating N, the necessary correction should be made by hand.

In Fig. 4.6, the value of the slope angle β ranges from 30° to 180°. It should be noted that for ordinary earth dams, $\beta < 90°$. However, if the dam is provided with a toe drain or a horizontal filter to arrest the seepage water, the value of β may be as high as 180°. This is illustrated in Fig. 4.7 below:

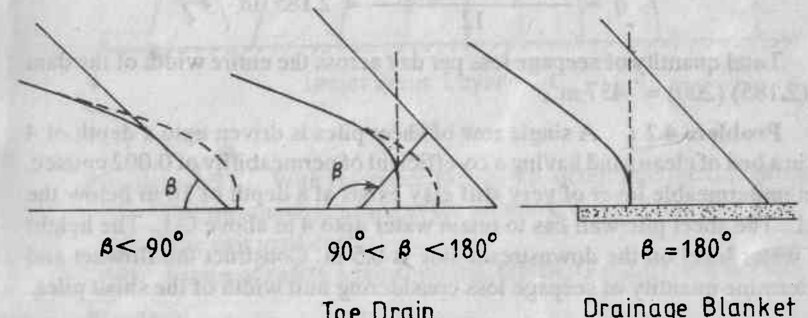


Fig. 4.7

It should also be noted that for an earth dam having a toe drain, chimney drain or horizontal drainage blanket, the beginning point of that drain on filter, and not the bottom corner of downstream face, should be taken into account while plotting the basic parabola.

EXAMPLES

Problem 4.1. A homogeneous earth dam, 30 m high, has a free board of 1.5 m. A flownet was constructed and the following results were noted:

No. of potential drops = 12

No. of flow channels = 3

The dam has a 18 m long horizontal filter at its downstream end. Calculate the seepage loss across the dam per day if the width of the dam be 200 m and the co-efficient of permeability of the soil be 3.55×10^{-4} cm/sec.

Solution: Using eqn. (4.4), the quantity of seepage loss across unit width of the dam is,

$$q = k.H. \frac{N_f}{N_d}$$

Here, $k = 3.55 \times 10^{-4}$ cm/sec

$$= \frac{(3.55 \times 10^{-4})(86400)}{100} \text{ m/day} = 0.3067 \text{m/day}$$

$$N_f = 3, \quad N_d = 12$$

As the downstream end is provided with a long horizontal filter, the downstream side should be dry.

$$\therefore \quad H = 30 - 1.5 = 28.5 \text{ m}$$

$$q = \frac{(0.3067)(28.5)(3)}{12} = 2.185 \text{ m}^3/\text{day}$$

Total quantity of seepage loss per day across the entire width of the dam = (2.185) (200) = 437 m^3.

Problem 4.2 : A single row of sheet piles is driven upto a depth of 4 m in a bed of clean sand having a co-efficient of permeability of 0.002 cm/sec. An impermeable leyer of very stiff clay exists at a depth of 10 m below the G.L. The sheet pile wall has to retain water upto 4 m above G.L. The height of water level on the downstream side is 0.5 m. Construct the flownet and determine quantity of seepage loss considering unit width of the sheet piles.

Solution : The flownet is given in Fig. 4.8.

Using eqn. (4.4),

$$q = k H \cdot \frac{N_f}{N_d}$$

Here, $k = 0.002 \text{ cm/sec} = \dfrac{(0.002)(86400)}{100} \text{ m/day} = 1.728 \text{ m/day}$

$$H = 4 - 0.5 = 3.5 \text{ m}$$

$$N_f = 7, \quad N_d = 12$$

$$\therefore \quad q = \frac{(1.728)(3.5)(7)}{12} = 3.53 \text{ m}^3/\text{day}$$

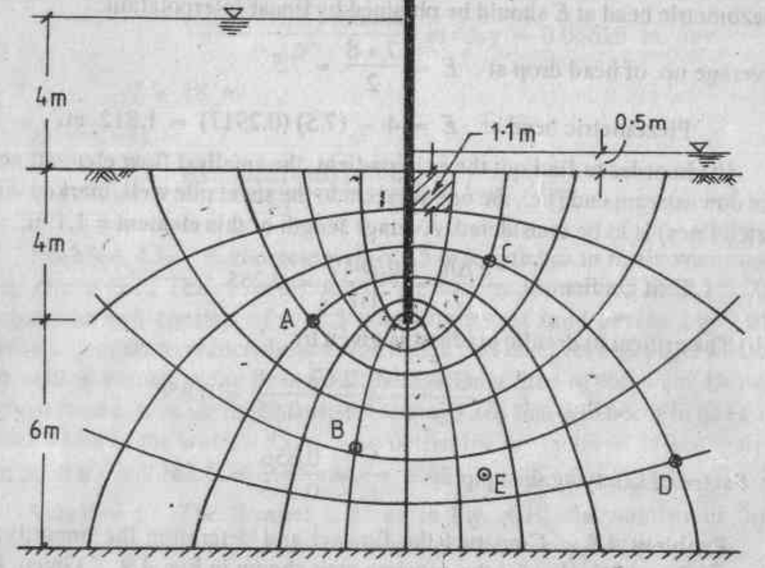


Fig. 4.8

Problem 4.3 : With reference to Fig. 4.8, determine the following:

(i) The piezometric heads at the points *A, B, C, D* and *E*.

(ii) The exit gradient

(iii) Factor of safety against piping. Given, $G = 2.67$, $e = 0.95$.

Solution:

(i) Initial piezometric head at the ground level on upstream side = 4 m.

$$\text{Head drop } \Delta H = \frac{\text{Head difference}}{\text{No. of head drops}} = \frac{4 - 0.5}{12} = 0.2917 \text{ m}$$

Now, number of head drops upto the point $A = 3$

$\therefore$ Head loss at $A = (3)(0.2917) = 0.875$ m.

Residual head at A = Initial head – head loss

$= 4 - 0.875 = 3.125$ m.

Similarly, the piezometric head at *B*, *C* and *D* are computed.

Piezometric head at $B = 4 - (5)(0.2917) = 2.542$ m.

at $C = 4 - (10)(0.2917) = 1.083$ m

$$\text{at } D = 4 - (10)(0.2917) = 1.083 \text{ m}$$

The point E lies in between the 5th and 6th flow lines. Hence, the piezometric head at E should be obtained by linear interpolation.

$$\text{Average no. of head drop at } \quad E = \frac{7+8}{2} = 7.5$$

$$\therefore \quad \text{Piezometric head at } \quad E = 4 - (7.5)(0.2917) = 1.812 \text{ m.}$$

(ii) In order to find out the exit gradient, the smallest flow element near the downstream end (i.e., the one adjacent to the sheet pile well, marked with hatch lines) is to be considered. Average length of this element = 1.1 m.

$$\therefore \quad \text{Exit gradient, } i_e = \frac{\Delta h}{l} = \frac{0.2917}{1.1} = 0.265$$

(iii) The critical hydraulic gradient is given by,

$$i_c = \frac{G-1}{1+e} = \frac{2.67-1}{1+0.95} = 0.856$$

$$\therefore \text{ Factor of safety against piping } = \frac{i_c}{i_e} = \frac{0.856}{0.265} = 3.23$$

Problem 4.4. Construct the flownet and determine the quantity of seepage loss in m^3/day for the concrete weir shown in Fig. 4.9. Given, $k = 6.5 \times 10^{-5}$ cm/sec.

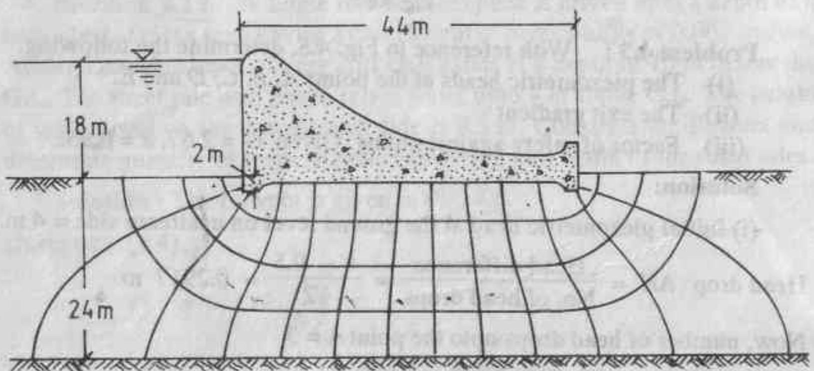


Fig. 4.9

Solution : Fig. 4.9 shows the flownet. From the figure we get,

No. of flow channels, $N_f = 4$.

No. of head drops, $N_d = 13$.

$$k = 6.5 \times 10^{-5} \text{ cm/sec}$$

$$= \frac{(6.5 \times 10^{-5})(86400)}{100} \text{ m/day} = 0.05616 \text{ m/day}$$

$$H = 18 \text{ m}$$

Using eqn (4.4),

$$q = \frac{(0.05616)(18)(4)}{13} = 0.311 \text{ m}^3/\text{day}$$

Problem 4.5. A concrete weir of 15 m length has to retain water upto 5 m above G.L. The cross-section of the weir is shown in Fig. 4.10. The foundation soil consists of a 12.5 thick stratum of sand having $k = 0.015$ cm/sec. In order to reduce the seepage loss, a 5 m deep vertical sheet pile cut off wall is placed at the bottom of the upstream face of the weir. Draw a flownet and determine the quantity of seepage loss that will occur in one day, if the width of the weir be 55 m. Also determine the factor of safety against piping if the soil has $G = 2.65$ and $e = 1.08$.

Solution : The flownet is given in Fig. 4.10. the number of flow channels is found to be 5, while the number of head drops = 16.

$$k = 0.015 \text{ cm/sec} = \frac{(0.015)(86400)}{100} \text{ m/day} = 12.96 \text{ m/day}$$

$$H = 5 \text{ m}$$

$$\therefore \quad q = \frac{(12.96)(5)(5)}{16} = 20.25 \text{ m}^3/\text{day}$$

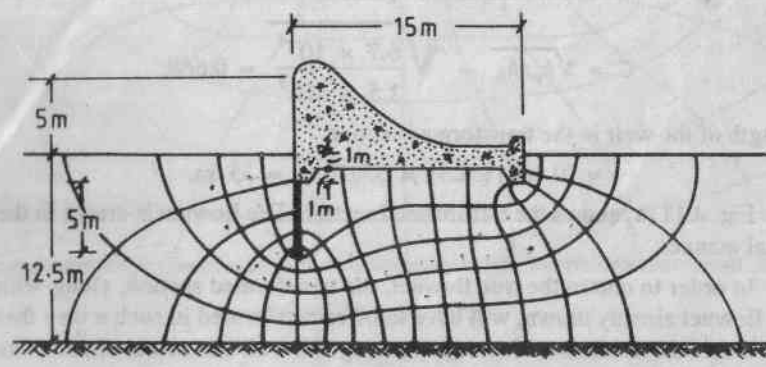


Fig. 4.10

This is the quantity of seepage loss across unit *width of the weir*. Considering the entire width of 55 m (on a plane perpendicular to that of the paper), total quantity of seepage loss per day = (20.25)(55) = 1113.75 m^3.

Again, the average length of the smallest flow element adjacent to the weir = 1.2 m.

$$\therefore \quad \text{Exit gradient} \quad i_e = \frac{\Delta H}{l} = \frac{H}{N_d \times l} = \frac{5}{(16)(1.2)} = 0.26$$

Critical hydraulic gradient,

$$i_c = \frac{G-1}{1+e} = \frac{2.65-1}{1+1.08} = 0.79$$

$$\therefore \quad \text{Factor of safety against piping} = \frac{0.79}{0.26} = 3.04$$

Problem 4.6. A concrete weir of 52.5 m length is founded at a depth of 2m in a deposit of fine sand for which the co-efficient of permeability in the horizontal and vertical directions are 1.5×10^{-3} cm/sec and 6.7×10^{-4} cm/sec respectively. The sand is underlain by a rock layer at a depth of 37 m below G.L. The high flood level on the upstream side is 18 m and the downstream side has a free standing water table upto 1.5 m above G.L. Draw the flownet and determine the quantity of seepage loss across unit width of the weir.

Solution: As the co-efficients of permeability of the soil in the horizontal and vertical directions are different, the cross-section of the weir should be transformed before constructing the flownet. All vertical dimensions of the transformed section will remain unchanged. But all horizontal dimensions should be multiplied by a constant factor *C*, where,

$$C = \sqrt{k_v/k_h} = \sqrt{\frac{6.7 \times 10^{-4}}{1.5 \times 10^{-3}}} = 0.668$$

Length of the weir in the transformed section

$$= (0.668)(52.5) = 35.07 \text{ m} \approx 35 \text{ m}.$$

Fig. 4.11 (a) shows the transformed section. The flownet is drawn in the usual manner.

In order to obtain the true flownet, the transformed section, along with the flownet already drawn, will have to be retransformed in such a way that all vertical dimensions will remain unchanged but all horizontal dimensions will be divided by a constant factor of 0.668.

In order to retransform the flownet, the location of all grid points (i.e., the intersection between flow lines and equipotential lines) should be

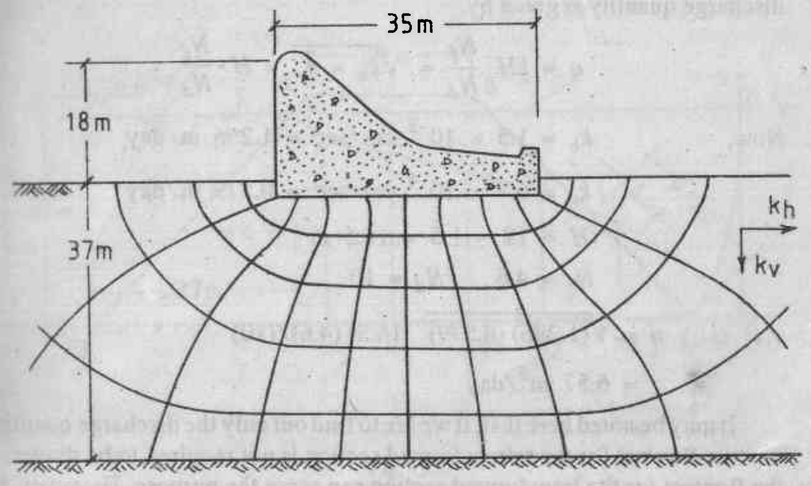


Fig. 4.11 (a)

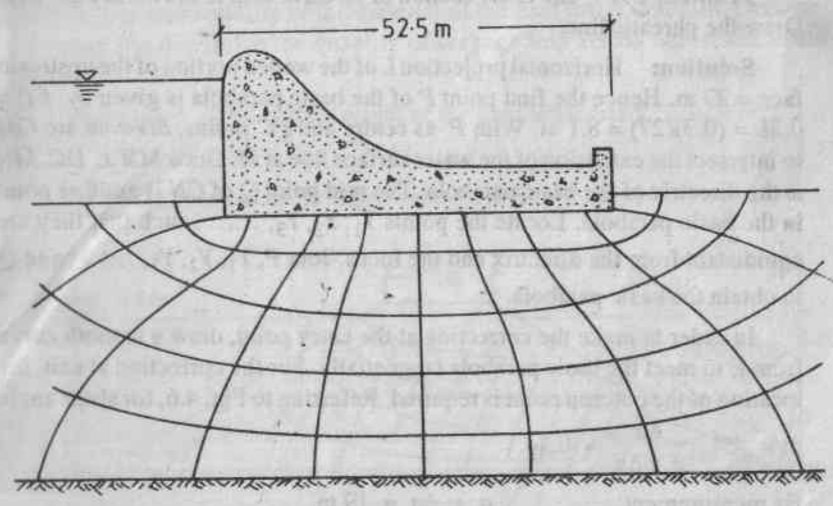


Fig. 4.11 (b)

determined. These grid points should then be located on the retransformed section. Joining these points in the appropriate order will give the true flownet in which all flow elements will be rectangular.

Fig. 4.11 (b) shows the retransformed section and the true flownet. The discharge quantity is given by,

$$q = kH \cdot \frac{N_f}{N_d} = \sqrt{k_h \times k_v} \times H \cdot \frac{N_f}{N_d}$$

Now, $k_h = 1.5 \times 10^{-3}$ cm/sec $= 1.296$ m/day

$k_v = 6.7 \times 10^{-4}$ cm/sec $= 0.579$ m/day

$H = 18 - 1.5 = 16.5$ m

$N_f = 4.6, \quad N_d = 10$

$\therefore \quad q = \sqrt{(1.296)(0.579)}\ (16.5)(4.6)/(10)$

$= 6.57\ m^3/day$

It may be noted here that, if we are to find out only the discharge quantity, the true flownet for the retransformed section is not required to be drawn, as the flownet for the transformed section can serve the purpose. However, for the determination of exit gradient, hydrostatic pressure heads, uplift pressures etc., the actual flownet has to be drawn.

Problem 4.7. The cross-section of an earth dam is shown in Fig. 4.12. Draw the phreatic line.

Solution: Horizontal projection L of the wetted portion of the upstream face = 27 m. Hence the first point P of the basic parabola is given by, $EP = 0.3L = (0.3)(27) = 8.1$ m. With P as centre and PC radius, draw an arc CM to intersect the extension of the water surface line at M. Draw $MN \perp DC$. MN is the directrix of the basic parabola. The mid point Q of CN is another point in the basic parabola. Locate the points Y_1, Y_2, Y_3, ... , such that they are equidistant from the directrix and the focus. Join P, Y_1, Y_2, Y_3, ... , and Q to obtain the basic parabola.

In order to make the correction at the entry point, draw a smooth curve from E to meet the basic parabola tangentially. For the correction at exit, the location of the outcrop point is required. Referring to Fig. 4.6, for slope angle

$$\beta = 45°, \quad \frac{\Delta a}{a + \Delta a} = 0.34.$$

By measurement, $a + \Delta a = 19$ m.

$\therefore \quad \Delta a = (0.34)(19) = 6.46$ m.

The distance CC' is laid off such that, $CC' = 6.46$ m. C' is the true outcrop point. Draw another smooth curve to meet tangentially the basic parabola at one end and the downstream face at C'. The curve EC' is then the required

phreatic line. The remaining portion of the basic parabola is shown with a broken line.

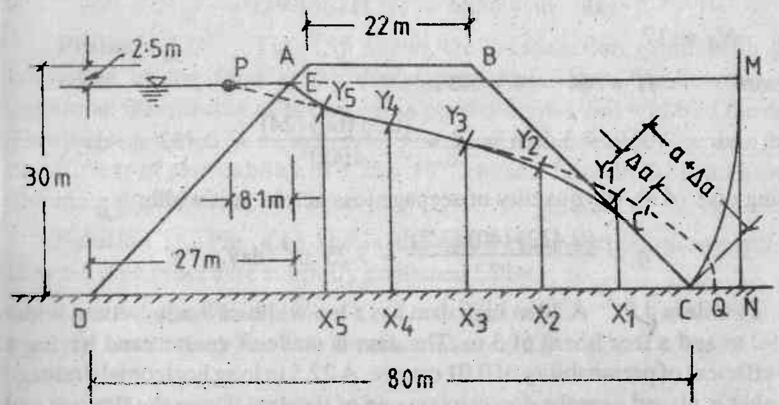


Fig. 4.12

Problem 4.8. A 44 m high earth dam has a top width of 28 m and a free board of 4 m. The upstream and downstream faces have equal slope angles of 30°. The dam is placed on an impervious foundation. The co-efficient of permeability of the material of the dam is 0.3 mm/min. Draw the flownet and determine the quantity of seepage loss across unit width of the dam.

Solution : The bottom width of the dam is given by,

$$B = 28 + 2\,(44/\tan 30°) = 180 \text{ m}.$$

The cross-section of the dam is shown in Fig. 4.13.

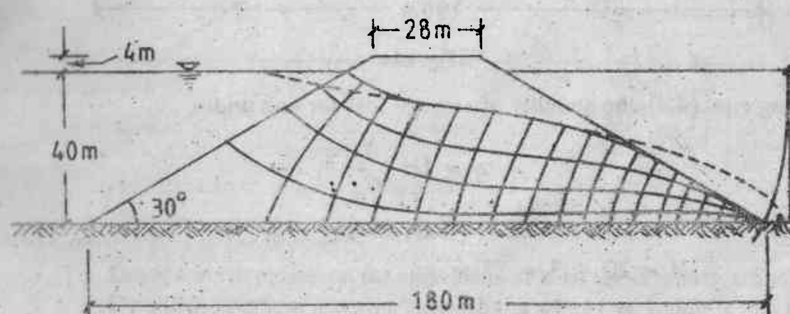


Fig. 4.13

The basic parabola and the phreatic line are drawn in the usual manner and the flownet is sketched. From Fig. 4.13 we obtain,

$N_f = 3.2$

$N_d = 17$

Again, $H = 44 - 4 = 40$ m.

and, $k = 0.3 \text{ mm/min} = \dfrac{(0.3)(60)(24)}{1000} = 0.432$ m/day.

Using eqn. (4.4), the quantity of seepage loss across unit width,

$$q = \frac{(0.432)(40)(3.2)}{17} = 3.25 \text{ m}^3\text{/day}.$$

Problem 4.9. A 20 m high dam has a top width of 8 m, a bottom width of 90 m and a free board of 3 m. The dam is made of coarse sand having a co-efficient of permeability of 0.01 cm/sec. A 22.5 m long horizontal drainage blanket is placed near the downstream end of the dam. Draw the flownet and determine the quantity of seepage loss if the width of the dam be 175 m.

Solution : The flownet is shown in Fig. 4.14.

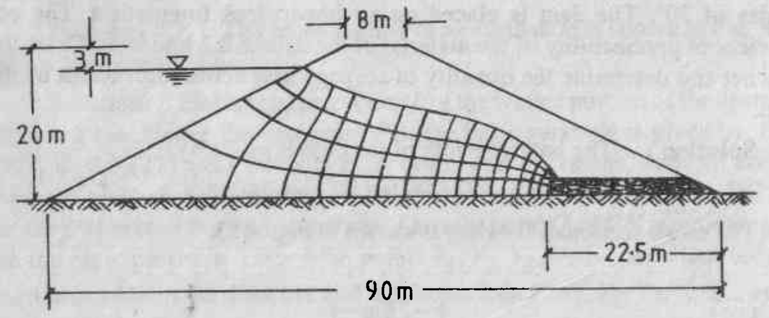


Fig. 4.14

Using eqn. (4.4), the quantity of seepage loss for unit width,

$$q = kH \cdot \frac{N_f}{N_d}$$

Here, $k = 0.01$ cm/sec $= 8.64$ m/day.

$H = 20 - 3 = 17$ m.

$N_f = 4, \quad N_d = 15$

$$\therefore \quad q = \frac{(8.64)(17)(4)}{15} = 39.168 \text{ m}^3\text{/day}.$$

Seepage loss across the entire dam

$$= (39.168)(175) = 6854.4 \text{ m}^3\text{/day}.$$

Problem 4.10. Fig. 4.15 shows the cross-section of an earth dam consisting of toe filter at the downstream end. Draw the flow-net and determine the quantity of seepage loss per day across unit width of the dam. The dam is founded on an impervious base and the material of the dam has a co-efficient of permeability of 3.28×10^{-3} cm/sec. Explain the procedure of obtaining the flownet.

Solution : Fig. 4.15 shows the given cross-section, alongwith the flownet. The procedure is briefly explained below:

(a) *Locating the phreatic line* :

(i) Locate the point D in the usual manner ($ED = 0.3L = 0.3 \times 37 = 11.1$ m).

(ii) The bottom left hand corner of the toe is taken as the focus of the basic parabola.

(iii) Draw the directrix and locate the point Q of the basic parabola.

(iv) Locate a number of points which are equidistant from the directrix and the focus.

(v) Join these points with a smooth curve to obtain the basic parabola.

(vi) Make the necessary corrections at the entry and the exit points.

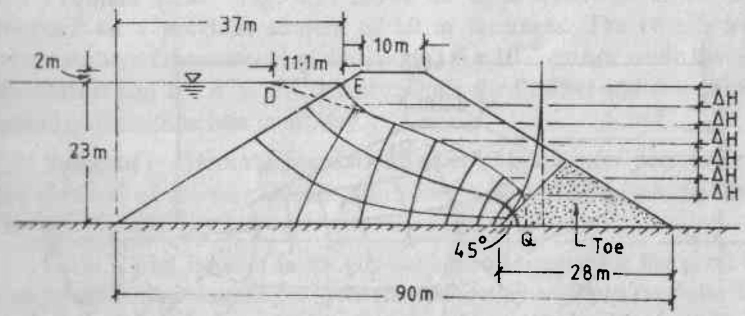


Fig. 4.15

(b) *Construction of the flownet* :

(i) Draw a vertical line on the right hand side of the downstream face. Divide the vertical distance between the water level and the outcrop point into any number of equal parts of the length ΔH.

(ii) Draw horizontal lines from each of these points and locate their points of intersection with the phreatic line.

(iii) Draw a number of equipotential lines from each of these intersection points. This will ensure that the head drops are equal.

(iv) Draw the flow lines. Adjust the flow lines and equipotential lines again and again until the flownet becomes orthogonally squared.

Using eqn. (4.4),

$$q = kH \cdot \frac{N_f}{N_d}.$$

Here, $k = 3.28 \times 10^{-3}$ cm/sec = 2.834 m/day.

$H = 23$ m.

$N_f = 2.3$

$N_d = 7$

$$\therefore \quad q = \frac{(2.834)(23)(2.3)}{7} = 21.42 \text{ m}^3/\text{day}.$$

Problem 4.11. A 30 m high earth dam having a top width of 20 m, a bottom width of 140 m and a free board of 3 m is founded on a 60 m thick deposit of a pervious soil which is underlain by an impermeable shale. The co-efficients of permeability of the material of the dam and the foundation soil are 0.003 cm/sec and 0.0003 cm/sec respectively. Draw the flownet and determine the quantity of seepage loss in m^3/day.

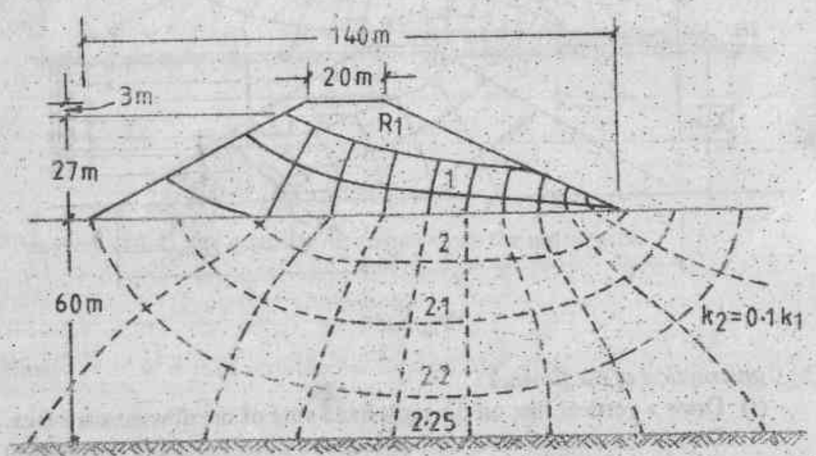


Fig. 4.16

Solution : The co-efficient of permeability of the material of the dam is 10 times that of the foundation soil. Hence the flownet in the earth dam is first constructed as if it were placed on an impervious foundation. It can be seen from Fig. 4.16 that the second flow line hits the interface between the dam and the foundation soil.

Now come down to the foundation soil and complete the flownet. The second flow line as well as all the equipotential lines will enter the foundation soil and will undergo a deviation at the interface. In the soil stratum, the flownet is drawn as orthogonally squared, as usual. However, as the k-value of the soil is only 1/10th of that of the material of the dam, each flow channel in the soil stratum carries a discharge equal to 1/10th of the discharge carried by a flow channel in the dam. Consequently, the flow channels in the soil are marked as 2.1, 2.2 etc.

From the figure we get,

total number of flow channels, $N_f = 2.25$

number of head drops, $N_d = 10$.

k for dam-material = 0.003 cm/sec = 2.592 m/day.

$H = 30 - 3 = 27$ m.

$$\therefore \quad q = \frac{(2.592)(27)(2.25)}{10} = 15.75 \text{ m}^3/\text{day}.$$

Problem 4.12. Fig. 4.17 shows the cross-section of an earth dam founded on a pervious stratum of 60 m thickness. The co-efficient of permeability of the material of the dam is 1.6×10^{-4} cm/sec while that of the foundation soil is 1.6×10^{-3} cm/sec. Draw the flownet and determine the quantity of seepage loss in m^3/day.

Solution : Here the foundation soil is 10 times more permeable than the material of the dam. Hence more emphasis will be given on seepage through this soil.

Draw a trial flownet in the foundation soil neglecting the earth dam. Extend all equipotential lines from the foundation soil into the dam. These lines should deviate from the interface, but this should be done only by hand and eqn. (4.11) need not be considered. Now draw the flow lines in the dam section and try to make the flow net orthogonally squared. This may necessitate certain readjustments, especially for the last flow line in the dam, which should enter into the foundation soil. All previously drawn flow lines in the foundation soil may have to be lowered. The final flownet is shown in Fig. 4.17.

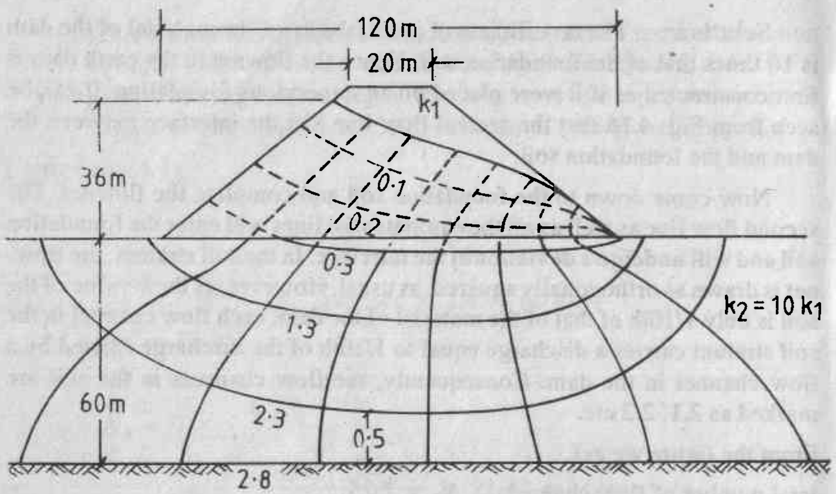


Fig. 4.17

As the co-efficient of permeability of the dam material is 1/10th of that of foundation soil, a flow channel in the dam is equivalent to 1/10th of a flow channel in the foundation soil. Consequently, the flow channels in the dam are marked as 0.1, 0.2, and 0.3. No. of full flow channels in the soil stratum = 2.5. Hence, $N_f = 0.3 + 2.5 = 2.8$.

$N_d = 8.$, $H = 36$ m

k for foundation soil $= 1.6 \times 10^{-3}$ cm/sec = 1.382 m/day.

$$\therefore \quad q = \frac{(1.382)\,(36)\,(2.8)}{8} = 17.41 \text{ m}^3/\text{day}.$$

Problem 4.13. Fig. 4.18 shows the cross-section of a zoned earth dam consisting of two zones. Zone I adjacent to the upstream face has $k = 0.001$ cm/sec while zone II adjacent to the downstream face has $k = 0.003$ cm/sec. Draw the flownet and determine the quantity of seepage loss in m^3/day.

Solution : The material of zone II is 3 times more permeable than that of zone I. Draw the phreatic line in zone I arbitrarily. From the interface between the two zones, the phreatic line should deviate downwards, as water can flow more easily in zone II. Draw this deviated phreatic line arbitrarily.

Now draw a vertical line and divide it into any number of equal parts of length ΔH. From each of these points draw horizontal lines to intersect the phreatic line. All these intersection points are springing points of the equipotential lines. In zone I, draw the flownet as orthogonally squared. Each flow line enters zone II after undergoing a deviation at the interface. However, the flow elements in zone II will not be squares but rectangles, for each of

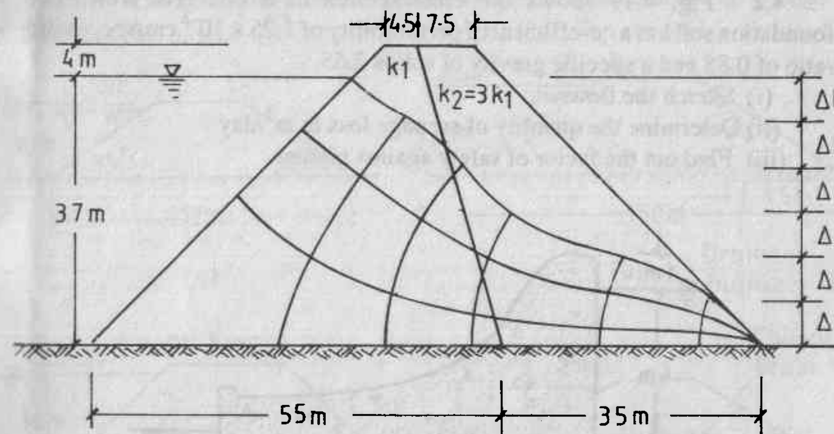


Fig. 4.18

which the average length in the direction of flow is 3 times (since $k_2/k_1 = 3$) the average width. In order to cope with this condition, the trial phreatic line drawn in zone II may have to be either raised or lowered and the flownet should be completed by trial and error.

Using eqn. (4.4),

$$q = k.H.\frac{N_f}{N_d}.$$

Now, $N_f = 2.5$, $N_d = 6$, $H = 37$m.

k for zone I = 0.001 cm/sec. = 0.864 m/day.

$$\therefore \quad q = \frac{(0.864)\,(37)\,(2.5)}{6} = 13.32 \text{ m}^3/\text{day}.$$

It may be noted that the methods employed in Problem 4.11 through 4.13, however crude they may seem to be, will yield results which are within ±10% of the results obtained by a more accurate and vigorous solution.

EXERCISE 4

4.1 On a waterfront, a sheet pile wall of 8 m height is embedded into the soil upto 6 m below G.L. The free board is 1 m while water on the downstream side stands upto 2 m above G.L. The foundation soil consists of a 15 m thick sand stratum (k = 0.009 cm/sec) which is underlain by an impervious layer. Draw the flownet and determine the quantity of seepage loss across a 1 m wide section of the sheet pile.

4.2 Fig. 4.19 shows the cross-section of a concrete weir. The foundation soil has a co-efficient of permeability of 1.25×10^{-3} cm/sec, a void ratio of 0.88 and a specific gravity of solids 2.65.

(i) Sketch the flownet.

(ii) Determine the quantity of seepage loss in m^3/day

(iii) Find out the factor of safety against piping.

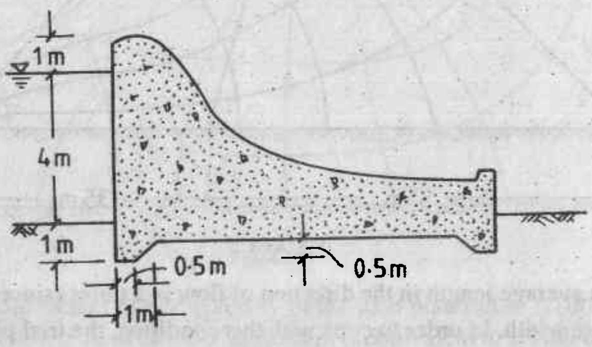


Fig. 4.19

4.3 (a) If a sheet pile cut-off wall of 3 m depth is introduced in the upstream end of the weir shown in Fig. 4.19, determine the percent reduction in the quantity of seepage loss.

(b) If in addition to this, another sheet pile of 2 m depth is placed at the downstream end, how will the seepage quantity change?

4.4 Fig. 4.20 shows the cross section of a concrete weir founded in an anisotropic soil mass. The co-efficient of permeability in the horizontal and vertical directions are respectively 5×10^{-4} mm/sec and 1.25×10^{-4} mm/sec. Sketch the flownet and determine the quantity of seepage loss in m^3/hr.

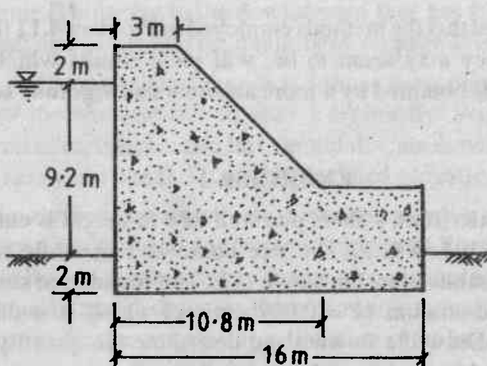


Fig. 4.20

4.5 Draw the phreatic lines for the dam sections shown in Fig. 4.21(a) through (d):

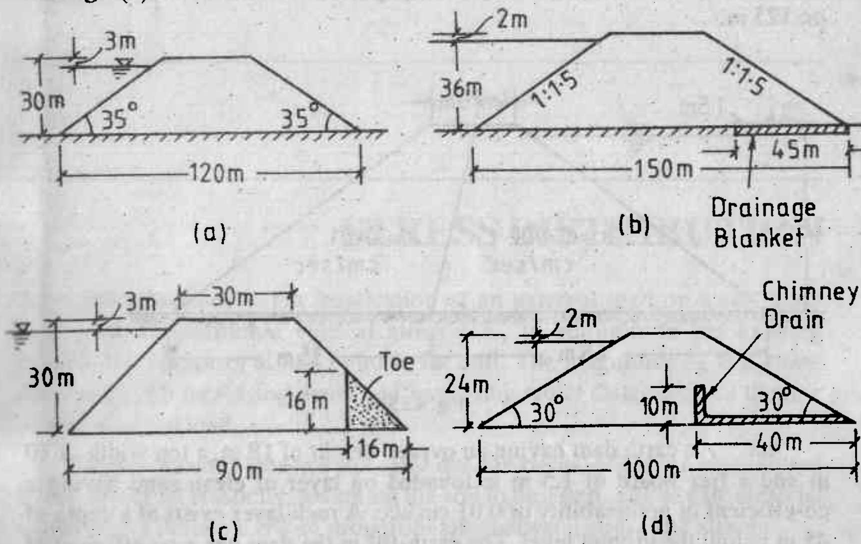


Fig. 4.21

4.6 Sketch the flownets and determine the quantity of seepage loss across unit width of the earth dams shown in Fig. 4.22 (a) through (c). All the dams are founded on impervious soils. Take $k = 0.002$ cm/sec.

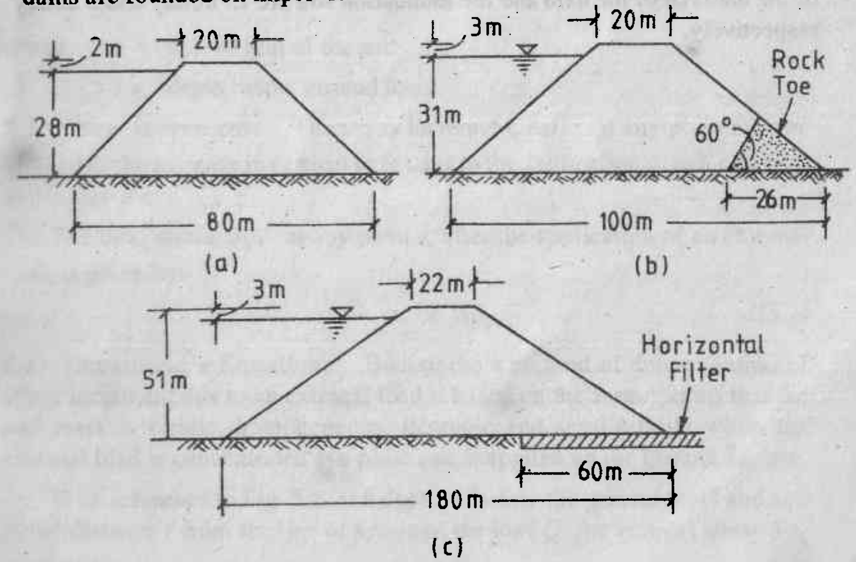


Fig. 4.22

4.7 Construct the flownet for the zone earth dam shown in Fig. 4.23. Hence compute the total quantity of seepage loss if the width of the reservoir be 125 m.

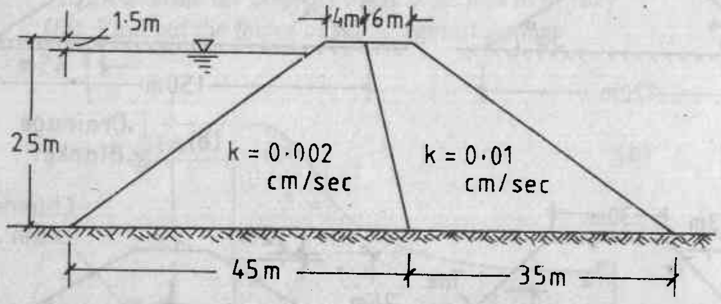


Fig. 4.23

4.8 An earth dam having an overall height of 18 m, a top width of 10 m and a free board of 1.5 m is founded on layer of clean sand having a co-efficient of permeability of 0.01 cm/sec. A rock layer exists at a depth of 45 m below the ground level. The earth-fill in the dam has a co-efficient of permeability of 0.002 cm/sec. Draw the flow net and determine the quantity of seepage loss in m^3/day.

4.9 Solve Problem 4.8 assuming that the co-efficient of permeability of the material of the dam and the foundation soil are 15 m/day and 5 m/day respectively.

5

STRESS DISTRIBUTION

5.1. Introduction: The application of an external load on a soil mass results in an additional vertical stress (i.e., in addition to the existing overburden pressure) at any point in the soil. The magnitude of this stress decreases with increasing depth and increasing radial distance from the line of action of the load.

The stress conditions in a soil mass due to external loads can be analysed by the theory of elasticity, assuming the soil to be a perfectly elastic material which obeys Hooke's law of proportionality between stress and strain.

5.2 Overburden Pressure: The overburden pressure at any point in a soil mass is defined as the initial vertical stress due to the self weight of the soil mass, and can be obtained from

$$\sigma_{z_o} = \gamma_z \qquad ...(5.1)$$

where, γ = unit weight of the soil

z = depth below ground level.

5.3 Stress Increment: The stress increment, $\Delta\sigma_z$, at any point may be defined as the increase in vertical stress due to the application of external load on the soil mess.

The total stress, σ_z, at any point z, after the application of an external load, is given by,

$$\sigma_z = \sigma_{z_o} + \Delta\sigma_z \qquad ...(5.2)$$

5.4. Boussinesq's Equation: Boussinesq's method of determination of stress increment due to an external load is based on the assumptions that the soil mass is elastic, homogeneous, isotropic and semi-infinite while the external load is concentrated at a point and is applied on the ground surface.

With reference to Fig. 5.1, at a depth z below the ground level and at a radial distance r from the line of action of the load Q, the vertical stress $\Delta\sigma_z$ is given by

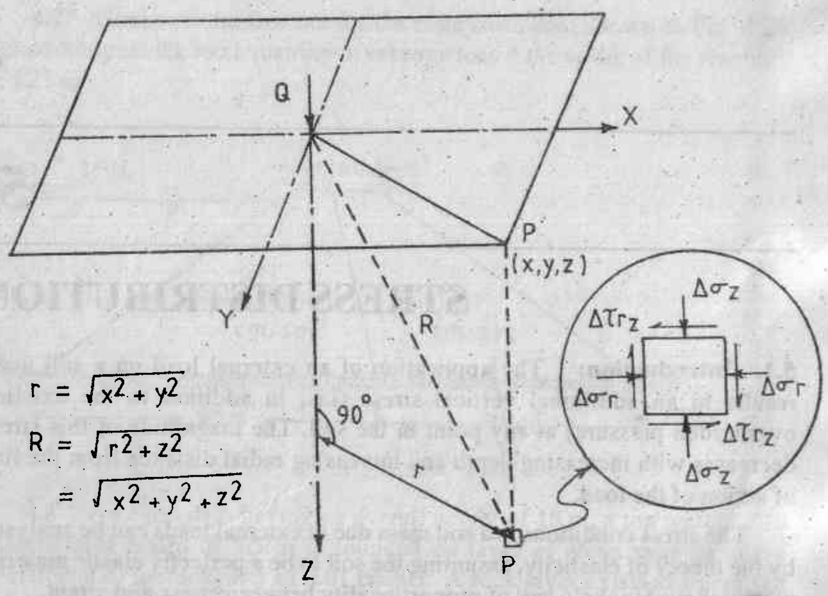


Fig. 5.1

$$\Delta\sigma_z = \frac{3Q}{2\pi}\cdot\frac{\cos^3\beta}{R^2} \qquad ...(5.3)$$

or,
$$\Delta\sigma_z = \frac{3Q}{2\pi z^2}\cdot\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad ...(5.4)$$

or,
$$\Delta\sigma_z = K_B\cdot\frac{Q}{z^2} \qquad ...(5.5)$$

where, K_B is called Boussinesq's influence factor and is given by,

$$K_B = \frac{3}{2\pi}\cdot\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} \qquad ...(5.6)$$

The tangential stress $\Delta\tau_{rz}$ and the radial stress $\Delta\sigma_r$ at the same point are given by

$$\Delta\tau_{rz} = K_B\cdot\frac{Q\cdot r}{z^3} \qquad ...(5.7)$$

and
$$\Delta\sigma_r = K_B\cdot\frac{Q\,r^2}{z^4} \qquad ...(5.8)$$

5.5. Westergaard's Equation: Westergaard assumed the soil not to be homogeneous but consisting of a number of closely spaced horizontal sheets of infinite rigidity but negligible thickness which restrain the soil from undergoing any lateral strain. According to Westergaard,

$$\Delta\sigma_z = \frac{Q}{2\pi z^2}\cdot\frac{\sqrt{(1-2\mu)/(2-2\mu)}}{\left[(1-2\mu)/(2-2\mu)+(r/z)^2\right]^{3/2}} \qquad ...(5.9)$$

where, μ = Poisson's ratio of the soil.

If for a given soil, $\mu = 0$, eqn. (5.9) reduces to

$$\Delta\sigma_z = \frac{Q}{\pi z^2}\cdot\left[\frac{1}{1+2\left(\frac{r}{z}\right)^2}\right]^{3/2} \qquad ...(5.10)$$

5.6. 2 : 1 Dispersion Method : This is an approximate method in which it is assumed that the effect of an external load is dispersed along straight lines inclined at $2V : 1H$.

With reference to Fig. 5.2, total load applied on the ground surface, $Q = q\,BL$.

At a depth z, this load is distributed over an area $(L+z)(B+z)$. Hence, stress intensity at this level,

$$\Delta\sigma_z = \frac{q\,B\,L}{(L+Z)(B+Z)} \qquad ...(5.11)$$

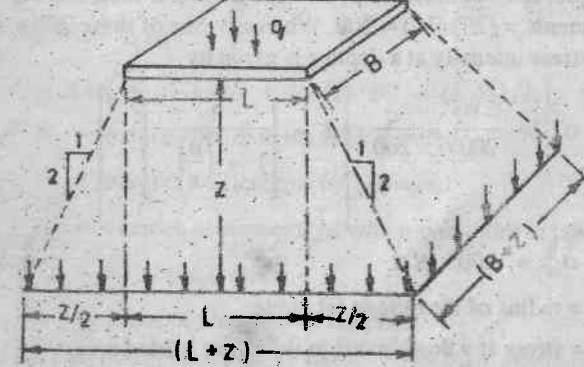


Fig. 5.2

5.7. Isobar: If the vertical stress intensities at various points in a soil mass due to an external load are computed, and the points having equal stress intensities are joined by a smooth curve, a number of stress contours, each having the shape of a bulb, are obtained. These are called isobars. The zone in the soil mass bounded by an isobar is called a pressure bulb. An infinite number of isobars can be drawn. The zone contained by the isobar corresponding to a stress intensity which is equal to 10% of the applied stress is taken to be the zone within which appreciable effects of the applied stress may occur. This zone is termed as the zone of influence.

5.8. Stress due to a Uniformly Loaded Circular Area: From Boussinesq's equation it can be proved that the stress intensity at a depth z below the centre of a circular area of radius a, which carries a uniformly distributed load q, is given by,

$$\Delta\sigma_z = q\left[1 - \left\{\frac{1}{1+\left(\frac{a}{z}\right)^2}\right\}^{3/2}\right] \qquad ...(5.12)$$

5.9. Newmark's Chart: The stress intensity at any point due to a uniformly loaded area of any shape can be determined with the help of Newmark's influence chart. It consists of a series of concentric circles of various radii and a series of radial lines drawn at regular angular intervals. The total area of the chart is thus divided into a number of elements. The elements may have different sizes, but each of them, when loaded with a given stress intensity, will give rise to the same vertical stress at a given point.

In order to prepare the chart, proceed as follows:

(1) Select the number of elements in which the chart should be divided, and determine the influence value for each element.

For example, if 10 concentric circles and 20 radial lines are drawn, the number of elements = (20) (10) = 200. When any one of these 200 elements is loaded, the stress intensity at a depth z is given by,

$$\sigma_{ze} = \frac{\Delta\sigma_z}{200} = \frac{q}{200}\cdot\left[1 - \left\{\frac{1}{1+\left(\frac{a}{z}\right)^2}\right\}^{3/2}\right] \qquad ...(5.13)$$

or, $$\sigma_{ze} = 0.005\ \Delta\sigma_z \qquad ...(5.14)$$

where, a = radius of the outermost circle

$\Delta\sigma_z$ = stress at a depth z due to the entire loaded area.

(2) Select an arbitrary value of z (say, z = 5 cm).

(3) Determine the radii of the circles from

$$1 - \left\{\frac{1}{1+\left(\frac{r_i}{z}\right)^2}\right\}^{3/2} = \frac{n_i}{n} \qquad ...(5.15)$$

where, n = total number of circles to be drawn

n_i = number of the circle whose radius is required

r_i = radius of that circle.

(4) Draw all the concentric circles.

(5) Determine the deflection angle of the radial lines from:

$$\delta = \frac{360°}{m} \qquad ...(5.16)$$

where, m = total number of radial lines to be drawn.

(6) Draw the radial lines with the deflection angles thus calculated.

With the help of the Newmark's chart (Fig. 5.15) the stress intensity at any point due to a uniformly loaded area of any given shape can be determined as follows:

(i) Adopt a drawing scale such that the depth at which the stress intensity is required is represented by the numerical value of z on the basis of which Newmark's chart is drawn.

(ii) Draw the plan of the loaded area on a tracing paper with this drawing scale. Locate the point P below which the stress is required.

(iii) Place the tracing paper on the chart in such a way that the point P on the tracing paper coincides with the centre of the circles.

(iv) Count the number of elements covered, fully or partly, by the plan of the area.

(v) Calculate $\Delta\sigma_z$ as:

$$\Delta\sigma_z = q\left[(i_f)N_1 + (i_f/2)\cdot N_2 + (i_f/3)N_3\right] \qquad ...(5.17)$$

where, N_1 = number of elements fully covered.

N_2 = number of elements half covered.

N_3 = number of elements of which one-third is covered.

i_f = influence factor

= 1/no. of elements of the chart.

5.10. Stress Due to Vertical Linear Load: The load resulting from a long but narrow wall, or a railway track, is an example of a vertical linear load

(Fig. 5.3 a). The vertical stress at a depth z and at a radial distance of r from the line of action of such a load of intensity q t/m, is given by,

$$\Delta\sigma_z = \frac{2qz^3}{\pi\left(r^2+z^2\right)^2} = \frac{2q}{\pi z}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^2 \quad \text{...(5.18)}$$

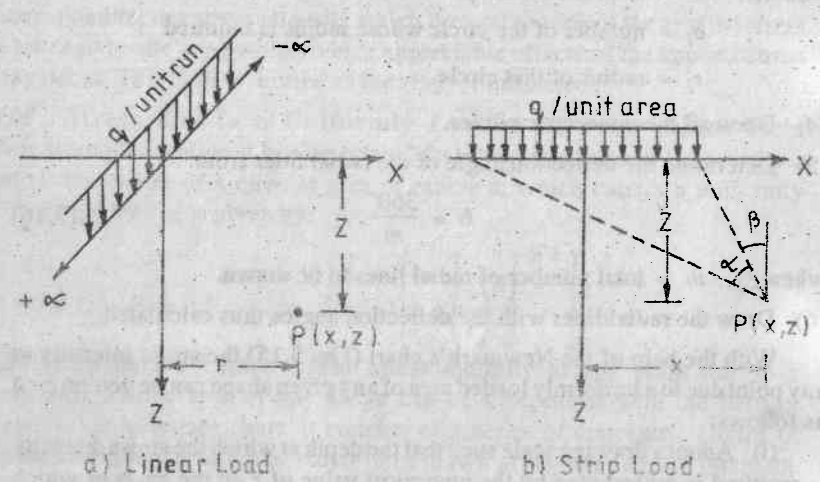


Fig. 5.3

5.11. Vertical Stress Due to a Strip Load: Loads uniformly distributed over an area of infinite length but a finite width are known as strip loads (e.g., continuous strip footings, highway pavements, etc.). The stress intensity at the point $P(x, z)$ due to a strip load of intensity q t/m^2 is given by (ref. Fig. 5.3 b)

$$\Delta\sigma_z = \frac{q}{\pi}\left[\alpha + \sin\alpha\cos(\alpha+2\beta)\right] \quad \text{...(5.19)}$$

5.12. Vertical Stress Due to Triangular Loads: Stresses due to triangular loading are required to be evaluated for analysing the effects of embankment loadings.

For the triangular load shown in Fig. 5.4, the stress intensity at the point $P(y, z)$ is given by :

$$\Delta\sigma_z = \frac{q}{2\pi}\left(\frac{y}{B}\cdot\alpha_1 - \sin 2\alpha_2\right) \quad \text{...(5.20)}$$

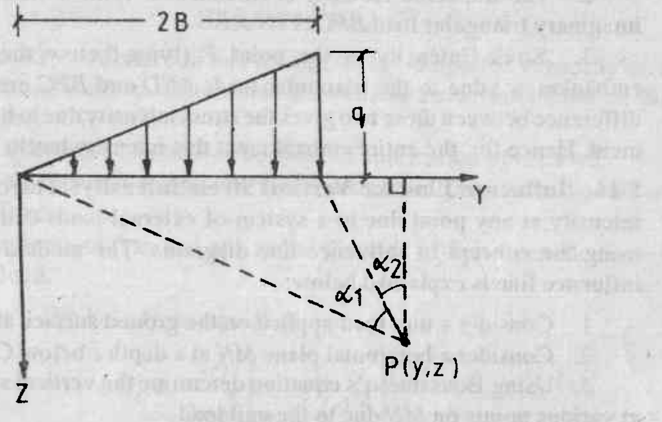


Fig. 5.4

5.13. Embankment Loading: Stresses in subsoils due to embankment loadings can be computed using eqn. (5.20).

Let it be required to compute the maximum vertical stress intensity at a depth z below an embankment having a top width $2b$, a base width $2(a+b)$ and a maximum intensity q t/m. The solution can be obtained in the following steps:

1. The embankment is divided into two equal parts as shown in Fig. 5.5. Two symmetrical trapezoidal loadings are obtained.

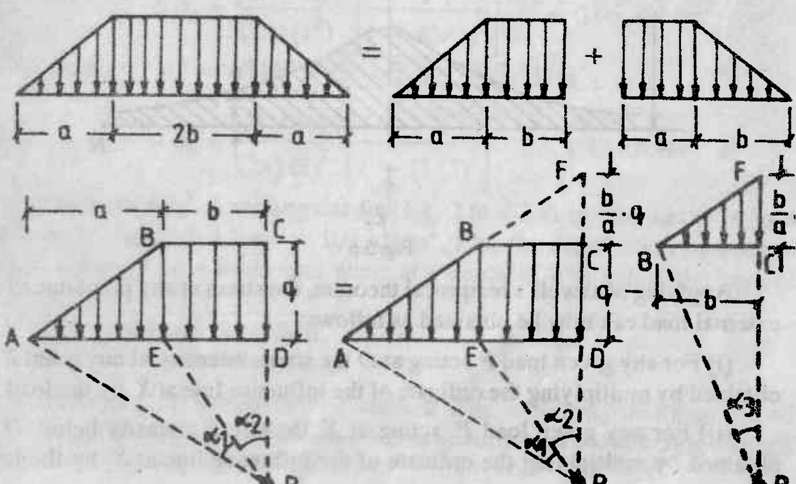


Fig. 5.5

2. The trapezoid *ABCDE* is extended to form Δ *AFD* by adding an imaginary triangular load *BFC*.

3. Stress intensity at the point *P* (lying below the centre of the embankment) due to the triangular loads *AFD* and *BFC* are obtained. The difference between these two gives the stress intensity due to half the embankment. Hence for the entire embankment this intensity has to be doubled.

5.14. Influence Line for Vertical Stress Intensity: The combined stress intensity at any point due to a system of external loads can be determined using the concept of influence line diagrams. The method of drawing an influence line is explained below:

1. Consider a unit load applied on the ground surface at *O*.
2. Consider a horizontal plane *MN* at a depth *z* below G.L.
3. Using Boussinesq's equation determine the vertical stress intensities at various points on *MN* due to the unit load.
4. Choose a vector scale and lay off the corresponding ordinates at the respective points to represent the computed stresses.
5. Join these ordinates with a smooth curve. This is the influence line.

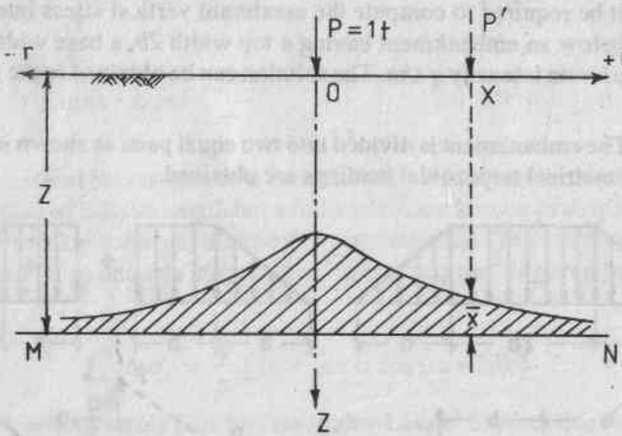


Fig. 5.6

Applying Maxwell's reciprocal theorem, the stress at any point due to an external load can now be obtained as follows:

(i) For any given load *P* acting at *O* the stress intensity at any point *X* is obtained by multiplying the ordinate of the influence line at *X* by the load *P*.

(ii) For any given load *P'* acting at *X*, the stress intensity below *O* is obtained by multiplying the ordinate of the influence line at *X* by the load *P'*.

EXAMPLES

Problem 5.1. A concentrated load of 40 kN is applied vertically on a horizontal ground surface. Determine the vertical stress intensities at the following points:

(i) At a depth of 2 m below the point of application of the load.

(ii) At a depth of 1 m and at a radial distance of 3 m from the line of action of the load.

(iii) At a depth of 3 m and at a radial distance of 1 m from the line of action of the load.

Solution: We have from eqn. (5.4),

$$\Delta \sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2}$$

(i) Here, $Q = 40$ kN, $z = 2$ m and $r = 0$

$\therefore$ $r/z = 0$

Hence eqn (5.4) gives

$$\Delta \sigma_z = \frac{3Q}{2\pi z^2} = \frac{(3)(40)}{(2\pi)(2^2)} = 4.77 \text{ kN/m}^2$$

(ii) In this case, $Q = 40$ kN, $z = 1$ m, $r = 3$ m

$$\therefore \quad \Delta \sigma_z = \frac{(3)(40)}{(2\pi)(1^2)} \cdot \left[\frac{1}{1+(3/1)^2}\right]^{5/2} = 0.06 \text{ kN/m}^2$$

(iii) Here, $Q = 40$ kN, $z = 3$ m, $r = 1$ m

$$\therefore \quad \Delta \sigma_z = \frac{(3)(40)}{(2\pi)(3)^2}\left[\frac{1}{1+(1/3)^2}\right]^{5/2} = 1.63 \text{ kN/m}^2$$

Problem 5.2. A rectangular footing, 2 m × 3 m in size, has to carry a uniformly distributed load of 100 kN/m². Plot the distribution of vertical stress intensity on a horizontal plane at a depth of 2 m below the base of footing by:

(i) Boussinesq's method.

(ii) 2 : 1 dispersion method.

Solution: (i) *Boussinesq's method.* The uniformly distributed load carried by the footing is to be considered as a concentrated load acting through the centre of gravity of the footing.

$$Q = q\,BL = (100)(2)(3) = 600 \text{ kN}.$$

Using eqn. (5.4), the stress intensity at a depth of 2 m and at a radial distance r from the line of action of Q is given by:

$$\Delta\sigma_z = \frac{3Q}{2\pi z^2}\cdot\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

$$= \frac{(3)(600)}{(2)(3.14)(2^2)}\cdot\left[\frac{1}{1+\left(\frac{r}{z}\right)^2}\right]^{5/2} = (71.62)\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

A number of points are chosen on the given plane and the stress intensity at each point is calculated. These are tabulated below:

Radial distance r *(m)*	r/z	$\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$	*Stress intensity* $\Delta\sigma_z$ *(kN/m²)*
0	0	1.0	71.62
± 1.0	± 0.5	0.572	40.97
± 2.0	± 1.0	0.177	12.68
± 3.0	± 1.5	0.052	3.72
± 4.0	± 2.0	0.018	1.28
± 5.0	± 2.5	0.007	0.50

(ii) *2:1 dispersion method*: Using eqn. (5.11),

$$\Delta\sigma_z = \frac{qBL}{(L+z)(B+z)} = \frac{(100)(2)(3)}{(2+2)(3+2)} = 30\ \text{kN/m}^2,$$

The distribution of stress intensities on the given plane, as obtained from Boussinesq's equation and 2 : 1 method, are shown in Fig. 5.7.

The following differences between the two stress distribution diagrams are to be noted :

(i) The maximum stress intensity obtained from Boussinesq's equation is 71.62 kN/m², which is nearly 2.4 times the maximum stress intensity given by 2 : 1 method.

(ii) In Boussinesq's method, the maximum stress intensity occurs directly below the point of application of the load. The stress intensity decreases rapidly with increasing radial distance, but except at an infinite distance, it never becomes zero. Whereas, the stress intensity given by 2 : 1 method remains constant over a radial distance of 2.5 m on either side of the centre line of the loaded area, and suddenly becomes non-existent beyond that limit.

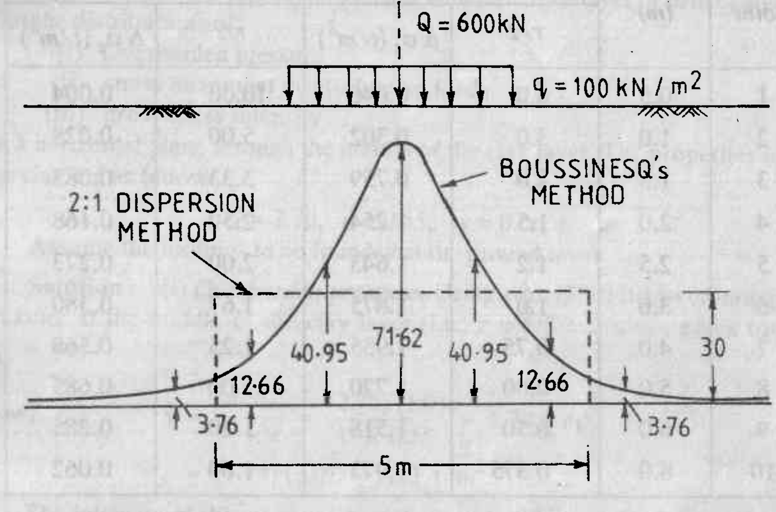


Fig. 5.7

(iii) Evidently, the pressure distribution given by Boussinesq's method is more logical and should be used in ordinary field problems.

Problem 5.3. A concentrated vertical load of 200 t is applied on the surface of a semi-infinite soil mass. Plot the distribution of vertical stress intensity on a vertical plane situated at a distance of (i) 3 m (ii) 5 m, from the line of action of the load.

Solution: In Fig. 5.8, let P be the point of application of the load. Let Y_1Y_1 and Y_2Y_2 be the given planes, located at radial distances of 3 m and 5 m respectively from P.

Using eqn. (5.4), the stress intensity at a depth z and radial distance r from the line of action of a 200 t load is,

$$\Delta\sigma_z = \frac{(3)(200)}{(2\pi)(z^2)}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} = \frac{95.49}{z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2} \quad ...(i)$$

The stress intensities at various points or the planes $Y_1\ Y_1$ and $Y_2\ Y_2$ may now be computed from eqn. (i). The results are shown in the following table.

No. of point	Depth (m)	Plane Y_1Y_1		Plane Y_2Y_2	
		r/z	$\Delta\sigma_z\ (t/m^2)$	r/z	$\Delta\sigma_z\ (t/m^2)$
1	0.5	6.0	0.046	10.00	0.004
2	1.0	3.0	0.302	5.00	0.028
3	1.5	2.0	0.759	3.33	0.083
4	2.0	1.5	1.254	2.50	0.168
5	2.5	1.2	1.643	2.00	0.273
6	3.0	1.0	1.875	1.67	0.380
7	4.0	0.75	1.955	1.25	0.568
8	5.0	0.60	1.770	1.00	0.685
9	6.0	0.50	1.518	1.20	0.285
10	8.0	0.375	1.073	1.60	0.062

The pressure distribution diagrams are shown in Fig. 5.8.

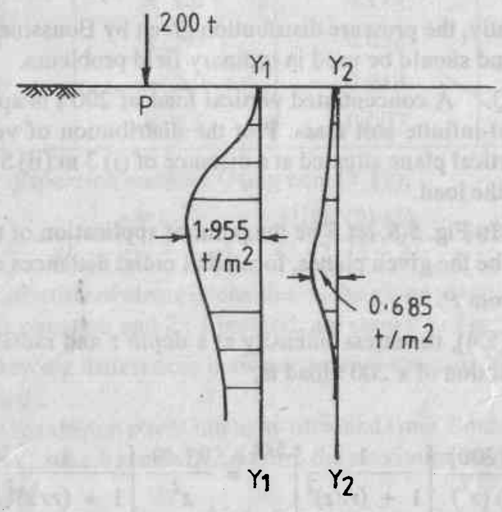


Fig. 5.8

Problem 5.4 The footings of three adjacent columns of a building lie on the same straight line and carry gross loads of 100 t, 150 t and 120 t respectively. The centre-to-centre distance between the first and second footing is 4 m while that between the second and the third is 3.5 m. The subsoil

consists of a 6 m thick clay layer which is underlain by a layer of dense sand. Plot the distributions of:

(i) Overburden pressure
(ii) stress increment due to footing loads
(iii) gross stress intensity

on a horizontal plane through the middle of the clay layer. The properties of the clay are as follows:

$$G = 2.70, \quad e = 0.55, \quad w = 0\ \%$$

Assume the footings to be founded at the ground level.

