

Stanisław Leśniewski

On the Foundations of Mathematics, Chapter VI

Source: Mereology OPM Notebook 1, pp. 30–35. Original in the Archives of the University of Notre Dame. Most likely this work was done between 1946 and 1949 when Sobociński was in Brussels.

Original publication: “Rozdział VI: Aksjomatyka ‘ogólnej teorji mnogości,’” pochadzająca z r. 1918. (Chapter VI: The axiomatization of the ‘general theory of sets’ from the year 1918), *Przegląd Filozoficzny*, 33 (1930), 77–81.

English translation: *Stanisław Leśniewski: Collected Works* (1992), pp. 315–320.

This chapter is entitled “The Axiomatization of the ‘General Theory of Sets’ from the year 1918.”

Leśniewski published a long paper entitled “O podstawach matematyki” (On the foundations of mathematics, briefly OPM), in XI chapters, containing several versions of his Mereology. At the time of publication, Leśniewski was not comfortable with formal notation and so expressed the theses and derivations in ordinary (Polish) language. In this notebook Sobociński has translated this material into symbolic form. Such a translation has never been published.

This development of Leśniewski’s mereology is based on *part* and four axioms. It’s main improvement over Chapter IV is there are no definitions interspersed among the axioms. In the original, these theses are labeled A through P, including the axioms. Sobociński has labeled them A1 through A11.

Brought to you by, Kursant V-VIII. Wosnik 33, zemt I-II
Mr. 77 i nust.

Kursant V.

Akademicka 22.1918 oparta na terminie „czyli”.

$$AI [S\Theta] : \beta_{\text{zu}}(\theta) \cdot \sigma \cdot \theta \cdot \tau(u(\beta))$$

$$AL [\beta\Theta u] : \beta_{\text{zu}}(\theta) \cdot \theta \cdot u(k) \cdot \sigma \cdot \beta_{\text{zu}}(k)$$

$$A3 [u\beta\theta] :: \lambda_{\text{fa}} : [v] : v_{\text{fa}}. \sigma : v = \beta \cdot v. v_{\text{zu}}(\beta) :: [v] : v_{\text{fa}}.$$

$$\Rightarrow v = \theta \cdot v. v_{\text{zu}}(\theta) :: [k] \quad k_{\text{zu}}(\beta) \cdot v \cdot k_{\text{zu}}(\theta) : \sigma : [\beta] :$$

$$\tau = k \cdot v. v_{\text{zu}}(k) : \beta_{\text{zu}} \cdot v \cdot [\beta] \cdot x_{\text{zu}}. \tau_{\text{zu}}(x) :: \sigma \cdot \beta_{\text{zu}}$$

$$A4 [Au] : A_{\text{zu}} \cdot \sigma \cdot [\beta] :: [\theta] : \beta_{\text{zu}} \cdot \sigma \cdot \theta = \beta \cdot v. v_{\text{zu}}(\beta) ::$$

$$[\theta] : \beta_{\text{zu}}(\beta) \cdot \sigma : [\beta] : \lambda = \theta \cdot v. v_{\text{zu}}(\theta) : \lambda_{\text{zu}} \cdot v \cdot [\beta] \cdot \tau_{\text{zu}}.$$

$$\lambda_{\text{zu}}(\beta)$$

$$DAI [S\theta] : \beta_{\text{zu}}(\theta). \equiv : \beta = \theta \cdot v. v_{\text{zu}}(\theta)$$

$$DA2 [Su] \quad \beta_{\text{zu}}(u). \equiv : S_{\text{zu}}. u \in d(\theta) : [\theta] : \theta_{\text{zu}}(\theta) \cdot \sigma \cdot [\beta]$$

$$\lambda_{\text{zu}}. \lambda_{\text{zu}}(\beta). \theta_{\text{zu}}(\theta)$$

Works, pp 315-320.

PF 33 (1930), 77-81

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OPM chapter VI.

$$A5[A, u, \beta, R] :: A_{\{u\}}[\theta] :: \theta_{\{u\}}. \triangleright : \theta = \beta \cdot v. \theta_{\{u\}}(\beta) :: [\theta] :: \theta_u \\ u(\beta). \triangleright : [g\beta] :: \beta = \theta \cdot v. \beta_{\{u\}} :: \beta_{\{u\}}. v. [g\beta]. \beta_{\{u\}}. \beta_{\{u\}}(\beta) :: \\ \kappa_{\{d\}}(\beta) :: \triangleright. [gv]. \kappa_{\{d\}}(v). \beta_{\{u\}}. \kappa_{\{d\}}(v)$$

