5th Class

Now the rules of procedure used in Ontology.

There are seven rules of procedure. They are very simple in practice.

The first two concern defs

A def is not an abbreviation, but a new thesis added to the system, according to the rules established by these two rules.

In some system $A =_{Df} B$

one gives proper construction of def

The other says if $\vdash \varphi(A)$ is true Then $\vdash \varphi(B)$. or vice versa.

We have no rules of use of defs in our system. The Defs are Theses & the other rules take care of this. We use defs in the

Luschai – see this book for an exploration of the rules.

same way as any other proved theorem.

All defs are constructed as equivalences.

$A \equiv B$

definiens definiendum.

def of false prop.

RI is a rule about Protothetical defs $[p] . p : \equiv . 0$

Protothetical def since

There need be no variable here. Then we defn a const & no gen quant is necessary

$[a_1 \ldots a_n] : A(a_0 \ldots a_n) : \equiv . \underline{\phi}(a_0 \ldots a_n)$

new defined const. & with its args it constitutes a prop fnct.

a wff which has a sense already in our language

eg in P.C.

$E\ NCpNq\ \ Kpq$ def of K.

we define a prop form functor.

Any variable in definiendum has to occur there only once. Any variable in definiendum is free there & must occur as a free

$0 \equiv 0 . \equiv . 1$ is a coll def with no variables at all.

variable in definiens. They may occur several times in definiens. Also several bound variables may occur in definiens.

Any expression in A must belong to a sem cat already occuring in our system. Such def can be unpredicative.

Any variable free in definiens & definiendum are bound by a general quant which precedes every thing.

No theses has a free variable.

Suppose you have n free variables in definiens & dums. (once only in ϕ). These variables have to be bound by parenthesis. The ϕ must have the form

$$\phi(a_1 \ldots a_n)$$

$a_1 \ldots a_n$ need not belong to the same sem cats. You can divide these variables in as many groups as you like & put them in pairs. The same can be done in PC & Protothetic.

Take eg conjunction for 3 variables. it is comm & sym. So we can order the variables in any way.

$Kpq . Kqr \;.\equiv.\; \Lambda(p\;q\;r)$

prop variables.

K is prop form funct for 2 prop args

Λ ——— 3 ———.

Thus K & Λ belong to different sem cats.

very easily we can define such combinations:

$\boxed{\boxed{\Lambda_1} \, t \, p \, q \, t} \, (r)$

$\Lambda_2 \, t \, p \, t \, t \, q \, t \, (r)$.

$\Lambda_3 \, t \, p \, t \, (q \, r)$

i.e you can cut our string of variables in as many pieces as you like & order them as we like — using different parenthesis.

Λ belongs to sem cat of prop form functor for 3 prop args.

$\Lambda_1 t p q t$ is prop form functor for 1 prop arg.

This is not a result — but some functor w two variable

Λ_1 is functor form functor for two prop args which together with these 2 args form a prop form functor for one prop arg.

Anyone can see that $\Lambda, \Lambda_1, \Lambda_2, \Lambda_3$
belong to diff sem cats.

This is used very often in mathematics.
He is investigating a funct of 3 args, but is
interested only in one arg. Then he
puts two variables in parameter.

To have a full theory of sem cats you
need the possibility of splitting the variable
of a functor. This is often times necessary
in practice. This device is used in math
& other branches of logic.

a sign
Schonthiec gave this device at the
1921.
same time as Lesniewski. He did it only
for functions. Schonflies ideas were used
by Church in his construction of the λ calculus
2 Princeton mimeographs

This λ-calculus is akin to combinatorial logic of Curry (which he will present in '66).

$\Lambda(pq)$ can be split only as

$$\Lambda_1 \{p\} (q)$$

or $$\Lambda_2 \{q\} (p)$$

But Λ_1 & Λ_2 are the same function since Λ is commutative.

φ is a new constant. Not necessarily belonging to a new sem cat. It can belong to an already defined sem cat.

if we define $K(pq)$ (using C, N as prim)

& later $A(pq)$

Then A & K belong to the same sem cat.

