Mathematical operations with matrices

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In the seventh chapter of the book **The House of Numbers** is studied n different topics d algebra, for example, the linear equations used to describe the relationship between two or more variables, which are represented by complex numbers (a b, a_{i+d}, x, y-n , and $_{mn...}$) and can be plotted in the Cartesian coordinate system. These equations are used in almost all sciences, especially for projections and other quantifications. In this new post on our blog, I will introduce the matrix which is another important tool of linear algebra.

A matrix is a two-dimensional table in which real numbers are presented (positive, negative, roots, coefficients of a system of equations...) sorted into rows and columns, which can be added, subtract and multiply.

$$\begin{cases} 2x + 5y = 12 \\ 4x - 6y = 2 \end{cases} \Rightarrow \begin{pmatrix} 2 & 5 & 12 \\ 4 & -6 & 2 \end{pmatrix} \quad \textbf{Matrix}$$

Matrices are used to describe systems of equations as in the previous example, as well as record data or elements, which are presented in vertical lines or columns and rows or lines horizontal lines, known as matrix dimensions. Each of these elements occupies a position in a row and in a column, so each element is represented by carrying two subscripts, for example, to rs (**a** is the element, the subscript **rs** that indicate the row and the column where the element is located).

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad (a_{rs} \quad a_{rs})$$

Rows 2 = m =2

Columns 2 = n = 2

Called matrix m*n = 2 x 2

Where: $a_{rs} = a_{11}, a_{12...}$ The elements

Examples:

$$B = \begin{pmatrix} 4 & 2 \\ 3 & 1 \end{pmatrix}$$

(4 is the element b_{11} y 2= $b_{12, ...}$) matrix 2x2

$$C = \begin{pmatrix} -4 & \sqrt{9} & 7 \\ 0 & 3 & 1 \end{pmatrix}$$

matrix m*n = 2x3

3 = element c_{22}

$$D = \begin{bmatrix} -2 & 8 & 4 \\ 5 & 3 & 1 \\ 0 & \sqrt{2} & 1/2 \\ -6 & 1 & 0 \end{bmatrix}$$

matrix 4 rows x 3 columns

-6 = element d_{41}

Types of matricess

Null matrix E = [all elements en the colunnas or rows are zeros]

Matrix row F = [has a single row] matrix 1xn F = (3 -5 1 7) 1x4

Colunna matrix G = [has a single colunna] matrix mx1 $G = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$ 2x1

Matrix row column H [a single element fulfills both functions] H = (-8) 1x1

Square matrix B = [the number of rows and colunnas is el same] matrix mxn example matrix B above

Rectangle matrix D = [the number of rows is different from the number of colunnas] matrix nxn, example matrix D and C

Principal diagonal (DP) of a matrix A **Trace of the matrix = ∑ of elements of the DP**

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$ **DP =** A_{11}, a_{22}, a_{33} **Trace (A) =** $a_{11} + a_{22} + a_{33}$

Secondary diagonal (DS) of a matrix A **DS** = a_{31}, a_{22}, a_{13} **Trace** = $a_{31} + a_{22} + a_{13}$

Top triangular matrix [all elements above DP are zeros] **In A** to 12, to **13**, **to23** **are zeros**
Lower triangular matrix [all elements below DP are zeros] **In A** a_{21}, a **31 and a 32** **are zeros**
Diagonal matrix (has elements outside the DP above and below that are zeros) **∴ is symmetric** because It contains an upper and lower triangular matrix inside. **DP =** a_{11}, a_{22}, a_{33} = 1, **3, 5**, the rest **zeros Unit matrix or identity (In)** the DP has all the elements = 1 the rest = 0, e.g. **in A:** to 11, to 22, **to33...** **= 1, 1, 1**
Transposed matrix A^T (The elements of matrix A are rearranged in such a way that the elements in the rows pass to the columns and the elements in the columns to the rows. When changes occur, the elements are presented in a new matrix with a superscript T indicating that they have the same elements and is denoted A^T, but the rows s of A are the columns s and the columns are the rows of the other matrix). Ex. matrix A = (2 -3 5) m x n = 1x3 and matrix B **m x n = 3x1** has the same elements in a column, **∴** B is matrix ^{AT} transposed or inverse of A. **.**

You can find other types of matrices that carry combined characteristics of the above line matrices such as conjugate, orthogonal, anti-Hermitian, adjoint or cofactor matrix among others that I will only mention so as not to extend this publication.

When working with matrices, we generally look for the ***determinant*** which is a number that relates all the elements in a matrix with the purpose of determining or indicating the sum of the possible size permutations of the scalar (increase or decrease) of the matrix. In a coordinate pair (example 3) the determinant refers to how much the base formed in a plane by the segments (x, y) has been compressed or stretched.