Solution : (i) *Overburden pressure*: Using eqn. (5.1), the overburden pressure at the middle of the clay layer (i.e., $z = 6/2 = 3$ m) is given by, $\sigma_{z_o} = \gamma z$

Now, $$\gamma = \frac{G \cdot \gamma_w}{1 + e} = \frac{(2.70)(1.0)}{1 + 0.55} = 1.74\ t/m^2$$

$\therefore$ $$\sigma_{z_o} = (1.74)(3.0) = 5.22\ t/m^2.$$

The intensity of this pressure over the horizontal plane XY through the middle of clay is constant.

(ii) In order to determine the stress increment due to footing loads at various points of XX, eqn. (5.4) can be used. The computed stresses are shown below:

No. of point	Vertical stress intensity due to : P1 (100 t)			P2 (150 t)			P3 (120 t)			$\Delta\sigma_z = \Delta\sigma_{z_1} + \Delta\sigma_{z_2} + \Delta\sigma_{z_3}$	σ_{z_o}	$\sigma_z = \sigma_{z_o} + \Delta\sigma_z$
	r (m)	r/z	$\Delta\sigma_{z_1}$ (t/m^2)	r (m)	r/z	$\Delta\sigma_{z_2}$ (t/m^2)	r (m)	r/z	$\Delta\sigma_{z_3}$ (t/m^2)	(t/m^2)	(t/m^2)	(t/m^2)
1	–2	–0.67	2.12	–6	–2.00	0.14	–9.5	–3.17	0.02	2.28	5.22	7.50
2	–1	–0.33	4.08	–5	–1.67	0.29	–8.5	–2.83	0.03	4.40	5.22	9.62
3	0	0	5.31	–4	–1.33	0.62	–7.5	–2.50	0.04	5.97	5.22	11.19
4	1	0.33	4.08	–3	–1.00	1.41	–6.5	–2.17	0.08	5.57	5.22	10.79
5	2	0.67	2.12	–2	–0.67	3.17	–5.5	–1.83	0.16	5.45	5.22	10.67
6	3	1.00	0.94	–1	–0.33	6.11	–4.5	–1.50	0.33	7.38	5.22	12.60
7	4	1.33	0.41	0	0	7.96	–3.5	–1.17	0.74	9.11	5.22	14.33
8	5	1.67	0.19	1	0.33	6.11	–2.5	–0.83	1.72	8.02	5.22	13.24
9	6	2.00	0.09	2	0.67	3.17	–1.5	–0.50	3.64	7.76	5.22	12.98
10	7	2.33	0.05	3	1.00	1.41	–0.5	–0.17	5.93	7.39	5.22	12.61
11	7.5	2.50	0.04	3.5	1.17	0.93	0	0	6.37	7.34	5.22	12.56
12	8.5	2.83	0.02	4.5	1.50	0.42	1	0.33	4.92	5.36	5.22	10.58
13	9.5	3.17	0.01	5.5	1.83	0.20	2	0.67	2.52	2.73	5.22	7.95

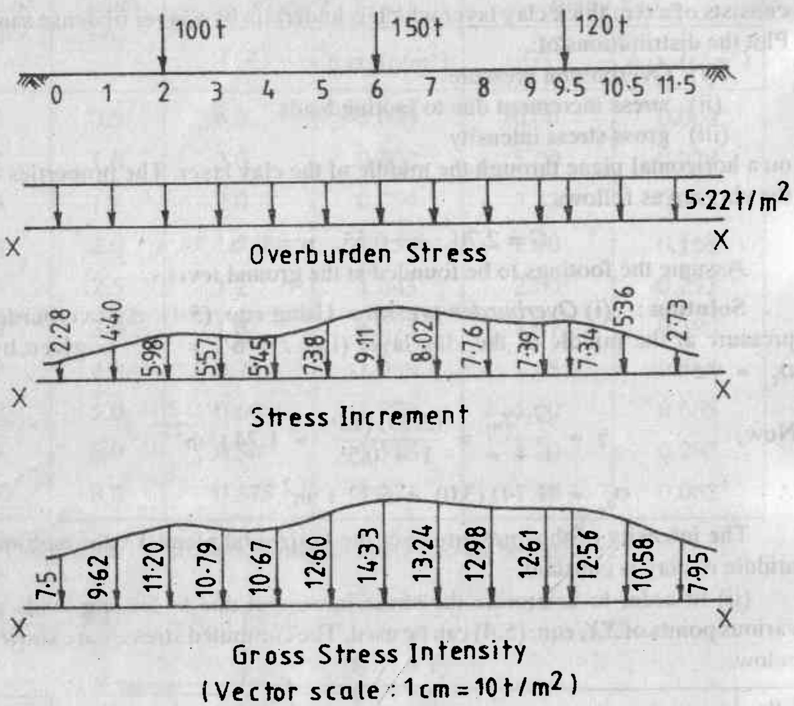


Fig. 5.9

(iii) *Total stress.* The total stress at any point is the sum of the over-burden pressure and the stress increment due to footing loads. These are tabulated in the last column of the above table.

The distribution of overburden pressure, stress increment and gross stress intensity are shown in Fig. 5.9.

Problem 5.5. Draw the influence lines for the vertical and shear stress intensities at a depth of 2.5 m below the ground level due to a unit vertical concentrated loads applied on the ground surface.

Solution: We have from eqns. (5.6) and (5.7),

$$\Delta \sigma_z = \frac{3Q}{2\pi z^2}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

and $$\Delta \tau_{rz} = \frac{3Qr}{2\pi z^3}\left[\frac{1}{1+(r/z)^2}\right]^{5/2}$$

Here, $Q = 1$ t, $z = 2.5$ m

$$\therefore \Delta \sigma_z = \frac{(3)(1)}{(2\pi)(2.5)^2}\left[\frac{1}{1+\dfrac{r^2}{(2.5)^2}}\right]^{5/2} = \frac{0.0764}{\left(1+0.16\,r^2\right)^{5/2}}$$

and, $$\Delta \tau_{rz} = \frac{(3)(1)(r)}{(2\pi)(2.5)^3}\left[\frac{1}{1+\dfrac{r^2}{(2.5)^2}}\right] = \frac{0.0306\,r}{\left(1+0.16\,r^2\right)^{5/2}}$$

The computed values of $\Delta \sigma_z$ and $\Delta \tau_{rz}$ at various points on the given horizontal plane are shown below:

No.	*r (m)*	$\Delta \sigma_z$ (t/m^2)	$\Delta \tau_{rz}$ (t/m^2)	*No.*	*r (m)*	$\Delta \sigma_z$ (t/m^2)	$\Delta \tau_{rz}$ (t/m^2)
1.	0	0.0764	0	8.	± 4.0	0.0032	± 0.0051
2.	± 0.25	0.0745	± 0.0075	9.	± 5.0	0.0014	± 0.0027
3.	± 0.50	0.0693	± 0.0139	10.	± 6.0	0.0006	± 0.0015
4.	± 1.0	0.0527	± 0.0211	11.	± 7.0	0.0003	± 0.0009
5.	± 1.5	0.0354	± 0.0213	12.	± 8.0	0.0002	± 0.0006
6.	± 2.0	0.0222	± 0.0178	13.	± 9.0	0.0001	± 0.0004
7.	± 3.0	0.0082	± 0.0099				

The distribution of $\Delta \sigma_z$ and $\Delta \tau_{rz}$ are shown in Fig. 5.10.

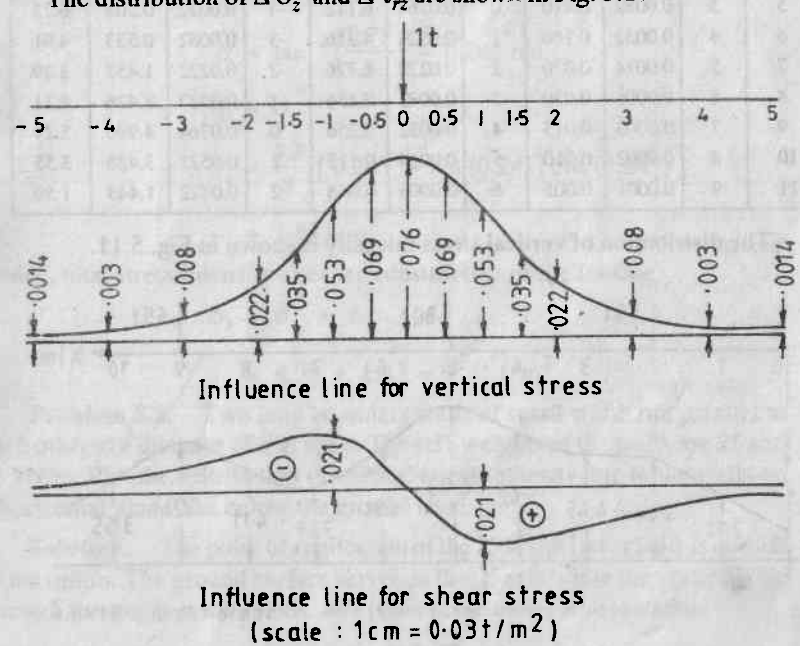


Fig. 5.10

Problem 5.6. Using the influence line plotted in Fig. 5.10 plot the distribution of vertical stress intensity on a horizontal plane through the middle of a 5 m thick clay layer due to the loading scheme shown in Fig. 5.11.

Solution: At any point on the given plane, the combined stress intensity can be obtained by summing up the stress intensities due to the individual loads, which, in turn, may be determined by the process explained in Art. 5.14.

The following table shows the computed values.

Dist. from the origin	Vertical stress intensity due to :-									
	P_1 (50 t)			P_2 (80 t)			P_3 (65 t)			
	Dist.	Ordinate of ILD	$\Delta\sigma_{z_1}$	Dist.	Ordinate of ILD	$\Delta\sigma_{z_2}$	Dist.	Ordinate of ILD	$\Delta\sigma_{z_3}$	$\Sigma\Delta\sigma_z$
(m)	(m)	(t/m²)	(t/m²)	(m)	(t/m²)	(t/m²)	(m)	(t/m²)	(t/m²)	(t/m²)
0	–2	0.0222	1.110	–5	0.0014	0.112	–9	0.0001	0.006	1.23
1	–1	0.0527	2.635	–4	0.0032	0.256	–8	0.0002	0.013	2.91
2	0	0.0764	3.720	–3	0.0082	0.656	–7	0.0003	0.019	4.40
3	1	0.0527	2.635	–2	0.0222	1.776	–6	0.0006	0.039	4.45
4	2	0.0222	1.110	–1	0.0527	4.216	–5	0.0014	0.091	5.42
5	3	0.0082	0.410	0	0.0764	6.112	–4	0.0032	0.208	6.73
6	4	0.0032	0.160	1	0.0527	4.216	–3	0.0082	0.533	4.91
7	5	0.0014	0.070	2	0.0222	1.776	–2	0.0222	1.433	3.29
8	6	0.0006	0.030	3	0.0082	0.656	–1	0.0527	3.426	4.11
9	7	0.0003	0.015	4	0.0032	0.256	0	0.0764	4.996	5.24
10	8	0.0002	0.010	5	0.0014	0.112	1	0.0527	3.426	3.55
11	9	0.0001	0.005	6	0.0006	0.048	2	0.0222	1.443	1.50

The distribution of vertical stress intensity is shown in Fig. 5.11.

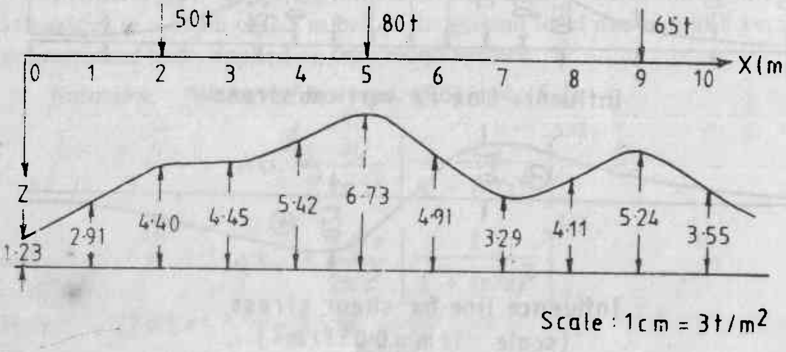


Fig. 5.11

Problem 5.7. It is proposed to construct a strip footing of 1.5 m width to carry a load of 12 t per metre run. The footing is to be placed at the ground level over a homogeneous deposit of sand having the following properties:

$$G = 2.65,\ e = 0.65,\ s = 10\%$$

Determine the vertical stress intensity at a depth of 3 m below the centre line of the footing, before and after its construction.

Solution: The bulk density of the sand,

$$\gamma = \frac{G + se}{1 + e} \cdot \gamma_w = \frac{2.68 + (0.10)(0.65)}{1 + 0.65}(1.0) = 1.66 \text{ t/m}^3$$

Before the construction of the footing, stress intensity at a depth of 3 m below the centre line of the footing is given by,

$$\sigma_{z_o} = \gamma z$$

$$= (1.66)(3.0) = 4.98 \text{ t/m}^2.$$

Stress increment at the same level due to the construction of the footing may be determined using eqn. (5.19).

$$\Delta\sigma_z = \frac{q}{\pi}[\alpha + \sin\alpha\cos(\alpha + 2\beta)]$$

The maximum stress intensity will occur directly below the centre line of the strip load. With reference to Fig. 5.3 (b).

$$\alpha = \tan^{-1}\left(\frac{0.75}{3}\right) = 0.245 \text{ rad.}$$

and $$\beta = 0$$

$\therefore$ $$\Delta\sigma_z = \frac{12}{\pi}[0.245 + \sin(0.245)\cos(0.245)]$$

$$= 1.83 \text{ t/m}^2$$

Hence, total stress intensity after the construction of the footing,

$$\sigma_z = \sigma_{z_0} + \Delta\sigma_z$$

$$= 4.98 + 1.83 = 6.81 \text{ t/m}^2$$

Problem 5.8. Two long boundary walls of small width run parallel to each other at a distance of 3 m apart. The self-weights of the walls are 25 and 15 kN/m. Plot the distribution of vertical stress intensity due to the walls on a horizontal plane 3 m below the ground level.

Solution: The point of application of the 25 kN/m linear load is chosen as the origin. The ground surface serves as the *Y*-axis while the vertical axis through the origin is the *Z*-axis. *MN* is the plane under consideration.

Using eqn. (5.18), the vertical stress intensity due to a line load q is given by:

$$\Delta \sigma_z = \frac{2q}{\pi z} \cdot \left[\frac{1}{1 + (r/z)^2} \right]^2$$

The stress intensities at various points on the plane are computed and are presented below in a tabular form:

No.	*Dist. from origin*	*Stress due to 25 kN/m load*			*Stress due to 15 kN/M load*			*Total stress* $\Delta\sigma_{z1} + \Delta\sigma_{z2}$
		r (m)	*r/z*	$\Delta\sigma_{z1}$ *(kN/m²)*	*r (m)*	*r/z*	$\Delta\sigma_{z2}$ *(kN/m²)*	*(kN/m²)*
1.	–2	–2	–0.67	2.53	–5	–1.67	0.22	2.75
2.	–1	–1	–0.33	4.31	–4	–1.33	0.42	4.73
3.	0	0	0	5.31	–3	–1.00	0.79	6.10
4.	1	1	0.33	4.31	–2	–0.67	1.52	5.83
5.	2	2	0.67	2.53	–1	–0.33	2.56	5.09
6.	3	3	1.00	1.33	0	0	3.18	4.51
7.	4	4	1.33	0.69	1	0.33	2.56	3.25
8.	5	5	1.67	0.37	2	0.67	1.52	1.89

The distribution of vertical stress is shown in Fig. 5.12.

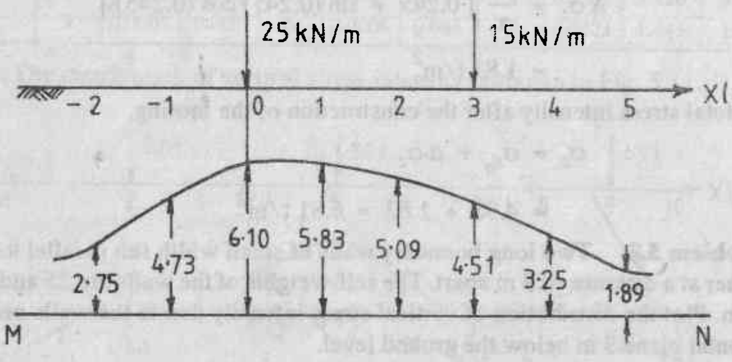


Fig. 5.12

Problem 5.9. A long flexible strip footing of 2.5 m width having a smooth base, is subjected to a uniformly distributed load of 80 kN/m run. Determine the vertical stress intensities at a depth of 2 m below:

(i) centre line of the footing

(ii) side face of the footing

(iii) a line parallel to the centre line of the footing at a distance of 3 m from it.

Solution: The cross-section of the given footing is presented in Fig. 5.12. The locations at which the stresses are to be determined are represented by the points A, B and C respectively.

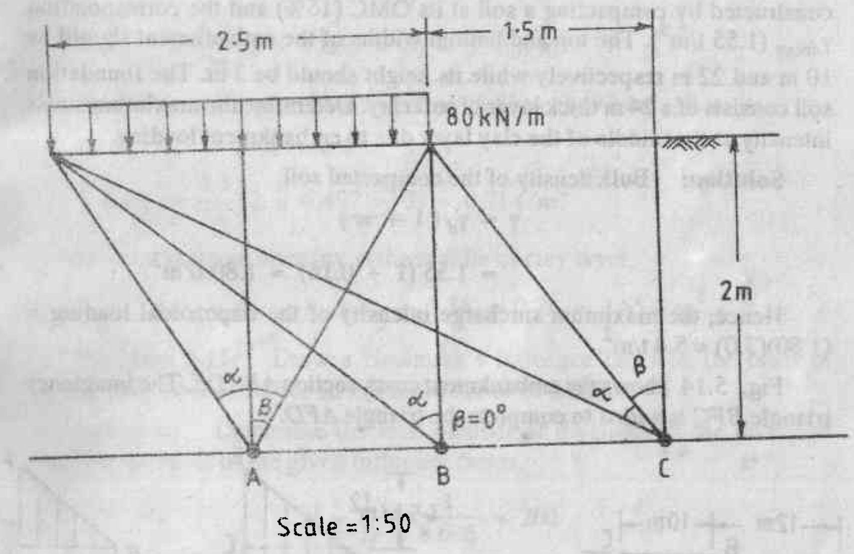


Fig. 5.13

(i) *Point A:* By measurement,

$$\alpha = 64° = 1.117 \text{ radian}$$

$$\beta = -32° = -0.558 \text{ radian}$$

$\therefore$ using eqn. (5.19),

$$\Delta \sigma_{z\,(A)} = \frac{80}{\pi} [1.117 + \sin(1.117) \cos\{1.117 + 2(-0.558)\}]$$

$$= 51.33 \text{ kN/m}^2.$$

(ii) *Point B:* Here, $\alpha = 51.5° = 0.899$ radian.

$\beta = 0°$

$$\therefore \quad \Delta\sigma_{z\,(B)} = \frac{80}{\pi}\,[\,0.899 + \sin(0.899)\cos(0.899 + 0)\,]$$

$$= 23.96 \text{ kN/m}^2.$$

(iii) *Point C:* $\alpha = 26° = 0.454$ radian.

$\beta = 36.5° = 0.637$ radian.

$$\therefore \quad \Delta\sigma_{z\,(C)} = \frac{80}{\pi}\,[\,0.454 + \sin(0.454)\cos(0.454 + 2 \times 0.637)\,]$$

$$= 9.81 \text{ kN/m}^2.$$

Problem 5.10. An embankment of trapezoidal cross-section is to be constructed by compacting a soil at its OMC (16%) and the corresponding γ_{dmax} (1.55 t/m^3). The top and bottom widths of the embankment should be 10 m and 22 m respectively while its height should be 3 m. The foundation soil consists of a 24 m thick layer of soft clay. Determine the maximum stress intensity at the middle of the clay layer due to embankment loading.

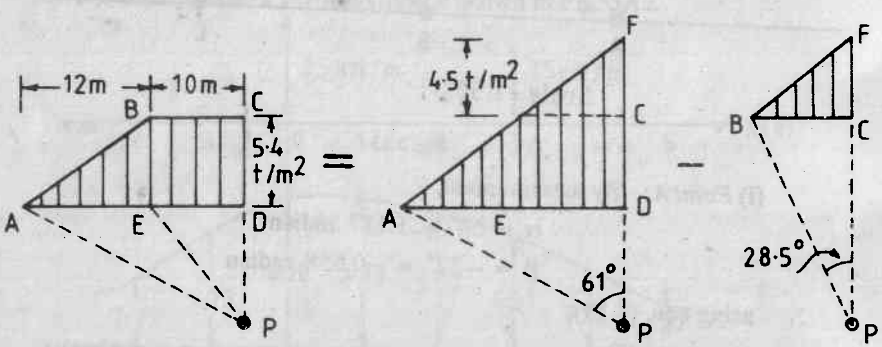
Solution: Bulk density of the compacted soil,

$$\gamma = \gamma_d\,(1 + w)$$

$$= 1.55\,(1 + 0.16) = 1.80 \text{ t/m}^3.$$

Hence, the maximum surcharge intensity of the trapezoidal loading = (1.80)(3.0) = 5.4 t/m^2.

Fig. 5.14 shows the embankment cross-section *ABCDE*. The imaginary triangle *BFC* is added to complete the triangle *AFD*.



Linear scale : 1:75 , Vector scale : 1cm = 0·2 t/m²

Fig. 5.14

Now, $\dfrac{FC}{BE} = \dfrac{BC}{AE}$,

or, $FC = \dfrac{BE \cdot BC}{AE} = \dfrac{(5.4)(2.5)}{3} = 4.5$ t/m.

$\therefore \quad FD = FC + BE = 4.5 + 5.4 = 9.9$ t/m.

Using eqn. (5.20), stress intensity due to triangular loading is given by,

$$\Delta\sigma_z = \frac{q}{2\pi}\left(\frac{y}{b}\cdot\alpha - \sin 2\beta\right).$$

For $\Delta\,AFD$, $q = 9.9$ t/m, $\alpha = 61° = 1.065$ rad., $\beta = 0$

$y = 2b$, i.e., $y/b = 2$.

$$\therefore \quad \Delta\sigma_{z_1} = \frac{9.9}{2\pi}(2 \times 1.065 - 0) = 3.36 \text{ t/m}^2.$$

For $\Delta\,BFC$, $q = 4.5$ t/m, $\alpha = 28.5° = 0.497$ rad., $\beta = 0$

$y/b = 2$

$$\therefore \quad \Delta\sigma_{z_2} = \frac{4.5}{2\pi}(2 \times 0.497 - 0) = 0.71 \text{ t/m}^2$$

$\therefore$ Net vertical stress intensity at the middle of clay layer,

$$\Delta\sigma_z = \Delta\sigma_{z_1} - \Delta\sigma_{z_2} = 3.36 - 0.71 = 2.65 \text{ t/m}^2.$$

Problem 5.11. Draw a Newmark's influence chart on the basis of Boussinesq's equation, for an influence factor of 0.005.

Solution: Determine the total number of divisions in the proposed chart on the basis of the given influence factor.

$$N = \frac{1}{I_f} = \frac{1}{0.005} = 200$$

If 10 concentric circles of appropriate radii are drawn, the area under the chart will be divided into 10 parts. It will then be further divided into smaller segments by drawing a number of radial lines. Evidently, the number of radial lines to be drawn = $\dfrac{200}{10} = 20$.

The angle between two consecutive radial lines = $\dfrac{360°}{20} = 18°$

Now, using eqn. (5.12), the vertical stress at a depth z below the centre of a circular area of radius r, carrying a uniformly distributed load q is given by:

$$\Delta\sigma_z = q\left[1 - \left\{\frac{1}{1 + (r/z)^2}\right\}^{3/2}\right]$$

or, $$\frac{\Delta\sigma_z}{q} = 1 - \left\{\frac{1}{1 + (r/z)^2}\right\}^{3/2}$$

or, $$\frac{1}{1 + (r/z)^2} = \left(1 - \frac{\Delta\sigma_z}{q}\right)^{2/3}$$

or, $$1 + (r/z)^2 = \left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3}$$

or, $$\frac{r}{z} = \sqrt{\left(1 - \frac{\Delta\sigma_z}{q}\right)^{-2/3} - 1} \qquad \text{...(5.21)}$$

For different values of $\Delta\sigma_z/q$ (ranging from 0.00 to 1.00 at the rate of 0.10) the corresponding values of r/z can be determined from eqn. (5.21). Consequently, the value of r can be obtained if z is known. While drawing the chart, we will arbitrarily take z = 2.5 cm. The computations are given below in a tabular form:

Circle No.	$\Delta\sigma_z/q$	r/z	z *(cm)*	r *(cm)*	*Circle No.*	$\Delta\sigma_z/q$	r/z	z *(cm)*	r *(cm)*
0	0.0	0.000	2.5	0.00	6	0.6	0.918	2.5	2.30
1	0.1	0.270	2.5	0.68	7	0.7	1.110	2.5	2.78
2	0.2	0.401	2.5	1.00	8	0.8	1.388	2.5	3.47
3	0.3	0.518	2.5	1.30	9	0.9	1.909	2.5	4.77
4	0.4	0.637	2.5	1.59	10	1.0	∞	2.5	∞
5	0.5	0.767	2.5	1.92					

Nine concentric circles are drawn with the radii shown in the table. A number of radial lines are then drawn from the centre at equal deflection angles of 18°.

The resulting Newmark's chart is shown in Fig. 5.15

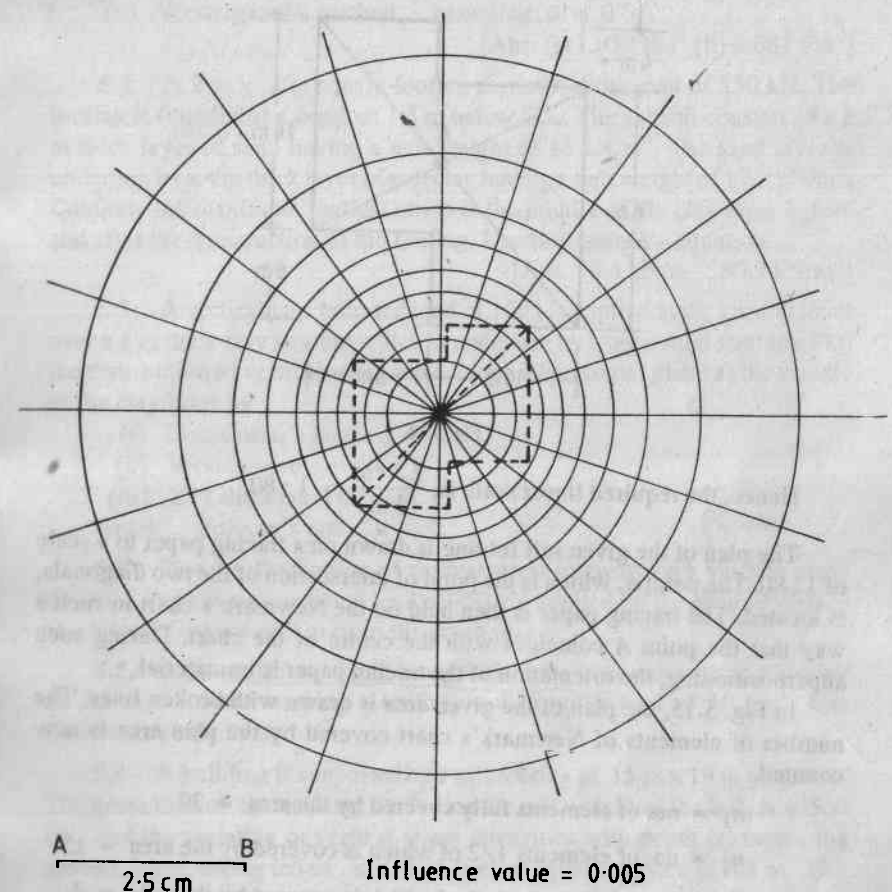


Fig. 5.15

Problem 5.12. Using the Newmark's chart prepared in Problem 5.11, determine the vertical stress intensity at a depth of 2 m below the point A of the raft footing shown in Fig. 5.16. The uniformly distributed load on the raft is 8.5 t/m^2.

Solution: In Problem 5.11 the Newmark's chart was prepared for z = 2.5 cm. In order to use this chart for the computation of vertical stress intensity at a depth of 2 m below any loaded area, the plan of the area is to be drawn in such a scale that a distance of 2.5 cm in the drawing may represent an actual distance of 2 m.

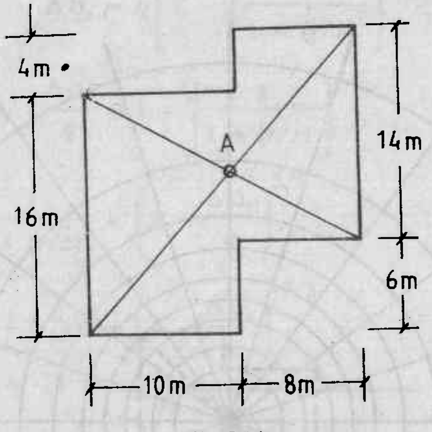


Fig. 5.16

Hence, the required linear scale is, $\frac{2.5 \text{ cm}}{2 \text{ m}} = 1 : 80$.

The plan of the given raft footing is drawn on a tracing paper to a scale of 1 : 80. The point *A*, which is the point of intersection of the two diagonals, is located. The tracing paper is then held on the Newmark's chart in such a way that the point *A* coincides with the centre of the chart. During such superpositioning, the orientation of the tracing paper is immaterial.

In Fig. 5.15, the plan of the given area is drawn with broken lines. The number of elements of Newmark's chart covered by the plan area is now counted.

n_1 = no. of elements fully covered by the area = 39

n_2 = no. of elements 1/2 of which is covered by the area = 15

n_3 = no. of elements 1/3 of which is covered by the area = 4

The stress intensity at a depth of 2 m below *A* is then given by,

$$\Delta \sigma_z = i_f \times n \times q$$

$$= (0.005)(39 + 15/2 + 4/3)(8.5)$$

$$= 2.03 \text{ t/m}^2$$

EXERCISE 5

5.1 A vertical concentrated load of 50 t is applied on the ground surface. Compute the vertical stress intensity at a point 3 m below the ground level and 2 m away from the line of action of the load by:

(i) Boussinesq's method
(ii) Westergaard's method, assuming $\mu = 0$

[Ans. (i) 1.058 t/m^2 (ii) 0.681 t/m^2]

5.2 A 2 m × 2 m square footing carries a gross load of 550 kN. The footing is founded at a depth of 1.5 m below G.L. The subsoil consists of a 2 m thick layer of sand having a unit weight of 18 kN/m^3. The sand layer is underlain by a 4 m thick layer of soft clay having a unit weight of 17.2 kN/m^3. Compute the maximum vertical stress at the middle of the clay layer before and after the construction of the footing. Use Boussinesq's equation.

[Ans. 70.4 kN/m^2 ; 80.9 kN/m^2]

5.3 A vertical concentrated load of 100 t is applied at the ground level over a 6 m thick clay stratum which is underlain by a deep sand stratum. Plot the distribution of vertical stress intensity on a horizontal plane at the middle of the clay layer by :

(i) Boussinesq's method.
(ii) Westergaard's method.
(iii) 2 : 1 dispersion method.

Given, Poisson's ratio, $\mu = 0$.

5.4 Plot the distribution of vertical stress intensity on a vertical plane due to a vertical concentrated load of 750 kN applied on the ground surface at a lateral distance of 3 m from the given plane.

5.5 Draw the isobars for 25% and 10% stress intensities due to a footing of 2.5 m × 2.5 m, carrying a uniformly distributed load of 10 t/m^2. Use Boussinesq's method.

5.6 A building is supported by a raft footing of 15 m × 18 m plan area. The gross load of the building, including the self weight of the raft, is 40500 kN. Plot the variation of vertical stress intensities with depth (z) below the ground level, taking $0.5 \text{ m} \le z \le 5.0$ m, at equal intervals of 0.5 m. Use 2 : 1 dispersion method.

5.7 Two adjacent footings of building, placed at a centre-to-centre distance of 4.5 m, have to carry gross loads of 750 kN each. Using Boussinesq's theory, plot the distribution of vertical stress intensity at a depth of 3 m below the base of the footings.

5.8 Three consecutive footings of a building are carrying gross loads of 80 t, 120 t and 110 t respectively. The centre-to-centre distance between the first and second footing is 3.0 m, while that between the second and third footing is 4.0 m. All the footings are founded at 1.5 m below G.L. Determine the maximum vertical stress intensity due to the footing loads at a depth of 3.5 m below G.L.

[Ans. 15.06 t/m^2]

5.9 (a) Draw the influence line for the vertical stress intensity at a depth of 2.0 m below the point of application of a unit load.

(b) Solve Problem 8 using the influence line thus drawn.

5.10 A strip footing of 2 m width carries a uniform load of 8 t/m^2. The footing is placed on the ground level over a homogeneous deposit of clay having the following properties :

$G = 2.72$, $e = 0.78$, $w = 12\%$.

Determine the initial and final overburden pressure at a depth of 3 m below the centre of the footing. [Ans. 5.49 t/m^2 ; 7.11 t/m^2]

5.11 Two long boundary walls run parallel to each other at a centre-to-centre distance of 1.5 m apart. The width and height of the first wall are 250 mm and 2000 mm respectively, while those of the second are respectively 125 mm and 3000 mm. Plot the distribution of vertical stress intensity due to the walls on a horizontal plane, 2 m below G.L. The walls have negligible depth of foundation and are made of brick masonry having a unit weight of 1920 kg/m^3.

5.12 Fig. 5.16. shows the cross-section of an earth dam. The unit weight of the earth-fill is 1.85 t/m^3. Determine the maximum stress intensity at a depth of 5 m below the base of the dam.

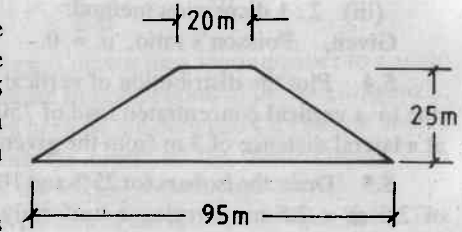


Fig. 5.17

5.13 Draw a Newmark's influence chart, on the basis of Boussinesq's equation, for an influence factor of 0.00556.

5.14 The plan of a raft footing supporting a multistoried building is shown in Fig. 5.18. The raft carries a u.d.l. of 15 t/m^2. Using the Newmark's chart given in Fig. 5.14, determine the vertical stress intensity at a depth of 3 m below point *A*.

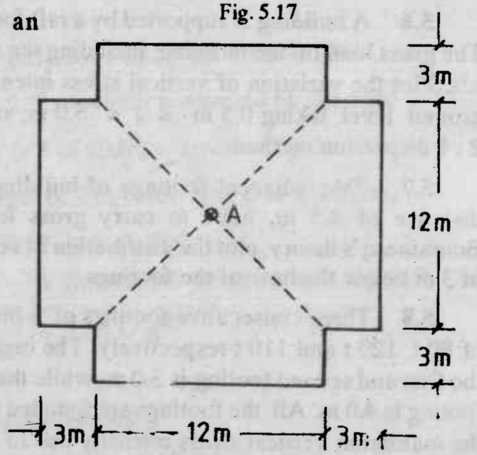


Fig. 5.18

6

CONSOLIDATION

6.1 Introduction: When an external static load is applied on a saturated soil mass, an excess pore water pressure is developed. As water is incompressible for the low stress ranges commonly encountered in foundation problems, this pore water now tries to escape from the void spaces. Such expulsion of water results in a decrease in the void ratio and, consequently, a reduction in the volume of the soil mass. This process is known as consolidation.

Consolidation is essentially a time-dependent process. In coarse-grained soils having high co-efficient of permeability the pore water escapes very rapidly. The time-dependent volume change of the soil mass, therefore, occurs only in less permeable fine-grained soils like clay and silt.

6.2 Definitions: The following terms are frequently used to express the compressibility characteristics of soils:

(i) *Co-efficient of compressibility (a_v):* It is defined as the change in void ratio per unit change in pressure.

i.e.,
$$a_v = -\frac{\Delta e}{\Delta p} = \frac{e_0 - e}{\sigma - \sigma_0} \qquad ...(6.1)$$

where, e_0 and e are the void ratios of a soil under the initial and final vertical stresses σ_0 and σ respectively.

(ii) *Co-efficient of volume change or volume compressibility (m_v):* It is defined as the change in volume of a soil mass per unit of its original volume due to unit change in pressure.

i.e.,
$$m_v = \frac{\Delta V}{V} \cdot \frac{1}{\Delta \sigma} \qquad ...(6.2)$$

Fig. 6.1 shows a soil mass having an initial void ratio e_0 . If the volume of solids be unity, then volume of voids is given by,

$$V_v = e_0 \cdot V_s = e_0 \cdot 1 = e_0$$

$\therefore$ Total volume, $V_0 = V_v + V_s$

$$= 1 + e_0$$

If the void ratio now decreases to e due to increase in pressure, then

$$V_1 = 1 + e.$$

or, change in volume $\Delta V = V_0 - V_1$

$$= 1 + e_0 - (1 + e)$$

$$= e_0 - e$$

$$= \Delta e.$$

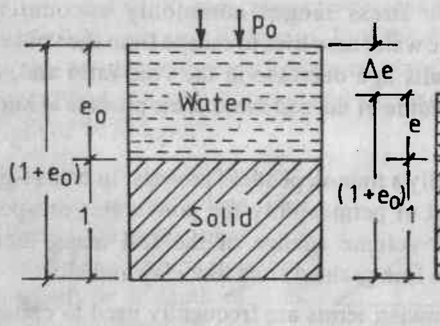


Fig. 6.1

From eqn. (6.2) we get,

$$m_v = \frac{\Delta e}{1 + e_0} \cdot \frac{1}{\Delta \sigma}$$

or,
$$m_v = \frac{\Delta e}{\Delta \sigma} \cdot \frac{1}{1 + e_0}$$

or,
$$m_v = \frac{a_v}{1 + e_0} \quad \text{...(6.3)}$$

The unit of both a_v and m_v is cm^2/kg.

(iii) *Compression index* (C_c): It is defined as the gradient of the virgin compression curve drawn from the results of a consolidation test performed on a soil.

Fig. 6.2. illustrates an e vs. $\log p$ curve.

By definition, C_c = gradient of AB

$$= \tan \theta$$

$$= \frac{AC}{BC}$$

But, $AC = e_0 - e = \Delta e$

and $BC = \log_{10} p - \log_{10} p_0$

$$\therefore \quad C_c = \frac{e_0 - e}{\log_{10} p - \log_{10} p_0} = \frac{\Delta e}{\log_{10} p/p_0} \quad \text{...(6.4)}$$

The value of the compression index may also be determined from the following empirical formulae:

For normally consolidated clays (sensitivity ≤ 4),

$$C_c = 0.009 (w_l - 10) \quad \text{...(6.5a)}$$

For remoulded clays, $C_c = 0.007 (w_l - 10)$ ...(6.5b)

where, w_l = liquid limit (%)

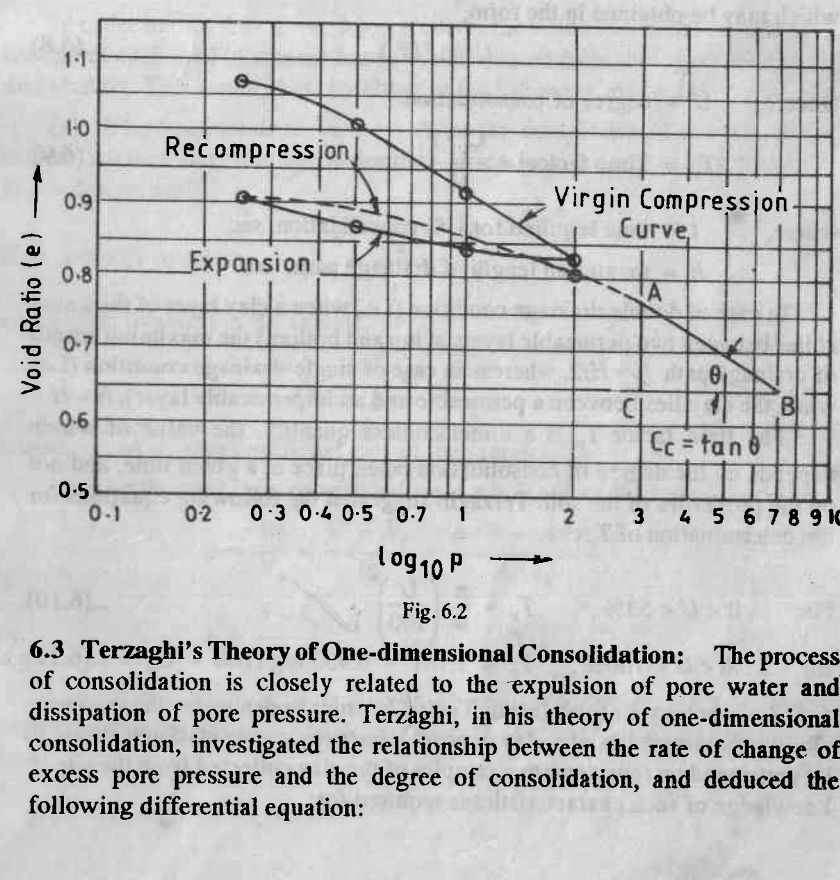


Fig. 6.2

6.3 Terzaghi's Theory of One-dimensional Consolidation: The process of consolidation is closely related to the expulsion of pore water and dissipation of pore pressure. Terzaghi, in his theory of one-dimensional consolidation, investigated the relationship between the rate of change of excess pore pressure and the degree of consolidation, and deduced the following differential equation:

$$\frac{\partial u}{\partial t} = C_v \cdot \frac{\partial^2 u}{\partial z^2} \qquad ...(6.6)$$

where, u stand for the excess pore pressure at a depth z

t stands for the time elapsed after the application of the load.

C_v = Co-efficient of consolidation, which is defined as:

$$C_v = \frac{k}{m_v \gamma_w} \qquad ...(6.7)$$

where, k = co-efficient of permeability, cm/sec

γ_w = unit weight of water, gm/cc

m_v = coefficient of volume change, cm^2/gm

The unit of C_v is cm^2/sec.

Equation (6.6) is a second order differential equation, the solution of which may be obtained in the form,

$$U = f(T_v) \qquad ...(6.8)$$

where, U = degree of consolidation.

$$T_v = \text{Time factor} = \frac{C_v \cdot t}{h^2} \qquad ...(6.9)$$

where, t = time required for U% consolidation, sec

h = maximum length of drainage path, cm.

In case of double drainage condition (*i.e.*, when a clay layer of thickness H lies between two permeable layers at top and bottom) the maximum length of drainage path $h = H/2$, whereas in case of single-drainage condition (*i.e.*, when the clay lies between a permeable and an impermeable layer), $h = H$.

The time factor T_v is a dimensionless quantity, the value of which depends on the degree of consolidation taken place at a given time, and not on the properties of the soil. Terzaghi suggested the following equations for the determination of T_v:

For $0 < U < 53\%$, $\quad T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 \qquad ...(6.10)$

for $53\% < U < 100\%$, $\quad T_v = 1.781 - 0.933 \log(100 - U) \qquad ...(6.11)$

6.4 Laboratory Consolidation Test: In order to determine the compressibility characteristics of a clay deposit, laboratory consolidation tests are to be performed on representative samples of the clay collected from the site. A knowledge of such characteristics is required for:

(i) Estimating the probable consolidation settlement of a proposed structure to be constructed on this soil.

(ii) To determine the time-rate of settlement.

The sample is placed in an oedometer between two porous stones and arrangements are made to keep the sample saturated throughout the test. The loading intensities are generally applied in the following order:

0.25, 0.5, 1.0, 2.0, 4.0, 8.0 and 16.0 kg/cm^2.

The vertical deformations of the sample under each loading intensity are measured with the help of a dial gauge. The readings are taken at elapsed times of :

0.25, 0.5, 1, 2, 4, 8, 15, 30, 60, 120, 240 and 1440 minutes.

From the results of the test, the following three curves are drawn:

(i) e vs. $\log_{10} p$ curve, to determine the value of C_c

(ii) Dial reading vs. $\log_{10} t$ curve
(iii) Dial reading vs. $\sqrt{t}$ curve
} to determine the value of C_v

In order to plot the e vs. $\log_{10} p$ curve, the void ratio of the sample at the end of each load increment has to be determined from the corresponding dial reading. This can be done by either of the following methods.

(a) *Height of solids method :* After the completion of the test, the sample is taken out from the oedometer, dried in oven and its dry weight W_d is determined.

Now, volume of solids, $\quad V_s = \frac{W_d}{\gamma_s} = \frac{W_d}{G\gamma_w}$

and, height of solids in the sample,

$$h_s = \frac{V_s}{A} = \frac{W_d}{G\gamma_w A} \qquad ...(6.12)$$

where, A = cross-sectional area of the sample.

Let e be the void ratio corresponding to a height h of the sample.

$$\therefore \quad e = \frac{V_v}{V_s} = \frac{V - V_s}{V_s} = \frac{A \cdot h - A \cdot h_s}{A \cdot h_s}$$

$$\text{or,} \quad e = \frac{h - h_s}{h_s} \qquad ...(6.13)$$

Thus if the value of h is known at any time during the test, the corresponding void ratio can be determined. The value of h may be obtained from:

$$h = H - R \cdot C$$

where, H = initial height of sample.

R = dial reading.

C = dial gauge constant

(b) *change in void ratio method :* With reference to Fig. 6.3, let,

V_1 = initial volume of the sample.

V_2 = volume of the sample at the end of compression under a loading intensity p

$\therefore \quad \Delta V$ = change in volume $= V_1 - V_2$

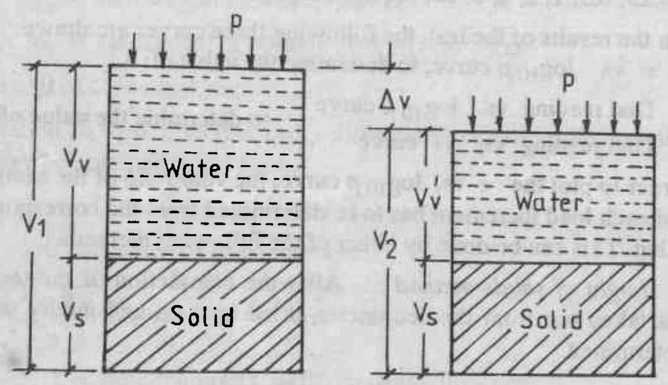


Fig. 6.3

Now $\quad V_1 = h_1 \cdot A$ and $V_2 = h_2 \cdot A$

$$\therefore \quad \frac{\Delta V}{V} = \frac{h_1 A - h_2 A}{h_1 A} = \frac{h_1 - h_2}{h_1} = \frac{\Delta h}{h_1}$$

If e_1 and e_2 be the void ratio corresponding to volume V_1 and V_2,

then, $\quad V_1 = V_v + V_s = e_1 V_s + V_s \quad [\because e_1 = V_v/V_s]$

$$= V_s(1 + e_1)$$

Similarly, $\quad V_2 = V_s(1 + e_2)$

$$\therefore \quad \Delta V = V_s(1 + e_1) - V_s(1 + e_2) = V_s(e_1 - e_2)$$

Therefore, $$\frac{\Delta h}{h} = \frac{\Delta V}{V} = \frac{V_s(e_1 - e_2)}{V_s(1 + e_1)}$$

or, $$\frac{\Delta h}{h} = \frac{\Delta e}{1 + e_1}$$

or, $$\Delta e = \frac{(1 + e_1)\Delta h}{h} \qquad ...(6.14)$$

Thus, knowing the values of e_1 and h, the change in void ratio at any given instant can be determined if the corresponding value of Δh is known.

6.5 Determination of C_v : For a given soil, the value of C_v is not constant but depends on the magnitude of the applied stress. In order to determine the degree of consolidation of clay layer under an external load, it is required to determine the initial and final pressures (σ_z and $\sigma_z + \Delta\sigma_z$ respectively) on the soil. If, for example, the initial and final pressure before and after the application of external load be 1 kg/cm^2 and 2 kg/cm^2, then the value of C_v must be obtained from this particular range of loading in the consolidation test.

The value of C_v may be determined from either of the following methods:

(a) Square root of time fitting method: $$C_v = \frac{(T_v)_{90} \times h^2}{t_{90}} \qquad ...(6.15)$$

(b) Logarithm of time fitting method: $$C_v = \frac{(T_v)_{50} \times h^2}{t_{50}} \qquad ...(6.16)$$

6.6 Computation of Settlement: The total settlement, S, of a footing is given by,

$$S = S_i + S_c + S_s \qquad ...(6.17)$$

where, S_i = immediate settlement

S_c = primary consolidation settlement

S_s = secondary consolidation settlement.

The secondary consolidation settlement is of importance only in case of highly organic soils and peats.

6.6.1 ***Immediate Settlement:*** The immediate settlement due to a vertical concentrated load Q at a depth z and radial distance r is given by,

$$S_i = \frac{Q}{2\pi E}\left[\frac{(1 + \mu) z^2}{\left(r^2 + z^2\right)^{3/2}} + \frac{2(1 - \mu^2)}{\left(r^2 + z^2\right)^{1/2}}\right] \qquad ...(6.18)$$

The immediate settlement due to a uniformly loaded area is given by,

$$S_i = q B . \frac{(1 - \mu^2)}{E} . I_f \qquad ...(6.19)$$

where, q = intensity of contact pressure

B = least lateral dimension of loaded area

μ = Poisson's ratio of soil

E = modulus of elasticity of soil

I_f = Influence factor, the value of which depends on:

(i) Type of the footing (i.e., whether it is rigid or flexible)
(ii) Shape of the footing
(iii) The location of the point below which settlement is required (i.e., the centre, corner or any other point of the footing)
(iv) Length to breadth ratio of the footing.

The value of I_f may be obtained from Table 6.1, while Table 6.2 gives the elastic properties of various soils.

Table 6.1 : Influence factors of various footings

Shape of loaded area	*Influence factor*			
	Flexible footings			*Rigid footings*
	Centre	*Corner*	*Average*	
Square	1.12	0.56	0.95	0.82
Circular	1.00	0.64	0.85	0.79
Rectangular:				
L/B = 1.5	1.36	0.68	1.20	1.06
L/B = 2.0	1.52	0.76	1.30	1.20
L/B = 2.5	2.10	1.05	1.83	1.70
L/B = 5.0	2.54	1.27	2.20	2.10
L/B = 10.0	3.38	1.69	2.96	3.40

Table 6.2 : Elastic properties of various soils

Type of soil	*Properties of soil*	*Void ratio*		
		0.41 to 0.50	*0.51 to 0.60*	*0.61 to 0.70*
1. Coarse sand (μ = 0.15)	ϕ (°) E (kN/m^2)	43 45200	40 39300	38 32400
2. Medium sand (μ = 0.2)	ϕ (°) E (kN/m^2)	40 45200	38 39300	35 32400
3. Fine sand (μ = 0.25)	ϕ (°) E (kN/m^2)	38 36600	36 27600	32 23500
4. Sandy silt (μ = 0.30 to 0.35)	ϕ (°) E (kN/m^2)	36 13800	34 11700	30 10000

6.6.2. *Consolidation Settlement:* Fig. 6.1 represents a soil sample subjected to an initial stress p_0 . Let e_0 be the void ratio of the soil. Due to a stress increment Δp, the void ratio reduces to e . The charge in void ratio,

$$\Delta e = e_0 - e$$

Again, let H_0 and H_1 be the initial and final thicknesses of the soil mass.

$$\therefore \quad \Delta H = H_0 - H_1$$

Now, by definition,

$$m_v = \frac{\Delta V}{V} \cdot \frac{1}{\Delta p}$$

For a laterally confined soil, area of cross-section A is constant.

$$\therefore \quad \frac{\Delta V}{V} = \frac{\Delta H \cdot A}{H_0 \cdot A} = \frac{\Delta H}{H_0}$$

$$\therefore \quad m_v = \frac{\Delta H}{H_0} \cdot \frac{1}{\Delta p}$$

or

$$\Delta H = m_v \cdot H_0 \cdot \Delta p \qquad ...(6.20)$$

The change in thickness of the soil mass, and hence the consolidation settlement, can be determined from eqn. (6.20).

Again, by definition.

$$C_c = \frac{\Delta e}{\log_{10} p_1/p_0}$$

or,

$$\Delta e = C_c \cdot \log_{10} \frac{p_1}{p_0}$$

Assuming unit volume of solids, the initial and final volume of the soil are,

$$V_0 = 1 + e_0, \quad \text{and} \quad V_1 = 1 + e,$$

$$\therefore \quad \Delta V = 1 + e_0 - (1 + e_0) = e_0 - e = \Delta e$$

or,

$$\frac{\Delta V}{V_0} = \frac{\Delta e}{1 + e_0}$$

But

$$\frac{\Delta H}{H_0} = \frac{\Delta V}{V_0}$$

Hence,

$$\frac{\Delta H}{H_0} = \frac{\Delta e}{1 + e_0} = \frac{C_c \cdot \log_{10} p_1/p_0}{1 + e_0} \qquad ...(6.21)$$

or, $$\Delta H = H_0 \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{p_0 + \Delta p}{p_0} \quad ...(6.21)$$

In eqn. (6.21) p_0 and p_1 represent the average initial and final pressure acting over the thickness H_0 of the soil. While computing the probable consolidation settlement of a clay stratum, generally it is assumed that, the average stresses are those acting at the mid-height of the clay stratum. However, this assumption is not correct because, as we have seen in chapter 5, the stress intensity due to an external load does not vary linearly with depth. If the thickness of the clay stratum is substantially high, this leads to erroneous results.

In order to determine accurately the probable consolidation settlement of a clay layer of finite thickness, the following steps should be followed.

(i) Divide the given clay layer into a number of sub-layers of small thickness.

(ii) Determine the effective overburden presssure and stress increment at the mid-height of each sub-layer.

(iii) Compute the consolidation settlement of each sub-layer using either eqn. (6.20) or eqn. (6.21).

(iv) The probable settlement of the clay stratum is then obtained by summing up the settlements of all sub-layers, *i.e.*,

$$S_c = \sum_{i=1}^{n} S_{c_i} \quad ...(6.22)$$

EXAMPLES

Problem 6.1. A normally consolidated clay stratum of 3 m thickness has two permeable layers at its top and bottom. The liquid limit and the initial void ratio of the clay are 36% and 0.82 respectively, while the initial overburden pressure at the middle of clay layer is 2 kg/cm². Due to the construction of a new building this pressure increases by 1.5 kg/cm². Compute the probable consolidation settlement of the building.

Solution: Using eqn. (6.21), consolidation settlement of the building,

$$S_c = H \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{p_0 + \Delta p}{p_0}$$

Again, using eqn. (6.5a)

$$C_c = 0.009\,(w_l - 10)$$
$$= 0.009\,(36 - 10) = 0.234$$
$$H = 3 \text{ m} = 300 \text{ cm}$$
$$e_0 = 0.82$$

$$p_0 = 2 \text{ kg/cm}^2$$
$$\Delta p = 1.5 \text{ kg/cm}^2$$
$$\therefore \quad S_c = (300) \cdot \frac{(0.234)}{(1 + 0.82)} \cdot \log_{10}\left[\frac{2 + 1.5}{2}\right] \text{cm}$$
$$= 9.37 \text{ cm}$$

Problem 6.2. A 3 m thick saturated clay layer is overlain by a 4 m thick sand layer and is underlain by rock. The unit weight of the sand and clay are 1.72 t/m³ and 1.85 t/m³ respectively. The clay has a liquid limit of 53% and a void ratio of 0.65. A concentrated load of 200 t is applied on the ground surface. Compute the probable consolidation settlement of the clay,

(i) considering the entire clay layer

(ii) dividing the clay layer into three sub-layers of equal thickness.

Solution: Compression index of clay,

$$C_c = 0.009\,(w_l - 10)$$
$$= 0.009\,(53 - 10) = 0.387.$$
$$H = 3 \text{ m} = 300 \text{ cm}$$
$$e_0 = 0.65$$

(i) Initial overburden pressure at mid-depth of clay layer,

$$p_0 = (4.0)\,(1.72) + (3.0/2)\,(1.85) = 9.65 \text{ t/m}^2 = 0.965 \text{ kg/cm}^2$$

From Boussinesq's eqn., the maximum stress intensity at the middle of clay layer,

$$\Delta p = \frac{(3)\,(200)}{(2)\,(3.14)\,(4 + 1.5)^2} = 3.16 \text{ t/m}^2 = 0.316 \text{ kg/cm}^2$$

$$\therefore \quad S_c = \frac{(300)\,(0.387)}{(1 + 0.65)} \cdot \log_{10} \frac{0.965 + 0.316}{0.965} = 8.66 \text{ cm}$$

(ii) In this case the clay layer is divided into three sub-layers of thickness 1 m each, as shown in Fig. (6.4).

The consolidation settlement of each sub-layer is estimated below:

Sub-layer I : Depth of the middle of layer 1 below G.L $= 4.0 + \frac{1.0}{2} = 4.5$ m

$$p_{01} = (4)\,(1.72) + (0.5)\,(1.85) = 7.805 \text{ t/m}^2 = 0.781 \text{ kg/cm}^2$$

$$\Delta p_1 = \frac{(3)\,(200)}{(2)\,(3.14)\,(4.5)^2} = 4.71 \text{ t/m}^2 = 0.471 \text{ kg/cm}^2$$

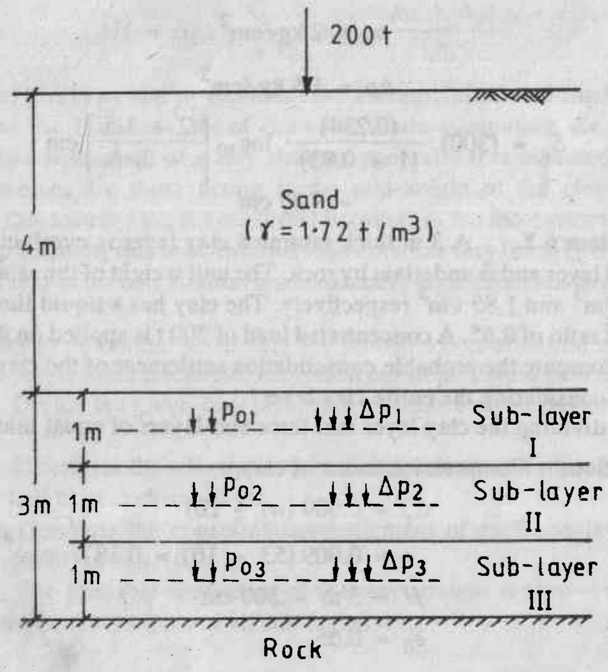


Fig. 6.4

$$\therefore \quad S_{c_1} = \frac{(100)\,(0.387)}{1\,+\,0.65} \cdot \log_{10} \frac{0.781\,+\,0.471}{0.781} = 4.81 \text{ cm.}$$

Sub-layer II: Depth of middle = 5.5 m.

$$p_{02} = (4)\,(1.72) + (1.5)\,(1.85) = 9.655 \text{ t/m}^2 = 0.965 \text{ kg/cm}^2.$$

$$\Delta p_2 = \frac{(3)\,(200)}{(2)\,(3.14)\,(5.5)^2} = 3.157 \text{ t/m}^2 = 0.316 \text{ kg/cm}^2$$

$$S_{c_2} = \frac{(100)\,(0.387)}{1\,+\,0.65} \log_{10} \frac{0.965\,+\,0.316}{0.965} = 2.89 \text{ cm}$$

Sub-layer III: Depth of middle = 6.5 m

$$p_{03} = (4)\,(1.72) + (2.5)\,(1.85) = 11.5 \text{ t/m}^2 = 1.15 \text{ kg/cm}^2$$

$$\Delta p_3 = \frac{(3)\,(200)}{(2)\,(3.14)\,(6.5)^2} = 2.26 \text{ cm}$$

$\therefore$ Total estimated settlement $= 4.81 + 2.89 + 2.26$

$= 9.96$ cm.

Problem 6.3. A 3 m thick layer of silty clay is sandwitched between two layers of dense sand. The effective overburden pressure at the centre of the silty clay layer is 2 kg/cm^2. However, due to the construction of a raft foundation, this pressure increases to 4 kg/cm^2.

Laboratory consolidation test was performed on a 2.5 cm thick sample of the silty clay. Under applied stresses of 2 kg/cm^2 and 4 kg/cm^2 the compressions of the sample were found to be 0.26 cm and 0.38 cm respectively. Compute the probable consolidation settlement of the raft.

Solution : Using eqn. (6.20),

$$S_c = m_v \cdot H_0 \cdot \Delta p$$

where, H_0 = initial thickness

= 2.5 cm for the soil sample and 300 cm for the soil in-situ.

Δp = change in effective pressure

$= 4 - 2 = 2 \text{ kg/cm}^2.$

m_v = co-efficient of volume change for the pressure range of 2 kg/cm^2 to 4 kg/cm^2

For the laboratory test:

Initial thickness of the sample = 2.5 cm

Thickness under a pressure of 2 kg/cm^2 = 2.5 – 0.26 = 2.24 cm.

Thickness under a pressure of 4 kg/cm^2 = 2.5 – 0.38 = 2.12 cm.

$\therefore$ Change in thickness when the pressure increases from 2 kg/cm^2 to 4 kg/cm^2 = 2.24 – 2.12 = 0.12 cm.

From eqn. (6.20)

$$0.12 = (m_v)\,(2.5)\,(2.0)$$

or, $$m_v = 0.024 \text{ cm}^2/\text{kg}.$$

Again, using eqn. (6.20), the consolidation settlement of the silty clay layer,

$$S_c = (0.024)\,(300)\,(2)$$

$$= 14.4 \text{ cm}$$

Problem 6.4. Due to the construction of a new structure the average vertical pressure at the centre of a 2.5 m thick clay layer increases from 1 kg/cm^2 to 2 kg/cm^2. A laboratory consolidation test was performed on a 2 cm thick undisturbed sample of the clay. Under applied stresses of 1 kg/cm^2 and 2 kg/cm^2 the equilibrium thicknesses of the sample were found to be 1.76 cm and 1.63 cm respectively. On removing the stress completely, the thickness increased to 1.88 cm. The final moisture content and the specific gravity of solids of the sample were found to be 29% and 2.71 respectively. Compute the probable consolidation settlement of the structure.

Solution: Let e_f and H_f be the final void ratio and thickness of the sample.

Then, $$e_f = \frac{wG}{s} = \frac{(0.29)(2.71)}{(1)} = 0.786$$

Again, $$\frac{\Delta H}{H_f} = \frac{\Delta e}{1 + e_f}$$

where, ΔH = change in thickness due a given stress

and, Δe = corresponding change in void ratio.

or, $$\Delta e = (1 + e_f) \cdot \frac{\Delta H}{H_f}$$

Here, $e_f = 0.786$, and $H_f = 1.88$ cm.

$$\therefore \quad \Delta e = (1 + 0.786) \cdot \frac{\Delta H}{(1.88)}$$

or, $$\Delta e = 0.95\,\Delta H \quad \text{...(i)}$$

when $\sigma = 2.0\text{ kg/cm}^2$, $\Delta H = 1.88 - 1.63 = 0.25$ cm

$\therefore$ $\Delta e = (0.95)(0.25) = 0.238$

Hence, void ratio at $\sigma = 2.0\text{ kg/cm}^2 = e_f - \Delta e$

$= 0.786 - 0.238 = 0.548.$

Again, when $\sigma = 1.0\text{ kg/cm}^2$, $\Delta H = 1.88 - 1.76 = 0.12$ cm

$\therefore$ $\Delta e = (0.95)(0.12) = 0.114$

Hence, void ratio at $\sigma = 1.0\text{ kg/cm}^2 = 0.786 - 0.114$

$= 0.672.$

Let m_v be the average value of the co-efficient of volume change in the pressure range of 1.0 to 2.0 kg/cm^2.

We have from eqn. (6.2),

$$m_v = \frac{\Delta e}{1 + e_0} \cdot \frac{1}{\Delta \sigma}$$

$$= \frac{(0.672 - 0.548)}{(1 + 0.548)} \cdot \frac{1}{(2.0 - 1.0)} = 0.08\text{ cm}^2/\text{kg}.$$

$\therefore$ Required consolidation settlement of the clay layer in the field

$$S_c = m_v H_0 \Delta\sigma$$

$$= (0.08)(250)(2 - 1) = 20\text{ cm}.$$

Hence, the required settlement of the structure = 20 cm

Problem 6.5. In a laboratory consolidation test, the void ratio of the sample reduced from 0.85 to 0.73 as the pressure was increased from 1 to 2 kg/cm^2. If the co-efficient of permeability of the soil be 3.3×10^{-4} cm/sec, determine:

(i) co-efficient of volume change

(ii) co-efficient of consolidation.

Solution : Using eqn. (6.2),

$$m_v = \frac{\Delta e}{1 + e_0} \cdot \frac{1}{\Delta p}$$

Here, $e_0 = 0.85$,

$\Delta e = 0.85 - 0.73 = 0.12$

$\Delta p = 2 - 1 = 1\text{ kg/cm}^2$

$$\therefore \quad m_v = \frac{(0.12)}{(1 + 0.85)} \cdot 1 = 0.065\text{ cm}^2/\text{kg}$$

Again, using eqn. (6.7),

$$C_v = \frac{k}{m_v \gamma_w}$$

Here, $m_v = 0.065\text{ cm}^2/\text{kg}$

$= 0.065 \times 10^{-3}\text{ cm}^2/\text{gm}$

$= 6.5 \times 10^{-5}\text{ cm}^2/\text{gm}$

$k = 3.3 \times 10^{-4}$ cm/sec.

