Ontology Notebook 3

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This is a small notebook, 12 by 19.5 cm, with a hard brown pasteboard cover.

It begins with 39 pages of definitions in Ontology. Many of these do not occur in Sobociński's classnotes on Ontology. They are more advanced than those ocuring in the first two Ontology Notebooks. An interesting feature is that he has numbered the semantical categories in the order that they are introduced and placed those numbers below the symbols to which they correspond.

After a gap of many pages there is a section of 35 unnumbered pages of notes on Rudolf Carnap's *Abriss der Logistik* (1929). This section is dated Bruksella, 9.IX.1948.

Immediately following is 22 pages of notes on an Analysis of Russell's Antinomy. Date at end: Bruksella, 10.IX.1948. Considering the contiguous dates, this material must have just been transcribed into this notebook (not developed as he wrote it) from the manuscript of his *Methodos* paper, which was received by the journal on 15 December 1948. This is only the formulas of the *Methodos* paper, not the text.

Summary prepared by V. Frederick Rickey, July 3, 2018

Ontologja

§1. Wykład aksjomatu:

T1. $[Aa] \therefore \varepsilon\{Aa\} \equiv : [\exists B] : \varepsilon\{AB\} . \varepsilon\{Ba\}$

T2. $[ABa] : \varepsilon\{AB\} . \varepsilon\{Ba\} \supset . \varepsilon\{Aa\}$ (T1)

T3. $[Aa] \therefore \varepsilon\{Aa\} . \supset : [\exists B] : \varepsilon\{AB\} . \varepsilon\{Ba\}$ (T1)

D1. $[ABa] : \varepsilon\{Aa\} . \varepsilon\{BA\} . \equiv . \varepsilon\{A * (Ba)\}$

T4. $[ABCa] : \varepsilon\{Aa\} . \varepsilon\{BA\} . \varepsilon\{CA\} . \supset . \varepsilon\{BC\}$

Dem.:

$[ABCa] :$

1) $\varepsilon\{Aa\} .$

2) $\varepsilon\{BA\} .$

3) $\varepsilon\{CA\} . \supset .$

4) $\varepsilon\{A * (Ba)\} .$ (D1, 1, 2)

5) $\varepsilon\{C * (Ba)\} .$ (T2, 3, 4)

$\varepsilon\{BC\}$ (D1, 5)

T5. $[Aa] : \varepsilon\{Aa\} . \supset \varepsilon\{AA\}$

Dem:

$[Aa] ::$

1) $\varepsilon\{Aa\} . \supset :$

$[\exists B] :$

2) $\varepsilon\{AB\} .$ $\Big\}$ $(T3, 1)$

3) $\varepsilon\{Ba\} .$

$\varepsilon\{AA\}$ $(T4, 2, 3)$

T5 $[ABa] : \varepsilon\{Aa\} . \varepsilon\{BA\} . \supset . \varepsilon\{AB\}$

Dem:

$[ABa] :$

1) $\varepsilon\{Aa\} .$

2) $\varepsilon\{BA\} . \supset .$

3) $\varepsilon\{AA\} .$ $(T5, 1)$

$\varepsilon\{AB\}$ $(T4, 1, 3, 2)$

D2. $[Aa] : \varepsilon\{Aa\} . \equiv . \varepsilon\{a\}\{A\}$

E1. $[ab] :. [C] : \varepsilon\{Ca\} . \equiv . \varepsilon\{Cb\} : \equiv : [\varphi] : \varphi\{a\} . \equiv . \varphi\{b\}$

T7. $[ABa] :: \varepsilon\{BA\} . \varepsilon\{Ba\} : [CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\} :. \supset . \varepsilon\{Aa\}$

Dem:

$[ABa] ::$

1) $\varepsilon\{BA\} .$

2) $\varepsilon\{Ba\} :.$

3) $[CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\} :. \supset :.$

4) $[C] : \varepsilon\{CA\} . \supset . \varepsilon\{CB\} :.$ $(3, 1)$

5) $[C] : \varepsilon\{CB\} . \supset . \varepsilon\{CA\} :.$ $(T2, 1)$

6) $[C] : \varepsilon\{CB\} . \equiv . \varepsilon\{CA\} :.$ $(5, 4)$

7) $[\varphi] : \varphi\{B\} . \equiv . \varphi\{A\} :.$ $(E1, 6)$

8) $\varepsilon\{a\}\{B\} .$ $(D2, 2)$

9) $\varepsilon\{a\}\{A\} .$ $(7, 8)$

$\varepsilon\{Aa\}$ $(D2, 9)$

T8. $[Aa] :: \varepsilon\{Aa\} . \equiv \therefore [\exists B] : \varepsilon\{BA\} . \varepsilon\{Ba\} \therefore [CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\}$

(T5, T4, T7)

T9. $[ABa] :: \varepsilon\{BA\} \therefore [C] : \varepsilon\{CA\} . \supset . \varepsilon\{Ca\} \therefore [CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\} \therefore \supset . \varepsilon\{Aa\}$

Dem.:

$[ABa] ::$

1) $\varepsilon\{BA\} \therefore$

2) $[C] : \varepsilon\{CA\} . \supset . \varepsilon\{Ca\} \therefore$

3) $[CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\} \therefore \supset .$

4) $\varepsilon\{Ba\} .$ (2, 1)

$\varepsilon\{Aa\}$ (T7, 1, 4, 3)

T10. $[Aa] :: \varepsilon\{Aa\} . \equiv \therefore [\exists B] . \varepsilon\{BA\} \therefore [C] : \varepsilon\{CA\} . \supset . \varepsilon\{Ca\} \therefore [CD] : \varepsilon\{CA\} . \varepsilon\{DA\} . \supset . \varepsilon\{CD\}$

(T5, T2, T4, T9)

§2 Terminy ontologiczne:

A) Surowe definicje pomocnicze:

D1. $[ABa] : \varepsilon\{Aa\} . \varepsilon\{BA\} . \equiv . \underset{2}{\varepsilon}\{\underset{1}{A} \underset{3}{*} \langle \underset{1}{B} \underset{1}{a} \rangle\}$

Inf. §1.

D2. $[Aa] : \varepsilon\{Aa\} . \equiv . \underset{4}{\varepsilon}\langle \underset{1}{a} \rangle\{\underset{1}{A}\}$

Inf. §1.

B) Elementarne terminy ontologiczne:

D3·1. $[Aa] : \varepsilon\{AA\} . \sim(\varepsilon\{Aa\}) . \equiv . \underset{2}{\varepsilon}\{\underset{1}{A} \underset{5}{\sim} \langle \underset{1}{a} \rangle\}$

A jest nie-a.

D3·2 $[Aa] : \sim(\varepsilon\{Aa\}) . \equiv . \underset{2}{\not\varepsilon}\{\underset{1}{A}\underset{1}{a}\}$

Nie jest prawdą, że A jest a; A nie jest a.

D3·3 $[a] \therefore [\exists A] . \varepsilon\{Aa\} : \equiv . \underset{6}{!}\{\underset{1}{a}\}$

Istnieje a; ex a.

$\mathfrak{D}3\cdot4\ [a]\therefore[AB]:\varepsilon\{Aa\}.\varepsilon\{Ba\}.\supset.\varepsilon\{AB\}:\equiv.$

$\underset{6}{\rightarrow}\{\underset{1}{a}\}$

A jest jedyne ; sol a

$\mathfrak{D}3\cdot5\ [A]:\varepsilon\{AA\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{1}{V}\}$

A jest przedmiotem

$\mathfrak{D}\text{-}3\cdot6\ [A]:\varepsilon\{AA\}.\sim(\varepsilon\{AA\}).\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{1}{\Lambda}\}$

A jest przedmiotem sprzecznym

$\mathfrak{D}\text{-}3\cdot7\ [AB]:\varepsilon\{AB\}.\varepsilon\{BB\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\,\underset{5}{\subset}(\underset{1}{B})\}$

A jest izos B

c) Terminy teorii równości

$\mathfrak{D}4\cdot1\ [AB]:\varepsilon\{AB\}.\varepsilon\{BA\}.\equiv.\underset{2}{=}\{\underset{1}{A}\underset{1}{B}\}$

A jest identyczne z B

$\mathfrak{D}4\cdot2\ [AB]:\varepsilon\{AA\}.\varepsilon\{BB\}.\sim(\varepsilon\{AB\}).\equiv.\underset{2}{\neq}\{\underset{1}{A}\underset{1}{B}\}$

A jest różne od B

$\mathfrak{D}4\cdot3\ [ab]\therefore[A]:\varepsilon\{Aa\}.\equiv.\varepsilon\{Ab\}:\equiv.\underset{2}{o}\{\underset{1}{a}\underset{1}{b}\}$

a równa się b

$\mathfrak{D}4\cdot4\ [ab]:\sim(o\{ab\}).\equiv.\underset{2}{\ominus}\{\underset{1}{a}\underset{1}{b}\}$

a nie równa się b

$\mathfrak{D}4\cdot5\ [ab]:!\{a\}.o\{ab\}.\equiv.\underset{2}{\square}\{\underset{1}{a}\underset{1}{b}\}$

a równa się egzystencjonalnie b

$D4{\cdot}6\ [ab]:\ !\{a\}.\ o\{ab\}.\equiv.\ \underset{2}{\rightrightarrows}\{\underset{1}{a}\underset{1}{b}\}$

a równa się [illegible] b

D) Terminy sylogistyczne:

$D5{\cdot}1\ [ab]\therefore [A]:\ \varepsilon\{Aa\}.\supset.\ \varepsilon\{Ab\}:\equiv.\ \underset{2}{\subset}\{\underset{1}{a}\underset{1}{b}\}$

Wszelkie a jest b; a sub b; subsumcja, inkluzja.

$D5{\cdot}2\ [ab]:\ !\{a\}.\subset\{ab\}.\equiv.\ \underset{2}{\sqsubset}\{\underset{1}{a}\underset{1}{b}\}$

Każde a jest b; a om b

$D5{\cdot}3.\ [ab]:\ \subset\{ab\}.\sim(\subset\{ba\}).\equiv.\ \underset{2}{\in}\{\underset{1}{a}\underset{1}{b}\}$

Inkluzja właściwa.

*276·03

$D5{\cdot}4\ [ab]:\ !\{a\}.\in\{ab\}.\equiv.\ \underset{2}{E}\{\underset{1}{a}\underset{1}{b}\}$

om właściwe.

$D5{\cdot}5\ [ab]\therefore [\exists A].\ \varepsilon\{Aa\}.\varepsilon\{Ab\}:\equiv.\ \underset{2}{\Delta}\{\underset{1}{a}\underset{1}{b}\}$

Pewne a ~~nie~~ jest b, a quod b.

$D5{\cdot}6\ [ab]:\sim(\Delta\{ab\}).\equiv.\underset{2}{\nabla}\{\underset{1}{a}\underset{1}{b}\}$

~~Żadne a nie jest b~~ Rozłączne jest a z b.

$D5{\cdot}7\ [ab]:\sim(\subset\{ab\}).\equiv.\underset{2}{\in}\{\underset{1}{a}\underset{1}{b}\}$

Pewne ~~Jakieś~~ a nie jest b

~~$D5{\cdot}8\ [ab]:!\{a\}.\sim(\subset\{ab\}).\equiv.\underset{2}{E}\{\underset{1}{a}\underset{1}{b}\}$~~

~~Pewne a nie jest b~~

~~$D5{\cdot}9\ [ab]:.[\varphi]:\varphi\{ab\}\equiv\varphi\{ba\}:\equiv.\underset{2}{Conv}\{\underset{1}{a}\underset{1}{b}\}$~~

~~Converse są a i b~~

$D5{\cdot}8\ [ab]:!\{a\}.\nabla\{ab\}.\equiv.\nabla\{ab\}$

Żadne ~~Każde~~ a nie jest b.

E). Terminy algebry logiki (zbiorów; klas):

$D6{\cdot}1\ [Aab]:\varepsilon\{Aa\}.\varepsilon\{Ab\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{\cap}(\underset{1}{a}\underset{1}{b})\}$

A jest a i b; koniunkcja międzynazwowa; iloczyn nazw.

$D6{\cdot}2\ [Aab]:.\varepsilon\{AA\}:\varepsilon\{Aa\}.\vee.\varepsilon\{Ab\}:\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{\cup}(\underset{1}{a}\underset{1}{b})\}$

A jest a lub b; alternatywa międzynazwowa; suma nazw.

$D6{\cdot}3\ [Aab]:\varepsilon\{Aa\}.\varepsilon\{A\sim(b)\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{-}(\underset{1}{a}\underset{1}{b})\}$

Różnica logiczna dwu nazw

$D6{\cdot}4\ [Aab]:\varepsilon\{A\sim(a)\}.\varepsilon\{Ab\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{dpa}(\underset{1}{a}\underset{1}{b})\}$

$D6{\cdot}5\ [Aab]:\varepsilon\{A\sim(a)\}.\varepsilon\{A\sim(b)\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{dpp}(\underset{1}{a}\underset{1}{b})\}$

$D6{\cdot}6\ [Aab]:.\varepsilon\{AA\}:\varepsilon\{Aa\}.\supset.\sim(\varepsilon\{Ab\}):\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\underset{3}{dpg}(\underset{1}{a}\underset{1}{b})\}$

$D6{\cdot}7\ [Aab] :\ \varepsilon\{AA\} \sim (\cancel{A}\{Aa\}.\vee.\varepsilon\{Ab\}). \equiv .\underset{2}{\varepsilon}\{\underset{1}{A}\ \underset{3}{\phi_{\varepsilon}}(\underset{1}{a}\underset{1}{b})\}$

I. t. d.

F.) Przyporządkowania elementarne:

$D7{\cdot}1.\ [abcd] :\ o\{ac\}.o\{bd\}. \equiv .\underset{6}{\downarrow}\{\underset{1}{a}\underset{1}{b}\}\{\underset{1}{c}\underset{1}{d}\}$

Para uporządkowana

$D7{\cdot}2\ [ABab] :\ \varepsilon\{Aa\}.\varepsilon\{Bb\}. \equiv .\underset{6}{\uparrow}\{\underset{1}{a}\underset{1}{b}\}\{\underset{1}{A}\underset{1}{B}\}$

$D7{\cdot}3\ [Aaby] :\ \varepsilon\{Aa\}.y\{Ab\}. \equiv .\underset{7}{\urcorner}[\underset{12}{\{ay\}}]\{\underset{1}{A}\underset{1}{b}\}$

$D7{\cdot}4\ [Aaby] :\ \varepsilon\{Aa\}.y\{bA\}. \equiv .\underset{7}{\ulcorner}[\underset{12}{\{ay\}}]\{\underset{1}{b}\underset{1}{A}\}$

$D7{\cdot}5\ [ABay] :\ \varepsilon\{Aa\}.\varepsilon\{Ba\}.y\{AB\}. \equiv .\underset{7}{\complement}[\underset{12}{\{ay\}}]\{\underset{1}{A}\underset{1}{B}\}$

$\mathfrak{D}7{\cdot}6\ [abAB\varphi]:\ \varepsilon\{Aa\}.\varepsilon\{Bb\}.\varphi\{AB\}.\equiv.\underset{20}{\int}\underset{1\,1\,2}{[ab\varphi]}^{c}\underset{11}{\{AB\}}$

8). Elementarne własności funkcji (zdaniotwórczej) od jednego argumentu

$\mathfrak{D}8{\cdot}1.\ [A\varphi]:\ \varepsilon\{AA\}.\varphi\{A\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\ \underset{7}{♀}\langle\underset{6}{\varphi}\rangle\}$

A spełnia funkcję φ ; A jest stsf ⟨φ⟩

$\mathfrak{D}8{\cdot}2\ [\varphi]:.\ [A]:\ \varphi\{A\}.\supset.\varepsilon\{AA\}:\equiv.\underset{8}{\Psi}\{\underset{6}{\varphi}\}$

φ jest relacją ; rel od φ

$\mathfrak{D}8{\cdot}3\ [A\varphi]:\ \varepsilon\{AA\}.\varphi\{A\}.\Psi\{\varphi\}.\equiv.\underset{2}{\varepsilon}\{\underset{1}{A}\ \underset{7}{♀}\langle\underset{6}{\varphi}\rangle\}$

A spełnia relację φ ; A jest sat ⟨φ⟩.

