St. Thomas Philosophy Seminar on the History of Logic

Sobociński's first position in the United States was in Saint Paul, Minnesota, at the College of Saint Thomas. He was only there for the spring semester of 1949–1950. During that semester he gave several talks.[1]

This manuscript is entitled "Philosophy Seminar" and is dated March 10, 1950. It is in three parts. The first two (pp. 1–10 and 11–21) are a double spaced typescript. The third (second pagination, pp. 1-5 is single spaced).

Sobociński wants to describe the relations between traditional and modern logic. He begins with a history of logic, which, he realizes is "a very young discipline." Yet he mentions numerous authors who have worked in this field (without detailed references). He discusses recent work on Aristotle's logic and on Stoic logic. His discussion of syllogistic naturally gives center place to his doctor-father, Łukasiewicz.

Stoic logic has given rise to propositional calculus. He distinguishes "schemes of reasoning" (rules) and "theses".

These three lectures display Sobociński's considerable knowledge of the history of logic.

[1] I have entitled the other manuscript "Lesniewski's Foundations of Mathematics." I

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PHILOSOPHY SEMINAR

Dr. Sobocinski

March 10, 1950

The object of this lecture will be the relations between the traditional logic and the (so-called) modern logic. I shall endeavour to show here what were the reasons which led to the creation of modern logic and what are its aims. Moreover, I shall try to prove that there are no fundamental differences between the traditional and the new logic. For better understanding I will precede my final conclusions by:

1) First, a short outline of the history of logic.

2) Second, by presenting some elementary theorems of modern logic.

From the history of logic I want to re-present some facts commonly known, established lately by contemporary historical research. This will help us to realize for what reasons and in what way the modern logic appeared. As we know, the history of logic is a very young discipline. It is true, that from the XIX century we inherited several works on this subject, but none of them has any scientific value. This is true first of all in respect to the enormous, monumental work "Geshichte der Logic im Abendlande" written by Karl Prantl in the middle of the XIX century (I ed. Leipzig, 1858, II ed. Berlin, 1927). The fact that the author belonged to the school of Hegel, combined with his profound hatred of Scholasticism and above all his ignorance of various logical problems has resulted in this voluminous work today being only a valuable collection of historical materials which Prantl collected and published, but which he did not really understand. One can say more or less the same about the book of Heinrich Maier "Die Syllogistik des Aristoteles," (Tubingen,1900) which was considered up till recently as the most fundamental book on this subject. Finally we must put into the same category, the book of J.N. Keynes "Studies and Exercises in Formal Logic" (4-th ed., London, 1906), which is by far the best text book of non-mathematical formal logic.

The critical and scientific history of logic treated as an independent branch of history emerged strictly speaking, almost 25 years ago, when this problem was at last understood by scientists who knew the methods both of historical and philological researches as well as the logical problems. During this time some important papers were published, as, for instance, "Zur Geschichte der Aussagenlogik" (Erkenntnis, 1935-36) by Dr. J. Łukasiewicz, former professor of Warsaw University, now professor of R.I. Academy of Dublin; "Geschichte der Logic" (Berlin, 1931), by Dr. H. Scholz, professor of Munster University, "Geshiedenis der Logica," (Den Haag, 1944) by Dr. E.W. Beth, professor of Amsterdam University. Besides these three authors I shall mention only by name such men as Fr. I. Bochenski, O.P., prelate M. Grabmann, Fr. K. Michalski, C.M., Fr. J. Salamucha, Fr. Stakelum, C.M., Fr. Boehner, O.F.M., Fr. R. Feys, Fr. A. Korcik, prof. Durr, prof St. Schayer and prof. Dopp. All these authors changed completely the picture we previously had of the history of logic. It is worth noticing that this profound revolution was effected principally by the Catholic scholars. These scholars proved irrevocably, that Kant's famous saying that logic has no history, as well as Prantl's opinion that there was one only logician in the world namely Aristotle, are entirely erroneous and due to ignorance. Although we are today still far from possessing the full picture of our subject and in particular the investigations of scholastic logic have hardly begun, nevertheless we can already affirm with total certitude, that the history of logic is more complicated than we supposed, and that the so-called traditional logic, is a result of the very complicated historical process. As Fr. Bochenski said, (see: Proceedings of the Tenth International Congress of Philosophy, Amsterdam 11-18 of August, 1948) "The history of logic can be pictured by an undulant curve: the periods of development are alternated by periods of profound decline, after which the work begins anew from the very foundations and usually without taking into account former achievements." Broadly speaking there were three periods in the history of the European thought when logic was at its height. The first one in antiquity, from Aristotle to the end of the middle Stoic school; the second in Middle Ages from the end of the XII century to the beginning of the Renaissance; and the third one in modern times since 1847. These periods are separated by epochs of deep fall and decline. Typical in this respect is the time of the late Middle Ages with an extremely high standard of logical considerations followed later by the period of the so-called "great" systems, from Descartes to Hegel with its complete stagnation in the field of logic.

Let us now recall the most important facts from the history of logic as this is the only way which will lead us to right conclusions.

As it has been established by the research made first by prof. Zukasiewicz there were in antiquity two different schools of logic which were independent of each other. The first of them was founded by Aristotle; the second one appeared some thirty to ninety years after the death of the author of the Organon. This second one grew on the bosom of the old Stoa. These two schools came into being independently of each other and were rival schools as they taught two different logical theories as different in fact as arithmetic is from geometry. Some time later, Antiquity resounded with logical quarrels between the followers of both schools. Eventually many centuries later towards the end of Antiquity this quarrel led to some sort of synthesis achieved by the so-called, late commentators of Aristotle. This synthesis in the form elaborated by Porfirios was transmitted to the early Middle Ages by the Christian martyr Boetius. Thus the Latin version of this synthesis worked out by Boetius became the basis for medieval logic.