The degree of difficulty to obtain the determinant depends on the length of the matrix, it will be easy in a matrix of short length or order, for example 2 x 2 (two rows and two columns), but it will be complicated as the matrix has more rows and columns (examples 1 and 2).

The determinant "det(A) or | A|" it is obtained by multiplying the elements of the DP and subtracting them from the result of the multiplication of the elements of the D S. Below is the arrangement of the elements to find the det(A):

Square matrix 2 x 2

$$\text{Det}(A) = |A| = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} = a_{11} \times a_{22} - a_{12} \times a_{21}$$

Example 1: Find the det (A) and det (B) of these matrices of order 2x2.

$$A = \begin{pmatrix} 4 & -2 \\ 3 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -5 & -3 \\ 2 & -1 \end{pmatrix} \longrightarrow$$

A = 4 x 1 – (-2) x 3 = 4 + 6 = 10 => det(A) o |A|

B = -5 x -1 – 2 x -3 = 5 – (2 x -3) = 5 + 6 = 11 => det(B) o |B|

If you want to find the determinant of a matrix of order 3. i.e. 3 x 3 apply this equation:

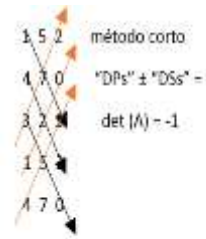
Example 2:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

det (A) = $a_{11} * a_{22} * a_{33} + a_{12}*a_{23}*a_{13} + a_{13}*a_{21}*a_{32} - a_{13}*a_{21}*a_{31}$

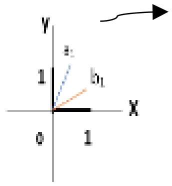
$$A = \begin{pmatrix} 1 & 5 & 2 \\ 4 & 7 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

det (A) = 1*7*1 + 5*0*2 + 2*4*2 – 2*4*3 = - 1



Operation on a 3x3 array can be simplified by typing the first two rows at the end To find their determinant, then multiply the elements diagonally towards down with positive sign (as if you had 3 DP and add them considering the sign of the element), then multiply from bottom to top with negative sign (as if you had 3 DS and subtracts them), finally adds and subtracts the found values.

Ejemplo 3:



Matrix R = $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ Y x

El det (R) = 1. If det decreases to 0.5 the position of segment a changes to $_1$ and **b** a **b_1** the area between the linear segments is compressed, what remains determined by multiplying det (R)*the initial area, ∴ as the DET value approaches 0 area ab may disappear at fully compress or convert a and b into a line, making each multiple or dependent segment of the other. If the det (R) is negative I would change the orientation of the area between segments A and B by moving it to another quadrant in the coordinates.

Matrix addition, subtraction, and multiplication operations

Sums: Only those with the same disposition are added **m x n** ∴ **(A + B) = m x n**

$$** \begin{bmatrix} 1 & 3 & 2 \\ 1 & 0 & 0 \\ 1 & 2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 5 \\ 7 & 5 & 0 \\ 2 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+0 & 3+0 & 2+5 \\ 1+7 & 0+5 & 0+0 \\ 1+2 & 2+1 & 2+1 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 7 \\ 8 & 5 & 0 \\ 3 & 3 & 3 \end{bmatrix}$$

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} + B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & a_{13}+b_{13} \\ a_{21}+b_{21} & a_{22}+b_{22} & a_{23}+b_{23} \\ a_{31}+b_{31} & a_{31}+b_{32} & a_{33}+b_{33} \end{pmatrix} = \begin{pmatrix} 1+0 & 3+0 & 2+5 \\ 1+7 & 0+5 & 0+0 \\ 1+2 & 2+1 & 2+1 \end{pmatrix} =$$

3 x 3 **3 x 3**

Subtraction: The process is similar to that of the sum the matrices must present the same arrangement m x n, ∴ we proceed to the subtraction of their elements **(A - B).**

** Subtracting the elements of matrices A and B above:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} - B = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} = \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & a_{13} - b_{13} \\ a_{21} - b_{21} & a_{22} - b_{22} & a_{23} - b_{23} \\ a_{31} - b_{31} & a_{31} - b_{32} & a_{33} - b_{33} \end{pmatrix} \quad A - B = \begin{pmatrix} 1-0 & 3-0 & 2-5 \\ 1-7 & 0-5 & 0-0 \\ 1-2 & 2-1 & 2-1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & -3 \\ -6 & -5 & 0 \\ -1 & 1 & 1 \end{pmatrix}$$

3 x 3 **3 x 3**

Multiplication:

A. The multiplication of matrices (A),(B),(C), for example: A(B + C) = AB + AC and if you multiply (AB)C ≠ (BC)
No commutative law. When a matrix multiplies another matrix (k B) that is multiplied by a number k, for example, A(k B) = k(AB) they multiply when sums and outputs can be realized.
If multiplication of A*B ∴ occurs, the transposed matrices can be multiplied $(AB)^T = A^T \times B^T$

