Stanisław Leśniewski

On the Foundations of Mathematics, Chapter IV

Source: Mereology OPM Notebook 1, pp. 1–28. Original in the Archives of the University of Notre Dame. Most likely this work was done between 1946 and 1949 when Sobociński was in Brussells.

Original publication: "Rozdział IV: O "Podstawach ogólney teoryi mnogości. I."," (Chapter IV: On foundations of the general theory of sets), *Przegląd Filozoficzny*, 31 (1928), 261–291.

English translation: *Stanisław Leśniewski: Collected Works* (1992), pp. 227–263.

This chapter is entitled "On "Foundations of the General Theory of Sets. I".

Leśniewski published a long paper entitled "O podstawach matematyki" (On the foundations of mathematics, briefly OPM), in XI chapters, containing several versions of his Mereology. At the time of publication, Leśniewski was not comfortable with formal notation and so expressed the theses and derivations in ordinary (Polish) language. In this notebook Sobociński has translated this material into symbolic form. Such a translation has never been published.

This is Leśniewski's first development of Mereology. It is based on *part* as a primitive term, with two axioms, two definitions, and then two more axioms. There are 48 theorems.

Mereologja

System Leśniewskiego (Przegląd Filozoficzny;
Rocznik 31, zeszyt III. Rok 1928)
str. 263 i nast. Rozdział IV.

Akzjomatyka:

I. $[P\theta]: P \varepsilon cz(\theta) . \supset . \theta \varepsilon \sim(cz(P))$

II. $[P\theta R]: P \varepsilon cz(\theta) . \theta \varepsilon cz(R) . \supset . P \varepsilon cz(R)$

D1. $[P\theta] \therefore P \varepsilon el(\theta) . \equiv : P = \theta . \vee . P \varepsilon cz(\theta)$ *)

D2. $[Pa] \therefore P \varepsilon Kl(a) . \equiv : P \varepsilon P . a \subset el(P) : [\cancel{R}\theta]:$
$\theta \varepsilon el(P) . \cancel{R \varepsilon el(\theta)} . \supset . [\exists RS] . S \varepsilon a . R \varepsilon el(S) . R \varepsilon el(\theta)$

III $[P\theta a]: P \varepsilon Kl(a) . \theta \varepsilon Kl(a) . \supset . P = \theta$

*) „$P \varepsilon el(\theta)$" w oryginale „$P \varepsilon$ ingrediensem(θ)"

Works, pp 227–263.
PF 31 (1928), 261–291.

CZ = CZĘŚCIĄ = part.

OPM Chapter IV

* The original, p 264 has ingredient! Sobo has updated it to "element", when did this change take place.

$\underline{IV}\quad [Sa] : S \varepsilon a . \supset . [\exists \Theta] . \Theta \varepsilon Kl(a)$

Twierdzenia

$\underline{T. 1.}\quad [S] : S \varepsilon V . \supset . S \varepsilon \sim(cz(S))$

Dem.:

$[S] :$

1) $S \varepsilon V . \supset .$

$S \varepsilon \sim(cz(S)) \qquad (I, \Theta | S)$

$T. 2.\quad [S] : S \varepsilon V . \supset . S \varepsilon el(S) \qquad (D1)$

$\underline{T. 3}\quad [S\Theta R] : R \varepsilon cz(S) . \Theta \varepsilon el(S) . \supset . [\exists s T] . s \varepsilon el(\Theta) . T \varepsilon cz(S) . s \varepsilon el(T)$

Dem.:

$[S\Theta R] :.$

1) $R \varepsilon cz(S) .$

2) $\Theta \varepsilon el(S) . \supset :$

3) $\Theta = S . \vee . \Theta \varepsilon cz(S) : \qquad (D1, 2)$

4) $K \varepsilon el(B).$ $(D1, 1)$

5) $K \varepsilon V.$ (4)

6) $K \varepsilon el(K).$ $(T2, 5)$

~~[illegible]~~

7) $\theta \varepsilon el(\theta).$ $(T2, 2)$

$[AT].$

8) $A \varepsilon el(B) . T \varepsilon \omega(B) . A \varepsilon el(T)$ $(4, 1, 6)$

$[AT] . A \varepsilon el(\theta) . T \varepsilon \omega(B) . A \varepsilon el(T)$ $(3; 8, 7)$

$\vdash T4\ [B\theta K] : B \varepsilon el(\theta) . \theta \varepsilon el(K) . \supset . B \varepsilon el(K)$

Dem.:

$[B\theta K] ::$

1) $B \varepsilon el(\theta).$

2) $\theta \varepsilon el(K) . \supset :$

3) $B = \theta . \vee . B \varepsilon \omega(\theta) :$ $(D1, 1)$

4) $\theta = K . \vee . \theta \varepsilon \omega(K) :$ $(D1, 2)$

5) $B = K . \vee . B \varepsilon \omega(K) :$ $(3; 4; \mathrm{II})$

$B \varepsilon el(K)$ $(D1, 5)$

– T5. $[\beta, \theta, \kappa] :: [\sigma] : \sigma \varepsilon el(\beta) . \supset . [\exists \tau] . \tau \varepsilon el(\sigma) . \tau \varepsilon el(\kappa) :$
$\theta \varepsilon el(\beta) : \supset . [\exists \upsilon \chi] . \upsilon \varepsilon el(\theta) . \chi \varepsilon el(\kappa) . \chi \varepsilon el(\beta) . \upsilon \varepsilon el(\chi)$