To make the picture quite clear I will recall some fundamental features characteristic of the Aristotelian logic, as well as those which characterize the logic of Stoics. This will help us to understand the origin of this synthesis which was transferred from the late Antiquity to the early Middle Ages.

Those principal points of the Aristotelian logic as set off by recent investigations can be presented as follows:

I. Aristotle conceived his syllogistics as a deductive system. Consequently he tried to give it the form of an axiomatic theory. For this purpose he posited without proof, that is as axioms, four syllogisms of the first figure. For him the reduction of other figures to the first one was nothing else than an attempt to prove, that it is possible to deduce all the syllogisms from Barbara, Celarent, Darii and Ferio, those four syllogisms being admitted by him a priori. Therefore we can state, that Aristotle, first in the world, understood the idea of the deductive system and tried to construct such a system even before the geometry of Euclid. Today we know that his axiomatics of syllogistics is not sufficient as a deductive basis for his system of logic. This however does not alter the fact that he is the first creator of the first deductive system in the world.

II. Aristotle is without doubt inventor of the idea of variables, many centuries before the Arab mathematicians. In his logical works he uses the letters α, β, γ et c., (which exactly correspond to our letters

P, S, M, in the same meaning), in which the letters are used in algebra. Moreover, Aristotle clearly distinguishes two different kinds of such variables. Those for which we can substitute only notions, and others for which we can substitute only declarative propositions. In other words, Aristotle distinguishes name variables and propositional variables, as can be clearly seen in A.Pr., A 15, 34 a, 22 a, etc. Knowing what is the importance of variables in the sciences especially in the deductive ones, we can not sufficiently stress the value of this discovery. Unfortunately, however, this was not understood by some of Aristotle's successors and followers.

III. Aristotle unmistakably limited the field of the development of the name variables. He allows in substituting for such variables only the unempty and common notions. It means, that if we want to use correctly his system of logic, we can substitute for the variables only such names, which denote the objects really existing and which have at least two designates. The second part of this condition (concerning common notions) does not interest us in our present considerations, but I would like to stress that it had been adopted by Aristotle in view of the specific properties of his philosophical system. However, the omission of this condition does not affect the correctness of the system of syllogistics.

The position is quite different in respect to the first part of the condition. This part declaring, that we can substitute for the variables only non-empty notions circumscribes the field of the Aristotelian logic. If we reject this condition, then we will have to reject several theses of the Aristotelian logic as incorrect. This was already perfectly known to some old scholastic logicians, and is mentioned by John of St. Thomas in his Cursus Philosophisus (vol.I, pp 32 ss.). The condition as stated above was also the principal cause of misunderstanding between the old and the new contemporary logic, because it often either is not understood or simply forgotten by some authors, We shall come back later to this topic when discussing the modern logic. At present let us only remember, that this is the principle which is, so to say, outside the proper, formal system of the Aristotelian logic. In order to be brief we shall call it "condition A."

IV. What we said in points 1, 2, and 3, circumscribes satisfactorily the sphere of Aristotelian logic, sufficiently, even if we did not know other declarations by Aristotle. His system of logic is a deductive theory, which establishes some connections between unempty notions, it is, to use a modern expression, a part of the calculus of terms.

V. Aristotle formulates the syllogisms constantly and persistently in a special manner. This is entirely different from that which is used by traditional logic, but it is in complete harmony with the requirements of the modern logic. The following example will explain this question. Everybody knows, that the syllogism Barbara has the following form in traditional logic:

$$\frac{\begin{array}{c} M\ a\ P \\ S\ a\ M \end{array}}{S\ a\ P}$$

In vain, however, we would look for such a form in Aristotle's writings. What can be found instead are only the following propositions as written down in A.Pr. A 4, 25 b, 38 s and 26 a, 34 ss, 37 ss, 26 b, 3 ss etc.:

Εἰ γὰρ τὸ α κατὰ παντος τοῦ β, καὶ τὸ β κατὰ παντος τοῦ γ ἀνάγκη τὸ α κατὰ παντος γ κατηγορεισθαι.

The exact translation of this sentence runs as follows:

> If α belongs to β in the whole extension and β belongs to γ in the whole extension, necessarily α is in the whole extension predicated γ.

That is, using our logical language:

> If every α is β and every β is γ, then every α is γ.

That is, using the traditional symbols:

> If S a M and M a P, then S a P.

It should be stressed, that all syllogisms were written by Aristotle in this manner, and in particular that the Minor comes first and the Major second. When we compare both formulas, that is the Aristotelian and that elaborated by the traditional logic we see at once the fundamental difference between them. In the traditional logic the syllogism is presented as a scheme of reasoning. It is a rule which declares, that having such and such two premises we can infer from them such and such conclusion. With Aristotle the syllogism has exactly the same form as every theorem of mathematics has, when correctly stated, for instance the following arithmetical thesis:

> If $a < b$ and $b < c$, then $a < c$.

Now let us state clearly in what exactly consists the difference.

The traditional syllogism is as we said, a verbal rule of reasoning. The well-known inscription:

$$\begin{array}{rl} \text{(if)} & \text{MaP} \\ \text{(and)} & \underline{\text{SaM}} \\ \text{(Then)} & \text{SaP} \end{array}$$

should in fact be read in the following way: "If we accept as true the statement that every M is P, and if we pass to the other statement which we also accept as true, namely that every S is M then we infer that every S is P."

