The Radicals

Sums, subtractions, multiplication and division with radicals

Guided Practice From the book THE HOUSE OF THE NUMBERS, A study guide to prepare for the math test included in the GED, HiSET or TASC exam.
CDEC – Chamber of Studies

I. Similar radicals

Radicals with the same indexes, as seen in the equation: a $\sqrt[n]{R}$ + b = $\sqrt[n]{R}$ (a + b)$\sqrt[n]{R}$

Examples:

1. $2\sqrt[3]{9} + 4\sqrt[3]{9} = 6\sqrt[3]{9}$
2. $5\sqrt[5]{20} - 3\sqrt[5]{20} + 2\sqrt[5]{20}$ => $5\sqrt[5]{20}$ - 3 + 2 = (5-3+2)$\sqrt[5]{20}$ = $4\sqrt[5]{20}$
3. $\sqrt[4]{4} + \sqrt[6]{8} - \sqrt[12]{64} = \sqrt[4]{2^2} + \sqrt[6]{2^3} - \sqrt[12]{2^6} =$

   Steps to make the indexes homogeneous:

   a) Find factors of each radicand to find equivalence to them (4, 8, and 64)

   => 4 = 2*2 = $\mathbf{2^2}$, 8 = 2*2*2= $\mathbf{2^3}$, 64 = 2*2*2*2*2*2 = $\mathbf{2^6}$ replace radicands

   $\sqrt[4]{4} + \sqrt[6]{8} - \sqrt[12]{64} = \sqrt[4]{2^2} + \sqrt[6]{2^3} - \sqrt[12]{2^6} =$

   b) Divide the indexes between the exponents o powers found: 4/2 = 2, 6/3 = 2, 12/6 = 2 to make the radicals homogeneous => $=\sqrt{2} + \sqrt{2} - \sqrt{2} = \sqrt{2}$

   c) Add and subtract 2 + 2 – 2 = 2 radicands, the answer is $\sqrt{2}$

4. $\sqrt{27} + 3\sqrt{3} - 2\sqrt{75} =$

   a) Find the factors of the radicand $\sqrt{27} = \sqrt{3^2 * 3} = 3\sqrt{3}$

   b) Replace in equation = > $3\sqrt{3}$ + $3\sqrt{3}$ - $2\sqrt{75}$ =

   c) Find the factors of the radicand $(2\sqrt{75}) = 2\sqrt{5^2 * 3} = 2 * 5\sqrt{3} = 10\sqrt{3}$ and

   d) Replace in equation = > $3\sqrt{3}$ + $3\sqrt{3}$ - $10\sqrt{3}$ = $6\sqrt{3}$ - $10\sqrt{3}$ = The answer = $4\sqrt{3}$

II. Multiplication of radicals with the same index: $\sqrt{a}.\sqrt{b} = \sqrt{a\ x\ b}$

When you must solve an exercise where the radicals have the same index, simply multiply the radicands and you will get the answer.

1. $\sqrt{3}.\sqrt{9} = \sqrt{3x9} = \sqrt{27} = \sqrt{3^2x3} = 3\sqrt{3}$ 2. $\sqrt{25}.\ \sqrt{9} = \sqrt{5^2}\ .\ \sqrt{3^2} = 5*3 = 15$

3. $\sqrt[3]{27} . \sqrt[3]{3375} = \sqrt[3]{3^3} . \sqrt[3]{15^3} = 3*15 = 45$

III. Multiplication of radicals with different indices

Here are three alternatives for finding the answers in radicals multiplication

Examples:

1. $\sqrt{25} . \sqrt[3]{9} . \sqrt[4]{27} = (\sqrt{5^2})(\sqrt[3]{3^2})(\sqrt[4]{3^3}) =$

a) Find the mcm between the indexes of the radicals (2, 3, 4) mcm = 12

2 3 4 | 2 b) Divide the mcm with each index (12/2 = 6), (12/3 = 4), (12/4 = 3)

1 3 2 | 2 c) Replace the index in the radical by 12, factor the radicands

1 3 1 | 3 and multiply each exponent by the values found 6, 4, and 3.

1 1 1 |

b) Replacing: $\sqrt{25} . \sqrt[3]{9} . \sqrt[4]{27} = \sqrt[12]{(5^2)^6} \text{ x } \sqrt[12]{(3^2)^4} \text{ x } \sqrt[12]{(3^3)^3} =$

solving $= \sqrt[12]{5^{12}} \text{ x } \sqrt[12]{3^8} \text{ x } \sqrt[12]{3^9} = 5\sqrt[12]{3^8 . 3^9}$

$= 5\sqrt[12]{3^{17}} = 5.3\sqrt[12]{3^5} = 15\sqrt[12]{3^5} = 15\sqrt[12]{243}$

c) If $\sqrt[12]{243} = 1.580521...$. In decimal format $15\sqrt[12]{243} = 15 \text{ x } 1.580521$
$= 23.70782876 = 15\sqrt[12]{243}$

2. $\sqrt{25} . \sqrt[3]{9}$

a) Find the mcm of the indices (2, 3) = 6

b) Replace the index on each radical by six and factor the radicands

c) Divide mcm between each index: 6/2 and 6/3 result (3, 2)

d) Multiply (3, 2) respectively for each power of the radicand

So, $\sqrt{25} . \sqrt[3]{9} = \sqrt[6]{5^2 . 3^2} = \sqrt[6]{5^6 . 3^4} = 5\sqrt[6]{3^4} = (5)(3^{4/6 = 2/3}) = (5)\sqrt[3]{3^2} = 5\sqrt[3]{9}$

3. $\sqrt[4]{4^2} . \sqrt[6]{8^2}$ => $4^{2/4} . 8^{2/6} = 4^{1/2} . 8^{1/3} = \sqrt{2^2} . \sqrt[3]{2^3} = 2*2 = 4$

IV. **Division of radicals** $\sqrt[n]{a/b}$ **root of a root** $\sqrt[n]{\sqrt[n]{a}}$ **or** $\sqrt[n]{\sqrt[n]{a}/b}$

1. $\sqrt{\frac{2^2}{5^2}} = \sqrt{4/25} = 2/5$ 2. $\sqrt{\sqrt{81}} = \sqrt[4]{81} = \sqrt[4]{3^4} = 3$