Dem.:

$[\beta \theta \kappa] ::$

1) $[\sigma] : \sigma \varepsilon el(\beta) . \supset . [\exists \tau] . \tau \varepsilon el(\sigma) . \tau \varepsilon el(\kappa) :$

2) $\theta \varepsilon el(\beta) : \supset .$

~~3~~ $[\exists \tau] .$

3) $\tau \varepsilon el(\theta) .$ $\Big\}$ $(1, 2)$

4) $\tau \varepsilon el(\kappa) .$

5) $\tau \varepsilon el(\beta) .$ $(T4, 3, 2)$

6) $\tau \varepsilon el(\tau) .$ $(T2, 3)$

$[\upsilon \chi] . \upsilon \varepsilon el(\theta) . \chi \varepsilon el(\kappa) . \chi \varepsilon el(\beta) . \upsilon \varepsilon el(\chi)$
$(3, 4, 5, 6)$

– T6 $[\beta \theta a] : \beta \varepsilon Kl(a) . \theta \varepsilon el(\beta) . \supset . [\exists \kappa] . \kappa \varepsilon el(\theta) . \sigma \varepsilon a . \sigma \varepsilon el(\beta) .$
$\kappa \varepsilon el(\sigma)$

[Dem.:

$[\beta \theta a] ::$

1) $B \varepsilon Kl(a)$.

2) $\theta \varepsilon el(B) . \supset :$

~~$[\exists k \theta]$~~

3) $[\theta] : \theta \varepsilon el(B) . \supset . [\exists k s] . s \varepsilon a . k \varepsilon el(s) . k \varepsilon el(\theta) :$ (D2, 1)

4) $a \sqsubset el(B) :$ (D2, 1)

$[\exists k s]$.

5) $s \varepsilon a$.

6) $k \varepsilon el(s)$. $\Bigg\}$ (3, 2)

7) $k \varepsilon el(\theta)$.

8) $! \{a\}$. (5)

9) $s \varepsilon el(B) :$ (4, 8, 5)

$[\exists k s] . k \varepsilon el(\theta) . s \varepsilon a . s \varepsilon el(B) . k \varepsilon el(s)$ (7, 5, 9, 6)

– T7 $[B] : B \varepsilon V . \supset . B \varepsilon Kl(el(B))$

Dem.:

$[B] ::$

1) $B \varepsilon V . \supset .$

2) $B \varepsilon el(B)$. (T2, 1)

3) $! \{el(B)\}$. (2)

4) $el(\mathcal{B}) \sqsubset el(\mathcal{B}):$ (3)

5) $[\Theta]:\Theta\, \varepsilon\, el(\mathcal{B}) . \supset . \Theta\, \varepsilon\, el(\Theta) . \Theta\, \varepsilon\, el(\mathcal{B}):$ (T2)

6) $[\Theta]:\Theta\, \varepsilon\, el(\mathcal{B}) . \supset . [\exists K J] . K\, \varepsilon\, el(\Theta) . J\, \varepsilon\, el(\mathcal{B}) . K\, \varepsilon\, el(J):$ (5)

$\mathcal{B}\, \varepsilon\, Kl(el(\mathcal{B}))$ (D2, 1, 4, 6)

$\vdash$ T8 $[\mathcal{B}]:\mathcal{B}\, \varepsilon\, V . \supset . \mathcal{B}\, \varepsilon\, Kl(\mathcal{B})$

Dem.:

$[\mathcal{B}]::$

1) $\mathcal{B}\, \varepsilon\, V . \supset .$

2) $\mathcal{B}\, \varepsilon\, \mathcal{B} .$ (1)

3) $\mathcal{B}\, \varepsilon\, el(\mathcal{B}):$ (T2, 1)

4) $[\Theta]:\Theta\, \varepsilon\, \mathcal{B} . \supset . \Theta\, \varepsilon\, el(\mathcal{B}):$ (3)

5) $\mathcal{B} \sqsubset el(\mathcal{B}):$ (4, 2)

6) $[\Theta]:\Theta\, \varepsilon\, el(\mathcal{B}) . \supset . \Theta\, \varepsilon\, el(\Theta) . \Theta\, \varepsilon\, el(\mathcal{B}):$ (T2)

7) $[\Theta]:\Theta\, \varepsilon\, el(\mathcal{B}) . \supset . [\exists K J] . K\, \varepsilon\, el(\Theta) . J\, \varepsilon\, \mathcal{B} . K\, \varepsilon\, el(\mathcal{B}):$ (6, 2)

$\mathcal{B}\, \varepsilon\, Kl(\mathcal{B})$ (1, 5, 7)

$-\mathrm{T}9\ [B\theta]:\theta\,\varepsilon\,\cancel{\mathrm{el}}\mathrm{cz}(B)\,.\supset.\,B\,\varepsilon\,\mathrm{Kl}(\mathrm{cz}(B))$

Dem.:

$[B\theta]::$

1) $\theta\,\varepsilon\,\mathrm{cz}(B).\supset.$

2) $B\,\varepsilon\,V.$ $(\mathrm{AI},1)$

3) $!\{\mathrm{cz}(B)\}.$ (1)