$\gamma_w = 1$ gm/cc.

$$\therefore \quad C_v = \frac{3.3 \times 10^{-4}}{(6.5 \times 10^{-5})(1)} = 5.07\text{ cm}^2/\text{sec}.$$

Problem 6.6. A 6 m thick clay layer is drained at both top and bottom. The co-efficient of consolidation of the soil is 5×10^{-4} cm^2/sec. Determine the time required for 50% consolidation of the layer due to an external load.

Solution : Using eqn. (6.9),

$$T_v = \frac{C_v \cdot t}{h^2}$$

or, $$t = \frac{T_v \cdot h^2}{C_v}$$

For 50% consolidation, $$T_v = \frac{\pi}{4}\left(\frac{U}{100}\right)^2 = \frac{\pi}{4}\left(\frac{50}{100}\right)^2 = 0.197.$$

For double drainage condition, $$h = \frac{H}{2} = \frac{600}{2} = 300 \text{ cm}.$$

and, $$C_v = 5 \times 10^{-4} \text{ cm}^2/\text{sec}.$$

$$\therefore \quad t = \frac{(0.197)(300)^2}{5 \times 10^{-4}} \text{ sec} = 3.546 \times 10^7 \text{ sec}$$

$$= \frac{3.546 \times 10^7}{86400} \text{ days} = 410 \text{ days}.$$

Problem 6.7 A raft footing is to be constructed on a 7.5 m thick clay layer which lies between two sand layers. In order to predict the time rate of settlement of the building, a 2.5 cm thick undisturbed sample of the soil was tested in the laboratory under double drainage condition. The sample was found to have undergone 50% consolidation in 12.5 minutes. Determine the time required for 50% settlement of the building.

Solution : We have from eqn. (6.9),

$$T_v = \frac{C_v \times t}{h^2}, \text{ or, } C_v = \frac{T_v \cdot h^2}{t}$$

In the laboratory test,

T_v = time factor for 50% consolidation = 0.197

t = 12.5 min.

$$h = \frac{H}{2} = \frac{2.5}{2} = 1.25 \text{ cm}$$

$$\therefore \quad C_v = \frac{(0.197)(1.25)^2}{(12.5)} = 0.0246 \text{ cm}^2/\text{min}.$$

In case of the actual building,

T_v = 0.197

$$h = \frac{(7.5)(100)}{(2)} = 375 \text{ cm}$$

$$\therefore \quad t = \frac{T_v \cdot h^2}{C_v} = \frac{(0.197)(375)^2}{0.0246} \text{ min}$$

$$= \frac{1126143.3}{60 \times 24} \text{ days}$$

$$= 782 \text{ days} = 2 \text{ years } 1 \text{ month and } 22 \text{ days}.$$

Problem 6.8 In a laboratory consolidation test, a 2.5 cm thick sample of clay reached 60% consolidation in 17 minutes under double drainage condition. Determine the time required for 60% consolidation of a layer of this soil in the field under the following conditions:

(i) when a 3 m thick layer of the given soil is sandwitched between two sand layers.

(ii) when a 5 m thick layer of the soil is overlain by a sand layer and underlain by a deep layer of intact shale.

Solution : Using eqn. (6.11), the time factor for 60% consolidation

$$T_v = 1.781 - 0.933 \log_{10}(100 - 60)$$

$$= 0.286$$

Again, using eqn. (6.9)

$$T_v = \frac{C_v \cdot t}{h^2}, \text{ or, } C_v = \frac{T_v \cdot h^2}{t}.$$

In the laboratory test,

t = 17 min.

h = 2.5/2 = 1.25 cm

$$\therefore \quad C_v = \frac{(0.286)(1.25)^2}{(17)} = 0.0263 \text{ cm}^2/\text{min}$$

(i) Here the soil layer is drained at both top and bottom

$$\therefore \quad H = \frac{(3)(100)}{(2)} = 150 \text{ cm}$$

$$t = \frac{(0.286)(150)^2}{0.0263} = 244800 \text{ min} = 170 \text{ days}$$

(ii) In this case the soil layer is drained at top only

$$\therefore \quad H = 5 \text{ m} = 500 \text{ cm}$$

$$t = \frac{(0.286)\,(500)^2}{0.0263} = 2718631 \text{ min} = 1888 \text{ days} = 5.17 \text{ years.}$$

Problem 6.9 The consolidation settlement of a new structure founded on a 5 m thick layer is estimated as 6.5 cm. The structure was found to have settled by 1.6 cm in 6 months after the completion of construction. If the clay layer is underlain by rock and overlain by a layer of coarse sand, determine:

(i) the time required for 50% consolidation to occur

(ii) the amount of settlement which will take place in the next six months.

Solution : Degree of consolidation occurred in the first six months

$$= \frac{1.6}{6.5} \times 100\% = 24.62\%$$

Time factor for $U = 24.62\%$

$$T_v = (\pi/4)\,(24.62/100)^2 = 0.048.$$

As single drainage condition is prevailing, $h = 5$ m.

Using eqn. (6.9),

$$C_v = \frac{(0.048)\,(52)}{(6)\,(30)} = 6.67 \times 10^{-3} \text{ m}^2/\text{day}.$$

(i) For 50% consolidation, time factor, $T_v = 0.197$.

Using eqn. (6.9),

$$t = \frac{T_v \times h^2}{C_v} = \frac{(0.197)\,(52)}{6.67 \times 10^{-3}} = 738.4 \text{ days}$$

$$= 2 \text{ years and } 8.4 \text{ days.}$$

(ii) Let U be the degree of consolidation that will take place in the next six months, *i.e.* at the end of 1 year since the completion of construction. We have already found that the time required for 50% consolidation is 2 years and 8.4 days. Thus, degree of consolidation occurred in 1 year must be less than 50%.

The corresponding time factor may be determined using eqn. (6.10),

$$T_v = (\pi/4)\,(U/100)^2 = \frac{\pi U^2}{40000}$$

Again, using eqn. (6.9),

$$T_v = \frac{(6.67 \times 10^{-3})\,(365)}{(5^2)} = 0.0974.$$

$$\therefore \quad \frac{\pi U^2}{40000} = 0.0974.$$

or, $$U = \sqrt{\frac{(40000)\,(0.0974)}{3.14}} = 35.22\%$$

If x be the amount of settlement, then

$$U = \frac{x}{6.5} \times 100$$

or, $$x = \frac{6.5\,U}{100} = \frac{(6.5)\,(35.22)}{100} = 2.29 \text{ cm}$$

Problem 6.10. Undisturbed samples were collected from a 3 m thick clay stratum which lies between two sand strata. A laboratory consolidation test was performed on a 2.5 cm thick sample of the clay. During the test, water was allowed to drain out only through the top of the sample. The time required for 50% consolidation was found to be 35 minutes. Determine the time required for 60% and 90% consolidation in the field.

Solution : As the sample was tested under single drainage condition, $h = H = 2.5$ cm

Again, for $U = 50\%$, we have $T_v = 0.197$.

Using eqn. (6.9),

$$C_v = \frac{(0.197)\,(2.5)^2}{(35)} = 0.035 \text{ cm}^2/\text{min}.$$

Now, for 60% consolidation, $T_{v_{60}} = 1.781 - 0.933 \log_{10}(100-60)$

$$= 0.286$$

For a double drainage condition,

$$h = \frac{H}{2} = \frac{3}{2} \text{ m} = 150 \text{ cm}.$$

$$\therefore \quad t_{60} = \frac{T_{v_{60}} \cdot h^2}{C_v}$$

$$= \frac{(0.286)\,(150)^2}{(0.035)}$$

$$= 183857 \text{ min}$$

$$= 127.7 \text{ days} \approx 128 \text{ days}$$

For 90% consolidation, $T_{v_{90}} = 1.781 - 0.933 \log_{10}(100-90)$

$$= 0.848.$$

$$\therefore \quad t_{90} = \frac{T_{v_{90}} \cdot h^2}{C_v}$$

$$= \frac{(0.848)\,(150)^2}{0.035} = 545143 \text{ min}$$

$$= 378 \text{ days.}$$

Problem 6.11. A flexible footing of 2 m × 2 m size carries a total load of 490 kN, inclusive of its self-weight. The footing rests on a sand layer having a modulus of elasticity of 40000 kN/m^2 and a Poisson's ratio of 0.38. Estimate the probable settlement below the centre and below any one corner of the footing.

Solution : We have, from eqn. (6.19),

$$S_i = q\,B\,\frac{(1-\mu^2)}{E}\,I_f.$$

Here, q = intensity of loading

$$= \frac{(490)}{(2)\,(2)} = 122.5 \text{ kN/m}^2$$

B = 2 m

$\mu = 0.38$, $E = 40000$ kN/m^2

The influence factor I_f may be obtained from table 6.1,

I_f(corner) = 0.56

I_f(centre) = 1.12.

∴ Immediate settlement below the centre,

$$S_{i(\text{centre})} = \frac{(122.5)\,(2)\,(1-0.38^2)}{(40000)} \cdot (1.12)$$

$$= 0.59 \text{ cm}$$

Immediate settlement below the corner

$$S_{i\,(\text{corner})} = \frac{(0.59)\,(0.56)}{(1.12)} = 0.295 \text{ cm.}$$

Problem 6.12. A 6 m thick clay stratum is overlain by a 8 m thick stratum of coarse sand and is underlain by an impermeable shale. A raft footing, supporting the columns of a building, is to be founded at a depth of 1.2 m below ground level. The size of the raft is 8.5 m × 13.6 m, and it is loaded uniformly with a stress intensity of 9.2 t/m^2. The water table is located

at 2 m below the ground level. The unit weight of sand above and below water table are 1.90 and 2.10 t/m^3. The properties of the clay are as follows:

Initial void ratio = 0.72

specific gravity of solids = 2.71

liquid limit = 42%

co-efficient of consolidation = 2.2 × 10^{-3} cm^2/sec.

Determine :

(i) Probable settlement of the raft.

(ii) The time required to undergo a settlement of 5 cm.

Solution : (i) The soil profile is shown in Fig. 6.5. The clay layer is divided into three sub-layers of thickness 2 m each. The settlement of each sub-layer may now be computed using eqn. (6.21),

$$\Delta H = H_0 \cdot \frac{C_c}{1+e_0} \cdot \log_{10} \frac{p_1}{p_0}.$$

The computation of settlement for the first sub-layer is shown below :

C_c = (0.009) (42 − 10) = 0.288

e_0 = 0.72

H_0 = 2 m = 200 cm.

Depth of middle of the sub-layer below G.L. = 8 + 2/2 = 9 m

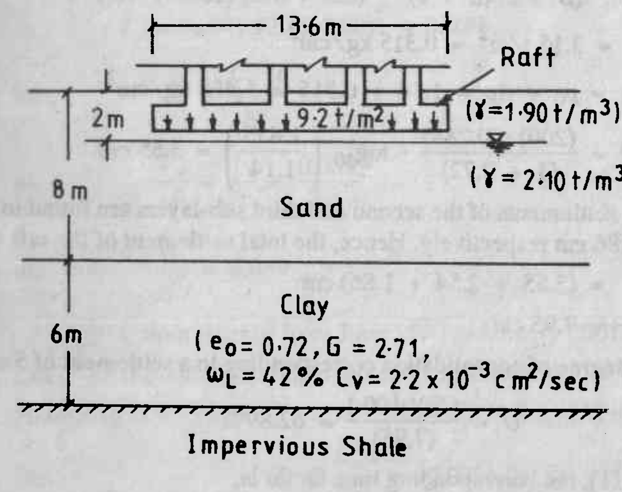


Fig. 6.5

Initial effective overburden stress at a depth of 9 m below G.L.

= stress due to sand above water table + stress due to sand below water table + stress due to clay

$$\therefore \quad p_0 = \gamma h_1 + \gamma_{sub} h_2 + \gamma_{clay} h_3.$$

Here, unit weights of sand are :

$$\gamma = 1.9 \text{ t/m}^3$$

and, $\gamma_{sat} = 2.1 \text{ t/m}^3$

$$\therefore \quad \gamma_{sub} = \gamma_{sat} - \gamma_w = 2.1 - 1 = 1.1 \text{ t/m}^3$$

Again, $\gamma_{clay} = \dfrac{G + e}{1 + e} \cdot \gamma_w$

$$= \frac{(2.71 + 0.72)}{(1 + 0.72)} \cdot (1) = 2.0 \text{ t/m}^3$$

$$\therefore \quad p_0 = (1.9)(2) - (1.1)(8 - 2) + (2.0 - 1.0)(1)$$

$$= 11.4 \text{ t/m}^2 = 1.14 \text{ kg/cm}^2$$

Again, depth of middle of this sub-layer below the base of footing = 9 – 1.5 = 7.5 m.

Using the 2 : 1 dispersion method,

$$\Delta p = \frac{qBL}{(B + z)(L + z)} = \frac{(9.2)(8.5)(13.6)}{(8.5 + 7.5)(13.6 + 7.5)}$$

$$= 3.15 \text{ t/m}^2 = 0.315 \text{ kg/cm}^2$$

$$\therefore \quad p_1 = p_0 + \Delta p = 1.14 + 0.315 = 1.455 \text{ kg/cm}^2$$

$$\therefore \quad \Delta H = \frac{(200) \cdot (0.288)}{(1 + 0.72)} \cdot \log_{10}\left(\frac{1.455}{1.14}\right) = 3.55 \text{ cm.}$$

Similarly, settlements of the second and third sub-layers are found to be 2.54 cm and 1.86 cm respectively. Hence, the total settlement of the raft

$$= (3.55 + 2.54 + 1.86) \text{ cm}$$

$$= 7.95 \text{ cm.}$$

(ii) The degree of consolidation corresponding to a settlement of 5 cm,

$$U = \frac{(5)(100)}{(7.95)} = 62.89\%$$

Using eqn. (6.11), the corresponding time factor is,

$$T_v = 1.781 - 0.933 \log_{10}(100 - 62.89)$$

$$= 0.317$$

As single drainage condition prevails at site,

$$h = H = 6 \text{ m} = 600 \text{ cm.}$$

Using eqn. (6.9),

$$t = \frac{T_v \cdot h^2}{C_v}$$

$$= \frac{(0.317)(600^2)}{(2.2 \times 10^{-3})} = 51872727 \text{ sec}$$

$$= 600 \text{ days}$$

Problem 6.13. The construction of a multistoreyed building started in January 1989 and was completed in June 1990. The total consolidation settlement of the building was estimated to be 8 cm. The average settlement of the building was measured in December 1991 and was found to be 2.2 cm. Compute the probable settlement of the building in January 2001.

Solution : Let C_v be the co-efficient of consolidation of the soil in the appropriate pressure range, and H be the effective length of drainage path.

Time elapsed from June 1990 to December 1991 = 1.5 years

Degree of consolidation occurred in 1.5 years.

$$U = \frac{(2.2)(100)}{(8)} \% = 27.5\%$$

$$\therefore \quad T_v = (\pi/4)\,(27.5/100)^2 = 0.059$$

But, $T_v = \dfrac{C_v \cdot t}{H^2}$

or, $\dfrac{C_v}{H^2} = \dfrac{T_v}{t} = \dfrac{0.059}{1.5}$

or, $\dfrac{C_v}{H^2} = 0.039$ ...(i)

Again, time elapsed from June 1990 to January 2001 = 10.5 years.

Let U be the corresponding degree of consolidation.

Assuming $U > 53\%$, $T_v = 1.781 - 0.933 \log_{10}(100 - U)$

But, $T_v = \dfrac{C_v t}{H^2} = (0.039)(10.5) \quad \left[\because \dfrac{C_v}{H^2} = 0.039\right]$

$$= 0.4095.$$

$\therefore \quad 1.781 - 0.933 \log_{10}(100 - U) = 0.4095$

or, $$\log_{10}(100 - U) = \frac{(1.781 - 0.4095)}{(0.933)} = 1.47$$

Taking antilog of both sides we get,

$$100 - U = 29.51$$

or, $$U = 100 - 29.51 = 70.49\%$$

$\therefore$ Amount of consolidation settlement in January 2001

$$= \frac{(8)(70.49)}{(100)} = 5.64 \text{ cm.}$$

Problem 6.14. A 2 m thick layer of saturated clay lies in between two permeable layers. The clay has the following properties :

liquid limit = 45%

co-efficient of permeability = 2.8×10^{-7} cm/sec

initial void ratio = 1.25

The initial effective overburden pressure at the middle of the clay layer is 2 kg/cm^2, and is likely to increase to 4 kg/cm^2 due to the construction of a new building. Determine :

(i) the final void ratio of the clay.
(ii) settlement of the proposed building.
(iii) time required for 50% consolidation.

Solution : (i) Compression index, $C_c = (0.009)(45 - 10) = 0.315$.

But, by definition,

$$C_c = \frac{\Delta e}{\log_{10} \dfrac{p_0 + \Delta p}{p_0}}$$

or, $$\Delta e = C_c \log_{10} \frac{p_0 + \Delta P}{p_0}$$

$\therefore$ $$\Delta e = (0.315) \log_{10}\{(2 + 2)/(2)\}$$

$$= 0.095$$

$\therefore$ Final void ratio $= e_0 - \Delta e$

$$= 1.25 - 0.095 = 1.155$$

(ii) Let ΔH be the consolidation settlement of the clay layer.

$\therefore$ $$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e_0}$$

or, $$\Delta H = H \cdot \frac{\Delta e}{1 + e_0}$$

$$= \frac{(2)(100)(0.095)}{(1 + 1.25)} = 8.44 \text{ cm}$$

(iii) In the pressure range of 2 to 4 kg/cm^2.

$$m_v = \frac{\Delta e}{1 + e_0} \cdot \frac{1}{\Delta p}$$

$$= \frac{(0.095)}{(1 + 1.25)} \cdot \frac{1}{(2)} = 0.021 \text{ cm}^2/\text{kg.}$$

Using eqn. (6.7), $$C_v = \frac{k}{m_v \gamma_w}$$

Here, $k = 2.8 \times 10^{-7}$ cm/sec.

$m_v = 0.021$ cm^2/kg

$\gamma_w = 1$ gm/cc $= 1 \times 10^{-3}$ kg/cc.

$\therefore$ $$C_v = \frac{(2.8 \times 10^{-7})}{(0.021 \times 10^{-3})} = 0.0133 \text{ cm}^2/\text{sec}$$

For 50% consolidation, we have, $T_v = 0.197$.

Using eqn. (6.9),

$$t = \frac{T_v \cdot h^2}{C_v} = \frac{(0.197)(200/2)^2}{(0.0133)} \text{ sec.}$$

$$= 1.71 \text{ days.}$$

Problem 6.15. A laboratory consolidation test was performed on a 2 cm thick sample of a silty clay, and the following results were obtained :

Pressure (kg/cm^2)	*Final dial gauge reading (mm)*	*Pressure (kg/cm^2)*	*Final dial gauge reading (mm)*
0	5.590	2.00	3.964
0.25	5.234	4.00	3.515
0.50	4.960	8.00	2.785
1.00	4.604	0	5.224

The final moisture content of the sample after swelling was found to be 32.5%. The specific gravity of solids = 2.70.

(i) Plot the e vs. $\log p$ curve.

(ii) Determine the compression index and the co-efficient of volume change of the soil.

Solution : In order to plot the e vs. $\log p$ curve, the final void ratios at the end of each pressure increment are to be determined.

The final void ratio of the sample at the end of swelling

$$e = \frac{wG}{s} = \frac{(0.325)(2.70)}{(1)} = 0.878.$$

The thickness of the sample at this stage,

$$H = H_0 - \Delta H$$
$$= 2.0 - (0.5590 - 0.5224) \text{ cm}$$
$$= 1.9634 \text{ cm.}$$

Now, we have,

$$\frac{\Delta H}{H} = \frac{\Delta e}{1 + e}$$

or,
$$\Delta e = \frac{\Delta H}{H}(1 + e)$$

Substituting the final values of e and H, we get,

$$\Delta e = \Delta H \cdot \frac{(1 + 0.878)}{1.9634} = 0.9565\, \Delta H \qquad \text{...(i)}$$

The change in void ratio, and hence the final void ratio after each load increment, are now determined by putting the corresponding values of ΔH in eqn. (i). The computed values are shown below in a tabular form :

Pressure range (kg/cm²)	*Pressure increment* Δp (kg/cm²)	*Increase in thickness* ΔH (cm)	*Change in void ratio* Δe	*Equilibrium void ratio*	$a_v \left(= -\frac{\Delta e}{\Delta p}\right)$ (cm²/kg)	$m_v \left(= \frac{a_v}{1 + e_0}\right)$ (cm²/kg)	$C_c \left[= -\frac{\Delta e}{\log_{10}(p_1/p_0)}\right]$
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
0 to 0.25	+0 .25	−0.0356	−0.034	0.879	0.136	0.072	—
0.25 to 0.50	+0.25	−0.0274	−0.026	0.863	0.104	0.055	0.086
0.50 to 1.00	+0.50	−0.0356	−0.034	0.829	0.068	0.036	0.113
1.00 to 2.00	+1.00	−0.0640	−0.061	0.768	0.061	0.019	0.203
2.00 to 4.00	+2.00	−0.0549	−0.053	0.715	0.027	0.014	0.176
4.00 to 8.00	+4.00	−0.0730	−0.070	0.645	0.015	0.008	0.199
8.00 to 0	−8.00	+0.2439	+0.233	0.878	—		

Note that, in column 5 of the above table, the final void ratio after the withdrawal of the load has been entered first ($e_f = 0.878$). The void ratios corresponding to the previous loadings were then determined by subtracting from it the appropriate values of Δe from col. 4.

For example, equilibrium void ratio corresponding to 8 kg/cm²

$$= e_f - \Delta e = 0.878 - 0.233 = 0.645,$$

and that corresponding to 4 kg/cm²

$$= 0.645 - (-0.070) = 0.715$$

The co-efficient of compressibility, a_v, and the co-efficient of volume change, m_v, are then computed for each pressure range, and are shown in col. 6 and 7 respectively. The computed values of C_c are given in col. 8.

The values of Δe (in ordinary scale) and Δp (in log scale) have been plotted along the Y and X-axis respectively on a semi-log graph paper, to obtain the e vs. $\log_{10} p$ curve. This is shown in Fig. 6.6

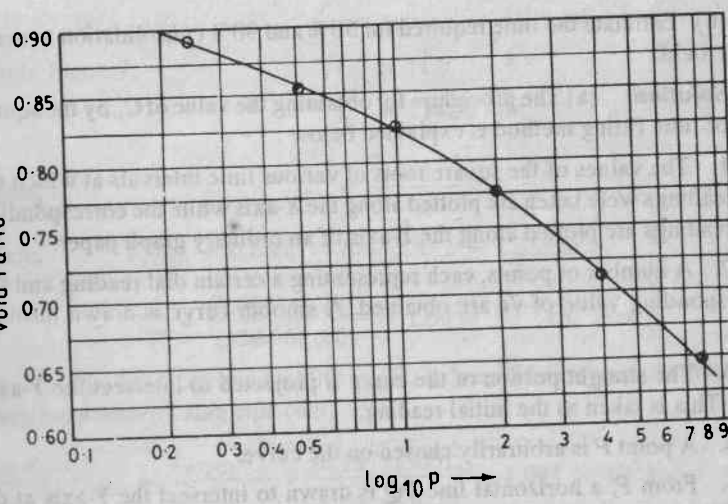


Fig. 6.6

Problem 6.16. A raft footing is to be founded in a 3 m thick layer of clay which is underlain by a highly permeable sand layer. The initial over burden pressure at the centre of the clay layer is 2.0 kg/cm² and this is likely to increase to 4.0 kg/cm² due to the construction of the raft. A 2.5 cm thick sample of this soil is tested in a consolidometer under double drainage condition. The following data were obtained when the pressure on the sample was increased from 2 to 4 kg/cm^2 :

Time (min)	Dial reading (Divisions)	Time (min)	Dial reading (Divisions)
0	1972	16.00	1727
0.25	1921	36.00	1642
1.00	1870	64.00	1555
2.25	1848	100.00	1491
4.00	1813	144.00	1449
9.00	1769		

The dial gauge constant is, 1 division = 0.002 min.

(a) Determine the co-efficient of consolidation of the soil by the square root of time fitting method.

(b) Estimate the time required for 50% and 90% consolidation to occur in the field.

Solution: (a) The procedure for obtaining the value of C_v by the square root of time fitting method is explained below :

1. The values of the square roots of various time intervals at which the dial readings were taken are plotted along the X-axis while the corresponding dial readings are plotted along the Y-axis of an ordinary graph paper.

2. A number of points, each representing a certain dial reading and the corresponding value of $\sqrt{t}$ are obtained. A smooth curve is drawn through them.

3. The straight portion of the curve is projected to intersect the Y-axis at R_c. This is taken as the initial reading.

4. A point P is arbitrarily chosen on the curve.

5. From P, a horizontal line PQ is drawn to intersect the Y-axis at Q. Let, $PQ = a$.

6. The point R is chosen on projected PQ, such that, $PR = 0.15a$.

7. R_0 and R are joined. The line R_0 is then projected to intersect the curve at S.

The dial reading corresponding to S represents 90% consolidation. Let t_{90} be the corresponding time required.

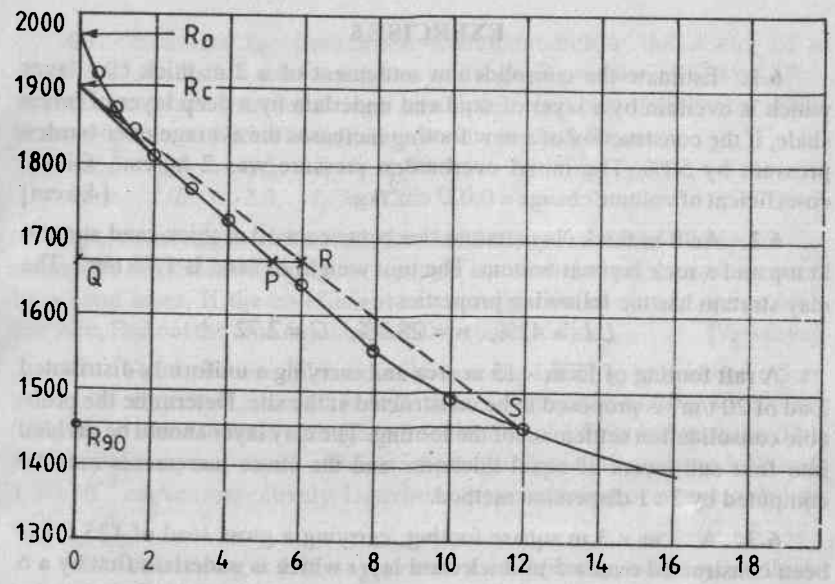


Fig. 6.7

From Fig. 6.7, $\sqrt{t_{90}} = 11.6$

$\therefore \quad t_{90} = 134.56$ min

Now, $$C_v = \frac{T_{v(90)} \cdot h^2}{t_{90}}$$

Here, $T_{v(90)} = 0.848, \quad h = 2.5/2 = 1.25$ cm

$$\therefore \quad C_v = \frac{(0.848)(1.25)^2}{(134.56)(60)} = 1.64 \times 10^{-4} \text{ cm}^2/\text{sec}$$

(b) The time required for 50% and 90% consolidation to occur in the field may be obtained using eqn. (6.9).

$$t_{50} = \frac{T_{v(50)}\, h^2}{C_v} = \frac{(0.197)(300)^2}{1.64 \times 10^{-4}} = 1.081 \times 10^8 \text{ sec}$$

= 1251 days = 3 years 5 months and 6 days.

$$t_{90} = \frac{T_{v(90)} \times h^2}{C_v} = \frac{(0.848)(300)^2}{1.64 \times 10^{-4}} = 4.654 \times 10^8 \text{ sec}$$

= 5386 days = 14 years 9 months and 6 days.

EXERCISE 6

6.1. Estimate the consolidation settlement of a 2 m thick clay layer which is overlain by a layer of sand and underlain by a deep layer of intact shale, if the construction of a new footing increases the average over-burden pressure by 50%. The initial overburden pressure was 2 kg/cm^2. Given, co-efficient of volume change = 0.023 cm^2/kg. [4.6 cm]

6.2. An 8 m thick clay stratum lies between a 10 m thick sand stratum at top and a rock layer at bottom. The unit weight of sand is 1.75 t/m^3. The clay stratum has the following properties:

$L.L. = 42\%, \quad w = 28.5\%, \quad G = 2.72$

A raft footing of 15 m × 15 m area and carrying a uniformly distributed load of 20 t/m^2 is proposed to be constructed at the site. Determine the probable consolidation settlement of the footing. The clay layer should be divided into four sub-layers of equal thickness and the stress increments may be computed by 2 : 1 dispersion method.

6.3. A 3 m × 3 m square footing, carrying a gross load of 125 t, has been constructed over a 5 m thick sand layer which is underlain first by a 6 m thick layer of soft clay and then a layer of impermeable shale. Compute the consolidation settlement of the footing by considering the clay layer (i) as a whole (ii) divided into three layers of equal thickness. Given,

unit weight of sand = 1.8 gm/cc
compression index of clay = 0.42
water content of clay = 32%
specific gravity of clay particles = 2.7 [(i) 3.6 cm (ii) 4.29 cm]

6.4. During a laboratory consolidation test, the void ratio of a soil sample decreased from 1.2 to 1.05 when the pressure on it increased from 2 to 4 kg/cm^2. Determine the co-efficient of compressibility and the co-efficient of volume change of the soil. Will these values remain the same if pressure increases from 4 to 8 kg/cm^2. [0.075 cm^2/kg; 0.034 cm^2/kg]

6.5. A consolidation test was performed on a sample of saturated clay in the laboratory. The liquid limit and the initial void ratio of the soil were 48% and 0.96 respectively. What will be the final void ratio of the soil if the pressure is increased from 0.25 kg/cm^2 to 1.0 kg/cm^2 ? [0.72]

6.6. Sample of a silty clay was subjected to a laboratory oedometer test. Under a vertical pressure of 2 kg/cm^2 the equilibrium void ratio was found to be 1.05. On increasing the pressure to 3 kg/cm^2, the final equilibrium void ratio reduced to 0.93. If the co-efficient of permeability of the soil be 1.2 × 10^{-7} cm/sec, determine the co-efficient of consolidation in m^2/day.

[1.77 × 10^{-2} m^2/day]

6.7. Estimate the immediate settlement below the centre of a 15 m × 25 m flexible raft footing carrying a gross pressure of 12 t/m^2. The raft rests on a sand stratum having a modulus of elasticity of 4080 t/m^2 and a Poisson's ratio of 0.25. The influence factors are as follows :

when $L/B = 1.5, \quad I_f = 1.36$
when $L/B = 2.0, \quad I_f = 1.52$ [5.84 cm]

6.8. A footing is to be constructed in a homogeneous bed of clay having an overall thickness of 3 m. The clay layer is underlain by rock and overlain by a sand layer. If the co-efficient of consolidation of clay be 9.5 × 10^{-4} cm^2/sec, find out the time required for 90% consolidation. [930 days]

6.9. The total consolidation settlement of a building founded on a 5 m thick silty clay layer, drained at both ends, is estimated to be 6.8 cm. The building is found to have undergone a settlement of 2.5 cm in 3 months. The initial void ratio and the co-efficient of permeability of the soil are 0.88 and 1.2 × 10^{-7} cm/sec respectively. Determine the co-efficient of compressibility of the soil. [0.265 cm^2/kg]

6.10. A building is to be supported by a raft footing laid in a 3 m thick bed of clay, which lies between two permeable layers. A 2.5 cm thick sample of the soil is found to have undergone 50% consolidation in 3 minutes under double drainage condition. Determine the time required for 90% consolidation of the building. [129.1 days]

6.11. In a laboratory consolidation test, a 2 cm thick clay specimen reached 50% consolidation in 12 minutes. The sample was drained at both top and bottom. A 2 m thick layer of this soil lies below a sand layer and above an impermeable layer of very stiff clay. Find out the degree of consolidation of the clay layers which will take place in 1 year. [27.5%]

6.12. An isolated footing of 2 m × 2 m plan area is constructed over a saturated sandy clay stratum of 5 m thickness. The soil has the following properties.

$E = 30600$ kN/m^2, $\mu = 0.36$, $C_c = 0.3$, $w = 35\%$, $G = 2.69$.

Estimate the probable settlement of the footing if it carries a gross load of 225 kN. [12.57 cm]

6.13. A 5 m thick layer of normally consolidated clay supports a newly constructed building. The weight of sand overlying the clay layer is 660 gm/cm^2 while the new construction increases the stress at the middle of the clay layer by 450 gm/cm^2. Compute the probable consolidation settlement of the building. Given,

$LL = 39\%, \quad G = 2.7, \quad w = 45\%.$ [4.39 cm]

6.14. The total consolidation settlement of a clay layer due to an imposed load is estimated to be 8.5 cm. A settlement of 2 cm took place in 15 days. Determine the time required for 50% and 90% consolidation.

[68 days; 292 days]

6.15. The results of a consolidation test are shown below :

Pressure (kg/cm²)	0	0.25	0.5	1.0	2.0	4.0	8.0
Final dial reading	2658	2699	2744	2804	2889	2966	3047

The sample had an initial height of 2 cm and an initial mass of 112.04 gm. After the completion of the test the oven-dried sample was found to weigh 81.39 gm. The specific gravity of solids was 2.71 and the dial gauge constant was: 1 divn. = 0.02 mm.

(a) Determine the equilibrium void ratio of the sample after each load increment.

(b) Determine the values of co-efficient of compressibility and co-efficient of volume change for various pressure ranges.

6.16. An undisturbed sample of saturated clay, collected from a depth of 5 m below G.L., was subjected to a laboratory consolidation test. The initial diameter and thickness of the sample were 7.5 cm and 2 cm respectively. The mass of the sample in the wet and dry states were 175.2 gm and 138.8 gm respectively. The final compressions under various pressures are shown below :

Pressure (kg/cm²)	0.25	0.50	1.00	2.00	4.00	8.00	16.00
Final compression (cm)	0.001	0.003	0.008	0.0168	0.0484	0.0901	0.1160

Plot the e vs $\log_{10} p$ curve and check whether the soil is over-consolidated. If so, determine the preconsolidation pressure. Given, $G = 2.67$.

7

COMPACTION

7.1 Introduction: Construction of structures on weak soils (e.g., soft clay, loose sand, etc) sometimes requires "stabilisation" of the soil mass, i.e., an artificial improvement of its engineering properties.

There are various methods of soil stabilisation, the most common one being the mechanical stabilisation, and the simplest technique of mechanical stabilisation is compaction.

A soil mass can be compacted by either a dynamic process or a static one. In the dynamic method the soil is compacted by repeated applications of a dead load, while in the static method compaction is done by a steadily increasing static load. Generally, the dynamic method gives better results in coarse-grained soils and the static compaction is suitable for less permeable fine-grained soils.

7.2 Moisture-density Relationships: While compacting a soil in the field, it is always desirable to compact the soil in such a way that its dry density is maximum. If a given soil is compacted under a specified compactive effort, its dry density will be the maximum at a certain moisture content, known as the optimum moisture content. Hence, before compacting a soil in the field, its optimum moisture content and the corresponding dry density must be determined in the laboratory. The test employed for this purpose is called Standard Proctor Test.

7.3 Standard Proctor Test: In this test, samples of the given soil are prepared at various moisture contents and are compacted in a cylindrical mould, 127.3 mm high and having an internal diameter of 100 mm. The sample is compacted in three layers of equal height, each layer being subjected to 25 blows of a compaction rammer having a self-weight of 2600 gm and a height of free fall of 310 mm.

Samples are compacted in the mould at increasing moisture contents. After each test, weight of the sample compacted is determined and its bulk and dry densities are computed.

A curve is then plotted to show the variation of dry density with moisture content (Fig. 7.1). The curve is usually parabolic in shape. Initially the dry

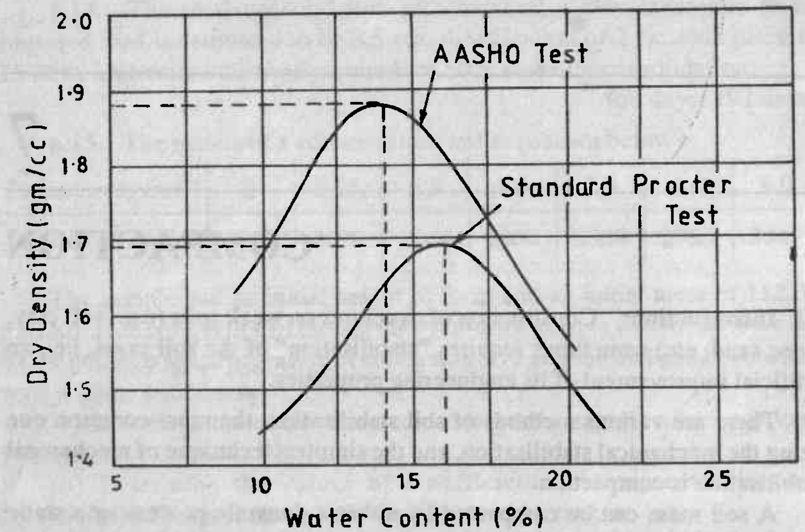


Fig. 7.1

density increases with increasing moisture content, until a certain peak value is reached. Further increase in moisture content results in a decrease in the dry density. The moisture content represented by the peak of the curve is the optimum moisture content (OMC) and the corresponding dry density is the maximum dry density of the soil under that particular compactive effort.

For heavier field compaction, the moisture-density relationship can be investigated by the modified AASHO test. The test procedure is similar to that of Proctor test except that a heavier rammer (weight = 4900 gm, free fall = 450 mm) is used and the soil is compacted in 5 layers.

Under heavier compaction, the moisture-density curve (Fig.7.1) is shifted upwards and simultaneously moves to the left, resulting in a lower OMC but a greater γ_{dmax}.

7.4 Zero Air Voids Line: Compaction is achieved by the expulsion of air from the voids. However, as the external load acts for a very short time, it is nearly impossible to drive out all the air from the voids. Thus, during compaction, a soil is not fully saturated. If the remaining air could be driven out, its void ratio would have been reduced and consequently, its dry density would have increased. The zero air voids line (Fig.7.2) is a theoretical curve which represents the relationship between water content and dry density of the soil when it is 100% saturated.

At any given moisture content, the dry density of a soil in the fully saturated condition can be derived as follows:

We have, $$\gamma_d = \frac{G\gamma_w}{1+e}$$

and $$wG = se$$

For a fully saturated soil, $s = 1$, $\therefore$ $wG = e$

$$\therefore \quad \gamma_d = \frac{G\gamma_w}{1+wG} \quad \text{... (7.1)}$$

From eqn (7.1) it is evident that, for a given soil, an increase in moisture content will always result in a decrease in γ_d. Hence the zero air voids line is always a steadily descending line.

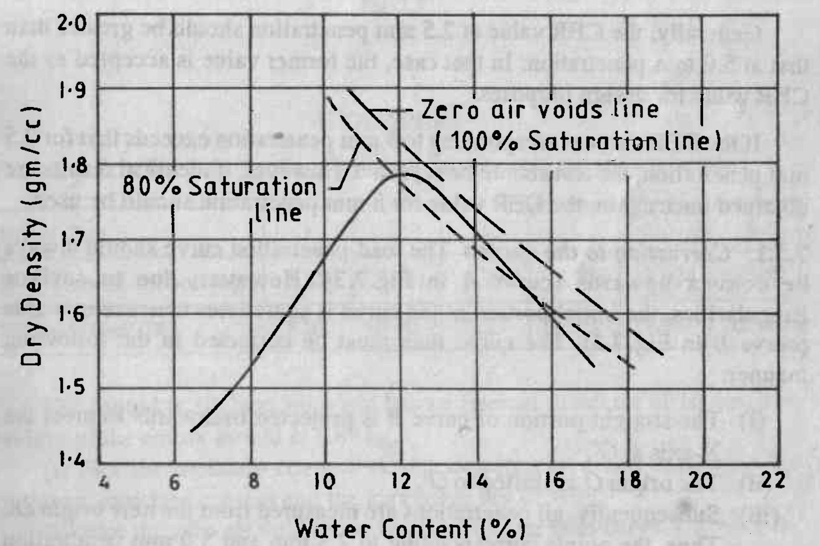


Fig. 7.2

7.5 California Bearing Ratio (CBR): The California bearing ratio test is of immense importance in the field of highway engineering. The CBR value of a soil or a paving material is a measure of its strength against probable rutting failure due to moving wheel loads.

The California bearing ratio is defined as the ratio of the force per unit area required to drive a cylindrical plunger of 50 mm diameter at the rate of

1.25 mm/min into a soil mass to that required to drive the same plunger at the same rate into a standard sample of crushed stone.

Thus, $$\text{CBR} = \frac{\text{Test load}}{\text{Standard load}} \times 100\% \quad \text{... (7.2)}$$

The test is performed by first compacting the given soil in the AASHO mould at the specified compactive effort as stated in Art. 7.3. The sample is compacted upto a height of 127 mm at the particular moisture content and density at which the CBR value is required. The plunger is then driven into the soil under a steadily increasing static load. The settlement of the plunger is measured with the help of a dial gauge while the corresponding load is obtained from the proving ring. From the results a load-settlement curve is plotted and the test loads for 2.5 mm and 5.0 mm penetration are determined. The values of unit standard loads corresponding to these two penetrations are 70 kg/cm^2 and 105 kg/cm^2 respectively. Therefore, the CBR-values at 2.5 mm and 5.0 mm penetrations can be determined.

Generally, the CBR value at 2.5 mm penetration should be greater than that at 5.0 mm penetration. In that case, the former value is accepted as the CBR value for design purposes.

If the CBR value corresponding to 5 mm penetration exceeds that for 2.5 mm penetration, the test should be repeated. However, if identical results are obtained once again, the CBR value for 5 mm penetration should be used.

7.5.1 *Correction to the curve* : The load-penetration curve should always be convex upwards (curve *A* in Fig.7.3). However, due to surface irregularities, the initial portion of the curve is sometimes concave upwards (curve *B* in Fig.7.3). The curve then must be corrected in the following manner:

(i) The straight portion of curve *B* is projected backwards to meet the X-axis at *O'*.

(ii) The origin *O* is shifted to *O'*.

(iii) Subsequently, all penetrations are measured from the new origin *O'*. Thus, the points corresponding to 2.5 mm and 5.0 mm penetration should be shifted towards the right by an amount equal to the shift of origin.

In order to simulate the worst possible field conditions, the CBR test is sometimes performed on soaked samples. After compacting the sample in the mould, the sample is kept submerged in water for a period of 4 days, after which the sample becomes almost saturated. The CBR test is then performed on this soaked sample.

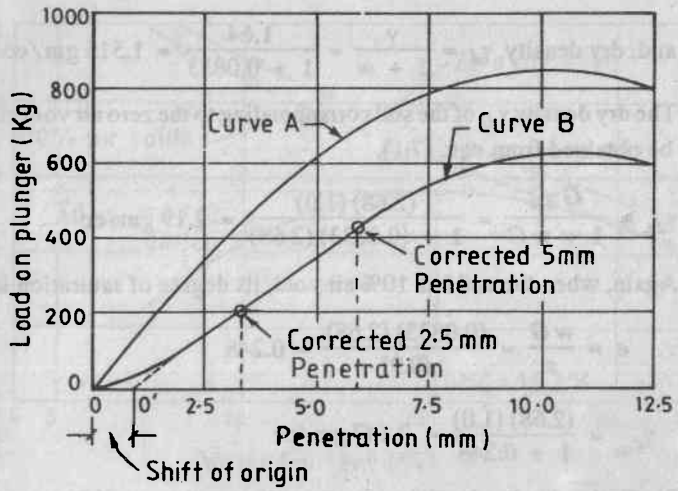


Fig. 7.3

EXAMPLES

Problem 7.1. The results of a laboratory Proctor test are shown below :

No. of Test	1	2	3	4	5	6
Wt. of mould and soil (kg.)	3.526	3.711	3.797	3.906	3.924	3.882
Water content (%)	8.33	10.40	12.23	16.20	17.92	20.39

The mould is 12.7 cm high and has an internal diameter of 10 cm. The weight of the empty mould is 1.89 kg.

(i) Plot the moisture content vs. dry density curve and determine the optimum moisture content and the maximum dry density.

(ii) Plot the zero air void curve and the 10% air void curve.

Given, $G = 2.68$.

Solution: Volume of the mould $= (\pi/4)(12.7)(10^2)$ cc

$= 997$ cc

In the first test, weight of soil $= 3.526 - 1.89$

$= 1.636$ kg $= 1636$ gm

$\therefore$ Bulk density, $\gamma = \frac{W}{V} = \frac{1636}{997} = 1.64$ gm/cc, and, dry density, $\gamma_d = \dfrac{\gamma}{1+w} = \dfrac{1.64}{1+0.0833} = 1.515$ gm/cc.

The dry density γ_{ds} of the soil corresponding to the zero air void condition may be obtained from eqn. (7.1).

$$\gamma_{ds} = \frac{G\gamma_w}{1+wG} = \frac{(2.68)(1.0)}{1+(0.0833)(2.68)} = 2.19 \text{ gm/cc}$$

Again, when the soil has 10% air void, its degree of saturation is 90%.

$$\therefore \quad e = \frac{wG}{s} = \frac{(0.0833)(2.68)}{(0.9)} = 0.248$$

$$\therefore \quad \gamma_{d_{90}} = \frac{(2.68)(1.0)}{1+0.248} = 2.147 \text{ gm/cc}$$

Similarly, the dry densities corresponding to the actual Proctor test, the zero air void condition, and the 10% air void condition are computed for the remaining five tests. Table 7.1 shows the results in a tabular form.

Table 7.1

No. of test	1	2	3	4	5	6
Wt. of mould and soil (kg.)	3.526	3.711	3.797	3.906	3.924	3.882
Water content (%)	8.33	10.40	12.23	16.20	17.92	20.39
Wt. of soil (gm)	1636	1821	1907	2016	2034	1992
Bulk density (gm/cc)	1.641	1.826	1.913	2.022	2.040	1.998
Dry density, γ_d (gm/cc)	1.515	1.654	1.705	1.740	1.730	1.660
Dry density for zero air void, γ_{ds} (gm/cc)	2.190	2.096	2.018	1.869	1.810	1.733
Dry density for 10% air void, γ_{90} (gm/cc)	2.147	2.046	1.964	1.808	1.747	1.667

The compaction curve is shown in Fig. 7.4. From the curve we find,

optimum moisture content = 15.2%

and maximum dry density = 1.76 gm/cc

The zero air void line and the 10% air void line also are shown in Fig. 7.4.

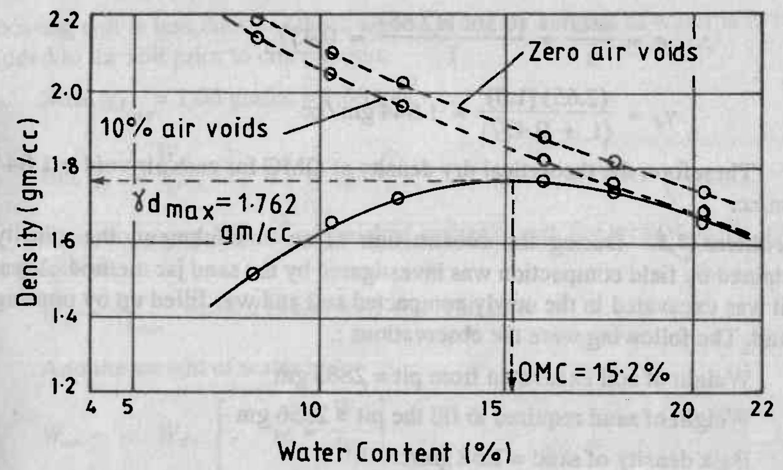


Fig. 7.4

Problem 7.2. The optimum moisture content of a soil is 16.5% and its maximum dry density is 1.57 gm/cc. The specific gravity of solids is 2.65. Determine:

(i) the degree of saturation and percentage of air voids of the soil at OMC.
(ii) the theoretical dry density at OMC corresponding to zero air voids.

Solution: (i) When the soil is at OMC, it has a moisture content of 16.5% and a dry density of 1.57 gm/cc.

Now, we have, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

$$\therefore \quad 1.57 = \frac{(2.65)(1.0)}{1+e}$$

or, $1 + e = \dfrac{2.65}{1.57} = 1.688$

or, $e = 0.688$

Again, $se = wG$, or, $s = \dfrac{wG}{e}$

$$\therefore \quad s = \frac{(0.165)(2.65)}{(0.688)} = 0.635 = 63.5\%$$

Hence, the required degree of saturation is 63.5% and the percentage of air void is (100 – 63.5)% = 36.5%

(ii) At zero air void the soil is fully saturated, i.e., $s = 1$.

$$\therefore \quad e = \frac{wG}{s} = \frac{(0.165)(2.65)}{1} = 0.437$$

$$\gamma_d = \frac{(2.65)(1.0)}{(1+0.437)} = 1.844 \text{ gm/cc}$$

Therefore, the theoretical dry density at OMC for zero air void = 1.844 gm/cc.

Problem 7.3. During the construction of an embankment, the density attained by field compaction was investigated by the sand jar method. A test pit was excavated in the newly compacted soil and was filled up by pouring sand. The following were the observations :

Weight of soil excavated from pit = 2883 gm

Weight of sand required to fill the pit = 2356 gm

Bulk density of sand = 1.52 gm/cc

Moisture content of embankment soil = 16%

Determine the dry density of the compacted soil.

Solution: The volume of sand required to fill up the pit,

$$V = \frac{W}{\gamma} = \frac{2356}{1.52} = 1550 \text{ cc.}$$

$\therefore$ Volume of the pit = 1550 cc.

But, weight of the soil excavated from the pit = 2883 gm

$\therefore$ In-situ bulk density of the soil, $\gamma = \dfrac{2883}{1550} = 1.86$ gm/cc

And, in-situ dry density of the soil,

$$\gamma_d = \frac{\gamma}{1+w} = \frac{1.86}{1+0.16} = 1.66 \text{ gm/cc.}$$

Problem 7.4. It is required to construct an embankment by compacting a soil excavated from nearby borrow areas. The optimum moisture content and the corresponding dry density of this soil were determined in the laboratory and were found to be 22.5% and 1.66 gm/cc respectively. However, the natural moisture content and bulk density of the soil were 9% and 1.78 gm/cc respectively.

Find out the quantity of soil to be excavated and the quantity of water to be added to it, for every 100 m^3 of finished embankment.

Solution: The embankment should be constructed by compacting the soil obtained from borrow area at the optimum moisture content and the corresponding maximum dry density. But the natural moisture content of the existing soil is less than its OMC. Hence, a certain amount of water is to be added to the soil prior to compaction.

Now, $\gamma_{d_{max}} = 1.66$ gm/cc = 1.66 t/m^3

But, $\gamma_d = \dfrac{W_d}{V}$, *or*, $W_d = \gamma_d \cdot V$

Thus, for every 100 m^3 of finished embankment, the weight of dry soil required is,

$$W_d = \gamma_{d_{max}} \cdot V = (1.66)(100) \text{ t} = 166 \text{ t.}$$

And the weight of water is,

$$W_w = w \cdot W_d \quad \left[\because \; w = \frac{W_w}{W_d}\right]$$

$$= (0.225)(166) = 37.35 \text{ t.}$$

The bulk density of the existing soil is 1.78 t/m^3 and its moisture content is 9%.

$\therefore$ Dry density of the existing soil, $\gamma_d = \dfrac{\gamma}{1+w}$

or, $$\gamma_d = \frac{1.78}{1+0.09} = 1.633 \text{ t/m}^3$$

The volume of soil, V_b, to be obtained from borrow area in order to obtain 166 t of dry soil is,

$$V_b = \frac{W_d}{\gamma_d} = \frac{166}{1.633} = 101.65 \text{ m}^3$$

Weight of water available from this soil,

$$W_{w_b} = W_d \cdot w_b = (166)(0.09) = 14.94 \text{ t}$$

$\therefore$ Quantity of water to be added = (37.35 – 14.94) t

Volume of water to be added= $\dfrac{\text{Weight of water}}{\text{Density of water}}$

But, density of water, $\gamma_w = 1$ gm/cc

$= 10^{-6}$ t/cc

$= (1000)(10^{-6})$ t/lit

$= 10^{-3}$ t/lit

$\therefore$ Volume of water to be added $= \dfrac{(22.41)}{(10^{-3})} = 22410$ litre.

Thus, 101.65 m^3 of soil is to be excavated from the borrow pit and 22410 litre of water is to be added to it.

Problem 7.5. An embankment was constructed by compacting a soil at a moisture content of 15.5% and a dry density of 1.72 gm/cc. If the specific gravity of soil solids be 2.68, determine the void ratio and degree of saturation of the embankment soil.

Solution: We have, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

Here, $\gamma_d = 1.72$ gm/cc, $G = 2.68$

$$\therefore \quad 1.72 = \frac{(2.68)(1.0)}{1+e}$$

$$\text{or,} \quad 1 + e = \frac{2.68}{1.72}$$

$$\text{or,} \quad e = 0.558.$$

Again, $se = wG$, or $s = \dfrac{wG}{e}$

$$\therefore \quad s = \frac{(0.155)(2.68)}{(0.558)} = 0.744 = 74.4\%$$

$\therefore$ The required degree of saturation is 74.4%.

Problem 7.6. In order to determine the relative density of a sand sample, its natural moisture content and bulk density were determined in the field and were found to be 7% and 1.61 gm/cc respectively. Samples of this soil were then compacted in a Proctor's mould of 1/30 cft capacity, at the loosest and the densest states. The following data were obtained:

Weight of empty mould = 2100 gm

Weight of mould + soil in the loosest state = 3363.6 gm

Weight of mould + soil in the densest state = 3857.4 gm

Moisture content of the sample used in tests = 11%

Determine the relative density of the sand and comment on its type.

Solution: Volume of the mould $= \dfrac{1}{30}$ cft

$$= \frac{(12^3)(2.54^3)}{(30)} \text{ cc} = 943.89 \text{ cc}$$

In the loosest state,

$$\text{bulk density} = \frac{(3363.6 - 2100)}{(943.89)} = 1.339 \text{ gm/cc}$$

$$\therefore \quad \text{dry density, } \gamma_{d_{min}} = \frac{\gamma}{1+w} = \frac{(1.339)}{(1+0.11)} = 1.206 \text{ gm/cc.}$$

In the densest state,

$$\text{bulk density} = \frac{(3857.4 - 2100)}{943.89} = 1.862 \text{ gm/cc}$$

$$\text{dry density, } \gamma_{d_{max}} = \frac{(1.862)}{(1+0.11)} = 1.677 \text{ gm/cc}$$

In-situ bulk density of the soil = 1.81 gm/cc and its natural moisture content = 7%

$$\therefore \quad \text{In-situ dry density, } \gamma_d = \frac{(1.61)}{(1+0.07)}$$

$$= 1.505 \text{ gm/cc}$$

$$\therefore \quad \text{Relative density, } R_D = \frac{\gamma_{d_{max}}}{\gamma_d} \cdot \frac{\gamma_d - \gamma_{d_{min}}}{\gamma_{d_{max}} - \gamma_{d_{min}}} \times 100\%$$

$$= \frac{(1.677)}{(1.505)} \cdot \frac{(1.505 - 1.206)}{(1.677 - 1.206)} (100)\ \%$$

$$= 70.74\ \%$$

Problem 7.7. It is required to construct an embankment having a total volume of 64000 cu.m. The required soil is to be collected from borrow pits. It was found that the existing soil has a moisture content of 14%, void ratio of 0.63 and specific gravity of solids of 2.68. Laboratory tests indicate that the OMC and maximum dry density of the soil are 19.5% and 1.72 gm/cc respectively. The soil is to be carried from the borrow pit to the construction site by trucks having average net carrying capacity of 5.5 t. Determine the total number of trips the trucks have to make for constructing the entire embankment. Also find out the quantity of water to be added to the borrowed soil before compaction.

Solution:

In-situ dry density of the soil, $\gamma_d = \dfrac{G\gamma_w}{1+e}$

$$= \frac{(2.68)(1.0)}{(1 + 0.63)} = 1.64 \text{ gm/cc}$$

$$= 1.64 \text{ t/m}^3$$

$\therefore$ In-situ bulk density, $\gamma = \gamma_d (1 + w) = (1.64)(1 + 0.14) = 1.87 \text{ t/m}^3$

Now, in 1 m^3 of borrowed soil, quantity of dry soil present is 1.64 t, and quantity of water present = (0.14) (1.64) [$\because W_w = w \cdot W_d$]

$$= 0.23 \text{ t}$$

While constructing the embankment, this soil has to be compacted at a moisture content of 19.5% and at a dry density of 1.72 t/m^3.

For 1 m^3 of finished embankment, dry soil required = 1.72 t,

and water required = (0.195)(1.72) t
= 0.335 t.

$\therefore$ For the entire embankment of 64000 m^3

quantity of dry soil required = (1.72) (64000) = 1,10,080 t

and, quantity of water required = (0.335) (64000) = 21,440 t

As the in-situ dry density of existing soil is 1.64 t/m^3, every cubic metre of excavation will produce 1.64 t of dry soil.

$\therefore$ Total volume of excavation required to be made $= \dfrac{1,10,080}{1.64} = 67,122 \text{ m}^3$

Gross weight of this soil = (1.87) (67,122) t = 1,25,518 t

$\therefore$ No. of trips to be made $= \dfrac{1,25,518}{5.5} = 22,812.5 \approx 22,822$

Weight of water obtained from 67,122 m^3 of borrowed soil

= (67,122) (0.23) t = 15,438 t

Weight of water finally required = 21,440 t.

$\therefore$ Quantity of water to be added before compaction

= (21,440 − 15,438) t = 6,002 t.

Problem 7.8. The rock content in a fill is 80% by dry weight. The rock can be compacted to a minimum void ratio of 0.73. The maximum dry unit weight to which the soil fraction can be compacted is 1.63 gm/cc. What is the maximum dry density to which the fill can be compacted ? Given, specific gravity of the rock = 2.56.

Solution: When the rock present in the fill is compacted to the densest state, its dry unit weight is given by,

$$\gamma_{d_{max}} = \frac{G\gamma_w}{1+e} = \frac{(2.56)(1.0)}{1 + 0.73} = 1.48 \text{ gm/cc.}$$

For the soil, $\gamma_{d_{max}} = 1.63$ gm/cc.

Let us now consider 1 gm of the given fill. According to the question, the weight of rock and soil present in the fill are 0.8 gm and 0.2 gm respectively.

Now, volume of 0.8 gm of rock $= \dfrac{0.8}{1.48}$ cc $= 0.54$ cc.

and, volume of 0.2 gm of dry soil $= \dfrac{0.2}{1.63} = 0.123$ cc.

$\therefore$ Total volume of 1 gm of fill = 0.54 + 0.123 = 0.663 cc.

$\therefore$ Dry unit weight of the fill $= \dfrac{\text{dry weight}}{\text{volume}}$

$$= \frac{1}{0.663} = 1.508 \text{ gm/cc.}$$

Problem 7.9. The results of a laboratory CBR test are shown below :

No. of test	1	2	3	4	5	6	7	8	9	10	11	12
Penetration (mm)	0	0.5	1.0	1.5	2.0	2.5	3.0	4.0	5.0	7.5	10.0	12.5
Load (kg)	0	19.8	52.5	93.7	132.1	171.9	207.0	288.8	322.2	401.7	431.8	458.3

Determine the CBR value of the soil. Given, unit standard loads for 2.5 mm and 5.0 mm penetrations are 70 and 105 kg/cm^2 respectively.

Solution: Fig. 7.5 shows the load vs. penetration curve. As the curve is initially concave upwards, an initial zero correction is required.

The straight portion of the curve is projected backwards to intersect the X-axis at O', which then becomes the new origin. Consequently, all points on the penetration axis are shifted to the right by an equal amount.

From Fig. 7.5 we obtain,

test load for corrected 2.5 mm penetration = 200 kg.

and, test load for corrected 5.0 mm penetration = 332 kg.

Area of CBR plunger $= \dfrac{\pi}{4}(5.0)^2 \text{ cm}^2 = 19.635 \text{ cm}^2$

$\therefore$ Unit test load for 2.5 mm penetration $= \dfrac{200}{19.635} = 10.19$ kg/cm^2

and, unit test load for 5.0 mm penetration $= \dfrac{332}{19.635} = 16.91$ kg/cm^2

$\therefore$ CBR value for 2.5 mm penetration $= \dfrac{10.19}{70} \times 100\% = 14.6\%$

CBR value for 5.0 mm penetration $= \dfrac{16.91}{105} \times 100\% = 16.1\%$

Thus, CBR value for 5.0 mm penetration is greater than that for 2.5 mm penetration. Therefore, the CBR test has to be repeated and if similar results are obtained once again, then the CBR value of 16.1% should be accepted.

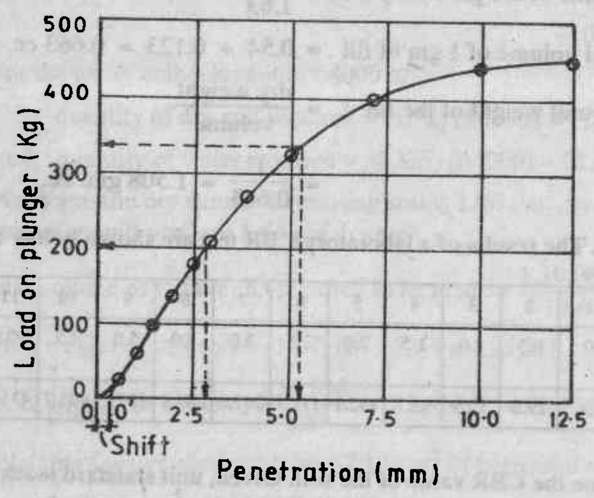


Fig. 7.5

EXERCISE 7

7.1. The following are the results of a Proctor compaction test performed on a soil sample.

Water Content (%)	9.2	12.7	15.5	18.3	20.2
Bulk Density (gm/cc)	1.524	1.749	1.949	2.049	2.019

(i) Plot the water content vs. dry density relationship and determine the optimum moisture content and the corresponding maximum dry density of the soil.

(ii) If the specific gravity of soil solids be 2.70, plot the zero air void line. [Ans. OMC = 17.6%, γ_{dmax} = 1.74 gm/cc]

7.2. The results of a standard Proctor test are shown below.

Water Content (%)	7.8	11.6	14.9	17.7	20.1	22.5
Wt. of soil and mould (gm)	3263.4	3523.28	3734.8	3852.9	3832.7	3765.1

The height and internal diameter of the mould are 12.6 cm and 10.1 cm respectively. The empty mould weighs 1950 gm. Plot the compaction curve and determine the optimum moisture content and the corresponding dry and bulk densities of the soil.

Also plot the zero air void line and the 80% saturation line.

Given, specific gravity of solids = 2.69. [Ans: OMC = 17%, γ_d = 1.6 gm/cc, γ = 1.87 gm/cc]

7.3. The in-situ density of a soil mass is being determined by the core cutter method. The height and internal diameter of the core are 12.7 cm and 10 cm respectively and its weight, when empty, is 1847 gm. When the core is filled with soil, it weighs 3674 gm. If the specific gravity of solids be 2.67 and the degree of saturation of the soil be 63%, determine the in-situ dry density of the soil. The in-situ void ratio of the soil is found to be 0.85. [Ans. 1.526 gm/cc]

7.4. An embankment of trapezoidal cross-section is to be constructed for a 2 km long highway. The embankment should have a height of 2.2 m and a top width of 10 m. The sides of the embankment are to be sloped at 2 H : 1 V. The soil obtained from the borrow area is tested in the laboratory and is found to have the following properties :

Natural moisture content = 12%

In-situ bulk density = 1.8 t/m^3

Optimum moisture content = 19%

Dry density at OMC = 1.65 t/m^3

Determine the quantity of soil to be excavated and the quantity of water to be added to it before constructing the embankment. [Ans: 65055 m^3; 7318 m^3]

7.5. Determine the magnitudes of compactive effort imparted to a soil during:

(i) Standard Proctor Test

(ii) Modified AASHO Test.

[Hints: Compactive effort = Wt. of rammer × height of fall × no. of blows/layer × no. of layers]

7.6. The specific gravity of solids of a soil is 2.65. Determine the quantity of dry soil and water required to compact the soil in a Proctor mould having $D = 10$ cm and $H = 12.7$ cm, at a void ratio of 0.6 and at a moisture content of 20%. [Ans: 1652 gm; 330 cc]

7.7. Three identical triaxial test samples of 7.5 cm height and 3.75 cm diameter are to be prepared at a moisture content of 15% and a dry density of 1.48 gm/cc. Determine the total quantity of oven-dried soil and volume of water required for the purpose . [Ans: 367.8 gm, 55.2 cc]

7.8. Determine the CBR value of a given soil from the following data obtained from a laboratory CBR test :

Load (kg)	0	19.8	50.1	81.8	120.0	170.1	421.7	605.9	699.3	662.8
Penetration (mm)	0	0.5	1.0	1.5	2.0	2.5	5.0	7.5	10.0	12.5

Plot the load-penetration curve and determine the CBR value of the soil. Comment on the test results. [Ans. 23.7%]

8

SHEAR STRENGTH

8.1 Introduction: When an external load is applied on a soil mass, shearing stresses are induced in it. If the shear stress developed on any plane in the soil exceeds a certain limiting value, failure of the soil occurs. The maximum shear stress which a given soil can withstand is called its shear strength.

The factors governing the shear strength of a soil are :

(i) internal friction, i.e., the resistance due to particle interlocking

(ii) cohesion, i.e., the resistance due to the internal structural bond which tends to hold the particles together.

According to Coulomb's law, the shear strength, τ, of a soil is given by:

$$\tau = c + \sigma \tan \phi \qquad ...(8.1)$$

where, σ = normal stress acting on the soil

c = cohesion

ϕ = angle of internal friction

The factors c and ϕ are called the shear parameters of a soil.

