Sobocinski

Spring 61

Class 10:30.

Seminar 2:00 Tues

Preliminaries

To prove logical Thms used in math - not elem Thms.

Lesniewski's Ontology

Functional Calc vs. Ontology.

They assume the existence of individuals x, y, z, —

& the variables represent really existing objects.

Sim Syl exists in Aristotelian logic.

it has assumptions not included in the syllog com.

even if you formalize it.

Aristotle built on nonempty general names

Aristotle does not hold for any other names

gen, indiv, empty names.

We do this philosophical assumption

under John Stuart

General mean

empty square-circle.

How to define such things properly

to avoid that we are misled by

John Stuart Mill

We distinguish connotation & denotation

connotation: the understanding we ascribe to

this word (name)

denotation: the name denotes certain objects

namely a name mean denotes objects

which are the men

that name name denotes certain objects

which are names

A name is general if it denotes more than one object. Thus Man is a general name John S (concrete & known to us) denotes only one object, namely object which is John Smith

& Empty name denotes no objects

In field of gen names we can make as many classes as you like. (eg name which denote 2, 3, ... objects)

Aristotle concisely built his syll for gen names. Thus in Aud syll for the namables which are occurring you cannot sub empty names & indeed

If you use Arabic names in syll There is no tragedy, but it is not Aristotle

Every man is mortal

Socrates is a man — The famous phil but not much hist of 6th Cent

Hence Socrates is mortal

This is not Quintelian syll. Since He

excluded Indiv names.

14th Cent introduced indiv names in the syll.

The sub of empty names under several

syllo invalid eg (Baroco)

This is the main misunderstanding between

traditional logicians & math logicians

You can use empty names in Boolean Algebras

Every a is b

is understood in a diff way in BA & Arist Logic

This causes the big trouble

Why did A limit his system to syllogistic general norms

It was certain philosophical reasons.

Since he was interested not in connections between individuals but between connection of genera.

Aristotle created logic - don't forget that he did it in a rather short time as he died young.

A similar assumption holds in pure calc. There is a pre logical pre axiomatic assumption - that there are individuals + certain kinds of variables, i.e. small latin letter

" It is built on a hierarchy of logical types

type 0. individual variables

1. functions whose args are of 0 order.

eg $f(x)$, $g(x,y)$, $h(\underbrace{x,y,\dots z}_{\text{finite}})$

each variable must be an individual

2. ~~prop~~ fnth $\varphi(\ldots)$ where at least one, args belong to the 1st order

eg $\varphi(f)$, $\varphi_1(f, x)$, $\varphi_2(f, g)$ ...

3. $\chi(\varphi)$ at least one arg must belong to the 2nd order but some may belong to the 0 or 1st order.

This hierarchy can be made as long as we like, but only a finite number of types.

In 0 order all variables are of the same type.

In 1st order f & g of $f(x)$ & $g(x, y)$ must be designated

i.e. functor for 1 arg

g is a functor for 2 args each of which are of type 0.

Now how to build functional calculus.

Use PC + make for prop. variables
sub of prop. fncts & later are applied
the well known rules of quants.

The rule of quants can be given in such a way.
Take 1st order prop. fncts. Over quants
bind only individual variables & you can
apply rules of quants only to indiv variables
you get pure calc of 1st order (FCI)
If f, g, h are not real variables, but
only symbols for fncts — abbreviations.
Then we have restricted pred calc of 1st order.
(we used this last sem in metalogic)
If you add to FCI a notion of identity =
Then we get FCI with =.

If rules abt quants say $\forall, \exists$ are variables (not abbrev) & can be quantified

Then you have Fnc calc of 2nd order (FCII).

If we say any variable of any order can be subjected to quant, Then you have Fnc Calc of ω order (FCω)

There is a big difference between FCI & FCII eg in FCI we have several Thms - Skolem-Lowenheim Thm

Godels Int Thm

Deduction Thm etc

Today we cannot prove any Thms for FCII.