Here, as we see, the formal scheme is mixed up with some directives telling us that we should <u>pass</u> from one premise to the other and then infer from both premises the conclusion. However, it is not explained what is meant by passing from one premise to the other and what is meant by inferring. In other words it is, as we said, a verbal rule of reasoning, and a rather clumsy one.

The Aristotelian syllogism is conceived in a completely different way. It is conceived by Aristotle as a thesis of a deductive system. Being part of this deductive system it is as every such thesis completely dead or inoperative; we cannot do anything with it, we cannot use it, unless we are provided with some directives, or as we say rules of procedure. In the case of the syllogism these rules are two, namely the rule of substitution and the rule of detachment.

It must be stressed that this conception, is quite explicit with Aristotle, that Aristotle realized what a deductive system is, although the rules of procedure were not fully developed and elaborated by him.

To sum up our last considerations we will say that both forms of syllogistics, the Aristotelian and the traditional express the same idea but in two different forms. They are both syllogistics, but they differ greatly from the formal point of view. Or we could say that they have different formal properties.

The syllogism as formulated by Aristotle allows the building of syllogistics as an axiomatic system.

The syllogism as formulated in the traditional logic cannot be used for this purpose.

The question of course arises as to what was the cause of this misunderstanding, change and confusion; how and why it happened that the Aristotelian form of syllogism was replaced by the traditional one. This is one of the most interesting and at the same time one of the most important questions of the history of logic. I shall try later to explain it. At present I mention only, that all great commentators of Aristotle in Antiquity up to Alexander of Aphrodisias inclusively, followed Aristotle.

VI. The Aristotelian form of syllogism as well as his other statements shows clearly, that Aristotle conceived the implication of the expression "If ..., then ..." in the wide, formal sense. It should be noted that, in his days a famous discussion was started by the school of Megara concerning the sense of "implication". Diodor Kronos (dead ⁄307), wanted to limit conditional proposition to those cases only where the antecedent and the consequent are bound together by their internal contents. Aristotle took the opposite view and of course he cannot be blamed for the fact that some of his followers preferred to follow on this point Diodor Kronos.

VII. One of the fundamental parts of modern logic is the so-called calculus of propositions. Aristotle did not discover it, but he was very close to this discovery. We find in his writings some rules of procedure which in fact are verbal paraphrases of certain theses of this calculus. Aristotle used these rules particularly often, when he dealt with the reduction of other figures of syllogism to the first one, as for instance in A.Pr.A.44, 50 a, 22 ss or A.Pr. B 8, 59 b 1 etc. One has the impression that he simply had not enough time for this discovery. He was particularly near this discovery in the period of "Topics." However, the honour of this discovery belongs to others. Aristotle's closest disciple Theophrast developed some parts of Aristotelian logic. Unfortunately, we have only few fragments of his works. However, even what was saved shows, that he was a great logician. Some of his ideas were recently discovered by the modern logicians independently of him. So for instance, was the concept of particular quantifier with its far-reaching consequences. Theophrast can be considered as a link which connects the Aristotelian period of the history of logic with the next period of Greek logic which, by the way, has not yet been sufficiently examined.

II.

About 300 years B.C., and over twenty years after the death of Aristotle, Zeno of Kition founded a new philosophical school - the school of Stoa. However, the real founder of the school was Chrysippos (scholarch 232 -205), the third ruler after Zeno. About him it was said, that without him Stoa would not exist. He was also the founder of the Stoic logic. Unfortunately only a few fragments were saved from the immense literature of old and middle Stoa (the new Stoa discussed only ethical problems and therefore it does not interest us here). Consequently, in order to reconstruct its logical doctrine it was not enough to use the classic collection of von Arnim (Veterum Stoicorum fragmenta), but it was also necessary to go through all the writings of Aristotle's commentators, of Sextus Empiricus as well as through some works of the Fathers of the Church. These difficulties explain why the Stoic logic became known only recently. It should be stressed that the solution of this historical problem threw much light on the genesis of the traditional logic.

Already at the first contact with the Stoic logic our attention is called to the fact, that it uses an entirely different terminology from the Aristotelian one. It is evident that Chrisipps and his followers did not know the language of Aristotle or of Theofrast. For instance when referring to their theory they use the term οἱ λόγοι a term unknown to Aristotle, and from which our "logic" is derived. For their logical formulas, they used the term οὐχ ἁπλᾶ ἀξιώματα (non-complete proposition) which we do not find in the Organon. It is from here that our term "axiom" is derived, although it has now a different meaning. However the difference between the Stoic and the Aristotelian logic is not limited to the terminology; it extends also to the contents and is decidedly of a fundamental character. It must be stressed that their logical formulas or axiomata are quite different from those of the Aristotelian syllogistic. A great number of these formulas is at present known to us and I shall write down a few of them in the same form as was used by the Stoics:

I.
$$\begin{array}{l} \text{If } \alpha', \text{ then } \beta' \\ \alpha' \\ \hline \text{therefore } \beta' \end{array}$$

II.
$$\begin{array}{l} \text{If } \alpha', \text{ then } \beta' \\ \text{non } \beta' \\ \hline \text{therefore non } \alpha' \end{array}$$

III.
$$\begin{array}{l} \alpha' \text{ or } \beta', \\ \text{non } \alpha' \\ \hline \text{therefore } \beta' \end{array}$$