$\mathfrak{D}8{\cdot}4\ [A\varphi]:\ \varepsilon\{A\ ♀\langle\varphi\rangle\}.\equiv.\underset{9}{♀}\{\underset{6}{\varphi}\}\{\underset{1}{A}\}$

stsf ⟨φ⟩ od A

$\mathfrak{D}8{\cdot}5\ [A\varphi]:\ \varepsilon\{A\ ♀\langle\varphi\rangle\}.\equiv.\underset{9}{♀}\{\underset{6}{\varphi}\}\{\underset{1}{A}\}$

sat ⟨φ⟩ od A

$D8{\cdot}6.\ [\varphi]\therefore[a]:\varphi\{a\}.\supset.!\{a\}:\equiv.\underset{8}{\lambda}\{\underset{6}{\varphi}\}$

φ jest funkcją egzystencjalną.

)

$D8{\cdot}7\ [\varphi]\therefore[\exists B]\therefore\iota\{BB\}:[A]:\varphi\{A\}.\equiv.=\{AB\}\therefore\equiv.$

$\underset{8}{\leftarrow}\{\underset{6}{\varphi}\}$

φ jest relacją jedyną

H). Elementarne funkcje wieloargumentowe:

$D2$ (str. 7)

$D9{\cdot}0\ [Aa]:\iota\{A\vee(a)\}.\equiv.\underset{4}{\vee}(\underset{1}{a})\{\underset{1}{A}\}$

$D9{\cdot}1\ [AB]:\iota\{A\,\iota(B)\}.\equiv.\underset{4}{\iota}(\underset{1}{B})\{\underset{1}{A}\}$

$D9{\cdot}2\ [AB]:=\{AB\}.\equiv.\underset{4}{=}(\underset{1}{B})\{\underset{1}{A}\}$

$D9{\cdot}3\ [AB]:\neq\{AB\}.\equiv.\underset{4}{\neq}(\underset{1}{B})\{\underset{1}{A}\}$

$D9{\cdot}4\ [ab]:o\{ab\}.\equiv.\underset{4}{o}(\underset{1}{b})\{\underset{1}{a}\}$

$D9{\cdot}5\ [ab]:\theta\{ab\}.\equiv.\underset{4}{\theta}(\underset{1}{b})\{\underset{1}{a}\}$

$D9{\cdot}6\ [ab]:\subset\{ab\}.\equiv.\underset{4}{\subset}(\underset{1}{b})\{\underset{1}{a}\}$

D9.7 $[ab]: \subset\{ab\}. \equiv . \underset{4}{\subset}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

D9.8 $[ab]: \sqsubset\{ab\}. \equiv . \underset{4}{\sqsubset}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

D9.9 $[ab]: E\{ab\}. \equiv . \underset{4}{E}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

D9.10 $[ab]: \Delta\{ab\}. \equiv . \underset{4}{\Delta}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

D9.11 $[ab]: \nabla\{ab\}. \equiv . \underset{4}{\nabla}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

D9.12 $[ab]: \mathrm{Conv}\{ab\}. \equiv . \underset{4}{\mathrm{Conv}}\underset{1}{\langle b\rangle}\underset{1}{\{a\}}$

I. Epsilony dla funkcji zdaniotwórczej od jednego argumentu nazwowego:

D10.1 $[\phi\varphi] \therefore \Upsilon\{\phi\} : [\exists A]. \phi\{A\}. \varphi\{A\} \therefore [BC]: \phi\{B\}. \phi\{C\}. \supset . =\{BC\} \therefore \equiv . \underset{10}{\varepsilon}\{\underset{6}{\phi}\underset{6}{\varphi}\}$

Ograniczony ε dla funkcji.

D10.2 $[\phi\varphi] \therefore [\exists a]. \phi\{a\}. \varphi\{a\} \therefore [bc]: \phi\{b\}. \phi\{c\}. \supset . =\{bc\} \therefore \equiv . \underset{10}{\varepsilon}\{\underset{6}{\phi}\underset{6}{\varphi}\}$

Ogólny ε dla funkcji

Definicje analogiczne do A – E i H:

D10.3 $[a\psi\varphi]: \varepsilon\{\psi\varphi\}. \psi\{a\}. \equiv . \underset{11}{*}[\underset{6}{\psi}\underset{6}{\varphi}]\underset{1}{\{a\}}$

Odpowiednik: $[\phi\psi\varphi]: \varepsilon\{\psi\phi\}. \varepsilon\{\phi\varphi\}. \equiv . \varepsilon\{\phi * [\psi\varphi]\}$
Inf. D1.

D10.4 $[\phi\varphi]: \varepsilon\{\phi\varphi\}. \equiv . \underset{12}{\varepsilon}\underset{6}{\langle\varphi\rangle}\underset{6}{\{\phi\}}$
inf. D2.

$D10{\cdot}5\ [a\varphi]: \sim(\varphi\{a\}) . \equiv . \underset{9}{\sim}\underset{6}{[E\varphi\exists]}\{\underset{1}{a}\}$

Odpowiednik: $[\varphi\psi]: \varepsilon\{\varphi\varphi\}.\sim(\varepsilon\{\varphi\psi\}) . \equiv . \varepsilon\{\varphi \sim E\psi\exists\}$

Cnf. D3·1

$D10{\cdot}6.\ [\varphi] :: [\exists\varphi] . \varepsilon\{\varphi\varphi\} : \equiv . \underset{8}{!}\{\underset{6}{\varphi}\}$

Odpowiednik: $[\varphi] :: [\exists a] . \varphi\{a\} : \equiv . !\{\varphi\}$

Cnf. D-3·3.

$D10{\cdot}7.\ [\varphi] :: [\varphi\psi] : \varepsilon\{\varphi\varphi\} . \varepsilon\{\psi\varphi\} . \supset . \varepsilon\{\varphi\psi\} : \equiv . \underset{8}{\rightarrow}\{\underset{6}{\varphi}\}$

Odpowiednik: $[\varphi] :: [ab] : \varphi\{a\} . \varphi\{b\} . \supset . o\{ab\} : \equiv . \rightarrow\{\varphi\}$

Cnf. D3·4.

$D10{\cdot}8.\ [a]: o\{aa\} . \equiv . \underset{6}{V}\{\underset{1}{a}\}$

Odpowiednik: $[\varphi]: \varepsilon\{\varphi\varphi\} . \equiv . \varepsilon\{\varphi V\}$

Cnf.: D3·5

$D10{\cdot}9\ [a]: \sim(o\{aa\}) . \equiv . \underset{6}{\Lambda}\{\underset{1}{a}\}$

Odpowiednik: $[\varphi]: \varepsilon\{\varphi\varphi\}.\sim(\varepsilon\{\varphi\varphi\}) . \equiv . \varepsilon\{\varphi\Lambda\}$

Cnf.: D3·6

$D10{\cdot}10\ [a\psi]: \psi\{a\} . \rightarrow\{\psi\} . \equiv . \underset{9}{\iota}\underset{6}{[E\psi\exists]}\{\underset{1}{a}\}$

Odpowiednik: $[\varphi\psi]: \varepsilon\{\varphi\psi\} . \varepsilon\{\psi\psi\} . \equiv . \varepsilon\{\varphi\ \iota E\psi\exists\}$

Cnf.: D3·7

$D10{\cdot}11\ [\varphi\psi] :: \varepsilon\{\varphi\psi\} . \varepsilon\{\psi\varphi\} . \equiv . \underset{10}{=}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Odpowiednik: $[\varphi\psi] :: [\exists a] . \varphi\{a\} . \psi\{a\} : [ab] : \varphi\{a\} . \psi\{b\} . \supset . o\{ab\} :: \equiv . =\{\varphi\psi\}$

Cnf.: D4·1

$D10{\cdot}12\ [\varphi\psi]: \varepsilon\{\varphi\varphi\} . \varepsilon\{\psi\psi\} . \sim(=\{\varphi\psi\}) . \equiv . \underset{10}{\neq}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Odpowiednik: $[\varphi\psi] :: \rightarrow\{\varphi\} . \rightarrow\{\psi\} : [\exists a] . \varphi\{a\} . \sim(\psi\{a\}) : [\exists b] . \psi\{b\} : \equiv . \underset{10}{\neq}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Cnf.: D4·2

$D10{\cdot}13\ [\varphi\psi] :: [\Phi]: \varepsilon\{\Phi\varphi\} . \equiv . \varepsilon\{\Phi\psi\} : \equiv . \underset{10}{o}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Odpowiednik: $[\varphi\psi] :: [a]: \varphi\{a\} . \equiv . \psi\{a\} : \equiv . o\{\varphi\psi\}$

Cnf.: D4·3

$D10{\cdot}14\ [\varphi\psi]:\sim(\circ\{\varphi\psi\}).\equiv.\underset{10}{\Theta}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Odpowiednik $[\varphi\psi]:\sim([a]:\varphi\{a\}.\equiv.\psi\{a\}).\equiv.\Theta\{\varphi\psi\}$

Inf.: D4·4 I.t.d.

$D10{\cdot}15\ [\varphi\psi]\therefore[\phi]:\varepsilon\{\phi\varphi\}.\supset.\varepsilon\{\phi\psi\}:\equiv.\underset{10}{\subset}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Odp.: $[\varphi\psi]\therefore[a]:\varphi\{a\}.\supset.\psi\{a\}:\equiv.\subset\{\varphi\psi\}$

Inf.: D5·1

$D10{\cdot}16\ [\varphi\psi]:!\{\varphi\}.\subset\{\varphi\psi\}.\equiv.\underset{10}{\sqsubset}\{\underset{6}{\varphi}\underset{6}{\psi}\}$

Inf.: D5·2

$D10{\cdot}16a\ [\varphi\chi]:\subset\{\varphi\chi\}.\sim(\subset\{\chi\varphi\}).\equiv.\underset{10}{\in}\{\underset{6}{\varphi}\underset{6}{\chi}\}$

Inf.: D5·3

$D10{\cdot}17\ [\varphi\chi]:!\{\varphi\}.\in\{\varphi\chi\}.\equiv.\underset{10}{E}\{\underset{6}{\varphi}\underset{6}{\chi}\}$

Inf.: D5·4

$D10{\cdot}18\ [\varphi\psi]\therefore[\exists\phi]:\varepsilon\{\phi\varphi\}.\varepsilon\{\phi\psi\}:\equiv.\underset{10}{\Delta}\{\underset{6}{\varphi}\underset{6}{\chi}\}$

Odp.: $[\varphi\psi]\therefore[\exists a].\varphi\{a\}.\chi\{a\}:\equiv.\Delta\{\varphi\chi\}$

Inf.: D5·5

$D10{\cdot}19\ [\varphi\chi]:\sim(\Delta\{\varphi\chi\}).\equiv.\underset{10}{\nabla}\{\underset{6}{\varphi}\underset{6}{\chi}\}$

Odp. D5·6

I.t.d.

$D10{\cdot}20\ [\varphi\psi a]:\varphi\{a\}.\psi\{a\}.\equiv.\underset{13}{\frown}[\underset{6}{\varphi}\underset{6}{\psi}]\{\underset{1}{a}\}$

Odp.: $[\phi\varphi\chi]:\varepsilon\{\phi\varphi\}.\varepsilon\{\phi\chi\}.\equiv.\varepsilon\{\phi\frown[\varphi\chi]\}$

Inf.: D6·1

$D10{\cdot}21\ [\varphi\psi a]:\varphi\{a\}.\vee.\psi\{a\}:\equiv.\underset{13}{\smile}[\underset{6}{\varphi}\underset{6}{\psi}]\{\underset{1}{a}\}$

Odp.: $[\phi\varphi\psi]\therefore\varepsilon\{\phi\phi\}:\varepsilon\{\phi\varphi\}.\vee.\varepsilon\{\phi\psi\}:\equiv.\varepsilon\{\phi\smile[\varphi\psi]\}$

Inf.: D·6·2.

$D10{\cdot}22\ [\varphi\psi a]:\varphi\{a\}.\sim[\psi]\{a\}.\equiv.\underset{13}{-}[\underset{6}{\varphi}\underset{6}{\psi}]\{\underset{1}{a}\}$

Inf. D·6·3

I.t.d.

$D10{\cdot}23\ [\phi\psi]:\varepsilon\{\phi\cdot[\psi]\}.\equiv.\underset{14}{\varepsilon}[\underset{6}{\psi}]\{\underset{6}{\phi}\}$

Inf.: D9·1

I.t.d.

$\mathfrak{D}10{\cdot}24\,[\psi x] \therefore [\phi] : \phi\{\psi x\} . \equiv . \phi\{x\psi\} : \equiv . \underset{10}{\mathrm{Conv}}\{\underset{6}{\psi}\,\underset{6}{x}\}$

Cmp.: $\mathfrak{D}5{\cdot}9$

$\mathfrak{D}10{\cdot}25\,[a\varphi] : [\exists\phi] . \phi\{a\} . \varphi[\phi] . \varepsilon\{\phi\phi\} .$

$\equiv . \cancel{\varepsilon\{\phi} \; \cancel{\mathrm{sq}} \, K\varphi\rangle\{a\}$

Odp.: $[\phi\varphi] : \varepsilon\{\phi\phi\} . \varphi[\phi] . \equiv . \varepsilon\{\phi\,\cancel{\mathrm{sq}}\,K\varphi\rangle\}$

Cf. $\mathfrak{D}$. ~~≡~~ 8·1.

7. Epsilony dla funkcji zdaniotwórczej od dwu argumentów nazwowych:

$\mathfrak{D}11{\cdot}1\,[\phi\varphi] \therefore [AB] : \phi\{AB\} . \supset . \varepsilon\{AA\} . \varepsilon\{BB\} : [\exists AB] .$

$\phi\{AB\} . \varphi\{AB\} \therefore [DEFG] : \phi\{DE\} . \phi\{FG\} . \supset .$

$= \{DF\} . = \{EG\} :: \equiv . \underset{15}{\varepsilon_1}\{\underset{2}{\phi}\,\underset{2}{\varphi}\}$

Wyznaczony ε dla funkcji.

$\mathfrak{D}11{\cdot}2\,[\phi\varphi] \therefore [\exists ab] . \phi\{ab\} . \varphi\{ab\} : . [cdef] : \phi\{cd\} .$

$\phi\{ef\} . \supset . \mathrm{o}\{ce\} . \mathrm{o}\{df\} :: \equiv . \underset{15}{\varepsilon}\{\underset{2}{\phi}\,\underset{2}{\varphi}\}$

Ogólny ε dla funkcji.