FCII is much more complicated.

From a logical pt of view if you like a field system (not complete) of logic

it is convenient to accept FCW

One can ask why necessity of types was introduced into logic. We must remark that before 1900 they built logic w/o precise notion of logical types.

Obviously they distinguished

x represents a nat. no

ϕ f is a variable of function of nat nos

for which you can substitute

They don't sub 5 for f.

Types were not introduced as a necessary condition for the building of logic & Math

Ernst Schröder, before Russell ordinary remarked that you need types.

Antinomy means a contradiction

but not an ordinary contra. They are

easy to get. eg to show a system is

incon. This actually increases our knowledge

We adopt the def given by Nelson (german phil)

neo-Kantian

(free school of phil)

Ant is a contra which you can

construct in a system stating ur

ops in which the truthes which you

believe & rules of procedure the

correctness of which we are convinced of

Ant. have very strong psychological

consequences.

An ordinary contradiction can be

a pleasant (or unpleasant) thing.

But Antinomies have an aspect of tragedy since we are convinced our rules & Axioms are true.

An Antinomy cannot be solved in a medical way — we cut out a part as we don't have a ✱. This does not solve the Antinomy, since our belief remains.

The amputation only solved the ✱ not our belief. We still have it.

The Ant is solved if we, by analysis of our assumptions, we convince ourselves that something was wrong in the beginning. eg we convince ourselves that one of our Axioms is wrong.

Now some easy examples of antinomies

The first contemporary one was by the Italian math + log C. Burali-Forti in 1897.

Phil + Logs did not recognize this

He used Cantor Naive set theory +

provided the following very simple arg

(it requires a very small knowledge of set theory)

There are ordinal numbers.

Construct the set of all ordinal numbers.

(If ords exist then this set exists).

But any set of ordinal nos is well

ordered (This is A-OK + true)

But if so then this set represents an

ordinal number not included in the set

Hence you have at once a $*$

Since we set was to include all ordinal nos

$A = \alpha \quad \alpha \notin A.$

2nd Class

Ant of Class of all classes. Formally this is very sim to B-F Ant. Defn class of all classes. it is a set. Use great Thm of Cantor. $A < 2^A$ 2^A is also a set & by def of A has to be included into A *.

The most famous ant published of which made a revolution in logic & found of math is Ant of Russell. It can be construed in several ways.

unformal. There are classes which not there own elts. eg class of men is not its own elt. Since this class is not a man. Some classes are there own elts. Class of all classes, if acceptable is such by def.

if it is its own elt. Define a class of classes which are not their own elts. Call it A.

If $A \notin A$, Then $A \in A$ by def of A.

if $A \in A$, Then $A \notin A$ $\quad -*-$.

It looks rather innocent.

But Russells reasoning was revolutionary.

This Ant can be formalized in different ways.

One is very simple which is presented in textbooks:

Df $[A]: A \in * . \equiv . \sim(A \in A)$.

Sub $*$ for A & get

$$* \in * . \equiv . \sim(* \in *)$$

Apply clavius twice & get

$$\sim(* \in *) . * \in *$$

by Duns Scotus you can prove any thing.

This formulation is not unique. You can still get it if you distinguish two diff notions of class.

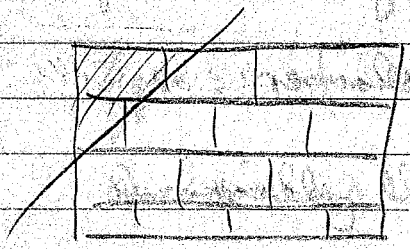
John is an elt of the class of men

This means nothing else than John is one of the elts of this class, i.e. John is a man.

There is a distributive class. This is an apparent notion. It can be reduced to purely logical notions.