IV.
$$\begin{array}{l} \text{non } /\alpha' \text{ and } \beta'/ \\ \alpha' \\ \hline \text{therefore non } \beta' \end{array}$$

V.
$$\begin{array}{l} \text{If } \alpha', \text{ then } \beta', \\ \text{If } \beta', \text{ then } \gamma' \\ \hline \text{therefore If } \alpha', \text{ then } \gamma \end{array}$$

VI.
$$\begin{array}{l} \text{non non } \alpha' \\ \hline \text{therefore } \alpha' \end{array}$$

VII.
$$\begin{array}{l} \text{If } \alpha', \text{ then } \beta', \\ \text{If non } \alpha', \text{ then } \beta' \\ \hline \text{therefore } \beta' \end{array}$$

Looking at these formulas we notice at once that they have the form used in traditional logic and that some of them are similar to the (so called) compound syllogisms. In order to understand why this is so, we must first consider what the Stoics intended to express by these formulas. As I mentioned before we know without any doubt that they wrote these formulas in the following way: If first, then second. First. Therefore second. And so on, and that they understood the expressions "first", "second", "third" and so on, as propositional variables, which they allowed to be replaced only by decla_rative propositions. Moreover, they understood the term "declarative proposition" in the same way as the modern logic, that is, in the widest sense. The position is exactly the same in respect to the expression "if..., then", that is in respect to the implication; this also they understood in the largest sense. What I said before describes and determines sufficiently the Stoic logic. In short, they discovered the calculus of propositions. We will have to consider later this theory; at present I would like to mention in a few words the strange history of this theory. There is no doubt that Aristotle and Theofrast anticipated it in some way, but, as I said before, it has been explicitly formulated only in the school of Stoa. However, their achievement was not really understood, was misinterpreted and deformed by later commentators and eventually, during the first centuries B.C., forgotten. This theory was (for the second time) discovered in India during the first centuries after Christ by some Buddhist philosophers, among whom the most prominent was Dinnaga. This discovery, it should be stressed, was made quite independently and without any influence of Greek philosophers, and it also was soon forgotten.

The late Scholastics discovered it for the third time in the history of human thought, and again as it seems, independently. With them these considerations were known by the title "De consequentiis". This time this theory has been more fully developed and more profoundly examined, than was the case with Stoics. However, the same causes which destroyed the continuity of the scientific development of the late Scholastics also put into oblivion their new logical considerations.

For the last time and definitely the calculus of propositions was discovered in 1879 by G. Frege who elaborated it in an almost perfect form. This time too it was discovered independently of all previous historical developments. The history of several repeated discoveries of this theory and the fact that they were repeatedly forgotten is the more strange in that during the same time, even during the periods when the calculus of propositions fell into complete oblivion several laws belonging to this theory were quite explicitly and clearly stated. A typical example in this respect is the theorem of Clavius from the XVI century as well as the famous "casus Sacceri" from the XVIII century. We will have to deal with both of them later.

Let us now go back to our analysis of Stoic Logic. Here I would like to mention following points:

I. The Stoic patterns were always given as schemes of reasoning and never as logical theses (as was the case of Aristotelian syllogisms). These Stoic schemes have exactly the same forms, as the syllogism of the traditional logic. This is an indubitable fact but historical documents which we possess do not explain the reasons, which induced Chrisippos to adopt such form.

If the Stoics wanted to formulate their patterns in the Aristotelian way, then for instance, the first of the above-mentioned should have the form of the following thesis:

If, if α, then β, and α, then β

The second:

If, if α, then β, and non β, then non α

The third:

If, α or β, and non α, then β

and so on

II. The Stoics did not develop their logic as an axiomatic theory.

III. Those remnants on which Stoic logis is based & which have reached us show that Stoics probably did not know the whole calculus of propositions. They show no trace of such important theorems, as for instance, those of Duns Scotus, of Clavius, of Pierce or the laws of De Morgan.

IV. However, they were perfectly aware, that their logic differs completely from the Aristotelian one and we have positive indications as to this point. Both these great schools of logic in Antiquity fought each other, and what seems rather strange, nobody in Antiquity realized that these two theories were not contradictory, but, on the contrary, complimentary. Taken together, the Aristotelian and the Stoic logic constitute a nearly complete system of logic.

After these considerations we realize how wrong was Prantl, when he described the Stoic logic as "grenzenlose Stüpiditat" (III, 472). It shows only that Prantl hardly understood anything of formal logic. It is true that the Stoics were inferior to Aristotle in their formal considerations, nonetheless, they created a new and fundamental part of logic, although the form they gave it was not satisfactory.

As I mentioned, there was in Antiquity a struggle between these two schools of logic. This we know from the writings of Aristotelian commentators. Originally these authors distinguish very well the Aristotelian logic from the Stoic theory. They not only understood very clearly the difference between Aristotelian syllogisms and Stoic formulas, but also they gave the logical laws of both schools in their authentic form:

the Aristotelian syllogisms as logical theses;

the Stoic patterns as schemes of reasoning.