4) $\mathrm{cz}(B)\sqsubset \mathrm{el}(B):$ $(\mathrm{D}1,1,3)$

5) $[\theta]:\theta\,\varepsilon\,\mathrm{el}(B).\supset.[\exists\kappa\iota].\kappa\,\varepsilon\,\mathrm{el}(\theta).\iota\,\varepsilon\,\mathrm{cz}(B).\kappa\,\varepsilon\,\mathrm{el}(\iota):$ $(\mathrm{T}3,1)$

$B\,\varepsilon\,\mathrm{Kl}(\mathrm{cz}(B))$ $(\mathrm{D}2,2,4,5)$

$-\mathrm{T}10\ [B\theta]:B\,\varepsilon\,\mathrm{el}(\theta).\supset.\theta\,\varepsilon\,\mathrm{Kl}(\mathrm{el}(\theta))$

Dem.:

$[B\theta]::$

1) $B\,\varepsilon\,\mathrm{el}(\theta).\supset:$

2) $B=\theta.\vee.B\,\varepsilon\,\mathrm{cz}(\theta):$ $(\mathrm{D}1,1)$

3) $\theta\,\varepsilon\,V.$ $(\mathrm{AI},2)$

$\theta\,\varepsilon\,\mathrm{Kl}(\mathrm{el}(\theta))$ $(\mathrm{T}7,3)$

D3 $[Pa] :. P \varepsilon zb(a) . \equiv . P \varepsilon V : [\Theta] : \Theta \varepsilon el(P) . \supset . [\exists K S] .$
$K \varepsilon el(\Theta) . S \varepsilon a . S \varepsilon el(P) . K \varepsilon el(S)$

T11 $[P \Theta a b] : P \varepsilon zb(a) . a \subset b . \Theta \varepsilon el(P) . \supset . [\exists K S] . K \varepsilon el(\Theta) .$
$S \varepsilon b . S \varepsilon el(P) . K \varepsilon el(S)$

Dem.:

$[P \Theta a b] :$

1) $P \varepsilon zb(a) .$

2) $a \subset b . \cancel{\supset} .$

3) $\Theta \varepsilon el(P) . \supset .$

$[\exists K S] .$

4) $K \varepsilon el(\Theta) .$

5) $S \varepsilon a .$

6) $S \varepsilon el(P) .$

7) $K \varepsilon el(S) .$

(4–7): (D3 $^{,3}_{,1}$)

8) $S \varepsilon b .$ (2, 5)

$[\exists K S] . K \varepsilon el(\Theta) . S \varepsilon b . S \varepsilon el(P) . K \varepsilon el(S)$ (4, 8, 6, 7)

T12. $[Pab] : P \varepsilon \mathrm{zb}(a) . a \subset b . \supset . P \varepsilon \mathrm{zb}(b)$

Dem.:

$[Pab] ::$

1) $P \varepsilon \mathrm{zb}(a) .$

2) $a \subset b . \supset :$

3) $[\Theta] : \Theta \varepsilon \mathrm{el}(P) . \supset . [\exists R S] . R \varepsilon \mathrm{el}(\Theta) . S \varepsilon b . S \varepsilon \mathrm{el}(P) . R \varepsilon \mathrm{el}(S) :$ (T11, 1, 2)

$P \varepsilon \mathrm{zb}(b)$ (D3, 1, 3)

T13 $[Pa] : P \varepsilon a . \supset . P \varepsilon \mathrm{zb}(a)$

Dem.:

$[Pa] ::$

1) $P \varepsilon a . \supset .$

2) $P \varepsilon \mathrm{el}(P) :$ (T2, 1)

3) $[\Theta] : \Theta \varepsilon \mathrm{el}(P) . \supset . \Theta \varepsilon \mathrm{el}(\Theta) . \Theta \varepsilon \mathrm{el}(P) :$ (T2)

4) $[\Theta] : \Theta \varepsilon \mathrm{el}(P) . \supset . [\exists R S] . R \varepsilon \mathrm{el}(\Theta) . S \varepsilon a . S \varepsilon \mathrm{el}(P) .$
$R \varepsilon \mathrm{el}(S) :$ (3, 1, 2)

$P \varepsilon \mathrm{zb}(a)$ (D3, 1, 4)

$\underline{T14}$ $[\beta a] : \beta \varepsilon \mathrm{Kl}(a) . \supset . \beta \varepsilon \mathrm{zb}(a)$

Dem.:

$[\beta a] :.$

1) $\beta \varepsilon \mathrm{Kl}(a) . \supset :$

2) $[\theta] : \theta \varepsilon \mathrm{el}(\beta) . \supset . [\exists \kappa \gamma] . \kappa \varepsilon \mathrm{el}(\theta) . \gamma \varepsilon a . \gamma \varepsilon \mathrm{el}(\beta) . \kappa \varepsilon \mathrm{el}(\gamma) :$ $(T6, 1)$

$\beta \varepsilon \mathrm{zb}(a)$ $(D3, 1, 2)$

$\underline{T15}$ $[\beta \theta a] : \beta \varepsilon \mathrm{Kl}(\mathrm{zb}(a)) . \theta \varepsilon \mathrm{el}(\beta) . \supset . [\exists \kappa \gamma] . \kappa \varepsilon \mathrm{el}(\theta) .$
$\gamma \varepsilon a . \kappa \varepsilon \mathrm{el}(\gamma)$

Dem.:

$[\beta \theta a] ::$

1) $\beta \varepsilon \mathrm{Kl}(\mathrm{zb}(a)) .$

2) $\theta \varepsilon \mathrm{el}(\beta) . \supset ::$

$[\exists \kappa \gamma] :.$

$$\left.\begin{array}{ll} 3) & \kappa \varepsilon \mathrm{el}(\theta) . \\ 4) & \gamma \varepsilon \mathrm{zb}(a) . \\ 5) & \kappa \varepsilon \mathrm{el}(\gamma) : \end{array}\right\} (D\underline{II}, 1, 2)$$

$[\exists T V].$

6) $T \varepsilon \mathrm{el}(K).$

7) $V \varepsilon a.$ $\Bigg\}$ $(D3, 4, 5)$

8) $T \varepsilon \mathrm{el}(V).$

9) $T \varepsilon \mathrm{el}(\theta) \therefore$ $(T4, 6, 3)$

$[\exists K S] . K \varepsilon \mathrm{el}(\theta) . S \varepsilon a . K \varepsilon \mathrm{el}(S)$ $(9, 7, 8)$

× $D4. \ [P\theta] \therefore P \varepsilon \mathrm{el}_1(\theta) . \equiv : [\exists a] . \theta \varepsilon \mathrm{Kl}(a) . P \varepsilon a$ *)

$T16 \ [P\theta] : P \varepsilon \mathrm{el}_1(\theta) . \supset . P \varepsilon \mathrm{el}(\theta)$

Dem.:

$[P\theta]:$

1) $P \varepsilon \mathrm{el}_1(\theta) . \supset .$

$[\exists a].$

2) $\theta \varepsilon \mathrm{Kl}(a).$ $\Big\}$ $(D4, 1)$

3) $P \varepsilon a.$

$P \varepsilon \mathrm{el}(\theta)$ $(D2, 2, 3)$

*) el_1 = „el" w oryginale = my.

—T17 $[B\theta]: B \varepsilon el(\theta). \supset . B \varepsilon el_1(\theta)$

Dem.:

$[B\theta]:$

1) $B \varepsilon el(\theta). \supset .$

2) $\theta \varepsilon Kl(el(\theta)).$ (T10, 1)

$. B \varepsilon el_1(\theta)$ (D4, 2, 1)

—T17a $[B\theta]: B \varepsilon el(\theta). \equiv . B \varepsilon el_1(\theta)$ (T16, T17) *)

—T18 $[B\theta]: B \varepsilon u(\theta). \supset . B \varepsilon el_1(\theta)$ (D1, T17)

—T19 $[B]: B \varepsilon V. \supset . B \varepsilon el_1(B)$ (T2, T17)

—T20 $[Ba]: B \varepsilon Kl(a). \supset . a \subset el_1(B)$ (D2, T17)

T21 $[Ba]: B \varepsilon ob(a). \supset . a \Delta el_1(B)$

*) W oryginale twierdzenia tego w tem miejscu niema.

Dem.:

$[Pa]:$

1) $P \varepsilon ub(a) . \supset .$

2) $P \varepsilon el(P) .$ $(T2, 1)$

$[\exists S] .$

3) $S \varepsilon a . S \varepsilon el(P) .$ $(D3, 1, 2)$

4) $S \varepsilon el_1(P) .$ $(T17, 3)$

$a \Delta el_1(P)$ $(3, 4)$

T22 $[P\theta] : P \varepsilon u(\theta) . \supset . \sim ([\exists R S a] : R \varepsilon el_1(S) . S \varepsilon ub(a) . \supset . R \varepsilon a)$

Dem.:

$[P\theta]:$

1) $P \varepsilon u(\theta) . \supset .$

2) $\theta \varepsilon \theta .$ $(T9, 1)$

3) $\theta \varepsilon ub(\theta) .$ $(T13, 2)$

4) $P \varepsilon el_1(\theta) .$ $(T18, 1)$

5) $\theta \varepsilon \sim (u(\theta)) .$ $(T1, 2)$

6) $P \neq Q .$ (1, 5)

7) $\sim(P \varepsilon Q) .$ (2, 6)

$\sim([R S a] : R \varepsilon el_1(S) . S \varepsilon zb(a) . \supset . R \varepsilon a)$ (3, 4, 7)

T23. $[PQR] : P \varepsilon el_1(Q) . Q \varepsilon el_1(R) . \supset . P \varepsilon el_1(R)$ (T4, T17a)

T24. $[Pa] : P \varepsilon Kl(zb(a)) . \supset . P \varepsilon Kl(a)$

Dem.:

$[Pa] :$

1) $P \varepsilon Kl(zb(a)) . \supset :$

2) $zb(a) \subset el(P) :$ (D2, 1)

$[\exists R] .$

3) $R \varepsilon zb(a) .$ (2)

$[\exists S] .$

4) $R \varepsilon a ::$ (T21, 3)

5) $[Q] : Q \varepsilon a . \supset . Q \varepsilon el(P) :$ (T13, 2)

6) $[Q] : Q \varepsilon el(P) . \supset . [\exists R S] . R \varepsilon el(Q) . S \varepsilon a . R \varepsilon el(S)$ (T15, 1)

$P \varepsilon Kl(a)$ (D2, 1, 4, 5, 6)

T$\underline{2}$5 $[B\,a]: B \varepsilon \mathrm{Kl}(a) . \supset . B \varepsilon \mathrm{Kl}(2b(a))$

Dem.:

$[B\,a]:$

1) $B \varepsilon \mathrm{Kl}(a) . \supset .$

2) $B \varepsilon 2b(a) .$ (T14, 1)