Another notion is the collective class. Collective class is not a logical notion & it is not an apparent notion. It cannot be dismissed using purely logical operations. Consider the class of men (w/o regard to time) if taken in the collective sense. then the class of men means a physical constituted

from these objects. To be united of a single
mass. Take Bricks from which this Bldg
is constructed. These bricks constitute



a certain object.

Take now

A is an elt of

From this it does not follow that
A is necessarily a brick. Any cat is a pet.
The coll class of men has more elts than
men. The set of men in this room are an elt.
GW's nose is an elt.

In the dist sense

A ∈ Els Then A is necessarily an object
from which the apparent class is built.

If we make such a distinction, then you cannot construct the Russell ant using the old classes of Mereology. We only discuss this ant in set classes.

Historically when Russell discovered this ant, there were two systems that found of math in preparation.

Naive set theory. - Cantor.

- Frege's system.

Russell set a letter to Frege when the Grundgesetze was in print saying his system is inconsistent. The same reasoning which Russell used could be applied to set theory.

The investigation of this ant & the

discovery of others & the rediscovery of the Cret Liar.

This Imp Ant belongs to a different type. In ancient greece a philosophy considered such a thing a Cretion said

"Any cretion who says someething lies."

if he lies he does not & v.v.

This Ant was famous in antiquity. One philosopher killed himself when he couldnt solve it. Paul to Titus a Timothy mentions it

The 14Th Century Scholastics solved it, but it was forgotten

This is different from the logical Ants This is a semantical ant. We have mixed several languages. Mixture of

the logic & the meta logic. At first the difference between the log & sem anty was not recognized. Two sols were presented to solve the known antinomies. They tried to solve all possible anty.

The purely logical system of Russell is given in Principia Mathematica.

The other sol was given by E. Zermelo in the set theory.

These sols are different & from this time there is some difference between logic & set theory.

Zermelo was only interested in log anty Russell tried both but died it only in a rather unsuccessful way.

Russell's def: take our def

of $[A]: A \in *_{-} \equiv \sim(A \in A)$

Russell says you cannot adopt such a

def & then cannot get the Cnt. You cannot

adopt this since you cannot distinguish

the logical types of subj & pred of $A \in A$.

There must be of a different type. We

construct a language in which type theory

was introduced mechanically - ramified

Theory of types. Subj & pred of undecided

prop belong to a different type

Then $A \in A$

has no sense at all.

So such a def is worthless.

The price of this set was that Russell excluded some single & obviously true things we used constantly in our language.

John is John

GW is 1st president of U.S.

They have to say

$A \in \dot{c}'A$

Means a class whose single elt is A.

Now we have to distinguish A & $\dot{c}'A$

Then John is an elt of the set whose only elt is John.

Dr has never met a set w one elt.

Another set was given by Zermelo.

His starting pt was that our notion of a set was too liberal.

You have to limit our possibility of constructing new sets. Thus he introduced his famous Ax of Comprehension.

$[a]:[\exists y]:.[x]:. x \in y. \equiv .\varphi(x).x \in a$

where φ is not a variable, but an abbrev of a certain wff.

a is an already existing set.

Hence we cannot construct any logical ent known to us. Set of all sets cannot be defined. But only a set of all sets previously obtained. No set of all ordinal nos, but only a set of all ordinals already existing in our system.

In both cases there was a limitation of the possibilities of defining new sets

or general nonsense. What is the use in set theory - By Zermelos idea you can distinguish good & bad sets. good means their existence is established by some Axioms in which we believe & bad sets which give a ✱ if we add one of them to the good sets. Also there is a lot of sets, about which we know nothing. You can try, by new axioms we try to enrich this branch as much as possible.

Eg we can add AC to gen set theory & also Gen Cont Hyp. You can try to add existence of inaccessible sets.

We don't know about these - probably they are bad.

MWF 10:30
Rm 339 O'shag
Seminar Tues 7:00. DR. S. Speaks 1st.