We must use different symbols

(we use K & A)

We can (in future) define a functor of a sem cat with analogous properties to a functor defined previously but of a diff sem cat. Then it is convenient to use the same symbol.

eg we write $\Lambda(pq)$ & $\Lambda(pqr)$

The same symbol Λ is convenient as both are conjunctions. We distinguish the two by the variables, parenthesis, their form, arrangement & number.

This enables us to establish the sem cat immediately.

$\Lambda(pq)$ & $\Lambda(p)(q)$

The two Λ's are different as the number of parenthesis are different.

This is the prototethetical type of def.

R. If such a thing is a well constructed def then we can add it (the thesis) to the system as a thesis.

These are called many-link defs.

The second type of defs are the Ontological def.

$[a_1, \ldots, a_n, X]: A(X, a_1 \ldots a_n) \circ B. \equiv X \epsilon \phi(a_1 \ldots a_n)$

All of the previous conditions for proto defs hold.

But we must have one more thing.

$a_1 \ldots a_n$ may again be arranged in many link fashion. Our definiendum defines not a prop funct, but a name funct.

A is a wff & either it is a conj or not.

x is a variable & it is used in the place of the subj of an individual proposition.

The add'l condition of construction of Ont def says that if A is a conj, then each x must occur as a subject of an individual prop which is a function of one prop. If not we must add a factor B of the form $x \varepsilon \alpha$.

eg $(x \varepsilon x)$ will do.

or $\exists a . x \varepsilon a$.

This must be done in order to avoid any logical antinomies.

This makes the Russell antinomy impossible.

D $[x] :. x \varepsilon * . \equiv : \sim (x \varepsilon x) . [\exists a] x \varepsilon a$

This Df is OK, but no * results.

The definiendum must not be stronger than the definiens. ie if you can establish something is an individual in one then you must be able to do this in the other also.

This condition is easy to apply.

This is our second rule.

6th Class.

Now a discussion of the remaining rules

III. Effectually similar to a rule in Protothetic &

a rule which is unused in our course.

A single rule concerning the use of Quants

All other rules of Quants follow from it, except

the rule of unemptyness.

$[a_1 \ldots a_n] : \varphi \equiv \varphi$

Say we have this already proved Thm

where $a_1 \ldots a_n$ (poss of diff sem cats) are in MGQ

The rule says you can distribute either

all or some of those variables in MGQ

across the $\equiv$ sign

& get $[a_1 \ldots a_n] . \varphi : \equiv . [a_1 \ldots a_n] \varphi$,

or get

$[a_{k+1} \ldots a_n] :: [a_1 \ldots a_k] . \varphi . \equiv . [a_1 \ldots a_k] . \varphi$

$k < n$

In this system the variables are unordered in the quants. There are no vacuous quants.

So we must have

$[a_{k+1} \cdots a_n] :: [a_1 \cdots a_{k-1}] : \varphi_{(-a_k)} \equiv [a_1 \cdots a_k]\ \varphi.$

if a_1 does not occur in φ

we can get

$[a_2 \cdots a_n] :.\ \varphi_{(-a_1)} \equiv . [a_1] . \varphi$

In this syst you have no particular quant.

$[\exists a]\ \varphi(a)$ is only an abbreviation

for $\sim([a] . \sim(\varphi(a)))$.

The particular quant is not introduced formally simply because really speaking we can adopt not only gen + part quant, but also inf many only

$[\exists a]\ \varphi(a)$ means φ holds for at least one a

Sin we could define which says f holds
at least for 2 a's.
But no-one has found how to define such
a thing successfully. Hence we use
part quant only as abbrev.

IV Rule Concerning Detachment.

Same as in Protothetic - it states it just for $\equiv$.
Later we prove it for $\supset$.

We state the rule here

$$\frac{\begin{array}{l}\vdash \alpha \equiv \beta \\ \vdash \alpha\end{array}}{\vdash \beta}$$

This rule does not hold under Quant. We must first distribute all variables in MGQ

As a metarule we have

$$\frac{\begin{array}{l}\vdash \alpha \supset \beta \\ \vdash \alpha\end{array}}{\vdash \beta}$$

The rule under quants is

$$[x_1 \ldots x_n].\ \alpha \supset \beta$$

$$\underline{[x_1 \ldots x_n].\ \alpha}$$

$$[x_1 \ldots x_n].\ \beta$$

Remember no vacuous quants

The rule of Prots or rule of dist of Quants is extremely powerful

V This causes difficulty where you have variables of different types or sem cats.