When expressed graphically, eqn. (8.1) can be represented by a straight line called the failure envelope. The general form of failure envelope for a cohesionless, a cohesive and a $c - \phi$ soil are shown in Fig. 8.1 (a), (b) and (c) respectively.

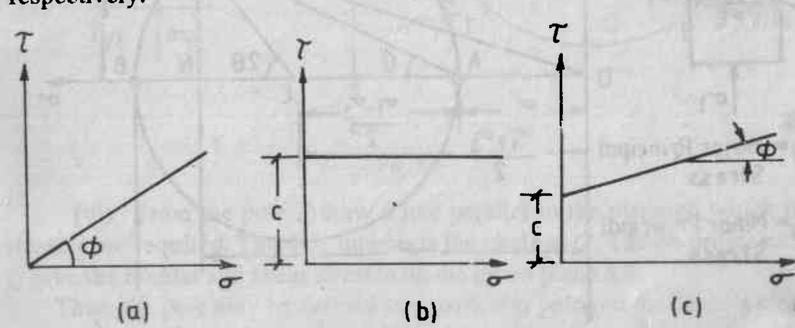


Fig. 8.1

The shear parameters of any soil depend not only on the nature of the soil but also on such factors like moisture content and loading conditions. At very low moisture content a cohesive soil may develop a certain amount of internal friction. Likewise at high moisture contents a cohesionless soil may show the signs of having an apparent cohesion.

8.2 Mohr's circle of stress: This is a graphical representation of the stress conditions in a soil mass which enables one to find out the stresses developed on any plane within the soil due to an external loading system.

In a stressed material, a plane which is subjected to only a normal stress, but no shear stress, is called a principal plane. Through any point in the material, two such planes exist. These planes are called the major and the minor principal planes, and are orthogonal to each other. If the principal stresses, σ_1 and σ_3, are known, the normal stress σ and shear stress τ on a plane inclined at an angle θ to the major principal plane is given by,

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta \quad \text{...(8.2)}$$

and, $$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta \quad \text{...(8.3)}$$

Equations (8.2) and (8.3) can be represented by a Mohr Circle, as illustrated in Fig. 8.2. The co-ordinates of any point on the circumference of the circle give the stress conditions on a particular plane represented by that point.

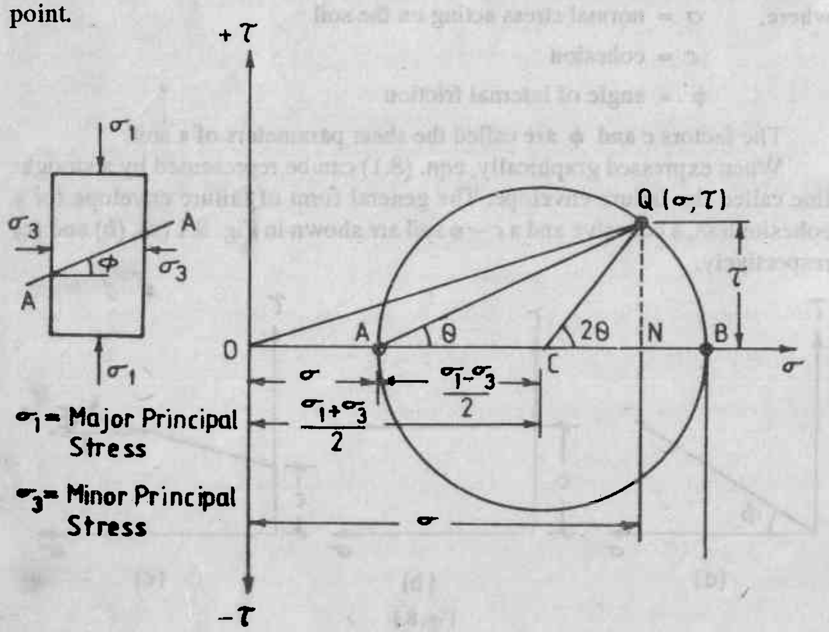


Fig. 8.2

8.3 Pole: The concept of the pole, or the origin of the planes, is very useful in such problems where the locations of the principal planes are not known.

Consider the soil element subjected to a system of external stresses as shown in Fig. 8.3. It is required to determine the normal and shear stresses acting on the plane *AA*, inclined at an angle θ to the horizontal.

Considering the free body diagram of the element it can be proved that the element can be in equilibrium only if, $\tau_{xy} = \tau_{yx}$.

The procedure for drawing the Mohr Circle and locating the pole are as follows :

(i) Choose the co-ordinate axes and select a vector scale.

(ii) Locate the points *A* and *B* such that they represent the stresses on the horizontal and vertical boundaries respectively, of the element.

(iii) Join *AB*. It intersects the σ-axis at *C*.

(iv) With *C* as centre and *CA* = *CB* as radius, draw the Mohr circle.

(v) The point *A* represents the stress conditions on the horizontal plane. From *A*, draw a straight line parallel to this plane. It intersects the circumference at *P*. Again, if from *B* a line is drawn parallel to the vertical plane (since the point *B* represents the stresses acting on this plane), it will intersect the circle at the same point *P*. This is the pole of Mohr's circle.

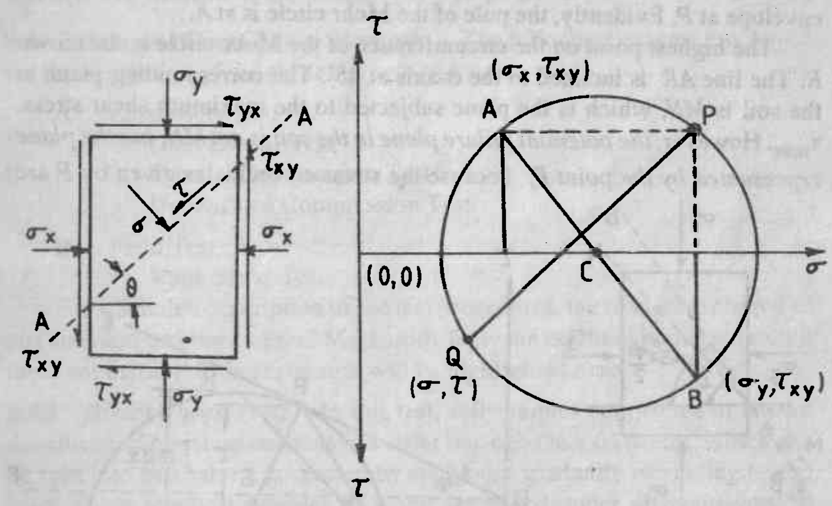


Fig. 8.3

(vi) From the pole *P* draw a line parallel to the plane on which the stresses are required. This line intersects the circle at *Q*. The co-ordinates of *Q* give the normal and shear stresses on the given plane *AA*.

Thus, the pole may be defined as a particular point on the Mohr's circle such that, if a line is drawn from this point making it parallel to any given plane within the soil mass, then, the co-ordinates of the point of intersection of this line with the circle will represent the stresses acting on that plane.

8.3.1 ***Sign convention:*** The following sign conventions are normally followed for plotting the stress co-ordinates :

Normal stress: Compressive stresses are taken as positive and tensile stresses as negative.

However, soils can with stand only compression and not tension. Hence the normal stress on any plane of a soil element which is in static equilibrium is always positive.

Shear stress: The sign of a shear stress is determined on the basis of the direction of its moment about any arbitrary point inside the soil mass. If the moment acts in the anti-clockwise direction, the shear stress is positive, whereas if it acts in the clockwise direction, the shear stress is negative.

8.3.2 ***Location of the failure plane :*** Fig. 8.4 represents a soil sample subjected to a major principal stress σ_1 and a minor principal stress σ_3. As the sample is on the verge of failure, the Mohr circle has touched the failure envelope at *P*. Evidently, the pole of the Mohr circle is at *A*.

The highest point on the circumference of the Mohr circle is the crown *R*. The line *AR* is inclined to the σ-axis at 45°. The corresponding plane in the soil is *MN*, which is the plane subjected to the maximum shear stress, τ_{max}. However, *the potential failure plane in the soil is not MN, but the plane represented by the point P,* because the stress co-ordinates given by *P* are

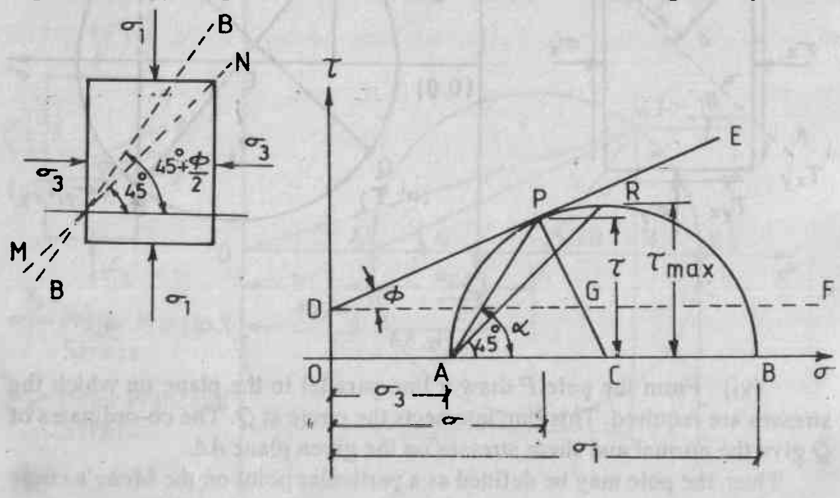


Fig. 8.4

such that coulomb's equation is satisfied as the point *P* lies on the failure envelope. In order to determine the location of this plane, join *PA* and *PC*.

Now, $\angle PCB = \angle PAC + \angle APC$

As $AC = PC$, $\angle APC = \angle PAC = \alpha$

$\therefore$ $\angle PCB = \alpha + \alpha = 2\alpha.$

Again, since $DF \parallel OB$, $\angle PGF = \angle PCB = 2\alpha$

In $\Delta\, PDG$, $\angle PGF = \angle PDG + \angle DPG$

or, $2\alpha = \phi + 90°$ [$\because PG \perp DE$, $\therefore \angle DPG = 90°$]

or, $$\alpha = 45° + \phi/2 \qquad ...(8.4)$$

In Fig. 8.4, the plane *BB*, drawn at (45° + φ/2) to the major principal plane, represents the failure plane.

It can be proved that, at failure the relationship between the two principal stresses is given by,

$$\sigma_1 = \sigma_3 \tan^2(45° + \phi/2) + 2c\tan(45° + \phi/2) \qquad ...(8.5)$$

or, $$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \qquad ...(8.6)$$

where, $$N_\phi = \text{flow value} = \tan^2(45° + \phi/2) \qquad ...(8.7)$$

8.4 Determination of Shear Strength: The following tests are employed for the evaluation of the shear strength of a soil :

A. Laboratory tests :
1. Direct Shear Test
2. Triaxial Compression Test
3. Unconfined Compression Test.

B. Field Test :
1. Vane Shear Test

For a detailed description of the test procedures, the reader is referred to any standard text book of Soil Mechanics. Only the essential points regarding the computation of shear strength will be highlighted here.

8.4.1 ***Direct Shear Test:*** In this test, soil samples compacted at known densities and moisture contents in a shear box of 6 cm × 6 cm size, which can be split into two halves, is sheared by applying a gradually increasing lateral load. Three identical samples of a soil are tested under different vertical compressive stresses and the corresponding shear stresses at failure are determined. A graph is then plotted between normal stress and shear stress. Results of each test are represented by a single point. Three points obtained from the three tests are joined by a straight line which is the failure envelope for the given soil. The slope of this line gives the angle of internal friction, while the intercept from the τ–axis gives the value of cohesion of the soil.

8.4.2 *Triaxial Compression Test:* In this test, cylindrical soil specimens of 3.8 cm diameter and 7.6 cm height, enclosed in an impermeable rubber membrane, are placed inside the triaxial cell. An all-round cell pressure, σ_3, is applied on the sample. Simultaneously, a gradually increasing vertical stress is applied until either the sample fails, or its axial strain exceeds 20%. Stress vs. strain curves are plotted to determine the normal stress at failure. This stress is called the deviator stress, σ_d. The major principal stress, σ_1, is obtained from the following relation (refer Fig. 8.5) :

$$\sigma_1 = \sigma_3 + \sigma_d \quad \text{....(8.8)}$$

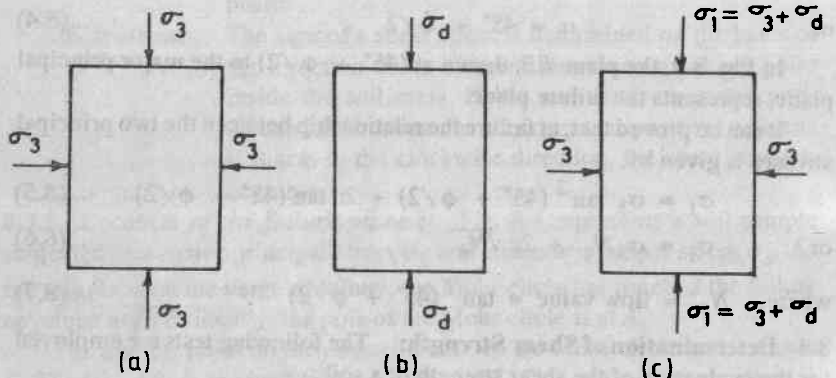


Fig. 8.5

Three samples of a soil are tested under different cell pressures. From the results, three Mohr circles are constructed, and a common tangent is drawn to them. This is the failure envelope.

The normal stress at any point during the test is determined by dividing the normal load obtained from the reading of the proving ring by the cross-sectional area of the sample. Due to the bulging of the sample during shear, the cross-sectional area should be modified using the following equation :

$$A_c = A_0/(1 - \varepsilon) \quad \text{...(8.9)}$$

where, A_c = corrected area

A_0 = initial area

ε = axial strain = $\Delta L/L$

where, ΔL = axial compression

L = initial length

In the drained triaxial tests, the volume of the sample may change during the test due to expulsion or absorption of water. In that case, the corrected area should be determined from :

$$A_c = \frac{V_1 \pm \Delta V}{L_1 - \Delta L} \quad \text{...(8.10)}$$

where, V_1 = initial volume of the specimen

ΔV = change in volume due to drainage.

L_1 = initial length of the specimen

ΔL = change in length of the specimen

8.4.3 *Unconfined Compression Test :* This is a special case of triaxial test in which $\sigma_3 = 0$. We have, from eqn. (8.5)

$$\sigma_1 = \sigma_3 \tan^2 (45° + \phi/2) + 2c \tan (45° + \phi/2)$$

As $\sigma_3 = 0$, for an unconfined compression test,

$$\sigma_1 = 2c \tan (45° + \phi/2) \quad \text{...(8.11)}$$

A number of tests on identical specimens will give the same value of σ_1. Thus, only one equation is available while two unknowns, viz., c and ϕ, are involved. Hence, eqn. (8.11) cannot be solved without having a prior knowledge of any one of the unknowns. Due to this reason, the unconfined compression test is employed to determine the shear parameters of purely cohesive soils only. For such soils, $\phi = 0°$, and hence,

$$\sigma_1 = 2c \tan 45° = 2c$$

The vertical stress σ_1 at failure, known as the unconfined compressive strength and denoted by q_u, is obtained by dividing the normal load at failure by the corrected area, as given by eqn. (8.9)

Thus, $q_u = 2c$

or, $$c = \frac{q_u}{2} \quad \text{...(8.12)}$$

8.4.4 *Vane Shear Test :* This is a field test used for the direct determination of the shear strength of a soil. Generally this test is conducted in soft clay situated at a great depth, samples of which are difficult to obtain.

The apparatus consists of four metal blades, called vanes, mounted on a steel rod, as shown in Fig. 8.6. The device is pushed slowly upto the desired depth

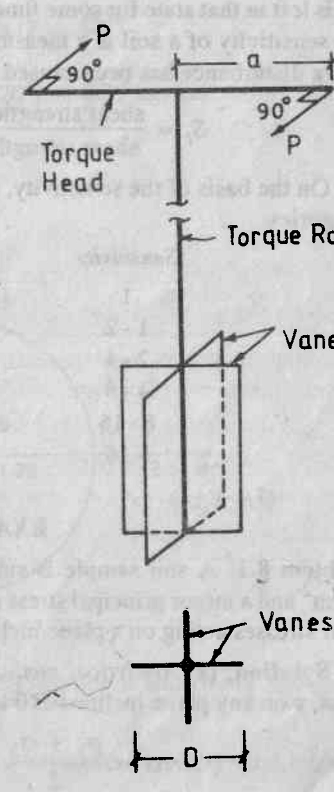


Fig. 8.6

and is rotated at a uniform speed by applying a torque through the torque rod. The amount of torque applied is recorded on a dial fitted to the rod. Failure occurs when the vane can be rotated without any further increase in the torque.

For a cohesive soil, $\phi = 0$. Hence coulomb's equation reduces to :

$$s = c$$

Thus, for a cohesive soil, the shear strength is equal to its cohesion. In a vane shear test, the cohesion, and hence the shear strength can be determined from:

$$c = \frac{T}{\pi D^2 \left(\frac{H}{2} + \frac{D}{6}\right)} \qquad ...(8.13)$$

where, T = torque applied (= $P.a$)

H = height of the vane

D = diameter of the vane.

8.5 Sensitivity: When the shear stresses developed in a soil exceeds its shear strength, the soil fails by shear and loses its strength. However, if the soil is left in that state for some time, it regains some of its original strength. The sensitivity of a soil is a measure of its capability of regaining strength after a disturbance has been caused in the soil. It is expressed as,

$$S_t = \frac{\text{shear strength in the undisturbed state}}{\text{shear strength in the remoulded state}} \qquad ...(8.14)$$

On the basis of the sensitivity, clayey soils are divided in the following categories:

Sensitivity	*Nature of clay*
1	Insensitive
1 - 2	Low sensitive
2 - 4	Medium sensitive
4 - 8	Sensitive
8 - 16	Extra sensitive
> 16	Quick clay

EXAMPLES

Problem 8.1. A soil sample is subjected to a major principal stress of 2 kg/cm^2 and a minor principal stress of 1.1 kg/cm^2. Determine the normal and shear stresses acting on a plane inclined at 30° to the major principal stress.

Solution: (a) *Analytical method* : The normal stress, σ and the shear stress, τ on any plane inclined at θ to the major principal plane is given by :

$$\sigma = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2} \cos 2\theta$$

and, $$\tau = \frac{\sigma_1 - \sigma_3}{2} \sin 2\theta$$

Here, $\sigma_1 = 2$ kg/cm^2 and $\sigma_3 = 1.1$ kg/cm^2

The given plane is inclined at 30° to the major principal stress. But the direction of major principal stress is perpendicular to the major principal plane. Hence the angle of inclination between the given plane and the major principal plane is,

$$\theta = 90° - 30° = 60°$$

$$\therefore \quad \sigma_1 = \frac{(2 + 1.1)}{2} + \frac{(2 - 1.1)}{2} . \cos(2 \times 60°)$$

$$= 1.55 + (0.45)(\cos 120°)$$

$$= 1.55 + (0.45)(-1/2)$$

$$= 1.325 \text{ kg/cm}^2$$

and $$\tau = \frac{(2 - 1.1)}{2} \sin(2 \times 60°)$$

$$= (0.45)(\sin 120°) = 0.39 \text{ kg/cm}^2$$

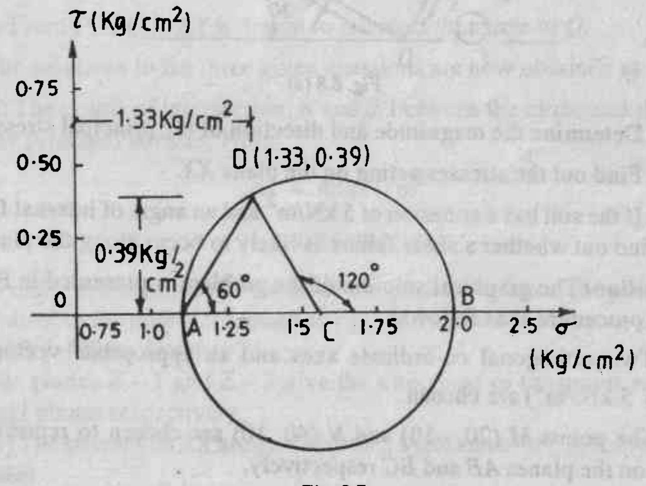


Fig. 8.7

(b) *Graphical method* : The graphical solution is shown in Fig 8.7. The procedure is stated below :

(i) The Mohr circle is drawn with $\sigma_1 = 2.0$ kg/cm^2 and $\sigma_3 = 1.1$ kg/cm^2

(ii) From the centre C of this circle, CD is drawn at an angle of 120° ($= 2\theta$) to the σ-axis. This line intersects the circle at D.

Alternatively, from the point A corresponding to σ_3, a straight line AD is drawn at an angle of 60° ($= \theta$) to the σ-axis. AD also intersects the circle at the same point D.

(iii) The co-ordinates of D give the normal and shear stresses acting on the given plane. From Fig. 8.7 we obtain,

$$\sigma = 1.33 \text{ kg/cm}^2$$

and $$\tau = 0.39 \text{ kg/cm}^2$$

Problem 8.2. The stresses acting on a soil element are shown in Fig. 8.8 (a).

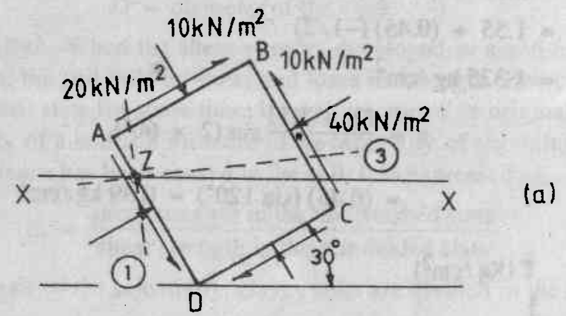


Fig. 8.8 (a)

(i) Determine the magnitude and direction of the principal stresses.

(ii) Find out the stresses acting on the plane XX.

(iii) If the soil has a cohesion of 5 kN/m^2 and an angle of internal friction of 25°, find out whether a shear failure is likely to occur along the plane XX.

Solution: The graphical solution of the problem is presented in Fig. 8.8 (b). The procedure is as follows:

1. Two orthogonal co-ordinate axes and an appropriate vector scale (1 cm = 5 kN/m^2) are chosen.

2. The points M (20, –10) and N (40, 10) are chosen to represent the stresses on the planes AB and BC respectively.

3. M and N are joined and the mid-point O of MN is located.

4. With O as centre and MN as diameter, the Mohr circle is drawn.

5. The point M represents the stresses on the plane AB. From M, a straight line MP is drawn parallel to AB, to intersect the circle at P. P is the pole.

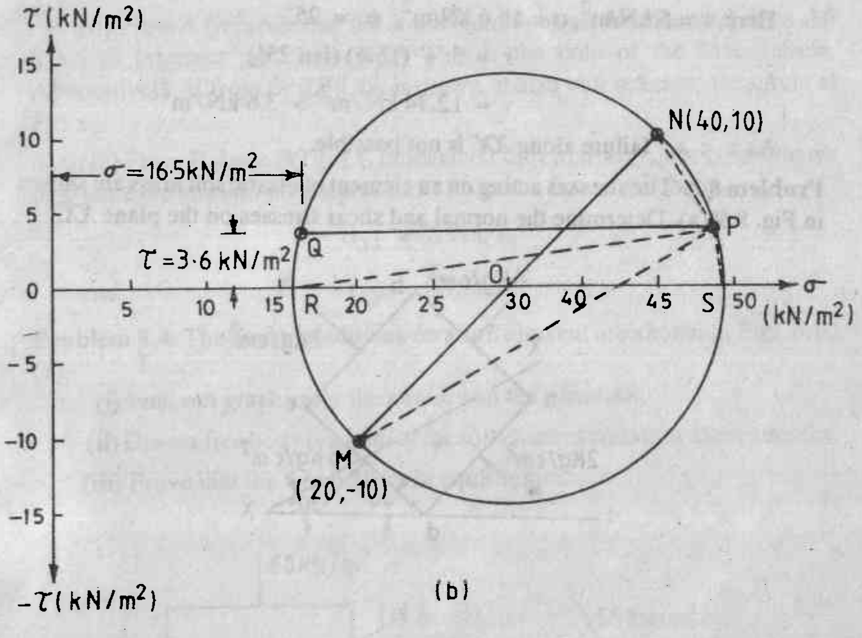


Fig. 8.8 (b)

6. From P, $PQ \parallel XX$ is drawn to intersect the circle at Q.

The solutions to the three given questions are now obtained as follows :

(i) The points of intersection, R and S, between the circle and the σ-axis give the principal stresses. Here,

$$\sigma_1 = 48 \text{ kN/m}^2$$

and $$\sigma_3 = 16.2 \text{ kN/m}^2$$

In order to locate the directions of the principal planes, the points R and S are joined to the pole P. Through any point Z in the soil element, $Z-1 \parallel PS$ and $Z-3 \parallel PR$ are drawn.

The planes $Z-1$ and $Z-3$ give the directions of the major and minor principal planes respectively.

(ii) The stresses on XX are given by the co-ordinates of Q. From the figure we obtain.

$$\sigma_{XX} = 16.5 \text{ kN/m}^2 \text{ and } \tau_{XX} = 3.6 \text{ kN/m}^2$$

(iii) The normal stress on XX is 16.6 kN/m^2. From coulomb's equation, the shear strength of a soil is given by,

$$s = c + \sigma \tan\phi$$

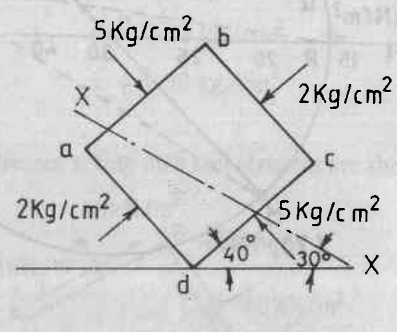
Here, $c = 5$ kN/m^2, $\sigma = 16.6$ kN/m^2, $\phi = 25°$

$\therefore \quad s = 5 + (16.6)(\tan 25°)$

$= 12.74$ kN/m^2 > 3.6 kN/m^2

As $\tau < s$, failure along XX is not possible.

Problem 8.3. The stresses acting on an element of elastic soil mass are shown in Fig. 8.9 (a). Determine the normal and shear stresses on the plane XX.

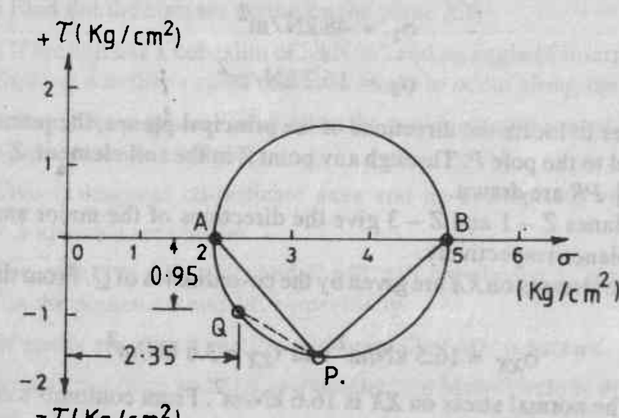


(a)

Fig. 8.9 (a)

Solution: Fig. 8.9 (b) shows the graphical solution. The solution is obtained in the following steps:

(i) Locate the points A (2, 0) and B (5, 0) which represent the principal stresses acting on the soil element. With AB as diameter, draw the Mohr's circle.



(b)

Fig. 8.9 (b)

(ii) From A (representing the stress conditions on the plane bc) draw $AP \parallel bc$, to intersect the circle at P. This is the pole of the Mohr circle. Alternatively, if from B, $BP \parallel ba$ is drawn, it also will intersect the circle at P.

(iii) From P, draw $PQ \parallel XX$. It intersects the circle at Q. The co-ordinates of Q give the stresses on the plane XX. From the figure we obtain,

$$\sigma_{XX} = 2.35 \text{ t/m}^2$$

and $$\tau_{XX} = 0.95 \text{ t/m}^2$$

Problem 8.4. The stress conditions on a soil element are shown in Fig. 8.10 (a).

(i) Find out graphically the stresses on the plane AA.

(ii) Draw a free body diagram of the soil element and show these stresses.

(iii) Prove that the free body is in equilibrium.

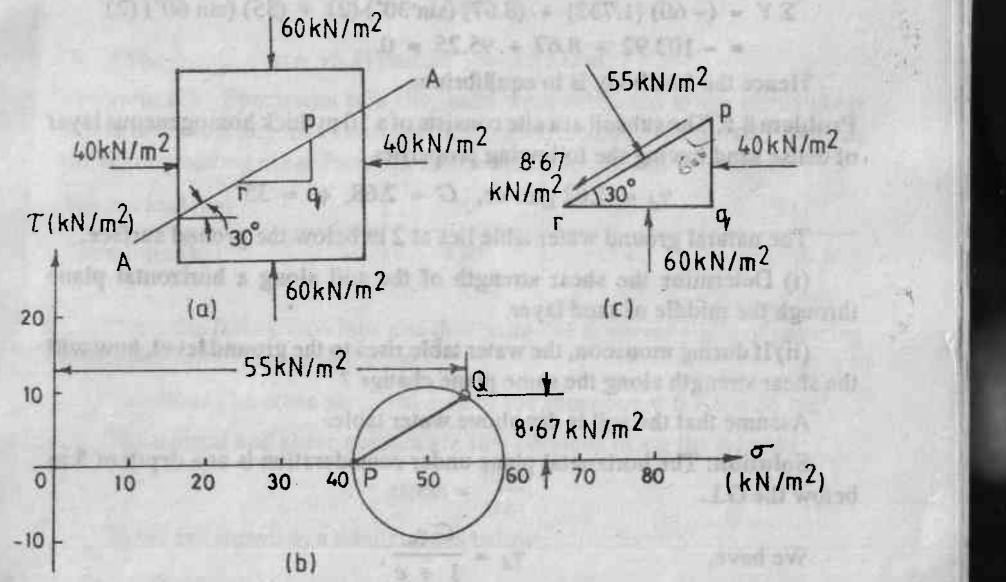


Fig. 8.10

Solution: (i) The graphical solution of the problem is shown in Fig. 8.10 (b), from which we get,

$$\sigma = 55 \text{ kN/m}^2$$

$$\tau = 8.67 \text{ kN/m}^2$$

(ii) The free body diagram of the soil element *pqr*, bounded by the vertical plane, the horizontal plane and the given plane *AA*, is shown in Fig. 8.10 (c).

(iii) The free body will be in equilibrium if the sum of the components of all forces acting on it along any two orthogonal axes separately be zero.

Let, $pq = 1$ **unit,**

$$\therefore \quad pr = \frac{pq}{\sin 30^\circ} = \frac{1}{0.5} = 2 \text{ units}$$

and, $$qr = \frac{pq}{\tan 30^\circ} = \frac{1}{1/\sqrt{3}} = \sqrt{3} = 1.732 \text{ units}$$

Considering unit thickness of the element,

$$\Sigma F_X = (-40)(1) - (8.67)(\cos 30^\circ)(2) + (55)(\cos 60^\circ)(2)$$
$$= -40 - 15 + 55 = 0.$$
$$\Sigma Y = (-60)(1.732) + (8.67)(\sin 30^\circ)(2) + (55)(\sin 60^\circ)(2)$$
$$= -103.92 + 8.67 + 95.25 = 0.$$

Hence the free body is in equilibrium.

Problem 8.5. The subsoil at a site consists of a 10 m thick homogeneous layer of dense sand having the following properties :

$$\gamma_d = 1.62 \text{ gm/cc}, \quad G = 2.68, \quad \phi = 35^\circ$$

The natural ground water table lies at 2 m below the ground surface.

(i) Determine the shear strength of the soil along a horizontal plane through the middle of sand layer.

(ii) If during monsoon, the water table rises to the ground level, how will the shear strength along the same plane change ?

Assume that the soil is dry above water table.

Solution: The horizontal plane under consideration is at a depth of 5 m below the G.L.

We have, $$\gamma_d = \frac{G\gamma_w}{1+e},$$

or, $$\frac{(2.68)(1.0)}{1+e} = 1.62$$

or, $$e = 0.654$$

Now, $$\gamma_{sat} = \frac{G+e}{1+e} \cdot \gamma_w$$
$$= \frac{2.68 + 0.654}{1 + 0.654}(1.0)$$
$$= 2.02 \text{ gm/cc} = 2.02 \text{ t/m}^3$$

(i) The normal stress on the given plane,

$$\sigma = \gamma_d \cdot z_1 + \gamma_{sub} \cdot z_2$$
$$= (1.62)(2) + (1.02)(3) = 6.3 \text{ t/m}^2$$

$\therefore$ **Shear strength of the soil at this plane,**

$$s = c + \sigma \tan \phi$$
$$= 0 + (6.3)(\tan 35^\circ) = 4.41 \text{ t/m}^2$$

(ii) In this case the entire soil mass is submerged.

$$\therefore \quad \sigma = \gamma_{sub} \cdot z = (1.02)(5) = 5.1 \text{ t/m}^2$$

and, $$s = (5.1)(\tan 35^\circ) = 3.57 \text{ t/m}^2$$

Problem 8.6 Specimens of a silty sand were subjected to the direct shear test in the laboratory, in a shear box of 6 cm × 6 cm size. The normal load and the corresponding shear forces at failure are shown below :

Normal load (kg)	10	20	30
Shear force (kg)	9.90	15.41	20.88

Draw the failure envelope and determine the apparent angle of shearing resistance and cohesion of the soil.

Solution: The cross-sectional area of the shear box = 6 × 6 = 36 cm^2.

The normal and shear stresses are first obtained using the relation,

$$\text{stress} = \frac{\text{load}}{\text{area}}$$

These are shown in a tabular form below :

Normal load (kg)	10	20	30
Shear force (kg)	9.90	15.41	20.88
Normal stress (kg/cm^2)	0.28	0.56	0.83
Shear stress (kg/cm^2)	0.275	0.428	0.580

In Fig. 8.11 the normal and shear stresses are plotted along the horizontal and vertical axes respectively. Three points thus obtained are then joined by a straight line. This is the failure envelope for the given soil.

The intercept of the failure envelope on the τ-axis represents the apparent cohesion, which is found to be 0.12 kg/cm^2. The apparent angle of shearing resistance is given by the angle of obliquity of the failure envelope to the horizontal, and is found to be 28.5°.

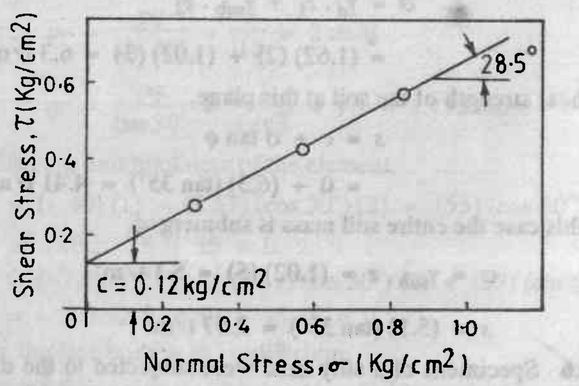


Fig. 8.11

Problem 8.7. A direct shear test was performed on a sample of dry sand. Under a normal stress of 1.5 kg/cm^2, failure occurred when the shear stress reached 0.65 kg/cm^2. Draw the Mohr circle and the failure envelope. Hence determine the orientation of the principal planes and the magnitude of the principal stresses.

Solution: The construction is shown in Fig 8.12. The procedure is as follows:

(i) Choose two orthogonal co-ordinate axes and a suitable vector scale. The scale chosen in this problem is : 1 cm = 0.4 kg/cm^2.

(ii) Locate the point Q corresponding to σ = 1.5 kg/cm^2 and τ = 0.65 kg/cm^2.

(iii) Since the soil is a dry sand, it should not have any apparent cohesion and the failure envelope should pass through the origin. Join the origin O and the point Q. OQ is the failure envelope.

(iv) The point Q represents the stresses on the failure plane. But in a direct shear test, the failure plane is always horizontal. Now, the point Q must lie on the Mohr circle and at this point the circle must touch the failure envelope. If Q can be joined to the centre of the circle the resulting line will

be the radius which must be perpendicular to OQ, since OQ is a tangent to the circle.

Thus, in order to locate the centre of Mohr circle, draw $QC \perp OQ$. QC meets the σ-axis at C, which then, is the centre of Mohr circle.

(v) With C as centre and CQ as radius, draw the Mohr circle. It intersects the σ-axis at A and B, which, then, represent the minor principal stress σ_3 and the major principal stress σ_1 respectively.

From Fig. 8.12, we obtain, σ_3 = 1.08 kg/cm^2, σ_1 = 2.47 kg/cm^2.

(vi) Draw a horizontal line PQ through Q. It intersects the circle at P. This is the pole of the Mohr circle.

(vii) Join PA and PB. These two lines are parallel to the directions of the planes on which σ_3 and σ_1, respectively, act. From the figure we obtain,

$$\angle PAB = 32.5° \quad \text{and} \quad \angle PBA = 57.5°$$

Hence the minor and the major principal planes are inclined to the horizontal at 32.5° and 57.5° respectively. The orientation of the planes are shown in Fig. 8.12 (b).

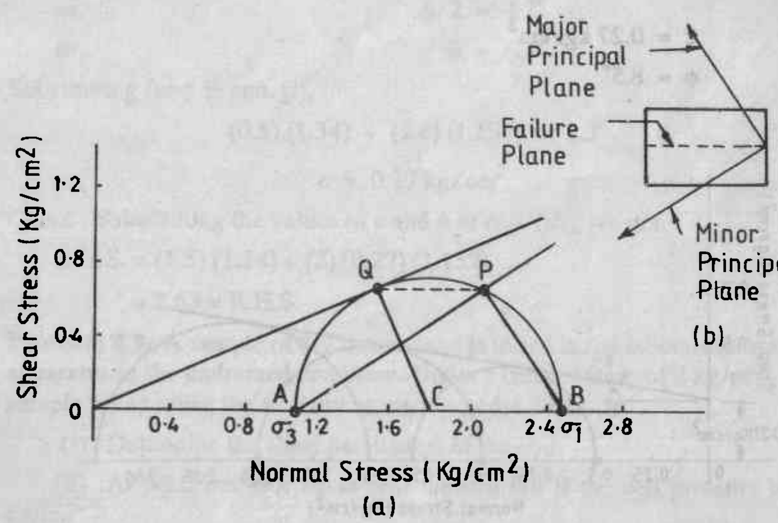


Fig. 8.12

Problem 8.8. Three identical specimens of a partially saturated clay were subjected to an unconsolidated undrained triaxial test and the following results were obtained:

Sample No.	Cell Pressure (kg/cm^2)	Deviator stress (kg/cm^2)
1.	0.5	0.80
2.	1.0	0.97
3.	1.5	1.13

Determine the shear parameters of the soil (i) graphically (ii) analytically.

Solution: In a triaxial test the cell pressure acts as the minor principal stress, while the major principal stress is the sum of the cell pressure and the deviator stress at failure. The values of σ_3 and σ_1 are shown below:

Sample No.	σ_3 (kg/cm^2)	σ_d (kg/cm^2)	σ_1 (kg/cm^2)
1.	0.5	0.80	1.30
2.	1.0	0.97	1.97
3.	1.5	1.17	2.67

(i) *Graphical solution* : Three Mohr circles are constructed and a common tangent is drawn through them (Fig. 8.13). The shear parameters are found to be :

$$c = 0.27 \text{ kg/cm}^2$$
$$\phi = 8.5°$$

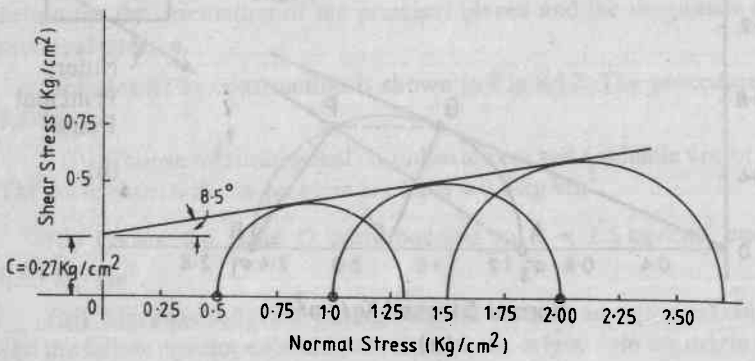


Fig. 8.13

(ii) *Analytical solution* : From eqn. (8.6) we have,

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

In case of the first sample, $\sigma_3 = 0.5$ kg/cm^2 and $\sigma_1 = 1.30$ kg/cm^2

Substituting in eqn. (8.6) we get,

$$0.5 N_\phi + 2c\sqrt{N_\phi} = 1.3 \quad \text{....(i)}$$

where $\quad N_\phi = \tan^2(45° + \phi/2)$

Similarly, for the second and third samples, the following equations are obtained:

$$N_\phi + 2c\sqrt{N_\phi} = 1.97 \quad \text{....(ii)}$$

and, $$1.5 N_\phi + 2c\sqrt{N_\phi} = 2.63 \quad \text{...(iii)}$$

subtracting (i) from (ii) we obtain,

$$0.5 N_\phi = 0.67, \quad \text{or,} \quad N_\phi = 1.34$$

or, $\tan^2(45° + \phi/2) = 1.34$

or, $\tan(45° + \phi/2) = 1.157$

or, $45° + \phi/2 = 49.2°$

or, $\phi/2 = 4.2°$

or, $\phi = 8.4°$

Substituting for ϕ in eqn. (i),

$$(0.5)(1.34) + (2c)(1.157) = 1.3$$

or, $c = 0.27$ kg/cm^2

Check : Substituting the values of c and ϕ in eqn. (iii), we get,

L.H.S. = (1.5) (1.34) + (2) (0.27) (1.157)

= 2.63 = R.H.S.

Problem 8.9. A sample of dry coarse sand is tested in the laboratory triaxial apparatus in the undrained condition. Under a cell pressure of 2 kg/cm^2, the sample failed when the deviator stress reached 4.38 kg/cm^2.

(i) Determine the shear parameters of the soil.

(ii) At what deviator stress will the soil fail if the cell pressure be 3 kg/cm^2 ?

Solution: Here, $\sigma_3 = 2$ kg/cm^2, $\sigma_d = 4.38$ kg/cm^2.

$$\therefore \quad \sigma_1 = \sigma_3 + \sigma_d = 2 + 4.38 = 6.38 \text{ kg/cm}^2.$$

With $\sigma_3 = 2$ kg/cm^2 and $\sigma_1 = 6.38$ kg/cm^2, a Mohr circle is drawn (Fig. 8.14).

Since the sample is made of coarse sand and since it is in the dry state, no apparent cohesion will develop and the failure envelope passes through the origin.

In order to locate the failure envelope, draw a tangent to the Mohr circle from the origin. By measurement, the angle of obliquity of this line is 31°. Hence, the shear parameters are:

$$c = 0, \phi = 31°$$

(ii) We have, from eqn. (8.6),

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

As $c = 0$, $\therefore \quad \sigma_1 = \sigma_3 N_\phi$

or, $$\sigma_1 = \sigma_3 \tan^2(45° + \phi/2) \quad ...(i)$$

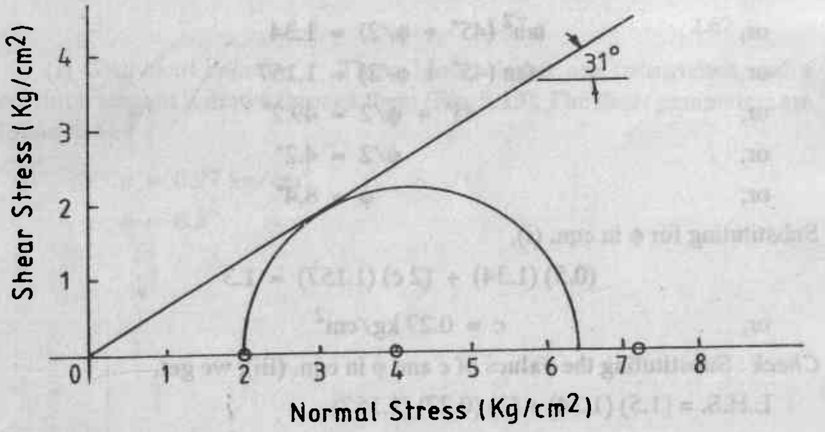


Fig. 8.14

Here, $\sigma_3 = 3$ kg/cm^2, $\phi = 31°$

$$\therefore \quad \sigma_1 = (3)[\tan(45° + 31°/2)]^2 = 9.37 \text{ kg/cm}^2$$

Deviator stress, $\sigma_d = \sigma_1 - \sigma_3$

$$= 9.37 - 3 = 6.37 \text{ kg/cm}^2$$

Problem 8.10. The following results were obtained from a laboratory triaxial test with arrangements for pore pressure measurements:

Sample No.	Cell pressure (kg/cm^2)	Deviator stress at failure (kg/cm^2)	Pore pressure at failure (kg/cm^2)
1.	1.0	2.02	0.41
2.	1.5	2.18	0.62
3.	2.0	2.37	0.70

Determine the shear parameters of the soil considering
(i) total stresses (ii) effective stressess.

Solution: The values of cell pressures and deviator stresses given in the problem are the total stress values. The corresponding effective stresses may be obtained from the relation :

$$\sigma' = \sigma - u$$

The major and minor principal stresses, considering the total stress analysis as well as effective stress analysis, are tabulated below :

Sample No.	σ_3 (kg/cm^2)	σ_d (kg/cm^2)	σ_1 (= $\sigma_3 + \sigma_d$) (kg/cm^2)	u (kg/cm^2)	σ'_3 (= $\sigma_3 - u$) (kg/cm^2)	σ'_1 (= $\sigma_1 - u$) (kg/cm^2)
1.	1.0	2.02	3.02	0.41	0.59	2.59
2.	1.5	2.18	3.68	0.62	0.88	3.06
3.	2.0	2.37	4.37	0.70	1.30	3.67

Total stress analysis: Three Mohr circles are drawn using the three sets of values of σ_1 and σ_3. In Fig. 8.15, these circles are shown by firm lines. A common tangent is drawn through them, which is the failure envelope for total stress analysis. From the figure we obtain.

$$c = 0.75 \text{ kg/cm}^2 \text{ and } \phi = 7.5°$$

Effective stress analysis: In this case the Mohr circles are drawn with the three sets of values of σ_1' and σ_3'. In Fig. 8.15 the effective stress circles are represented by broken lines. The values of the corresponding shear strength parameters are,

$$c' = 0.65 \text{ kg/cm}^2 \text{ and } \phi' = 13°$$

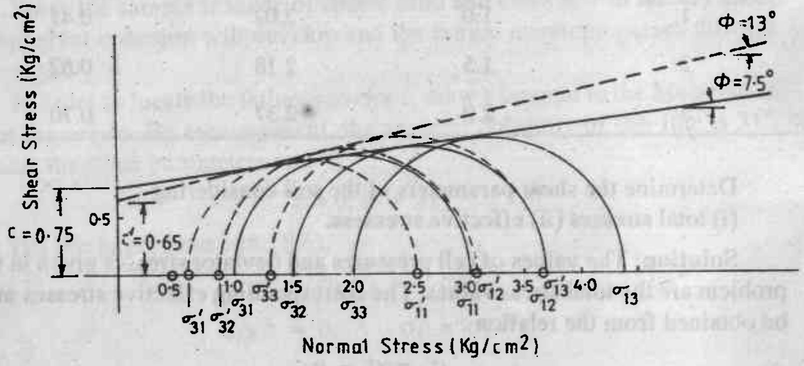


Fig. 8.15

Problem 8.11 The shear strength parameters of a given soil are, $c = 0.26$ kg/cm^2 and $\phi = 21°$. Undrained triaxial tests are to be carried out on specimens of this soil. Determine :

(i) deviator stress at which failure will occur if the cell pressure be 2.5 kg/cm^2.

(ii) the cell pressure during the test, if the sample fails when the deviator stress reaches 1.68 kg/cm^2.

Solution:

(i) We have from eqn. (8.6).

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi}$$

For the given soil, $c = 0.26$ kg/cm^2 and $\phi = 21°$

$$\therefore \quad N_\phi = \tan^2(45° + \phi/2) = \tan^2(45° + 21°/2) = 2.117.$$

and $\sqrt{N_\phi} = \sqrt{2.117} = 1.455$

Hence, eqn. (8.6) reduces to :

$$\sigma_1 = 2.117\,\sigma_3 + (2)(0.26)(1.455)$$

or, $\quad \sigma_1 = 2.117\,\sigma_3 + 0.757 \qquad$ ....(i)

When $\quad \sigma_3 = 2.5$ kg/cm^2

$$\sigma_1 = (2.117)(2.5) + 0.757$$

$$= 6.05 \text{ kg/cm}^2$$

Now, $\quad \sigma_d = \sigma_1 - \sigma_3$

$$= 6.05 - 2.5 = 3.55 \text{ kg/cm}^2$$

Hence the required deviator stress at failure is 3.55 kg/cm^2.

(ii) Let the required cell pressure be x kg/cm^2.

$\therefore \quad \sigma_1 = \sigma_d + \sigma_3$

or, $\quad \sigma_1 = 1.68 + x \qquad$ ...(ii)

Substituting for σ_1 and σ_3 in eqn. (i), we get

$$1.68 + x = 2.117x + 0.757$$

or, $\quad 1.117x = 0.923$

or, $\quad x = 0.83$

$\therefore$ The required cell pressure is 0.83 kg/cm^2.

Problem 8.12 The following are the results of a set of drained triaxial tests performed on three identical specimens of 38 mm diameter and 76 mm height:

Sample No.	Cell pressure (kN/m^2)	Deviator load at failure (kN)	Change in Volume (cc)	Axial Deformation (mm)
1.	50	0.0711	– 0.9	5.1
2.	100	0.0859	– 1.3	7.0
3.	150	0.0956	– 1.6	9.1

Determine the shear parameters of the soil.

Solution: The deviator loads at failure corresponding to each cell pressure are given. In order to determine the corresponding deviator stresses, these loads are to be divided by the corrected area of the sample, which can be obtained from

$$A_c = \frac{V_1 \pm \Delta V}{L_1 - \Delta L}$$

Here, $\quad V_1$ = Initial volume of the specimen

$$= (\pi/4)(3.8^2)(7.6) \text{ cc}$$

$$= 86.19 \text{ cc}$$

$$L_1 = 7.6 \text{ cm}$$

For the first sample, $\Delta V = -0.9$ cc and $\Delta L = 5.1$ cm

$$\therefore \quad A_c = \frac{86.19 - 0.9}{7.6 - 0.51} = 12.03 \text{ cm}^2 = 12.03 \times 10^{-4} \text{ m}^2$$

$$\therefore \quad \sigma_d = \frac{0.0711}{12.03 \times 10^{-4}} = 59.10 \text{ kN/m}^2$$

$$\text{and, } \sigma_1 = \sigma_3 + \sigma_d = 50 + 59.10 = 109.10 \text{ kN/m}^2$$

The major principal stresses for two other samples are computed in a similar manner. The results are tabulated below :

Sample No.	σ_3 (kN/m²)	F_d (kN)	ΔV (cc)	ΔL (cm)	A_c (cm²)	σ_d (kN/m²)	σ_1 (kN/m²)
1	50	0.0711	– 0.9	5.1	12.03	59.10	109.10
2	100	0.0859	– 1.3	7.0	12.36	69.50	169.50
3	150	0.0956	– 1.6	9.1	12.65	75.61	225.61

Three Mohr circles are constructed and their common tangent is drawn. This is the failure envelope of the soil (Fig. 8.16).

By measurement we obtain,

$c = 25$ kN/m², $\phi = 3.8°$

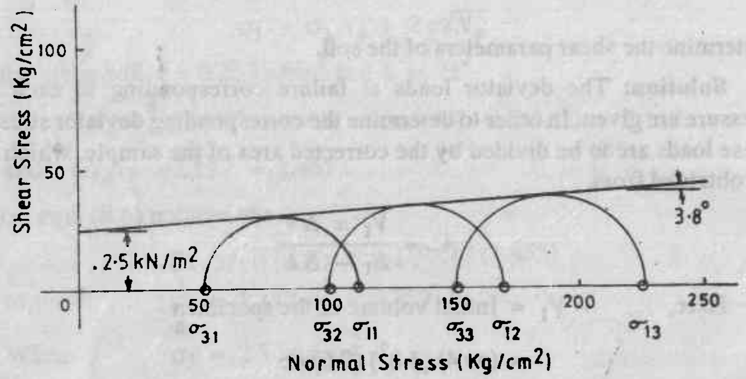


Fig. 8.16

Problem 8.13. An unconfined compression test was performed on an undisturbed sample of normally consolidated clay, having a diameter of 3.75 cm and 7.5 cm high. Failure occurred under a vertical compressive load of 116.3 kg. The axial deformation recorded at failure was 0.9 cm. A remoulded sample of the same soil failed under a compressive load of 68.2 kg, and the corresponding axial compression was 1.15 cm.

Determine the unconfined compressive strength and cohesion of the soil in the undisturbed as well as remoulded state.

Also determine the sensitivity of the soil and hence classify it accordingly.

Solution: (a) *Undisturbed state* :

Initial area of cross-section of the sample,

$$A_0 = (\pi/4)(3.75)^2 = 11.04 \text{ cm}^2$$

$$\text{Axial strain at failure, } \varepsilon = \frac{\Delta L}{L} = \frac{0.9}{7.5} = 0.12$$

$$\therefore \quad \text{Corrected area, } A_c = \frac{A_0}{1 - \varepsilon}$$

$$= \frac{11.04}{1 - 0.12} = 12.55 \text{ cm}^2$$

$$\text{Normal stress at failure} = \frac{116.3}{12.55} = 9.27 \text{ kg/cm}^2$$

$$\therefore \quad \text{Unconfined compressive strength, } q_u = 9.27 \text{ kg/cm}^2$$

$$\text{and, cohesion } c = \frac{q_u}{2} = \frac{9.27}{2} = 4.64 \text{ kg/cm}^2$$

(b) *Remoulded state* :

$$\varepsilon = \frac{1.15}{7.5} = 0.153$$

$$A_c = \frac{11.04}{1 - 0.153} = 13.03 \text{ cm}^2$$

$$\therefore \quad q_u = \frac{68.2}{13.03} = 5.23 \text{ kg/cm}^2$$

$$\text{or, } \quad c = \frac{q_u}{2} = \frac{5.23}{2} = 2.62 \text{ kg/cm}^2$$

$$\text{Sensitivity} = \frac{\text{strength in the undisturbed state}}{\text{strength in the remoulded state}}$$

$$= \frac{9.27}{5.23} = 1.77$$

As the value of sensitivity lies between 1 and 2, the soil is classified as a low sensitive soil.

Problem 8.14. In a *CU* triaxial test, a soil sample was consolidated at a cell pressure of 2 kg/cm^2 and a back pressure of 1 kg/cm^2, for 24 hours. On the next day, the cell pressure was increased to 3 kg/cm^2. This resulted in the development of a pore pressure of 0.08 kg/cm^2. The axial stress was then gradually increased to 4.5 kg/cm^2, which resulted in a failure of the soil. The pore pressure recorded at failure was 0.5 kg/cm^2. Determine Skempton's pore pressure parameters *A* and *B*.

Solution: We have

$$\Delta u = B[\Delta\sigma_3 + A(\Delta\sigma_1 - \Delta\sigma_3)],$$ where *A* and *B* are **Skempton's pore pressure parameters.**

In the first case, $\Delta\sigma_3 = 3 - 2 = 1$ kg/cm^2, $\Delta\sigma_1 = 0$

$\therefore$ $$0.08 = B[1 + A(0 - 1)]$$

or, $$B(1 - A) = 0.08 \qquad \text{...(i)}$$

In the second case,

$$\Delta\sigma_1 = 4.5 - 1 = 3.5 \text{ kg/cm}^2, \quad \Delta\sigma_3 = 0$$

$\therefore$ $$0.50 - 0.08 = B[0 + A(3.5 - 0)]$$

or, $$0.42 = 3.5AB \qquad \text{...(ii)}$$

Dividing (i) by (ii), we get,

$$\frac{1 - A}{3.5A} = \frac{0.08}{0.42}$$

or, $$\frac{1 - A}{A} = 0.67$$

or, $$1 - A = 0.67A, \text{ or, } A = 0.6$$

Substituting this value for *A* in (i), we obtain

$$B(1 - 0.6) = 0.08$$

or, $$B = \frac{0.08}{0.4} = 0.2$$

Problem 8.16. A vane shear test was carried out in the field to determine the shearing strength of a deep-seated layer of soft clay. The vane was 11.25 cm high and 7.5 cm across the blades. The equivalent torque recorded at the torque head at failure was 417.5 kg-cm. The vane was then rotated very rapidly in order to completely remould the soil. It was found that the remoulded soil can be sheared by applying a torque of 283.2 kg-cm.

Determine the shear strength of the soil in the undisturbed and remoulded states and its sensitivity.

Solution: We know that,

$$s = \frac{T}{\pi D^2\left(\frac{H}{2} + \frac{D}{6}\right)}$$

Here, $H = 11.25$ cm and $D = 7.5$ cm,

$\therefore$ $$s = \frac{T}{(\pi)(7.5^2)(11.25/2 + 7.5/6)}$$

or, $$s = \frac{T}{1113.67}$$

In the undisturbed state, $T = 417.5$ kg-cm,

$\therefore$ $$s = \frac{417.5}{1113.67} = 0.37 \text{ kg/cm}^2$$

In the remoulded state, $T = 283.2$ kg-cm,

$\therefore$ $$s = \frac{283.2}{1113.67} = 0.25 \text{ kg/cm}^2$$

$$\text{Sensitivity} = \frac{0.37}{0.25} = 1.48$$

EXERCISE 8

8.1. The normal stresses acting on two orthogonal planes of a soil sample are 250 kN/m^2 and 110 kN/m^2. Find out the normal and shear stresses on a plane inclined at 60° to the direction of the major principal stress.

[Ans. σ = 215 kN/m^2, τ = 60.6 kN/m^2]

8.2. The stress conditions on a soil element are shown in Fig. 8.17. Determine:

(i) The orientation and magnitude of the principal stresses.

(ii) The stresses acting on the horizontal and the vertical planes.

[Ans. (i) σ_1 = 2.76 kg/cm^2 at 98.5° with horizontal; σ_3 = 0.83 kg/cm^2 at 8.5° with horizontal (ii) σ_H = 0.77 kg/cm^2, τ_H = − 0.3 kg/cm^2; σ_V = 2.72 kg/cm^2, τ_V = 0.3 kg/cm^2]

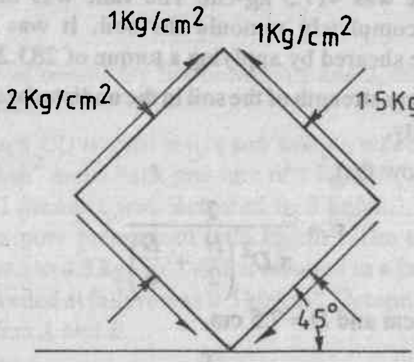


Fig. 8.17

8.3. Fig. 8.18 illustrates the stress conditions on a soil element.

(i) Determine the normal and shear stresses on the plane *X-X*.

(ii) Draw a free body diagram of the element bounded by plane *X-X* and show these stresses.

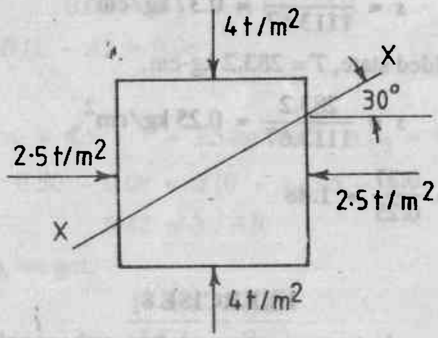


Fig. 8.18

(iii) Prove that the free body is in equilibrium.

[Ans : (i) $\sigma = 3.63$ kg/cm^2, $\tau = 0.65$ kg/cm^2]

8.4. The subsoil at a site consists of a 5 m thick stratum of a cohesionless soil which is underlain by a rock layer. A surcharge of 5 t/m^2 is placed on the ground level. The properties of the soil are as follows:

$$G = 2.68, e = 0.7, w = 6\%, \phi = 30°$$

Determine the shear strength of the soil on a horizontal plane at a depth of 2 m below the G.L. [Ans : 4.82 t/m^2]

8.5. In problem 8.4, if the water table rises from a great depth to the ground surface so that the soil becomes fully saturated and its natural moisture content increases to 19%, how will the shear strength on the given plane change ? [Ans. Reduced by 0.85 t/m^2]

8.6. The stress conditions on an infinitely small soil element are shown in Fig. 8.19. Find out the magnitude and direction of the principal stresses.

[Ans: σ_1 = 1.68 kg/cm^2 at 12° to the horizontal
σ_3 = 0.47 kg/cm^2 at 102° to the horizontal]

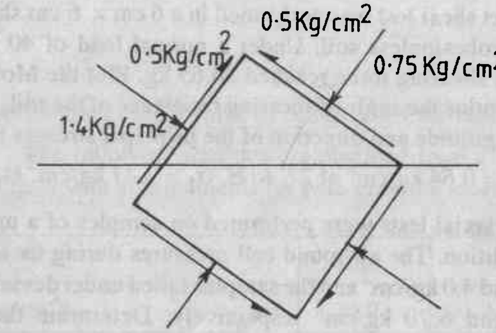


Fig. 8.19

8.7. The results of a direct shear test performed on a soil sample in a shear box of 6 cm × 6 cm size are given below:

Normal load (kg.)	30	40	50	60
Shear force at failure (kg.)	19.5	26.3	32.4	39.9

Plot the failure envelope for the soil and determine its shear parameters.

[Ans. $c = 0$, $\phi = 33°$]

8.8. A given soil has a unit cohesion of 2 t/m^2 and an angle of internal friction of 28°. Samples of the soil were tested in the laboratory in a triaxial apparatus under the undrained condition. Determine :

(i) Deviator stress at failure when the cell pressure is 1.5 kg/cm^2.

(ii) The applied cell pressure, if the sample fails under a total vertical pressure of 5.09 kg/cm^2. [Ans. (i) 3.32 kg/cm^2, (ii) 2.5 kg/cm^2]

8.9. A set of triaxial tests were performed on three samples of a soil. The cell pressures and the deviator stresses at failure are given below:

Sample No.	Cell Pr. (kN/m^2)	Deviator stress (kN/m^2)
1	200	690
2	300	855
3	400	1030

Plot Mohr's circles of stress and determine the apparent cohesion and angle of internal friction. [Ans. $c = 112$ kN/m^2, $\phi = 27°$]

8.10. A direct shear test was performed in a 6 cm × 6 cm shear box on a sample of dry, cohesionless soil. Under a normal load of 40 kg, failure occurred when the shearing force reached 26.65 kg. Plot the Mohr strength envelope and determine the angle of shearing resistance of the soil. Determine graphically the magnitude and direction of the principal stresses at failure.

[Ans. $\phi = 36°$; $\sigma_3 = 0.64$ kg/cm^2 at 27° to H, $\sigma_1 = 2.47$ kg/cm^2 at 117° to H]

8.11. Two triaxial tests were performed on samples of a moist soil in an undrained condition. The all-round cell pressures during these two tests were 2.5 kg/cm^2 and 4.0 kg/cm^2 and the samples failed under deviator stresses of 4.85 kg/cm^2 and 6.70 kg/cm^2 respectively. Determine the apparent cohesion and the apparent angle of shearing resistance of the soil (i) analytically (ii) graphically.

Do you expect to obtain the same values of the shear parameters if the samples were tested in a drained condition ? Explain your answer with reasons. [Ans. $c = 0.59$ kg/cm^2, $\phi = 22.4°$]

8.12. Laboratory triaxial tests were performed on three soil samples of 3.8 cm diameter and 7.6 cm height. The following results were obtained:

Sample No.	Cell Pr. (kg/cm^2)	Deviator load at failure (kg)	Change in volume (cc)	Axial Deformation (cm)
1	0.5	45	1.1	0.92
2	1.0	52	1.5	1.15
3	2.0	79.5	1.7	1.22

Plot Mohr's circles and determine the apparent values of shear parameters of the soil. [Ans. $c = 1$ kg/cm^2, $\phi = 18.7°$]

8.13. A set of triaxial tests were performed on three samples of a fine-grained soil. The height and diameter of each sample were 75 mm and 37.5 mm respectively. The following are the results:

Sample No.	Cell Pr. (kg/cm^2)	Deviator load (kg)	Axial Deformation (cm)
1	1.45	29.5	0.98
2	2.70	37.9	1.13
3	?	42.8	1.16

Determine the missing value of cell pressure in test no. 3.

8.14. The following results were obtained from a set of consolidated undrained tests with arrangements for pore pressure measurements:

Test No.	1	2	3
Cell Pr. (kg/cm^2)	1.0	2.0	3.0
Deviator Stress (kg/cm^2)	1.31	1.62	1.89
Pore pressure (kg/cm^2)	0.18	0.42	0.86

Determine the shear parameters of the soil, considering (i) total stress (ii) Effective stress.

[Ans. (i) $c = 0.46$ kg/cm^2, $\phi = 6.5°$

(ii) $c' = 0.42$ kg/cm^2, $\phi' = 9.8°$]

8.15. An unconfined compression test was performed on a silty clay sample of 4 cm diameter and 8 cm height. The sample failed under a compressive load of 23 kg and the deformation recorded at failure was 1.42 cm. A triaxial test was performed on an identical sample of the same soil. The all round cell pressure was 1 kg/cm^2 and the sample failed under a deviator load of a 39.5 kg. The axial deformation recorded at failure was 1.18 cm. Find out the apparent values of shear parameters (i) graphically and (ii) analytically. [Ans $c = 0.70$ kg/cm^2, $\phi = 4.5°$]

8.16. A 21.5 cm long cylindrical soil sample having a diameter of 10 cm was subject to an increasing vertical compressive load. Failure occurred when the load reached 151 kg, and the corresponding axial deformation was 2 cm. The sample was made of clay and had the following properties :

$$G = 2.67, e = 0.69, w = 26\ \%$$

Determine the shear parameters of the soil

[Ans. $\phi = 0°$, $c = 0.77$ kg/cm^2]

8.17. An unconfined compression test was performed on a cylindrical soil sample having a diameter of 37.5 mm and a height of 75 mm. The sample failed at a vertical compressive load of 23.5 kg. The axial strain recorded at failure was 1.16 cm and the failure plane was observed to be inclined at 53° to the horizontal Determine the apparent shear parameters of the soil.