3rd Class
Feb 13, 1964

The Ants gave two new ways of creating sets. Both of the sets are artificial. They never showed why the ants could be constructed to begin with.

Russell's limitation says

$x \in \alpha$ Russell has no form like this

$\varphi(x)$ x has a property φ.

This gives a prop fnct & we can operate with it as in prev calc.

Since $\varphi(x)$ is an extremely unusual form & not used in our current language Russell uses

$x \in \alpha \equiv . \varphi(x)$.

He gets the individual prop this way.

In some mysterious way this property φ is changed into a class, a name.

R & W clearly indicate that clearly they have only $\mathcal{P}(X)$ ... Thus $\in$ is only an apparent symbol — but constantly used.

Let us analyze this indecided prop.

First he assumes the only 2 pred belong to diff log types. This is how he destroys the antinomy

Before we know that $\mathcal{P}$, X in $\mathcal{P}(X)$ were of different types. Hence he destroys the ant since $\sim(X \in X)$ has no sense in the language.

Peter is 1st pope

John Smith is Peter Mueller (alias)

These are expelled from Russells language.

There are languages in which (with $\mathcal{P}(X)$ & $X \in b$ can be used & almost in the same sense.

In Polish this can be done (but is used rarely).

In set theory any Axiom of form

x is some newly created set ⟶ (1)

is rep by Ax of Comprehension

$[a]:[\exists y]:[x]:. x \in y . \equiv . \varphi(x) . x \in a$

a set; an abbreviation wff; a previously defined set.

There is yet another approach & it is the most natural for Dr S.

Analyze the meaning of the word "to be"

what does "A is b" mean.

You can establish certain (3) conditions which explain the meaning of the word "is"

In english we have articles so this makes things complicated. The Latin (or esp w/o articles)

A is b

John Smith is man

true if this fofils the following 3 condetion.

① A is unempty

② A is unique

③ anything which is A is also b

In our example if JS does not exit our intuition are saying that this is not a true prop.

eg Hamlet is a man.
of Shakespeare

false since the Shakespearian Hamlet never existed.

A is a man. We must know that A really exists.

J.S. must also be unique.

man is mortal

is not correct use of the word "is".

It is only an abbrev of

every man is mortal.

A must be an individual.

In our language Hamlet is a man

is not a true prop. Sin Hamlet is an empty name,
ie false

It is an object of our thought, but we speak

not of objects of our thought, but of objects which

really exist in the world.

A true prop is

if Hamlet exists then Hamlet is a man.

So we can express our thoughts in this language.

man is mortal

is false since there are more than one man

(There are 4 in this room now).

every man is mortal is OK.

Our spoken language is not an ideal language.

Our new language is.

A ⊂ B

every angels are mortal

not true since anything which is an angel is

not mortal

George Wash is mortal

is perfectly correct since ①, ②, ③ are sty.

With english speaking people & any people

using a language w articles there is some

trouble, namely:

GW is The first president

"is" means just "A = B" identity.

A augustus

A is a roman emperor.

From this does not follow that

A is identical with a Roman emperor.

to do this we must show A was the 1st emperor.

This means the 1st emperor was an object.

while a roman emperor is a general name.

A is b, in this system, in a certain way

is a definition of an individual.

We started with names in general & are able

able to distinguish individual names. It is not

the task of logic to establish which objects exist

in the world. Our task is to establish a correct

use of some formulas, using which you

can learn that certain names are individuals.

eg we cannot find the existence of an individual.

if A is b then neither A nor b is an empty name.

$[\exists x, y]: x \in$ Roman Emperor $. y \in RE. \supset. x = y$

This previous Roman Emperor is an individual.

general, individ, empty names. There are the three kinds of names. To show something is an individ we must show uniqueness & existence. There is only one empty name

Propositions & the square-circle are the something. Whatever can be said about one can be said about the other.