The situation began to change in late Antiquity. The gradual decadence of the antique civilization, the loss of Stoic writings and the complete disappearance of the interest in the Aristotelian philosophy characterize this period of Antiquity. We know that in those times, while almost nobody followed the philosophy of Aristotle, his scientific authority grew considerably. Later on, the Scholars, deprived of the ancient scientific culture and possessing no adequate invention, tried to melt down the remnants of the antique knowledge into some sort of synthesis, the elements of various authors and schools being often put together mechanically. Such procedure can be observed in various branches of knowledge and frequently we are unable to decipher what were the original sources on which they were based. The result of such a procedure was that the uniformity and homogeneity of the synthesis achieved in this way was only apparent. With the whole expressiveness we can trace very distinctly this process in the logic of late Antiquity. With older commentators, for instance, with Alexander of Aphrodisias or with Galien (c. 199 B.C.) we find a correct exposition of both systems of logic. Later these two systems are mixed together. Various authors cease to understand the difference, existing between the Aristotelian and the Stoic logic. They do not distinguish the name variables of the Aristotelian syllogistics from the propositional variables of the Stoic formulas and often confuse the terminology of both schools, taking no account of the obvious meaning.

This process is particularly clear in the writings of Philopon, one of the latest Aristotelian commentators, who lived at the end of the V and the beginning of the VI century. With this Neoplatonic author the confusion of ideas and terminologies reaches its climax. Aristotelian syllogisms receive the form of Stoic schemes, and these are interpreted as laws of the logic of terms. The following example will show us how it happened.

Let us remember the first Stoic formula, which we earlier mentioned:

$$\begin{array}{c} \text{If } \alpha\text{, then } \beta \\ \alpha \\ \hline \text{therefore } \beta \end{array}$$

and let us compare this with the syllogism Barbara, in the form as used by Aristotle namely:

If every α is β and every β is γ then every α is γ

(see my first lecture, page 7)

In Greek the Stoic formula has the following form

$$\frac{\begin{array}{c} \varepsilon\grave{\iota}\ \tau\grave{o}\ \alpha,\ \tau\grave{o}\ \beta \\ \tau\grave{o}\ \alpha \end{array}}{\check{\alpha}\rho\alpha\ \tau\grave{o}\ \beta}$$

The word "εἰ" means exactly the same as English "If ..., then..."
The word "ἄρα" corresponds to the Latin "ergo" or English "therefore."
Now it should be stressed that the word "ἄρα" is very seldom used by Aristotle, on the other hand it is a typical and a necessary element in the Stoic logic.

The Aristotelian syllogism is a thesis having always the form of an implication, it is a conditional proposition, and as such, it is always preceded by the words "εἰ" or "ἐπεί", that is the English "if". For the variables of the Aristotelian syllogism only the unempty notions can be substituted.

The Stoic "formula" is a scheme of reasoning and as such it never has the form of a conditional proposition. For the variables of such scheme only declarative propositions can be substituted. This can be proved not only by explicit historical declarations of some Stoic philosophers, but also corroborated by obvious reasons. So for instance, if we take, the third of the above-mentioned Stoic formulas:

$$\frac{\begin{array}{c} \alpha \text{ or } \beta \\ \text{non } \alpha \end{array}}{\text{therefore } \beta}$$

and if we substitute for variables and any notions, say "man" and "horse", we obtain the following expression:

$$\frac{\begin{array}{c} \text{MAN or HORSE} \\ \text{non MAN} \end{array}}{\text{therefore HORSE}}$$

which is obviously meaningless.

On the contrary we obtain an entirely correct reasoning, if we substitute for variables α and β any declarative propositions, for instance, "After this lecture I shall go immediately to St. Paul" and "After this lecture I shall go immediately home." In that case our formula assumes the following form:

> (After this lecture I shall go immediately to St. Paul.) or (After this lecture I shall go immediately home.)
>
> non (After this lecture I shall go immediately home ~~St. Paul.~~)
>
> ---
>
> therefore (After this lecture I shall go immediately to St. Paul.)

As we see we have here three propositions.

The first consists of two propositions united by the word "or". It states, that one of two eventualities will materialize. The second proposition states, that it is not true, that the first eventuality will materialize. Therefore it is true, as stated by the third proposition, that the second eventuality will materialize.

We see that it is a correct reasoning and that it is different from the syllogistic one.

The synthesis, produced by the late antiquity confused and mixed together everything and the writings of Philopon give evidence on this point. First of all, the terminology is entirely haphazard. Our science, created by Aristotle, receives the name "logic" which is of Stoic provenience and besides several Stoic expressions enter into the logical language. Moreover, what was worse, in this synthesis the unfortunate Stoic conception, namely the scheme of reasoning, replaced the correct Aristotelian conception of a logical thesis. At the same time the idea of Stoic logic became completely disfigured and eventually people ceased even to realize that those were two greatly different systems of logic. Therefore, the Stoic word "ἄρα", our "ergo" or "therefore", seldom used by Aristotle, was introduced into every logical formula, no matter whether of Stoic or Aristotelian provenience. On the other hand, the Aristotelian words "εἰ" or "ἐπεί" were dropped. From now on, the Aristotelian syllogisms have lost their primitive, correct form. They cease to be theses having the form of implication, and contrary to the intention of Aristotle they assume the form of Stoic schemes of reasoning. The homogeneous Aristotelian thesis, for instance, Barbara:

If SaM and MaP, then SaP

after the expression "if ..., then ...", split in three separate parts: "SaM", "MaP", "SaP" and after the addition of the Stoic term "ἄρα" acquired the Stoic form

M a P
S a M
ἄρα Therefore S a P

the form so well known from traditional logic.