$[\exists \Theta] .$

3) $\Theta \varepsilon \mathrm{Kl}(2b(a)) .$ $(\mathrm{A}\underline{\mathrm{IV}}, 2)$

4) $\Theta \varepsilon \mathrm{Kl}(a) .$ (T24, 3)

5) $B \varepsilon \Theta .$ $(\mathrm{A}\underline{\mathrm{III}}, 1, 4)$

$B \varepsilon \mathrm{Kl}(2b(a))$ (5, 3)

T$\underline{2}$6 $[B\,a]: B \varepsilon 2b(a) . \supset . B \varepsilon \mathrm{el}(\mathrm{Kl}(a))$

Dem.:

$[B\,a]:$

1) $B \varepsilon 2b(a) . \supset .$

$[\exists K] .$

2) $K \varepsilon a .$ (T21, 1)

$[\exists S]$.

3) $S \, \varepsilon \, Kl(a)$. $(A\underline{\overline{IV}}, 2)$

4) $S \, \varepsilon \, Kl(zb(a))$. $(T25, 3)$

5) $P \, \varepsilon \, el(S)$. $(D2, 4, 1)$

6) $S = Vl(a)$. $(A\underline{\overline{III}}, 3)$

~~$P \, \varepsilon \, Kl(zb(a))$~~

$P \, \varepsilon \, el(Kl(a))$ $(5, 6)$

$T\underline{27}$ $[PR] \therefore P \, \varepsilon \, V : [S] : S \, \varepsilon \, el(P) . \supset . [\exists \Theta] . \Theta \, \varepsilon \, el(S) . \Theta \, \varepsilon \, el(R) :$
$\supset . P \, \varepsilon \, el(R)$

Dem::

$[PR] \therefore$

1) $P \, \varepsilon \, V :$

2) $[S] : S \, \varepsilon \, el(P) . \supset . [\exists \Theta] . \Theta \, \varepsilon \, el(S) . \Theta \, \varepsilon \, el(R) : \supset :$

3) $[\Theta] : \Theta \, \varepsilon \, el(P) . \supset . [\exists \upsilon \chi] . \upsilon \, \varepsilon \, el(\Theta) . \chi \, \varepsilon \, el(R) . \chi \, \varepsilon \, el(P) . \upsilon \, \varepsilon \, el(\chi) :$
$(T5, 2)$

4) $P \, \varepsilon \, zb(el(R))$. $(D3, 1, 3)$

5) $P \, \varepsilon \, el(Kl(el(R)))$. $(T26, 4)$

$[\exists T]$.

6) $T \varepsilon el(K)$. $(T21, 4)$

~~7)~~ 7) $K \varepsilon Kl(el(K))$. $(T10, 6)$

8) $Kl(el(K)) \varepsilon V$. $(T10, 5)$

9) $K = Kl(el(K))$. $(7, 8)$

$B \varepsilon el(K)$ $(5, 9)$

D5 $[B\theta] : B \varepsilon el_2(\theta) . \equiv . B \varepsilon V . el_1(B) \subset el_1(\theta)$ *)

T28 $[B\theta] : B \varepsilon el_2(\theta) . \supset . B \varepsilon el_1(\theta)$

Dem.:

$[B\theta]$:

1) $B \varepsilon el_2(\theta) . \supset .$

2) $B \varepsilon el_1(B)$. $(D5, 1, T19)$

$B \varepsilon el_1(\theta)$ $(D5, 1, 2)$

*) el_2 = „podzbiór" w oryginale = ing = el = el_1

$\mathrm{T}29\ [B\theta]: B\,\varepsilon\,\mathrm{el}_1(\theta) . \supset . B\,\varepsilon\,\mathrm{el}_2(\theta)$

Dem.:

$[B\theta] ::$

1) $B\,\varepsilon\,\mathrm{el}_1(\theta) . \supset :$

2) $B\,\varepsilon\,\mathrm{el}_1(B) :$ $(\mathrm{T}19, 1)$

3) $[K]: K\,\varepsilon\,\mathrm{el}_1(B) . \supset . K\,\varepsilon\,\mathrm{el}_1(\theta) :$ $(\mathrm{T}23, 1)$

$B\,\varepsilon\,\mathrm{el}_2(\theta)$ $(\mathrm{D}5, 2, 3)$

$\mathrm{T}29a\ [B\theta]: B\,\varepsilon\,\mathrm{el}_2(\theta) . \equiv . B\,\varepsilon\,\mathrm{el}_1(\theta)$ $(\mathrm{T}28, \mathrm{T}29)$ *)

$\mathrm{T}29b\ [B\theta]: B\,\varepsilon\,\mathrm{el}_2(\theta) . \equiv . B\,\varepsilon\,\mathrm{el}(\theta)$ $(\mathrm{T}29a, \mathrm{T}17a)$ *)

$\mathrm{T}30\ [B\theta]: B\,\varepsilon\,a(\theta) . \supset . B\,\varepsilon\,\mathrm{el}_2(\theta)$ $(\mathrm{T}18, \mathrm{T}\overset{19?}{29})$

$\mathrm{D}6\ [B\theta]: B\,\varepsilon\,\mathrm{zw}(\theta) . \equiv . B\,\varepsilon\,V . [\exists K] . K\,\varepsilon\,\mathrm{el}(\theta) . \mathrm{el}(\theta) \subset \sim(\mathrm{el}(B))$ **)

---

*) W oryginale tych twierdzeń w tem miejscu niema.

