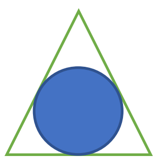
Book The House of Numbers,
GED Math test

Ricardo Giraldo C.

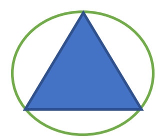
Geometric figures inscribed or circumscribed to a circumference

These are two opposing concepts that tend to confuse the student when solving geometry problems. A geometric figure is inscribed to a circumference when its vertices touch her sides, for example, the triangle ABC is inscribed on the circumference because his vertices touch points of her sides like is observed in the second example below. The opposite situation occurred when the circumference touch points on the sides of a geometric figure, the first example show this circumstance where the circumference circumscribed a triangle.
Examples:



The triangle is circumscribed to the circle

(Triangle outside …)

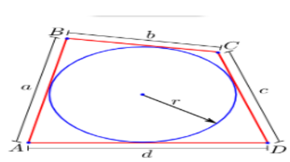


The triangle is inscribed in the circle

(Triangle inside …)

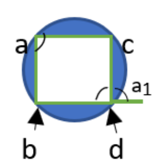
When in a test asking to determine which of these figures can be inscribed to the other, answer based on these concepts. The circle has no vertices, it cannot be inscribed to the triangle, but the circle can touch the walls of the triangle circumscribing him to her. Another example, a quadrilateral is enclosing in a circle, but its vertices do not touch a point in the circle, Is it inscribed to the circle? The answer is not for definition. In the same test presenting a problem a trapezium with one unknow measurement side = (X), you have to determine if the quadrilateral can be inscriptible, or it is circumscribed to a circumference already.

To give an answer can be based on (TPITOT) the theorem of Pitot (1695-1771) French engineer and physicist that says "A quadrilateral circumscribes a circumference if the sum of its opposite sides It's the same a + c = b + d." In this case, let assume the side AB = 7, side BC = 6, side CD = 10 and side DA = x applying equality 7 + 10 = 6 + x => x = 17 – 6 => x =11 ∴ 7 + 10 = 6 + 11 The quadrilateral is circumscribed to the circumference.



More theorems about angles in inscribed quadrilaterals (TAC)

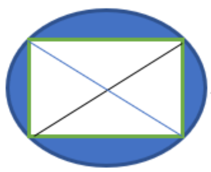
When a quadrilateral is inscribed on a circle, its opposite angles are supplementary adding to 180^0, so in the figure shown on the side $\measuredangle$ a + $\measuredangle$ d = 180^0. Another theorem, in a quadrilateral an inner angle is equal to its outer opposite angle, so $\measuredangle$ a = $\measuredangle$ a_1 therefore $\measuredangle$ a_1 + $\measuredangle$ d = 180^0 because are part of a flat angle; The same goes for the other angles inscribed on the circumference => $\measuredangle$ b + $\measuredangle$ c = 180^0. Finally, All the $\measuredangle_s$ α, ∅, β … inscribed to a circumference or a circle create an arc that measures twice the angle ($2\widehat{\alpha}$, $2\widehat{\emptyset}$, $2\widehat{\beta}$ …), for example, in B the $\measuredangle$ β has the arc $\widehat{AD}$ = 2 β ∴ in the figure, the ∑ of the arcs of opposites angles in the quadrilateral inscribed is = 360^0



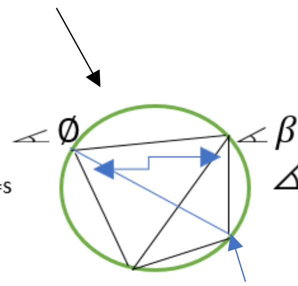
Conclusion 1: $\widehat{cb} + \widehat{bc} = 360^0$ y $\measuredangle$ a = $\measuredangle$ a_1 porque $\measuredangle$d + $\measuredangle$ a_1 = 180^0 un $\measuredangle$ llano

The diagonals in a quadrilateral inscribed divide their internal angles, accordingly the opposite sides determinate angles with equal measurement, for example, $\measuredangle\beta$ and $\measuredangle\emptyset$.

Conclusion 2:



∡s =s

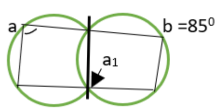


∡s different (angles can be repeated, opposite angles ∑ 180^0 ...)