[Ans. $c = 0.68$ kg/cm^2, $\phi = 16°$]

8.18. A triaxial test was performed on a sample of dry sand having an apparent ϕ-value of 36°. Initially, a chamber pressure of 5 kg/cm^2 was applied and the deviator stress was gradually increased to 3 kg/cm^2. Keeping this deviator stress unchanged, the cell pressure was then gradually reduced. At what value of cell pressure the sample will fail?

[Ans. 1.05 kg/cm^2]

8.19. Determine the minimum lateral pressure required to prevent failure of a soil subjected to a total vertical stress of 10 kg/cm^2. The shear parameters of the soil are given as : $c = 0.3$ kg/cm^2, $\phi = 17.5°$.

[Ans. 4.94 kg/cm^2]

8.20. A laboratory vane shear test was performed in an undisturbed sample of soft clay. The diameter and height of the vane were 6.3 mm and 11.3 mm respectively. The sample failed under an applied torque of 110 gm cm. The sample was then completely disturbed by rotating the vane rapidly. The remoulded soil failed under a torque of 45 gm-cm. Determine the undrained shear strength of the soil in the undisturbed and remoulded states and compute its sensitivity. [Ans. 0.55 and 0.22 kg/cm^2 respectively; 2.5]

8.21. If a field vane shear test is performed on the soil mentioned in above problem, with a vane of 11.3 cm height and 7.5 cm diameter, determine the torques required to fail the soil in the undisturbed and remoulded states.

[Ans. 670.6 kg-cm; 268.2 kg-cm]

9

EARTH PRESSURE

9.1. Introduction: It is often required to maintain a difference in the elevation level of the ground on the left and right hand sides of a vertical section. Such situations call for the construction of an earth-retaining structure, e.g., a retaining wall or a sheet-pile wall. The earth retained by such a structure exerts a lateral thrust which is of paramount importance in the design of the retaining structure.

Depending on the conditions prevailing at the site, the lateral earth pressure may be divided into the following three categories:

(i) Earth pressure at rest.

(ii) Active earth pressure.

(iii) Passive earth pressure.

9.2. Earth Pressure at Rest: Fig. 9.1 (a) shows a retaining wall, embedded below the ground level upto a depth *D*, and retaining earth upto a height *H*. If the wall is perfectly rigid, no lateral movement of the wall can occur. And hence, no deformation of the soil can take place. The lateral pressure exerted by the soil is then called the earth pressure at rest.

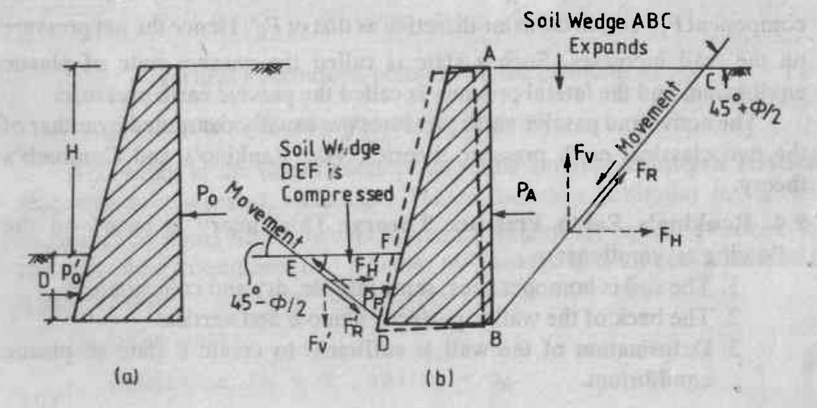


Fig. 9.1

The conjugate relationship between the lateral earth pressure and the vertical overburden pressure is given by:

$$\sigma_h = K_0 \cdot \sigma_v, \text{ or } \sigma_h = K_0 \cdot \gamma z \qquad ...(9.1)$$

where K_0 = co-efficient of earth pressure at rest.

γ = unit weight of soil

z = depth at which lateral pressure is measured.

The value of K_0 depends on the properties of the soil and its stress history, and is given by:

$$K_0 = \frac{\mu}{1 - \mu} \qquad ...(9.2)$$

where, μ = Poisson's ratio of the soil.

9.3. Active and Passive Earth Pressures: In reality, a retaining wall is not rigid, but flexible, i.e., it is free to rotate about its base. In Fig. 9.1(a), let P_0 and P_0' be the at-rest lateral thrusts acting on the back and front faces of the wall respectively. Due to the difference in elevation levels, $P_0 > P_0'$. Hence, a flexible wall will yield away from the backfill. The soil wedge ABC will then tend to slide down along the potential sliding surface BC. This condition is illustrated in Fig. 9.1 (b). The frictional resistance F_R against such movement will act upward along BC. Its horizontal component F_H will act in the opposite direction to that of P_0. Thus the net pressure on the wall will decrease. Such a state is called the active state of plastic equilibrium, and the lateral pressure is called the active earth pressure.

Simultaneously, the soil wedge DEF in front of the wall gets compressed. The frictional resistance F_R' in this case acts along ED and its horizontal component F_H' acts in the same direction as that of P_0'. Hence the net pressure on the wall increases. Such a state is called the passive state of plastic equilibrium and the lateral pressure is called the passive earth pressure.

The active and passive earth pressures are usually computed by either of the two classical earth pressure theories, viz., Rankine's and Coulomb's theory.

9.4. Rankine's Earth Pressure Theory: This theory is based on the following assumptions:

1. The soil is homogeneous, semi-infinite, dry and cohesionless.
2. The back of the wall is perfectly smooth and vertical.
3. Deformation of the wall is sufficient to create a state of plastic equilibrium.

4. On any vertical plane in the soil adjacent to the wall a conjugate relationship exists between the lateral earth pressure and the vertical overburden pressure.

This theory was later extended by other investigators to take into account cohesive backfills and walls with battered backface.

The equation governing the relationship between the major and minor principal stresses, acting on a soil element, is given by,

$$\sigma_1 = \sigma_3 N_\phi + 2c\sqrt{N_\phi} \qquad ...(9.3)$$

where, $N_\phi = (45° + \phi/2)$

ϕ = angle of internal friction

c = cohesion.

Let us consider an infinitesimally small soil element at a depth Z below the ground level, adjacent to a retaining wall, as shown in Fig. 9.2.

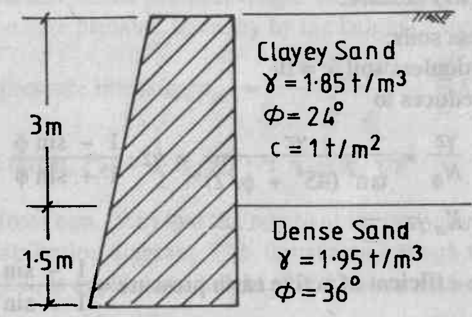


Fig. 9.2

σ_v = vertical overburden pressure on the element.

σ_h = lateral earth pressure on the element.

According to the fourth assumption stated above, a conjugate relationship exists between σ_v and σ_h. The relationship is similar to the one expressed by eqn. (9.3). However, the exact form of the equation depends on the prevailing conditions, i.e., whether the backfill is in an active state or in a passive state.

(i) *Active state* :

In this case, $\sigma_1 = \sigma_v$, and $\sigma_3 = \sigma_h$.

But, $\sigma_v = \gamma z$

and, σ_h = active pressure intensity = p_a.

$\therefore$ Eqn. (9.3) gives,

$$\gamma z = p_a \cdot N_\phi + 2c\sqrt{N_\phi}$$

or, $$p_a = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} \quad ...(9.4)$$

(ii) *Passive state* :

Here, $\sigma_1 = \sigma_h$ and $\sigma_3 = \sigma_v$

But, $\sigma_v = \gamma z$

and, σ_h = passive pressure intensity = p_p

$\therefore$ Eqn. (9.3) gives,

$$p_p = \gamma z N_\phi + 2c\sqrt{N_\phi} \quad ...(9.5)$$

9.4.1. ***Computation of Earth Pressure Using Rankine's Theory*** **:**

(A) *Active Earth Pressure*:

(a) Cohesionless soils:

For a cohesionless soil, $c = 0$.

$\therefore$ Eqn (9.4) reduces to

$$p_a = \frac{\gamma z}{N_\phi} = \frac{\gamma z}{\tan^2(45° + \phi/2)} = \gamma z \cdot \frac{1 - \sin\phi}{1 + \sin\phi}$$

or, $$p_a = K_a \gamma z \quad ...(9.6)$$

where, K_a = co-efficient of active earth pressure = $\frac{1 - \sin\phi}{1 + \sin\phi}$ ...(9.7)

Eqn. (9.6) and (9.7) can be used to compute the active earth pressure for various backfill conditions, as discussed below:

(i) *Dry or Moist Backfill with Horizontal Ground Surface*:

Fig. 9.3 (a) shows a retaining wall supporting a homogeneous backfill of dry or moist soil, upto a height H.

At any depth z below the top of the wall.

$$p_a = K_a \gamma z$$

At the top of the wall $(z = 0)$, $p_a = 0$

At the base of the wall $(z = H)$, $p_a = K_a \gamma H$

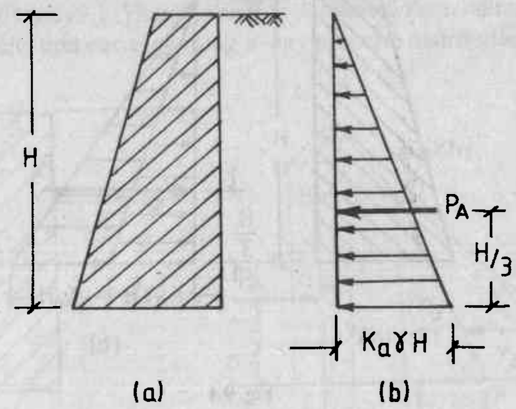


Fig. 9.3

Fig. 9.3 (b) shows the distribution of active pressure intensity. The magnitude of resultant thrust per unit length of wall may be obtained by multiplying the average pressure intensity by the height of the wall.

$$\text{Average pressure intensity, } p_{av} = \frac{0 + K_a \gamma H}{2} = \frac{1}{2} K_a \gamma H$$

$$\therefore \text{ Resultant thrust, } P_A = \frac{1}{2} K_a \gamma H \cdot H = \frac{1}{2} K_a \gamma H^2 \quad ...(9.8)$$

It is evident from eqn. (9.8) that the resultant thrust is given by the area of the pressure distribution diagram. This thrust acts through the centroid of the triangle ABC, i.e., is applied at a height of $H/3$ above the base of the wall.

(ii) *Fully Submerged Backfill*:

This condition is shown in Fig. 9.4 (a). As the soil is fully submerged, its effective unit weight is,

$$\gamma' = \gamma_{sat} - \gamma_w$$

At any depth z below the top of the wall, the total active pressure is the sum of pressures exerted by the soil and water. According to Pascal's law, a fluid exerts equal pressure in all directions at any given depth.

Hence, at a depth z,

$$p_a = K_a \gamma' z + \gamma_w z \quad ...(9.9)$$

The corresponding pressure distribution diagram is shown in Fig. 9.4 (b)

(iii) *Partially Submerged Backfill*:

(a) Backfill having similar properties above and below water table:

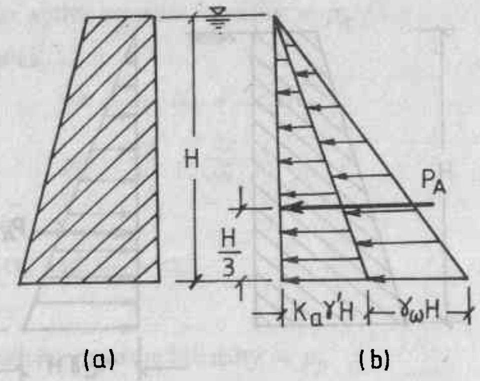


Fig. 9.4

In Fig. 9.5 (a), the retaining wall has to retain earth upto a height H. The ground water table is located at a depth h_1 below ground level. The active pressure intensities are given by:

Above ground water table: $p_a = K_a \gamma z \ (0 \le z \le h_1)$

Below ground water table: $p_a = K_a \gamma h_1 + K_a \gamma' z + \gamma_w z \ (0 \le z \le h_2,$ z being measured from G.W.T.)

Fig. 9.5 (b) shows the corresponding pressure distribution diagram. The resultant active thrust per unit run of the wall is given by the entire area of this diagram. It is easier to determine the area by dividing it into a number of triangle and rectangles. In Fig. 9.5 (b).

$P_1 = \Delta ABD$, P_2 = area of $BCED$

$P_3 = \Delta DEF$, $P_4 = \Delta DFG$.

$\therefore$ Resultant active thrust,

$$P_A = P_1 + P_2 + P_3 + P_4 = \sum_{i=1}^{n} P_i \qquad \text{...(9.10)}$$

The point of application of P_A can be determined by taking moments of individual pressure areas about the base of the wall. Thus,

$$P_A \cdot \bar{y} = P_1 y_1 + P_2 y_2 + P_3 y_3 + P_4 y_4$$

or,
$$\bar{y} = \frac{\sum_{i=1}^{n} P_i \cdot y_i}{\sum_{i=1}^{n} P_i}$$

Eqns. (9.10) and (9.11) may be used to determine the resultant thrust and its point of application corresponding to any pressure distribution diagram.

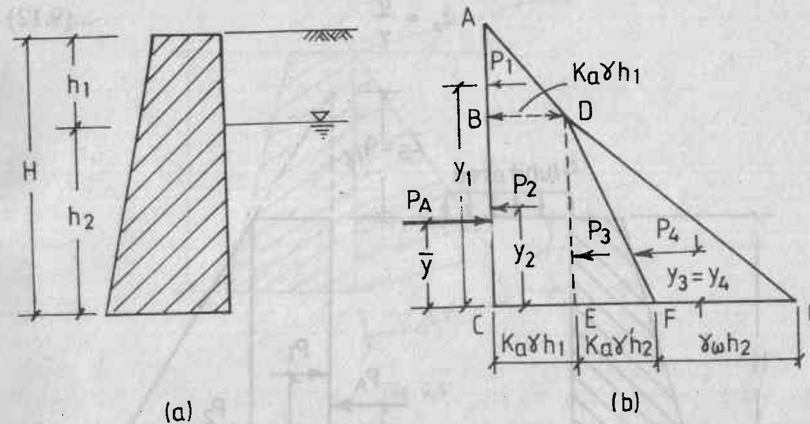


Fig. 9.5

(b) Backfill having different properties above and below water table:

Fig. 9.6 (a) and (b) illustrate this backfill condition and the corresponding pressure distribution diagram.

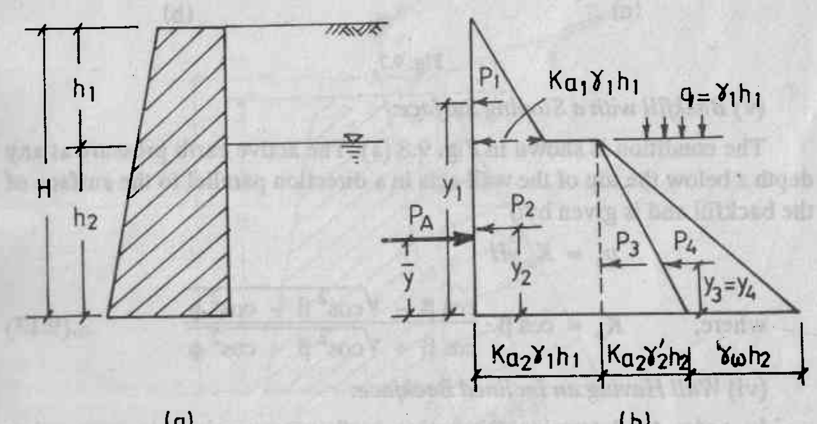


Fig. 9.6

(iv) *Backfill with Uniform Surcharge*:

Fig. 9.7 (a) illustrates a retaining wall supporting a backfill loaded with a uniform surcharge q. The corresponding pressure distribution diagram is shown in Fig. 9.7 (b). From the figure it is evident that the effect of the surcharge is identical to that of an imaginary backfill having a height z_s, placed above G.L., where,

$$z_s = \frac{q}{\gamma} \qquad ...(9.12)$$

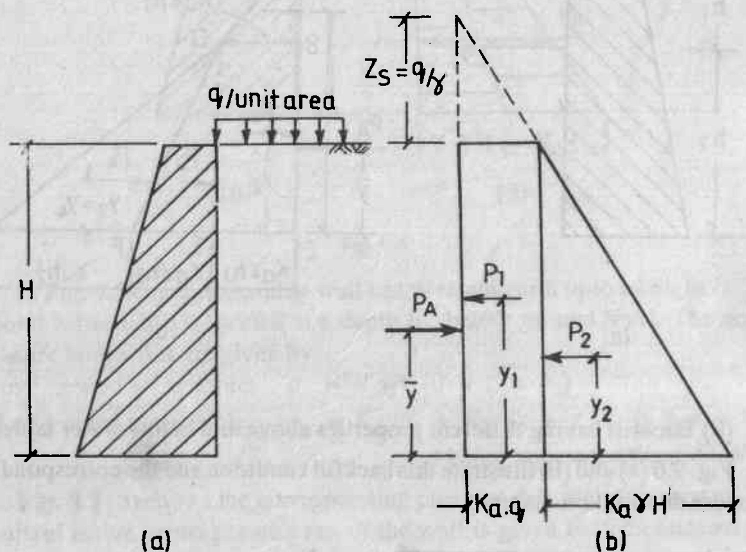


Fig. 9.7

(v) *Backfill with a Sloping Surface*:

The condition is shown in Fig. 9.8 (a). The active earth pressure at any depth z below the top of the wall acts in a direction parallel to the surface of the backfill and is given by:

$$p_a = K_a \gamma H$$

where,

$$K_a = \cos\beta \cdot \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}} \qquad ...(9.13)$$

(vi) *Wall Having an Inclined Backface*:

In order to determine the active earth pressure in this case using Rankine's theory, the following steps should be followed (Ref. Fig. 9.9)

(i) Draw the wall section and the ground line.

(ii) Draw a vertical line through the base of the wall to intersect the ground line at *c*.

(iii) Compute the length *BC* from:

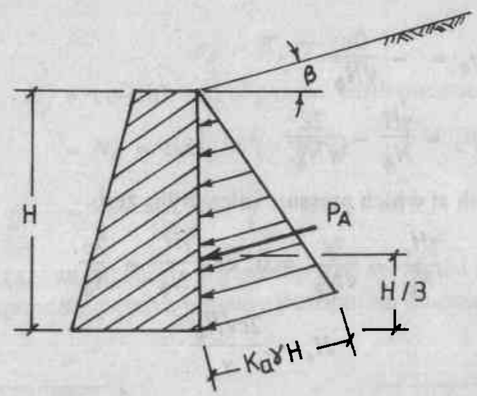


Fig. 9.8

$$BC = H(1 + \tan\theta\tan\beta) \qquad ...(9.14)$$

(iv) Determine the active pressure on this imaginary plane *BC*, using eqn. (9.13).

(v) For designing the wall, compute the self-weight of the soil wedge *ABC* and consider its effect on the stability of the wall separately.

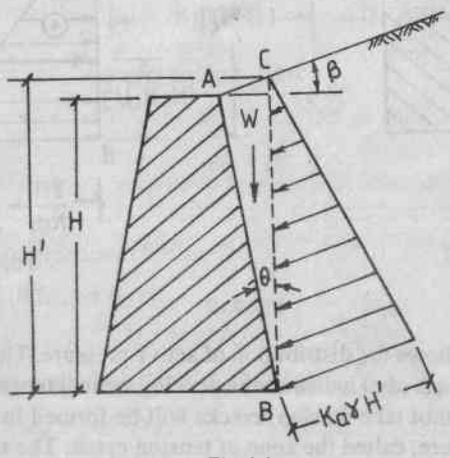


Fig. 9.9

(b) Cohesive-frictional Soils:

From eqn. (9.4), the active earth pressure at a depth z is given by,

$$p_a = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

At $z = 0$, $p_a = -\dfrac{2c}{\sqrt{N_\phi}}$

At $z = H$, $p_a = \dfrac{\gamma H}{N_\phi} - \dfrac{2c}{\sqrt{N_\phi}}$.

Let H_c be the depth at which pressure intensity is zero.

$$\therefore \quad \frac{\gamma H_c}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = 0, \text{ or, } \frac{\gamma H_c}{N_\phi} = \frac{2c}{N_\phi}$$

or,
$$H_c = \frac{2c\sqrt{N_\phi}}{\gamma} \qquad ...(9.15)$$

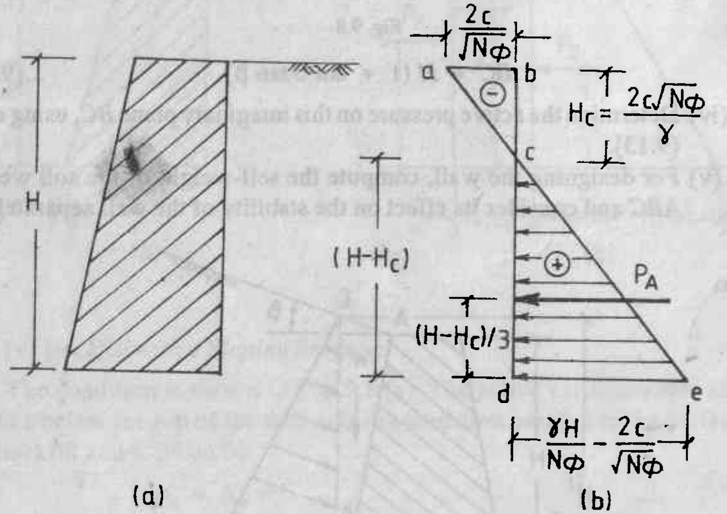


Fig. 9.10

Fig. 9.10 (b) shows the distribution of active pressure. The negative side of this diagram (i.e., $\Delta\, abc$) indicates the development of tension upto a depth H_c. Since soils cannot take tension, cracks will be formed in this zone. The depth H_c is, therefore, called the zone of tension crack. The resultant lateral thrust is obtained by computing the area of the positive side of the diagram (i.e. $\Delta\, cde$).

(B) *Passive Earth Pressure*:

(a) Cohesionless soils:

For a cohesionless soil, eqn. (9.5) reduces to:

$$p_p = \gamma z N_\phi$$

or, $\quad p_p = K_p \gamma z$

where, $\quad K_p$ = co–efficient of passive earth pressure

$$= N_\phi = \tan^2(45° + \phi/2)$$

$$= \frac{1 + \sin\phi}{1 - \sin\phi} = \frac{1}{K_a}$$

Fig. 9.11 (a) and (b) shows a retaining wall subjected to a passive state, and the corresponding passive pressure distribution diagram.

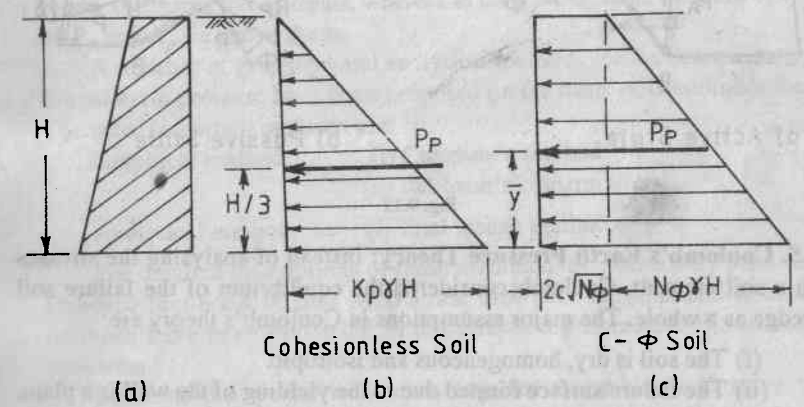


Fig. 9.11

(b) *Cohesive-frictional Soils*:

From eqn. (9.5), we have,

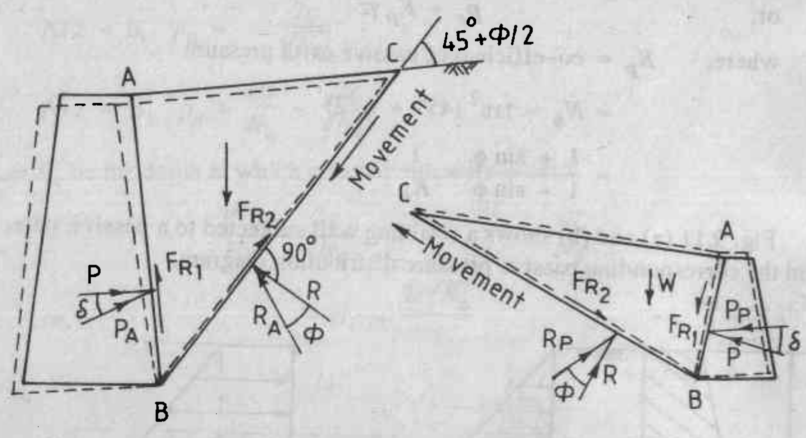
$$p_p = \gamma z N_\phi + 2c\sqrt{N_\phi}$$

For the retaining wall shown in Fig. 9.11 (a),

at $z = 0$, $\quad p_p = 2c\sqrt{N_\phi}$

at $z = H$, $\quad p_p = \gamma H N_\phi + 2c\sqrt{N_\phi}$

The corresponding pressure distribution diagram is shown in Fig. 9.11 (c).



a) Active State b) Passive State

Fig. 9.12

9.5. Coulomb's Earth Pressure Theory: Instead of analysing the stresses on a soil element, Coulomb considered the equilibrium of the failure soil wedge as a whole. The major assumptions in Coulomb's theory are:

(i) The soil is dry, homogeneous and isotropic.

(ii) The failure surface formed due to the yielding of the wall is a plane surface.

(iii) The failure wedge is a rigid body.

(iv) The backface of the wall is rough.

(v) The resultant thrust acts on the backface of the wall at one-third height and is inclined to the normal on the wall at this point at an angle δ , where,

$$\delta = \text{angle of wall friction.}$$

Based on this theory, the lateral earth pressure can be determined by the trial and error method. As the location of the actual failure surface is not known, a number of potential failure surfaces are chosen and the lateral earth pressure is determined for each of them. The one for which the lateral thrust reaches a certain extreme value (minimum for active state and maximum for passive state) is accepted as the true failure surface, and the corresponding lateral thrust is accepted as the active or passive thrust, as the case may be.

9.5.1 ***Wall friction*** **:** The concept of wall friction is illustrated in Fig. 9.12 (a) and (b).

In the active state, the wall moves away from the backfill and the failure wedge *ABC* tends to move downwards. As it slides down, frictional resistances act upward along the backface of the wall (soil-wall friction) and the failure plane (soil-to-soil friction). In absence of the frictional force F_{R_1}, the active thrust P would have been acting normally on the backface. But now the resultant P_A of P and F_{R_1} is inclined at an angle δ to the normal on the backface. Due to similar reasons, the soil reaction R_A will also be inclined at an angle ϕ to the normal on the failure surface.

The same arguments lead us to the conclusion that in a passive state also, P_p and R_p will be inclined at angles δ and ϕ respectively to the normals on AB and BC. However, in the active state, the lines of action of P_A and R_A lie below the respective normals, whereas in the passive state, the lines of action of P_p and R_p lie above them.

A number of graphical and analytical methods for the determination of lateral earth pressure have been proposed on the basis of Coulomb's theory. The most important methods are:

Graphical method: (i) Culmann's method
(ii) Rebhann's construction

Analytical method: (i) Trial wedge method.

For detailed descriptions of these methods, the reader may refer to any standard text-book of Soil Mechanics. However the application of these methods have been illustrated in this chapter by a number of worked-out problems.

Some of the special techniques required to enable us to solve more complex problems involving external loads, or irregularities in the shape of the wall or the ground surface. have also been dealt with.

EXAMPLES

Problem 9.1. A 5 m high rigid retaining wall has to retain a backfill of dry, cohesionless soil having the following properties:

$$\phi = 30°, \; e = 0.74, G = 2.68, \mu = 0.36.$$

(i) Plot the distribution of lateral earth pressure on the wall.

(ii) Determine the magnitude and point of application of the resultant thrust.

(iii) Compute the percent change in the lateral thrust if the water table rises from a great depth to the top of the backfill.

Solution: (i) Bulk density of the dry backfill,

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{(2.68)(1.0)}{1+0.74} = 1.54 \text{ t/m}^3.$$

As the wall is rigid, the lateral pressure exerted by the backfill is earth pressure at rest.

Co-efficient of earth pressure at rest,

$$K_0 = \frac{\mu}{1-\mu} = \frac{0.36}{1-0.36} = 0.5625$$

At the top of the wall ($z = 0$), $p_0 = 0$

At the base of the wall ($z = 5$ m), $p_0 = K_0 \gamma z$

$$= (0.5625)(1.54)(5.0)$$

$$= 4.33 \text{ t/m}^2$$

The distribution of lateral earth pressure is shown in Fig. 9.13.

(ii) Resultant lateral thrust on the wall (considering unit width),

$$P_0 = \frac{1}{2} K_0 \gamma H^2$$

$$= (1/2)(0.5625)(1.54)(5.0)^2$$

$$= 10.83 \text{ t per m run}$$

The resultant thrust is applied at a height of 5/3 = 1.67 m above the base of the wall.

(iii) If the water table rises to the top of the backfill, the soil will get fully submerged.

$$\gamma_{sub} = \frac{G-1}{1+e}\cdot\gamma_w = \left(\frac{2.68-1}{1+0.74}\right)(1) = 0.965 \text{ t/m}^2$$

Resultant thrust

$$= \frac{1}{2} K_0 \gamma_{sub} H^2 + \frac{1}{2}\gamma_w H^2$$

$$= (1/2)(5.0)^2 [(0.965)(0.5625) + 1]$$

$$= 19.28 \text{ t per m run}$$

Per cent increase in lateral thrust

$$= \frac{19.28 - 10.83}{10.83} \times 100\%$$

$$= 78\%.$$

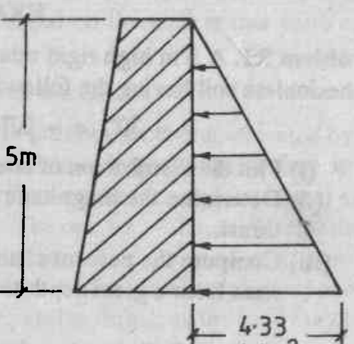


Fig. 9.13

Problem 9.2. A retaining wall with a smooth, vertical backface has to retain a sand backfill upto a height of 4.5 m. A uniform surcharge of 5 t/m^2 is placed over the backfill. The water table is at 2 m below G.L. The specific gravity of solids and the void ratio of the backfill are 2.68 and 0.82 respectively. The soil above the water table has a degree of saturation of 10%. The angle of internal friction of the soil, both above and below water table, is 30°.

Determine the magnitude and point of application of the resultant active thrust on the wall.

Solution: Bulk density of the soil above water table,

$$\gamma = \frac{G+se}{1+e}\gamma_w$$

$$= \frac{2.68 + (0.10)(0.82)}{1+0.82}(1) = 1.517 \text{ t/m}^3$$

Submerged density of the soil below water table,

$$\gamma_{sub} = \frac{G-1}{1+e}\gamma_w = \frac{2.68-1}{1+0.82}(1) = 0.923 \text{ t/m}^3$$

Co-efficient of active earth pressure,

$$K_a = \frac{1-\sin 30°}{1+\sin 30°} = \frac{1}{3}.$$

Active pressure due to surcharge $= K_a q = \left(\frac{1}{3}\right)(5.0) = 1.67$ t/m^2

Active pressure at B due to moist soil above water table

$$= K_a \gamma z = \left(\frac{1}{3}\right)(1.517)(2) = 1.01 \text{ t/m}^2.$$

Active pressure at C due to submerged soil

$$= K_a \gamma_{sub} z = \left(\frac{1}{3}\right)(0.923)(2.5) = 0.77 \text{ t/m}^2.$$

Lateral pressure exerted by water

$$= \gamma_w z = (1)(2.5) = 2.5 \text{ t/m}^2.$$

The pressure distribution diagram is shown in Fig. 9.14.

The resultant active thrust is equal to the area *abcde*. For convenie this area is divided into a number of triangles and rectangles. Considering width of the wall,

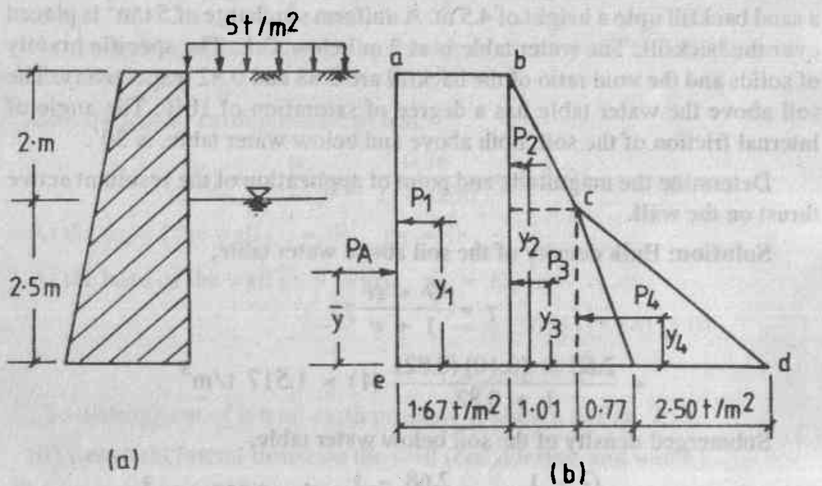


Fig. 9.14

$P_1 = (1.67)(4.5) = 7.51$ t $\qquad y_1 = 4.5/2 = 2.25$ m

$P_2 = \left(\frac{1}{2}\right)(1.01)(2) = 1.01$ t $\qquad y_2 = 2.5 + 2/3 = 3.17$ m

$P_3 = (1.01)(2.5) = 2.52$ t $\qquad y_3 = 2.5/2 = 1.25$ m

$P_4 = \left(\frac{1}{2}\right)(0.77 + 2.5)(2.5) = 4.09$ t $\qquad y_4 = 2.5/3 = 0.83$ m

$\therefore$ Resultant thrust $P_A = P_1 + P_2 + P_3 + P_4$

$= 15.13$ t per m run.

The point of application of this thrust above the base of the wall may be obtained from eqn. (9.11).

$$\bar{y} = \frac{(7.51)(2.25) + (1.01)(3.17) + (2.52)(1.25) + (4.09)(0.83)}{15.13}$$

$$= \frac{26.64}{15.13} = 1.76 \text{ m.}$$

Problem 9.3. A 5 m high masonry retaining wall has to retain a backfill of sandy soil having a unit weight of 1.82 gm/cc and an angle of internal friction of 32°. The surface of the backfill is inclined at an angle of 10° to the horizontal. Determine the magnitude and point of application of the active thrust on the wall.

Solution: Co-efficient of active earth pressure,

$$K_a = \cos\beta \, \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

$$= (\cos 10°)\frac{\cos 10° - \sqrt{\cos^2 10° - \cos^2 32°}}{\cos 10° + \sqrt{\cos^2 10° - \cos^2 32°}} = 0.296$$

Resultant thrust, $P_A = \frac{1}{2} K_a \gamma H^2$

$$= \left(\frac{1}{2}\right)(0.296)(1.82)(5)^2$$

$$= 6.734 \text{ t/m}$$

This thrust is inclined at 10° to the horizontal (i.e., acts parallel to the ground surface) and is applied at a height of 5/3 = 1.67 m above the base of the wall.

Problem 9.4. A retaining wall with a smooth vertical back has to retain a backfill of cohesionless soil upto a height of 5 m above G.L. The soil has a void ratio of 0.83 and the specific gravity of soil solids is 2.68. The water table is located at a depth of 2 m below the top of the backfill. The soil above the water table is 20% saturated. The angle of internal friction of the soil above and below water table are found to be 32° and 28° respectively. Plot the distribution of active earth pressure on the wall and determine the magnitude and point of application of the resultant thrust.

Solution: Bulk density of the soil above water table,

$$\gamma = \frac{G + se}{1 + e} \cdot \gamma_w$$

$$= \frac{2.68 + (0.2)(0.83)}{1 + 0.83}(1) = 1.55 \text{ t/m}^3$$

Submerged density of the soil below water table,

$$\gamma_{sub} = \frac{G - 1}{1 + e} \cdot \gamma_w = \frac{2.68 - 1}{1 + 0.83}(1) = 0.92 \text{ t/m}^3$$

Active earth pressure above water table:

Co-efficient of active earth pressure, $K_{a_1} = \frac{1 - \sin 32°}{1 + \sin 32°} = 0.307$

At A ($z = 0$), $p_A = 0$

At B ($z = 2$ m), $p_B = K_{a_1} \gamma z_1 = (0.307)(1.55)(2) = 0.95$ t/m^2

Active pressure below water table: In this case the upper layer (i.e., the moist soil above water table) should be treated as a uniform surcharge, for which the intensity q is equal to the self-weight of the layer.

$$\therefore \quad q = \gamma z_1 = (1.55)(2) = 3.10 \text{ t/m}^2$$

$$\text{Now,} \quad K_{a_2} = \frac{1 - \sin 28^\circ}{1 + \sin 28^\circ} = 0.361$$

At B ($z' = 0$), $p_B = K_{a_2} q = (0.361)(3.10) = 1.12 \text{ t/m}^2$

At C ($z' = 3$ m), $p_C = K_{a_2} q + K_{a_2} \gamma_{sub} z' + \gamma_w z'$

$$= 1.12 + (0.361)(0.92)(3) + (1)(3)$$
$$= 1.12 + 0.99 + 3$$
$$= 5.11 \text{ t/m}^2$$

The pressure distribution diagram is shown in Fig. 9.15 (b)

Now, $P_1 = \left(\frac{1}{2}\right)(2)(0.95) = 0.95$ t/m $\quad y_1 = 3 + 2/3 = 3.67$ m

$P_2 = (1.12)(3) = 3..36$ t/m $\quad y_2 = 3/2 = 1.5$ m

$P_3 = \left(\frac{1}{2}\right)(0.99 + 3)(3) = 5.98$ t/m $y_3 = 3/3 = 1$ m

$\therefore$ Resultant thrust $P_A = P_1 + P_2 + P_3$

$$= 10.29 \text{ t per m run.}$$

$$\bar{y} = \frac{(0.95)(3.67) + (3.36)(1.5) + (5.98)(1)}{10.29}$$

$$= 1.41 \text{ m}$$

$\therefore$ The resultant thrust of 10.29 t per m run is applied at 1.41 m above the base of the wall.

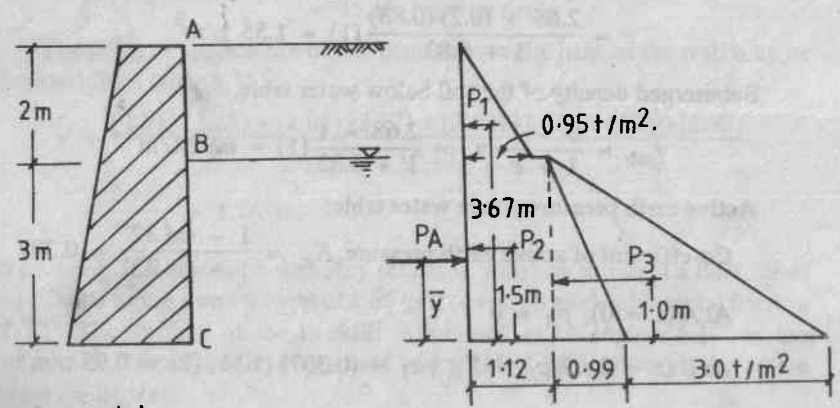


Fig. 9.15

Problem 9.5. For the retaining wall shown in Fig. 9.16 (a), plot the distribution of active earth pressure and determine the magnitude and point of application of the resultant active thrust.

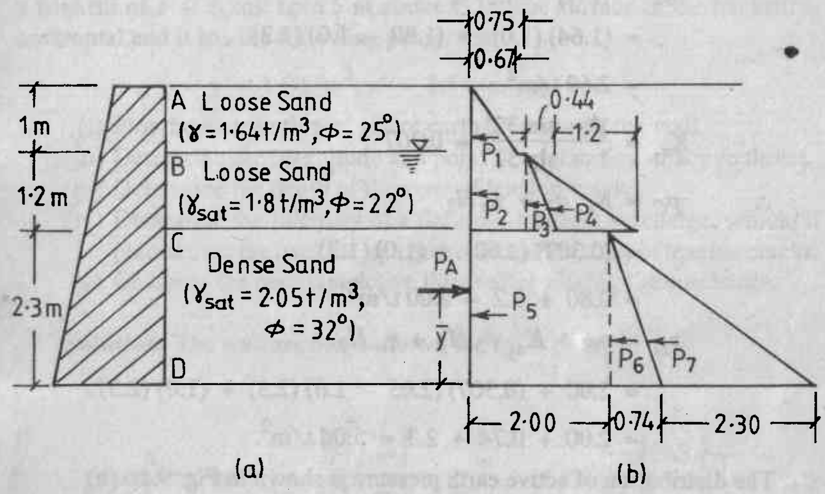


Fig. 9.16

Solution: Active pressures exerted by various strata are as follows:

Stratum I:

$$K_{a_1} = \frac{1 - \sin 25^\circ}{1 + \sin 25^\circ} = 0.406$$

$$p_A = 0$$

$$p_B = K_{a_1} \gamma_1 H_1 = (0.406)(1.64)(1.0) = 0.67 \text{ t/m}^2$$

Stratum II: This stratum is fully submerged. While computing the active earth pressure in this region, stratum I is to be treated as a uniform surcharge of intensity q_1, where,

$$q_1 = \gamma_1 H_1 = (1.64)(1.0) = 1.64 \text{ t/m}^2.$$

$$\text{Now,} \quad K_{a_2} = \frac{1 - \sin 22^\circ}{1 + \sin 22^\circ} = 0.455$$

$$\therefore \quad p_B = K_{a_2} q_1 = (0.455)(1.64) = 0.75 \text{ t/m}^2.$$

$$p_C = K_{a_2} q_1 + K_{a_2} \gamma'_2 H_2 + \gamma_w H_2$$

$$= 0.75 + (0.455)(1.80 - 1.0)(1.2) + (1.0)(1.2)$$

$= 0.75 + 0.44 + 1.2 = 2.39 \text{ t/m}^2$

Stratum III: Equivalent surcharge

$$q_2 = \gamma_1 H_1 + \gamma'_2 H_2$$
$$= (1.64)(1.0) + (1.80 - 1.0)(1.2)$$
$$= 2.60 \text{ t/m}^2.$$
$$K_{a_3} = \frac{1 - \sin 32°}{1 + \sin 32°} = 0.307$$
$$\therefore \quad p_C = K_{a_3} q_2 + \gamma_w H_2$$
$$= (0.307)(2.60) + (1.0)(1.2)$$
$$= 0.80 + 1.2 = 2.00 \text{ t/m}^2.$$
$$p_D = p_C + K_{a_3} \gamma'_3 H_3 + \gamma_w H_3$$
$$= 2.00 + (0.307)(2.05 - 1.0)(2.3) + (1.0)(2.3)$$
$$= 2.00 + 0.74 + 2.3 = 5.04 \text{ t/m}^2.$$

The distribution of active earth pressure is shown in Fig. 9.16 (b)

Computation of forces and lever arms:

$P_1 = (0.5)(1.0)(0.67) = 0.335$ t/m $\quad y_1 = 3.5 + 10/3 = 3.83$ m

$P_2 = (1.2)(0.75) = 0.90$ t/m $\quad y_2 = 2.3 + 1.2/2 = 2.90$ m

$P_3 = (0.5)(1.2)(0.44) = 0.264$ t/m $\quad y_3 = 2.3 + 1.2/3 = 2.70$ m

$P_4 = (0.5)(1.2)(1.2) = 0.72$ t/m $\quad y_4 = 2.3 + 1.2/3 = 2.70$ m

$P_5 = (2.3)(2.0) = 4.6$ t/m $\quad y_5 = 2.3/2 = 1.15$ m

$P_6 = (0.5)(2.3)(0.74) = 0.851$ t/m $\quad y_6 = 2.3/3 = 0.77$ m

$P_7 = (0.5)(2.3)(2.3) = 2.645$ t/m $\quad y_7 = 2.3/3 = 0.77$ m

$$\therefore \quad P_A = \sum_{i=1}^{n} P_i = 10.315 \text{ t/m}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} (P_i \times y_i)}{\sum_{i=1}^{n} P_i} = \frac{14.53}{10.315} = 1.409 \text{ m}$$

Hence the resultant active thrust of 10.315 t per m run is applied at 1.409 m above the base of the wall.

Problem 9.6. A retaining wall with a smooth vertical backface has to retain a backfill of $c - \phi$ soil upto 5 m above G.L. The surface of the backfill is horizontal and it has the following properties:

$$\gamma = 1.8 \text{ t/m}^3, \; c = 1.5 \text{ t/m}^2, \; \phi = 12°$$

(i) Plot the distribution of active earth pressure on the wall.
(ii) Determine the magnitude and point of application of active thrust.
(iii) Determine the depth of the zone of tension cracks.
(iv) Determine the intensity of a fictitious uniform surcharge, which, if placed over the backfill, can prevent the formation of tension cracks.
(v) Compute the resultant active thrust after placing the surcharge.

Solution: The wall section is shown in Fig. 9.17 (a)

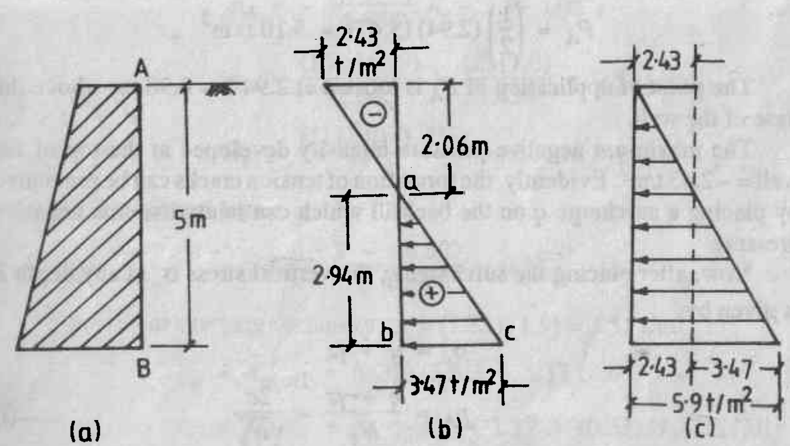


Fig. 9.17

For a $c - \phi$ soil, the intensity of active earth pressure at any depth z is given by:

$$p_a = \frac{\gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}.$$

Here, $\quad N_\phi = \tan^2 (45° + 12°/2) = \tan^2 51° = 1.525$

and, $\quad \sqrt{N_\phi} = 1.235$

At the top of the wall ($z = 0$),

$$p_A = -\frac{2c}{\sqrt{N_\phi}} = -\frac{(2)(1.5)}{1.235} = -2.43 \text{ t/m}^2.$$

At the base of the wall ($z = 5$ m),

$$p_B = \frac{(1.8)(5)}{1.525} - \frac{(2)(1.5)}{1.235} = 3.47 \text{ t/m}^2.$$

The pressure distribution diagram is shown in Fig. 9.17 (b).

The depth of the zone of tension crack is given by,

$$H_c = \frac{2c\sqrt{N_\phi}}{\gamma},$$

or, $$H_c = \frac{(2)(1.5)(1.235)}{1.8} = 2.06 \text{ m}.$$

The resultant active thrust is given by the part *abc* of the pressure distribution diagram.

$$P_A = \left(\frac{1}{2}\right)(2.94)(3.47) = 5.10 \text{ t/m}^2$$

The point of application of P_A is located at 2.94/3 = 0.98 m, above the base of the wall.

The maximum negative pressure intensity developed at the top of the wall = −2.43 t/m². Evidently, the formation of tension cracks can be prevented by placing a surcharge q on the backfill which can neutralise this negative pressure.

Now, after placing the surcharge q, the vertical stress σ_v, at any depth Z is given by,

$$\sigma_v = q + \gamma z$$

$$\therefore \quad p_A = \frac{q + \gamma z}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} \quad \text{...(i)}$$

At $z = 0$, $$p_A = \frac{q}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

But the magnitude of q is such that, at $z = 0$, $p_A = 0$,

$$\therefore \quad \frac{q}{N_\phi} - \frac{2c}{\sqrt{N_\phi}} = 0$$

or, $$q = 2c\sqrt{N_\phi} = (2)(1.5)(1.235) = 3.7 \text{ t/m}^2.$$

Again, at $z = H$, $$p_A = \frac{q + \gamma H}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}$$

$$= \frac{3.7 + (1.8)(5)}{1.525} - \frac{(2)(1.5)}{1.235} = 5.9 \text{ t/m}^2$$

The pressure distribution diagram after placing the surcharge is shown in Fig. 9.17 (c). The resultant active thrust in this case is given by,

$P_A = (0.5)(5.9)(5) = 14.75$ t/m, applied at a height of 5/3 = 1.67 m above the base.

Problem 9.7. A retaining wall of 5 m height has to retain a stratified backfill as shown in Fig. 9.18 (a). Find out the magnitude of total active thrust on the wall and locate its point of application.

Solution: (i) Sandy silt layer:

$$N_\phi = \tan^2(45° + 20°/2) = 2.04$$

$$\sqrt{N_\phi} = 1.438$$

$$p_A = -\frac{(2)(1.0)}{1.438} = -1.39 \text{ t/m}^2.$$

$$p_B = \frac{(1.85)(1.9)}{2.04} - \frac{(2)(1.0)}{1.438} = 0.33 \text{ t/m}^2.$$

$$H_C = \frac{(2)(1.0)(1.438)}{1.85} = 1.55 \text{ m}$$

(ii) Loose sand layer:

$$K_{a_2} = \frac{1 - \sin 30°}{1 + \sin 30°} = 0.33$$

Equivalent surcharge intensity, $q_1 = (1.85)(1.9) = 3.51$ t/m²

$$p_B = K_{a_2} q_1 = (0.33)(3.51) = 1.17 \text{ t/m}^2.$$

$$p_C = K_{a_2} q_1 + K_{a_2}\gamma_2 H_2 = 1.17 + (0.33)(1.72)(1.0)$$

$$= 1.17 + 0.57 = 1.74 \text{ t/m}^2.$$

(iii) Dense sand layer:

$$K_{a_3} = \frac{1 - \sin 36°}{1 + \sin 36°} = 0.26$$

Equivalent surcharge intensity, $q_2 = (1.85)(1.9) + (1.72)(1.0) = 5.23$ t/m²

$$p_C = K_{a_3} q_2 = (0.26)(5.23) = 1.36 \text{ t/m}^2$$

$$p_D = K_{a_3} q_2 + K_{a_3}\gamma_3 H_3$$

$$= 1.36 + (0.26)(1.88)(1.6)$$

$$= 1.36 + 0.78 = 2.14 \text{ t/m}^2$$

Computation of forces and lever arms:

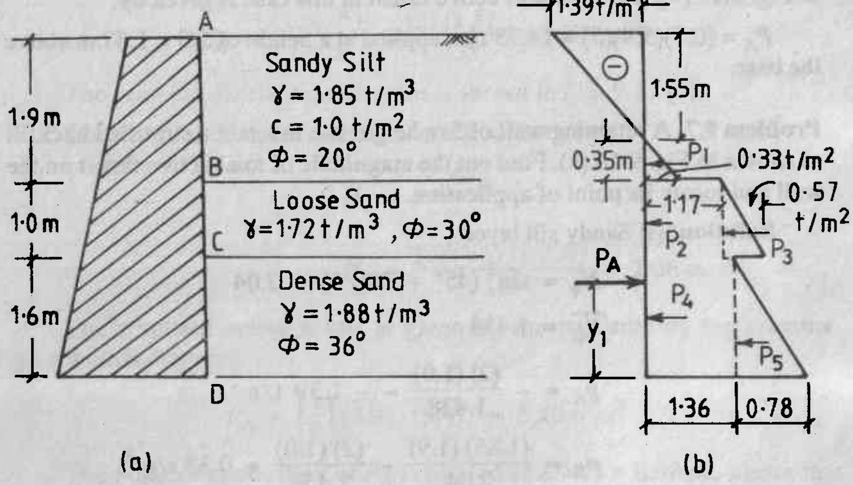


Fig. 9.18

$P_1 = (0.5)(0.35)(0.33) = 0.06 \text{ t/m}, \; y_1 = 2.6 + 0.35/3 = 2.72 \text{ m}$

$P_2 = (1.17)(1.0) = 1.17 \text{ t/m}, \quad y_2 = 1.6 + 1.0/2 = 2.10 \text{ m}$

$P_3 = (0.5)(1.0)(0.57) = 0.29 \text{ t/m}, \quad y_3 = 1.6 + 1.0/3 = 1.93 \text{ m}$

$P_4 = (1.36)(1.6) = 2.18 \text{ t/m}, \quad y_4 = 1.6/2 = 0.80 \text{ m}$

$P_5 = (0.5)(1.6)(0.78) = 0.62 \text{ t/m}, \quad y_5 = 1.6/3 = 0.53 \text{ m}$

$$\sum_{i=1}^{n} P_i = 4.32 \text{ t/m}$$

$$\bar{y} = \frac{\sum_{i=1}^{n} (P_i \times y_i)}{\sum_{i=1}^{n} P_i} = \frac{5.25}{4.32} = 1.216 \text{ m}$$

Problem 9.8. A 4 m high retaining wall has a backface inclined at a positive batter angle of 8°. The backfill ($\gamma = 1.78 \text{ t/m}^3$, $\phi = 30°$) is inclined upwards

at 10° to the horizontal. The angle of wall friction is 20°. Determine the total lateral pressure exerted by the backfill, using:

(a) Culmann's method

(b) Rebhann's method.

Solution: (a) Culmann's method: Fig. 9.19 illustrates the solution of the problem by Culmann's method. The procedure is explained below:

(i) The backface AB is drawn to a scale of 1 : 100.

(ii) The ground line AC, ϕ line BC and ψ line BX are drawn. Here,

$$\psi = 90° - (\delta + \theta) = 90° - (20° + 8°) = 62°.$$

(iii) The points $D_1, D_2, ..., D_8$ are chosen on AC at equal intervals of 1 m. $BD_1, BD_2, ..., BD_8$ are joined.

(iv) From B, $BN \perp AC$ is drawn. Its length is measured and is found to be 4.06 m.

$$\text{Alternatively, } BN = BA \cdot \cos(\alpha - \beta) = H \cdot \frac{\cos(\alpha - \beta)}{\cos \alpha}$$

$$= \frac{(4.0)\cos(10° - 8°)}{\cos 10°} = 4.06 \text{ m}$$

(v) Considering unit width of the wall, the self-weights of various wedges are computed. For example,

$$\text{Weight of the wedge } ABD_1 = W_1 = \frac{1}{2} \cdot AD_1 \cdot BN \cdot \gamma$$

$$= (0.5)(1.0)(4.06)(1.78) = 3.61 \text{ t per m.}$$

Weight of the wedge

$$ABD_2 = W_2 = 2W_1 = (2)(3.61) = 7.22 \text{ t/m.}$$

Similarly, $W_3 = 10.83$ t/m, $W_4 = 14.44$ t/m, $W_5 = 18.05$ t/m

$W_6 = 21.66$ t/m, $W_7 = 25.27$ t/m, $W_8 = 28.88$ t/m.

(vi) Using a vector scale of 1 cm = 3.61 t/m, the weights of various wedges are plotted along BC, and the points $C_1, C_2, ..., C_8$ are obtained.

(vii) From C_1, $C_1E_1 \parallel BX$ is drawn to intersect BD_1 at E_1.

(viii) Similarly, a number of lines are drawn parallel to the ψ-line BX from the points $C_2, C_3, ..., C_8$, and the corresponding intersection points $E_2, E_3, ..., E_8$ with $BD_2, BD_3, ..., BD_8$ respectively are located.

(ix) The pressure curve is obtained by joining $E_1, E_2, ..., E_8$ by a smooth curve.

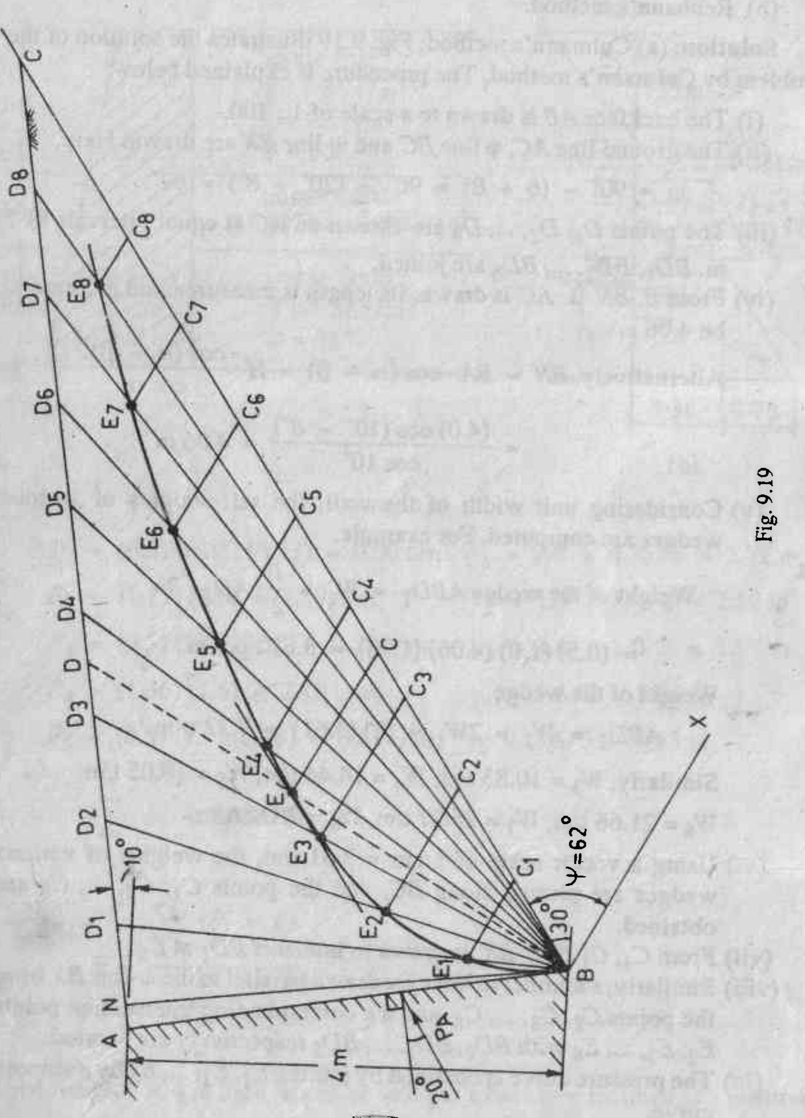


Fig. 9.19

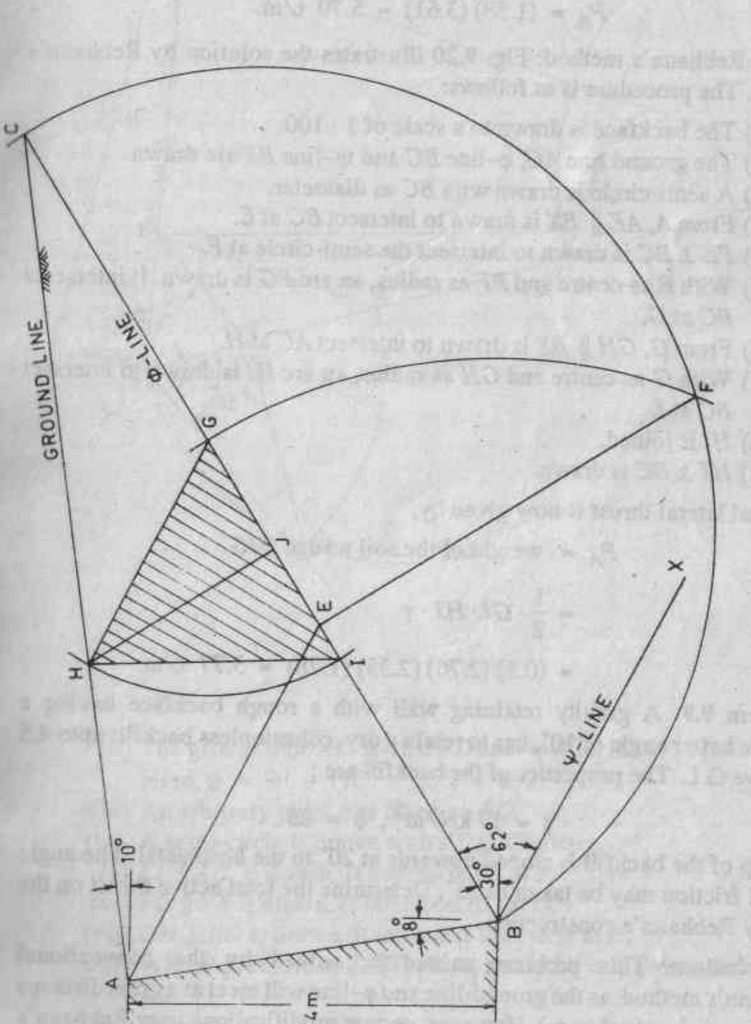


Fig. 9.20

(ix) A tangent to this curve is drawn at E, such that CE denotes the maximum ordinate of the pressure curve. BE is joined and extended to intersect the ground line at D. BD is the failure plane.

The magnitude of the resultant active thrust is given by the distance CE, the length of which is found to be 1.58 cm.

$$\therefore \quad P_A = (1.58)(3.61) = 5.70 \text{ t/m.}$$

(b) Rebhann's method: Fig. 9.20 illustrates the solution by Rebhann's method. The procedure is as follows:

(i) The backface is drawn to a scale of 1 : 100.
(ii) The ground line AC, ϕ–line BC and ψ–line BX are drawn.
(iii) A semi-circle is drawn with BC as diameter.
(iv) From A, $AE \parallel BX$ is drawn to intersect BC at E.
(v) $FE \perp BC$ is drawn to intersect the semi-circle at F.
(vi) With B as centre and BF as radius, an arc FG is drawn. It intersects BC at G.
(vii) From G, $GH \parallel BX$ is drawn to intersect AC at H.
(viii) With G as centre and GH as radius, an arc HI is drawn to intersect BC at I.
(ix) HI is joined.
(x) $HJ \perp BC$ is drawn.

The total lateral thrust is now given by,

$$P_A = \text{weight of the soil wedge } HIG$$

$$= \frac{1}{2} \cdot GI \cdot HJ \cdot \gamma$$

$$= (0.5)(2.76)(2.35)(1.78) = 5.77 \text{ t/m.}$$

Problem 9.9. A gravity retaining wall with a rough backface having a positive batter angle of 10°, has to retain a dry, cohesionless backfill upto 4.5 m above G.L. The properties of the backfill are :

$$\gamma = 17 \text{ kN/m}^3, \; \phi = 25$$

The top of the backfill is sloped upwards at 20° to the horizontal. The angle of wall friction may be taken as 15°. Determine the total active thrust on the wall by Rebhann's construction.

Solution: This problem cannot be solved by the conventional Rebhann's method, as the ground-line and ϕ–line will meet at a great distance ($\because$ β is nearly equal to ϕ). However, certain modifications over Rebhann's method will enable us to solve the problem. The solution is presented in Fig. 9.21, while the procedure is explained below :

(i) The backface of the wall, AB, is drawn to a scale of 1 : 80.

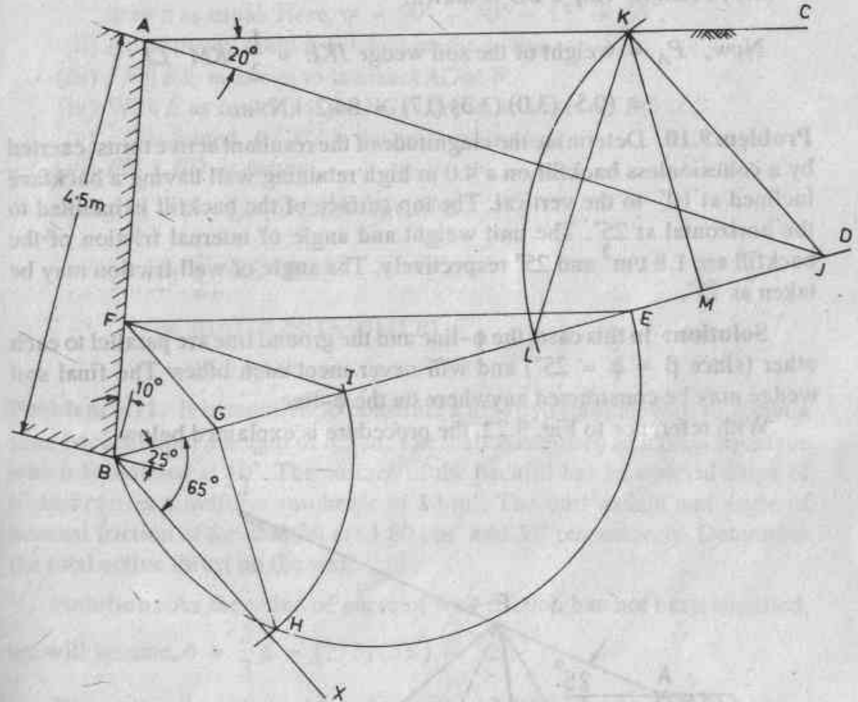


Fig. 9.21

(ii) The ground-line AC, ϕ–line BD and ψ–line BX are drawn. Here, $\psi = 90 - (10° + 15°) = 65°$
(iii) An arbitrary point E is taken on BD.
(iv) A semi-circle is drawn with BE as diameter.
(v) $EF \parallel AC$ is drawn. It intersects AB at F.
(vi) $FG \parallel BX$ is drawn. It intersects BD at G.
(vii) $GH \perp BD$ is drawn. It intersects the circle at H.
(viii) With B as centre and BH radius, an arc HI is drawn to intersect BD at I.
(ix) FI is joined.
(x) $AJ \parallel FI$ is drawn. AJ intersects AD at J.
(xi) From J, $JK \parallel BX$ is drawn to intersect AC at K.
(xii) With J as centre and JK as radius, an arc KL is drawn to intersect BD at L.