B. By multiplying a matrix by a scalar, for example a real number (J) which is a constant. The product will be another matrix of equal order: matrix z A = (a_{rs})x J = a matrix of = order. In this case, multiply the first row by the constant and then the second row by J to get the new matrix.
Example:

$$B = \begin{pmatrix} 3 & 6 & -2 \\ 7 & -4 & 5 \end{pmatrix} \times 5 = \begin{pmatrix} 15 & 30 & -10 \\ 35 & -20 & 25 \end{pmatrix}$$

Can j be a fraction?
Yes, J =1/3 The product shall be: $\begin{pmatrix} 1 & 2 & -2/3 \\ 2\,1/3 & -4/3 & 1\,2/3 \end{pmatrix}$ matrix 2x3

C. To multiply an array by another array, the condition is that the number of columns in the first array must be equal to the number of rows in the second array. The multiplication begins with the elements of (A) in the first row by the elements of (B) in the first column: $a_{11} \times b_{11}$, $a_{12} \times b_{21}$, $a_{13} \times b_{31}$, $a_{14} \times b_{41}$ and continues, the elements of (A) in the second row by those of the second column in (B) to obtain the product of multiplication that will be the matrix (C).

Example: $A = \begin{pmatrix} 3 & 6 & -2 & 4 \\ 7 & -4 & 5 & -1 \end{pmatrix} \times B = \begin{pmatrix} 1 & 2 & 3 \\ 1 & 0 & 0 \\ 2 & -3 & 8 \\ 1 & 4 & -6 \end{pmatrix}$ =>

2x4 4 x3
m x **n** **m** x n

Multiplication process to find the elements of matrix C:

C_{11} = 3*1 + 6*1 + -2*2 + 4*1 = 9
C_{12} = 3*2 + 6*0 + -2*-3 + 4*4 = 28
C_{13} = 3*3 + 6*0 + -2*8 + 4*-6 = -31
C_{21} = 7*1 + -4*1 + 5*2 + -1*1 = 12
C_{22} = 7*2 + -4*0 + 5*-3 + -1*4 = - 5
C_{23} = 7*3 + -4*0 + 5*8 + -1*-6 = 67

$$C = \begin{pmatrix} 9 & 28 & 31 \\ 12 & -5 & 67 \end{pmatrix}$$

A X B = C (OK)
B X A = C (Commutative law does not apply to matrix multiplication)

Conclusions:

* The product of multiplying two square matrices D x EA is another square matrix.
* When you multiply the matrices A x B of order 3x3 the product will be a matrix of 3 x 3.
* If you swap the position of rows or columns in a matrix with each other, you will have another equivalent matrix.
* When multiplying an identity matrix, A by another matrix B, the product will be the same matrix B.
* If you multiply a matrix A by its inverse $^{A-1}$ the product will be the neutral element or identity matrix.
* Matrices cannot be divided e.g., Matrix A/Matrix B only add, subtract and multiply.

* A determinant will be zero if the matrix has a row or column with zeros
* When the elements of a row or column matrix are multiplied by an R number ∴ | A|*R
  and if a pair of rows or columns in an array are swapped, the det(A) changes the sign.
* In a triangular matrix the det(A) is the product of its diagonals
* Removing a column or row from an array creates a new det(A) with the elements that
  They remained and this takes the name of minor determinant.

Remember when clearing an unknown in a matrix equation, e.g. A*x = B to find the value of x no you can divide A/B and say x= A/B but you could use the inverse matrix of $^{A-1}$ on both sides of the equation (with det ≠0), so on the one hand eliminates the matrix A: A^{-1} *A what is made equal to the identity ∴ x = A^{-1}*B.

Practice:

1. Find the determinant of a 3x3 matrix with rows (1 5 2)(470)(321)
2. In a matrix All elements of a row or column are zeros How much is the determinant worth?
3. Use a short method or trick to find the det of a 3x3 matrix with (5 3 -3), (3 -1 0) and (4 2 -3). The answer the determinant must be equal to 12.
4. multiply matrix A of order mx1 that has a column (-1 0 1) and matrix B of order 1xn that has a row (1 1 1). Is the answer zero? V F
5. Create and multiply two matrices A of order 3x2 and B of order 2x3, the matrices must contain two elements = 0.5 located between rows and in different columns (one decimal is positive and another has a negative sign). Is the answer a 3x3 order matrix with zero elements? V F
6. The commutative law does not apply to matrix multiplication if you multiply a 2x3 matrix and another of 3x2 the product will be a matrix of a) 3x3 b) 2x2 (Try creating a matrix exercise).
7. To add or subtract two matrices which condition must be met:
   (a) The two operations must have only positive elements
   (b) The two operations must have the same arrangement m x n
   c) The two operations can be performed with matrices of different order or arrangement.