∡ α opposite supplementary to ∡ Ø, ∑ 180^0

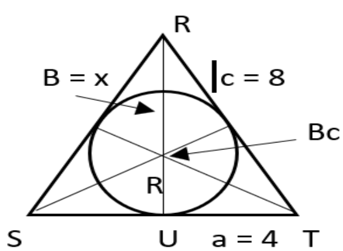
Guided practice

1. If you have a problem with two secant circles (cutting off) and have a quadrilateral inscribed among these, the problem can be solved by taking in consideration the theorems described previously, for example: find the measure of ∡ A = x knowing that ∡ B = 85^0



1. Join the point where the circles touch, divide the quadrilateral in two.
2. Applying theorems: ∡ a =∡ a_1 => ∡ a_1 + ∡ b = 180^0 opposites supplementary in a quadrilateral inscribed on a circumference ∑ 180^0
3. ∴ if ∡ a_1 + b = 180^0 = > a = 180 – 85 = 95^0 as a = a_1 => value of ∡ a = 95^0

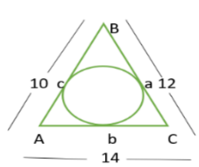
2. Find the radius (inradius) of l circle in the circumscribed equilateral triangle, with sides of 8 inches.



Solution:

1. Plot the medians of the polygon, which are cut at a point called Barycenter (Bc). By plotting the medians, the sides have are divided into equal segments.
2. The barycenter divides the median into two parts, where the smaller part is equivalent to the radius (r) of the circumference and the part greater is twice the measurement of the radius (2r).
   ∴ to find the radius, you can form the right triangle RUT with the hypotenuse c = 8in and the unknown, the opposite side RU o (b) = x.
3. Solve using the Pythagorean equation:
   C 2 = a 2 + b 2 => 8 ² = => b = = = 6.93 = 7 = UK $a^2 + b^2 => b = \sqrt{8^2 - 4^2}\sqrt{64 - 16}\sqrt{48}$
   La median is = r + 2r = 3r, ∴ r = 1/3 of b (UK) the opposite side of the right triangle.
   $\therefore \frac{1}{3} * b$ = 6.93/3 = 2. 31 The measurement of the radio. This data could be also uses to find the circle area, for example, the same circle A = π*r^2 => 3.1416 $(2.31)^2$ = 16.73 inches^2

4. In the triangle ABC circumscribed to a circle, a, b and c are the tangent points of the circumference (tangent sides AB, BC, and AC) measuring AB = 10, BC = 12, AC = 14, respectively. Calculate the measurement of segment Ba in the triangle.



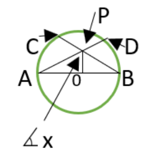
Solution:

You need to calculate the half perimeter of the triangle represented by the letter (p) or (s) (half of the perimeter) and use the PCT property concurrency of tangents that says: When two tangents are drawn from an outer point (B in this example) to the circumference, the distance between the points of tangency is always the same or congruent, thus Ab = Ac Cb = Ca and Bc = Ba. Follow these steps to find segment Ba.

1. Find the half-perimeter (p) = (a + b + c)/2 => p = (12 + 14 + 10)/2 = 18
2. Apply the property of tangents: Ab = p – side a = > 18 – 12 = 6, Bc = p – side b => 18 – 14 = 4 and Ca = p - side c = > 18 – 10 = 8 ∴ if Bc = Ba by the property of tangents => Ba = 4.

Practice:

1. Amplify the attached figure and follow the steps given to find the measurement of the ∡ APO = x, knowing that the segment $\overline{OP}$ = $\overline{PD}$, $\overline{AB}$ is the diameter and the arcs $\widehat{CD}$ = $\widehat{BD}$ are the arcs of s ∡ A and B respectively, which measure twice the value of these angles. Steps for the solution draw the picture, diameter, angles… and follow these steps:



a. On the circumference there are two $∡_s$ inscribed A and B
b. Use the vectors or segments of the angles to form geometric shapes within of the circumference that help you to identify other angles could be serve as reference to calculate the value of the ∡ APO = x, for example, when joining the dots OD form the isosceles triangles AOD, OPD... with 2 $∡_{\text{equal s}}$, or by joining the BD points forms inside the circumference the quadrilateral POBD as well.

c. Apply theorems about the angles inscribed in a quadrilateral, for example, the ∑ of two opposite internal angles is = 180^0, and begin to identify and angle of $90∡^0$ taking as antecedent the measure of its arc in the circumference....

d. Draw the diagonals of the quadrilateral and apply the properties of its angles (opposite, external...) to identify similarity and differences between angles, you could use Greek letters α, $\emptyset, \beta$ ... to identify each angle. Finally, you can see could form the right triangle ADB and taking the results on the angles that correspond to the other figures contrasts them, this will allow you to arrive at the equation $\theta + 2\theta + 90 = 180$ and find the value of the angle $∡_{\text{apo}} = x = 60^0$.