(xiii) KL is joined.
(xiv) From K, $KM \perp BD$ is drawn.

Now, P_A = weight of the soil wedge $JKL = \frac{1}{2} \cdot KM \cdot LJ \cdot \gamma$

$= (0.5)(3.0)(3.3)(17) = 84.2$ kN/m.

Problem 9.10. Determine the magnitude of the resultant active thrust exerted by a cohesionless backfill on a 4.0 m high retaining wall having a backface inclined at 10° to the vertical. The top surface of the backfill is inclined to the horizontal at 25°. The unit weight and angle of internal friction of the backfill are 1.8 t/m^2 and 25° respectively. The angle of wall friction may be taken as 15°.

Solution: In this case, the ϕ–line and the ground line are parallel to each other (since $\beta = \phi = 25°$) and will never meet each other. The final soil wedge may be constructed anywhere on the ϕ–line.

With reference to Fig. 9.22, the procedure is explained below :

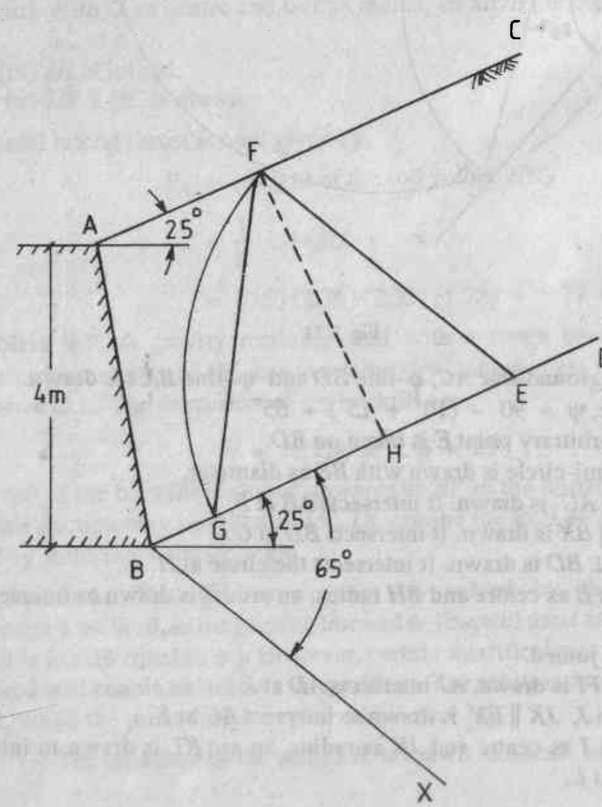


Fig. 9.22

(i) The backface AB, ground line AC, ϕ–line BD and ψ–line BX are drawn as usual. Here, $\psi = 90° - 10° - 15° = 65°$.
(ii) An arbitrary point E is taken on the ϕ–line.
(iii) $EF \parallel BX$ is drawn to intersect AC at F.
(iv) With E as centre, an arc FG is drawn to intersect BD at G.
(v) FG is joined. $\Delta\, EFG$ is the pressure triangle.
(vi) $FH \perp BD$ is drawn.

Now, P_A = weight of the wedge EFG

$= \frac{1}{2} \cdot GE \cdot FH \cdot \gamma$

$= (0.5)(4.28)(3.90)(1.8)$

$= 15.02$ t/m run.

Problem 9.11. It is required to construct a gravity retaining wall to retain a sand backfill upto a height of 3.5 m. The wall is required to have a backface which is battered at 10°. The surface of the backfill has an upward slope of 8° and carries a uniform surcharge of 1 t/m^2. The unit weight and angle of internal friction of the backfill are 1.80 t/m^3 and 33° respectively. Determine the total active thrust on the wall.

Solution: As the value of angle of wall friction has not been supplied, we will assume, $\delta = \frac{2}{3}\phi = (2/3)(33°) = 22°$

The uniform surcharge placed over the backfill may be replaced by an equivalent soil layer which will exert the same vertical stress on the backfill. The height of this equivalent soil layer is,

$$h_e = \frac{q}{\gamma} = \frac{1.0}{1.80} = 0.56 \text{ m.}$$

The problem can now be solved by Rebhann's method, with a few modifications as explained below :

(i) The backface AB, ground line AC, ϕ–line BC and ψ–line BX are drawn as usual.
(ii) From A, a vertical line AA' is drawn and the distance AA_1 is laid off from it such that, $AA_1 = h_e = 0.56$ m.
(iii) Through A_1, A_1C_2 is drawn parallel to the ground line to intersect the ϕ -line at C_2. AB and A_1C_2 are projected backwards to meet at A_2. A_2B and A_2C_2 will now be treated as the modified backface and the ground line respectively.

The problem is then solved by Rebhann's construction in the usual manner. With reference to Fig. 9.23, the resultant active thrust,

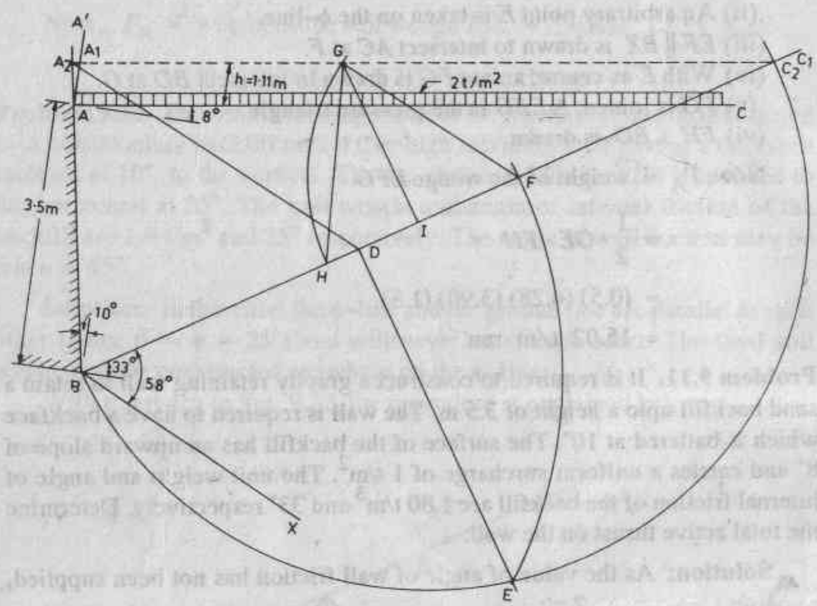


Fig. 9.23

P_A = weight of the wedge $FGH = \frac{1}{2} \cdot FH \cdot GI \cdot \gamma$

= (1/2) (2.73) (2.32) (1.80) = 5.70 t per m run.

Problem 9.12. A retaining wall has to support a homogeneous backfill upto 3.6 m above G.L. A long brick wall, 375 mm thick and 3.5 m high, runs parallel to the retaining wall at a horizontal distance of 2.8 m from the top corner of the back of the wall. Determine the total lateral thrust on the wall, given,

$\gamma = 1.8\ t/m^3$, $\phi = 30°$, $c = 0$, $\alpha = 78°$, $\beta = 6°$, $\delta = 15°$.

Solution: Unit weight of brick masonry = 1920 kg/m^3.

∴ Self-weight of the brick wall per metre run

= (0.375) (3.5) (1920) = 2.52 t.

The brick wall acts as a concentrated linear load on the backfill. The problem can be solved by Culmann's graphical method. The solution is presented in Fig. 9.24 and the step-by-step procedure is stated below :

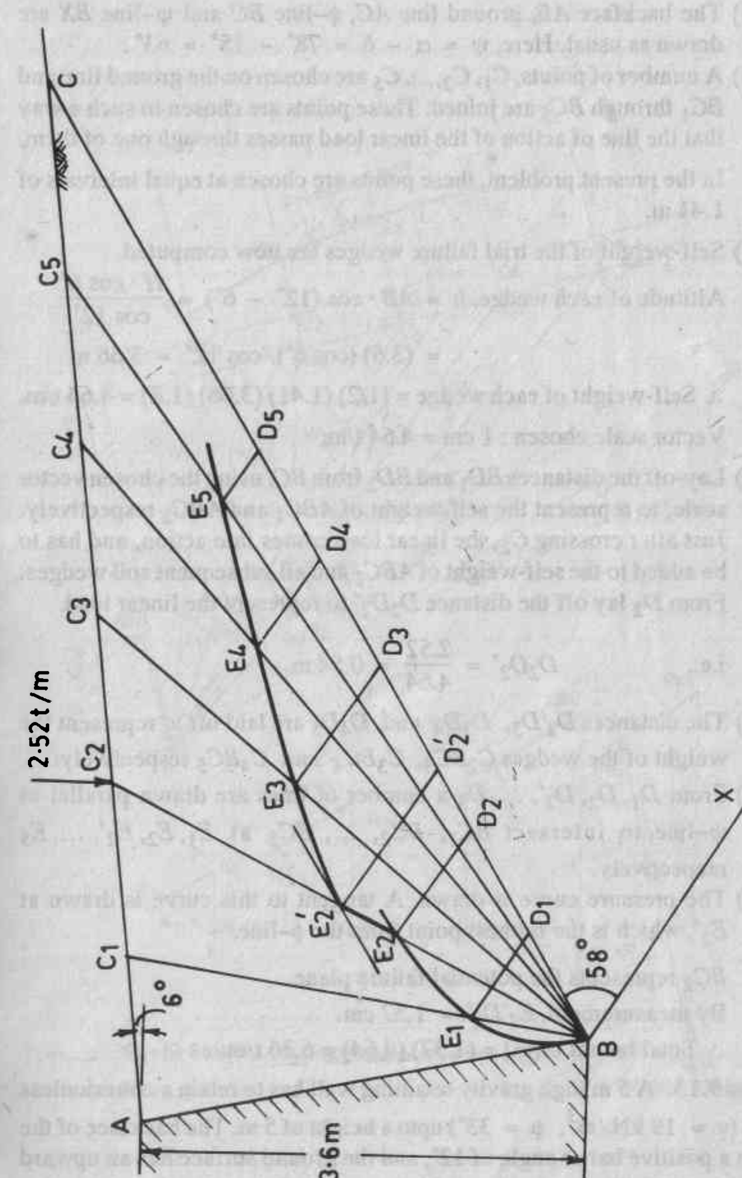


Fig. 9.24

(i) The backface AB, ground line AC, ϕ–line BC and ψ–line BX are drawn as usual. Here, $\psi = \alpha - \delta = 78° - 15° = 63°$.

(ii) A number of points, $C_1, C_2, ..., C_5$ are chosen on the ground line and BC_1 through BC_5 are joined. These points are chosen in such a way that the line of action of the linear load passes through one of them.

In the present problem, these points are chosen at equal intervals of 1.41 m.

(iii) Self-weight of the trial failure wedges are now computed.

Altitude of each wedge, $h = AB \cdot \cos(12° - 6°) = \dfrac{H \cdot \cos 6°}{\cos 12°}$

$= (3.6)(\cos 6°)/\cos 12° = 3.66$ m

$\therefore$ Self-weight of each wedge = (1/2) (1.41) (3.66) (1.8) = 4.64 t/m.

Vector scale chosen : 1 cm = 4.64 t/m.

(iv) Lay-off the distances BD_1 and BD_2 from BC, using the chosen vector scale, to represent the self-weight of ABC_1 and ABC_2 respectively. Just after crossing C_2, the linear load comes into action, and has to be added to the self-weight of ABC_2 and all subsequent soil wedges. From D_2 lay off the distance D_2D_2' to represent the linear load.

i.e., $D_2D_2' = \dfrac{2.52}{4.64} = 0.54$ m.

(v) The distances $D_2'D_3$, D_3D_4 and D_4D_5 are laid off to represent the weight of the wedges C_2BC_3, C_3BC_4 and C_4BC_5 respectively.

(vi) From $D_1, D_2, D_2', ..., D_5$ a number of lines are drawn parallel to ψ–line to intersect BC_1, BC_2, ..., BC_5 at $E_1, E_2, E_2', ..., E_5$ respectively.

(vii) The pressure curve is drawn. A tangent to this curve is drawn at E_2', which is the farthest point from the ϕ–line.

BC_2 represents the potential failure plane.

By measurement, $E_2'D_2' = 1.37$ cm.

$\therefore$ Total lateral thrust = (1.37) (4.64) = 6.36 t/m.

Problem 9.13. A 5 m high gravity retaining wall has to retain a cohesionless backfill ($\gamma = 19$ kN/m^3, $\phi = 33°$) upto a height of 5 m. The backface of the wall has a positive batter angle of 12°, and the ground surface has an upward inclination of 15°. The angle of wall friction is 20°. Determine the total active thrust by the trial wedge method.

Solution: Fig. 9.25 (a) shows the section of the wall, drawn to scale. The ground line AC and the ϕ-line BD are drawn.

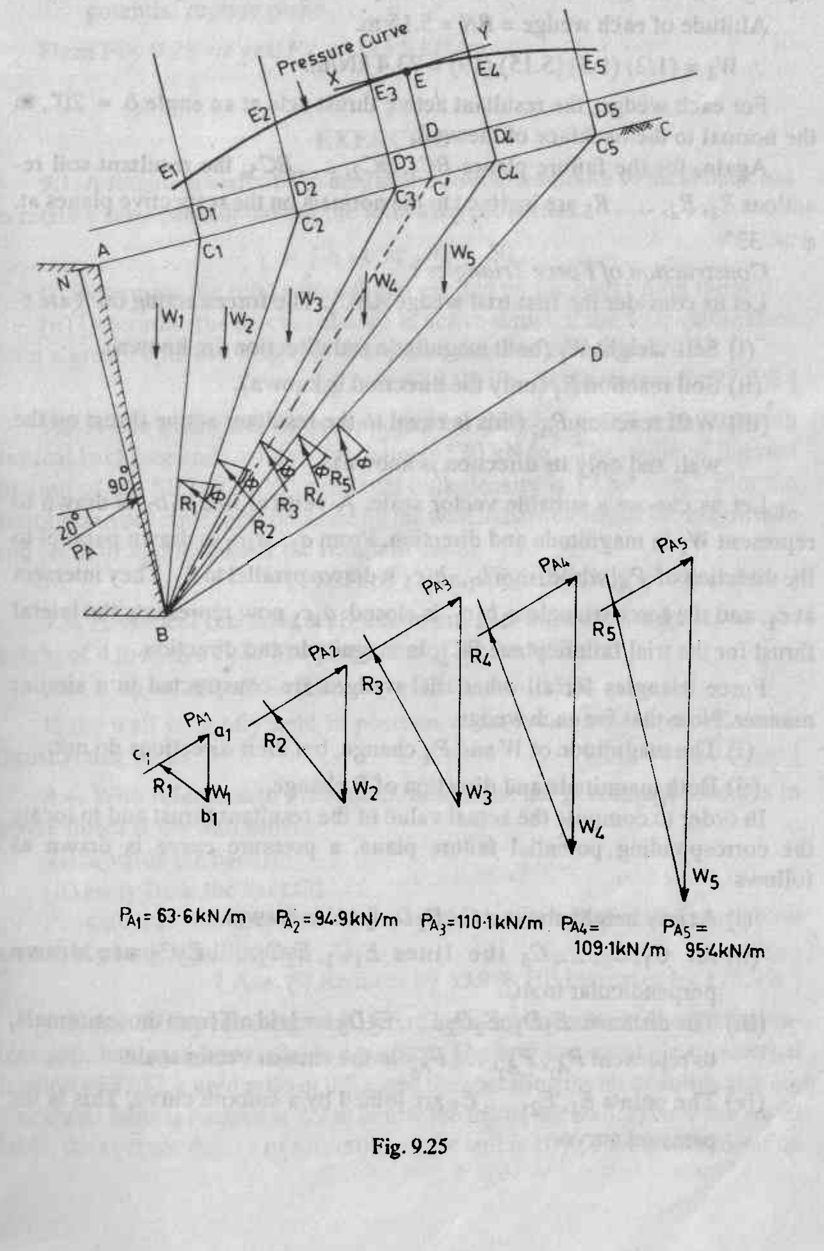


Fig. 9.25

Equal distances $AC_1 = C_1C_2 = \ldots = C_4C_5 = 1.5$ m are laid off from AC. The lines $BC_1, BC_2, \ldots, BC_5$ are joined. These are the trial failure lines. Let $W_1, W_2, \ldots, W_5$ be the self-weights of the wedges $ABC_1, C_1BC_2, \ldots, C_4BC_5$. According to the construction, $W_1 = W_2 = \ldots = W_5 = W$ (say).

Altitude of each wedge = BN = 5.15 m.

$\therefore W_1 = (1/2)(1.5)(5.15)(19) = 73.4$ kN/m.

For each wedge, the resultant active thrust acts at an angle $\delta = 20°$, to the normal to the backface of the wall.

Again, for the failure planes $BC_1, BC_2, \ldots, BC_5$, the resultant soil reactions $R_1, R_2, \ldots, R_5$ are inclined to the normals on the respective planes at, $\phi = 33°$.

Construction of Force Triangles :

Let us consider the first trial wedge ABC_1. The forces acting on it are :

(i) Self-weight W_1 (both magnitude and direction are known).

(ii) Soil reaction R_1 (only the direction is known).

(iii) Wall reaction P_{A_1} (this is equal to the resultant active thrust on the wall and only its direction is known).

Let us choose a suitable vector scale. A vertical line a_1b_1 is drawn to represent W_1 in magnitude and direction. From a_1, a_1c_1 is drawn parallel to the direction of P_A, while from b_1, b_1c_1 is drawn parallel to R_1. They intersect at c_1, and the force triangle $a_1b_1c_1$ is closed. a_1c_1 now represents the lateral thrust for the trial failure plane BC_1, in magnitude and direction.

Force triangles for all other trial wedges are constructed in a similar manner. Note that for each wedge,

(i) The magnitude of W and P_A change, but their directions do not.

(ii) Both magnitude and direction of R change.

In order to compute the actual value of the resultant thrust and to locate the corresponding potential failure plane, a pressure curve is drawn as follows:

(i) At any height above AC, $D_1D_5 \parallel AC$ is drawn.

(ii) At $C_1, C_2, \ldots, C_5$ the lines $E_1C_1, E_2C_2, \ldots, E_5C_5$ are drawn perpendicular to AC.

(iii) The distances $E_1D_1, E_2D_2, \ldots, E_5D_5$ are laid off from those normals, to represent $P_{A_1}, P_{A_2}, \ldots, P_{A_5}$ to the chosen vector scale.

(iv) The points $E_1, E_2, \ldots, E_5$ are joined by a smooth curve. This is the pressure curve.

(v) At E, a tangent XEY is drawn to the curve, making it parallel to AC. The distance of this tangent from DE gives the maximum value of P_A.

(vi) From E, draw $EC' \perp AC'$. Join BC', which now represents the potential rupture plane.

From Fig. 9.25 we get, $P_A = 113.5$ kN/m.

EXERCISE 9

9.1. A retaining wall of 4 m height and having a smooth vertical back has to retain a sand backfill having the following properties:

$$\gamma = 1.85 \text{ t/m}^3, \ \phi = 30°.$$

(i) Determine the total active thrust exerted by the backfill on the wall.

(ii) Determine the percent change in active thrust, if the water table rises from a great depth to a height of 2 m above the base of the wall.

[Ans. (i) 4.93 t/m (ii) Increases by 27.2%]

9.2. A 6 m high earth fill is supported by a retaining wall with a smooth vertical backface and carries a surcharge of 30 kN/m^2. The angle of internal friction of the fill soil is 30°, while its bulk density is 17.5 kN/m^3. Plot the distribution of active earth pressure on the wall. Also determine the magnitude and point of application of the resultant thrust.

[Ans. 165 kN/m, applied at 2.36 m above base]

9.3. A vertical retaining wall has to retain a horizontal backfill upto a height of 4 m above G.L. The properties of the backfill are :

$$c = 0, \ \phi = 28°, \ G = 2.68, \ w = 11\%, \ s = 55\%, \ \mu = 0.5$$

If the wall is rigidly held in position, what is the magnitude of active thrust acting on it? [Ans. 15.5 t/m]

9.4. With reference to Problem 3, determine the percentage changes in active thrust if the wall moves :

(i) towards the backfill

(ii) away from the backfill

Assume that, the lateral movement of the wall is sufficient to bring about a state of plastic equilibrium.

[Ans. (i) Reduces by 63.9% (ii) Increases by 176.9%]

9.5. A masonry retaining wall, 5.5 m high, retains a backfill of cohesionless soil, having a horizontal top surface. The soil has an angle of internal friction of 27.5°, a void ratio of 0.83, and the specific gravity of solids is 2.65. The water table is located at 2.2 m below the top of the wall. Above the water table, the average degree of saturation of the soil is 10%. Plot the distribution

of active earth pressure and compute the magnitude and point of application of the resultant thrust. [Ans. 12.56 t/m applied at 1.58 m above the base]

9.6. A cohesionless backfill, retained by a 5 m high retaining wall with a smooth vertical back, is bounded by a horizontal surface. The water table is at 2 m below the top of the wall. Above the water table, the angle of internal friction and bulk density of the soil are 18 kN/m^3 and 30° respectively. Below the water table, the bulk density increases by 10% while the friction angle decreases by 20%. Determine the resultant active pressure on the wall.

[Ans. 97.9 kN/m]

9.7. A retaining wall having a smooth vertical back retains a dry, cohesionless backfill. State, giving reasons, how the active earth pressure exerted by the backfill will change in each of the following cases:

(a) the backfill becomes saturated due to capillary water, while the ground water table remains below the base of the wall.

(b) the ground water table rises above the base, but there is no capillary water.

(c) the given backfill is replaced by a cohesionless soil having :

(i) same unit weight but greater angle of internal friction.

(ii) same angle of internal friction but greater unit weight.

(iii) same unit weight and angle of internal friction, but having a small apparent cohesion.

9.8. Compute the total active thrust and its point of application for the retaining wall shown in Fig. 9.26. The wall has a smooth backface.

[Ans. 3.6 t/m, 0.90 m above the base]

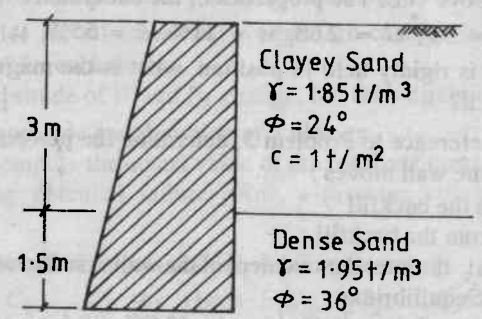


Fig. 9.26

9.9. A smooth vertical retaining wall has to retain a backfill of cohesionless soil upto a height of 4 m above G.L. The properties of the backfill are :

$$\gamma = 19\ \text{kN/m}^3,\ \phi = 36°$$

(a) Determine the active thrust on the wall if the backfill has a horizontal top surface.

(b) Determine the percent change in the active thrust if, instead of being horizontal, the backfill is now sloped upwards at an angle of 15° to the horizontal. [Ans. (a) 39.52 kN/m. (b) Increases by 8.85%]

9.10. A masonry wall has to retain a cohesive backfill having an unconfined compressive strength of 4 t/m^2 and a bulk density of 1.72 gm/cc. The overall height of the wall is 6 m. Determine :

(i) the depth upto which tension cracks will be extended.

(ii) the magnitude and point of application of the active thrust.

[Ans. (i) 2.32 m (ii) 11.63 kN/m at 1.23 m above base]

9.11. With reference to Problem 9.10, determine the minimum intensity of a uniform surcharge, which when placed over the backfill, will prevent the formation of tension cracks. [Ans. 3.08 t/m]

9.12. A 5 m high masonry retaining wall with a vertical backface retains a horizontal backfill of dry sand having $\gamma = 20$ kN/m^3 and $\phi = 32°$. Compute the resultant active thrust on the wall by :

(i) Rankine's theory

(ii) Coulomb's theory, using the trial wedge method.

Which one of the results is more realistic and why?

[Ans. (i) 76.75 kN/m (ii) 79.3 kN/m, assuming $\delta = \frac{2\phi}{3}$]

9.13. An R.C.C. retaining wall, having a backface inclined to the vertical at 10°, has to retain a horizontal backfill of dry sand upto a height of 5.2 m. The soil has a unit weight of 17.5 kN/m^3 and an angle of internal friction of 28°. The angle of friction between soil and concrete may be taken as 18°. Determine the point of application, direction and magnitude of the active thrust. Use the trial wedge method.

[Ans. 98 kN/m, at 18° to the normal on the backface]

9.14. Solve Problem 9.13 graphically, using :

(i) Culmann's method.

(ii) Rebhann's construction.

9.15. A gravity retaining wall has to retain a 6 m high backfill of dry, cohesionless soil ($\gamma = 19$ kN/m^3, $\phi = 36°$) having a surcharge angle of 8°. The back of the wall has a positive batter angle of 10°. The backfill carries a linear load of 5 t/m, running parallel to the wall, at a distance of 3 m from the top of the backface, measured along the ground. Compute the total active thrust on the wall by Culmann's method. Locate the point of application and direction of this thrust. Assume, $\delta = \frac{2}{3}\phi$. [Ans. 155 kN/m]

9.16. Compute the total active thrust exerted by the backfill on the retaining wall system shown in Fig. 9.27. Locate the position of the potential rupture surface.

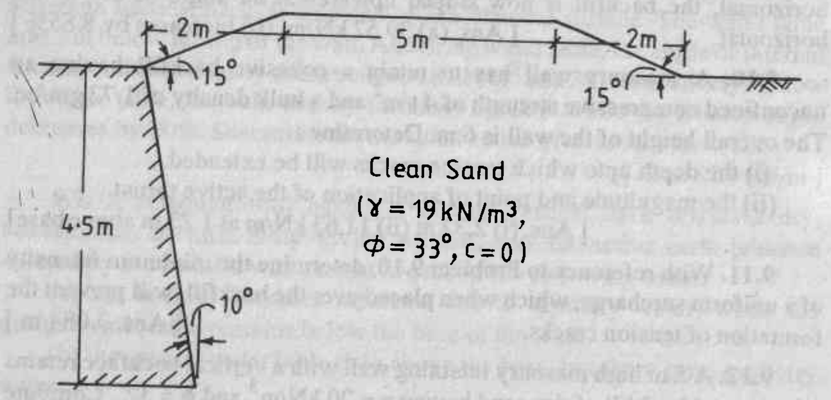


Fig. 9.27

9.17. The backfill placed behind a 5 m high masonry retaining wall consists of a partially saturated clayey silt, having the following properties:

unit weight = 18.5 kN/m^3
cohesion = 10 kN/m^2
angle of internal friction = 21°
angle of wall friction = 12°
adhesion between soil and wall = 8 kN/m^2
surcharge angle of backfill = 10°

The back of the wall is inclined to the horizontal at 80°. Determine the magnitude and direction of the active thrust by the trial wedge method. Also determine the depth to which tension cracks will be extended.

[Ans. 27.5 kN/m run ; 1.57 m]

9.18. A retaining wall, 4.5 m high and having a positive batter angle of 15°, has to retain a cohesionless backfill having a unit weight of 1.95 t/m^3 and an angle of internal friction of 31°. Using Rebhann's method, determine the magnitude of lateral thrust on the wall, if the surcharge angle of the backfill is:

(i) 10° (ii) 25° (iii) 31°.

9.19. A 4 m high retaining wall with a vertical backface was constructed to retain a backfill of loose sand with a horizontal top surface flushed to the top of the wall. Laboratory investigations revealed that the sand had the following properties:

$$\phi = 25°,\ G = 2.65,\ e = 1.05,\ s = 0$$

The back of the wall is relatively smooth. Compute the total active earth pressure exerted by the backfill using any suitable theory.

A few months after construction, the backfill was thoroughly compacted and consequently, its ϕ -value increased to 32°. However, the top surface of the backfill was depressed by 80 cm. Determine the percent change in the total active earth pressure.

9.20. A 4 m high earth-retaining structure having a smooth vertical backface retains a backfill having the following properties:

$$c = 2\ t/m^2,\ \phi = 22°,\ \gamma = 1.85\ t/m^2$$

Plot the distribution of passive pressure on the wall and determine the magnitude and point of application of the total lateral force.

[Ans. 56.3 t/m ; 1.61 m above the base]

10

STABILITY OF SLOPES

10.1 Introduction: A slope in a soil mass is encountered when the elevation of the ground surface gradually changes from a lower level to a higher one. Such a slope may be either natural (in hilly region) or man-made (in artificially constructed embankment or excavations).

The soil mass bounded by a slope has a tendency to slide down. The principal factor causing such a sliding failure is the self-weight of the soil. However, the failure may be aggravated due to seepage of water or seismic forces. Every man-made slope has to be properly designed to ascertain the safety of the slope against sliding failure.

Various methods are available for analysing the stability of slopes. Generally these methods are based on the following assumptions :

1. Any slope stability problem is a two-dimensional one.
2. The shear parameters of the soil are constant along any possible slip surface.
3. In problems involving seepage of water, the flownet can be constructed and the seepage forces can be determined.

10.2 Stability of Infinite Slopes: In Fig. 10.1, *X-X* represents an infinite slope which is inclined to the horizontal at an angle β. On any plane *YY* (*YY* || *XX*) at a depth z below the ground level the soil properties and the overburden pressure are constant. Hence, failure may occur along a plane parallel to the slope at some depth. The conditions for such a failure may be analysed by considering the equilibrium of the soil prism *ABCD* of width b.

Considering unit thickness, volume of the prism $V = z\,b\cos\beta$

and, weight of the prism, $W = \gamma z\,b\cos\beta$

Vertical stress on *YY* due to the self-weight.

$$\sigma_z = \frac{W}{b} = \gamma z\cos\beta \qquad ...(10.1)$$

This vertical stress can be resolved into the following two components :

$$\sigma = \sigma_z\cos\beta = \gamma z\cos^2\beta \qquad ...(10.2)$$

and, $\tau = \sigma_z\sin\beta = \gamma z\cos\beta\sin\beta \qquad ...(10.3)$

Failure will occur if the shear stress τ exceeds the shear strength τ_f of the soil. The factor of safety against such failure is given by,

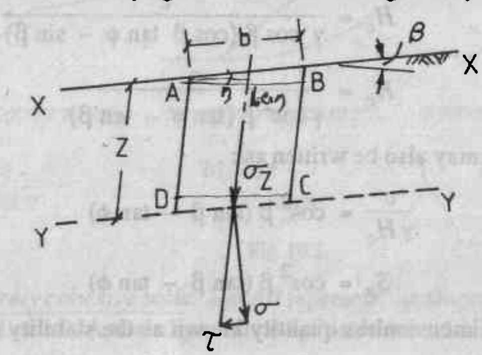


Fig. 10.1.

$$F = \frac{\tau_f}{\tau} \qquad ...(10.4)$$

(i) *Cohesionless soils*: We have from Coulomb's equation,

$$\tau_f = c + \sigma\tan\phi$$

For a cohesionless soil, $c = 0$,

$\therefore \qquad \tau_f = \sigma\tan\phi$

Substituting in eqn. (10.4)

$$F = \frac{\sigma\tan\phi}{\tau}$$

Again, substituting the expressions for σ and τ.

$$F = \frac{\gamma z\cos^2\beta\cdot\tan\phi}{\gamma z\cos\beta\sin\beta} = \frac{\tan\phi}{\tan\beta} \qquad ...(10.5)$$

When $\phi = \beta$, $F = 1$. Thus a slope in a cohesionless soil is stable till $\beta \le \phi$, provided that no external force is present.

(ii) $c - \phi$ *soils*: In this case, the factor of safety against slope failure is given by,

$$F = \frac{c + \sigma\tan\phi}{\tau}$$

or, $$F = \frac{c + \gamma z\cos^2\beta\tan\phi}{\gamma z\cos\beta\sin\beta} \qquad ...(10.6)$$

Let H_c be the critical height of the slope for which $F = 1$ (i.e, $\tau_f = \tau$)

$$\therefore \quad \gamma H_c \cos\beta \sin\beta = c + \gamma H_c \cos^2\beta \tan\phi$$

or,
$$H_c = \frac{c}{\gamma \cos\beta (\cos\beta \tan\phi - \sin\beta)}$$

or,
$$H_c = \frac{c}{\gamma \cos^2\beta (\tan\phi - \tan\beta)} \quad ...(10.7)$$

Eqn. (10.7) may also be written as :

$$\frac{c}{\gamma H_c} = \cos^2\beta (\tan\beta - \tan\phi) \quad ...(10.8)$$

or,
$$S_n = \cos^2\beta (\tan\beta - \tan\phi) \quad ...(10.9)$$

where, S_n is a dimensionless quantity known as the stability number and is given by :

$$S_n = \frac{c}{\gamma H_c} \quad ...(10.10)$$

If a factor of safety F_c is applied to the cohesion such that the mobilised cohesion at a depth H is,

$$c_m = \frac{c}{F_c} \quad ...(10.11)$$

Then,
$$S_n = \frac{c_m}{\gamma H} = \frac{c}{F_c \gamma H} \quad ...(10.12)$$

From eqns. (10.10) and (10.12), we get,

$$\frac{c}{\gamma H_c} = \frac{c}{F_c \gamma H}$$

or,
$$F_c = \frac{H_c}{H} = F_H.$$

Hence, the factor of safety against cohesion, F_c, is the same as the factor of safety with respect to height, F_H.

10.3 Stability of Finite Slopes: In case of slopes of limited extent, three types of failure may occur. These are: face failure, toe failure and base failure (Fig. 10.2 *a*, *b* and *c* respectively).

Various methods of analysing the failure of finite slopes are discussed below.

10.4 Swedish Circle Method: In this method, the surface of sliding is assumed to be an arc of a circle.

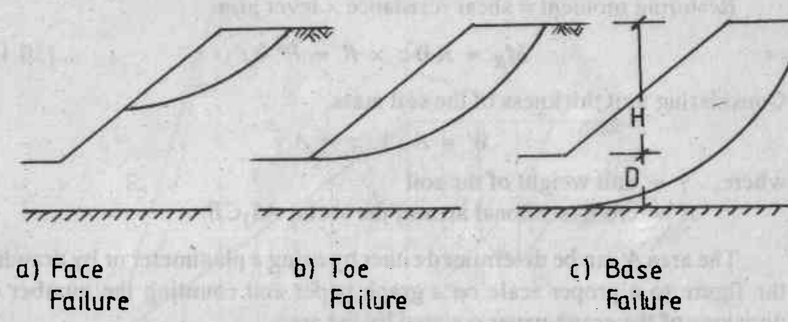


Fig. 10.2

(a) Purely cohesive soils: Let AB represent the slope whose stability has to be investigated. A trial slip circle AS_1C is drawn with O as centre and $OA = OC = R$ as radius.

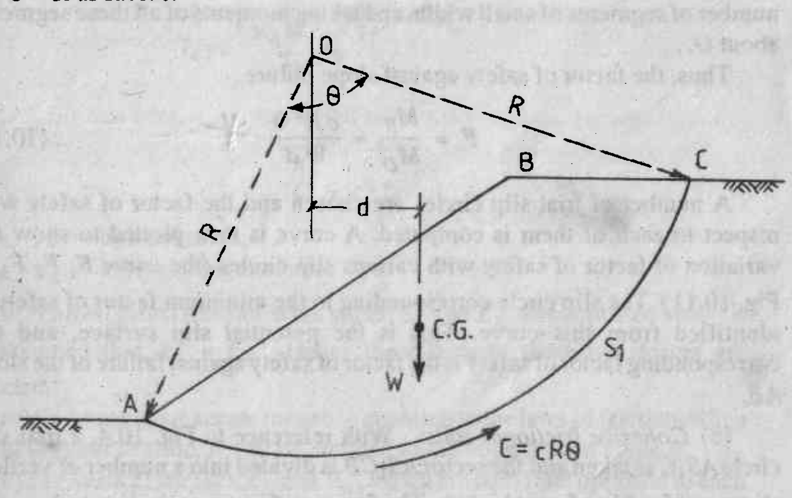


Fig. 10.3

Let W be the weight of the soil mass AS_1CB acting vertically downwards through the centre of gravity and c be the unit cohesion of the soil. The self-weight tends to cause the sliding while the shear resistance along the plane AS_1C counteracts it.

Now, arc length $\quad AS_1C = R \cdot \theta$

where, $\theta = \angle AOC$ (expressed in radians)

$\therefore$ Total shear resistance along the plane $AS_1C = R\,\theta\,c$

Restoring moment = shear resistance × lever arm

or $$M_R = R\,\theta\,c \times R = R^2\,\theta\,c \qquad ...(10.13)$$

Considering unit thickness of the soil mass,

$$W = A \cdot 1 \cdot \gamma = A\,\gamma$$

where, γ = unit weight of the soil

A = cross–sectional area of the sector AS_1CB.

The area A can be determined either by using a planimeter or by drawing the figure to a proper scale on a graph paper and counting the number of divisions of the graph paper covered by the area.

Now, disturbing moment, $M_d = W \cdot d$

where, d = lever arm of W with respect to O.

The distance d may be determined by dividing the area into an arbitrary number of segments of small width, and taking moments of all these segments about O.

Thus, the factor of safety against slope failure,

$$F = \frac{M_R}{M_D} = \frac{c\,R^2\,\theta}{W\,d} \qquad ...(10.14)$$

A number of trial slip circles are chosen and the factor of safety with respect to each of them is computed. A curve is then plotted to show the variation of factor of safety with various slip circles (the curve $F_1\,F_2\,F_3$ in Fig. 10.11). The slip circle corresponding to the minimum factor of safety is identified from this curve. This is the potential slip surface, and the corresponding factor of safety is the factor of safety against failure of the slope AB.

(*b*) *Cohesive frictional soils*: With reference to Fig. 10.4, a trial slip circle AS_1C is taken and the sector AS_1CB is divided into a number of vertical slices, preferably of equal width. The forces acting on each slice are:

(i) Self-weight, W, of the slice, acting vertically downwards through the centre of gravity. Considering unit thickness of the slice,

$$W = \gamma \times b_a \times l_a \qquad ...(10.15)$$

where, b_a and l_a represent the average height and length of the slice respectively.

(ii) The cohesive force, C, acting along the arc in a direction opposing the probable motion of the sliding soil.

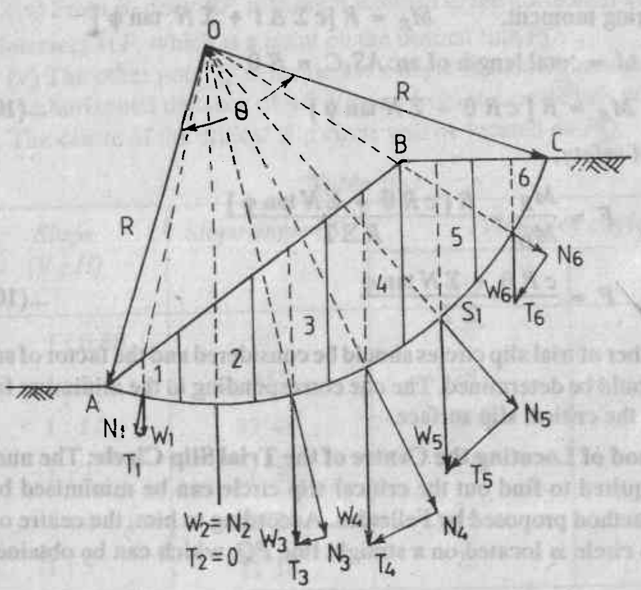


Fig. 10.4

$$C = c \cdot l_a$$

where, c = unit cohesion,

l_a = average length of slice

(iii) Lateral thrust from adjacent slices, E_L and E_R. In simplified analysis it is assumed that, $E_L = E_R$. Hence the effects of these two forces are neglected.

(iv) Soil reaction R across the arc. According to the laws of friction, when the soil is about to slide, R will be inclined to the normal at an angle ϕ.

(v) The vertical stresses, V_L and V_R, which are equal and opposite to each other and hence need not be considered.

The weight W is resolved into a normal component N and a tangential component T. For some of the slices T will enhance the failure, for the others it will resist the failure. The algebraic sum of the normal and tangential components are obtained from :

$$\Sigma T = \Sigma (W \sin \alpha) \qquad ...(10.17)$$

and, $$\Sigma N = \Sigma (W \cos \alpha) \qquad ...(10.18)$$

Now, driving moment, $$M_D = R\,\Sigma T \qquad ...(10.19)$$

and, Restoring moment, $M_R = R\,[c\,\Sigma\,\Delta l + \Sigma N \tan\phi\,]$

But $\Sigma\,\Delta l$ = total length of arc $AS_1C = R\,\theta$

$$\therefore \quad M_R = R\,[\,c\,R\,\theta + \Sigma N \tan\phi\,] \qquad ...(10.20)$$

$\therefore$ Factor of safety,

$$F = \frac{M_R}{M_D} = \frac{R\,[\,c\,R\,\theta + \Sigma N \tan\phi\,]}{R\,\Sigma\,T}$$

or,
$$F = \frac{c\,R\,\theta + \Sigma N \tan\phi}{\Sigma\,T} \qquad ...(10.21)$$

A number of trial slip circles should be considered and the factor of safety for each should be determined. The one corresponding to the minimum factor of safety is the critical slip surface.

10.5 Method of Locating the Centre of the Trial Slip Circle: The number of trials required to find out the critical slip circle can be minimised by an empirical method proposed by Fellenius. According to him, the centre of the critical slip circle is located on a straight line *PQ*, which can be obtained as follows :

(i) Draw the given slope *AB* and determine the slope angle, β.

(ii) Determine the values of the angles α_1 and α_2 (Fig. 10.5) from Table 10.1.

(iii) From *A*, draw *AP* at an angle of α_1 to *AB*.

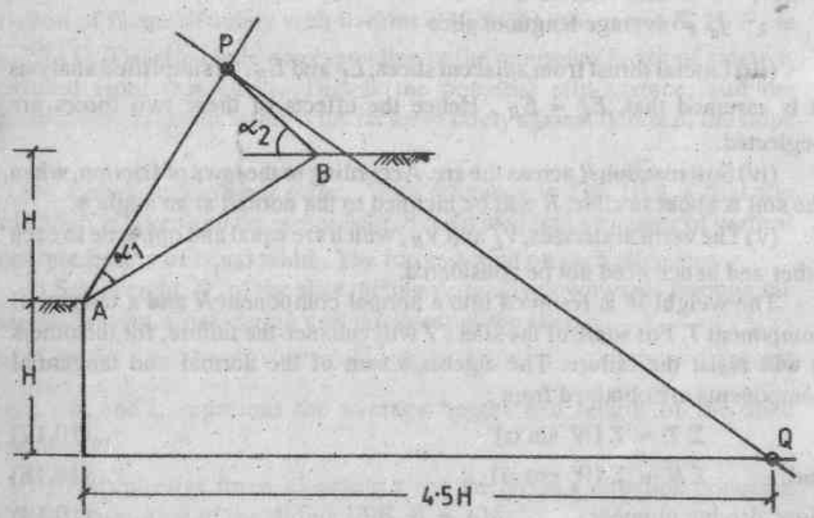


Fig. 10.5

(iv) From *B*, draw *BP*, making it inclined to the horizontal at α_2. *BP* and *AP* intersect at *P*, which is a point on the desired line *PQ*.

(v) The other point *Q* is located at a depth *H* below the toe of the slope and at a horizontal distance of 4.5 *H* away from it. Locate this point and join *PQ*. The centre of the critical slip circle will be located on *PQ*.

Table 10.1

Slope (*V* : *H*)	*Slope angle* (β)	*Values of angles*	
		α_1	α_2
1 : 0.58	60°	29°	40°
1 : 1	45°	28°	37°
1 : 1.5	33°48′	26°	35°
1 : 2	26°36′	25°	35°
1 : 3	18°24′	25°	35°
1 : 5	11°18′	25°	27°

10.8 Friction Circle Method: This method is based on the assumption that the resultant force *R* on the rupture surface is tangential to a circle of radius $\gamma = R \sin\phi$ which is concentric with the trial slip circle. Various steps involved are given below :

1. Draw the given slope to a chosen scale.

2. Select a trial slip circle of radius *R*, the centre of which is located at *O* (Fig. 10.6 a)

3. Compute r (= $R \sin\phi$) and draw another circle of radius *r*, with *O* as the centre.

4. Now consider the equilibrium of the sliding soil mass under the following forces :

(i) Self-weight *W* of the sector *ABCD*.

(ii) The cohesive force *C* along the plane *ADC*, the magnitude and direction of which can be computed as follows :

Let *c* be the unit cohesion. The arc *ADC* is divided into a number of small elements. Let $C_1, C_2, ..., C_n$ be the mobilised cohesive forces along them.

The resultant *C* of these forces can be determined by drawing a force polygon.

Now, the mobilised unit cohesion, c_m', is given by :

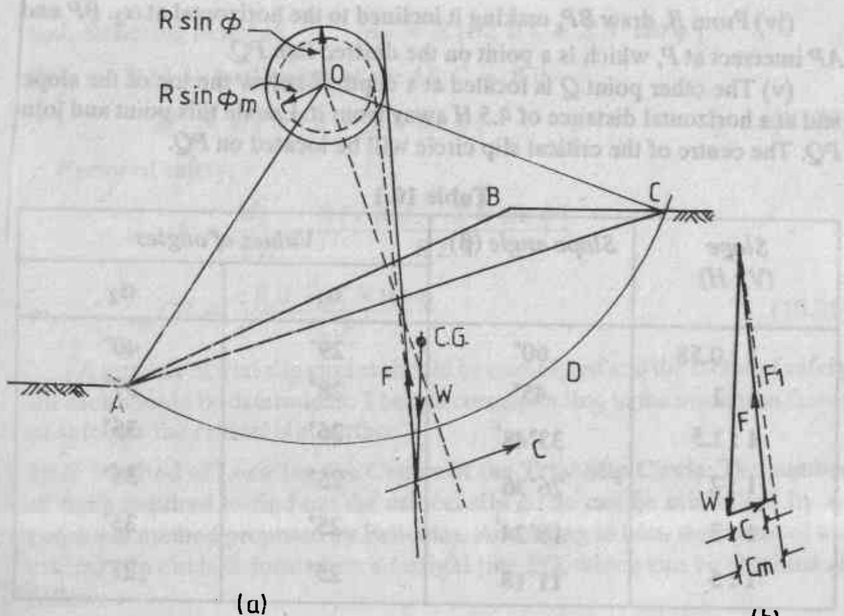


Fig. 10.6

$$c_m' = \frac{c'}{F_c} \quad \text{...(10.22)}$$

where, F_c = factor of safety with respect to cohesion.

The cohesive force is given by

$$C = C_m' L_c = \frac{c' L_c}{F_c} \quad \text{...(10.23)}$$

But, summing up the moments of all forces about o and equating to zero, we get,

$$C \cdot L_a \cdot R = C \cdot L_c \cdot a \quad \text{...(10.24)}$$

where, a = perpendicular distance of line of action of C from the centre of the slip circle.

$$\therefore \quad a = \frac{L_a}{L_c} \cdot R \quad \text{...(10.25)}$$

(iii) The other force is the soil reaction F, which is assumed to be **tangential** to the friction circle.

5. Draw the triangle of forces in the following manner :

(i) Draw a vertical line ab to represent W (Fig. 10.6 (b)).

(ii) From a draw ac, making it parallel to the line of action of F_R.

(iii) From b drop a perpendicular bd on ac. The line bd now represents, in magnitude and direction, the cohesive force C_R required to maintain the equilibrium of the soil mass $ABCD$ along the chosen slip circle.

6. Determine the unit cohesion c_r required for stability from :

$$c_r = \frac{c}{L_c} \quad \text{...(10.26)}$$

7. The factor of safety w.r.t. cohesion is now obtained from :

$$F_c = \frac{\text{actual cohesion}}{\text{required cohesion}} = \frac{c}{c_r} \quad \text{...(10.27)}$$

8. The factor of safety w.r.t. shear strength can be obtained as follows:

(i) Assume a certain factor of safety with respect to the angle of internal friction. Let it be F_ϕ. The mobilised angle of internal friction is then given by:

$$\tan \phi_m = \frac{\tan \phi}{F_\phi} \quad \text{...(10.28)}$$

(ii) Draw a new friction circle with O as centre and r' as radius, where,

$$r' = R \sin \phi_m \quad \text{...(10.29)}$$

(iii) The factor of safety w.r.t. cohesion F_c is then obtained by forming another triangle of forces. Compare F_C and F_ϕ. If they are different, go for another trial.

(iv) In this manner, adjust the radius of the circle until F_ϕ and F_c become equal to each other. This value is then accepted as the factor of safety for shear strength of the soil w.r.t. the given trial slip circle.

10.9 Taylor's Stability Number: Taylor carried out stability analysis of a large number of slopes having various heights, slope angles and soil properties. On the basis of the results, he proposed a simple method by which the factor of safety of a given finite slope can be easily determined with reasonable accuracy. Taylor introduced a dimensionless parameter, called Taylor's Stability Number, which is given by,

$$S_n = \frac{c}{F_c \gamma H} \quad \text{...(10.30)}$$

The value of S_n may be obtained from Fig. 10.7.

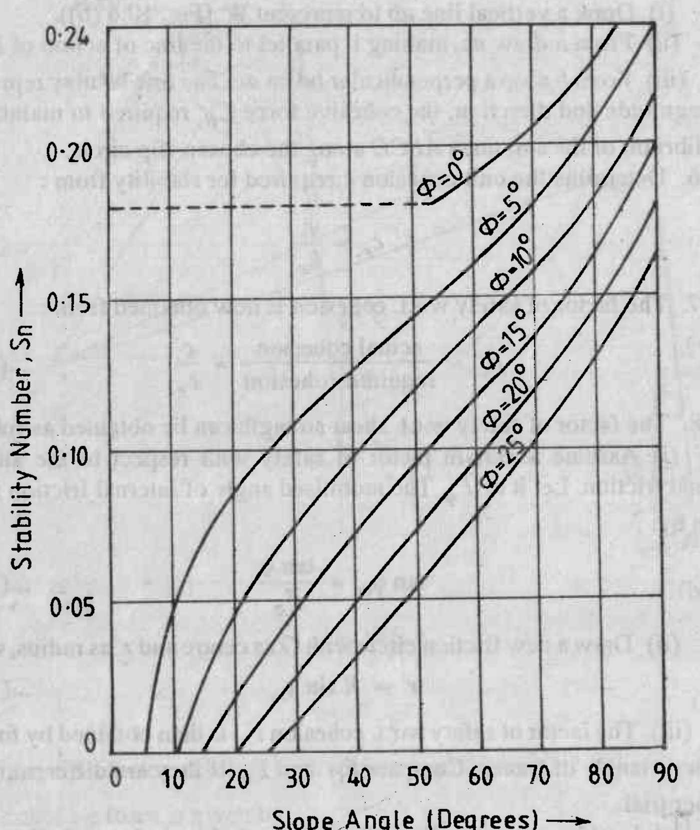


Fig. 10.7

The stability numbers are obtained for factor of safety w.r.t. cohesion, while the factor of safety w.r.t. friction, F_ϕ is initially taken as unity.

The values of S_n obtained from Fig. 10.7 are applicable for slip circles passing through the toe. However for slopes made in cohesive soils of limited depth and underlain by a hard stratum, the critical slip circle passes below the toe. In such cases, the value of S_n should be obtained from Fig. 10.8. In this figure, the depth factor plotted along the x-axis is defined as :

$$n_d = \frac{D + H}{H} \quad ...(10.31)$$

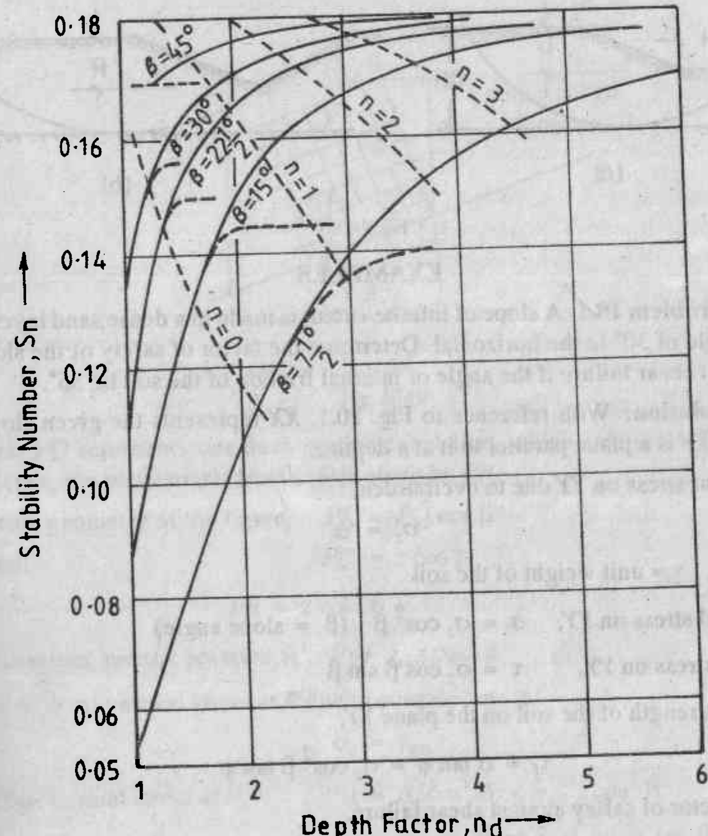


Fig. 10.8

where, D = Depth of hard stratum below toe

H = Height of slope above toe.

Fig. 10.8 consists of a family of curves for various slope angles. Each curve consists of two parts. The portions drawn with firm lines are applicable to field conditions illustrated in Fig. 10.9 (a), while the portions drawn with broken lines are meant for the conditions shown in Fig. 10.9 (b).

The figure also consists of a third set of curves, shown with broken lines, for various values of n, where n represents the distance x of the rupture circle from the toe, as illustrated in Fig. 10.9 (a), and is given by,

$$n = \frac{x}{H}$$

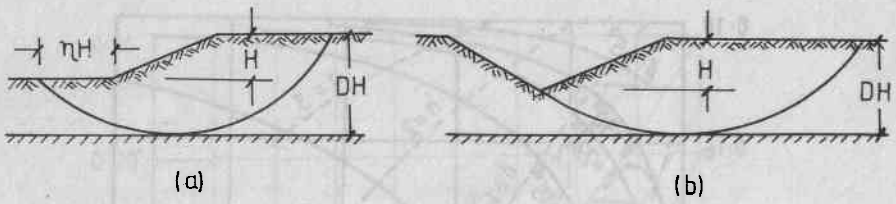


Fig. 10.9

EXAMPLES

Problem 10.1 A slope of infinite extent is made in a dense sand layer at an angle of 30° to the horizontal. Determine the factor of safety of the slope against shear failure if the angle of internal friction of the soil be 36°.

Solution: With reference to Fig. 10.1, *XX* represents the given slope, while *YY* is a plane parallel to it at a depth *z*.

Vertical stress on *YY* due to overburden,

$$\sigma_z = \gamma z$$

where, γ = unit weight of the soil

Normal stress on *YY*, $\sigma = \sigma_z \cos^2\beta$ (β = slope angle)

Shear stress on *YY*, $\tau = \sigma_z \cos\beta \sin\beta$

Shear strength of the soil on the plane *YY*,

$$\tau_f = \sigma \tan\phi = \sigma_z \cos^2\beta \tan\phi .$$

But, factor of saftey against shear failure,

$$F_S = \frac{\tau_f}{\tau} = \frac{\sigma_z \cos^2\beta \tan\phi}{\sigma_z \cos\beta \sin\beta} = \frac{\tan\phi}{\tan\beta}$$

$$= \frac{\tan 36°}{\tan 30°} = 1.258 .$$

Problem 10.2 A slope inclined at 16° to the horizontal is to be made in a cohesionless deposit having the following properties :

$G = 2.70$, $e = 0.72$, $\phi = 35°$,

Determine the factor of safety of the slope against shear failure if water percolates in a direction parallel to the surface of the slope.

Solution: The given conditions are shown in Fig. 10.10.

YY is a plane located at a depth *z* below the slope. As water percolates in a direction parallel to the slope, all flow lines must be parallel to the slope. Therefore, all equipotential lines should be perpendicular to the slope. The

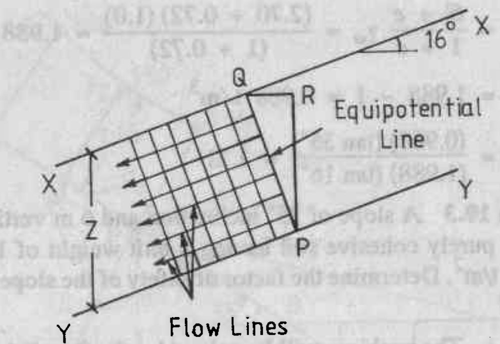


Fig. 10.10

line *PQ* represents one such equipotential line, which intersects *YY* at *P*. Hence, the piezometric head at *P* is given by *PR*.

From geometry of the figure, $PR = PQ \cos\beta$

and, $PQ = z\cos\beta$

$\therefore$ $PR = z\cos^2\beta$

Therefore, neutral pressure at $P = \gamma_w z \cos^2\beta$

Now, total vertical stress at *P* due to overburden,

$$\sigma_z = \gamma_{sat}\, z$$

Total normal stress at *P*, $\sigma = \sigma_z \cos^2\beta = \gamma_{sat}\, z \cos^2\beta$

Shear stress at *P*, $\tau = \sigma_z \cos\beta \sin\beta = \gamma_{sat}\, z \cos\beta \sin\beta$

Effective normal stress at *P* = total normal stress – neutral stress

or, $\sigma' = \gamma_{sat} \cdot z \cos^2\beta - \gamma_w z \cos^2\beta$

$$= z\cos^2\beta\,(\gamma_{sat} - \gamma_w) = \gamma_{sub}\, z \cos^2\beta .$$

However, the shearing stress is entirely intergrannular.

$\therefore$ Shear strength of the soil on *YY*,

$$\tau_f = \sigma' \tan\phi = \gamma_{sub}\, z \cos^2\beta \tan\phi$$

$\therefore$ Factor of safety against shear failure,

$$F_S = \frac{\tau_f}{\tau} = \frac{\gamma_{sub}\, z \cos^2\beta \tan\phi}{\gamma_{sat}\, z \cos\beta \sin\beta} = \frac{\gamma_{sub} \tan\phi}{\gamma_{sat} \tan\beta}$$

Now, $\gamma_{sat} = \dfrac{G + e}{1 + e} \gamma_w = \dfrac{(2.70 + 0.72)(1.0)}{(1 + 0.72)} = 1.988 \text{ t/m}^3$

$\therefore \quad \gamma_{sub} = 1.988 - 1 = 0.988 \text{ t/m}^3$

$\therefore \quad F_S = \dfrac{(0.988)(\tan 35°)}{(1.988)(\tan 16°)} = 1.21$

Problem 10.3 A slope of 35° inclination and 6 m vertical height is to be made in a purely cohesive soil having a unit weight of 1.85 t/m^3 and a cohesion of 6 t/m^2. Determine the factor of safety of the slope against sliding failure.

Solution : The problem will be solved by the Swedish circle method. The solution is presented in Fig. 10.11 and the procedure is explained below:

(i) The given slope AB is drawn to a scale of 1 : 200.

(ii) The values of α_1 and α_2 for $\beta = 35°$ are determined from Table 10.1 by making linear interpolation between $\beta = 33°48'$ and $\beta = 45°$. The following values are obtained :

$$\alpha_1 = 26.2°, \ \alpha_2 = 35°$$

(iii) The point Q, lying at a depth of $H = 6$ m below A and at a linear distance of 4.5 H = 27 m from A is located.

(iv) From A and B, two straight lines AP and BP are drawn such that,

$$\angle PAB = 26.2°, \text{ and } \angle HBP = 35°$$

AP and BP intersect at P.

(v) PQ is joined. The centre of the critical slip circle should be located on this line.

(vi) PB is measured and found to be 4. 6 m. On projected PQ, two more points P' and P'' are taken such that, $PP' = P'P'' = \dfrac{4.6}{2} = 2.3$ m.

(vii) Three trial slip circles are drawn with P, P' and P'' as centres and PA, $P'A$ and $P''A$ respectively as radius. The factor of safety with respect to each circle is determined separately.

Fig. 10.11 shows the determination of F_S with respect to the first trial slip circle, having its centre at P.

The procedure is stated below :

(i) The area under the slope and the slip surface is divided into 7 slices. The first 6 slices have a width of 2 m each while the width of the 7th slice is 2.2 m.

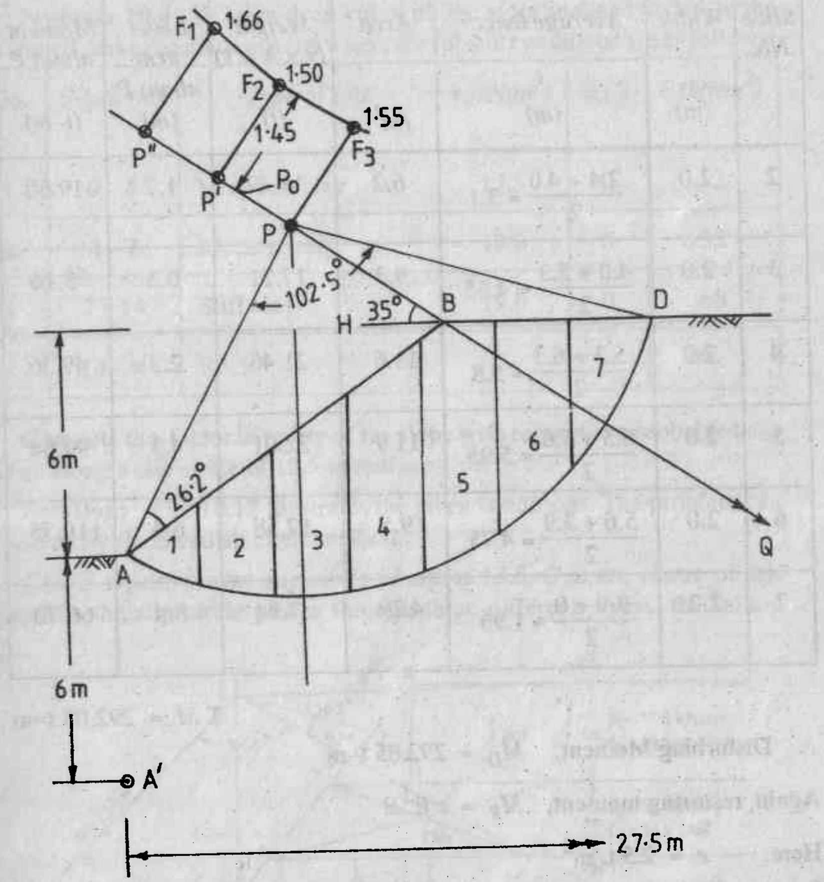


Fig. 10.11

(ii) Considering unit thickness, the area, weight and the disturbing moment for each slice are determined. These are tabulated below :

Slice No.	Width (m)	Average length (m)	Area (m^2)	Weight ($\gamma \times A \times 1$) (t)	Lever arm about P (m)	Moment about P (t–m)
1	2.0	$\dfrac{0 + 2.2}{2} = 1.1$	2.2	4.07	3.7	–15.06

Slice No.	*Width* (m)	*Average length* (m)	*Area* (m^2)	*Weight* $(\gamma \times A \times 1)$ (t)	*Lever arm about P* (m)	*Moment about P* (t–m)
2	2.0	$\frac{2.4+4.0}{2} = 3.1$	6.2	11.47	1.7	–19.50
3	2.0	$\frac{4.0+5.3}{2} = 4.65$	9.3	17.21	0.3	5.16
4	2.0	$\frac{5.3+6.3}{2} = 5.8$	11.6	21.46	2.3	49.36
5	2.0	$\frac{6.3+5.6}{2} = 5.95$	11.9	22.01	4.3	94.64
6	2.0	$\frac{5.6+3.9}{2} = 4.75$	9.5	17.58	6.3	110.75
7	2.2	$\frac{3.9+0}{2} = 1.95$	4.29	7.94	8.4	66.70

$$\Sigma M = 292.05 \text{ t-m}$$

∴ Disturbing Moment, $M_D = 292.05$ t–m

Again, restoring moment, $M_R = c R^2 . \theta$

Here, $c = 2.5 \text{ t/m}^2$

$R = PA = 9.9$ m

$\theta = \angle APD = 102.5° = 1.789$ radian

∴ $M_R = (2.5)(9.9^2)(1.789) = 438.35$ t–m

∴ Factor of safety $= \frac{M_R}{M_D} = \frac{438.35}{292.05} = 1.50$

In a similar manner, the factor of safety of the slope w.r.t. the two other slip circles (having their centres at P' and P'') are determined and are found to be 1.55 and 1.66 respectively. A curve representing the variation of factor of safety is then plotted. The minimum factor of safety of the slope, as obtained from this curve, is 1.45. The corresponding critical slip circle will have its centre located at P_o.

Problem 10.4 A 10 m deep cut, with the sides inclined at 50° to the horizontal, has to be made at a site where the subsoil conditions are as follows:

No.	*Depth (m)*	*Type of soil*	γ *(kN/m³)*	ϕ *(°)*	*c (kN/m²)*
1	0 - 4	Very soft clay	17.5	0	12
2	4 - 7	Medium clay	18.0	0	35
3	7 - 14	Stiff clay	19.0	0	68
4	14 - ∞	Rock	—	–	–

Compute the factor of safety of the slope with respect to a probable base failure along a slip circle of 13.5 m radius.

Solution: Fig. 10.12 illustrates the given conditions. The problem can be solved by the Swedish circle method.