Definicje analogiczne do A-E:

$\mathfrak{D}11{\cdot}3\,[ab\psi\varphi] : \varepsilon\{\psi\varphi\} . \psi\{ab\} . \equiv . \underset{16}{*}\{\underset{2}{\psi}\,\underset{2}{\varphi}\}\{\underset{1}{a}\,\underset{1}{b}\}$

Cmp.: $\mathfrak{D}1$ i $\mathfrak{D}10{\cdot}3$

$\mathfrak{D}11{\cdot}4\,[\phi\varphi] : \varepsilon\{\phi\varphi\} . \equiv . \underset{17}{\varepsilon}\{\underset{2}{\varphi}\}\{\underset{2}{\phi}\}$

Cmp.: $\mathfrak{D}2$

D11·5 $[ab\varphi]:\sim(\varphi\{ab\}).\equiv.\underset{18}{\nu}\underset{2}{[\varphi]}\underset{1\,1}{\{ab\}}$

Inf.: D3·2

D11·6 $[\varphi]::[\exists\phi].\varepsilon\{\phi\varphi\}:\equiv.\underset{19}{!}\underset{2}{\{\varphi\}}$

Inf.: D3·3

D11·7 $[\varphi]::[\phi\psi]:\varepsilon\{\phi\varphi\}.\varepsilon\{\psi\varphi\}.\supset.\varepsilon\{\phi\psi\}:\equiv.\underset{19}{\rightarrow}\underset{2}{\{\varphi\}}$

Inf.: D3·4

D11·8 $[ab]:o\{aa\}.o\{bb\}.\equiv.\underset{2}{V}\underset{1\,1}{\{ab\}}$

Inf.: D3·5

D11·9 $[ab]:\sim(o\{aa\}.o\{bb\}).\equiv.\underset{2}{\Lambda}\underset{1\,1}{\{ab\}}$

Inf.: D3·6

D11·10 $[ab\varphi]:\varphi\{ab\}.\rightarrow\{\varphi\}.\equiv.\underset{18}{\iota}\underset{2}{[\varphi]}\underset{1\,1}{\{ab\}}$

Inf.: D3·7

D11·11 $[\phi\psi]:\varepsilon\{\phi\psi\}.\varepsilon\{\psi\phi\}.\equiv.\underset{15}{=}\underset{2\,2}{\{\phi\psi\}}$

Inf.: D4·1

D11·12 $[\phi\psi]:\varepsilon\{\phi\phi\}.\varepsilon\{\psi\psi\}.\sim(\varepsilon\{\phi\psi\}).\equiv.\underset{15}{\neq}\underset{2\,2}{\{\phi\psi\}}$

Inf.: D4·2.

D11·13 $[\varphi\psi]::[\phi]:\varepsilon\{\phi\varphi\}.\equiv.\varepsilon\{\phi\psi\}:\equiv.\underset{15}{o}\underset{2\,2}{\{\varphi\psi\}}$

Inf.: D4·3.

D11·14 $[\varphi\psi]:\sim(o\{\varphi\psi\}).\equiv.\underset{15}{\ominus}\underset{2\,2}{\{\varphi\psi\}}$

Inf.: D4·4.

D11·15 $[\varphi\psi]::[\phi]:\varepsilon\{\phi\varphi\}.\supset.\varepsilon\{\phi\psi\}:\equiv.\underset{15}{\subset}\underset{2\,2}{\{\varphi\psi\}}$

Inf.: D5·1.

D11·16 $[\varphi\psi]:!\{\varphi\}.\subset\{\varphi\psi\}.\equiv.\underset{15}{\sqsubset}\underset{2\,2}{\{\varphi\psi\}}$

Inf.: D5·2

D11·16a $[\varphi\chi]:\subset\{\varphi\chi\}.\sim(\subset\{\chi\varphi\}).\equiv.\underset{15}{\underline{\subset}}\underset{2\,2}{\{\varphi\chi\}}$

Inf.: D5·3

D11·17 $[\varphi\chi]:!\{\varphi\}.\in\{\varphi\chi\}.\equiv.\underset{15}{\mathrm{E}}\underset{2\,2}{\{\varphi\chi\}}$

Inf.: D5·4

D11·18 $[\varphi\psi]::[\exists\phi].\varepsilon\{\phi\varphi\}.\varepsilon\{\phi\psi\}:\equiv.\underset{15}{\Delta}\underset{2\,2}{\{\varphi\psi\}}$

Inf.: D5·5

$D11{\cdot}19\ [\varphi x] : \sim(\Delta\{\varphi x\}) . \equiv . \underset{15}{\nabla}\{\underset{2\,2}{\varphi x}\}$

Inf.: D5·6

I.t.d.

$D11{\cdot}20\ [\varphi\psi ab] : \varphi\{ab\} . \psi\{ab\} . \equiv . \underset{16}{\wedge}[\underset{2\,2}{\varphi\psi}]\{\underset{1\,1}{ab}\}$

Inf.: D6·1

$D11{\cdot}21\ [\varphi\psi ab] :. \varphi\{ab\} . \vee . \psi\{ab\} : \equiv . \underset{16}{\vee}[\underset{2\,2}{\varphi\psi}]\{\underset{1\,1}{ab}\}$

Inf.: D6·2

$D11{\cdot}22\ [\varphi\psi ab] : \varphi\{ab\} . \sim[\psi]\{ab\} . \equiv . \underset{16}{-}[\underset{2\,2}{\varphi\psi}]\{\underset{1\,1}{ab}\}$

Inf.: D6·3

I.t.d.

$D11{\cdot}23 . [\varphi\chi] :. [\phi] : \varphi\{\phi\chi\} . \equiv . \phi\{\varphi\chi\} : \equiv . \underset{15}{\mathrm{Conv}}\{\underset{2\,2}{\varphi\chi}\}$

Inf.: D5·9

K. Elementarne własności funkcji zdaniotwórczej od dwu argumentów nazwowych:

$D12{\cdot}1\ [AB] : \varepsilon\{AA\} . \varepsilon\{BB\} . \varphi\{AB\} . \equiv . \underset{18}{\mathrm{sat}}[\underset{2}{\varphi}]\{\underset{1\,1}{AB}\}$

A i B spełniają funkcję φ

$D12{\cdot}2\ [\varphi] :. [AB] : \varphi\{AB\} . \supset . \varepsilon\{AA\} . \varepsilon\{BB\} : \equiv .$

$\underset{19}{\mathrm{Y}}\{\underset{2}{\varphi}\}$

φ jest relacją

$D12{\cdot}3\ [\varphi] :. [Aa] : \varphi\{Aa\} . \supset . \varepsilon\{AA\} : \equiv . \underset{19}{\mathrm{Y}}\{\underset{2}{\varphi}\}$

φ jest relacją ze względu na pierwszy argument

$D12{\cdot}4\ [\varphi] :. [aB] : \varphi\{aB\} . \supset . \varepsilon\{BB\} : \equiv . \underset{19}{\mathrm{Y}}\{\underset{2}{\varphi}\}$

φ jest relacją ze względu na drugi argument

D12.5 $[\varphi]\therefore[ab]:\varphi\{ab\}.\supset.!\{a\}.!\{b\}:\equiv.\underset{19}{\lambda}\{\underset{2}{\varphi}\}$

φ jest funkcją egzystencjonalną.

D12.6 $[\varphi]\therefore[ab]:\varphi\{ab\}.\supset.!\{a\}:\equiv.\underset{19}{\lambda}\{\underset{2}{\varphi}\}$

φ jest funkcją egzystencjalną ze względu na pierwszy argument.

D12.7 $[\varphi]\therefore[ab]:\varphi\{ab\}.\supset.!\{b\}:\equiv.\underset{19}{\lambda}\{\underset{2}{\varphi}\}$

φ jest funkcją egzystencjalną ze względu na drugi argument.

D12.7 $[\varphi]\therefore[\exists CD]\therefore\varepsilon\{CC\}.\varepsilon\{DD\}:[AB]:\varphi\{AB\}.$
$\equiv.=\{AC\}.=\{BD\}\therefore\equiv.\underset{19}{\leftharpoondown}\{\underset{2}{\varphi}\}$

φ jest relacją jedyną

D12.8 $[AB\varphi]:\Psi\{\varphi\}.\varphi\{AB\}.\equiv.\underset{18}{\mathring{|}}[\underset{2}{\varphi}]\{\underset{1}{A}\underset{1}{B}\}$

Sat φ od A i B

D12.9 $[\varphi ab]:\varphi\{ab\}.\equiv.\underset{18}{\Upsilon}[\underset{2}{\varphi}]\{\underset{1}{a}\underset{1}{b}\}$

Konwers funkcji φ.

D12.10 $[\varphi]\therefore[abc]:\varphi\{ab\}.\varphi\{ac\}.\supset.o\{bc\}:\equiv.\underset{19}{\rightrightarrows}\{\underset{2}{\varphi}\}$

Funkcja wielojednoznaczna.

D12.11 $[\varphi]\therefore[abc]:\varphi\{ac\}.\varphi\{bc\}.\supset.o\{ac\}:\equiv.\underset{19}{\leftleftarrows}\{\underset{2}{\varphi}\}$

Funkcja jednowieloznaczna

D12.13 $[\varphi]:\rightrightarrows\{\varphi\}.\leftleftarrows\{\varphi\}.\equiv.\underset{19}{\rightleftarrows}\{\underset{2}{\varphi}\}$

Funkcja jednojednoznaczna

D12.14 $[\varphi]\therefore[ab]:o\{ab\}.\supset.\varphi\{ab\}:\equiv.\underset{19}{Arw}\{\underset{2}{\varphi}\}$

Funkcja absolutnie samozwrotna.

Fil. 3278 Arch. lub Kom. egram.
Fol. 2410

Bestwa 5/5
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$\mathfrak{D}12\cdot15\ [\varphi]:[\exists a].\varphi\{aa\}.\equiv.\underset{19}{\mathrm{Zw}}\{\underset{2}{\varphi}\}$

Funkcja zwrotna.

$\mathfrak{D}12\cdot16\ [\varphi]:[a].\sim(\varphi\{aa\}).\equiv.\underset{19}{\mathrm{ANzw}}\{\underset{2}{\varphi}\}$

Funkcja niezwrotna absolutnie.

$\mathfrak{D}12\cdot17\ [\varphi]:[\exists a].\sim(\varphi\{aa\}).\equiv.\underset{19}{\mathrm{Nzw}}\{\underset{2}{\varphi}\}$

Funkcja niezwrotna

$\mathfrak{D}12\cdot18\ [\varphi]\therefore[ab]:\varphi\{ab\}.\supset.\varphi\{ba\}:\equiv.\underset{19}{\mathrm{Sym}}\{\underset{2}{\varphi}\}$

Funkcja symetryczna

$\mathfrak{D}12\cdot19\ [\varphi]:[\exists ab].\varphi\{ab\}.\sim(\varphi\{ba\}).\equiv.\underset{19}{\mathrm{Nsym}}\{\underset{2}{\varphi}\}$

Funkcja niesymetryczna

$\mathfrak{D}12\cdot20\ [\varphi]\therefore[ab]:\varphi\{ab\}.\supset.\sim(\varphi\{ba\}):\equiv.\underset{19}{\mathrm{Asym}}\{\underset{2}{\varphi}\}$

Funkcja asymetryczna

$\mathfrak{D}12\cdot21\ [\varphi]\therefore[abc]:\varphi\{ab\}.\varphi\{bc\}.\supset.\varphi\{ac\}:\equiv.\underset{19}{\mathrm{Przech}}\{\underset{2}{\varphi}\}$

Funkcja przechodnia.

$\mathfrak{D}12\cdot22\ [\varphi]\therefore[\exists abc]:\varphi\{ab\}.\varphi\{bc\}.\sim(\varphi\{ac\}):\equiv.\underset{19}{\mathrm{Nprzech}}\{\underset{2}{\varphi}\}$

Funkcja nieprzechodnia.

$\mathfrak{D}12\cdot23\ [\varphi]\therefore[abc]:\varphi\{ab\}.\varphi\{bc\}.\supset.\sim(\varphi\{ac\}):\equiv.\underset{19}{\mathrm{Aprzech}}\{\underset{2}{\varphi}\}$

Funkcja aprzechodnia.

$\mathfrak{D}12\cdot24\ [\varphi]\therefore[ab]:\varphi\{ab\}.\vee.\varphi\{ba\}:\equiv.\underset{19}{\mathrm{Spjt}}\{\underset{2}{\varphi}\}$

Funkcja spójna.

$\mathfrak{D}12\cdot25.\ [\varphi]\therefore[ab]:\varphi\{ab\}.\supset.[\exists c].\varphi\{ac\}.\varphi\{cb\}:\equiv.\underset{19}{\mathrm{Zwt}}\{\underset{2}{\varphi}\}$

Funkcja zwarta. * 270·01

§3. Funkcje równościowe.

T1. $[\varphi]\therefore[a].\varphi\{aa\}:\equiv:[ab]:o\{ab\}.\supset.\varphi\{ab\}$

T2. $[\varphi]:[a].\varphi\{aa\}:\equiv.\mathrm{Arw}\{\varphi\}$

(T1, D12·14)

Df. funkcji absolutnie samozwrotnej.

Df. funkcji symetrycznej = D12·18

Df. funkcji przechodniej = D12·21

Df. funkcji równościowej:

D12·26. $[\varphi]\therefore[ab]\therefore\varphi\{ab\}.\equiv:[c]:\varphi\{cb\}.\supset.\varphi\{ca\}\therefore\equiv.\mathrm{Równ}.\{\varphi\}$

T2. $[\varphi]:\mathrm{Równ}\{\varphi\}.\equiv.\mathrm{Arw}\{\varphi\}.\mathrm{Sym}\{\varphi\}.\mathrm{Przech}\{\varphi\}$

(D12·26; T1; D12·18; D12·21)

Df. funkcji absolutnie samozwrotnej i przechodniej:

D12·27 $[\varphi]\therefore[ab]\therefore\varphi\{ab\}.\equiv:[c]:\varphi\{ca\}.\supset.\varphi\{cb\}\therefore\equiv\mathrm{Asamprzech}\{\varphi\}$.

T3. $[\varphi]:\mathrm{Asamprzech}\{\varphi\}.\equiv.\mathrm{Arw}\{\varphi\}.\mathrm{Przech}\{\varphi\}$.

(D12·17; T1; D12·21)

Df. funkcji absolutnie samozwrotnej i ~~przechodniej~~ symetrycznej:

D12·28 $[\varphi]\therefore[ab]\therefore\sim(\varphi\{aa\}).\vee.\varphi\{ab\}:\supset.\varphi\{ba\}\therefore\equiv.\mathrm{Absamsym}\{\varphi\}$

T4. $[\varphi]:\mathrm{Absamsym}\{\varphi\}.\equiv.\mathrm{Arw}\{\varphi\}.\mathrm{Sym}\{\varphi\}$

(D12·28; T1; D12·18)

Df. funkcji symetrycznej i przechodniej:

D12·29 $[\varphi]\therefore[ab]\therefore\varphi\{ab\}.\vee.\varphi\{ba\}:\varphi\{cb\}:\supset.\varphi\{ac\}\therefore\equiv.\mathrm{Symprzech}\{\varphi\}$

T5 $[\varphi]:\mathrm{Symprzech}\{\varphi\}.\equiv.\mathrm{Sym}\{\varphi\}.\mathrm{Przech}\{\varphi\}$

D12·30. $[\varphi]\therefore[ab]:\varphi\{ab\}.\sim(a\,o\,b).\supset.\varphi\{ba\}:\equiv.\mathrm{napół.sym}\{\varphi\}$

Df. funkcji napółsymetrycznej.

Df. 12·31. $[\varphi] \therefore [abc] : \varphi\{ab\} . \varphi\{bc\} . \sim(a \circ c) . \supset . \varphi\{ac\} : \equiv . \text{Alioprzech}\{\varphi\}$

Df. Funkcja alioprzechodnia.