But, the fate of the Stoic logic was even worse. The authors, we mentioned before did not understand the difference between the name-- and the propositional--variables. They felt, that the Stoic formulas cannot be treated, as a part of the Aristotelian logic. They became aware that Stoic schemes in which symbols are interpreted as name--variables lead to nonsense, as shown by the above-mentioned example of man and horse. Therefore, they tried to interpret them in a different way and came to the conclusion that those ciphers (which as we know were symbols of propositional variables) stand for predicates of some categorical proposition whose subject and copula has been lost or omitted e.g. by a careless copyist. That e.g. the original inscription was " is P", or "Every S is P", or "Some S is P" and that for some reasons eventually only the last part remained. It goes without saying that such an explanation is contrary to all intentions of Stoic philosophers.

Applying this interpretation they obtained the following well-known formulas:

I. If S is P, then S is N
S is P

Therefore S is N

II. If S is P, then S is N
S is not N

Therefore S is not P

These formulas, which we know so well from the traditional logic, where they are called Compound and Disjunctive Syllogisms, are merely substitutions of the above-mentioned authentic Stoic formulas. In order to understand better the situation thus created in logic, let us imagine that in arithmetic the general thesis:

$$a + b = b + a$$

has been rejected and only its particular substitutions, e.g.:

$$(c \times a) \; (c : b) = (c : b) \; (c \times a)$$

were accepted.

We will agree that the result of such procedure would be a great weakening of our arithmetic system.

This is exactly what happened to the Stoic formulas which lost their strength and importance, and becoming weak substitutions were added to the Aristotelian logic as modest and unimportant supplement.

I have presented in a short and schematic way the historical origin of the traditional logic. This history shows clearly that the so-called traditional logic came into being as the result of an inept confusion of the Aristotelian logic with the Stoic one.

Besides in the course of this process, both these systems underwent some deformations. This genesis of the traditional logic was completely forgotten, so that, when in the late Middle Ages, scholastics discovered anew the calculus of propositions they were unaware of its connection to such an extent that they treated their considerations of compound syllogisms quite separately from their studies of the calculus of propositions. The first were a part of normal logical compendiums, the second were developed in special treatises, called "De consequentiis".

I hope that what I have said explains sufficiently the fundamental points from the history of the antique logic.

Evidently I was obliged to omit many important facts as e.g. the whole problem of the Platonic logic or the interesting fact that the fourth figure of the syllogism was discovered many years before Galien. However, I would like to mention the indubitable fact that the first logical antinomy, namely "The Liar" has been discovered by the school of Megara. From Rustow's classical work on this subject (Der Lügner, Leipzig 1910) we know what a great importance was attributed to this problem by Antiquity. We know also that Antiquity did not solve this problem. For a solution it had to wait till the XIV-th and then again till the XX-th century.

III.

§3. In ~~my previous lectures~~ §§ 1-2 I gave a short explanation of the genesis of the traditional logic, which began in later antiquity. The inheritance received by the early Middle Ages was exceptionally poor. Originnally Aristotle's one logical writing namely "De interpretatione" was known and only from X century "De categoriis". The logic was known principally from some writings of Porfirios, of Boecius, of Cossiodorus, of Martianus Capella and from "De dialectica" of St. Augustin. In the year 1128 Jacob of Venice first translated the remaining parts of the "Organon". In this period, from the end of Antiquity to the middle of the XII century, scholasticism principally developed its own Scientific language. In this time, also the well know terminology of traditional logic was created and this was done on the basis of the writings of Boecius; St. Augustine, and through him Plato, also greatly influenced the rising language, as well as some logical conceptions. However this period probably did not contribute anything to formal logic. The authors of that time having at their disposal little fragments only of the Aristotelian logic, could only prepare the way for the future development and pose some problems. Their whole effort was directed towards constructing a uniform language and terminology for the use of theology and philosophy. This was the task which had not been accomplished by the Patristic period. In conformity with the scholastic tradition, adopted also by Prantl the logic of this period is called "logica vetus".

More or less from the middle of the XII century to the end of the XIII century, or to the death of Duns Scotus /+1303/, we have a new period of scholastic logic, called "Logica nova". Its characteristic features are:

1. All the writings of Aristotle were gradually discovered. Moreover the works of some commentators of Aristotle and as well as these of some Byzantine and Arabian authors became known. Consequently, the whole Aristotelian logic was adopted by the scholastics, but it was formulated in the terminology established by "logica vetus". Hence as this last one dependent on Boetius and late Aristotle's commentators, the formal logic of this second period was understood in the same sense as it had been in late Antiquity. That is, the syllogisms are treated as the rules of reasoning and the remnants of Stoic logic, as an unimportant part of the Aristotelian logic of terms.

II. In this period we meet the first scholastic compendiums of logic, for instance "Summulae logicales" of Petrus Hispanus.*) This famous text book, written by the later Pope John the XXI /+1277/, had a career similar to the "Sententiae" of Petrus Lombardius. There were in this time other manuals of logic, perhaps better ones, but because of the clarity and the simplicity of exposition the "Summulae logicales" became the standard logical book for all the later Middle Ages. This fact greatly influenced the history of medieval logic and above all, the later scholastic philosophy. In the book of Petrus Hispanus formal logic is poorly presented, and besides his manual was strongly influence by the problematic of Aristotle's Topics. This had a rather disastrous effect. As Petrus Hispanus' work was a principal source of logical knowledge during the XIV and XV centuries, the modes of discussion expounded in Topics in the form which is not too precise were recognized to have an importance equal to the strict laws of formal logic, which were the only ones used by the great scholastics of the XIII century. This fact was one of the reasons for the critical attitude of the philosophy of the XIV century.

III. In this period the first logical symbolism appeared, namely letters A,I,E,O were adopted as symbols of logical functors: "every..is...", some...is..." etc. This, we know, was an idea of M.Psellos, but we do not know by which way it because known in the West. However this question implying the problem of Psellos' influence on Western logic, first raised by Prantl, has been never staisfactorily answered.