CDGE represents the slip circle of radius 13.5. *O* is the centre of this circle. As the slip circle passes through three different layers, the failure

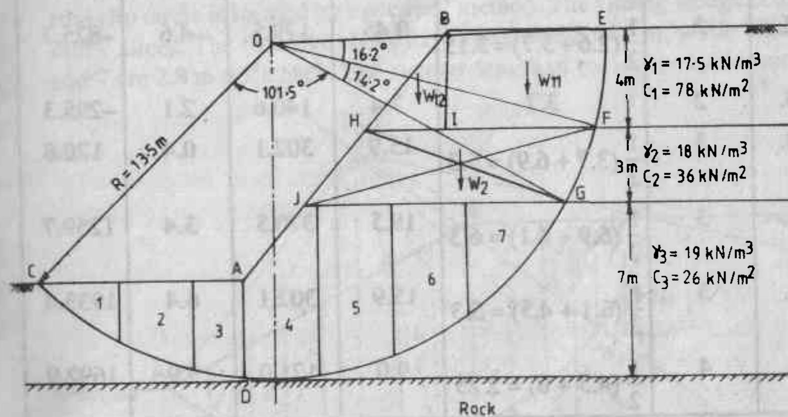


Fig. 10.12

wedge consists of three different zones. Let W_1, W_2 and W_3 be the weights of the three zones and x_1, x_2 and x_3 be the corresponding lever arms about *O*.

From *B*, draw $BI \perp HF$. Zone 1 may now be divided into the triangle *BHI* and the rectangle *BIFE*

Weight of $\Delta BHI = (0.5)(17.5)(3.3)(4.0)$ kN $= 115.5$ kN.

Weight of sector $BIFE$ = (17.5) (6.5) (4.0) kN = 455 kN.

$\therefore \quad W_1 = 115.5 + 455 = 560.5$ kN.

$$x_1 = \frac{(115.5)(7.2 - 3.3/3) + (455)(7.2 + 6.5/2)}{560.5} = 9.74 \text{ m.}$$

The second zone *HFGJ* is assumed to be a parallelogram, the centroid of which lies at the intersection of the diagonals.

$$W_2 = (18)(10.6)(3) = 572.4 \text{ kN}$$

$$x_2 = 7.5 \text{ m} \quad \text{(by measurement)}$$

In order to find out W_3 and x_3, zone III is divided into 7 slices. The area, lever arm and moment of each slice about O are determined. These are tabulated below :

Slice No.	*Width (m)*	*Average length (m)*	*Area (m^2)*	*Weight (kN)*	*Lever arm about O (m)*	*Moment about O (kN - m)*
1.	3.5	$\frac{1}{2}(0 + 2.6) = 1.3$	4.55	96.3	7.6	–731.8
2.	3	$\frac{1}{2}(2.6 + 3.7) = 3.15$	9.45	179.5	4.6	–825.7
3.	2	3.7	7.4	140.6	2.1	–295.3
4.	3	$\frac{1}{2}(3.7 + 6.9) = 5.3$	15.9	302.1	0.4	120.8
5.	3	$\frac{1}{2}(6.9 + 6.1) = 6.5$	19.5	370.5	3.4	1259.7
6.	3	$\frac{1}{2}(6.1 + 4.5) = 5.3$	15.9	302.1	6.4	1933.4
7.	4	$\frac{1}{2}(4.5 + 0) = 2.25$	9.0	171.0	9.9	1692.9

$\Sigma M = 3153.9$

$$\therefore \quad W_3 \times x_3 = 3153.9 \text{ kN} - \text{m}$$

Now, total disturbing moment $= W_1 x_1 + W_2 x_2 + W_3 x_3$

$= (560.5)(9.74) + (572.4)(7.5) + 3153.9$

$= 12906.19$

As the slip circle passes through three different soil layers, the resisting force consists of the cohesive forces mobilised along the three segments of the slip circle. The corresponding angles are shown in the figure.

Therefore, total restoring moment

$$= c_1 R^2 \theta_1 + c_2 R^2 \theta_2 + c_3 R^2 \theta_3$$

$$= R^2 (c_1 \theta_1 + c_2 \theta_2 + c_3 \theta_3)$$

$$= (13.5^2)\,[(26)(101.5) + (36)(14.2) + (78)(16.2)]\,\frac{\pi}{180}$$

$$= 14039.69 \text{ kN–m.}$$

$\therefore$ Factor of safety along the given slip circle

$$= \frac{14039.69}{12906.19} = 1.09$$

Problem 10.5 A slope of 1 V : 2 H is to be made in a silty clay having an angle of internal friction of 5° and a cohesion of 0.25 kg/cm^2. The unit weight of the soil is 1.85 gm/cc, and the depth of cut is 8 m. Compute the factor of safety of the slope by the Swedish circle method.

Solution: The given slope is shown in Fig. 10.13 (a). The centre of a trial slip circle is located by Fellenius' method. The sliding wedge is divided into 7 slices. The first five slices have equal width of 4 m, while slice no. 6 and 7 are 2.8 m wide each. The average length of each slice is measured.

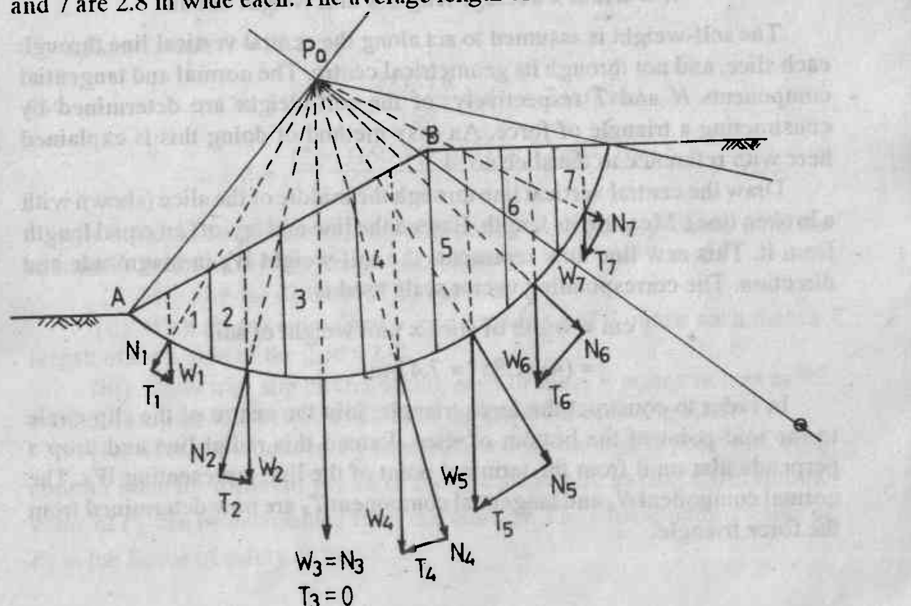


Fig. 10.13 (a)

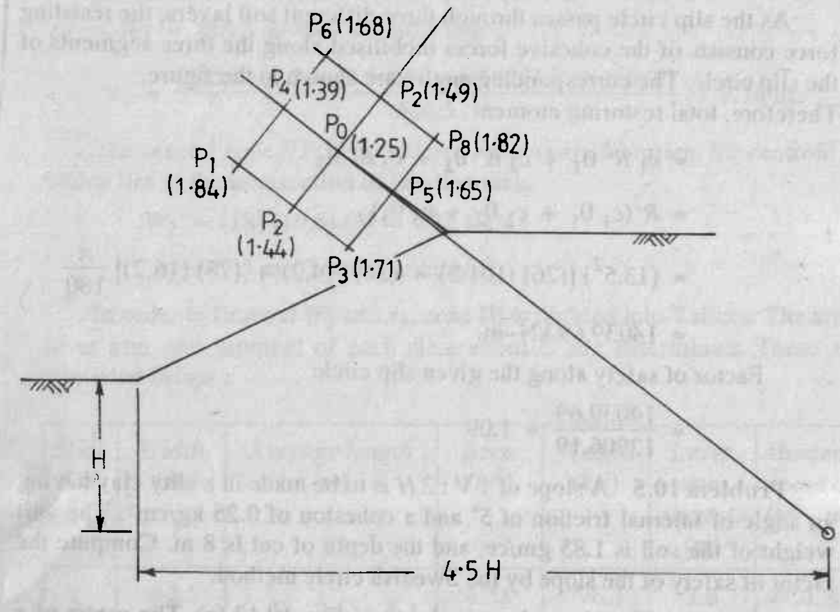


Fig. 10.13 (b)

The weight of any slice may now be determined as :

$$W = \text{width} \times \text{average length} \times \text{unit weight of soil}$$

The self-weight is assumed to act along the central vertical line through each slice, and not through its geometrical centre. The normal and tangential components N and T respectively, of the self-weight are determined by constructing a triangle of force. An easy method of doing this is explained here with reference to the slice no. 4.

Draw the central vertical line through the middle of the slice (shown with a broken line). Measure its length. Extend the line and lay-off an equal length from it. This new line now represents the self-weight W_4 in magnitude and direction. The corresponding vector scale used is

$$1 \text{ cm} = \text{width of slice} \times \text{unit weight of soil}$$

$$= (4)\,(1.85) = 7.4 \text{ t/m}$$

In order to construct the force triangle, join the centre of the slip circle to the mid-point of the bottom of slice. Extend this radial line and drop a perpendicular on it from the terminal point of the line representing W_4. The normal component N_4 and tangential component T_4 are now determined from the force triangle.

In this manner, the normal and tangential components for each slice are determined. The results are tabulated below :

Slice No.	*Width (m)*	*Average length (m)*	*Weight (t)*	*N (t)*	*T (t)*
1	4	2.4	17.76	14.7	–10.1
2	4	6.2	45.88	44.7	–9.9
3	4	8.7	64.38	64.4	0
4	4	10.3	76.22	73.8	19.4
5	4	9.65	71.41	62.3	34.6
6	2.8	8.4	43.51	30.1	31.2
7	2.8	3.6	18.7	10.1	15.9

$$\Sigma W = 337.9 \text{ t}, \ \Sigma N = 255.4 \text{ t}, \ \Sigma T = 81.1 \text{ t}$$

It should be noted that, as the width of slices 6 and 7 are 70% of that of the other slices, the length of the vertical lines representing W_7 and W_6 are 70% of the average length of slice no. 6 and 7 respectively.

The factor of safety of the slope w.r.t. the slip circle under consideration may now be determined using eqn. (10.24) :

$$F_s = \frac{c R \theta + \Sigma N \tan \phi}{\Sigma T}$$

By measurement, $R = 15.8$ m,

and, $$\theta = 115° = \frac{(115)\pi}{180} \text{ rad.} = 2.007 \text{ rad.}$$

$$\therefore \quad F_S = \frac{(2.5)\,(15.8)\,(2.007) + (255.4)\,(\tan 5°)}{81.1} = 1.25$$

In order to locate the critical slip circle, i.e., the slip circle with the minimum factor of safety, proceed as follows :

(i) Measure the distance P_0B, Let it be L.

(ii) With P_0 as centre form a grid consisting of 9 points such that the length of each side of the grid = $L/2$.

(iii) Draw trial slip circles taking each of these 9 points in turn as the centre. Compute the factor of safety of the slope for each slip circle.

(iv) Plot the values of F_S thus obtained for each grid point and draw contour lines for different values of F_S. The slip circle having the minimum value of F_S can be determined from this contour. The corresponding value of F_S is the factor of safety of the slope.

The process is illustrated in Fig. 10.13 (b). It is found that the slip circle having the minimum factor of safety is the one drawn with P_0 as the centre. Thus, Fellenius' method yields an accurate result in this case. The factor of safety of the slope is found to be 1.25.

Problem 10.6 A 12 m high embankment has side slopes of 1 V : 2 H. The soil has a unit weight of 1.8 t/m^3, cohesion of 1.5 t/m^2 and angle of internal friction of 15°. Determine the factor of safety of the slope with respect to any chosen slip circle. Use the friction circle method.

Solution: The slope is drawn in Fig. 10.14. A trial slip circle AEC is drawn with a radius R = 20.5 m. The chord AC is joined and its length is found to be 32 m. Let D be the mid point of AC.

The centre of the slip circle P is joined to D and PD is extended. It intersects the slope at F and the slip circle at E. The mid-point G of EF may be taken as the centre of gravity of the area $ABCE$.

Now, area $ABCE$ = ΔABC + area $ADCE$

$$= \frac{1}{2} . BH . AC + \frac{2}{3} AC . DE$$

$$= \frac{1}{2} (2.2)(32) + \frac{2}{3}(32)(7.45)$$

$$= 194.1 \text{ m}^2$$

Considering unit width of the slope, weight of the soil wedge

$$ABCE = (194.1)(1)(1.8) \text{ t}$$

$$= 349.38 \text{ t}.$$

Now, deflection angle δ = 102° = 1.78 radian

$\therefore$ Arc length of AEC = L = $R\,\theta$ = (20.5) (1.78) = 36.49 m.

The lever arm l_a of the cohesive force with respect to P is given by,

$$l_a = \frac{L}{L_c} \cdot R$$

$$= \frac{36.49}{32}(20.5) = 23.38 \text{ m}.$$

At a distance of 23.38 m from P, draw a line parallel to the chord AC. This gives the direction of the cohesive force C. Again, through G, draw a vertical line to represent the self-weight of the soil wedge W. The lines of action of W and C intersect at Q.

Now, radius of the friction circle,

$$r = R \sin\phi = (20.5)(\sin 10°) = 3.56 \text{ m}.$$

Through the point of intersection of W and C, draw a straight line making it tangent to the friction circle. This line represents the third force F.

Choose a vector scale and draw a straight line to represent W in magnitude and direction. Using the known lines of action of C and F, complete the force triangle and determine the magnitude of C from it.

The value of C obtained here is 41 t.

$\therefore$ Mobilised cohesion, $$c_m = \frac{C}{L} = \frac{41}{36.49} = 1.12 \text{ t/m}^2$$

The factor of safety with respect to cohesion is,

$$F_c = \frac{c}{c_m} = \frac{1.5}{1.12} = 1.34$$

F_c = 1.34 when the factor of safety with respect to friction, F_ϕ = 1.0. However, these two factors of safety should be so adjusted that they are equal to one another.

As a first trial, let F_ϕ = 1.20

$$\therefore \quad \tan\phi_m = \frac{\tan\phi}{1.20} = \frac{\tan 10°}{1.20}$$

or, ϕ_m = 8.36°.

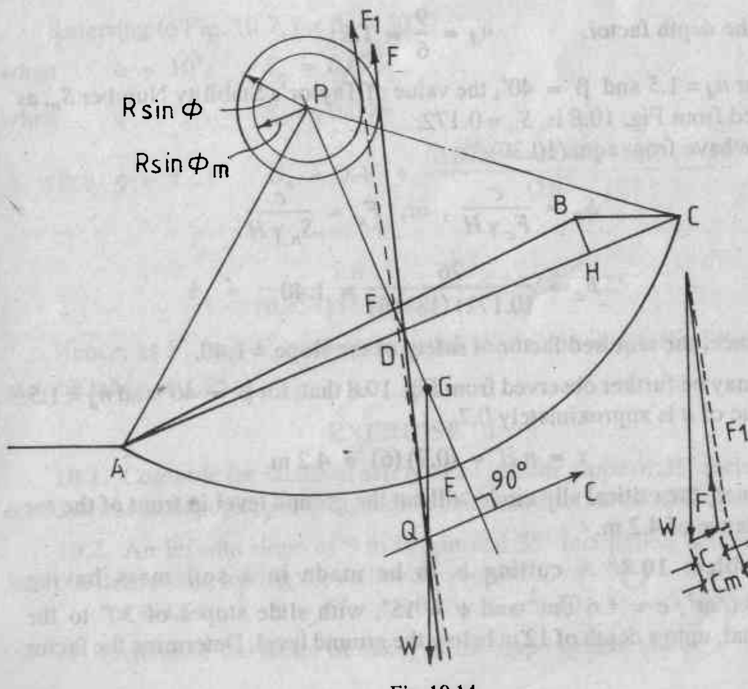


Fig. 10.14

The new radius of the friction circle is,

$$r' = R \sin \phi_m = (20.5)(\sin 8.36°) = 2.98 \text{ m}$$

Draw another friction circle with this radius.

The direction of F slightly changes. A new force triangle is constructed. The value of C obtained from it is 46.7 t.

$\therefore$ Mobilised cohesion, $c_m = \frac{C}{L} = \frac{46.7}{36.49} = 1.27 \text{ t/m}^2$

Factor of safety w.r.t cohesion, $F_c = \frac{1.5}{1.27} = 1.18$

$$\therefore \quad F_c \approx F_\phi$$

Hence the factor of safety of the given slope for the slip circle under consideration is 1.18.

Problem 10.7 It is required to make a 6 m deep excavation in a stratum of soft clay having $\gamma = 18 \text{ kN/m}^3$ and $c = 26 \text{ kN/m}^2$. A rock layer exists at a depth of 9 m below the ground level. Determine the factor of safety of the slope against sliding if the slope angle be 40°.

Solution: The problem will be solved by Taylor's method.

Here, the depth factor, $n_d = \frac{9}{6} = 1.5$

For $n_d = 1.5$ and $\beta = 40°$, the value of Taylor's Stability Number S_n, as obtained from Fig. 10.8 is, $S_n = 0.172$.

But, we have from eqn. (10.30),

$$S_n = \frac{c}{F_c \gamma H}, \text{ or, } F_c = \frac{c}{S_n \gamma H}$$

$$\therefore \quad F_c = \frac{26}{(0.172)(18)(6)} = 1.40$$

Hence, the required factor of safety of the slope = 1.40.

It may be further observed from Fig. 10.8 that, for $\beta = 40°$ and $n_d = 1.5$, the value of n is approximately 0.7.

$$\therefore \quad x = nH = (0.7)(6) = 4.2 \text{ m}$$

Hence, the critical slip circle will cut the ground level in front of the toe at a distance of 4.2 m.

Problem 10.8 A cutting is to be made in a soil mass having $\gamma = 1.8 \text{ t/m}^3$, $c = 1.6 \text{ t/m}^2$ and $\phi = 15°$, with slide slopes of 30° to the horizontal, upto a depth of 12 m below the ground level. Determine the factor of safety of the slope against shear failure. Assume that friction and cohesion are mobilised to the same proportion of their ultimate values.

Solution: In case of full mobilisation of friction (i.e., $F_\phi = 1$), the value of Taylor's Stability Number for $\phi = 15°$ and $\beta = 30°$, as obtained from Fig. 10.7, is, $S_n = 0.046$.

Using eqn. (10.30)

$$S_n = \frac{c}{F_c \gamma H}, \text{ or, } F_c = \frac{c}{S_n \gamma H}$$

or, $$F_c = \frac{1.6}{(0.046)(1.8)(12)} = 1.61$$

However, as friction will not be fully mobilised, the actual value of F_c will be less than this, and is to be found out by trials.

Let $$F_\phi = 1.25$$

$$\therefore \quad \tan \phi = \frac{\tan 15°}{1.25} = 0.2143$$

or, $$\phi = 12.1°$$

Referring to Fig. 10.7, for $\beta = 30°$,

when $\phi = 10°$, $S_n = 0.075$

when $\phi = 15°$, $S_n = 0.046$

$\therefore$ when $\phi = 12.1°$, $S_n = 0.46 + \frac{(0.075 - 0.046)(12.1 - 10)}{(15 - 10)}$

$$= 0.058.$$

$$F_c = \frac{1.6}{(0.058)(1.8)(12)} = 1.277 \approx 1.25$$

Hence, as F_c and F_ϕ are nearly equal, the factor of safety of the slope may be taken as 1.25.

EXERCISE 10

10.1. Compute the factor of safety of an infinite slope of 35° inclination made in a sand deposit having an angle of internal friction of 40°. [Ans. 1.2]

10.2. An infinite slope of 6 m height and 35° inclination is made in a layer of dense sand having the following properties :

$$c = 4.5 \text{ t/m}^2, \ \phi = 5°, \ e = 0.85, \ G = 2.70, \ w = 0\%$$

(a) Determine the factor of safety of the slope against sliding.

(b) How will the factor of safety change if the slope gets fully submerged? [Ans. (a) 1.25 (b) 1.98]

10.3. Determine the factor of safety of the slope *AB* with respect to the given slip circle shown in Fig. 10.15. The soil has a unit weight of 18.5 kN/m^3 and a cohesion of 42 kN/m^2. Use the Swedish circle method. [Ans : 1.42]

10.4. A 12 m deep cut is made in a silty clay with side slopes of 30°. The soil has the following properties :

$$\gamma = 1.9 \text{ gm/cc}, \quad c = 0.25 \text{ kg/cm}^2, \quad \phi = 8°.$$

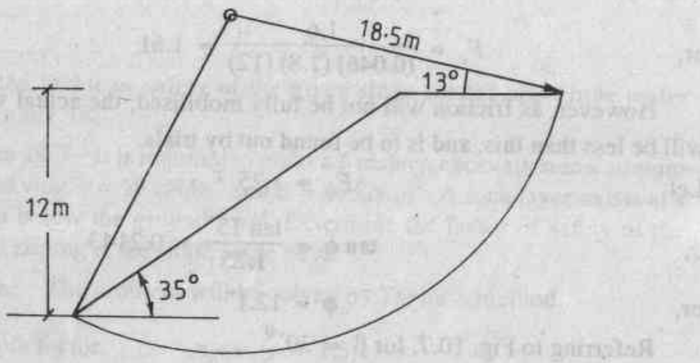


Fig. 10.15

Locate the centre of the critical slip circle by Fellenius' method and determine the factor of safety of the slope against sliding failure by the Swedish circle method. [Ans. 1.45]

10.5. Determine the factor of safety of the slope shown in Fig. 10.16 with respect to the given friction circle by the standard method of slices.

10.6. A 10 m deep cut is to be made in a soil with side slopes of 1 *V* : 1 *H*. The unit weight of the soil is 1.8 gm/cc and the soil has an unconfined compressive strength of 0.63 kg/cm^2. Determine the factor of safety of the slope against sliding,

(i) neglecting tension cracks

(ii) considering tension cracks

10.7. Compute the factor of safety of the slope shown in Fig. 10.17 with respect to the given slip circle by the friction circle method.

10.8. An unlined irrigation canal has a depth of 8 m and a side slopes of 1 : 1. The properties of the soil are as follows :

$$c = 2.0 \text{ t/m}^2, \quad \phi = 15°, \quad \gamma = 1.8 \text{ t/m}^3$$

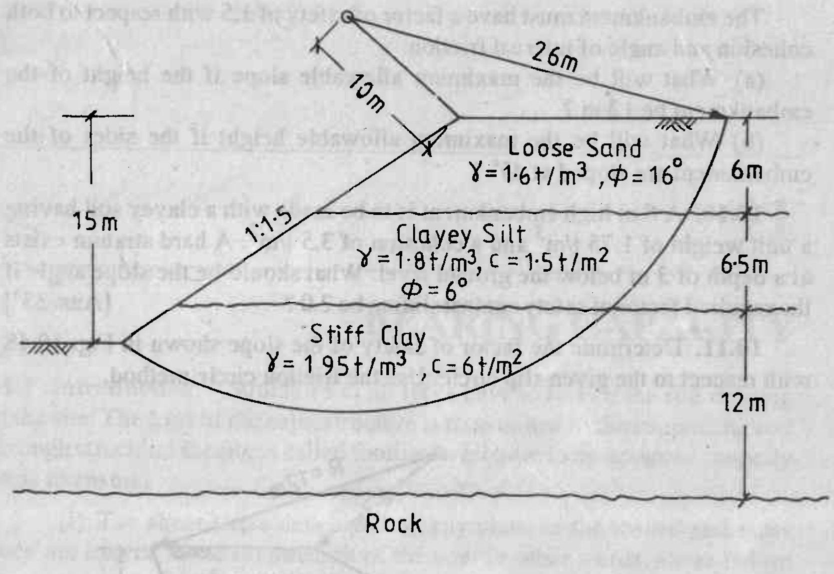


Fig. 10.16

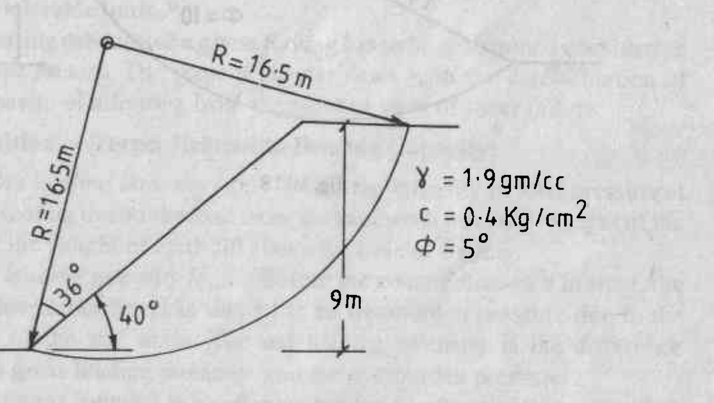


Fig. 10.17

Determine the factor of safety of the side slopes of the canal against sliding by Taylor's method. [Ans. 1.25]

10.9. An embankment is constructed with a c-ϕ soil having the following properties :

$$c = 2.5 \text{ t/m}^2, \quad \phi = 12°, \quad \gamma = 1.85 \text{ t/m}^3$$

The embankment must have a factor of safety of 1.5 with respect to both cohesion and angle of internal friction.

(a) What will be the maximum allowable slope if the height of the embankment be 12 m ?

(b) What will be the maximum allowable height if the sides of the embankment are sloped at 45° ?

10.10. A 6 m high embankment is to be made with a clayey soil having a unit weight of 1.75 t/m^3 and a cohesion of 3.5 t/m^2. A hard stratum exists at a depth of 3 m below the ground level. What should be the slope angle if the required factor of safety against sliding be 2.0 ? [Ans. 33°]

10.11. Determine the factor of safety of the slope shown in Fig. 10.18 with respect to the given slip circle. Use the friction circle method.

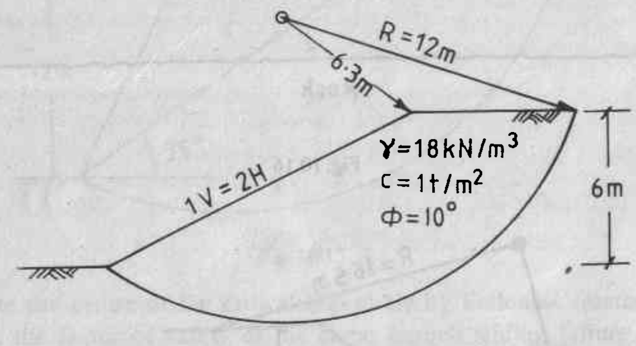


Fig. 10.18

11

BEARING CAPACITY

11.1 Introduction: Structures of all types have to rest on the soil existing at the site. The load of the superstructure is transmitted to the supporting soil through structural members called footings which are to be designed properly so as to ensure:

(i) The shear stress developed on any plane in the loaded soil mass does not exceed the shear strength of the soil. In other words, shear failure does not occur.

(ii) The settlement of the footing due to the applied load does not exceed the tolerable limit.

The bearing capacity of a given footing has to be determined considering both of these factors. The present chapter deals with the determination of bearing capacity of a footing from the point of view of shear failure.

11.2 Definition of Terms Related to Bearing Capacity:

1. *Gross loading intensity (q)*: This is the intensity of total pressure at the base of footing due to the load from the superstructure, self-weight of the footing and the weight of earth fill above the base of footing.

2. *Net loading intensity (q_n)*: Before the construction of a footing, the soil at the foundation level is subject to an overburden pressure due to the self-weight of the soil mass. The net loading intensity is the difference between the gross loading intensity and the overburden pressure.

If a footing is founded in a soil mass having a unit weight γ at a depth D below the ground level, then,

$$q_n = q - \gamma D \quad ...(11.1)$$

3. *Ultimate bearing capacity (q_u)*: This is the minimum intensity of loading at the base of the foundation which will cause a shear failure of the soil.

4. *Net ultimate bearing capacity (q_{nu})*: This is the minimum net pressure intensity due to the applied load (i.e., excluding the existing overburden pressure) which will cause a shear failure.

5. *Net safe bearing capacity* (q_{ns}): The minimum net pressure intensity at the base of footing with respect to a specified factor of safety against shear failure, i.e.,

$$q_{ns} = \frac{q_{nu}}{F} \quad ...(11.2)$$

6. *Safe bearing capacity* (q_s): The maximum gross loading intensity which the footing will safely carry without the risk of shear failure, irrespective of the magnitude of settlement.

Thus, $$q_s = q_{ns} + \gamma D \quad ...(11.3)$$

or, $$q_s = \frac{q_{nu}}{F} + \gamma D \quad ...(11.4)$$

7. *Allowable bearing capacity* (q_a): This is the net intensity of loading which the foundation will carry without undergoing settlement in excess of the permissible value but not exceeding the net safe bearing capacity.

11.3 Types of Shear Failure: The shear failure of a soil mass supporting a structure may take place in either of the following modes:

(i) General shear failure
(ii) Local shear failure
(iii) Punching shear failure

In dense sands and stiff clays, when the loading intensity exceeds a certain limit, the footing generally settles suddenly into the soil and well defined slip surfaces are formed. The shear strength of the soil is fully mobilised along these surfaces. This is called a general shear failure.

In relatively loose sands and in medium clays, the footing settles gradually. The failure planes are not so well defined and the shear strength of the soil is not fully mobilised. No heaving of soil takes place above the ground level. This type of failure is called local shear failure.

In very loose sands and soft saturated clays, a footing is often found to virtually sink into the soil. No failure plane is formed at all. Such a failure is due to the shear failure along the vertical face around the perimeter of the base of the footing. The soil beyond this zone remains practically unaffected. This type of failure is called punching shear failure.

The type of shear failure expected to occur at a site has a direct bearing on the theoretical computation of bearing capacity.

11.4 Terzaghi's Theory: This theory is an extension of the concept originally developed by Prandtl. The mode of general shear failure of a footing is illustrated in Fig. 11.1 (a). Considering the critical equilibrium of the soil wedge *xyz* under the forces shown in Fig. 11.1 (b), Terzaghi derived

the following expression for the ultimate bearing capacity of a footing of width *B*, placed at a depth *D* below G.L.:

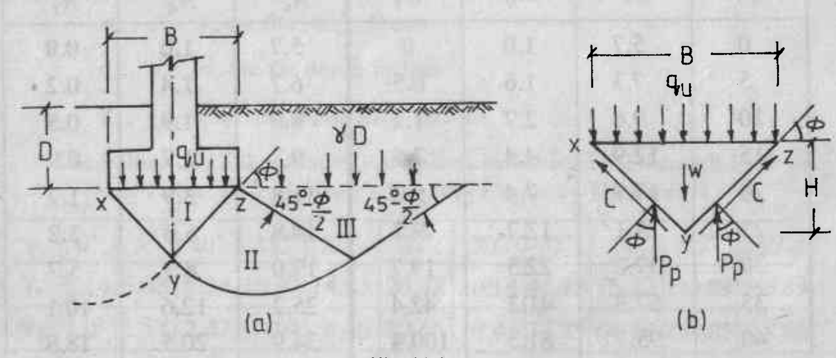


Fig. 11.1

$$q_n = c N_c + \gamma D N_q + 0.5 \gamma B N_\gamma \quad ...(11.5)$$

where, N_c, N_q and N_γ are bearing capacity factors which depend on the angle of internal friction of the soil.

Eqn. (11.5) is applicable to general shear failure. For local shear failure, the following equation is to be used:

$$q_n = c' N_c' + \gamma D N_q' + 0.5 \gamma B N_\gamma' \quad ...(11.6)$$

where, $$c' = \frac{2}{3} c \quad ...(11.7)$$

and, N_c', N_q' and N_γ' are the bearing capacity factors obtained from ϕ',

where, $$\phi' = \tan^{-1}\left(\frac{2}{3}\tan\phi\right) \quad ...(11.8)$$

Eqn. (11.5) is meant for strip footings. However, for square and circular footings the following modified equations should be used, which take into account the shape factors:

For square footings,

$$q_n = 1.3 c N_c + \gamma D N_q + 0.4 \gamma B N_\gamma \quad ...(11.9)$$

For circular footings,

$$q_n = 1.3 c N_c + \gamma D N_q + 0.3 \gamma B N_\gamma \quad ...(11.10)$$

where, *B* stands for the width of a square footing or the diameter of a circular footing.

The values of Terzaghi's bearing capacity factors are given in Table 11.1.

Table 11.1: Terzaghi's Bearing Capacity Factors

$\phi°$	N_c	N_q	N_γ	N_c'	N_q	N_γ'
0	5.7	1.0	0	5.7	1.0	0.0
5	7.3	1.6	0.5	6.7	1.4	0.2
10	9.6	2.7	1.2	8.0	1.9	0.5
15	12.9	4.4	2.5	9.7	2.7	0.9
20	17.7	7.4	5.0	11.8	3.9	1.7
25	25.1	12.7	9.7	14.8	5.6	3.2
30	37.2	22.5	19.7	19.0	8.3	5.7
35	57.8	41.3	42.4	25.2	12.6	10.1
40	95.7	81.3	100.4	34.9	20.5	18.8
45	172.3	173.3	297.4	51.2	34.1	37.7
50	347.5	415.1	1153.2	81.3	65.6	87.1

11.5 Skempton's Equation: This equation is applicable to footings founded on cohesive soils. The net ultimate bearing capacity of such a footing is given by:

$$q_{nu} = c\,N_c \qquad ...(11.11)$$

where, c = cohesion.

N_c = Bearing capacity factor which depends on the shape of the footing as well as on the depth of foundation.

The ultimate bearing capacity is given by:

$$q_n = c\,N_c + \gamma D \qquad ...(11.12)$$

Skempton suggested the following values of N_c :

(i) when $D = 0$ (i.e., when the footing is at the ground level)

for strip footings, $N_c = 5.14$

for square and circular footings, $N_c = 6.20$

(ii) when $D/B < 2.5$:

$$N_c = (1 + 0.2\,D/B)\,N_{c\,(surface)} \qquad ...(11.13)$$

(iii) when $D/B > 2.5$:

$$N_c = 1.5 \times N_{c\,(surface)} \qquad ...(11.14)$$

(iv) For rectangular footings:

$$N_c = (1 + 0.2\,B/L)\,N_{c\,(strip)} \qquad ...(11.15)$$

11.6 Brinch Hansen's Equation: According to J. Brinch Hansen, the ultimate bearing capacity of a footing is given by,

$$q_n = c\,N_c\,s_c\,d_c\,i_c + q\,N_q\,s_q\,d_q\,i_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma \qquad ...(11.16)$$

where, $q = \gamma D$

s_c, s_q, s_γ are the shape factors

d_c, d_q, d_γ are the depth factors

i_c, i_q, i_γ are the inclination factors

The values of all these factors are given in Tables 11.2 through 11.5:

Table 11.2 Hansen's Bearing Capacity Factors

ϕ	0°	5°	10°	15°	20°	25°	30°	35°	40°	45°	50°
N_c	5.14	6.48	8.34	10.97	14.83	20.72	30.14	46.13	75.32	133.89	266.89
N_q	1.0	1.57	2.47	3.94	6.40	10.66	18.40	33.29	64.18	134.85	318.96
N_γ	0.0	0.09	0.47	1.42	3.54	8.11	18.08	40.69	95.41	240.85	681.84

Table 11.3: Shape Factors for Hensen's Equation

Shape of footing	s_c	s_q	s_γ
Continuous	1.00	1.00	1.00
Rectangular	$1 + 0.2\,B/L$	$1 + 0.2\,B/L$	$1 - 0.4\,B/L$
Square	1.3	1.2	0.8
Circular	1.3	1.2	0.6

Table 11.4: Depth Factors for Hensen's Equation

d_c	d_q	d_γ
$1 + 0.35\,D/B$	For $\phi = 0°$, $d_q = 1.0$	1.0
(for all values of ϕ)	For $\phi > 25°$, $d_q = d_c$	(for all values of ϕ)

Table 11.5: Inclination Factors for Hansen's Equation

i_c	i_q	i_γ
$1 - \dfrac{H}{2\,c\,B\,L}$	$1 - 0.5\,\dfrac{H}{V}$	i_q^2

Valid upto : $H \le V \tan \delta - c_{\phi} \cdot B L$ ... (11.17)

where, H and V are the horizontal and vertical components of the resultant load acting on the footing.

L = length of footing parallel to H.

c_{ϕ} = cohesion between footing and soil.

δ = angle of friction between footing and soil.

11.7 Bearing Capacity Equation as per IS Code: Hansen's bearing capacity equation was later modified by Vesic. In IS: 6403 - 1981, the following equations were proposed, which incorporated Vesic's modifications:

For general shear failure:

$$q_u = c N_c s_c d_c i_c + q N_q s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma W' \quad ...(11.18)$$

for local shear failure.

$$q_u = \frac{2}{3} c N_c' s_c d_c i_c + q N_q' s_q d_q i_q + 0.5 \gamma B N_\gamma s_\gamma d_\gamma i_\gamma W' \quad ...(11.19)$$

The shape factors s_c, s_q and s_γ are the same as those used in Brinch Hansen's equation and can be obtained from Table 11.3.
The depth factors are given by:

$$d_c = 1 + 0.2 (D/B) \cdot \sqrt{N_\phi}$$

$$d_q = d_\gamma = 1 \quad \text{for } \phi < 10^\circ$$

$$d_q = d_\gamma = 1 + 0.1 (D/B) \cdot \sqrt{N_\phi} \quad \text{for } \phi > 10^\circ$$

The inclination factors are given by:

$$i_c = i_q = (1 - \alpha/90)^2$$

$$i_\gamma = \left(1 - \frac{\alpha}{\phi}\right)^2$$

where, $N_\phi = \tan^2 (45^\circ + \phi/2)$

and α = angle of inclination of the resultant force on the footing.

In eqns. (11.18) and (11.19), W' = correction factor for water table.

The N_c and N_q - values are similar to those given in Table 11.2, while the N_γ-values are slightly different.

11.8 Effect of Water Table on Bearing Capacity: In Terzaghi's bearing capacity equation, the second and third terms are dependent on the unit weight

of the soil. When the soil is fully submerged, the submerged density γ_{sub} should be used in place of γ. But if the water table is at the base of the footing, only the third term is affected. The general bearing capacity equation is, therefore, modified as:

$$q_u = c N_c + \gamma D_f N_q W_1 + 0.5 \gamma B N_\gamma W_2 \quad ...(11.20)$$

where, W_1 and W_2 are the correction factors.

For most soils, γ_{sub} is nearly equal to half the value of γ. Hence, the correction factors are given by (Refer to Fig. 11.2):

$$W_1 = 0.5 (1 + z_1/D_1) \quad ...(11.21)$$

When the water table is at G.L., $W_1 = 0.5$
and when it is at the base of footing, $W_1 = 1.0$

$$W_2 = 0.5 (1 + z_2/B) \quad ...(11.22)$$

When the water table is at the base of footing, $W_2 = 0.5$

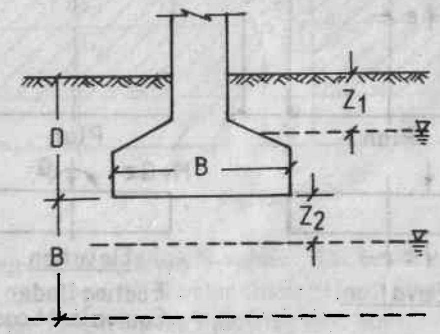


Fig. 11.2

When it is at a depth B below the base of footing, $W_2 = 1.0$.

Here it is assumed that, if the water table is at a depth equal to or greater than B below the base, the bearing capacity remains unaffected.

IS : 6403-1981 recommends the use of a single correction factor W' to be used in the third term of equations (11.18) and (11.19). The value of W' is as follows:

(a) If the water table is at or below a depth of $D + B$ beneath the G.L., then $W' = 1$.

(b) If it is at a depth D or above, $W' = 0.5$

(c) If the depth of water table is such that, $D < D_w < (D+B)$, the value of W' should be obtained by linear interpolation.

11.9 Eccentrically Loaded Footings: A footing is said to be eccentrically loaded if the resultant load on it is applied away from the centre of gravity of the load. Such footings may be designed by either of the following methods:

(a) *Method I*: In this method the load Q of eccentricity e is replaced by an equal concentric load Q and a balancing moment of magnitude $M = Q.e$.

Stress distribution diagrams due to the concentric load as well as the balancing moment are plotted (Fig. 11.3). The maximum stress intensity of the superimposed diagram should be less than the allowable bearing capacity of the footing.

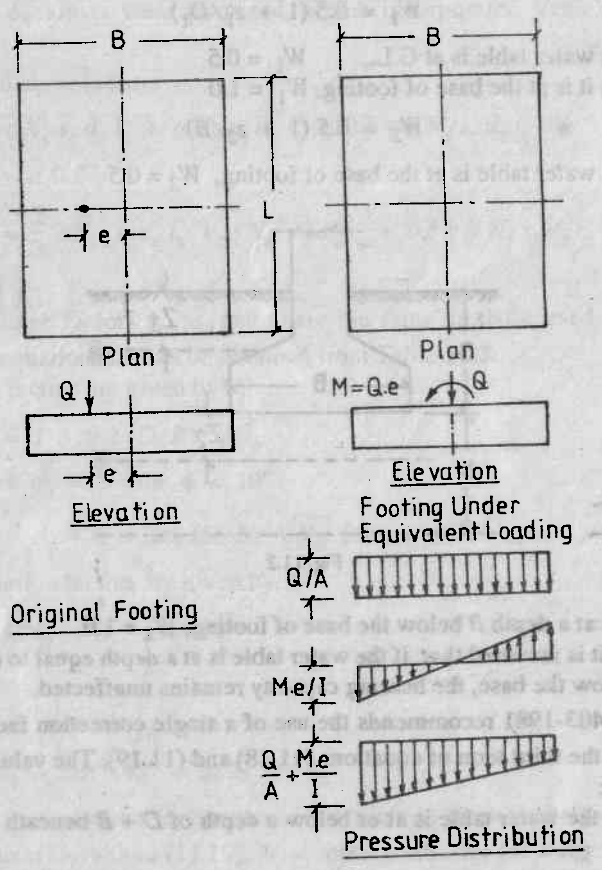


Fig. 11.3

(b) *Method II (Meyerhof's method)*: In this method if a footing is exposed to an eccentric load, only a portion of the plan area of the footing is considered to be useful. This area is termed as the effective area.

In case of single eccentricity (Fig.11.4a) the effective dimension of the footing in the direction of eccentricity is reduced by $2e$.

i.e., $$B' = B - 2e$$

$\therefore$ $$A' = L(B - 2e) \quad ...(11.23)$$

In case of double eccentricity the dimensions in both directions are reduced as follows:

$$L' = L - 2e_L \text{ and } B' = B - 2e_B$$

$\therefore$ $$A' = L' \times B' = (L - 2e_L)(B - 2e_B) \quad ...(11.24)$$

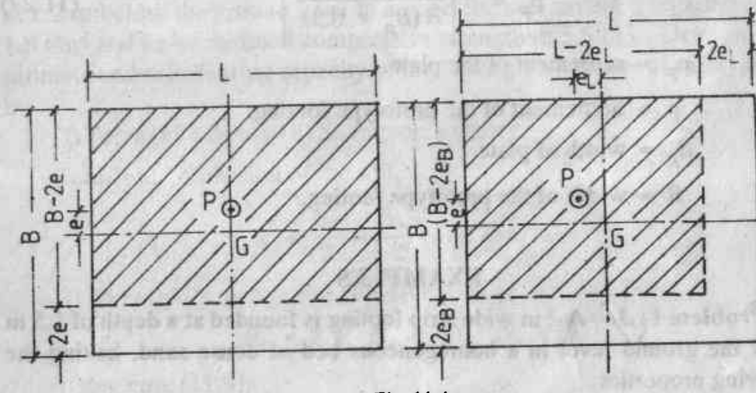


Fig. 11.4

11.10 Bearing Capacity from N-value: The bearing capacity of a footing may be determined from the N-value obtained from Standard Penetration Test carried out in the field from the following equations:

For strip footings:

$$q_{nu} = 0.785\,(100 + N^2)\,D\,W_1 + 0.471\,N^2\,B\,W_2 \quad ...(11.25)$$

For square footings:

$$q_{nu} = 0.943\,(100 + N^2)\,D\,W_1 + 0.314\,N^2\,B\,W_2 \quad ...(11.26)$$

where, N = average corrected blow count.

D = depth of footing.

B = width of footing.

W_1, W_2 = correction factors for water table.

q_{nu} = net ultimate bearing capacity in kN/m^2.

11.11 Bearing Capacity from Plate Load Test: The bearing capacity of a footing to be placed on a soil mass may be assessed from the results of a plate load test carried out at the site at the desired depth. However, the process has got several limitations.

The method of computing the bearing capacity of a prototype footing from the plate load test data is illustrated in Problem 11.10.

The settlement of the prototype footing, when founded on granular soils, is given by the following relationship suggested by Terzaghi and Peck.

$$\rho_p = \rho \left[\frac{B_p (B + 0.3)}{B (B_p + 0.3)} \right]^2 \quad \text{...(11.27)}$$

where, ρ_p = settlement of the plate.

ρ = Settlement of the prototype footing.

B_p = width of plate.

B = width of the prototype footing.

EXAMPLES

Problem 11.1. A 2 m wide strip footing is founded at a depth of 1.5 m below the ground level in a homogeneous bed of dense sand, having the following properties:

$$\phi = 36°, \quad \gamma = 1.85 \text{ t/m}^3.$$

Determine the ultimate, net ultimate, net safe and safe bearing capacity of the footing. Given, for $\phi = 36°$

$$N_c = 60, \quad N_q = 42, \quad N_\gamma = 47.$$

Assume a factor of safety of 3.0.

Solution: As $\phi = 36°$, a general shear failure is likely to occur.

(i) Ultimate bearing capacity:

$$q_u = c N_c + \gamma D N_q + 0.5 \gamma B N_\gamma$$

Here, $c = 0$, $\gamma = 1.85$ t/m^3, $D = 1.5$ m, $B = 2.0$ m.

$N_q = 42$ and $N_\gamma = 47$

$$\therefore \quad q_u = (1.85)(1.5)(42) + (0.5)(1.85)(2.0)(47)$$

$$= 186.5 \text{ t/m}^2.$$

(ii) Net ultimate bearing capacity:

$$q_{nu} = q_u - \gamma D$$

$$= 186.5 - (1.85)(1.5) = 183.7 \text{ t/m}^2.$$

$$= 186.5 - (1.85)(1.5) = 183.7 \text{ t/m}^2.$$

(iii) Net safe bearing capacity:

$$q_{ns} = \frac{q_{nu}}{F_s} = \frac{183.7}{3.0} = 61.2 \text{ t/m}^2.$$

(iv) Safe bearing capacity:

$$q_s = q_{ns} + \gamma D_f$$

$$= 61.2 + (1.85)(1.5) = 64 \text{ t/m}^2.$$

Problem 11.2. A square footing of 2.5 m × 2.5 m size has been founded at 1.2 m below the ground level in a cohesive soil having a bulk density of 1.8 t/m^3 and an unconfined compressive strength of 5.5 t/m^2. Determine the ultimate and safe bearing capacity of the footing for a factor of safety of 2.5, by

(i) Terzaghi's theory (ii) Skempton's theory.

Solution: Cohesion of the soil,

$$c = \frac{q_u}{2} = \frac{5.5}{2} = 2.75 \text{ t/m}^2$$

(i) *Terzaghi's theory*: For cohesive soils ($\phi = 0$) we have,

$$N_c = 5.7, \quad N_q = 1.0, \quad N_\gamma = 0.$$

Using eqn. (11.9),

$$q_u = (1.3)(2.75)(5.7) + (1.8)(1.2)(1.0)$$

$$= 22.54 \text{ t/m}^2$$

$$q_s = \frac{q_u - \gamma D}{F_s} + \gamma D = \frac{22.54 - (1.2)(1.0)}{2.5} + (1.8)(1.2)$$

$$= 10.31 \text{ t/m}^2.$$

(ii) *Skempton's method*:

Here, $$D/B = \frac{1.2}{2.5} = 0.48 < 2.5.$$

∴ Eqn. (11.13) is applicable.

$$N_c = (1 + 0.2\, D/B)\, N_{c\,(\text{surface})}$$

But for square footings, $N_{c\,(\text{surface})} = 6.20$.

$$\therefore \quad N_c = \left\{ 1 + \frac{(0.2)(1.2)}{2.5} \right\} (6.20) = 6.79$$

$$\therefore \quad q_{nu} = c\,N_c$$

$$= (2.75)(6.79) = 18.67\ t/m^2$$

$$q_u = q_{nu} + \gamma D = 18.67 + (1.8)(1.2) = 20.83\ t/m^2$$

$$q_s = \frac{q_{nu}}{F_s} + \gamma D$$

$$= \frac{18.67}{2.5} + (1.8)(1.2) = 9.63\ t/m^2.$$

Problem 11.3. Determine the safe load that can be carried by a square footing of 2.2 m × 2.2 m size, placed at a depth of 1.6 m below G.L. The foundation soil has the following properties:

$$\gamma = 1.65\ t/m^3, \quad c = 1.2\ t/m^2, \quad \phi = 20°,$$

Assume a factor of safety of 2.5. Given, for $\phi = 20°$,

$$N_c = 17.7, \quad N_q = 7.4, \quad N_\gamma = 5.0$$

$$N_c' = 11.8, \quad N_q' = 3.8, \quad N_\gamma' = 1.3$$

Solution: The low value of unit weight of the soil suggests that the soil is in the loose state. Moreover, $\phi = 20° < 28°$. Hence a local shear failure is likely to occur. Using eqn. (11.26), the net ultimate bearing capacity of the footing is given by,

$$q_{nu} = 1.3\,c'\,N_c' + \gamma D\,(N_q' - 1) + 0.4\,\gamma B\,N_\gamma'$$

Here, $c' = \frac{2}{3}c = (2/3)(1.2) = 0.8\ t/m^2$

$$N_c' = 11.8, \quad N_q' = 3.8, \quad N_\gamma' = 1.3$$

$$\therefore \quad q_{nu} = (1.3)(0.8)(11.8) + (1.65)(1.6)(3.8 - 1) + (0.4)(1.65)(2.2)(1.3)$$

$$= 12.27 + 7.39 + 1.89$$

$$= 21.55\ t/m^2$$

The safe bearing capacity of the footing:

$$q_s = \frac{q_{nu}}{F_s} + \gamma D = \frac{21.55}{2.5} + (1.65)(1.6) = 11.26\ t/m^2$$

∴ Gross safe load to be carried by the footing,

$$= q_s \times \text{Area of footing}$$

$$= (11.26)(2.2)^2 = 54.5\ t.$$

Problem 11.4. A square footing of 2 m × 2 m size is subject to a gross vertical load of 180 t. The depth of foundation is 1 m. The foundation soil consists of a deposit of dense sand having a bulk density of 1.85 t/m^3 and an angle of internal friction of 36°. Determine the factor of safety against shear failure.

Solution: We have, for $\phi = 36°$

$$N_c = 60, \quad N_q = 42, \quad N_\gamma = 47$$

Using eqn. (11.9) and noting that the first term vanishes as $c = 0$,

$$q_u = (1.85)(1.0)(42) + (0.4)(1.85)(2.0)(47)$$

$$= 147.3\ t/m^2.$$

$$\therefore \quad q_{nu} = 147.3 - (1.85)(1.0) = 145.5\ t/m^2.$$

Now, actual bearing pressure at the base of footing,

$$q_b = \frac{Q}{A} = \frac{180}{(2)(2)} = 45\ t/m^2.$$

But, $q_b = \frac{q_{nu}}{F_s} + \gamma D_f$

or, $F_s = \frac{q_{nu}}{q_b - \gamma D_f} = \frac{145.5}{45 - (1.85)(1.0)} = 3.37.$

Problem 11.5. A column of a building, carrying a net vertical load of 125 t, has to be supported by a square footing. The footing is to be placed at 1.2 m below G.L. in a homogeneous bed of soil having the following properties:

$$\gamma = 1.82\ gm/cc, \quad \phi = 30°$$

Determine the minimum size of the footing required to have a factor of safety of 2.5 against shear failure. Use Terzaghi's formula.

Solution: Net load on column from superstructure = 125 t.

Add 10% for the self weight of the footing = 12.5 t

137.5 t

Gross load = 137.5 t = 138 t (say).

Now, the safe bearing capacity of a square footing on a cohesionless soil is given by

$$q_s = \frac{1}{F}\left[1.3\,c\,N_c + \gamma D\,(N_q - 1) + 0.4\,\gamma B\,N_\gamma\right] + \gamma D$$

From table 11.1, for $\phi = 30°$, $N_c = 37.2$, $N_q = 22.5$, $N_\gamma = 19.7$

$$\therefore \quad q_s = \frac{1}{2.5}[(1.82)(1.2)(22.5-1) + (0.4)(1.82)(B)(19.7)] + (1.82)(1.2)$$

$$= 18.78 + 5.74B + 2.18 = 20.96 + 5.74B$$

The safe load that can be carried by the footing,

$$Q = q_s \times A$$

$$= (20.96 + 5.74B)B^2$$

$$= 5.74B^3 + 20.96B^2$$

$$\therefore \quad 5.74B^3 + 20.96B^2 = 138,$$

or, $$B^3 + 3.65B^2 = 24.04$$

Solving the above equation by trial and error, we obtain,

$$B = 2.06 \text{ m} = 2.10 \text{ m (say)}$$

Hence, the required size of the footing = 2.10 m

Problem 11.6. If the size of the footing in Problem 11.5 has to be restricted to 1.75 m × 1.75 m , at what depth the footing should be placed?

Solution: The bearing capacity of a footing placed in a cohesionless soil increases with depth. In Problem 11.5, the depth of the footing was specified as 1.2 m. The corresponding size required for supporting a gross load of 138 t was found to be 2.45 m × 2.45 m. However, if the size of the footing has to be restricted to 1.75 m × 1.75 m (such restrictions are sometimes necessary for avoiding encroachment on adjacent land) and if the column still has to withstand the same gross load, its depth has to be increased. Let d be the required depth.

Now, $$q_s = \frac{1}{F}\left[\gamma D(N_q - 1) + 0.4\gamma B N_\gamma\right] + \gamma D$$

$$\therefore \quad q_s = \frac{(1.82)d(22.5-1) + (0.4)(1.82)(1.75)(19.7)}{2.5} + (1.82)d$$

or, $$q_s = 17.472d + 10.039$$

Again, actual contact pressure, $$q_c = \frac{138}{(1.75)(1.75)} = 45.061 \text{ t/m}^2$$

$$\therefore \quad 17.472d + 10.039 = 45.061$$

or, $$d = 2.00 \text{ m.}$$

∴ The footing has to be founded at a depth of 2.00 m below G.L.

Problem 11.7. An R.C.C. column footing of 1.8 m × 1.8 m size is founded at 1.5 m below G.L. The subsoil consists of a loose deposit of silty sand having the following properties:

$$\gamma = 1.75 \text{ t/m}^3, \quad \phi = 20°, \quad c = 1.1 \text{ t/m}^2$$

Determine the ultimate bearing capacity of the footing when the ground water table is located at:

(i) ground level (ii) 0.6 m below ground level.

(iii) 2.0 m below the base of footing (iv) 4.0 m below the base of footing.

Given, for $\phi = 20°$, $N_c' = 11.8$, $N_q' = 3.8$, $N_\gamma' = 1.3$.

Solution: Assuming a local shear failure, the ultimate bearing capacity of a square footing is given by,

$$q_u = 1.3c'N_c' + \gamma D N_q' W_1 + 0.4\gamma B N_\gamma' W_2$$

Here, $$c' = \frac{2}{3}c = (2/3)(1.1) = 0.73 \text{ t/m}^2.$$

$$\gamma = 1.75 \text{ t/m}^3, \quad D = 1.5 \text{ m}, \quad B = 1.8 \text{ m}$$

$$\therefore \quad q_u = (1.3)(0.73)(11.8) + (1.75)(1.5)(3.8)W_1 + (0.4)(1.75)(1.8)(1.3)W_2$$

or, $$q_u = 11.2 + 9.97W_1 + 1.64W_2 \quad \text{...(i)}$$

(i) When the water table is at the ground level, $z_1 = 0$.

Using eqn. (11.21), $W_1 = 0.5(1 + 0) = 0.5$.

W_2 is not applicable (i.e., $W_2 = 1$).

$$\therefore \quad q_u = 11.2 + (9.97)(0.5) + 1.64$$

$$= 17.82 \text{ t/m}^2$$

(ii) When the water table is at 0.6 m below the ground level, $z_1 = 0.6$ m,

$$\therefore \quad W_1 = 0.5(1 + 0.6/1.5) = 0.7$$

W_2 is again not applicable

$$\therefore \quad q_u = 11.2 + (9.97)(0.7) + 1.64$$

$$= 19.82 \text{ t/m}^2$$

(iii) When the water table is at 2.0 m below G.L.,

$$z_2 = 2.0 - 1.5 = 0.5 \text{ m}$$

$$\therefore \quad W_2 = 0.5(1 + 0.5/2.0) = 0.625$$

Here W_1 is not applicable

$$\therefore \quad q_u = 11.2 + 9.97 + (1.64)(0.625)$$

$$= 22.19 \text{ t/m}^2$$

(iv) When the ground water table is at 4 m below the base of footing, no correction due to ground water table is necessary. In other words, the ultimate bearing capacity is not affected by the ground water table.

$$\therefore \quad q_u = 11.2 + 9.97 + 1.64 = 22.81 \text{ t/m}^2$$

It is evident from the above results, that, the bearing capacity of a footing increases with increasing depth of the ground water table.

Problem 11.8. Two adjacent columns of a building, carrying a vertical load of Q tonne each, are supported by a combined footing of 2 m × 3.5 m size, founded at 1.2 m below the ground level. Determine the maximum allowable value of Q if the foundation soil consists of a deep, homogeneous stratum of :

(i) Saturated silty clay ($\gamma = 1.9 \text{ t/m}^3$, $c = 4.6 \text{ t/m}^2$)

(ii) Partially saturated inorganic silt

$$(\gamma = 1.84 \text{ t/m}^3, \ \phi = 10°, \ c = 1.6 \text{ t/m}^2)$$

Solution: Total column load to be carried by the combined footing :

(a) load from the columns $= Q + Q = 2Q$

(b) self-weight of footing (say 10% of column loading) $= 0.2\,Q$

gross load $= 2.2\,Q$

(i) In this case, as the foundation soil is purely cohesive, Skempton's formula may be applied.

Here, $D/B = 1.2/2.0 = 0.6 < 2.5$

Using eqns. (11.11), (11.13) and (11.15), the net ultimate bearing capacity of a rectangular footing is given by,

$$q_{nu} = 5.14\,(1 + 0.2\,D/B)\,(1 + 0.2\,B/L)\,c$$

$$= (5.14)\,[1 + (0.2)(0.6)]\,[1 + (0.2)(2.0/3.5)]\,(4.6)$$

$$= 29.51 \text{ t/m}^2$$

Considering a factor of safety of 2.5, the safe bearing capacity,

$$q_s = \frac{29.51}{2.5} = 11.8 \text{ t/m}^2$$

$\therefore$ Safe gross load on the footing $= q_s \cdot A$

$$= (11.8)(2.0)(3.5) = 82.6 \text{ t}$$

$$\therefore \quad 2.2\,Q = 82.6$$

$$\text{or,} \quad Q = 37.5 \text{ t}$$

Hence, the required safe load on each column = 37.5 t.

(ii) In this case, Brinch Hansen's equation is expected to yield a more reliable result. We have from eqn. (11.16),

$$q_u = c\,N_c\,s_c\,d_c\,i_c + \gamma\,D\,N_q\,s_q\,d_q\,i_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma$$

For $\phi = 10°$, referring to table 11.2,

$N_c = 8.34$, $N_q = 2.47$, $N_\gamma = 0.47$

The shape factors, depth factors and inclination factors are obtained from tables 11.3, 11.4 and 11.5.

For a rectangular footing of 2.0 m × 3.5 m size, founded at a depth of 1.2 m below G.L., we get,

$$s_c = s_q = 1 + (0.2)(2.0/3.5) = 1.114$$

$$s_\gamma = 1 - (0.4)(2/3.5) = 0.77$$

$$d_c = 1 + (0.35)(1.2/2.0) = 1.21$$

when $\phi = 0°$, $d_q = 1.0$ and when $\phi = 25°$, $d_q = d_c = 1.21$

By linear interpolation, for $\phi = 10°$, $d_q = 1.0 + \dfrac{(1.21 - 1.0)}{25}(10)$

$$= 1.084$$

Since the loading is vertical, $i_c = i_q = i_\gamma = 1$

$$\therefore \quad q_u = (1.6)(8.34)(1.114)(1.21)(1.0) + (1.84)(1.2)(2.47)(1.114)(1.084)(1.0) + (0.5)(1.84)(2.0)(0.47)(0.77)(1.0)(1.0)$$

$$= 17.99 + 6.59 + 0.67 = 25.25 \text{ t/m}^2$$

$$\therefore \quad q_s = \frac{25.25 - (1.84)(1.2)}{2.5} + (1.84)(1.2) = 11.42 \text{ t/m}^2$$

Total gross load on footing = (11.42) (2.0) (3.5) = 79.97 t.

$$\therefore \quad 2.2\,Q = 79.97 \text{ t}$$

$$\text{or,} \quad Q = 36.35 \text{ t}$$

$\therefore$ Safe load on each column = 36.35 t

Problem 11.9. A rectangular footing of 2.4 m × 3.5 m size is to be constructed at 1.5 m below G.L. in a c–ϕ soil having the following properties:

$$\gamma = 1.75 \text{ t/m}^3, \quad \phi = 20°, \quad c = 1.0 \text{ t/m}^2.$$

The footing has to carry a gross vertical load of 70 t, inclusive of its self-weight. In addition, the column is subject to a horizontal load of 11 t applied at a height of 3.3 m above the base of the footing. Determine the factor of safety of the footing against shear failure :

(i) using Brinch Hansen's method.

(ii) As per IS : 6403 – 1981.

Solution: The loading condition of the column and the footing is shown in Fig.11.5. Due to the presence of the horizontal force, the resultant load on the column is inclined, and the footing becomes eccentrically loaded. Let e be this eccentricity.

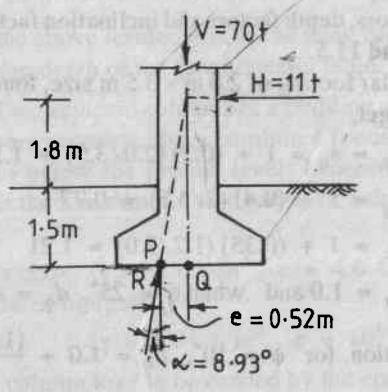


Fig. 11.5

Let R be the resultant soil reaction, applied at P, which can be resolved into two components, R_V and R_H.

$\Sigma H = 0$ gives, $R_H = 11$ t

$\Sigma V = 0$ gives, $R_V = 70$ t

Summing up the moments of all forces about the mid-point of the base Q, we get,

$$R_V \times PQ = 11 \times z.$$

or, $$PQ = \frac{(11)(3.3)}{70} = 0.52 \text{ m}$$

$\therefore$ $$e = 0.52 \text{ m}$$

Effective length of the footing, $L' = L - 2e$

$$= 3.5 - (2)(0.52) = 2.46 \text{ m}$$

Effective width, $B' = B = 2.4$ m

$\therefore$ Effective area $A' = L'B' = (246)(2.4) = 5.9 \text{ m}^2$

(i) *Brinch Hansen's eqn*: We have from eqn. (11.16),

$$q_{ult} = c\,N_c\,s_c\,d_c\,i_c + \gamma\,D\,N_q\,s_q\,d_q\,i_q + 0.5\,\gamma\,B\,N_\gamma\,s_\gamma\,d_\gamma\,i_\gamma.$$

For $\phi = 20°$, $N_c = 14.83$, $N_q = 6.40$, $N_\gamma = 3.54$

$$s_c = s_q = 1 + (0.2)(2.4/3.5) = 1.137$$

$$s_\gamma = 1 - (0.4)(2.4/3.5) = 0.714$$

$$d_c = 1 + (0.35)(1.5/2.4) = 1.219$$

For $\phi = 20°$, $d_q = 1.0 + \dfrac{(1.328 - 1.0)}{25}(20) = 1.262$

$d_\gamma = 1.0$,

$$i_c = 1 - H/2\,cBL = 1 - 11/(2 \times 1.0 \times 2.4 \times 3.5)$$

$$= 0.345$$

$$i_q = 1 - 0.5\,\frac{H}{V} = 1 - \frac{(0.5)(11)}{70} = 0.921$$

$$i_\gamma = i_q^2 = (0.921)^2 = 0.848$$

$\therefore$ $$q_u = (1.0)(14.83)(1.137)(1.219)(0.345) + (1.75)(1.5)(6.4)(1.137)(1.262)(0.921) + (0.5)(1.75)(2.4)(3.54)(0.714)(1.0)(0.848)$$

$$= 33.79 \text{ t/m}^2$$

Safe bearing capacity,

$$q_s = \frac{q_u - \gamma' D}{F_s} + \gamma D$$

or, $$q_s = \frac{33.79 - (1.75)(1.5)}{F_s} + (1.75)(1.5)$$

or, $$q_s = \frac{31.165}{F_s} + 2.625$$

Actual contact pressure due to the given loading,

$$q_c = \frac{70}{5.9} = 11.86 \text{ t/m}^2$$

$\therefore$ $$\frac{31.165}{F_s} + 2.625 = 11.86$$

or, $$F_s = 3.37$$

(ii) As per IS:6403-1981:

For $\phi = 20°$, $N_c = 14.83$, $N_q = 6.40$, $N_\gamma = 5.39$

Values of s_c, s_q and s_γ are the same as those obtained for Brinch Hansen's method.