T6. $[\varphi\chi] : \subset\{\varphi\chi\} . \text{aprzech}\{\chi\} . \supset . \text{aprzech}\{\varphi\}$

T7 $[\varphi\chi] : \subset\{\varphi\chi\} . \text{asym}\{\chi\} . \supset . \text{asym}\{\varphi\}$

1965

ε_1 — wyróżniony

D1. $[\phi\varphi] \therefore [ab] : \phi\{a\} . \phi\{b\} . \supset . a \circ b : \Psi\{\phi\} . \Psi\{\varphi\} . [\exists A] . \phi\{A\} . \varphi\{A\} : \equiv . \varepsilon_1\{\phi\varphi\}$

D2. $[\varphi] : \Psi\{\varphi\} . [\exists A] . \varphi\{A\} . \equiv . !_1\{\varphi\}$

D3. $[\varphi] \therefore [AB] : A \varepsilon_1 \text{sat}\langle\varphi\rangle . B \varepsilon_1 \text{sat}\langle\varphi\rangle . \supset . A = B : \equiv . \longrightarrow\{\varphi\}$

D4. $[\varphi A] \therefore A \varepsilon_1 A : \sim(\varphi\{A\}) . \vee . \sim(\Psi\{\varphi\}) : \equiv . \sim_1 E\varphi \exists\{A\}$

81 blank pages follow.

Dr Rudolf Carnap

Abriss der Logistik
mit besonderer Berücksichtigung der Relationstheorie und ihrer Anwendungen

Wien, Verlag von Julius Springer. 1929.

$$D7{\cdot}01 \quad x = y . =Df. (\varphi) . \varphi x \supset \varphi y \quad \{\neq, I, J$$

$$D7{\cdot}02 \quad E!(\iota x)(\varphi x) . =Df: (\exists u): \varphi x . \equiv_x . x = u$$

$$L7{\cdot}1 \quad E!(\iota x)(\varphi x) . \supset: (z) . \varphi z \supset \psi (\iota x)(\varphi x)$$

$$L7{\cdot}2 \quad \psi(\iota x)(\varphi x) . \supset . E!(\iota x)(\varphi x)$$

$$D8{\cdot}01 \quad x \,\varepsilon\, \hat{z}(\varphi z) . =Df. \varphi x$$

$$D8{\cdot}021 \quad x, y \,\varepsilon\, \alpha . =Df. x \,\varepsilon\, \alpha . y \,\varepsilon\, \alpha$$

$$D8{\cdot}022 \quad x \sim\varepsilon\, \alpha . =Df. \sim(x \,\varepsilon\, \alpha)$$

$$L8{\cdot}1 \quad \varphi x \equiv_x \psi x . \equiv . \hat{z}(\varphi z) = \hat{z}(\psi z)$$

$$D10{\cdot}01 \quad \alpha \subset \beta . =Df: x \,\varepsilon\, \alpha . \supset_x . x \,\varepsilon\, \beta$$

D10·021 $\alpha \frown \beta = Df . \hat{x}(x \in \alpha . x \in \beta)$

D10·022 $\alpha \smile \beta = Df\ \hat{x}(x \in \alpha . \vee . x \in \beta)$

D10·023 $-\alpha = Df\ \hat{x}(x \sim \in \alpha)$

D10·024 $\alpha - \beta = Df\ \alpha \frown - \beta$

L10·1 $\alpha \subset \beta . \beta \subset \alpha . \equiv . \alpha = \beta$

L10·21 $\alpha \frown \beta = \beta \frown \alpha$

L10·22 $(\alpha \frown \beta) \frown \gamma = \alpha \frown (\beta \frown \gamma)$

L10·23 $(\alpha \frown \beta) \smile (\alpha \frown \gamma) = \alpha \frown (\beta \smile \gamma)$

L10·31 $\alpha \subset \beta . \beta \subset \gamma . \supset . \alpha \subset \gamma$

L10·32 $\alpha \subset \beta . x \in \alpha . \supset . x \in \beta$

D10·031 $V = Df\ \hat{x}(x = x)$

D10·032 $\Lambda = Df - V$

D10·033 $\exists ! \alpha = Df . (\exists x) . x \in \alpha$

D10·034 $\alpha\ Fr\ \beta . = Df . \alpha \frown \beta = \Lambda$

„Fr" = „Fremd"

L10·41 $\alpha \subset V$

L10·42 $\Lambda \subset \alpha$

L10·43 $\exists ! \alpha . \equiv . \alpha \neq \Lambda$

L10·44 $\alpha\ Fr\ \beta . \equiv . \alpha \subset - \beta$

D10·041 $p'\kappa = Df\ \hat{x}(\alpha \in \kappa . \supset_\alpha . x \in \alpha)$

D10·042 $s'\kappa = Df\ \hat{x}\{(\exists \alpha) . \alpha \in \kappa . x \in \alpha\}$

„$p'\kappa$" = „Durchschnitt" oder „Produkt"

„$s'\kappa$" = „Vereinigung" oder „Summe"

D10·051 $Cl'\alpha = Df\ \hat{\beta}(\beta \subset \alpha)$

„$Cl'\alpha$" = „Potenzmenge"

D10·052 $Cl\,ex'\alpha = Df\ \hat{\beta}(\beta \subset \alpha . \exists ! \beta)$

D11·01 $u\ \hat{x}\hat{y}(\varphi x y)\ v . = Df . \varphi u v$

L11·1 $\varphi u v . \equiv_{u,v} . \psi u v : \equiv . \hat{x}\hat{y}(\varphi x y) = \hat{x}\hat{y}(\psi x y)$

D12·01 $R \subset S . \equiv Df\ x R y . \supset_{x,y} . x S y$

D12·021 $R \dot{\frown} S = Df\ \hat{x}\hat{y}(x R y . x S y)$

D12·022 $R \dot\cup S . =_{Df} \hat{x}\hat{y}\,(xRy . \lor . xSy)$

D12·023 $\dot{-} R =_{Df} \hat{x}\hat{y}\,\{\sim(xRy)\}$

D12·024 $R \dot{-} S =_{Df} R \dot\cap \dot{-} S$

L12·1 $R \subset S . S \subset R . \equiv . R = S$

L12·21 $R \dot\cap S = S \dot\cap R$

L12·22 $(R \dot\cap S) \dot\cap T = R \dot\cap (S \dot\cap T)$

L12·23 $(R \dot\cap S) \dot\cup (R \dot\cap T) = R \dot\cap (S \dot\cup T)$

L12·31 $R \subset S . S \subset T . \supset . R \subset T$

L12·32 $R \subset S . xRy . \supset . xSy$

D12·033 $\dot\exists ! R . =_{Df} . (\exists x, y) . xRy$

L12·43 $\dot\exists ! R . \equiv . R \neq \dot\Lambda$

D14·01 $R'y =_{Df} (\imath x)(xRy)$

D14·02 $R'S'y =_{Df} R'(S'y)$

L14·1 $x = R'y . \equiv : (z) : zRy . \equiv . z = x$

L14·2 $E! R'y . \equiv :. (\exists x) :. (z) : zRy . \equiv . z = x$

D14·03 $\overrightarrow{R}'y =_{Df} \hat{x}\,(xRy)$ $\quad \overrightarrow{R} = Sg'R$

D14·04 $\overleftarrow{R}'x =_{Df} \hat{y}\,(xRy)$ $\quad \overleftarrow{R} = Gs'R$

D14·05 $R''\beta =_{Df} \hat{x}\{(\exists y) . y \in \beta . xRy\}$

D14·06 $R_\epsilon \beta =_{Df} R''\beta$

L14·3 $x \in \overrightarrow{R}'y . \equiv . xRy$

L14·4 $y \in \overleftarrow{R}'x . \equiv . xRy$

D15·01 $\breve{R} =_{Df} \hat{x}\hat{y}\,(yRx)$ $\quad \{ Cnv'R$

L15·11 $x \breve{R} y . \equiv . yRx$

L15·12 $\breve{\breve{R}} = R$

D15·02 $sym =_{Df} \hat{R}(\breve{R} \subset R)$

D15·03 $as =_{Df} \hat{R}(\breve{R} \subset \dot{-} R)$

Relacje: symetryczna (sym), asymetryczna (as), niesymetryczna —

D15·041 $D'R =_{Df} \hat{x}\{(\exists x) . xRy\}$

D15·042 $G'R =_{Df} \hat{y}\{(\exists x) . xRy\}$

D15·043 $C'R =_{Df} D'R \cup a'R$

D15·044 $F =_{Df} \hat{x}\hat{R}(x \in C'R)$

L15·21 $a'R = D'\breve{R}$

L15·22 $a'R \subset D'R .\equiv. D'R = C'R$

L15·23 $D'R \subset a'R .\equiv. a'R = C'R$

D15·051 $\alpha \upharpoonleft R =_{Df} \hat{x}\hat{y}(x \in \alpha . xRy)$

D15·052 $R \upharpoonright \beta =_{Df} \hat{x}\hat{y}(xRy . y \in \beta)$

D15·053 $\alpha \upharpoonleft R \upharpoonright \beta =_{Df} \hat{x}\hat{y}(x \in \alpha . xRy . y \in \beta)$

D15·054 $R \upharpoonright\!\!\upharpoonright \alpha =_{Df} \alpha \upharpoonleft R \upharpoonright \alpha$

D15·06 $\alpha \uparrow \beta =_{Df} \hat{x}\hat{y}(x \in \alpha . y \in \beta)$

„$\alpha \upharpoonleft R$" = „R vorbeschränkt auf α"; „$R \upharpoonright \beta$" = „R nachbeschränkt auf β"; „$\alpha \upharpoonleft R \upharpoonright \beta$" = „$R$ beschränkt auf α und β"; „$R \upharpoonright\!\!\upharpoonright \alpha$" = „$R$ beschränkt auf γ"

D16·01 $R|S =_{Df} \hat{x}\hat{z}\{(\exists y) . xRy . ySz\}$

„$R|S$" = „Verkettung" oder „Relationsprodukt"

D16·02 $R^0 =_{Df} I \upharpoonright C'R$

„$R^2 = R|R$; $R^2|R = R^3$ etc. ...)

L16·11 $Cnv'(R|S) = \breve{S}|\breve{R}$

L16·12 $(P|Q)|R = P|(Q|R)$

L16·21 $P|(Q \dot{\cup} R) = P|Q \dot{\cup} P|R$

L16·22 $(P \dot{\cup} Q)|R = P|R \dot{\cup} Q|R$

L16·23 $P|(Q \dot{\cap} R) \subset P|Q \dot{\cap} P|R$

L16·24 $(P \dot{\cap} Q)|R \subset P|R \dot{\cap} Q|R$

L16·31 $\exists ! a'R \cap D'S .\equiv. \dot{\exists} ! R|S$

L16·32 $D'(R|S) \subset D'R . a'(R|S) \subset a'S$

L16·33 $E!P'Q'z .\supset. P'Q'z = (P|Q)'z$

L16·34 $(R|S)''\gamma = R''S''\gamma$

L16·41 $R^0|R = R|R^0 = R$

L16·42 $\breve{\varepsilon}|\varepsilon = \dot{-}\,Fr$

D16·031 $trans =_{Df} \hat{R}(R^2 \subset R)$

„trans" = „transitiv"

$\mathcal{D}16\cdot032 \quad \mathrm{intr} =_{Df} \hat{R}(R^2 \subset -R)$

„intr" = „intransitiv"

„nicht-transitiv" —

„aliotransitiv" — $\hat{R}(R^2 - J \subset R)$ §16b

$\mathcal{L}16\cdot5 \quad \mathrm{Cnv}``\mathrm{trans} \subset \mathrm{trans}$

$\mathcal{D}16\cdot041 \quad \mathrm{refl} =_{Df} \hat{R}(R^0 \subset R)$ reflexiv

$\mathcal{D}16\cdot042 \quad \mathrm{reflex} =_{Df} \hat{R}(J \subset R)$ totalreflexiv

$\mathcal{D}16\cdot043 \quad \mathrm{irr} =_{Df} \hat{R}(R \subset \bar{J})$ irreflexiv

„nicht-reflexiv" —

$\mathcal{L}16\cdot61 \quad R \in \mathrm{refl} . \equiv . R^0 \subset R . \equiv :[x]: x \in C`R . \supset . xRx$

$\mathcal{L}16\cdot62 \quad R \in \mathrm{reflex} . \equiv . R \in \mathrm{refl} . C`R = V . \equiv . J \subset R .$

$\equiv . (x) . xRx$

$\mathcal{L}16\cdot63 \quad \mathrm{trans} \cap \mathrm{sym} \subset \mathrm{refl}$

$\mathcal{L}16\cdot64 \quad R \in \mathrm{as} . \supset . R^2 \in \mathrm{irr}$

$\mathcal{L}16\cdot65 \quad \mathrm{as} \subset \mathrm{irr}$

$\mathcal{L}16\cdot66 \quad \mathrm{trans} \cap \mathrm{as} = \mathrm{trans} \cap \mathrm{irr}$

Operatory (§17)

$\mathcal{D}17\cdot01 \quad x\mathring{f} =_{Df} \hat{u}\hat{y}(u = x\mathring{f}y)$

$\mathcal{D}17\cdot02 \quad \mathring{f}y =_{Df} \hat{u}\hat{x}(u = x\mathring{f}y)$

$\mathcal{L}17\cdot11 \quad u(x\mathring{f})y . \equiv . u = x\mathring{f}y$

$\mathcal{L}17\cdot12 \quad u(\mathring{f}y)x . \equiv . u = x\mathring{f}y$

$\mathcal{L}17\cdot13 \quad x\mathring{f}`y = \mathring{f}y`x = x\mathring{f}y$

$\mathcal{L}17\cdot21 \quad u \in x\mathring{f}``\alpha . \equiv . (\exists y) . y \in \alpha . u = x\mathring{f}y$

$\mathcal{L}17\cdot22 \quad u \in \mathring{f}y``\alpha . \equiv . (\exists x) . x \in \alpha . u = x\mathring{f}y$

$\mathcal{D}17\cdot03 \quad \alpha \underset{''}{\mathring{f}} y . =_{Df} \mathring{f}y``\alpha$

$\mathcal{L}17\cdot31 \quad \alpha \underset{''}{f} y . = \hat{u}\{(\exists x) . x \in \alpha . u = x\mathring{f}y\}$

$\mathcal{L}17\cdot32 \quad \alpha \underset{''}{\mathring{f}}`y = \underset{''}{\mathring{f}}y`\alpha = \alpha \underset{''}{\mathring{f}} y$

$\mathcal{L}17\cdot33 \quad \alpha \underset{''}{\mathring{f}}``\beta = \hat{\gamma}\{(\exists y) . y \in \beta . \gamma = \alpha \underset{''}{\mathring{f}} y\} =$

$= \hat{\gamma}\{(\exists y) . y \in \beta . \gamma = \mathring{f}y``\alpha\}$

$\mathcal{L}17\cdot34 \quad s`(\alpha \underset{''}{\mathring{f}}``\beta) = \hat{z}\{(\exists x, y)\, x \in \alpha . y \in \beta . z = x\mathring{f}y\}$

Relacje wieloelementowe (§18)

D18·01 $\hat{x}\hat{y}\hat{z}(q x y z)(u v w) . =_{Df} q u v w$

D18·021 $x M(y, z) . =_{Df} M(x, y, z)$

D18·022 $M'(y, z) =_{Df} (\imath x)\{M(x, y, z)\}$

D18·023 $\overrightarrow{M}'(y z) =_{Df} \hat{x}\{M(x, y, z)\}$

D18·031 $D_1{}'M =_{Df} \hat{x}\{(\exists y z) . M(x y z)\}$

Analogicznie dla $D_2{}'M$ i $D_3{}'M$

D18·032 $C'M =_{Df} D_1{}'M \cup D_2{}'M \cup D_3{}'M$

D18·041 ${}^1M =_{Df} \hat{y}\hat{z}\{(\exists x) . M(x y z)\}$, anal. ${}^2M$, ${}^3M$

D18·042 ${}^1M_u =_{Df} \hat{x}\hat{y}\{M(u, y z)\}$, anal ${}^2M_u$, ${}^3M_u$

D18·051 $M^{132} =_{Df} \hat{x}\hat{z}\hat{y}\{M(x y z)\}$

D18·052 $M^{213} =_{Df} \hat{y}\hat{x}\hat{z}\{M(x y z)\}$, anal. M^{231}, M^{312}, M^{321}

D18·051–052 analogon „konwersji" = „Diversen"

D19·011 $\iota' x = [x] =_{Df} \hat{y}(y = x)$

D19·012 $[x, y] =_{Df} [x] \cup [y]$ i t. d.