**) „zw" = zewnętrzny

T31 $[A] : A \varepsilon V . \supset . A \varepsilon \sim(ex(A))$

Dem.:

$[A] :$

1) $A \varepsilon V . \supset .$

2) $\sim(A \varepsilon ex(A)) .$ (D6)

$A \varepsilon \sim(ex(A))$ (1,2)

T32 $[AB] : A \varepsilon ex(B) . \supset . B \varepsilon ex(A)$

Dem.:

$[AB] ::$

1). $A \varepsilon ex(B) . \supset :$

$[\exists K] .$

2). $K \varepsilon el(B) .$ (D6,1)

3) $B \varepsilon V :$ (T10,2)

4) $[K] : K \varepsilon el(A) . \supset . K \varepsilon \sim(el(B)) :$ (D6,1)

5) $A \varepsilon el(A) .$ (T2,1)

$B \varepsilon ex(A)$ (D6,3,5,4)

D7 $[PQR]\therefore P\,\varepsilon\,\mathrm{dop}(QR).\equiv.Q\,\varepsilon\,el_2(R)$. ~~$[\exists\alpha]$~~ ∴ ~~$P\,\varepsilon\,Kl(el_1(R))$~~ :

: ~~$[V]:V\,\varepsilon\,el_1(R).\supset V\,\varepsilon\,el_1(R).V\,\varepsilon\,ent(Q)$~~ *) $\overset{*}{}P\,\varepsilon\,Kl(el(R)\cap zw(Q))$

T33 $[PQR]:P\,\varepsilon\,\mathrm{dop}(QR).\supset.Q\,\varepsilon\,el(R)$ (D7, T2)

T34 $[PQRS]:P\,\varepsilon\,\mathrm{dop}(QR).S\,\varepsilon\,el(P).\supset.[\exists VX].V\,\varepsilon\,el(S).X\,\varepsilon\,el_1(R).X\,\varepsilon\,zw(Q).V\,\varepsilon\,el(X)$

Dem.:

$[PQRS]::$

1) $P\,\varepsilon\,\mathrm{dop}(QR).$

2) $S\,\varepsilon\,el(P).\supset\therefore$

~~$[\exists\alpha]$~~ ∴

3) $P\,\varepsilon\,Kl(el_1(R)\cap zw(Q))$ (D7, 1)

~~4) $[V]:V\,\varepsilon\,el_1(R).\supset.V\,\varepsilon\,el_1(R).V\,\varepsilon\,zw(Q)\therefore$~~

4) $[Q]:Q\,\varepsilon\,el(P).\supset.[\exists KS].$ ~~$S\,\varepsilon\,el_1(R)$~~ $K\,\varepsilon\,el(S).K\,\varepsilon\,el(Q)\therefore$
$S\,\varepsilon\,el_1(R)\cap zw(Q)$ (D2, 3)

*) „dop" = dopełnienie

D7. dop = dopełnienie

$[\exists v x]$.

5) $x \varepsilon el_1(\kappa)$.

6) $v \varepsilon el(x)$.

7) $v \varepsilon el(P)$. $\Bigg\}$ (5, 2)

~~9) $x \varepsilon el_1(\kappa)$.~~

8) $x \varepsilon nw(\theta)$. $\Bigg\}$ ~~(4, 6)~~

$[\exists v x] . v \varepsilon el(s) . x \varepsilon el_1(\kappa) . x \varepsilon nw(\theta) . v \varepsilon el(x)$

(8, 9, 10, 7)

T35 . $[P \theta \kappa s] : P \varepsilon dyp(\theta \kappa) . s \varepsilon el(P) . \supset . [\exists v] . v \varepsilon el(s) . v \varepsilon el(\kappa)$

Dem.:

$[P \theta \kappa s]$:

1) $P \varepsilon dyp(\theta \kappa)$.

2) $s \varepsilon el(P) . \supset$.

$[\exists v x]$.

3) $v \varepsilon el(s)$.

4) $x \varepsilon el_1(\kappa)$. $\Bigg\}$ (T34, 1, 2)

5) $v \varepsilon el(x)$.

6) $X \varepsilon el(R)$. (T16, 4)

7) $V \varepsilon el(R)$. (T4, 5, 6)

$[\exists V] . V \varepsilon el(S) . V \varepsilon el(R)$ (3, 7)

T36 $[P \theta R] :: P \varepsilon dqz(\theta R) . \supset . P \varepsilon zw(\theta)$

Dem.:

$[P \theta R] ::$

1) $P \varepsilon dqz(\theta R) . \supset .$

2) $\theta \varepsilon el(\theta) :$ (T33, 1)

3) $[S] : S \varepsilon el(P) . \supset . [\exists T V] . T \varepsilon zw(\theta) . V \varepsilon el(S) . V \varepsilon el(T) :$

4) $[S] : S \varepsilon el(P) . \supset . [\exists V] . V \varepsilon el(S) . V \varepsilon \sim(el(\theta)) :$ (T34, 1)

(D6, 3)

5) $[S] : S \varepsilon el(P) . \supset . \sim(S \varepsilon el(\theta) :$ (T4, 4)

$P \varepsilon zw(\theta)$ (D6, 1, 2, 5)

T37 $[P R] : P \varepsilon V . \supset . P \varepsilon \sim(dqz(P R))$ (T31, T36)