Dem:

$$[A, u, \beta, R] ::$$

- 1) $A_{\{u\}} ::$
- 2) $[\theta] :: \theta_{\{u\}}. \triangleright : \theta = \beta \cdot v. \theta_{\{u\}}(\beta) ::$
- 3) $[\theta] :: \theta_{\{u\}}(\beta). \triangleright : [g\beta] :: \beta = \theta \cdot v. \beta_{\{u\}} :: \beta_{\{u\}}. v. [g\beta]. \beta_{\{u\}}. \beta_{\{u\}}(\beta) ::$
- 4) $\kappa_{\{d\}}(\beta) :: \triangleright ::$
- 5) $\kappa = \beta \cdot v. \kappa_{\{u\}}(\beta) :: \quad (\text{DAI}, 4)$
- 6) $A = \beta \cdot v. A_{\{u\}}(\beta) :: \quad (2, 1)$

$$[g\beta] ::$$

- 7) $\beta = \kappa \cdot v. \beta_{\{u\}}(v) :: \quad (\text{DAI}) \quad \{(5; 6; 1; 3)\}$
- 8) $\beta_{\{u\}}. v. [g\beta]. \beta_{\{u\}}. \beta_{\{u\}}(\beta) :: \quad (\text{DAI}, 5)$
- 9) $\beta_{\{d\}}(v) :: \quad (\text{DAI}, 7)$
- 10) $\beta_{\{d\}}(\beta) :: \quad (\text{DAI}, 9)$

$$[g\beta] ::$$

11)
12)

$$\begin{aligned} \mathcal{T}_{\text{eu}}: \\ \mathcal{T}_{\text{el}}(\beta) := \end{aligned} \quad \left\{ \begin{array}{l} (8; 10; \text{DAI}) \\ (9; 11; 12) \end{array} \right.$$

$$[\exists \forall x]. \forall_{\text{rel}(R)} x \exists a. \forall_{\text{rel}(X)} (9; 11; 12)$$

A6 $[\exists a]: \exists_{\text{el}(\alpha)} \Rightarrow a \in \text{el}(\beta)$

Dem.:

$[\exists a]:$

1)

$\exists_{\{\text{Kl}(a)\}} \Rightarrow$

2)

$\exists_{\text{el}(\beta)}$.

(DAI, 1)

$[\exists \beta].$

3)

\mathcal{T}_{eu} .

(DAL, 1, 2)

4)

$!\{a\}:$

(3)

5)

$[\emptyset]: \theta_{\text{eu}} \Rightarrow \theta_{\text{el}(\beta)}$:

(DAL, 1)

$a \in \text{el}(\beta)$

(4, 5)

A7 $[\exists_{\text{Ku}}]: \exists_{\{\text{Kl}(a)\}} \text{Ku}(\beta) \Rightarrow [\exists \beta]: \beta = R \cdot V \cdot \mathcal{T}_{\text{eu}}(R) :$

$\mathcal{T}_{\text{eu}} \cdot V \cdot [\exists \beta] \cdot \mathcal{T}_{\text{eu}} \cdot \mathcal{T}_{\text{eu}}(\beta)$

Dem.:

$[\exists_{\text{Ku}}]:$

- 1) $\beta_3 \text{Kl}(a)$.
 2) $\kappa_{\text{rel}}(\beta) \cdot ?$:
 3) $\kappa_{\text{rel}}(\beta) \cdot \quad (\text{DAI}, 2)$
 $[\exists s]:$
 4) $\theta_{\text{rel}}(k)$.
 5) ι_{su} . $\left\{ \begin{array}{l} (\text{DAZ}; 1; 3) \\ \vdots \theta_{\text{rel}}(s) \end{array} \right.$
 6) $\theta = k \cdot v \cdot \theta_{\text{rel}}(k) \cdot \quad (\text{DAI}; 4)$
 7) $\theta = \iota \cdot v \cdot \theta_{\text{rel}}(s) \cdot \quad (\text{DAI}; 6)$
 8) $\theta_{\text{su}} \cdot v \cdot [\exists v] \cdot \iota_{\text{su}} \cdot \theta_{\text{rel}}(v) \cdot (3, 5)$
 9) $[\exists s]: \iota = k \cdot v \cdot \iota_{\text{su}}(k) \cdot \iota_{\text{su}} \cdot v \cdot [\exists s] \cdot \iota_{\text{su}} \cdot \iota_{\text{su}}(s) \quad (7, 9)$

A8 $[\beta_a] := \beta_3 \text{Kl}(a) \cdot ? \cdot !\{a\} \cdot [\theta] \cdot \theta_{\text{su}} \cdot \vdots \theta = \beta \cdot v \cdot \theta_{\text{rel}}(\beta) \in \{\text{A6, DAI}\}$