D19·02 $x \downarrow y =_{Df} [x] \uparrow [y]$

D19·031 $0 =_{Df} [\Lambda]$

D19·032 $1 =_{Df} \hat{\alpha}\{(\exists x) . \alpha = [x]\}$

D19·033 $2 =_{Df} \hat{\alpha}\{(\exists x y) . x \neq y . \alpha = [x, y]\}$

L19·11 $\alpha \varepsilon 1 . \equiv :. \exists ! \alpha :. (x, y) : x, y \varepsilon \alpha . \supset . x = y$

L19·12 $[x] \varepsilon 1$

L19·13 $x \varepsilon \alpha . \equiv . [x] \subset \alpha$

L19·14 $[x y] \varepsilon 2 . \equiv . x \neq y$

D19·041 $1 \to Cls =_{Df} \hat{R}\{(x y z) : x R z . y R z . \supset . x = y\}$

D19·042 $Cls \to 1 =_{Df} \hat{R}\{(x y z) : x R y . x R z . \supset . y = z\}$

D19·043 $1 \to 1 =_{Df} (1 \to Cls) \cap (Cls \to 1)$

Kolejność w książce:

D19·040 $\alpha \to \beta =_{Df} \hat{R}(\overrightarrow{R}{}'' G' R \subset \alpha . \overleftarrow{R}{}'' D' R \subset \beta)$

L19·20 $R \varepsilon \alpha \to \beta . \equiv :. (y) : y \varepsilon G' R . \supset . \overrightarrow{R}' y \varepsilon \alpha :.$
$(x) : x \varepsilon D' R . \supset . \overleftarrow{R}' x \varepsilon \beta$

L19·21 $R \varepsilon 1 \to Cls . \equiv : (y) : y \varepsilon G' R . \equiv . E ! R' y$ (26)

L19·22 $R \varepsilon 1 \to Cls . \equiv : (x y) : x R y . \equiv . x = R' y$

$L19\cdot23$ $Cnv''(1\to cls) = cls \to 1$

$L19\cdot24$ $Cnv''(1\to 1) = 1 \to 1$

$L19\cdot31$ $Rl''(1\to cls) \subset 1 \to cls$ (bo(?) dla $cls\to1$ i $1\to1$)

$L19\cdot32$ $R, S \,\epsilon\, 1\to cls \,.\supset.\, R|S \,\epsilon\, 1\to cls$ (" " " i ")

§20 Zasada abstrakcji

$D20\cdot01$ $sim =_{Df} sym \cap refl$ (Ähnlichkeiten, similitas)

$D20\cdot02$ $aeq =_{Df} trans \cap sym$ (Gleichheiten, aequalitas)

$D20\cdot03$ $aeqt =_{Df} aeq \cap reflex$ (Totalgleichheiten)

$L20\cdot11$ $aeq \subset refl$

$L20\cdot12$ $aeq \subset sim$

$D20\cdot04$ $Sim'R =_{Df} \hat{\alpha}\{\alpha \uparrow \alpha \subset R :. (x) : \alpha \subset \vec{R}'x \,.\supset.\, x \,\epsilon\, \alpha\}$ (Ähnlichkeitskreis)

$D20\cdot05$ $Aeq =_{Df} Sim \restriction aeq$ (Abstraktionsklassen)

$L20\cdot2$ $\kappa = Aeq'R \,.\equiv.\, \kappa = Sim'R \,.\, R \,\epsilon\, aeq$

$L20\cdot3$ $R \,\epsilon\, aeq \,.\supset.\, Aeq'R = D'\vec{R} - [\Lambda]$

$L20\cdot4$ $a \,\epsilon\, Aeq'R \,.\, x \,\epsilon\, a \,.\equiv.\, R \,\epsilon\, trans \cap sym \,.\, x \,\epsilon\, C'R \,.$
$a = \vec{R}'x$

$L20\cdot5$ $J \restriction Aeq'R \in Fr$

$L20\cdot6$ $1 = Aeq'J$

§21 Liczby kardynalne

$D21\cdot01$ $\alpha\, \overline{sm}\, \beta =_{Df} 1\to1 \cap D'\alpha \cap \text{Ɑ}'\beta$

$D21\cdot02$ $Sm =_{Df} \hat{\alpha}\hat{\beta}\,(\exists!\, \alpha\, \overline{sm}\, \beta)$

$L21\cdot11$ $Sm \,\epsilon\, aeq$

$L21\cdot12$ $Sm \,\epsilon\, reflex \,.\, Sm \,\epsilon\, aeqt$

$L21\cdot13$ $\alpha\, Sm\, \gamma \,.\, \beta\, Sm\, \delta \,.\, \gamma \subset \beta \,.\, \delta \subset \alpha \,.\supset.\, \alpha\, Sm\, \beta$

(Inaczej: $Sm|\mathbf{C} \cap \breve{\mathbf{C}}|Sm \in Sm$; Schröder-Bernstein)

$D21\cdot03$ $Nc'\alpha =_{Df} \vec{Sm}'\alpha$

$D21\cdot04$ $nc =_{Df} D'Nc$ (liczba kardynalna)

$L21\cdot21$ $nc = Aeq'Sm$

$L21\cdot22$ $\alpha \,\epsilon\, Nc'\alpha$

$L21\cdot23$ $\alpha \,\epsilon\, Nc'\beta \,.\equiv.\, \beta \,\epsilon\, Nc'\alpha \,.\equiv.\, \alpha\, Sm\, \beta$

$L21\cdot24$ $\alpha\, Sm\, \beta \,.\supset.\, Nc'\alpha = Nc'\beta$

$L21\cdot25$ $\mu \,\epsilon\, nc \,.\equiv.\, (\exists\alpha) \,.\, \mu = Nc'\alpha$

L21·31 $0 = Nc'\Lambda$

L21·32 $\exists!\, 0$

L21·33 $Nc'\gamma = 0 \,.\equiv.\, \gamma = \Lambda$

L21·34 $1 = Nc'[x]$

L21·35 $x \neq y \,.\supset.\, 2 = Nc'[x,y]$

L21·36 $0, 1, 2 \,\epsilon\, nc$

D21·05 $\mu + \nu =_{Df} \hat{\xi}\{(\exists \gamma,\delta) . \gamma \epsilon \mu . \delta \epsilon \nu . \gamma \,\mathrm{Fr}\, \delta . \xi = \gamma \cup \delta\}$

D21·06 $N =_{Df} \breve{+}1$ (Inf. 17·02)

§22 Izomorfja; Liczby porządkowe

D22·01 $S ; Q =_{Df} S | Q | \breve{S}$ — analogon $S''\beta$

~~Obraz Q~~ S-wy obraz Q

D22·02 $S \dagger Q =_{Df} S ; Q$ analogon $S_\epsilon{}'\beta$

D22·03 $P \;\mathrm{smor}\; Q =_{Df} \hat{S}\{S \epsilon 1 \to 1 . C'Q = \mathrm{Q}'S . P = S ; Q\}$

D22·04 $\mathrm{Smor} =_{Df} \hat{P}\hat{Q}\{\exists!\, P \;\mathrm{smor}\; Q\}$ Izomorfja

L22·11 $\mathrm{Smor} \,\epsilon\, \mathrm{aeqt}$

L22·12 $P \;\mathrm{smor}\; Q \,.\supset.\, C'P \;\mathrm{sm}\; C'Q$

L22·13 $S \epsilon P \;\mathrm{smor}\; Q \,.\equiv.\, S \epsilon (C'P)\, \overline{\mathrm{sm}}\, (C'Q) . P = S ; Q$

D22·05 $Nr'P =_{Df} \overrightarrow{\mathrm{Smor}}'P$

D22·06 $nr =_{Df} D'Nr$ (liczba porządkowa)

L22·21 $nr = \mathrm{Aeq}'\mathrm{Smor}$

L22·22 $P \,\epsilon\, Nr'P$

L22·23 $P \epsilon Nr'Q \,.\equiv.\, Q \epsilon Nr'P \,.\equiv.\, P \;\mathrm{smor}\; Q$

L22·24 $P \;\mathrm{smor}\; Q \,.\equiv.\, Nr'P = Nr'Q$

L22·25 $\lambda \epsilon nr \,.\equiv.\, [\exists P] . \lambda = Nr'P$

D22·07 $\mathrm{struc} =_{Df} \hat{\alpha}(\mathrm{Smor}''\alpha \subset \alpha)$ (Strukturellen Eigenschaften)

L22·31 $\alpha \epsilon \mathrm{struc} \,.\equiv:\, (P,Q) : P \epsilon \alpha . P \;\mathrm{smor}\; Q \,.\supset.\, Q \epsilon \alpha$

L22·32 trans, intr, refl, reflex, irr, sym, as, $1 \to$ cls, cls $\to 1$, $1 \to 1 \,\epsilon\,$ struct

(a także: prog, connex, ser, bord, Ω, ded, dicht, dedek, η, ϑ)

§ 23 Łańcuchy i grupy

Bei α heißt „R erblich" $= \breve{R}''\alpha \subset \alpha$

D23·01 $R_* =_{Df} \hat{x}\hat{y}\{x \epsilon C'R :. (\xi) : \breve{R}''\xi \subset \xi . x \epsilon \xi . \supset . y \epsilon \xi\}$

28

D23·02 $\breve{R}_* =_{Df} \mathrm{Cnv}'R_*$

$x^?$ $y^?$

L23·11 $x R_* y . \equiv :. x \in C'R :. (\zeta) : R''\zeta \subset \zeta . y \in \zeta . \supset . x \in \zeta$

D23·03 $R_{po} =_{Df} \hat{x}\hat{y}\{(\zeta) : \breve{R}''\zeta \subset \zeta . \overleftarrow{R}'x \subset \zeta . \supset . y \in \zeta\}$

L23·12 $R^0 \in R_* . R \in R_* . R^2 \in R_*$ (it. d.)

L23·13 $R_* | R \in R_*$

L23·21 $R \in R_{po} . R^2 \in R_{po}$ (it. d.)

L23·22 $R_{po} | R \in R_{po}$

L23·23 $R_*, R_{po} \in \mathrm{trans}$

L23·24 $R \in \mathrm{trans} . \supset . R_{po} = R . R_* = R \dot\cup R^0$

L23·25 $R_* = R^0 \dot\cup R_{po}$

D23·04 $\mathrm{Potid}'R =_{Df} (|\vec{R})_*'R^0$

D23·05 $\mathrm{Pot}'R =_{Df} (|\vec{R})_*'R$

L23·31 $\mathrm{Potid}'R = \mathrm{Pot}'R \cup [R^0]$

L23·32 $R_* = \dot{s}'\mathrm{Potid}'R$

L23·33 $R_{po} = \dot{s}'\mathrm{Pot}'R$

D23·06 $\mathrm{gru} =_{Df} \hat\Lambda\{(P,Q) : P, Q \in \Lambda . \supset . P|Q \in \Lambda . \breve{P} \in \Lambda :.$

$I \upharpoonright s'C''\Lambda \in \Lambda\}$ (Grupa logiczna;

$I \upharpoonright s'C'\Lambda \in \Lambda$ = „Einheitselement")

D23·07 $\mathrm{abel} =_{Df} \hat\Lambda\{\Lambda \in \mathrm{gru} :. (P,Q) : P, Q \in \Lambda . \supset . P|Q = Q|P\}$

D23·08 $\mathrm{Zykl}'P =_{Df} \mathrm{Potid}'P \cup \mathrm{Cnv}''\mathrm{Potid}'P$ (cykliczna grupa P)

D23·09 $\mathrm{zykl} =_{Df} \mathrm{D}'\mathrm{Zykl}$

§24 Endlich und Unendlich

Kolejność autora:

L24·10 $\epsilon_\Delta'\kappa = (1 \to \mathrm{Cls}) \cap \mathrm{Rl}'\epsilon \cap \overleftarrow{\mathrm{D}}'\kappa$

D24·01 $P_\Delta'\kappa = (1 \to \mathrm{Cls}) \cap \mathrm{Rl}'P \cap \overleftarrow{\mathrm{D}}'\kappa$ (Selektor)

24b $(\kappa) : \Lambda \sim\in \kappa . \supset . \exists ! \epsilon_\Delta'\kappa$ (Aks. wyboru)

D24·02 $\mathrm{Nc\ induct} =_{Df} \overleftarrow{N}_*'0$ (liczby indukcyjne)

D24·03 $\mathrm{Cls\ induct} =_{Df} s'\mathrm{Nc\ induct}$

L24·2 $\alpha \in \mathrm{Nc\ induct} . \equiv :. (\mu) :. (\xi) : \xi \in \mu . \supset . \xi + 1 \in \mu :$

$0 \in \mu . \supset . \alpha \in \mu$

29

D24·04 $\mathrm{Cls\ refl} =_{Df} \hat\alpha\{(\exists\beta) . \beta \subset \dot\cap \mathrm{sm} \dot\cap \mathrm{J} \alpha\}$

D24·05 $\mathrm{Nc\ refl.} =_{Df} \mathrm{Nc}''\mathrm{Cls\ refl}$ (liczby paradoksalne)

L24·31 $\xi \in nc\ refl. \equiv . \exists ! \xi . \xi = \xi + 1$

L24·32 nc induct Fr nc refl

L24·33 Cls induct Fr Cls refl

24c $(\mu) : \mu \in nc\ induct . \supset . \exists ! \mu$
(Unendlichkeitsaxiom)

§25 Verschiedene Zerlegungen einer Relation

D25·01 $B = _{Df} \hat{x}\hat{P}(x \in D'P - Ɑ'P)$ (Anfangsglieder; $\vec{B}'\breve{P} =$ „Endglieder")

D25·021 $\mathrm{Min}_P = \mathrm{Min}(P) = _{Df} \hat{x}\hat{\alpha}(x \in \alpha \cap C'P - \breve{P}''\alpha)$

D25·022 $\mathrm{Max}_P = \mathrm{Max}(P) = _{Df} \mathrm{Min}(\breve{P})$

L25·1 $\mathrm{Min}_P \subset \epsilon . \mathrm{Max}_P \subset \epsilon$

D25·031 $I_R'x = _{Df} \overleftarrow{R}_*'x \cap \hat{z}(z R_{po} z)$

D25·032 $J_R'x = _{Df} \overleftarrow{R}_*'x - I_R'x$

($\overleftarrow{R}_* x =$ „R-Nachkommenschaft" von x)

D25·033 $\overleftrightarrow{R}'x = _{Df} \vec{R}'x \cup ([x] \cap C'R) \cup \overleftarrow{R}'x$

($\overleftarrow{R}$-Vorfahrenschaft $= \breve{R}$-Nachkommenschaft)

L25·2 $\overleftrightarrow{R}_*'x = \vec{R}_*'x \cup \overleftarrow{R}_*'x$ (R-Familien)
25c

Sind alle Glieder einer R-Familie zirkulär, so heisst die Familie selbst „zirkulär". Sind alle R-Glieder zirkulär, so heisst die Relation R selbst zirkulär. Hat R kein zirkuläres Glied, so heisst R „offen" ($R_{po} \subset J$, $R_{po} \in irr$)

Gilt $z R_{po} z$, so heisst z ein „zirkuläres" R-Glied

D25·041 $P(x - y) = _{Df} \overleftarrow{P}_{po}'x \cap \vec{P}_{po}'y$

D25·042 $P(x \dashv y) = _{Df} \overleftarrow{P}_{po}'x \cap \vec{P}_*'y$

D25·043 $P(x \vdash y) = _{Df} \overleftarrow{P}_*'x \cap \vec{P}_{po}'y$

D25·044 $P(x \vdash\!\dashv y) = _{Df} \overleftarrow{P}_*'x \cap \vec{P}_*'y$

P-Intervalle

D25·05 $P_\nu = _{Df} \hat{x}\hat{y}(Nc'P(x \vdash\!\dashv y) = \nu + 1)$ P-Schnitte

L25·31 $P_0 \subset P^0$

L25·32 $P \in (1 \to Cls) \cup (Cls \to 1) . P_{po} \subset J . \supset . P_1 = P . P_2 = P^2 . P_3 = P^3$ i.t.d.