T38 $[P\theta R S] : P \varepsilon \mathrm{dop}(\theta R) . S \varepsilon \mathrm{dop}(\theta R) . \supset . P \varepsilon S$

Dem.:

$[P\theta R S] :.$

1) $P \varepsilon \mathrm{dop}(\theta R) .$

2) $S \varepsilon \mathrm{dop}(\theta R) . \supset :$

~~$[\exists a]$~~ $:$

3) $P \varepsilon \mathrm{Kl}(\mathrm{el}(R) \cap \mathrm{nw}(\theta)) .$ (D7, 1)

4) $S \varepsilon \mathrm{Kl}(\mathrm{el}(R) \cap \mathrm{nw}(\theta)) .$ (D7, 2)

$P \varepsilon S$ (A3, 3, 4)

T39 $[P\theta R] : P \varepsilon \mathrm{dop}(\theta R) . \supset . P \varepsilon \mathrm{el}(R)$

Dem.:

$[P\theta R] :.$

1) $P \varepsilon \mathrm{dop}(\theta R) . \supset :$

2) $[S] : S \varepsilon \mathrm{el}(P) . \supset . [\exists V] . V \varepsilon \mathrm{el}(S) . V \varepsilon \mathrm{el}(R) :$ (T35, 1)

$P \varepsilon \mathrm{el}(R)$ (T27, 1, 2)

T40 $[P\theta K s]: P \varepsilon \mathrm{dop}(\theta K) . s \varepsilon \mathrm{el}_1(K) . \sim(s \varepsilon \mathrm{el}(\theta)) . \supset . \sim(s \varepsilon \mathrm{nw}(P))$

Dem.:

$[P\theta K s]::$

1) $P \varepsilon \mathrm{dop}(\theta K).$

2) $s \varepsilon \mathrm{el}_1(K).$

3) $\sim(s \varepsilon \mathrm{el}(\theta)) . \supset :$

4) $P \varepsilon \mathrm{Kl}(\mathrm{el}(K) \cap \mathrm{w}(\theta)).$ (D7, 1)

~~5) $[V]: V \varepsilon \mathrm{el}_1(K) . \supset . V \varepsilon \mathrm{nw}(\theta):$~~

5) $\theta \varepsilon \mathrm{el}(\theta).$ (T33, 1)

$[\exists T].$

6) $T \varepsilon \mathrm{el}(s).$ (T27, 2, 3)

7) $\sim([\exists V] . V \varepsilon \mathrm{el}(T) . V \varepsilon \mathrm{el}(\theta)).$

8) $T \varepsilon \mathrm{el}_1(K).$ (T17, 7)

9) $T \varepsilon \mathrm{el}_1(K).$ (T23, 8, 2)

10) $T \varepsilon \mathrm{nw}(\theta).$ (D6, 7, 5, 8)

11) $T \varepsilon \mathrm{el}(P).$ (D2, 4, 9, 10)

$\sim(s \varepsilon \mathrm{nw}(P))$ (D6, 11, 6)

$T\varepsilon\, el(v) . v\varepsilon\, \theta \cup$

T41 $[P\theta K S] : P\varepsilon\, dopr(\theta K) . S\varepsilon\, el(K) . \supset . [\exists T] . T\varepsilon\, el(S)$. ~~$T\varepsilon\, dopr(\theta\cup P)$~~

Dem.:

$[P\theta K S] \therefore$

1) $P\varepsilon\, dopr(\theta K)$.

2) $S\varepsilon\, el(K) . \supset :$

3) $S\varepsilon\, el_1(K) :$ (T17, 2)

4) $S\varepsilon\, el(\theta) . v . \sim(S\varepsilon\, zw(P)) :$ (T40, 1, 2)

5) $S\varepsilon\, el(S)$. (T2, 2)

6) $P\varepsilon\, el(P) :$ (T2, 1)

7) $el(S)\, \Delta\, el(\theta) . v . el(S)\, \Delta\, el(P) :$ (4, 5, D6, 6)

8) $\theta\varepsilon\, \theta\cup P$. (D7, 1)

9) $P\varepsilon\, \theta\cup P$. (1)

$[\exists T] . T\varepsilon\, el(S) . T\varepsilon$ ~~$el(\theta\cup P)$~~ $el(v) . v\varepsilon\, \theta\cup P$ (7, 8, 9)

T42 $[P\theta] : P\varepsilon\, cz(\theta) . \supset . [\exists K] . K\varepsilon\, dopr(P\theta)$

Dem.:

$[P\theta] :$

1) $P\varepsilon\, cz(\theta) . \supset .$

2) $P \varepsilon el_2(\Theta)$. (T30, 1)

3) $\Theta \varepsilon \sim(ex(P))$. (AI, 1)

4) $\Theta \neq P$. (3, 1)

5) $\sim(\Theta \varepsilon el(P))$. (DI, 4, 3)

6) $P \varepsilon el(P)$. (T2, 1)

$[\exists S]$.

7) $S \varepsilon el(\Theta)$. (T27, 3, 5)

8) $\sim([\exists T].T \varepsilon el(P).T \varepsilon el(S))$.

9) $S \varepsilon el_1(\Theta)$. (T17, 7)

10) $S \varepsilon nw(P)$. (D6, 7, 6, 8)

11) $S \varepsilon el_1(\Theta) \cap nw(P)$. (9, 10)

$[\exists R]$.