A9 $[\beta_a] := \beta_3 \text{Kl}(a) \equiv \beta_3 \beta \cdot a \subseteq \text{el}(\beta) \cdot [\theta] \cdot \theta_{\text{rel}}(\beta) \cdot ?$
 $[\exists v], \iota_{\text{su}}, \kappa_{\text{rel}}(v), \kappa_{\text{rel}}(\theta) \quad (\text{DAZ, A6})$

A10 $[\beta \theta a] := \beta_3 \text{Kl}(a) \cdot \theta_3 \text{Kl}(a) \cdot ? \cdot \beta_3 \theta$

Demo:

$[\beta \theta a] :=$

- 1) $\beta_{\{Kl(a)\}}$
- 2) $\theta_{\{Kl(a)\}} \Rightarrow$
- 3) $\{\{a\}\} \quad (\text{A8;1})$
- 4) $[g] \because \beta_{\{u\}} \circ g = \theta \cdot v \cdot \beta_{\{u\}}(g) \therefore (\text{A8;1})$
- 5) $[1] \because \beta_{\{u\}} \circ 1 = \theta \cdot v \cdot \beta_{\{u\}}(1) \therefore (\text{A8;2})$
- 6) $[k] \because R_{\{u\}}(g) \cdot v \cdot R_{\{u\}}(h) \Rightarrow [gh] \circ 1 = R \cdot v \cdot \beta_{\{u\}}(R) \circ \beta_{\{u\}}(v) \circ \beta_{\{u\}}(h)$
 $\beta_{\{u\}} \circ v \cdot [gh] \circ \beta_{\{u\}}(h) \quad (\text{A7;1;2})$
 $\beta_{\{u\}} \quad (\text{A3;3;4;5;6})$

All $[Bu]$: $\beta_{\{u\}} \circ [g \circ \theta] \circ \theta_{\{Kl(u)\}}$

Defn:

$[Bu] \circ$

1) $\beta_{\{u\}} \circ \circ$

$[g \circ \theta] \circ$

2) $[k] \circ R_{\{u\}} \circ R = \theta \cdot v \cdot R_{\{u\}}(k) \circ \in \mathbb{F}$

3) $[k] \circ R_{\{u\}}(\theta) \circ [g \circ 1] \circ 1 = k \cdot v \cdot \beta_{\{u\}}(k) \circ \beta_{\{u\}}(v) \circ \beta_{\{u\}}(1) \quad (\text{A7;1})$

$v \cdot [g \circ 1] \circ \beta_{\{u\}}(v) \circ \beta_{\{u\}}(1) \circ$

4) $[k] = R_{\{u\}} \circ R_{\{u\}}(\theta) \circ (\theta, \text{DAI})$

5)

$[A]: \text{Ned}(0) \rightarrow [E15]. \text{ Ned}(K). \text{ Ned}(S):$
 $(A5; 1; 2; 3)$

~~E15~~

6)

$\leftarrow \dots \beta = 0 \cdot v \cdot \beta_{\text{zu}}(0): (1; 2)$

7)

$\theta_1 \theta: \cancel{\text{A1}}$ (6; A1)

$[E \theta]. \theta_{\text{zu}}(u)$ (DAR; f; 4; 5)

$\{I, II, D1, D2, III, IV\} \Leftrightarrow \{A1, A2, A3, A4\}$

A1 jst rwmknttnre \approx I

A2 jst rwmknttnre \approx II

A3 jst rwmknttnre \approx zw 70 \approx rwmknttnre V

A4 mynka verpckniv \approx zw 102 \approx rwmknttnre ~~V~~

DA1 jst rwmknttnre \approx df. I

DA2 jst rwmknttnre \approx zw 57 \approx rwmknttnre V

{DA2} mynka \approx {DL, TS}

A5 [A, u, B, R]

u(B). \Rightarrow E

R $\not\approx$ el(B).

Denn:

[AuB R];

1)

2)

3)

[G].S

4)

5)

6)

7)

8)

9)

10)

f31 (focus 31)

1)
2)
3)

4)
5)
6)
7)
8)
9)

A8 [Ba]

A9 jst. riwokratthe 2 D 2

A9 [Ba]
[gns].

A10 jst. riwokratthe 2 III

A10 [Ba]

Dem.:

[Ba]

f 33

1)
2)
3)
4)
5)
6)

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All jut röwnkettutne 2 IV

All $[S_u]$:

Dem:

$[S_u]$

1)
2)
3)

V-5

4)

f 34