D25·06 $\nu_P = _{Df} \breve{P}_{\nu \dot{-} 1}{}' B' P$ Gliednummern

D25·07 $\mathrm{sect}'R = _{Df} \hat{\alpha}(\alpha \subset C'R . R''\alpha \subset \alpha)$

„Abschnitt" oder „Section"

§26 Progressionen

D26·01 $prog =_{Df} (1 \to 1) \cap \hat{R}(D'R = \overleftarrow{R_*}'B'R)$

L26·11 $R \,\varepsilon\, prog . \equiv : R \,\varepsilon\, (1 \to 1) . E!\, B'R : (x) : x \,\varepsilon\, D'R . \equiv . x \,\varepsilon\, \overleftarrow{R_*}'B'R$

L26·12 $R \,\varepsilon\, prog . \supset . \mathrm{Q}'R \subset D'R . C'R = D'R$

L26·2 $R \,\varepsilon\, prog . \supset . R_{po} \in J$ (offen §25c)

L26·3 $N \,\varepsilon\, prog$

L26·41 $prog \,\varepsilon\, struct$

L26·42 $prog \,\varepsilon\, nc$

L26·43 $R \,\varepsilon\, prog . \equiv . R \; smor \; N$

D26·02 $\aleph_0 =_{Df} D''prog$

L26·5 $\alpha \,\varepsilon\, \aleph_0 . \equiv . \alpha \; sm \; nc \; induct$

L26·6 $\aleph_0 \,\varepsilon\, nc$

§27 Reihen

D27·01 $connex =_{Df} \hat{R}(J \restriction C'R \subset R \cup \breve{R})$

„zusammenhängend" = „verbunden" = „connex"

L27·11 $R \,\varepsilon\, connex . \equiv : (x,y) : x,y \,\varepsilon\, C'R . x \neq y . \supset . x \, R \cup \breve{R} \, y : \equiv : (x) : x \,\varepsilon\, C'R . \supset . \overleftrightarrow{R}'x = C'R$

L27·12 $connex \,\varepsilon\, struct$

L27·13 $R \,\varepsilon\, connex . \supset . \overrightarrow{B}'R, \overrightarrow{B}'\breve{R}, \overrightarrow{Max_R}'\alpha, \overrightarrow{Min_R}'\alpha \,\varepsilon\, 0 \cup 1$

D27·02 $ser =_{Df} irr \cap trans \cap connex$ (series)

L27·21 $R \,\varepsilon\, ser . \equiv . R \,\varepsilon\, irr \cap trans \cap connex . \equiv . R \,\varepsilon\, as \cap trans \cap connex . \equiv . R \subset J . R^2 \subset R : J \restriction C'R \subset R . \equiv . R \,\varepsilon\, connex . R^2, R^3 \,\varepsilon\, irr . \equiv . R \,\varepsilon\, connex . R^6 \,\varepsilon\, irr . \equiv . R \,\varepsilon\, connex . R_{po} \,\varepsilon\, irr$

L27·22 $Cnv''ser \subset ser$

L27·23 $ser \,\varepsilon\, struct$

L27·24 $R \,\varepsilon\, ser . \supset . R \restriction \alpha \,\varepsilon\, ser$

§28 Grenzbegriffe

D28·01 $seq_R = seq(R) =_{Df} \hat{x}\hat{\alpha}\{x \, Min_R \, p'\overleftarrow{R}''(\alpha \cap C'R)\}$

„sequens"

$\mathrm{D28{\cdot}02}\quad \mathrm{Praec}_R = \mathrm{Praec}(R) =_{Df} \mathrm{Seq}(\breve{R})$ Präcedens

$\mathrm{L28{\cdot}11}\quad x\,\mathrm{Praec}_R\,\alpha .\equiv. x\,\mathrm{Max}_R \cdot p'\overrightarrow{R}''(\alpha \cap C'R)$

$\mathrm{L28{\cdot}12}\quad x\,\mathrm{Seq}_R\,\alpha .\supset. x \in C'R - \alpha$

$\mathrm{D28{\cdot}03}\quad \mathrm{Lt}_R = \mathrm{Lt}(R) =_{Df} \mathrm{Seq}_R \restriction (-\mathrm{a}'\mathrm{Max}_R)$ obere Grenze

$\mathrm{D28{\cdot}04}\quad \mathrm{Tl}_R = \mathrm{Tl}(R) =_{Df} \mathrm{Praec}_R \restriction (-\mathrm{a}'\mathrm{Min}_R)$ untere Grenze

$\mathrm{L28{\cdot}21}\quad \mathrm{Tl}_R = \mathrm{Lt}(\breve{R})$

$\mathrm{L28{\cdot}22}\quad R \in \mathrm{ser} .\supset: (x,\alpha): x = \mathrm{Lt}_R{}'\alpha .\equiv. x \in C'R - \alpha .$
$\overrightarrow{R}'x = R''\alpha$

$\mathrm{D28{\cdot}05}\quad \mathrm{Limax}_R = \mathrm{Limax}(R) =_{Df} \mathrm{Max}_R \dot\cup \mathrm{Lt}_R$

$\mathrm{D28{\cdot}06}\quad \mathrm{Limin}_R = \mathrm{Limin}(R) =_{Df} \mathrm{Min}_R \dot\cup \mathrm{Tl}_R$

$\mathrm{L28{\cdot}31}\quad \mathrm{Limin}_R = \mathrm{Limax}(\breve{R})$

$\mathrm{L28{\cdot}32}\quad R \in \mathrm{connex} .\supset. \overrightarrow{\mathrm{Seq}_R}'\alpha, \overrightarrow{\mathrm{Praec}_R}'\alpha, \overrightarrow{\mathrm{Lt}_R}'\alpha, \overrightarrow{\mathrm{Tl}_R}'\alpha,$
$\overrightarrow{\mathrm{Limax}_R}'\alpha, \overrightarrow{\mathrm{Limin}_R}'\alpha \in 0 \cup 1$

$\mathrm{L28{\cdot}33}\quad \exists!\,\overrightarrow{\mathrm{Max}_R}'\alpha .\supset. \overrightarrow{\mathrm{Limax}_R}'\alpha = \overrightarrow{\mathrm{Max}_R}'\alpha$

$\mathrm{L28{\cdot}34}\quad \overrightarrow{\mathrm{Max}_R}'\alpha = \Lambda .\supset. \overrightarrow{\mathrm{Limax}_R}'\alpha = \overrightarrow{\mathrm{Seq}_R}'\alpha =$
$\overrightarrow{\mathrm{Lt}_R}'\alpha$

§29 Stetigkeit

$\mathrm{D29{\cdot}01}\quad \mathrm{bord} =_{Df} \hat{R}(\mathrm{Clex}'C'R \subset \mathrm{a}'\mathrm{Min}_R)$

$\mathrm{D29{\cdot}02}\quad \Omega =_{Df} \mathrm{ser} \cap \mathrm{bord}$

$\mathrm{D29{\cdot}03}\quad \mathrm{no} =_{Df} \mathrm{Nr}''\Omega$

$\mathrm{L29{\cdot}1}\quad \mathrm{bord}, \Omega \in \mathrm{struct}$

$\mathrm{D29{\cdot}04}\quad \mathrm{ded} =_{Df} \hat{R}\{(\alpha).\alpha \in \mathrm{a}'\mathrm{Max}_R \cup \mathrm{a}'\mathrm{Seq}_R\}$

„dedekindisch"

$\mathrm{L29{\cdot}21}\quad R \in \mathrm{ded} .\equiv. (\alpha).\alpha \in \mathrm{a}'\mathrm{Limax}_R$

$\mathrm{L29{\cdot}22}\quad R \in \Omega . E!\,B'\breve{R} .\supset. R \in \mathrm{ded}$

$\mathrm{L29{\cdot}23}\quad \mathrm{ded} \in \mathrm{struct}$

$\mathrm{L29{\cdot}24}\quad R \in \mathrm{ded} .\supset. \exists!\,\overrightarrow{B}'R . \exists!\,\overrightarrow{B}'\breve{R}$

29c $\quad R \subset R^2$ = dicht

$R \restriction \alpha \subset R^2$ = in α dicht

R heisst „nirgends dicht", wenn R in keinem Intervall dicht ist.

$\mathrm{D29{\cdot}05}\quad \mathrm{dedek} =_{Df} \mathrm{ded} \cap \hat{R}(R^2 = R)$

κ heißt „von dedekindischer Stetigkeit"

L29·31 $\kappa \,\varepsilon\, ser. \supset : \kappa \,\varepsilon\, dedek. \equiv . \alpha' Max_\kappa = - \alpha' seq_\kappa$

L29·32 $\hat{\kappa}(\kappa \subset \kappa^2) \,\varepsilon\, struct$ (dicht)

L29·33 $dedek \,\varepsilon\, struct$

D29·06 $\eta =_{Df} \hat{\kappa}\{\kappa \,\varepsilon\, ser. \kappa \subset \kappa^2 . C'\kappa \,\varepsilon\, \aleph_0 . D'\kappa = \alpha'\kappa\}$

L29·41 $\eta \,\varepsilon\, struct$

L29·42 $\kappa, \lambda \,\varepsilon\, \eta \supset . \kappa \; smor \; \lambda$

L29·43 $\eta \,\varepsilon\, nr$

D29·07 $Med =_{Df} \hat{\lambda}\hat{\kappa}\{\lambda \subset C'\kappa . \kappa \subset \kappa \upharpoonright \lambda | \kappa\}$

„Zwischenklasse" = Med

D29·08 $\vartheta =_{Df} ser \cap ded \cap \breve{Med}'' \aleph_0$

ϑ = „stetig" = „von Cantorscher Stetigkeit"

L29·51 $\vartheta \subset dedek$

L29·52 $\vartheta \subset struct$

L29·53 $\kappa, \lambda \,\varepsilon\, \vartheta . \supset . \kappa \; smor \; \lambda$

L29·54 $\vartheta \,\varepsilon\, nr$

§ 31. Aksjomatyka Teorji mnogości

Terminy: El = „ε" = $x \, El \, y$ = die Menge x ist Element der Menge y.

$me =_{Df} C'El$ = Die Mengen

$Kl =_{Df} \overrightarrow{El}$; $Kl'x$ Klasse der Elemente der Menge x

Teilmenge von: $Tl =_{Df} \breve{Kl} | \subset | Kl$

Gleichheit: $Gl =_{Df} Tl \cap \breve{Tl}$

„Elementefremd zueinander" $Fre =_{Df} \breve{Kl} | \overline{Fl} | Kl$

$fr =_{Df} \hat{x}\{J \upharpoonright Kl'x \subset Fre\}$

$ex =_{Df} \alpha' El$

A1. $Gl | El \subset El$

$Pr =_{Df} \hat{x}\hat{u}\hat{v}\{Kl'x = [u, v]\}$; $x = Pr'(u, v)$

heißt: x ist die Paarmenge $\{u, v\}$

A2 $J \upharpoonright me \subset {}^1 Pr$

$Ver =_{Df} \hat{x}\hat{y}\{Kl'x = s'Kl''Kl'y\}$ oder

$Ver =_{Df} \breve{Kl} | s | Kl_\varepsilon | Kl$

A3 $ex \subset \alpha' Ver$ (Existenz der Vereinigungsmenge)

$Po =_{Df} \breve{Kl} | \overrightarrow{El}$ Potenzmenge

A4 $me \subset \alpha' Po$

$Aus =_{Df} \hat{x}\hat{y}\hat{\hat{P}}\hat{\hat{Q}} \{x, y \in me . Kl'x = Kl'y \wedge \hat{u}$

$(P'u \; El \; Q'u)\}$ Aussonderung

A5 $(y, P, Q) : y \in me . P, Q \in fkt . \supset . {}^{1}Aus(y, P, Q)$

$Prod'x =_{Df} (\breve{Kl})^2 {}'' D'' El_{\Delta}{}' Kl'x$

Auswahlaxiom:

A6 $(x) : x \in ex \wedge fr . Kl'x \subset ex . \supset . Prod'x \in ex$

A7a $\exists ! \, me$

Die Nullmenge: $nu =_{Df} B' El$

A7b $\exists ! \, me : (\exists x) . nu \, El \, x . Einr'' Kl'x \subset Kl'x$

(lepiej $nu \, El \, x = E ! \, B' El . \supset . (B' El) El \, x$.

$Ers =_{Df} \hat{x}\hat{y}\hat{k} \{k \in fkt . Kl'x = k'' Kl'y\}$

A8 $(x, k) : x \in me . k \in fkt . \supset . E ! \, Ers'(xk)$

Die Menge 2: $2a =_{Df} \overrightarrow{Einr_{*}}{}' nu$

Beschränktheitsaxiom:

$me \subset (sy' s' dofkt)' (nu, 2a)$

„fkt" i „dofkt" nie są zdefiniowane

§ 32 Aksjomatyka teorji liczb naturalnych.

I) $0 = nu$, liczba – $2a$, ly – Nf.