12) $R \varepsilon Kl(el_1(\Theta) \cap nw(P))$. (AIV, 11)

$[\exists R].R \varepsilon dopl(P\Theta)$ (D7, 2, 12)

T43 $[P\Theta R]: P \varepsilon dopl(\Theta R). \supset . P \varepsilon ex(R)$

Dem.:

$[P\Theta R]$:

1) $P \varepsilon \mathrm{dsp}(\Theta R) . \supset .$

2) $P \varepsilon \mathrm{el}(R) .$ (T39, 1)

3) $P \varepsilon \mathrm{nw}(\Theta) .$ (T36, 1)

$[\exists S] .$

4) $S \varepsilon \mathrm{el}(\Theta) . S \varepsilon \sim(\mathrm{el}(P)) .$ (D6, 3)

5) $S \varepsilon \mathrm{el}_1(\Theta) . S \varepsilon \sim(\mathrm{el}_1(P)) .$ (4, T17, T16)

6). $\sim(\Theta \varepsilon \mathrm{el}_2(P)) .$ (D5, 5)

7). $\Theta \varepsilon \mathrm{el}_2(R) .$ (D7, 1)

8) $P \neq R .$ (6, 7)

$P \varepsilon \mathrm{cr}(R)$ (D1, 2, 8)

T44 $[P\Theta] : P \varepsilon V . \supset . P \varepsilon \sim(\mathrm{dsp}(\Theta P))$ (T1, T43)

T45 $[P\Theta R] : P \varepsilon \mathrm{dsp}(\Theta R) . \supset . \Theta \varepsilon \mathrm{dsp}(P R)$

Dem.:

$[P\Theta R] :.$

1) $P \varepsilon \mathrm{dsp}(\Theta R) . \supset :$

2) $P \varepsilon \mathrm{cr}(R) .$ (T43, 1)

3) $\mathcal{P}\,\varepsilon\,\mathrm{el}_2(K).$ (T30, 2)

4) $\Theta\,\varepsilon\,\mathrm{el}_2(K).$ (D7, 1)

5) $\Theta\,\varepsilon\,\mathrm{el}_1(K).$ (T28, 4)

6) $\mathcal{P}\,\varepsilon\,\mathrm{zw}(\Theta).$ (T36, 1)

7) $\Theta\,\varepsilon\,\mathrm{zw}(\mathcal{P}):$ (T32, 6)

8) $[S]:S\,\varepsilon\,\mathrm{el}_1(K).\,S\,\varepsilon\,\mathrm{zw}(\mathcal{P}).\supset.\,S\,\varepsilon\,\mathrm{el}(\Theta):$ (T40, 1)

9) $[S]:S\,\varepsilon\,\mathrm{el}(\Theta).\supset.[\exists VT].\,V\,\varepsilon\,\mathrm{el}(S).\,T\,\varepsilon\,\mathrm{el}_1(K).\,T\,\varepsilon\,\mathrm{zw}(\mathcal{P}).$
$V\,\varepsilon\,\mathrm{el}(T):$ (T2, 5, 7)

10) $\Theta\,\varepsilon\,$ ~~dop~~ $\mathrm{Kl}(\mathrm{el}(K)\cap\mathrm{zw}(\mathcal{P})).$ (D2, 5, 7, 8, 9)
$\Theta\,\varepsilon\,\mathrm{dop}(\mathcal{P}K)$ (D7, 3, 10)

T46 $[\mathcal{P}\Theta K]:\mathcal{P}\,\varepsilon\,\mathrm{dop}(\Theta K).\supset.\,\Theta\,\varepsilon\,\mathrm{cz}(K)$ (T45, T43)

T47 $[\mathcal{P}\Theta]:\mathcal{P}\,\varepsilon\,V.\supset.\,\mathcal{P}\,\varepsilon\,\sim(\mathrm{dop}(\Theta\Theta))$

Dem.:

$[\mathcal{P}\Theta]:$

1) $\mathcal{P}\,\varepsilon\,V.\supset.$

2) $\sim(\Theta\,\varepsilon\,\mathrm{cz}(\Theta))$ (A1)

~~5~~ $P \varepsilon \sim(\mathrm{dopl}(\theta\theta))$ (T46, 1, 2)

T48 $[P\theta K]: P \varepsilon \mathrm{dopl}(\theta K) . \supset . K \varepsilon \mathrm{Kl}(\theta \cup P)$

Dem.:

$[P\theta K]::$

1) $P \varepsilon \mathrm{dopl}(\theta K) . \supset :$

2) $\theta \varepsilon \mathrm{cz}(K) .$ (T46, 1)

3) $\theta \varepsilon \mathrm{el}(K) .$ (D I, 2)

4) $K \varepsilon V .$ (T10, 3)

5) $P \varepsilon \theta \cup P .$ (1)

6) $P \varepsilon \mathrm{el}(K) :$ (T39, 1)

7) $[S]: S \varepsilon \theta \cup P . \supset . S \varepsilon \mathrm{el}(K) :$ (3, 6)

8) $[S]: S \varepsilon \mathrm{el}(K) . \supset . [\exists V T] . T \varepsilon \mathrm{el}(S) . T \varepsilon \mathrm{el}(V)$

~~$T \varepsilon \mathrm{el}(\theta \cup P) : T \varepsilon \mathrm{el}(T) .$~~ (T41, 1)

$K \varepsilon \mathrm{Kl}(\theta \cup P)$ (D II, 4, 5, 7, 8)