In our language A & b or A is b belong to the same type - semantical category

This is the sem cat of names, (not sets)

$A = B .\equiv: A \text{ is } B . B \text{ is a } A$

In any lang such a def of is can be adopted. We use ε for is (not $\in$)

$A \varepsilon b$ only if A & b belong to the same sem cat

This allows us to sub $*/x$ & get Russell and (provided the def is OK – & it's not),

An incorrect def can give us $*$

Df $x \in * \equiv \sim(x \in x)$.

$* \in * . \equiv . \sim(* \in *)$

This Def is incorrect since from definiendum we know x is an individual, whereas in the definiens you cannot deduce that x is an individual. So one of them is false. so x is not necess an indiv. So this is a true def as something stronger in essential in the definiendum.

To get a correct def we must say that x is an individual

We must always have the condition $[\exists A]\ x \in A$

or what is the same we must always

have $x \in \lambda$.

Df $\quad x \in \lambda\ .\equiv .\sim(x \in x)\ .\ x \in x$.

This gives no $\to\leftarrow$. But the sub of λ for x

is invalid.

4th Class

$A \varepsilon b . \equiv .$ 1. A is energetic.

2. A is unique

3. Anything which is A is also b.

if we try to express these three conditions with the same ε we get

formula B.

$[Ab]: A \varepsilon a . \equiv : [\exists B] . B \varepsilon A : [B C] . B \varepsilon A . C \varepsilon A . \supset$

$B \varepsilon C : [B] : B \varepsilon A . \supset . B \varepsilon a$

This is the single axiom of ontology

This syst is stronger than F C w.

This was the 1st Axiom established for ontology.

why Cop & small latin letters?

We have no given individual variables & constants.

Rather individuals are defined & to be the

sibly of an indiv prop.

We use caps for individuals.

This is a custom — not a prescription.

If we know something is an indiv we use
caps. If we know it's a gen name we use
a small letter (the type Caps are used
for Proper names — though some poets
only like small letters). (This is opposite
of the custom in set theory).

This theory of logic (of logic of names)
which has the primitive symbol ε
Other things could be adopted

$a \subset b$ boolean inclusion
can do it.

$a \sqsubset b$ aristotelian inclusion
also can be used as prim term

$a \cap b$ join
$a \cup b$ meet } also can be used as prim term — but we don't have an axiom system.
$a - b$

There are several differences between functional calculus & ontology

In FC you have no prim term. Only several rules of quantification.

It is not true that a full free calc has only rules about 4 cons. There are also infinitely many axioms.

By a special rule you have to add to FC an inf seq of formulas about extentionality. You need one for every logical type.

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} also can be used as prim terms — but we don't have an axiom system.

There are several differences between functional calculus & ontology.

In FC you have no prim term. Only several rules of quantification.

It is not true that a full func calc has only rules about proofs. There are also infinitely many axioms.

By a special rule you have to add to FC an inf seq of formulas about extentionality. You need one for every logical type.

In FC., if you want it to be full, regardless of order, you need to assume an axiom of pseudo-definitions. It is akin to ~~the~~ of comprehension in set theory. It can be formulated in the following way.

$[\exists y]: [x]: x \in y . \equiv . \varphi(x) . \quad x \in a$

(φ allows of opt) (drop since we have a type Theory)

such an inf seq of p-defs is added to full FC of any degree (if the syst is to be full (not complete)).

This is not necessary in ontology.

Even in FC defs have a creative role.

Dr does not know a proof of Lehrsatz Fermtes Thm which does not use either a real def or p-def.

In unrestricted FC you have expressions like

$$\varphi(x)$$

where φ are variable functors for some type of arg.
We only have variables for names in Ontology.
Our rule of substitution is to make such things:
if you have an expression which is a const
for certain logical type, — sem cat —
you can use from that moment, the
variables belonging to this type.
In FC you have, before the A.S.
description of all possible types which can occur
in this system. Not in Ontology.
Here we have the following method. In Carnap
of logical types we have semantical categories.
It is connected with the sense of the formulas.