A1. $nu \in 2a$

A2. $(x) : x \in 2a . \supset . Nf'x \in 2a$

A3 $(x, y) : x, y \in 2a . Nf'x = Nf'y . \supset . x = y$

(czyli $Nf \in (els \rightarrow 1)$)

A4 $(x) : x \in 2a . \supset . Nf'x \neq nu$

(czyli: $nu \sim \in D'Nf$)

A5 $(a) :: nu \in a :. (x) : x \in a . \supset . Nf'x \in a :.$

$\supset . 2a \subset a$

(czyli: $(a) : nu \in a . Nf'a \subset a . \supset . 2a \subset a$)

II Vg – Vorgänger

A1' $\nabla_y \in 1 \to 1$

A2' $E!\, B'\nabla_y$

D1' $2a =_{Df} C'\nabla_y$

A3' $E!\, B'\nabla_y . \supset . 2a = \overleftarrow{\nabla_{y*}}' B'\nabla_y$

A4' $G'\nabla_y \subset D'\nabla_y$

III ∇_y

A1'' $\nabla_y \in 1 \to 1$ $(= A1')$

A2'' $D'\nabla_y = \overleftarrow{\nabla_{y*}}' B'\nabla_y$ $(= A2'.3'.4')$

§ 33 Aksjomatyka Topologji

„$\alpha U x$" – α ist eine Umgebung von x

Punkt D1 $pu =_{Df} G'U$

A1a $D'U \subset Cl'\,pu$

A1b $U \in \breve{\varepsilon}$

A2 $(\alpha\beta x) : \alpha U x . \beta U x . \supset . [\exists \gamma] . \gamma U x . \gamma \subset \alpha \cap \beta$

A3 $(\alpha y) : \alpha \in D'U . y \in \alpha . \supset . (\exists \gamma) . \gamma U y . \gamma \subset \alpha$

A4 $(xy) : x, y \in pu . x \neq y . \supset . (\exists \alpha\beta) . \alpha U x . \beta U y .$
$\alpha\ Fr\ \beta$

D2 $Inn =_{Df} \breve{U} | C$

D3 $Kp =_{Df} \varepsilon \dot{-} Inn$

D4 $geb =_{Df} - G'Kp$

D5 $rdm =_{Df} - G'Inn$

D6 $Grp =_{Df} \hat{x}\hat{\alpha}\{x\, Kp\, \alpha . v . x\, Kp\, -\alpha\}$

D7 $Ber =_{Df} \hat{x}\hat{\alpha}\{(\beta) : \beta U x . \supset . \exists!\, \alpha \cap \beta\}$

D8 $Hf =_{Df} \hat{x}\hat{\alpha}\{(\beta) : \beta U x . \supset . \alpha \cap \beta \in \text{cls refl}\}$

D9 $Verd =_{Df} \hat{x}\hat{\alpha}\{(\beta) : \beta U x . \supset .$
$Nc'(\alpha \cap \beta) > \aleph_0\}$

D10 $I_s =_{Df} \varepsilon \dot{-} Hf$

Df logiczne, mające zastosowania w topologji

Hausdorffschen Umgebungssysteme:

35

D11 $\text{hausd} =_{Df} \hat{R}\{D'R \subset Cl'Cl'R . R \in \breve{\xi}$:

: $(\alpha, \beta) . \overleftarrow{R}'\alpha \cap \overleftarrow{R}'\beta \subset s'\overleftarrow{R}''Cl'(\alpha\cap\beta) : \xi \upharpoonright D'R$

$\in \breve{R}|C . J \upharpoonright Cl'R \subset \breve{R}|Fr|R\}$

D12 $\text{Umr}'\Theta =_{Df} \hat{\alpha}\hat{x}\{(\exists n\kappa) . n \in nc\ induct.$

$\kappa = \hat{\beta}[(\exists m) . m \leqq n . \beta = \overrightarrow{\Theta^{m}}'x] . \alpha = s'\kappa\}$

Umgebungsrelation

D13 $\text{Umgr}'\Theta =_{Df} \hat{\alpha}\hat{x}\{(\exists n) . n \in nc\ induct.$

$\alpha = \overrightarrow{\Theta^{n}}'x\}$

D14 $\text{Hf} =_{Df}$ ■ $\hat{\alpha}\hat{\mathfrak{U}}\{\mathfrak{U} \in \text{hausd} : (\beta) : \beta\mathfrak{U}x . \supset .$

$\alpha \cap \beta \in \text{clsrefl}\}$

D15 $\text{Berg} =_{Df} \hat{\delta}\hat{\beta}\hat{\mathfrak{U}}\{\delta = \overrightarrow{\text{Hf}}'(\beta\mathfrak{U}) - \beta\}$

D16 $\text{Drph} =_{Df} \hat{n}\hat{\alpha}\hat{x}\hat{\mathfrak{U}}\{\mathfrak{U} \in \text{hausd} :$

$x \in Cl'\mathfrak{U} . \alpha \subset Cl'\mathfrak{U} : . (\beta) : \beta\mathfrak{U}x . \supset .$

$[\exists\gamma] . \gamma\mathfrak{U}x . \gamma \subset \beta . Dz(n-1, \alpha \cap \text{Begr}(\gamma\mathfrak{U}),$

$\mathfrak{U})\}$

D17 $\text{Dzp} =_{Df} \hat{n}\hat{\alpha}\hat{x}\hat{\mathfrak{U}}\{\text{Dzph}(n, \alpha, x, \mathfrak{U}) .$

$\sim \text{Dzph}(n-1, \alpha, x, \mathfrak{U})\}$

D18 $\text{Dz} =_{Df} \hat{n}\hat{\alpha}\hat{\mathfrak{U}}\{(x) : x \in \alpha . \supset . \text{Dzp}'(\alpha, x, \mathfrak{U})$

$\leqq n : . \ ^{3}\text{Dzp}(n, \alpha, \mathfrak{U})\}$

D19 $\text{Dzhom} =_{Df} \hat{n}\hat{\alpha}\hat{\mathfrak{U}}\{(x) : x \in \alpha . \supset . \text{Dzp}(n, \alpha, x, \mathfrak{U})\}$

D20 $\text{Dzhomum} =_{Df} \hat{n}\hat{\Theta}\{\Theta \in \text{sim} . \text{Dzhom}\{n,$

$C'\Theta, \text{Umgr}'\Theta)\}$

§ 34 Aksjomatyka geometrji

Die Geraden als Klassen: (Pieri)

ger = die Klasse der Geraden

Die Punkte D1 $\text{pu} =_{Df} s'\text{ger}$

A1 $\dot{\exists}! J \upharpoonright \text{pu}$ (P. II, III)

Die Gerade durch x und y:

D2 $\overline{xy} =_{Df} (\imath\alpha)(\alpha \in \text{ger} . x \neq y . x, y \in \alpha)$

A2 $(\alpha) : \alpha \in \text{ger} . \supset . (\exists xy) . \alpha = \overline{xy}$ (P. IV)

A3 $(x,y): x,y \in pu . x \neq y . \supset . E!\overline{xy}$ (𝔅 $\underline{IX}, \underline{X}$)

A4 $(x,y): E!\overline{xy} . \supset . \exists! \overline{xy} - [x,y]$ (𝔅 $\underline{VIII}$)

A5 $(x,y): E!\overline{xy} . \supset . \exists! pu - \overline{xy}$ (𝔅 $\underline{XI}$)

A6 $(xyzuv): x \in pu - \overline{yz} . u \in \overline{yz} - \overline{[yz]} .$

$v \in \overline{xz} - [xz] . \supset . \exists! \overline{xu} \cap \overline{yv}$ (𝔅 $\underline{XII}$)

Der Schnittpunkt zweier Geraden:

D3 $\overline{xu} \times \overline{yv} =_{Df} \iota'(\overline{xu} \cap \overline{yv})$

Die Ebene durch x,y,z:

D4 $\overline{xyz} =_{Df} s'\hat{\alpha}\{(\exists u). x \sim\in \overline{yz} . u \in \overline{yz} .$

$\alpha = \overline{xu}\}$

Die Ebenen:

D5 $eb =_{Df} \hat{\gamma}\{(\exists xyz). \gamma = \overline{xyz}\}$

Vier harmonische Punkte:

D6 $Ha =_{Df} \hat{x}\hat{y}\hat{z}\hat{w}\{(\exists u,v) :. z,w \in \overline{xy} .$

$uv \sim\in \overline{xy} . z \in \overline{uv} .:. (zt) : n = \overline{xu} \times$

$\overline{yv} . t = \overline{xv} \times \overline{yu} . \supset . w \in \overline{zt}$

A7 (𝔅 $\underline{XIII}$) $(x,y,z): x \in pu - \overline{yz} . \supset . \exists! pu -$

$\overline{xyz}$

A8 (𝔅 $\underline{XIV}$) $(x,y,z): x \in \overline{yz} - [y,z] . \supset .$

$Ha'(x,y,z) \neq x$

Projektive Definition der Strecke x,y,z:

D7 $str'(x,y,z) =_{Df} \hat{u}\{(\exists v,w) . Ha(x,z,$

$v,w) . Ha(u,y,v,w)$

A9 (𝔅 $\underline{XV}$) $(x,y,z,u): y \in \overline{xz} - [xz] . u \in$

$\overline{xz} - str'(x,y,z) - [x,z] . \supset . u \in str'(y,z,x)$

A10 (𝔅 $\underline{XVI}$) $(x,y,z): x \in \overline{yz} - [y,z] . \supset .$

$str'(y,z,x) \cap str'(z,x,y) \subset - str'(x,y,z)$

A11 (𝔅 $\underline{XVII}$) $(x,y,zu): x \in \overline{yz} - [yz] .$

$u \in str'(x,y,z) . \supset . str'(x,u,z) \subset str'(x,y,z)$

D8 $Str_{x,y,z} =_{Df} \hat{u}\hat{v}\{u,v \in str'(x,y,z) .$

$u \in \text{tr}'(z,x,v)\}$

Dedekindsche Stetigkeit:

A12 (§ XVIII) $(x,y,z,\gamma,\alpha) : \gamma = \text{tr}'(x,y,z) .$
$\exists!\alpha . \exists!\gamma - \alpha . \supset . (\exists w) . w \in \gamma . \gamma \cap \overrightarrow{Vor}_{x,y,z}{}'w \subset \alpha . \gamma \cap \overleftarrow{Vor}_{x,y,z}{}'w \subset \gamma - \alpha$

§ 35 Aksjomatyka geometrji.

K = die Geraden als Relationen

Die Punkte:

D1. $pu =_{Df} s'C''K$

A1 $\dot{\exists}!\, J'pu$ (wie in 34)

A2 $K \subset sym$

A3 $K \subset irr$

A4 $K \subset connex$

A5 $(R) : R \in K . \supset . R^2 \dot{-} I \subset R$ (aliotransitiv)

A6 $(x,y) : x,y \in pu . x \neq y . \supset . E!(\iota R)(R \in K . xRy)$

A7 $(R) : R \in K . \supset . \exists!\, pu - C'R$

Die Gerade durch x,y:

D2 $\overline{xy} =_{Df} (\iota\alpha)\{(\exists R) . R \in K . xRy . \alpha = C'R\}$

Dalszy ciąg jak w § 34.

Dalszy ciąg nie przedstawia ciekawych rzeczy.

Bruxelles, 9.IX.1948.

Analiza antynomji Russella.

§1. Założenia najprostsze.

A1. $[a].[\exists A].A \varepsilon Kl(a)$

A2. $[A B a b]: A \varepsilon Kl(a). A \varepsilon Kl(b). B \varepsilon b. \supset . B \varepsilon a$

D1. $[A] :. A \varepsilon * . \equiv : A \varepsilon A : [a] : A \varepsilon Kl(a). \supset . \sim(A \varepsilon a)$

A3 $[A] : A \varepsilon Kl(*). \supset . \sim(A \varepsilon *)$ (D1, $a|*$)

A4 $[A a] : A \varepsilon Kl(*). A \varepsilon Kl(a). \supset . \sim(A \varepsilon a)$ (A2, A3)

A5 $[A] : A \varepsilon Kl(*). \supset . A \varepsilon *$

Dem.:

$[A]:$

1) $A \varepsilon Kl(*). \supset :$

2) $A \varepsilon A :$ (1)

3) $[a] : A \varepsilon Kl(a). \supset . \sim(A \varepsilon a)$ (A4, 1)

$A \varepsilon *$ (D1, 2, 3)

A6. $[A].\sim(A \varepsilon Kl(*))$ (A3, A5)

$\{A1\}$ jest sprzeczne z $\{A6\}$

$\{A1, A2\}$ prowadzi do sprzeczności

§2. Założenia równoważne A2.

B1 $[AaB]: A \varepsilon Kl(a) . B \varepsilon a . \supset . B \varepsilon el(A)$

B2 $[AaB]: A \varepsilon Kl(a) . B \varepsilon el(A) . \supset . B \varepsilon a$

I. $\{B1, B2\} \rightarrow \{A2\}$

A2 $[AaBb]: A \varepsilon Kl(a) . A \varepsilon Kl(b) . B \varepsilon b . \supset . B \varepsilon a$

Dem.:

$[AaBb]:$

1) $A \varepsilon Kl(a).$

2) $A \varepsilon Kl(b).$

3) $B \varepsilon b . \supset .$

4) $B \varepsilon el(A).$ (B1; 2; 3)

$B \varepsilon a$ (B2; 1; 4)

II $\{A2\} \rightarrow \{B1, B2\}$

D2 $[A, B]: B \varepsilon el(A) . \equiv . [\exists a] . A \varepsilon Kl(a) . B \varepsilon a$

B1 $[AaB]: A \varepsilon Kl(a) . B \varepsilon a . \supset . B \varepsilon el(A)$ (D2)

B2 $[AaB]: A \varepsilon Kl(a) . B \varepsilon el(A) . \supset . B \varepsilon a$

Dem.:

$[AaB]:$

1) $A \varepsilon Kl(a).$

2) $B \varepsilon el(A) . \supset$

$[\exists b].$

3) $A \varepsilon Kl(b).$

4) $B \varepsilon b.$ (D2, 2)

$B \varepsilon a$ (A2, 1, 3, 4)

III $\{A2\} \rightleftarrows \{B1, B2\}$ (I, II)

A więc: $\{A1, A2\} \rightleftarrows \{A1, B1, B2\}$

§3. Zastąpienie założenia A1.

{A1} jest nieintuicyjne. Zastępujemy go przez

C1 $[Ba]: B \varepsilon a . \supset . [\exists A] . A \varepsilon Kl(a)$

I. $\{A1\} \rightarrow \{C1\}$, ale nie odwrotnie.

II. C1, A2 oraz

C2 $[\exists AB] . A \varepsilon A . B \varepsilon B . \sim(A = B)$

daje pewność:

C1.

A2.

C2.

D1.

A3. (D1)

A4. (A2, A3)

A5. (D1, A4)

A6. (A3, A5)

A7 $[B] . \sim(B \varepsilon *)$ (C1, A6)

A8 $[Aa]: A \varepsilon Kl(a) . \supset . A \varepsilon a$

Dem.:

$[Aa]:$

1) $A \varepsilon Kl(a) . \supset .$

2) $A \varepsilon A .$ (1)

$[\exists b] .$

3) $A \varepsilon Kl(b) .$ (D1; A7; 2)

4) $A \varepsilon b .$ (D1; A7; 2)

$A \varepsilon a$ (A2; 1; 3; 4)

A9 $[ABC]: A \varepsilon A . C \varepsilon Kl(A \circ B) . B \varepsilon C . \supset . A = B$

Dem.:

$[ABC]:$

1) $A \varepsilon A .$

2) $C \varepsilon Kl(A \circ B) .$

3) $B \varepsilon C . \supset .$

4) $B \varepsilon B.$ (3)

5) $A \varepsilon A \circ B.$ (1)

$[\exists D].$

6) $D \varepsilon Kl(B).$ (C1, 4)

7) $D \varepsilon B.$ (A8, 6)

8) $D \varepsilon C.$ (7, 3)

9) $D \varepsilon Kl(A \circ B).$ (8, 2)

10) $A \varepsilon B.$ (A2, $\cancel{C1}$, 6, 9, 5)

$A = B$ (10, 4)

A10 $[AB]: A \varepsilon A . B \varepsilon B . \supset . A = B$

Dem.:

$[AB]:.$

1) $A \varepsilon A.$

2) $B \varepsilon B . \supset :$

3) $A \varepsilon (A \circ B):$ (1)

$[\exists C]:$

4) $C \varepsilon Kl(A \circ B).$ (C1; 3)

5) $C \varepsilon (A \circ B):$ (A8; 4)

6) $C \varepsilon A . \vee . C \varepsilon B:$ (5)

7) $A \varepsilon C . \vee . B \varepsilon C:$ (6, 1, 2)

8) $C \varepsilon Kl(B \circ A):$ (4)

$A = B$ (7; A9; 2; 8; A9; 1; 4)

{A10} jest sprzeczne z {C2}

{C1, A2, C2} prowadzi do sprzeczności

§ 4 Osłabienie przez Fregego założenia A2.

Frege zastąpił A2 przez

E1 $[A a B b]: A \varepsilon Kl(a) . A \varepsilon Kl(b) . B \varepsilon b . \sim(B \varepsilon Kl(b)) .$
$\supset . B \varepsilon a$

I układ:

E1.

E1.

E2 $[Aab]: A \varepsilon Kl(a) . B \varepsilon Kl(a) . \supset . A = B$

E3 $[\exists ABC]: A \varepsilon A . B \varepsilon B . C \varepsilon C . \sim(A=B) . \sim(A=C) . \sim(B=C)$

prawdzi do pewnemoście.