2. Calculate the radius and the area of a circumference inscribed on an isosceles trapezoid (two equal sides) whose bases They measure 20 cm. and 40 cm. Steps to follow draw the figure, identify the parts, their measurements and the Unknowns. Find the equal sides using Pitot's theorem and some artifice such as introducing another or other geometric figures inside the trapezoid, for example, a quadrilateral, a triangle, for Calculate the value of the radius = x the unknown. (answer the sides 30 cm and radius 14.2 cm)

3. In a triangle ABC circumscribed to a circumference, the tangent points of the circumference are a, b, and c corresponding to tangent sides measuring AB = 10, BC = 12, AC = 14, respectively. You were asking to calculate the measurement of segment Ab. (answer the segment measure 6 cm)

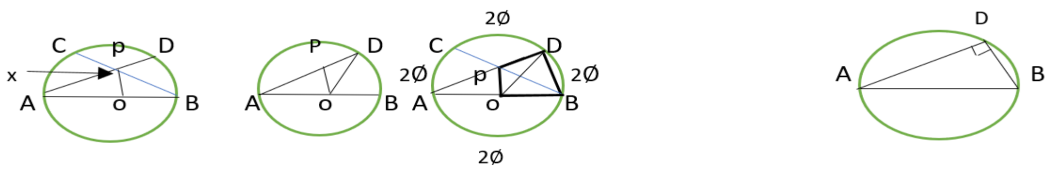
4. In a circumscribed equilateral triangle with sides of 8 inches. Find the inradius and the area of a circle. (the answer is r = 2. 31 cm and the area A =16.73 inches^2).

5. Two secant circles (cutting off each other) having the quadrilateral ABCD inscribed among these. Draw the picture and find the measure of ∡ α in A = x knowing that ∡ β in B = 74^0.

6. Determine whether or not the quadrilateral with sides measure of side AB = 7, side BC = 6, side CD = 10 and side DA = (X) can be circumscribed to a circle (Apply the Pitot’s theorem to find the answer)

Solution to the practice, problems one and two

1. Graphing and analyzing:



∡ apo = x

2△ ∡s a, d, o = ∅

1▱ OBDP ∡s cbd y obc = $\frac{2\emptyset}{2} = \emptyset$

∴ ∡ pod = ∅ y ∡s opuestos d + o = 180°

T∡s ▱ (TAC)

1 ◺ => ∡ adb = 1/2 $\overset{\frown}{AB}$ = 180/2 =

∡ adb = 90^0 , ∡ dab = ∅ (∡ △)

∡ abd = 2∅ (Σ▱ ∡s cbd + obc)

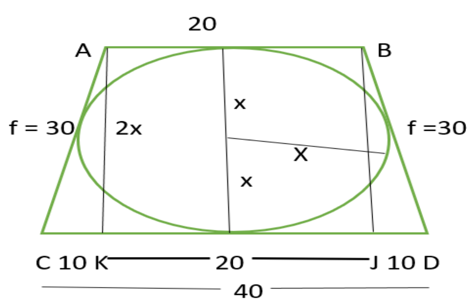
$\therefore\ \emptyset + 2\emptyset + 90 = 180^0$

$\emptyset = \frac{180-90}{3} = 30^0$

Conclusion: taking the third circle, in the quadrilateral the obd = ∡Σ of angles cbd + obc
2∅ = 60^0 equivalent to ∡ abd = 2 =∅ 60° in the right triangle on the fourth circle.

Finally, with this information return to the third circumference and apply the MAC postulate that says in a quadrilateral the measure of the internal angle is equal to the external angle, in this case ∡ obd = apo = x = ∡ > x on the first circle = ∡ 60^0.

2. Graphing:



Analyzing:

a) T-PITOT: f + f = 20 + 40
2 f = 60 => f = 60/2 = 30 = f

b) Plot circumference diameter= 2x or 2 radii

c) Form the AKJB rectangle and are on the sides two right triangles with an unknown 2x the side opposite to hypotenuse, using the formula of Pythagoras will find the value of x.

∴ 30 2 = 10 2 + $(2x)^2$ => $2x = \sqrt{30^2 - 10^2}$

=> 2x = => 2x = 28.3 $\sqrt{800}$ x = 28.3/2

Answer: The radius measures 14.2 cm

The rest of the answers are between examples and guide practices developed in this document.