IV. The greatest scholastics of the XIII century did not treat logic as their principal subject-matter. No one of them composed any work devoted primarily to logical problems: we have only their commentaries on the logical writings of Aristotle and Boecius. Besides their logical observations are scattered all over their philosophical and theological writings. In order to reconstruct the logical doctrine of St. Thomas, for instance, one should collect and analyze all logical passages from his writings. This work has still to be done, however modern investigations made up till now show that St. Thomas' logical remarks and considerations included in his later, mature works go considerably beyond those of his commentaries. In both "Summae" he uses often and consciously the logical forms which were known to the traditional logic. In many places quite unexpectedly we meet passages containing profound logical considerations, which unfortunately were never developed into special dissertations.

The third period of scholastic logic, with the so called "logica modernorum" includes the XIV and XV centuries. Logic of this period attains a very high standard in its development, but being in close connection with the contemporary philosophy, it suffered the fate of the later scholasticism. Logica modernorum went into complete oblivion together with the whole philosophy of Ockham and his successors and followers, and only recently, owing to the modern research work we begin to realize how original was this philosopphy and that its standard was very high. ~~First of all logica modern research work we begin to realize how original was this philosophy and that its standard was very high.~~ First of all logica modernorum rebuilt the calculus of propositions, forgotten since the Antiquity. Besides its analysis of language and related problems led to conclusions which are very close to the modern semantical investigations. This, for instance, for the first time the famous antinomy of "the liar" was solved correctly by those logiciens of the XIV century. They showed that it arose by the inadmisible mixture and confusion of two various languages, namely of the language which we speak, with the language which we speak about.

These are the principal data and the fundamental features of the scholastic logic. A full and detailed presentation of this question is impossible because the investigations of the scholastic logic conducted up till now are completely insufficient and inadequate. As Fr. Bocheński observed "we know practically nothing" about this Logic.[12] We can show only the main lines of the historical progress of scholastic logic, and we can indicate some of the most interesting points, which are being discovered now. We have neither adequate investigations of the Arabian logic and its influences on the various medievel authors, which makes impossible any scientific work in this field. We have not even any monograph on logic by such well known scholastic personalities as Albertus Magnus, Duns Scotus and not even on St. Thomas. As far as St. Thomas is concerned we have only few and small contributions, but even from them it is evident that the Angelic Doctor went considerably beyond the considerations of the traditional logic. I will mention here only a few points.

The formulation of the famous definition of the term "equal" often attributed without foundation to Leibniz, was discovered really by St. Thomas. Burali-Forti first ~~discovered~~, that St. Thomas defines this term in the following way: "Quaecumque sunt idem ita se habent quod quidquid praedicatur de uno, praedicatur et de alio". ./S.Th.I^{a} 40, 1, 1^{m}/.[19] Leibniz expresses exactly the same idea: "Eadem sunt, quorum unum potest substitutui alteri salva veritate".[20] Both these formulas translated into the present logical or mathematical language, have exactly the same meaning: "For any a and b, a is equal with b, if and only if for any φ, φ for a, if and only if φ for b", which we can write in the following symbolical form:

I. $[ab] : a = b . \equiv : [\varphi] : \varphi(a) \equiv \varphi(b)$

This formula proved that at we can define this fundamental mathematical term by logical notions only, that is, that we can express this term, as a purely logical term. This will become still more evident if we note that on the strength of the Aristotelian condition "A" mentioned before the expression:

$$[y] : y(a) \equiv y(b)$$

is equivalent to the following formula: Every a is b and every b is a.

Therefore, using for the functor "every ... is ..., a symbol "⊏", we obtain the following logical thesis:

II. $[ab] : a = b . \equiv . a \sqsubset b , b \sqsubset a$

Consequently, the term "equal" can be defined in the field of syllogistics.

Everyone with an elementary knowledge of the foundations of mathematics and its history will understood that the history of logic would have been different if this idea of St. Thomas had been understood properly. However, if with some restrictions we add the first thesis to the syllogistic, then we can obtain the whole contemporary elementary calculus of terms. Another point, which I would like to mention is St. Thomas' opinion on the construction and meaning of the universal propositions, which comes very close to some ideas of modern logic, and this will become obvious to everyone who will compare St. Th. S.T. 1ª 13, 12 and Principia Mathematica I,*10.[20]

These points which we mentioned, as well as others show clearly that the logical considerations of St. Thomas cannot be squeezed within the narrow frame of traditional logic. For the construction of his great and compact philosophical and theological system, St. Thomas was obliged to use stronger logical instruments, than these which could be supplied by the traditional logic. A logical analysis showed that his various reasonings, absolutely correct from the logical point of view, are not contained in the modern logic. This is true, for instance, in respect to the proofs of the existence of God[22] and of immorality of the human soul.[23]

Unfortunately, St. Thomas did not collect,[24] develop and build his logical considerations into a uniform system, but even scattered all over his works as they reached us they show early that he did not treat logic as a dead and closed knowledge. From where St. Thomas was standing there is only one small step needed to the position of modern logic.

Although St. Thomas used various forms of reasoning, which in fact belong to the calculus of propositions, he did not formulate any law of this theory.

The first thesis from this field, which was constructed consciously in the Middle Ages was formulated by Duns Scotus. This famous law unknown to the Stoics, declares:[25]

If we adopt in the same time two contradictory propositions, then we must adopt any proposition.