Dem.:

E4 $[aB]: B \varepsilon a . \supset . [\exists A] . A \varepsilon Kl(Kl(a))$

Dem.:

$[aB]::$

1) $B \varepsilon a . \supset :$

$[\exists A].$

2) $A \varepsilon Kl(a):$ $(C1, 1)$

$[\exists A] . A \varepsilon Kl(Kl(a))$ $(C1, 2)$

E5 $[AaBb] :. B \varepsilon b . A \varepsilon Kl(a) . A \varepsilon Kl(b) . \supset : A = B . \vee . B \varepsilon a$

Dem.:

$[AaBb] :.$

1) $B \varepsilon b .$

2) $A \varepsilon Kl(a) .$

3) $A \varepsilon Kl(b) . \supset :$

4) $B \varepsilon Kl(b) . \vee . B \varepsilon a :$ $(E1; 2; 3; 1)$

$A = B . \vee . B \varepsilon a$ $(4; E2; 3)$

E6. $[aBbC] :. B \varepsilon a . C \varepsilon Kl(Kl(a)) . C \varepsilon Kl(Kl(b)) . \supset :$

$[\exists A] : A \varepsilon Kl(a) : C = A . \vee . A \varepsilon Kl(b)$

Dem.:

$[aBbC] ::$

1) $B \varepsilon a .$

2) $C \varepsilon Kl(Kl(a)) .$

3) $C \varepsilon Kl(Kl(b)) . \supset :.$

$[\exists A].$

4) $A \varepsilon Kl(a):$ $(C1, 1)$

5) $C = A . \vee . B \varepsilon Kl(b) :.$ $(E5; 4; 3; 2)$

$[\exists A] : A \varepsilon Kl(a) : C = A . \vee . B \varepsilon Kl(b)$ $(4, 5)$

$E7\ [ABbC]: B \varepsilon b . A \varepsilon Kl(Kl(b)) . C \varepsilon Kl(b) . A = C . \supset . A \varepsilon b$

Dem.:

$[ABbC]:$

1) $B \varepsilon b .$

2) $A \varepsilon Kl(Kl(b)) .$

3) $C \varepsilon Kl(b) .$

4) $A = C . \supset .$

5) $A \varepsilon Kl(b) .$ $\quad (3, 4)$

6) $B \varepsilon Kl(b) .$ $\quad (E1; 2; 5; 1)$

7) $A = B .$ $\quad (E2; 5; 6)$

$A \varepsilon b$ $\quad (1, 7)$

$E8\ [aBbC]: B \varepsilon a . C \varepsilon Kl(Kl(a)) . C \varepsilon Kl(Kl(b)) . C \varepsilon b . \supset . C \varepsilon a$

Dem.:

$[aBbC] ::$

1) $B \varepsilon a .$

2) $C \varepsilon Kl(Kl(a)) .$

3) $C \varepsilon Kl(Kl(b)) .$

4) $C \varepsilon b . \supset ::$

$[\exists A] ::$

5) $A \varepsilon Kl(a) :$

6) $C = A . \vee . A \varepsilon Kl(b) :$ $\quad \Big\} (E6; 1; 2; 3)$

7) $C = A . \vee . C \varepsilon a ::$ $\quad (6; E5; 4; 5)$

$C \varepsilon a$ $\quad (7; E7; 1; 2; 5)$

$D3\ [A] :: A \varepsilon ! . \equiv :. A \varepsilon A :. [a] : A \varepsilon Kl(Kl(a)) . \supset . \sim(A \varepsilon a)$

$E9\ [AB]: A \varepsilon Kl(Kl(!)) . \supset . \sim(B \varepsilon !)$

$\uparrow \ulcorner * \urcorner$ in print
$*$

Dem.:

$[AB] ::$

1) $A \varepsilon Kl(Kl(!)) . \supset :$

2) $A \varepsilon A .$ $\quad (1)$

3) $\sim(A \varepsilon !) :$ $\quad (D3, 1)$

$[\exists a]:$

4) $A \varepsilon Kl(Kl(a)).$
5) $A \varepsilon a:$ $\Big\{ (D3; 3; 2)$

$\sim(B \varepsilon !)$ $(E8; 1; 4; 5; 3)$

E10. $[B] . \sim (B \varepsilon !)$ $(E4; E9)$

E11. $[aB] : B \varepsilon a . \supset . [\exists A] . A \varepsilon Kl(Kl(a)) . A \varepsilon a$

Dem.:

$[aB] ::$

1) $B \varepsilon a . \supset ::$

$[\exists A] ::$

2) $A \varepsilon Kl(Kl(a)) :$ $(E4; 1)$

$[\exists b] :$

3) $A \varepsilon Kl(Kl(b)) .$
4) $A \varepsilon b :$ $\Big\{ (D3; 2; E1)$

5) $A \varepsilon a ::$ $(E8; 1; 2; 3; 4)$

$[\exists A] . A \varepsilon Kl(Kl(a)) . A \varepsilon a$ $(2; 5)$

E12. $[BDE] : B \varepsilon B . \sim (B = D) . E \varepsilon Kl(Kl(B \circ D)) .$ $D \varepsilon E . \supset . B \varepsilon Kl(D)$

Dem.:

$[BDE] ::$

1) $B \varepsilon B .$
2) $\sim (B = D) .$
3) $E \varepsilon Kl(Kl(B \circ D)) .$
4) $D \varepsilon E . \supset ::$
5) $\sim (B \varepsilon D) ::$ $(2, 4)$

$[\exists C] ::$

6) $C \varepsilon Kl(Kl(D)) .$
7) $C \varepsilon D .$ $\Big\{ (E11; 4)$
8) $C \varepsilon E .$ $(7; 4)$

9) $C \varepsilon Kl(Kl(B \cup D)) ::$ (8, 3)

$[\exists A]:$

10) $A \varepsilon Kl(D):$ (E6; 7; 8; 9)

11) $C = A . \vee . A \varepsilon Kl(B \cup D) ::$

$[\exists F]:$

12) $F \varepsilon Kl(B \cup D):$ (E6; 1; 4; 6)

13) $C = F . \vee . F \varepsilon Kl(D):$

14) $A = F.$ (11; 13; E2; 12; 10)

15) $F \varepsilon Kl(D).$ (10; 14)

16) $F = B ::$ (E5; 1; 15; 12; 5)

$B \varepsilon Kl(D)$ (15; 16)

E13 $[BDE]: B \varepsilon B . \sim(B = D) . E \varepsilon Kl(Kl(B \cup D)) . D \varepsilon E . \supset . D \varepsilon Kl(B)$

Dem.:

$[BDE] .:$

1) $B \varepsilon B.$

2) $\sim(B = D).$

3) $E \varepsilon Kl(Kl(B \cup D)).$

4) $D \varepsilon E . \supset :$

5) $B \varepsilon Kl(D):$ (E13; 1; 2; 3; 4)

$[\exists A]:$

6) $A \varepsilon Kl(Kl(B)).$ (E12; 1)

7) $A \varepsilon B.$

8) $A \varepsilon Kl(D).$ (7; 5)

9) $B = A.$ (E2; 5; 8)

10) $\sim(A = D):$ (9; 2)

$D \varepsilon Kl(B)$ (E5; 4; 6; 8; 10)

E14 $[BD]: B \varepsilon B . D \varepsilon D . \sim(B = D) . \supset . B \varepsilon Kl(D)$

Dem.:

$[BD] ::$

1) $B \varepsilon B.$

2) $D \varepsilon D.$

3) $\sim(B = D) . \supset .:$

$[\exists E]:$

4) $E \varepsilon Kl(Kl(B \cup D)).$ $\Big\}$ $(E12;1)$

5) $E \varepsilon B \cup D:$

6) $B \varepsilon E . \vee . D \varepsilon E :.$ $(5;1;2)$

$B \varepsilon Kl(D)$ $(6;E13;2;3;4;E12;1)$

E15 $[ABD]: A \varepsilon A . B \varepsilon B . C \varepsilon C . \sim(B=A) . \sim(B=C) . \supset . A=C$

Dem.: $[ABD]:$

1) $A \varepsilon A.$

2) $B \varepsilon B.$

3) $C \varepsilon C.$

4) $\sim(B=A).$

5) $\sim(B=D) . \supset :$

6) $B \varepsilon Kl(D).$ $(E14;2;3;5)$

7) $B \varepsilon Kl(A).$ $(E14;2;1;4)$

8) $A \varepsilon D.$ $(E5;1;6;7;4)$

$A=D$ $(8,3)$

$\{E15\}$ jest sprzeczne z $\{E3\}$

$\{C1, E1, E2, E3\}$ prowadzi do sprzeczności.

II. Układ:

A1, E1, E2 i C2 prowadzi do sprzeczności:

D3

C1 (A1)

E4 – E15 (C1, E1, E2, D3)

F1 $[ABC]: A \varepsilon A . B \varepsilon B . C \varepsilon Kl(!) : C=A . \supset . A=B$

Dem.:

$[ABC]:.$

1) $A \varepsilon A.$

2) $B \varepsilon B.$

3) $C \varepsilon Kl(!).$

4) $C=A . \supset :$

5) $A=B . \vee . A \varepsilon Kl(B):$ $(E14;1;2)$

6) $A \varepsilon Kl(!).$ $(3;4)$

$A = B$ $(5, E5, 2, 6, E10)$

$\mathfrak{F}2\ [AB] : A \varepsilon A . B \varepsilon B . \supset . A = B$

Dem.:

$[AB] :$

1) $A \varepsilon A .$

2) $B \varepsilon B . \supset :$

$[\exists C] :$

3) $C \varepsilon Kl(1) :$

4) $C = A . \vee . C = B . \vee . A = B :$ $(E15; 1; 3; 2)$

$A = B$ $(4; \mathfrak{F}1; 1; 2; 3)$

$\{\mathfrak{F}2\}$ jest pewne z $\{C2\}$

$\{A1, E1, E2, C2\}$ prowadzi do sprzeczności.

§5 $\{C1, E1, E2,$ [illegible]$\}$ nie prowadzi do sprzeczności, jeżeli nie prowadzą do sprzeczności założenia:

§1 $C \varepsilon C$

§2 $E \varepsilon E$

§3 $\sim(C = E)$

E15

$D4\ [Aa] :. A \varepsilon Kl_1(a) . \equiv : A \varepsilon A : A = C . E \varepsilon a . \vee . C \varepsilon a . \sim(A \varepsilon a)$

§4 $[aB] :. B \varepsilon a . C = B . \supset : C \varepsilon Kl_1(a) . \vee . E \varepsilon Kl_1(a)$

Dem.:

$[aB] :$

1) $B \varepsilon a .$

2) $C = B . \supset :$

3) $C \varepsilon a :$ $(1, 2)$

$C \varepsilon Kl_1(a) . \vee . E \varepsilon Kl_1(a)$ $(D4; 3; §1)$

§5 $[aB] : B \varepsilon a . E = B . \supset . C \varepsilon Kl_1(a)$

Dem.:

$[aB] :$

1) $B \varepsilon a .$

2) $E = B . \supset .$

3) $E \varepsilon a$ $(1, 2)$

$C \varepsilon Kl_1(a)$ $(D4, §1, 3)$

$\dot{C}1.\ [aB] : B \varepsilon a . \supset . [\exists A] . A \varepsilon Kl_1(a)$

Dem.:

$[aB] :.$

1) $B \varepsilon a . \supset :$

2) $C = B . \vee . E = B :$ $(E15; S2; S1; 1; S3)$

3) $C \varepsilon Kl_1(a) . \vee . E \varepsilon Kl_1(a) :$ $(2; S4; S5)$

$[\exists A] . A \varepsilon Kl_1(a)$ (3)

§6 $[AaB] : B \varepsilon B . A = C . E \varepsilon A . \sim(A = B) . \supset . B \varepsilon a$

Dem.:

$[AaB] :$

1) $B \varepsilon B .$

2) $A = C .$

3) $E \varepsilon A .$

4) $\sim(A = B) . \supset .$

5) $\sim(C = B) .$ $(2, 4)$

6) $E = B .$ $(E15; S2; S1; 1; S3; 5)$

$B \varepsilon a$ $(3, 6)$

§7 $[AaB] : A \varepsilon A . B \varepsilon B . C \varepsilon a . \sim(A \varepsilon a) . \sim(A = B) . \supset . B \varepsilon a$

Dem.:

$[AaB] :$

1) $A \varepsilon A .$

2) $B \varepsilon B .$

3) $C \varepsilon a .$

4) $\sim(A \varepsilon a) .$

5) $\sim(A = B) . \supset .$

6) $\sim(C \varepsilon A) .$ $(3; 4)$

7) $C = B .$ $(E15; S1; 1; 2; 6; 5)$

$B \varepsilon a$ $(3; 7)$

$\dot{E}1\ [AaBb] : A \varepsilon Kl_1(a) . A \varepsilon Kl_1(b) . B \varepsilon b . \sim(B \varepsilon Kl_1(b)) . \supset . B \varepsilon a$

Dem.:

$[AaBb] :.$

1) $A \varepsilon Kl_1(a) .$

2) $A \varepsilon Kl_1(b) .$

3) $B \varepsilon b .$

4) $\sim(B \varepsilon Kl_1(b)) . \supset :$

5) $A = C . E \varepsilon a . \vee . C \varepsilon a . \sim(A \varepsilon a) :$ $(D4; 1)$

6) $\sim(A = B) .$ $(2, 4)$

$B \varepsilon a$ $(5; S6; 3; 6; S7; 1)$

$\dot{E}2\ [AaB] : A \varepsilon Kl_1(a) . B \varepsilon Kl_1(a) . \supset . A = B$

Dem.:

$[AaB] :.$

1) $A \varepsilon Kl_1(a) .$

2) $B \varepsilon Kl_1(a) . \supset :$

3) $A = C . E \varepsilon a . \vee . C \varepsilon a . \sim(A \varepsilon a) :$ $(D4; 1)$

4) $B = C . E \varepsilon a . \vee . C \varepsilon a . \sim(B \varepsilon a) :$ $(D4; 2)$

$A = B$ $(3; 4; S6; 1; 2; S7)$

$\dot{C}2\ [\exists AB] . A \varepsilon A . B \varepsilon B . \sim(A = B)$ $(S1, S2, S3)$

$\{\dot{C}1, \dot{E}1, \dot{E}2, \dot{C}2\}$ jest równokształtny $\{C1, E1, E2, C2\}$

§6 $\{A1, E1, E2\}$ nie prowadzi do sprzeczności, jeśli nie prowadzi do sprzeczności $\{S1, A10\}$

D5 $[Aa] : A \varepsilon Kl_2(a) . \equiv . A \varepsilon A . a \subset a$

$\ddot{A}1\ [a] . [\exists A] . A \varepsilon Kl_2(a)$ $(D5; S1)$

49

$H1\ [ABb]: A\,\varepsilon\,Kl_2(b) . B\,\varepsilon\,b . \supset . B\,\varepsilon\,Kl_2(b)$

Dem ∴

$[ABb]:$

1) $A\,\varepsilon\,Kl_2(b) .$

2) $B\,\varepsilon\,b . \supset .$

3) $A = B .$ $(A10; 1; 2)$

$B\,\varepsilon\,Kl_2(b)$ $(1, 3)$

$\ddot{E}1\ [AaBb]: A\,\varepsilon\,Kl_2(a) . A\,\varepsilon\,Kl_2(b) . B\,\varepsilon\,b . \sim(B\,\varepsilon\,Kl_2(b)) .$

$\supset . B\,\varepsilon\,a$ $(H1)$

$\ddot{E}2\ [AaB]: A\,\varepsilon\,Kl_2(a) . B\,\varepsilon\,Kl_2(a) . \supset . A = B$ $(A10)$

Są to rozwiązania Leśniewskiego (394-6 z 2.1938) dowodzące konieczności rozróżnienia klas dystrybutywnych i kolektywnych oraz odrzucenia tez Ak i Bk

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Bruksella, 10.IX.1948.

End of Notebook

50