Now $\sqrt{N_\phi} = \tan(45° + 20°/2) = 1.428$

$$\therefore \quad d_c = 1 + \frac{(0.2)(1.5)(1.428)}{2.4} = 1.18$$

$$d_q = d_\gamma = 1 + \frac{(0.1)(1.5)(1.428)}{2.4} = 1.09$$

Angle of inclination of the resultant load,

$$\alpha = \tan^{-1}\frac{H}{V} = \tan^{-1}\frac{11}{70} = 8.93°$$

$$\therefore \quad i_c = i_q = \left(1 - \frac{8.93}{90}\right)^2 = 0.811$$

$$i_\gamma = \left(1 - \frac{8.93}{20}\right)^2 = 0.306$$

$$\therefore q_u = (1.0)(14.83)(1.137)(1.18)(0.811) + (1.75)(1.5)(6.4)(1.137)(1.09)(0.811) + (0.5)(1.75)(2.4)(5.39)(0.714)(1.09)(0.306)$$

$$= 35.72 \text{ t/m}^2$$

Safe bearing capacity, $q_s = \dfrac{35.72 - (1.75)(1.5)}{F_s} + (1.75)(1.5)$

or, $q_s = \dfrac{33.095}{F_s} + 2.625$

$$\therefore \quad \frac{33.095}{F_s} + 2.625 = \frac{70}{5.9}$$

or, $F_s = 3.58$

Problem 11.10. The following results were obtained from a plate load test performed on a square plate of 30 cm × 30 cm size at a depth of 1.2 m below the ground level in a homogeneous bed of sand:

Applied Load (kg/cm²)	0.25	0.5	1	2	3	4	5
Settlement (mm)	0.45	0.85	1.80	3.45	5.60	8.75	14.50

(i) Plot the load vs settlement curve and determine the ultimate bearing capacity of the plate.

(ii) Determine the ultimate load which a footing of 1.5 m × 1.5 m, placed at 1.2 m below G.L. in the same soil, will carry if the allowable settlement is 2 cm.

Solution: (i) The load-settlement curve is shown in Fig. 11.6. In order to determine the ultimate bearing capacity of the plate, two tengents were drawn to the load-settlement curve as shown in the figure. The load corresponding to the intersection point of these two tangents is found to be 3.75 kg/cm².

$$\therefore \quad q_{u\,(\text{plate})} = 3.75 \text{ kg/cm}^2 = 37.5 \text{ t/m}^2$$

(ii) Using eq. (11.27), the settlement of the prototype footing is given by:

$$\rho = \rho_p \left[\frac{B(B_p + 30.5)}{B_p(B + 30.5)}\right]^2$$

Here, $\rho = 2$ cm $= 20$ mm, $B = 1.5$ m $= 150$ cm $B_p = 30$ cm.

$$20 = \rho_p \left[\frac{150(30 + 30.5)}{30(150 + 30.5)}\right]^2 = 2.809\,\rho_p$$

or, $\rho_p = \dfrac{20}{2.809} = 7.12$ mm.

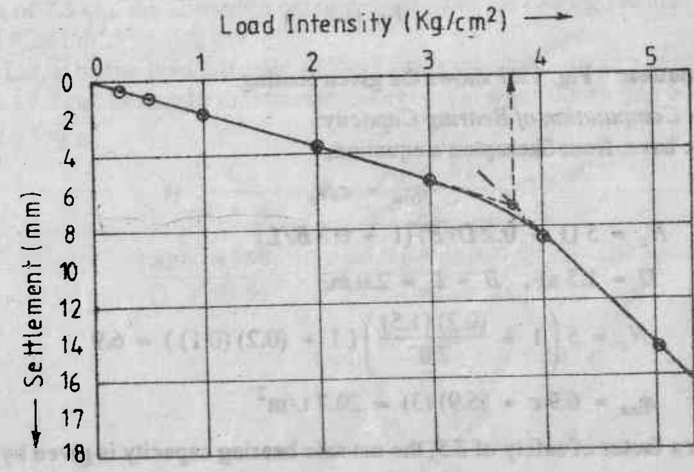


Fig. 11.6

From the load-settlement curve we get, for a settlement of 7.12 mm of the plate, the corresponding load on the plate

$$= 3.70 \text{ kg/cm}^2 = 37 \text{ t/m}^2.$$

$\therefore$ Ultimate bearing capacity of the prototype footing = 37 t/m^2.

$\therefore$ Ultimate load = (37)(1.5) (1.5) = 83.25 t.

Problem 11.11. Determine the allowable bearing capacity of a 2 m × 2 m square footing founded at a depth of 1.5 m below the ground level in a deep stratum of silty clay having the following average properties:

$$\gamma = 1.8 \text{ t/m}^3, \quad c = 3 \text{ t/m}^2, \quad \phi = 0°, \quad C_c = 0.259, \quad e_o = 0.85$$

The maximum permissible settlement of the footing is 7.5 cm. The highest position of the water table at the site is at a depth of 1.0 m below G.L.

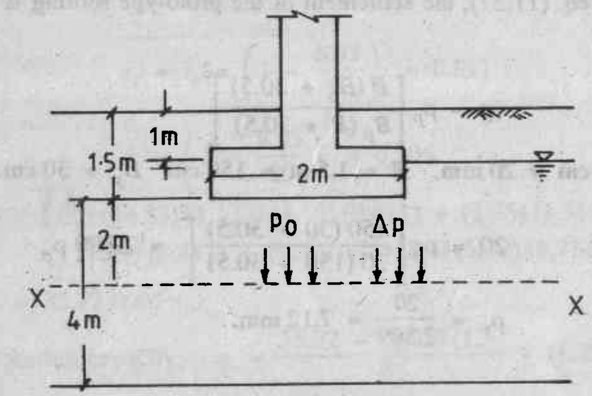


Fig. 11.7

Solution: Fig. 11.7 shows the given footing.

(i) *Computation of Bearing Capacity*:

We have, from Skempton's equation,

$$q_{nu} = c N_c$$

where, $N_c = 5(1 + 0.2 D/B)(1 + 0.2 B/L)$

Here, $D = 1.5$ m, $B = L = 2.0$ m.

$$\therefore \quad N_c = 5\left(1 + \frac{(0.2)(1.5)}{2.0}\right)(1 + (0.2)(0.1)) = 6.9$$

$$\therefore \quad q_{nu} = 6.9\, c = (6.9)(3) = 20.7 \text{ t/m}^2$$

For a factor of safety of 2.5, the net safe bearing capacity is given by,

$$q_{ns} = \frac{q_{nu}}{F_s} = \frac{20.7}{2.5} = 8.28 \text{ t/m}^2.$$

(ii) *Computation of Settlement*: As the underlying soil is saturated silty clay, only consolidation settlement will take place. The zone of influence below the base of footing is extended to a maximum depth of twice the width of the footing, i.e., 4 m below the base. In Fig. 11.7, *X-X* is a horizontal plane through the middle of this consolidating layer.

Now, initial effective overburden pressure on *X-X*,

$$p_0 = \gamma z_1 + \gamma_{sub} z_2$$

$$= (1.8)(1.0) + (1.8 - 1)(0.5 + 2.0)$$

$$= 3.8 \text{ t/m}^2 = 0.38 \text{ kg/cm}^2$$

Using 2 : 1 dispersion method, stress increment at *X-X*,

$$\Delta p = \frac{(8.28)(2.0)(2.0)}{(2.0 + 2.0)^2} = 2.07 \text{ t/m}^2 = 0.207 \text{ kg/m}^2$$

(assuming the footing to be loaded with 8.28 t/m^2).

$\therefore$ Consolidation settlement,

$$\rho_c = H \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{p_0 + \Delta p}{p_0}$$

$$= \frac{(400)(0.259)}{(1 + 0.85)} \cdot \log_{10} \frac{0.38 + 0.207}{0.38} = 10.58 \text{ cm.}$$

As the estimated settlement is greater than the maximum permissible limit of 7.5 cm, the allowable bearing capacity of the footing should be less than 8.28 t/m^2.

Let, q be the load intensity on the footing which results in a settlement of just 7.5 cm. Let Δp be the stress intensity on *X-X* when the footing is loaded with q t/m^2.

$$\therefore \quad H \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{p_0 + \Delta p}{p_0} = 7.5$$

$$\text{or,} \quad \frac{(400)(0.259)}{(1 + 0.85)} \cdot \log_{10} \frac{p_0 + \Delta p}{p_0} = 7.5$$

$$\text{or,} \quad \log_{10} \frac{p_0 + \Delta p}{p_0} = 0.1339$$

$$\text{or,} \quad \frac{p_0 + \Delta p}{p_0} = 1.1433.$$

But the value of p_0 at X-X is constant, and is equal to 0.38 kg/cm².

$$\therefore \quad \frac{0.38 + \Delta p}{0.38} = 1.3612$$

Solving, we get,

$$\Delta p = 0.1372 \text{ kg/cm}^2 = 1.372 \text{ t/m}^2$$

$$\text{But,} \quad \Delta p = \frac{q B L}{(B + z)(L + z)} = \frac{q B^2}{(B + Z)^2}$$

$$\therefore \quad 1.372 = \frac{q (2^2)}{(2 + 2)^2}$$

$$\text{or,} \quad q = 5.49 \text{ t/m}^2 \approx 5.5 \text{ t/m}^2$$

Hence, a loading intensity of 5.5 t/m² will result in a consolidation settlement of 7.5 cm. Therefore, the required allowable bearing capacity of the footing = 5.5 t/m².

EXERCISE 11

11.1. Determine the ultimate bearing capacity of the following footings placed at 1.2 m below the ground level in a homogeneous deposit of firm soil having $\gamma = 1.8 \text{ t/m}^3$, $\phi = 20°$ and $c = 1.8 \text{ t/m}^2$.

(i) a strip footing of 2 m width
(ii) a square footing of 2 m × 2 m size
(iii) a circular footing of 2 m diameter.

given, for $\phi = 20°$, $N_c = 17.7$, $N_q = 7.4$, $N_\gamma = 5.0$

[Ans. (i) 56.84 t/m² (ii) 65.12 t/m² (iii) 62.80 t/m²]

11.2. A 2.5 m × 2.5 m square footing is founded at a depth of 1.5 m below G.L. in a loose soil deposit having the following properties:

$$\gamma = 1.65 \text{ t/m}^3, \quad c = 0.2 \text{ kg/cm}^2, \quad \phi = 15°$$

Determine:

(i) the ultimate bearing capacity
(ii) the net ultimate bearing capacity
(iii) the net safe bearing capacity
(iv) the safe bearing capacity.

The factor of safety should be taken as 3.0. Given, for $\phi = 15°$,

$N_c = 12.9$, $N_q = 4.4$, $N_\gamma = 2.5$, $N_c' = 9.7$, $N_q' = 2.7$, $N_\gamma' = 0.9$.

[Ans. (i) 24.98 t/m² (ii) 22.51 t/m² (iii) 7.50 t/m² (iv) 9.98 t/m²]

11.3. A circular footing of 2.5 m diameter rests at 1.3 m below G.L. in a soil mass having an average cohesion of 10 kN/m², an angle of internal

friction of 28° and a bulk density of 18 kN/m³. The water table is located at a great depth. Determine the safe bearing capacity of the footing. Assume a general shear failure. Given, for $\phi = 28°$, $N_c = 32.5$, $N_q = 18.8$ and $N_\gamma = 15.7$. The factor of safety should be taken as 3.0. [Ans. 373.7 kN/m²]

11.4. In Problem 11.3, if the water table rises to the ground level due to flooding, determine the percent change in the safe bearing capacity of the footing. [Ans: Decreases by 18.6%]

11.5. A square footing of 2.2 m × 2.2 m size is founded at a depth of 1.2 m below G.L. in a homogeneous bed of dry sand having a unit weight of 1.95 t/m³ and an angle of internal friction of 36°. Determine the safe load the footing can carry with respect to a factor of safety of 3.0 against shear failure. Given, for $\phi = 36°$, $N_c = 65.4$, $N_q = 49.4$, $N_\gamma = 54$.

11.6. A 2.0 m wide strip footing is required to be founded in a bed of dense sand having a bulk density of 2.0 t/m³ and an angle of shearing resistance of 35°. Plot the variation of ultimate bearing capacity of the footing with depth of foundation, D_f, for $0 \leq D_f \leq 3.0$ m. Given, for $\phi = 35°$, $N_c = 58$, $N_q = 41.5$, $N_\gamma = 42.4$.

11.7. Determine the safe load a circular footing of 5 m diameter founded at a depth of 1.0 m below G.L. can carry. The foundation soil is a saturated clay having an unconfined compressive strength of 6 t/m² and a unit weight of 1.75 t/m³. Assume a factor of safety of 2.5. Use Skempton's and Terzaghi's methods and compare the results. State, giving reasons, which one is more reliable.

[Ans: Terzaghi: 154.92 t, Skempton: 131.48 t, Skempton's method]

11.8. A strip footing has to carry a gross load of 120 kN per metre run. The footing is placed at 1.25 m below G.L. in a homogeneous sand stratum. The unit weight and angle of internal friction of the sand are 19 kN/m³ and 32° respectively. Determine the minimum width of the footing required in order to have a factor of safety of 3.0. Given, for $\phi = 32°$, $N_c = 44$, $N_q = 29$, $N_\gamma = 26$. [Ans: 2.18 m]

11.9. The size of square footing must be restricted to 1.5 m × 1.5 m. The footing has to carry a net load of 150 t coming from the superstructure. The foundation soil has the following properties:

$$\gamma = 1.91 \text{ gm/cc}, \quad c = 0, \quad \phi = 36°$$

$$\text{For } \phi = 36°, \quad N_c = 65, \quad N_q = 49, \quad N_\gamma = 54.$$

Determine the minimum depth at which the footing has to be placed in order to have a factor of safety of 2.5 against shear failure. [Ans: 1.10 m]

11.10. Complete shear failure of an RCC footing took place under a gross load of 62450 kg. The dimensions of the footing were 2.25 m × 2.25 m and the depth of foundation was 1.4 m. The subsoil consisted of a deep stratum of medium clay (γ = 1.8 t/m^3). Find out the average unit cohesion of the clay. [Ans: c = 3.5 t/m^2]

11.11. The footing of a column is 1.5 m × 1.5 m in size, and is founded at a depth of 1.25 m below the ground level. The properties of the foundation soil are:

$$c = 0.1 \text{ kg/cm}^2, \quad \phi = 15°, \quad \gamma = 1.75 \text{ gm/cc.}$$

Determine the safe load the footing can carry with a factor of safety of 2.5, when the water table is at:

(i) 0.5 m below the ground level.

(ii) 0.5 m below the base of footing. [Ans: (i) 24.99 t (ii) 28.29 t]

11.12. The subsoil at a site consists of a homogeneous bed of normally consolidated soil having the following properties:

$$\gamma = 1.85 \text{ t/m}^3, \quad c = 3.5 \text{ t/m}^2, \quad \phi = 10°$$

A 2 m × 3.5 m footing is to be founded on this soil at a depth of 1.5 m. Determine the safe load the footing can carry with a factor of safety of 2.5. Use Brinch Hansen's method.

Given, for ϕ = 10°, N_c = 8.34, N_q = 2.47, N_γ = 0.47.

[Ans: 152.44 t]

11.13. Redo Problem 11.12 using the method recommended by IS: 6403-1981. Given, for ϕ = 10°, N_c = 8.35, N_q = 2.47, N_γ = 1.22.

[Ans: 152.08 t]

11.14. Determine the factor of safety against shear failure of a 1.5 m wide strip footing located at a depth of 1 m below the ground level in a bed of dense sand having γ = 1.9 t/m^3 and ϕ = 40°, if it carries a uniformly distributed load of 22 t per metre run. Use Terzaghi's equation. Given, for ϕ = 40°, N_c = 75.32, N_q = 64.18, and N_γ = 95.41. [Ans: 2.61]

11.15. An R.C.C. column is subject to a vertical force of 900 kN acting through its centre line and a horizontal thrust of 120 kN acting at 2.7 m above G.L. The column is supported by a square footing of 2.5 m × 2.5 m size, placed at a depth of 1.2 m below G.L. The foundation soil has an angle of internal friction of 35° and a bulk density of 18.5 kN/m^3. Assuming a factor of safety of 3.0, determine the safe load. Use:

(i) Brinch Hansen's method

(N_c = 46.12, N_q = 33.3, N_γ = 40.69)

(ii) Recommendation of IS: 6403 - 1981

(N_c = 46.12, N_q = 33.3, N_γ = 48.03)

[Ans: (i) 3458 kN (ii) 2687 kN]

11.16. In order to assess the bearing capacity of a 2.5 m square footing, a plate load test was conducted at a site with a square plate of 60 cm × 60 cm size. The following results were obtained:

Applied load (kg)	180	360	720	1080	1440	1800
Settlement (mm)	0.82	1.78	2.71	3.62	5.40	9.30

If the allowable settlement of the footing be 1.5 cm, find out the allowable load on the footing. [Ans: 284.4 t]

12

PILE FOUNDATIONS

12.1 Introduction: According to Terzaghi, a foundation is called a deep foundation if its width is less than its depth (i.e., $D/B > 1$). Various types of deep foundations are:

1. Pile foundations
2. Well foundations or open caissons.
3. Pier foundations or drilled caissons.

12.2 Pile Foundations: Piles are generally used to transfer the load of a structure to a deep-seated, strong soil stratum. The other applications of piles are as follows:

(i) to compact a loose soil layer (compaction piles)
(ii) to hold down structures subject to uplift or overturning forces (tension piles)
(iii) to provide anchorage against horizontal pull applied on earth-retaining structures (anchor piles)
(iv) to protect waterfront structures from the impact of marine vessels (fender piles)
(v) to resist oblique compressive loads (batter piles).

12.3 Classification of Piles According to Load Dispersal Characteristics: On the basis of the mode of load dispersion, piles can be classified into the following two categories:

(i) *Bearing piles*: When a pile passes through a weak stratum but its tip penetrates into a stratum of substantial bearing capacity, the pile transfers the load imposed on it to the stronger stratum. Such a pile is called a bearing pile.

(ii) *Friction pile*: When a pile is extended to a considerable depth in a stratum of poor bearing capacity, it derives its load carrying capacity from the friction of the soil mass on the sides of the pile. Such a pile is called a friction pile.

12.4 Bearing Capacity of Piles: The bearing capacity of a pile may be defined as the maximum load which can be sustained by a pile without producing excessive settlement.

The bearing capacity of an individual pile may be determined by the following methods:

(i) Dynamic formula
(ii) Static formula
(iii) Pile load test

12.5 Dynamic Formulae: The dynamic formulae are based on the concept that a pile derives its bearing capacity from the energy spent in driving it.

The following dynamic formulae are most widely used:

1. *Engineering News Formula*: According to this formula, the safe bearing capacity of a pile is given by:

$$Q = \frac{WH}{F(s+c)} \quad ...(12.1)$$

where, Q = safe load in kg
W = weight of hammer in kg
H = fall of hammer in cm
s = average penetration of the pile in the last n blows in cm
For drop hammers, $n = 5$
for steam hammers, $n = 20$
c = additional penetration of the pile which would have taken place had there been no loss of energy in driving the pile.
For drop hammers, $c = 2.5$ cm.
for steam hammers, $c = 0.25$ cm.

Eqn. (12.1) gives the general form of the Engineering News Formula. The specific forms of this formula for different types of hammers are given below:

(i) For drop hammer: $$Q = \frac{Wh}{6(s+2.5)} \quad ...(12.2)$$

(ii) For single acting steam hammer: $$Q = \frac{Wh}{6(s+0.25)} \quad ...(12.3)$$

(iii) For double acting steam hammer: $$Q = \frac{(W+ap)h}{6(s+0.25)} \quad ...(12.4)$$

where, a = effective area of the piston in cm^2
p = mean effective steam pressure in kg/cm^2.

2. *Modified Hiley Formula*: IS : 2911 (Part 1) - 1964 recommends the following formula based on an expression originally derived by Hiley:

$$Q_u = \frac{\eta_h \cdot W \cdot H \cdot \eta_b}{s + c/2} \quad ...(12.5)$$

where, Q = ultimate load on pile (kg)

W, H, s and c have the same meaning as in eqn. (12.1)

η_h = efficiency of hammer.

η_b = efficiency of hammer blow

= the ratio of energy after impact to the striking energy of the ram.

When $W > eP$, $$\eta_b = \frac{W + e^2 P}{W + P} \quad ...(12.6)$$

when $W < eP$, $$\eta_b = \frac{W + e^2 P}{W + P} - \left[\frac{W - eP}{W + P}\right]^2 \quad ...(12.7)$$

where, P = weight of the pile alongwith anvil, helmet, etc

e = co-efficient or restitution, the value of which may vary between 0 and 0.5, depending on the driving system as well as the material of the pile.

In eqn. (12.5), C represents the temporary elastic compression, which is given by,

$$C = C_1 + C_2 + C_3, \quad ...(12.8)$$

where, C_1, C_2 and C_3 represent the elastic compressions of the dolly and packing, the pile and the soil respectively. Their values may be obtained from:

$$C_1 = 1.77 \frac{Q_u}{A_P} \quad ...(12.9)$$

$$C_2 = 0.657 \frac{Q_u L}{A_P} \quad ...(12.10)$$

$$C_3 = 3.55 \frac{Q_u}{A_P} \quad ...(12.11)$$

where, A_P = cross-sectional area of the pile, cm

L = length of pile, m

The safe load on a pile may be obtained from:

$$Q_s = \frac{Q_u}{F_s} \quad ...(12.12)$$

The value of F_s should lie between 2 and 3.

12.6 Static Formulae: The static formulae are based on the concept that the ultimate load bearing capacity (Q_u) of a pile is equal to the sum of the total skin friction acting on the surface area of the embedded portion of the pile (Q_f) and the end bearing resistance acting on the pile tip (Q_b), as illustrated in Fig. 12.1.

$$\therefore \quad Q_u = Q_f + Q_b \quad ...(12.13)$$

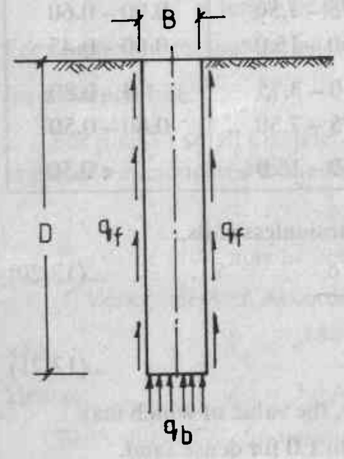


Fig. 12.1

But, $Q_f = q_f \cdot A_f$ and $Q_b = q_b \cdot A_b$

$$\therefore \quad Q_f = q_f \cdot A_f + q_b \cdot A_b \quad ...(12.14)$$

where, q_f = average unit skin friction

q_b = point bearing resistance of the pile tip

A_f = surface area of the pile on which the skin friction acts.

A_b = c/s area of the pile at its tip.

The methods of evaluating q_f and q_b are explained below:

1. *Cohesive Soils*:

Average unit skin friction, $q_f = \alpha c$ ...(12.15)

where, c = unit cohesion

α = adhesion factor, which depends on the consistency of the soil and may be determined from Table 12.1

Average point bearing resistance

$$q_b = c N_c \quad ...(12.16)$$

According to Skempton, for deep foundations, $N_c = 9$

$$\therefore \quad q_b = 9c \quad ...(12.17)$$

$$\therefore \quad Q_u = \alpha c A_f + 9 c A_b \quad ...(12.18)$$

For a pile of diameter B and embedded depth D.

$$A_b = \frac{\pi}{4} B^2 \text{ and } A_f = \pi B D$$

Eqn. (12.14) therefore reduces to:

$$Q_u = \pi B D c\alpha + 2.25\,\pi B^2 c \quad \text{...(12.19)}$$

Table 12.1: Adhesion Factors

Pile material	*Consistency*	*Cohesion (t/m²)*	*Adhesion factor α*
Timber & Concrete	soft	0 – 3.75	1 – 0.90
	medium	3.75 – 7.50	0.90 – 0.60
	stiff	7.50 – 15.0	0.60 – 0.45
Steel	soft	0 – 3.75	1.0 – 0.80
	medium	3.75 – 7.50	0.80 – 0.50
	stiff	7.50 – 15.0	< 0.50

2. *Cohesionless Soils*: For piles driven in cohesionless soils,

$$q_f = \overline{q_a}\, K_s \tan\delta \quad \text{...(12.20)}$$

where, $\overline{q_a}$ = average overburden pressure

i.e., $\overline{q_a} = \gamma' \cdot z$ ...(12.21)

K_s = co-efficient of earth pressure, the value of which may vary from 0.5 for loose sand to 1.0 for dense sand.

δ = friction angle of the soil on the pile, which depends on the angle of internal friction ϕ of the soil.

The value of δ may be obtained from Table 12.2.

Table 12.2: Friction Angle

Pile material	*Surface condition*	*Value of $\frac{\delta}{\phi}$*	
		Dry sand	*Saturated sand*
Steel	Smooth (polished)	0.54	0.64
	Rough (rusted)	0.76	0.80
Wood	Parallel to grain	0.76	0.85
	Perpendicular to grain	0.88	0.89
Concrete	Smooth (made in metal form work)	0.76	0.80
	Grained (made in timber form work)	0.88	0.80
	Rough (cast on ground)	0.98	0.90

For a purely cohesionless soil, $c = 0$. Hence, the point bearing resistance is given by,

$$q_b = \gamma D N_q s_q + \gamma B N_\gamma s_\gamma \quad \text{...(12.22)}$$

where, N_q, N_γ = Bearing capacity factors.

s_q, s_γ = Shape factors

B = width or diameter of pile

D = length of pile

For a square or rectangular pile, $s_\gamma = 0.5$

for a circular pile, $s_\gamma = 0.3$

For piles of small diameter or width, the second term of eqn. (12.22) is negligible as compared to the first term. Thus, for all practical purposes,

$$q_b = \gamma D N_q s_q \quad \text{...(12.23)}$$

The value of N_q may be determined by the following methods:

(i) *Vesic's method*: According to Vesic, $s_q = 3$,

and, $N_q = \varepsilon^{3.8\,\phi \tan\phi} \cdot \tan^2(45° + \phi/2)$

Hence, $q_b = 3\, q\, N_q$ ...(12.24)

The values of N_q for various values of ϕ are given in Table 12.3.

Table 12.3: Bearing Capacity Factors

φ (degrees)	N_q	*φ (degrees)*	N_q
0	1.0	30	9.5
5	1.2	35	18.7
10	1.6	40	42.5
15	2.2	45	115.0
20	3.3	50	422.0
25	5.3		

(ii) *Berezantsev's method*: According to Berezantsev the N_q values depend on the *D/B* ratio of the pile and the angle of internal friction of the soil. The N_q-value may be obtained from Fig. 12.2.

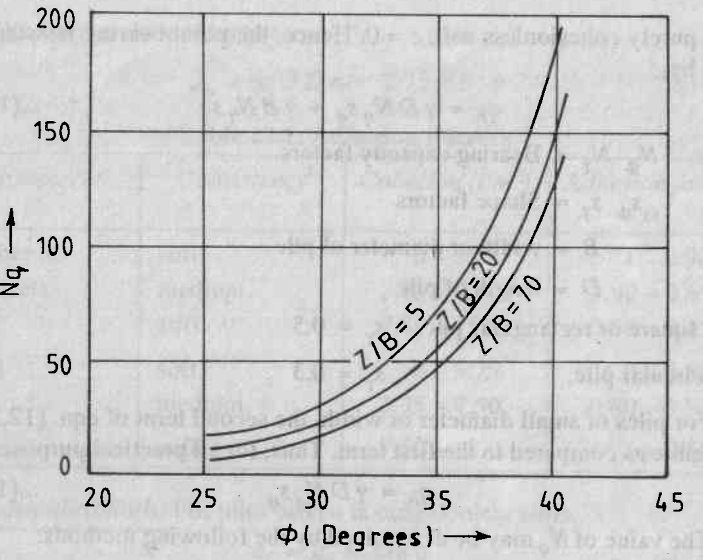


Fig. 12.2

12.7 Pile Capacity from Penetration Tests: The pile capacity can also be determined from the results of the Standard Penetration Test or Static Cone Penetration Test performed in the field, using the following equations:

(i) *Standard Penetration Test*:

$$Q_u = 4NA_b + 0.02NA_f \qquad ...(12.25)$$

where, Q_u = ultimate bearing capacity of pile in kg

N = blow count (without overburden correction)

A_b = base area of pile in cm^2

A_f = surface area of pile in cm^2

However, for a bored pile,

$$Q_u = 1.33NA_b + 0.02NA_f \qquad ...(12.26)$$

(ii) *Static cone penetration test*:

$$Q_u = q_c A_b + \frac{1}{2} q_c A_f \qquad ...(12.27)$$

where, q_c = cone resistance at tip.

12.8 Group Action in Piles: A pile foundation consists of a number of closely spaced piles, known as a pile group. Due to the overlapping in the stressed zone of individual piles, the bearing capacity of a group of friction

piles is generally less than the product of capacity of a single pile and the number of piles in the group. In order to determine the bearing capacity of a pile group, Q_g, a correction factor η_g is required to be used.

$$Q_g = nQ_u\eta_g \qquad ...(12.28)$$

where, n = number of piles in the group

Q_u = ultimate bearing capacity of each pile

η_g = efficiency of the pile group

The value of η_g may be obtained from the following empirical formulae:

(i) *Converse-Labarre' Formula*:

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right] \qquad ...(12.29)$$

where, m = number of rows of pile in the group

n = number of piles in each row

$\theta = \tan^{-1}\frac{d}{s}$

where, d = diameter of each pile

s = spacing of the piles

(ii) *Los Angeles formula*:

$$\eta_g = 1 - \frac{d}{\pi mns}\left[m(n-1) + n(m-1) + \sqrt{2}(m-1)(n-1)\right] \qquad ...(12.30)$$

12.9 Design of a Pile Group: The piles in a group are connected to a rigid pile cap so that the group of piles behaves as a unit. The group capacity may be determined by the efficiency equation (eqn. 12.24). A more rational

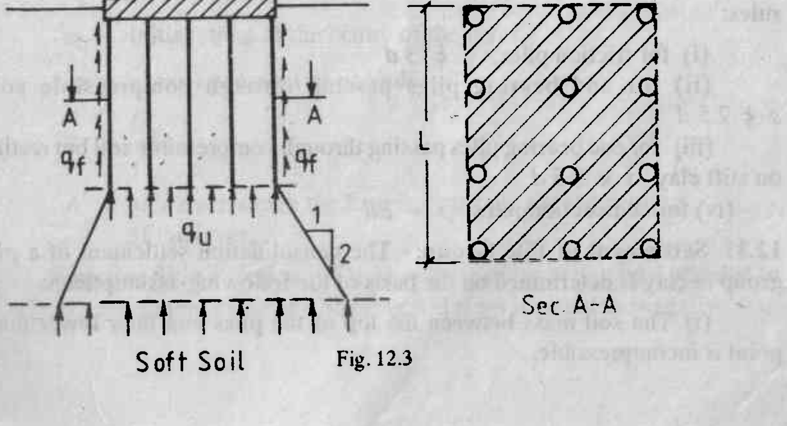


Fig. 12.3

method is the rigid block method recommended by Terzaghi and Peck. According to this method the ultimate bearing of a pile group equals the sum of the ultimate bearing capacity of block occupied by the group and the shearing resistance mobilised along the perimeter of the group. With reference to Fig. 12.3.

$$Q_g = q_u B L + D_f (2B + 2L) s - \gamma D_f B L \quad \text{...(12.31)}$$

where, Q_g = ultimate bearing capacity of the pile group.

q_u = ultimate bearing capacity per unit area of the stressed area at a depth D_f

B, L = width and length of pile group

γ = unit weight of soil

s = average shearing resistance of soil per unit area between ground surface and the bottom of pile

D_f = depth of embedment of piles.

The safe load on the pile group is given by,

$$Q_{sg} = \frac{Q_g}{F_s} \quad \text{...(12.32)}$$

The minimum value of F_s should be taken as 3.0.

The above equations are applicable to cohesive soils. For end bearing piles on hard rock (irrespective of the spacing) and on dense sand (with spacing greater than 3 times pile diameter) the group capacity equals the sum of individual capacities. i.e.,

$$Q_g = N \cdot Q_u \quad \text{...(12.33)}$$

12.10 Spacing of Piles: As per the recommendations of IS : 2911 (Part 1)-1964, the spacing of piles may be obtained from the following general rules:

(i) for friction piles, $s \nless 3\,d$

(ii) for end bearing piles passing through compressible soil, $s \nless 2.5\,d$

(iii) for end bearing piles passing through compressible soil but resting on stiff clay, $s \simeq 3.5\,d$

(iv) for compaction piles, $s = 2\,d$

12.11 Settlement of Pile Group: The consolidation settlement of a pile group in clay is determined on the basis of the following assumptions:

(i) The soil mass between the top of the piles and their lower third point is incompressible.

(ii) The load on the pile group is effectively transmitted to the soil at this lower one-third point.

(iii) The presence of pile below this level is ignored.

(iv) The transmitted load is dispersed as 60° to the horizontal.

With reference to Fig. 12.4, the settlement of the group is given by:

$$\rho = H \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{\sigma_0 + \Delta\sigma}{\sigma_0} \quad \text{...(12.34)}$$

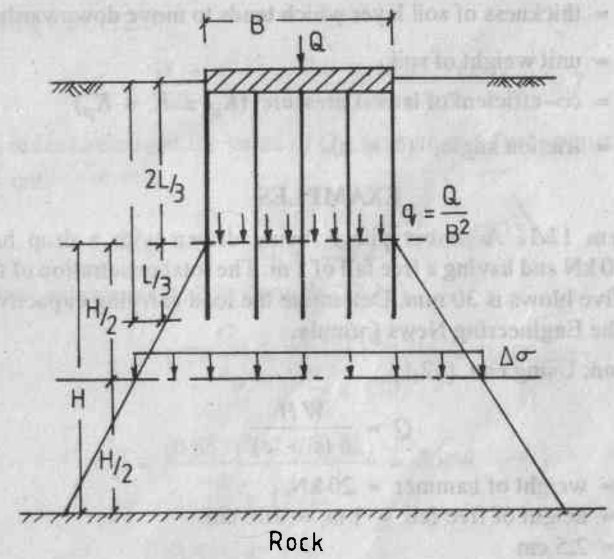


Fig. 12.4

where, H = thickness of the layer

C_c = compression index, e_0 = initial void ratio

σ_0 = initial stress at the centre of the layer

$\Delta\sigma$ = stress increment due to piles

$= \dfrac{Q_g}{A'}$

A' = area over which the load is distributed at the centre of the layer.

12.12 Negative Skin Friction: The downward drag acting on a pile due to the relative movement of the surrounding soil mass is called the negative skin friction. This tends to reduce the load carrying capacity of the pile. Its magnitude can be determined from:

(i) for cohesive soils: $Q_{nf} = p \cdot c \cdot L_f$ ...(12.35)

(ii) for cohesionless soils: $Q_{nf} = \frac{1}{2} L_f^2\, p\, \gamma\, K \tan \delta$ ...(12.36)

where, p = perimeter of the pile
c = average cohesion of the soil
L_f = thickness of soil layer which tends to move downwards
γ = unit weight of soil
K = co-efficient of lateral pressure $(K_a \le K \le K_p)$
δ = friction angle, $(\delta \le \phi)$

EXAMPLES

Problem 12.1. A timber pile is being driven with a drop hammer weighing 20 kN and having a free fall of 1 m. The total penetration of the pile in the last five blows is 30 mm. Determine the load carrying capacity of the pile using the Engineering News formula.

Solution: Using eqn. (12.1),

$$Q = \frac{WH}{6(s + c)}$$

Here, W = weight of hammer = 20 kN.
H = height of free fall = 1 m = 100 cm.
c = 2.5 cm
s = average penetration for the last 5 blows

$$= \frac{30}{5} = 6 \text{ mm} = 0.6 \text{ cm}$$

$$\therefore \quad Q = \frac{(20)(100)}{6(0.6 + 2.5)} = 107.5 \text{ kN}$$

Problem 12.2. Determine the safe load that can be carried by a pile having a gross weight of 1.5 t, using the modified Hiley's formula. Given,

weight of hammer = 2.0 t
height of free fall = 91 cm
hammer efficiency = 75%
average penetration under the last 5 blows = 10 mm
length of pile = 22 m
diameter of pile = 300 mm.
co-efficient of restitution = 0.55

Solution: From eqn. (12.5), the ultimate load on pile,

$$Q_u = \frac{\eta_h \cdot W H \cdot \eta_b}{s + c/2}$$

Here, $W = 3.0$ t, $H = 91$ cm, $\eta_h = 75\% = 0.75$
$s = 10$ mm $= 1.0$ cm

Now, $eP = (0.55)(1.5) = 0.825$ t

$\therefore \quad W > eP$

Using eqn. (12.6),

$$\eta_b = \frac{W + e^2 P}{W + P} = \frac{2.0 + (0.55^2)(1.5)}{2.0 + 1.5} = 0.7$$

In order to find out the value of Q_u, assume as a first approximation, $c = 2.5$ cm.

$$\therefore \quad Q_u = \frac{(0.75)(2.0)(91)(0.7)}{1.0 + 2.5/2} = 42.47 \text{ t.}$$

Now, using eqns. (12.9) through (12.11),

$$C_1 = 1.77 \frac{Q_u}{A_p} = \frac{(1.77)(42.47)}{\frac{\pi}{4} \times (30)^2} = 0.106 \text{ cm}$$

$$C_2 = \frac{(0.657)(42.47)(22)}{\frac{\pi}{4} \times (30)^2} = 0.868$$

$$C_3 = \frac{(3.55)(42.47)}{\frac{\pi}{4} \times (30)^2} = 0.213 \text{ cm}$$

$$\therefore \quad C = C_1 + C_2 + C_3 = 1.187 \text{ cm} < 2.5 \text{ cm.}$$

Let $Q_u = 50$ t, $\therefore C = \frac{(1.187)(50)}{42.47} = 1.397$ cm

$$\therefore \quad Q_u = \frac{(0.75)(2.0)(91)(0.7)}{1.0 + 1.397/2} = 56.25 \text{ t}$$

Let $Q_u = 55$ t, $\therefore C = \frac{(1.187)(55)}{42.47} = 1.537$

$$\therefore \quad Q_u = \frac{(0.75)(2.0)(91)(0.7)}{1.0 + 1.537/2} = 54 \text{ t}$$

In the third iteration the assumed and computed values of Q_u are quite close. Hence, the ultimate load bearing capacity of the pile is 54 t.

Consequently, the safe bearing capacity

$$Q_s = \frac{Q_u}{F_s} = \frac{54}{2.5} = 21.6 \text{ t.}$$

Problem 12.3. An RCC pile of 18 m overall length is driven into a deep stratum of soft clay having an unconfined compressive strength of 3.5 t/m^2. The diameter of the pile is 30 cm. Determine the safe load that can be carried by the pile with a factor of safety of 3.0.

Solution: From eqn. (12.14),

$$Q_u = q_f \cdot A_f + q_b \cdot A_b.$$

As the pile is driven into a cohesive soil,

$$q_f = \alpha \cdot c$$

The value of adhesion factor α may be obtained from Table 12.1. For a soft clay having $c = \frac{q_u}{2} = \frac{3.5}{2} = 1.75$ t/m^2, α may be taken as 0.95.

Again, we have, $q_b = 9\,c$

A_b = c/s area of pile tip

$$= \frac{\pi}{4} \times \left(\frac{30}{100}\right)^2 = 0.07 \text{ m}^2$$

A_f = surface area of the pile

$$= \pi\,(0.30)\,(18) = 16.96 \text{ m}^2$$

$$\therefore \quad Q_u = (0.95)\,(1.75)\,(16.96) + (9)\,(1.75)\,(0.07)$$

$$= 28.2 + 1.1 = 29.3 \text{ t}$$

$$\therefore \text{ Safe load, } Q_s = \frac{Q_u}{F_s} = \frac{29.3}{3.0} = 9.76 \text{ t.}$$

Problem 12.4. A smooth RCC pile of 40 cm diameter and 15 m length is driven into a deep stratum of dry, loose sand having a unit weight of 1.6 t/m^3 and an angle of internal friction of 25°. Determine the safe load which can be carried by the pile. Given, for $\phi = 25°$, Vesic's bearing capacity factor $N_q = 5.3$.

Solution: Using eqn. (12.20),

$$q_f = \overline{q_a}\, K_s \tan\delta$$

Here, $\overline{q_a}$ = average overburden pressure

$$= \gamma z = \frac{\gamma H}{2}$$

$$= \frac{(1.6)\,(15)}{2} = 12 \text{ t/m}^2$$

For loose sand, $K_s = 0.5$

The value of δ may be obtained from Table 12.2. For a smooth RCC pile embedded in dry sand,

$$\delta/\phi = 0.76, \quad \text{or,} \quad \delta = (0.76)\,(25°) = 19°$$

$$\therefore \quad q_f = (12)\,(0.5)\,(\tan 19°)$$

$$= 2.066 \text{ t/m}^2$$

Using eqn. (12.24),

$$q_b = 3\,q\,N_q$$

$$= (3)\,(1.6)\,(15)\,(5.3) = 381.6 \text{ t/m}^2$$

$$A_f = \pi B D = \pi\,(0.40)\,(15) = 18.85 \text{ m}^2$$

$$A_b = \frac{\pi}{4} B^2 = (\pi/4)\,(0.40^2) = 0.126 \text{ m}^2$$

$$\therefore \quad Q_u = (2.066)\,(18.85) + (381.6)\,(0.126)$$

$$= 38.94 + 48.08$$

$$= 87.02 \text{ t} = 87 \text{ t}$$

$$\therefore \quad Q_s = \frac{Q_u}{F_s} = \frac{87}{3} = 29 \text{ t.}$$

Problem 12.5. A bored concrete pile of 400 mm diameter and having an overall length of 12.5 m is embedded in a saturated stratum of $c - \phi$ soil having the following properties:

$$c = 15 \text{ kN/m}^2, \quad \phi = 20°, \quad \gamma_{sat} = 18 \text{ kN/m}^3$$

Determine the safe bearing capacity of the pile. Given, for $\phi = 20°$, the bearing capacity factors are:

$$N_c = 26, \quad N_q = 10, \quad N_\gamma = 4.$$

Assume reasonable values for all other factors.

Solution: For piles embedded in a $c - \phi$ soil,

$$q_b = c N_c + \gamma' D\,(N_q - 1) + 0.5\,\gamma' B N_\gamma$$

$$= (15)(26) + (18 - 10)(12.5)(10 - 1) + (0.5)(18 - 10)(0.40)(4)$$

$$= 390 + 900 + 6.4$$

$$= 1296.4$$

$$q_f = \alpha c + \overline{q_a} K_s \tan \delta.$$

Assume, $\alpha = 0.5$, $K_s = 1$, $\delta/\phi = 0.80$.

$$\delta = (0.80)(20^\circ) = 16^\circ$$

$$\therefore \quad q_f = (0.5)(15) + (18 - 10)(12.5/2)(1.0)(\tan 16^\circ)$$

$$= 21.84 \text{ kN/m}^2$$

Again $\quad A_f = \pi (0.4)(12.5) = 15.71 \text{ m}^2$

and, $\quad A_b = \frac{\pi}{4}(0.40)^2 = 0.126 \text{ m}^2$

$$\therefore \quad Q_u = (21.84)(15.71) + (1296.4)(0.126)$$

$$= 343.1 + 163.3$$

$$= 506.4 \text{ kN.}$$

$$\therefore \quad Q_s = \frac{506.4}{3} = 168.8 \text{ kN} \approx 168 \text{ kN.}$$

Problem 12.6. The column of a footing is founded at a depth of 1.5 m below G.L. and is supported by a number of piles each having a length of 10 m. The subsoil consists of three layers, the properties of which are given below:

Layer I : $c = 3 \text{ t/m}^2$, $\gamma = 1.85 \text{ t/m}^3$, $\phi = 0^\circ$, $H = 6.5$ m

Layer II : $c = 5 \text{ t/m}^2$, $\gamma = 1.90 \text{ t/m}^3$, $\phi = 0^\circ$, $H = 3$ m

Layer III : $c = 0$, $\gamma = 1.80 \text{ t/m}^3$, $\phi = 30^\circ$, $H = 15$ m

Determine the safe load on each pile if the diameter of the piles be 500 mm and the required factor of safety be 2.5. Assume, adhesion factor $\alpha = 0.80$.

Solution: The depth of embedment of the piles in the three layers are respectively, 5 m, 3 m and 2 m.

For the first layer, $q_{f_1} = \alpha c_1 = (0.80)(3) = 2.4 \text{ t/m}^2$.

and, $A_{f_1} = \pi (0.5)(5) = 7.85 \text{ m}^2$

For the second layer, $q_{f_2} = \alpha c_2 = (0.80)(5) = 4 \text{ t/m}^2$

and, $A_{f_2} = \pi (0.5)(3.0) = 4.71 \text{ m}^2$

For the third layer, the skin friction may be neglected.

Again, using eqn. (12.24),

$$q_b = 3 q N_q = (3)(1.85 \times 5 + 1.9 \times 3 + 1.8 \times 2)(9.5)$$

$$= 528.67 \text{ t/m}^2$$

and $\quad A_b = \frac{\pi}{4} \cdot (0.5)^2 = 0.196 \text{ m}^2$

$$\therefore \quad Q_u = (2.4)(7.85) + (4)(4.71) + (528.67)(0.196)$$

$$= 18.84 + 18.84 + 103.62$$

$$= 141.3 \text{ t}$$

$$\therefore \quad Q_s = \frac{141.3}{3} = 47.1 \text{ t} \approx 47 \text{ t.}$$

Problem 12.7. A raft foundation is supported by a pile group consisting of 15 piles arranged in 3 rows. The diameter and length of each pile are 300 mm and 15 m respectively. The spacing between the piles is 1.2 m. The foundation soil consists of a soft clay layer having $c = 3.2 \text{ t/m}^2$ and $\gamma = 1.9 \text{ t/m}^3$. Determine the capacity of the pile group.

Solution: (i) Considering individual action of piles:

$$q_f = \alpha c$$

$$= (0.9)(3.2) \quad [\text{Assuming } \alpha = 0.90]$$

$$= 2.88 \text{ t/m}^2.$$

$$A_f = \pi (0.30)(15) = 14.14 \text{ m}^2$$

$$q_b = 9c = 9(3.2) = 28.8 \text{ t/m}^2$$

$$A_b = \frac{\pi}{4}(0.30^2) = 0.071 \text{ m}^2$$

∴ Individual capacity of each pile,

$$Q_u = (2.88)(14.14) + (28.8)(0.071)$$

$$= 42.77 \text{ t}$$

∴ Group capacity, $Q_{ug} = (15)(42.77) = 641.55$ t

(ii) Considering group action of piles: Assuming a block failure, the capacity of the pile group may be obtained from eqn. (12.31):

$$Q_g = q_b B L + D_f (2B + 2L) s - \gamma D_f B L$$

With reference to Fig. 12.3,

width of the block, $B = 2(1.2) + 2(0.15) = 2.7$ m

length of the block, $L = 4(1.2) + 2(0.15) = 5.1$ m

depth of the block, $D_f = 15$ m

$$q_b = 9c = (9)(3.2) = 28.8 \text{ t/m}^2$$

$$s = q_f = \alpha c = (0.9)(3.2) = 2.88 \text{ t/m}^2$$

$$\therefore Q_g = (28.8)(2.7)(5.1) + 15(2 \times 2.7 + 2 \times 5.1)(2.88) - (1.9)(15)(2.7)(5.1)$$

$$= 678.05 \text{ t} > 641.55 \text{ t}$$

Hence, the ultimate bearing capacity of the pile group is 641.55 t.

∴ Safe bearing capacity w.r.t. a factor of safety of 2.5,

$$Q_{sg} = \frac{641.55}{2.5} = 256.62 \text{ t} \approx 256 \text{ t.}$$

Problem 12.8. A group of 12 piles, each having a diameter of 500 mm and 30 m long, supports a raft foundation. The piles are arranged in 3 rows and spaced at 1.25 m c/c. The properties of the foundation soil are as follows:

$$\gamma' = 11 \text{ kN/m}^3, \quad q_u = 75 \text{ kN/m}^2, \quad \phi = 0°.$$

Assuming $\alpha = 0.80$ and $F_s = 2.5$, determine the capacity of the pile group.

Solution: (i) Considering individual action of piles:

$$q_f = \alpha c = (0.80)(75/2) = 30 \text{ kN/m}^2$$

$$q_b = 9c = (9)(75/2) = 337.5 \text{ kN/m}^2$$

$$A_f = \pi(0.50)(30) = 47.12 \text{ m}^2$$

$$A_b = \frac{\pi}{4}(0.50^2) = 0.196 \text{ m}^2$$

∴ Capacity of each pile,

$$Q_u = (30)(47.12) + (337.5)(0.196)$$

$$= 1479.75 \text{ kN}$$

∴ Group capacity = (12) (1479.75)

$$= 17757 \text{ kN}$$

(ii) Considering group action of piles: Assuming a block failure, width of block, $B = 2(1.25) + 2(0.50/2)$

$$= 3 \text{ m.}$$

length of block, $L = 3(1.25) + 2(0.50/2) = 4.25$ m

depth of block, $D_f = 30$ m.

$$q_f = 30 \text{ kN/m}^2, \; q_b = 337.5 \text{ kN/m}^2$$

∴ Group capacity, $Q_g = (337.5)(3)(4.25) + 30(2 \times 3 + 2 \times 4.25)(30) - (11)(30)(3)(4.25)$

$$= 13145.6 \text{ kN} < 17757 \text{ kN}$$

Hence, group action governs the capacity of the pile group.

$$\therefore \quad Q_g = 13145.6 \text{ kN.}$$

$$\therefore \quad Q_s = \frac{13145.6}{2.5} = 5258.2 \text{ kN} \approx 5258 \text{ kN.}$$

Problem 12.9. A group of 20 piles, each having a diameter of 40 mm and 10 m long, are arranged in 4 rows at a spacing 1.0 m c/c. The capacity of each pile is 380 kN. Determine the group capacity of the piles.

Solution: Using eqn. (12.28), the capacity of the pile group,

$$Q_g = n \cdot Q_u \cdot \eta_g.$$

Here, $n = 20$, $Q_u = 380$ kN.

The efficiency of the pile group, η_g, may be determined by either of the following formula:

(i) Converse - Labarre Formula: Using eqn. (12.29),

$$\eta_g = 1 - \frac{\theta}{90}\left[\frac{(n-1)m + (m-1)n}{mn}\right]$$

Here, $m = 4$, $n = 5$,

$$\theta = \tan^{-1}\frac{d}{s} = \tan^{-1}\left(\frac{0.40}{1.0}\right) = 21.8°$$

$$\therefore \quad \eta_g = 1 - \frac{21.8}{90}\left[\frac{(5-1)4 + (4-1)5}{(4)(5)}\right] = 0.624 = 62.4\%$$

(ii) Los Angeles formula: Using eqn. (12.30),

$$\eta_g = 1 - \frac{d}{\pi m n s}\left[m(n-1) + n(m-1) + \sqrt{2}(m-1)(n-1)\right]$$

$$= 1 - \frac{0.40}{\pi\,(4)\,(5)\,(1)}\,[4\,(5-1) + 5\,(4-1) + \sqrt{2\,(4-1)\,(5-1)}\,]$$

$$= 0.771 = 77.1\%$$

The lower value should be used. Hence, the capacity of the pile group

$$Q_g = (20)\,(380)\,(0.624)$$

$$= 4742.4 \text{ kN} \approx 4742 \text{ kN}.$$

Problem 12.10 It is proposed to drive a group of piles in a bed of loose sand to support a raft. The group will consist of 16 piles, each of 300 mm diameter and 12 m length. The results of standard penetration tests performed at the site at various depths are given below:

Depth (m)	2.0	4.5	7.5	9.0	12.0
Blow count	8	10	8	11	9

Estimate the capacity of the pile group, if the spacing of the piles be 1.5 m c/c.

Solution: Average N–value $= \frac{8 + 10 + 8 + 11 + 9}{5} = 9.2 \approx 9$

Using eqn. (12.25), the capacity of a driven pile,

$$Q_u = 4\,N\,A_b + 0.02\,N\,A_f$$

Here, the average value of $N = 9$

$$A_b = \frac{\pi}{4}\,(30)^2 = 706.86 \text{ cm}^2$$

$$A_f = \pi\,(30)\,(12) = 1130.97 \text{ cm}^2$$

$$\therefore \quad Q_u = (4)\,(9)\,(706.86) + (0.02)\,(9)\,(1130.97) \text{ kg}$$

$$= 25650 \text{ kg} = 25.65 \text{ t}.$$

As the spacing of piles is as high as 5 D, it can be assumed that there is no overlapping of stressed zones.

$\therefore$ Group capacity, $Q_g = n \cdot Q_u$

$$= (16)\,(25.65) \text{ t}$$

$$= 410 \text{ t}$$

Prpblem 12.11 A raft foundation has to be supported by a group of concrete piles. The gross load to be carried by the pile group is 250 t, inclusive of the weight of the pile cap. The subsoil consists of a 25 m thick stratum of normally consolidated clay having an unconfined compressive strength of 4.8

t/m^2 and an effective unit weight of 0.9 t/m^3. Design the pile group with a factor of safety of 3 against shear failure.

Solution: Let us use 16 Nos. of 400 mm ϕ R.C.C. piles in a square formation. Let the spacing s be equal to 3 d,

i.e. $$s = (3)\,(0.40) = 1.2 \text{ m}$$

Let L be the length of each pile.

Now, $$c = \frac{q_u}{2} = \frac{4.8}{2} = 2.4 \text{ t/m}^2$$

$$q_f = \alpha\,c = (0.9)\,(2.4) = 2.16 \text{ t/m}^2. \quad \text{[Assuming } \alpha = 0.90]$$

$$q_b = 9\,c = (9)\,(2.4) = 2.16 \text{ t/m}^2$$

$$A_f = \pi\,B\,L = (0.40)\,\pi\,L = 1.257\,L \text{ m}^2$$

$$A_b = \frac{\pi}{4} \cdot (0.40)^2 = 0.126 \text{ m}^2$$

$\therefore$ Capacity of each pile,

$$Q_u = (2.16)\,(1.257\,L) + (21.6)\,(0.126)$$

or, $$Q_u = 2.715\,L + 2.722$$

Safe bearing capacity of each pile,

$$Q_S = \frac{Q_u}{3} \approx 0.905\,L + 0.907$$

Actual load to be carried by each pile $= \frac{250}{16} = 15.625$ t.

$$\therefore \quad 0.905\,L + 0.907 = 15.625$$

or, $$L = 16.27 \text{ m} \approx 16.5 \text{ m}$$

Check for group action: Considering the shear failure of a block of dimension, $B \times L \times D$,

$$B = L = 3\,s + d = 3\,(1.2) + 0.4 = 4 \text{ m}$$

$$D = 16.5 \text{ m}$$

$\therefore$ Capacity of the pile group, $Q_g = (21.6)\,(4^2) + (16.5)\,(2)\,(4 + 4)\,(2.16)$

$$- (0.9)\,(16.5)\,(4^2)$$

$$= 894.24 \text{ t}$$

Safe bearing capacity of the pile group

$$Q_{sg} = \frac{894.24}{3} = 298 \text{ t} > 250 \text{ t}.$$

Hence the designed group of piles is safe from the consideration of block failure.

Problem 12.12. A raft footing founded at a depth of 1.5 m below G.L. in a 19.5 thick stratum of normally consolidated clay underlain by a dense sand layer, is to be supported by a group of 16 piles of length 12 m and diameter 400 mm arranged in a square formation. The gross load to be carried by the pile group (including the self-weight of pile cap) is 350 t. The piles are spaced at 1.2 m c/c. The water table is located at the ground level. The properties of the foundation soil are:

$w = 32\%$, $G = 2.67$, $L.L = 41\%$

Estimate the probable consolidation settlement of the pile group.

Solution: With reference to Fig. 12.4, the load from the pile group is assumed to be transmitted to the foundation soil at the lower one-third point, i.e., at a depth of $\frac{2}{3} \times 12 = 8$ m below the pile cap and $8 + 1.5 = 9.5$ m below G.L. The thickness of the clay layer undergoing consolidation settlement = 10 m. Let us divide this zone into three sub-layers of thickness 3 m, 3 m and 4 m respectively.

The settlement of each sub-layer may be obtained from:

$$\rho_c = H \cdot \frac{C_c}{1 + e_0} \cdot \log_{10} \frac{\sigma_0 + \Delta\sigma}{\sigma_0}$$

Now, we have, $wG = se$, or, $e = \frac{wG}{s}$

$$\therefore \quad e_0 = \frac{(0.32)(2.67)}{1.0} = 0.854$$

$$C_c = 0.009(LL - 10) = 0.009(41 - 10) = 0.279$$

Again, $\gamma_{sat} = \frac{G + e}{1 + e} \gamma_w = \frac{2.67 + 0.854}{1 + 0.854} (1) = 1.90 \text{ t/m}^3$

and, $\gamma_{sub} = 1.90 - 1.00 = 0.90 \text{ t/m}^3$

Settlement of the first sub-layer:

σ_0 = initial overburden pressure at the middle of the layer

$$= \gamma' z = (0.90)(1.5 + 8.0 + 3.02) = 9.9 \text{ t/m}^2$$

Dimensions of the block of piles,

$$L = B = 3s + d = 3(1.2) + 0.4 = 4 \text{ m}$$

Assuming the load to be dispersed along straight lines inclined to the horizontal at 60°, the area over which the gross load is distributed at the middle of the first layer,

$$A_1 = (L + 2H_1/2 \cdot \tan 30°)(B + 2H_1/2 \cdot \tan 30°)$$

$$= (B + H_1 \tan 30°)^2$$

$$= (4 + 3 \tan 30°)^2 = 32.86 \text{ m}^2$$

$$\therefore \quad \Delta\sigma = \frac{Q}{A_1} = \frac{350}{32.86} = 10.65 \text{ t/m}^2$$

$$\therefore \quad \rho_{c_1} = \frac{(300)(0.279)}{1 + 0.854} \cdot \log_{10} \frac{9.9 + 10.65}{9.9} = 14.32 \text{ cm.}$$

Settlement of the second sub-layer:

$$\sigma_0 = (0.90)(1.5 + 8.0 + 3.0 + 3.0/2) = 12.6 \text{ t/m}^2$$

$$A_2 = (4 + 2 \times 4.5 \times \tan 30°)^2 = 84.57 \text{ m}^2$$

$$\Delta\sigma = \frac{Q}{A_2} = \frac{350}{84.57} = 4.14 \text{ t/m}^2$$

$$\rho_{c_2} = \frac{(300)(0.279)}{1 + 0.854} \cdot \log_{10} \frac{12.6 + 4.14}{12.6} = 5.57 \text{ cm}$$

Settlement of the third sub-layer:

$$\sigma_0 = (0.90)(1.5 + 8.0 + 6.0 + 4.0/2) = 15.75 \text{ t/m}^2$$

$$A_3 = (4 + 2 \times 8 \times \tan 30°)^2 = 175.23 \text{ m}^2$$

$$\Delta\sigma = \frac{350}{175.23} = 1.997 \text{ t/m}^2$$

$$P_{c_3} = \frac{(400)(0.279)}{1 + 0.854} \cdot \log_{10} \frac{15.75 + 1.997}{15.75} = 3.12 \text{ cm}$$

$\therefore$ Total settlement, $\rho_c = \rho_{c_1} + \rho_{c_2} + \rho_{c_3}$

$$= 14.32 + 5.57 + 3.12$$

$$= 23 \text{ cm.}$$

EXERCISE 12

12.1. Determine the safe load carrying capacity of an RCC pile driven by a drop hammer weighing 3 t and having a free fall of 1.5 m, if the average penetration for the last five blows be 12 mm. [Ans. 20.3 t]

12.2. An RCC pile having a diameter of 400 mm and a length of 10 m is being driven with a drop hammer weighing 30 kN, with a height of free fall of 1.2 m. The average penetration for the last few blows has been recorded as 9 mm. If the efficiency of the hammer be 70% and the co-efficient or restitution 0.50, determine the safe load the pile can carry using modified Hiley's formula. Given, unit weight of RCC = 24 kN/m^3. Assume a factor of safety of 3.0. [Ans. 200 kN]

12.3. A 22 m long pile having a diameter of 500 mm is driven into a deep stratum of soft clay having an unconfined compressive strength of 5.6 t/m^2. Determine the static load bearing capacity of the pile with respect to a factor of safety of 2.5. [Ans. 40 t]

12.4. A concrete pile of 30 cm diameter is embedded in a stratum of soft clay having γ = 1.7 t/m^3, q_u = 4.2 t/m^2. The thickness of the clay stratum is 8 m and the pile penetrates through a distance of 1.2 m into the underlying stratum of dense sand, having γ = 1.85 t/m^3 and ϕ = 36°. Determine the safe load carrying capacity of the pile with a factor of safety of 3.

Given, δ = 0.80 ϕ and for ϕ = 36°, Vesic's bearing capacity factor N_q = 23, α = 1, K_s = 1. [Ans. 32.3 t]

12.5. A smooth steel pile of 8 m length and 400 mm diameter is driven into a cohesionless soil mass having the following properties:

$$\gamma_{sat} = 1.8 \text{ t/m}^3, \quad \phi = 30°$$

The water table is located at the ground level. If δ = 0.60 ϕ and Vesic's bearing capacity factor N_q for ϕ = 30° be 9.5, determine the safe capacity of the pile with a factor of safety of 2.5. Given, K_s = 0.7. [Ans. 12.1 t]

12.6. A 12 m long pile having a diameter of 300 mm is cast-in-situ at a site where the sub-soil consists of the following strata:

Stratum I: thickness = 5 m, γ' = 10 kN/m^3, ϕ = 30°, c = 10 kN/m^2

Stratum II: thickness = 16 m, γ' = 9 kN/m^3, ϕ = 0°, c = 60 kN/m^2

Determine the safe load on the pile with a factor of safety of 2.0. Assume reasonable values for all other data.

12.7. A 16 m long bored concrete pile having a diameter of 500 mm is embedded in a saturated stratum of sandy silt having the following properties

$$\gamma_{sat} = 19.5 \text{ kN/m}^3, \quad c = 11 \text{ kN/m}^2, \quad \phi = 20°$$

Determine the safe load carrying capacity of the pile with a factor of safety of 3.0. Given,

adhesion factor = 0.75
co-efficient of earth pressure = 0.85

friction angle = 16°
for ϕ = 20°, N_c = 26, N_q = 10, N_γ = 4. [Ans. 279 kN]

12.8. Determine the ultimate load capacity of an RCC pile of 500 mm diameter supporting the footing of a column. The sub-soil conditions are sketched in Fig. 12.5. Given,

adhesion factor for soft clay = 0.9
and that for clayey silt = 0.7

Vesic's bearing capacity factor N_q for ϕ = 30° is 9.5. The water table is located at a great depth. Skin friction in sand may be neglected. [Ans. 232 t]

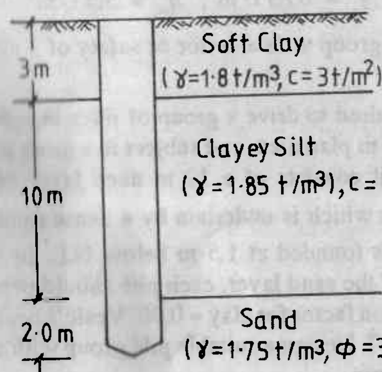


Fig. 12.5

12.9. A pile group consists of 42 piles arranged in 6 rows with a centre-to-centre spacing of 1.5 m in each direction. Each pile is 22 m long and 500 mm in diameter. Find out the group capacity of the pile using:

(i) Converse-Labarre formula
(ii) Los Angeles formula.

Given, load bearing capacity of each pile = 78 t.

[Ans. (i) 2142 t (ii) 2624 t]

12.10. A pile group consisting of 25 piles arranged in a square formation is to support a raft footing. The length and diameter of each pile are 15 m and 300 mm respectively, while their spacing is 85 cm c/c. The foundation soil is a normally consolidated clay having c = 5 t/m^2 and γ = 1.85 t/m^3. Determine the safe load bearing capacity of the pile group. Take α = 0.85 and F_s = 3.0. [Ans. 527 t]

12.11. A multistoried building is to be supported by a raft footing placed on a pile foundation. The pile group supporting the raft consists of 96 piles of 26 m length and 400 mm diameter, with a spacing of 2.0 m c/c. The water table is located near the ground surface and the properties of the foundation soil are as follows:

$$\gamma_{sat} = 2.0 \text{ t/m}^3, \quad c = 3.6 \text{ t/m}^2, \quad \phi = 0°.$$

The adhesion factor may be taken as 0.95.

Determine the capacity of the pile group with a factor of safety of 3.0.

12.12. Design a pile group to support a raft footing of 8 m × 12 m size and carrying a gross load of 760 t. The self weight of the pile cap may be assumed as 20% of the gross load on footing. The subsoil consists of a homogeneous layer of soft clay, extending to a great depth and having the following properties:

$$\gamma' = 0.85 \text{ t/m}^3, \quad q_u = 5.7 \text{ t/m}^2$$

Design the pile group with a factor of safety of 3 against shear failure. Given, $\alpha = 0.85$.

12.13. It is required to drive a group of piles in order to support a raft footing of 10 m × 10 m plan area, and subject to a gross pressure intensity of 15 t/m^2. The subsoil consists of a 12 m deep layer of soft clay ($\gamma = 1.8$ t/m^3, $q_u = 4.5$ t/m^2) which is underlain by a dense sand layer ($\gamma = 2$ t/m^3, $\phi = 35°$). The raft is founded at 1.5 m below G.L. In order to utilize the bearing resistance of the sand layer, each pile should penetrate through it at least 4D. The adhesion factor for clay = 0.90. Vesic's bearing capacity factor N_q for $\phi = 35°$ is 18.7. Design a suitable pile group with a factor of safety of 2.5 against shear failure.

Assume that the self weight of pile cap = 25% of pressure intensity on the raft.

12.14. A raft footing is founded at a depth of 3.5 m below G.L. in a 24 m thick stratum of soft clay having the following properties:

$$\gamma_{sat} = 2.05 \text{ t/m}^3, \quad C_c = 0.3$$

The gross load to be carried by the pile group, including the self weight of the pile cap, is 800 t. The group consists of 81 piles of 400 mm ϕ, arranged in a square formation, and extended to a depth of 12 m below the pile cap. The spacing of the piles is 1.25 m. The water table is located at the ground level. Compute the probable consolidation settlement of the pile group.