An exact symbolism of Ontology which is very precise but very inconvenient as any function is written before the args, but they are in parenthesis (reason later.)

$\varepsilon\{Aa\}$ is the elementary unit of our $A \times B$ + quants + P.C.

This is a prop fnct for two nome arguments

ε is a functn which from two nomes constructs a proposition. i.e. prop form fnct for 2 nome args.

We start with only two sem cats.

They are primitive. Any other sem cat has to be introduced by way of def.

If by our rule of def we can construct a const of a new sem cat, then from this moment we introduce this sem cat in the syst.

& henceforth we can use the variables belonging to this sem cat

eg

D [a]:[∃A]:A ε a .≡. ! {a}

definiendum

We defined the notion a is an unempty name. This is a new constant "!". It is a functor which constructs a prop fnct from one argument. This gives us a new (3rd) sem cat, namely sem cat of prop form functors of 1 name arg. After this def was added we can use also variables of this sem cat. Thus the expression f(a) is now a senseful expression in our system, where f is a variable.

Our Ax alone enabled us to use

$f'\{a\ b\}$ prop form just for 2 name args

These two sem cats are different.

How can we destinguish f & f'.

In an easy way: # of args included

in parenthesis & form of parenthesis enable

us to decide. The fourth is then

case are the same as the enclosed

args are of the same sem cat.

$f''\{a\ b\ c\ d\}$ is of yet another sem cat:

sem cat of prop form just for 4 name args.

Theoretically then when we intro

a def we must decide whether or not

we need parenthesis of a different shape

(to do this practically it is something else)

We have the freedom to use any many forms of parenthesis.

We could drop parenthesis if we use the following

$\varepsilon\{A\,a\}$ under a variable denote its sem cat

$\frac{P}{n\,n}$ λ n

Then we can erase the parenthesis.

But this is inconvenient. Our scales almost parenthesis look awful but they are actually very convenient.

We get certain analogs w FC

$!\{a\}$

$\varphi\{a\}$ $\varphi\{x\}$

φ φ

This is a general name. in FC this is always an individual

We can add some rectulum to our system so that "a" will be a individual in 9(a)

see Spring of '63

Now something about Quants.

We can quantify variables of any sem set belonging to our system.

Really speaking we have not rules.

This can be done in two ways.

This system is based not on PC but on Protothetic of Protothetic has such a property: If some variable belongs to sem set which has a sense for us in any system based on protothetic, then any rules concerning quant can be used on them variables.

prototheti

B... our sole Axiom

$\varphi\{a\}$ has a sense for us. (Cy D p 41)

In prototheti you can easily prove

$[\varphi p]:.[a]:.\ p \supset \varphi\{a\} :\equiv: p \supset [a].\ \varphi(a)$

This is nothing else than one of the rules of quant. (p is a prop variable).

This formula has a sense in our language.

Now a sketch of this in Prototheti - w/o Ontology.

p is a prop variable, so prove

1. $[\varphi]:.[a]:.\ 0 \supset \varphi(a) :\equiv:.\ 0 \supset [a].\ \varphi(a)$

This is obvious since $[p].\ 0 \supset p$

2. $[\varphi]:.[a]:.\ 1 \supset \varphi(a) :\equiv:.\ 1 \supset [a].\ \varphi(a)$

This is equiv to $1 \supset q.\equiv q$

$[\varphi].\ \varphi(1).\ \varphi(0).\supset.[p].\ \varphi(p)$

Law of Leibniz.

This can be proved in Prototheta.

So our desired rule of quant follows.

Thus we don't need the rules of quants in Ontology.

Suppose we would like to build FC on Ont. Then instead of then formulas about quants for each order you can accept that expressions which are occuring in FC have a sense for us. Then the full FC can be construed as our syst. w/o additional rules of quants.