In the symbolic form, we can express it in the following way:

$$p \supset . \sim p \supset q$$

The logical considerations of Duns Scotus are not yet satisfactorily examined. We know neither in which way nor to what extent did they influence subsequent generations of philosophers.

It is a fact, for instance, that W. Ockham knew a number of thesis belonging to the calculus of propositions and that he was fully conscious of the difference between them and the traditional logic. But we do not know whether he was or was not influenced in this respect by Duns Scotus. It is quite possible that Pseudo-Duns Scotus, the unknown author of the "Quaestiones disputate super philosophiam", was here an intermediate link, but this point requires a historical investigation.[26].

With W. Ockham began the new period of scholastic logic, the period of "logica modernorum". As nearly the whole philosophy of the XIV century, so the logica modernorum was unknown up to the present time. Only recently we discovered that the XIV century was a period in which scientific investigation was equal to those of the XVI and XVII centuries. Thus, for instance, Nicolaus of Oresme formulated his geometry of coordinates, which is nothing else than the analytical geometry of Descartes. In the year 1370 the same author established exactly the same laws of mechanics which later were formulated by Galileo. As for the logic of the XIV century, from what we know already, we can say that it reached a high degree of development.

The logicians of the XIV century knew the calculus of propositions better than the Stoics. For instance, they discovered important laws, discovered again independently in the XIX c. by De Morgan:

$$a/ \sim(p . q) \;.\equiv.\; \sim p \vee \sim q$$

$$b/ \sim(p \vee q) \;.\equiv.\; \sim p . \sim q$$

four theses of transposition:

$$c/ \quad p \supset q . \supset . \sim q \supset \sim p$$
$$d/ \quad p \supset \sim q . \supset . q \supset \sim p$$
$$e/ \quad \sim p \supset q . \supset . \sim q \supset p$$
$$f/ \quad \sim p \supset \sim q . \supset . q \supset p$$

and many other theses.

However, they were not able to connect these considerations with the Aristotelian logic, and treated them, as separate branch of their logic. The theses of this calculus were considered separately from each other and never presented in the form of an axiomatic system. They also never used this calculus to mathematical investigations. The result was, that, when the philosophy and science of the XIV century sank into complete oblivion, the above mentioned logical discoveries did not leave any trace in the traditional logic.

The second important field of logical investigations of the XIV century was semantics, which as developed in that period came very close to similar contemporary considerations. Many problems which have been posed recently, were discussed and often solved correctly 600 years ago. The interest in semantical problems increased still after the death of Ockham /+1350/, but unfortunately, we cannot enter here into this problem, so important and interesting.[27)]

We must, however, mention one more fact, namely that the famous cardinal Peter d'Ailly /1350-1420/ intended for some philosophical purposes, to adopt some sort of a many value system of logic.[28)] This one fact only, shows convincingly how highly developed were the logical considerations in this period.

The decline of the medievel scholasticism entailed also the end of logica modernorum. In the XV century we find mainly unimportant commentaries and trite text-books, as for instance, the large one written by Paul of Venice, or a small, well written one by Savonarola. This last one was often reprinted during the XVI century. The logic of this period, like the scholastic philosophy of the time of the Renaissance has not yet been examined.

Summing up our considerations of the scholastic logic we must stress that:

I. In the Middle Ages the logic never was considered as a dead and closed knowledge, to which nothing could be added. On the contrary, the scholastic logic is treated, as normal living science in which new facts and new laws can be discovered.

II. The most important considerations of the medieval logicians must be sought not in the official and customary logical commentaries and manuals, but in the passages scattered all over philosophical and theological writings.

III. Various logical considerations and discoveries, as, for instance, the definition of the term "equal" by St. Thomas and calculus of propositions rebuilt by logica modernorum, resulted in the fact that medieval logic is closer to the modern logic, than the contemporary traditional logic.

IV. Therefore we must distinguish in our terminology following different notions:

a/ The ancient logic dating from the Antiquity.

b/ The medieval logic, which includes: logica vetus, logica nova, and logica modernorum.

c/ The traditional logic, which is a logical doctrine formulated principally on the basis of customary comentaries and manuals of ligica nova, but which does not include these creative innovations, which the medieval logic had developed contrary to the common opinion. The spirit in which the traditional logic is usually treated and conceived points rather to some conceptions of the Renaissance and of Descartes, than to the attitude of the scholastic philosophy.

This is the result of various historical causes, of which, I shall mention only the most important:

I. The decline of the scholastic philosophy in the time of the Renaissamce resulted in nearly the entire scholastic literary production being neglected and forgotten. Consequently almost nothing was published in print and even now its greatest part is still in manuscripts. Only the writings of the greatest authors and the official text-books were known and read, but rather from the theological point of view. Hence, when in XVI and XVII centuries it came to the revival of the scholastic philosophy, those text books were accepted in the same manner, as the philosophical or theological writings of St. Thomas or Duns Scotus. The whole problematic of logica modernorum remained unknown in this period.

II. The naturalistic interpretation of the aristotelian philosophy in the Renaissance and the development of natural sciences resulted the fact, that the main stress was put on mathematics, still very primitive at that time. The logic began to be viewed as closed and accomplished knowledge, without any possibility of new problems.

III. The philosophy of Descartes did not understand the significance of logic. The famous text-book of logic of Port-Royale / La logique de Port-Royal/ published by Arnould and his adherents is a classical expression of this tendency. This book became a base for the future text-books of traditional logic. Its evil influence on the whole traditional logic, can be traced down, even to the writings of neoscholastic authors.