Rule of Substitution in PC is very simple

if p is a variable in a wff φ of PC then for p we can sub any constant, or prop funct (where it is built of things already in the system).

Formulation of this rule causes no diff.

If we have variable functor we have trouble.

$\varphi(p)$

↑ functor is a variable.

or $\varphi(x)$ from F.C.

We want to transform $\varphi(x)$ into $x = y$

FC does this in the following way

Start w $\varphi(x)$ & sub so as to get $x = y$

They say "$= y$" is an incomplete symbol

(named thus since such an expression is neither a formula nor belonging to no sem cat. it really has no sense).

to get $y = x$ we need to sub "$y =$" for φ

This is also an incomplete symbol

& $= y$ & $y =$ are different incomplete symbols.

In some system the formulation of this rule is extremly complicated.

In Prot + Ontology the Int point is that you may sub for variable occurring in MGQ only expressions belonging to the same sem cat as this variable.

We do not have incomplete symbols. (Theoretically)

(sometimes we use them in informal proofs)

How do we go from $\varphi(x)$ to $x=y$.

To this end we use many limb functors

Say we have

1 $[f, x] :: \quad \varphi(f\{x\})$

a Thesis proved already

a part of our Thesis

2. We wish to get $\varphi(x=y)$.

any connected symbol may be used as a variable up to the moment it is defined as a constant.

We need to use an auxiliary definition, Namely:

Df $[xy]: x = y. \equiv . \boxed{\phi_\alpha\, \vdash y \dashv}\{x\}$

This means a constant (we shall never use it again)

This is a many links def with in the 1st link the argument(s) $\boxed{}$ belongs to the same sem cat as f

So we may sub

$$f / \phi_\alpha \vdash y \dashv$$

By sub we can get

$$[xy].\ \psi(\underbrace{\phi_\alpha \vdash y \dashv \{x\}})$$

This expression is either free (i.e. not under some quantified function).

Then we can use our def of ϕ_α to get $[xy].\ \psi(x = y)$

or it is an argument of some quantified functor

Then we must use the next two rules (Extentionality)

One more thing on sub. If we have say

$[x].\ \phi([\exists y].\ \psi(xy))$.
or $[y]$

i.e. our free variable x in ψ is also under an interior quant.

Then we cannot for x substitute a variable y or a formula wherein a variable equiform to y

If we do we get

$[y].\ \phi([\exists y].\ \psi(yy))$

Then we have a great confusion

& you can make a ~~*~~ at once.

This Banal mistake should never be made. It is impossible as our alphabet is unlimited.

This is a rule in any system which possesses interior quants. The exact formulation of this rule is not easy & it is a long question, but these remarks & a few days of practice make things clear.

Now take FC I. (even not restricted) Here we have no rule of extensionality or a limited rule (say for individuals)

To build a full FC of any order, then for any type you must accept as axiomatic a suitable rule of ext.

If you have accepted in our system a new sem cat then you can accept a theor which guarantees extensionality for all expressions belonging to this sem cat.

Ontology is a much richer system of types than P.C. We have not only types which contain prop form functors. But we also have name form functors. (Just as we have 2 kinds of defs) We would like to have extent for all sem cats. So we must divide the rule in two parts. Extensionality for Ontological forms & for protothetical forms.

In Proto if we have

$p \equiv q$ then to get

$$\frac{\phi(\alpha) \quad \alpha \equiv \beta}{\phi(\beta)}$$

we must accept the Axiom

$$p \equiv q \,.\supset.\, [f]: f(p) \equiv f(q)$$

A peculiar property of protothetic is that this thesis is equiv (over the field of PC) to

$f(0) \supset . f(1) \supset [p] f(p)$

ie bivalency.

Such a formula gives 2 kinds of variables prop form funct for 1 prop arg & props.

To construct such a thesis we must introduce two sem cats. We can do this by introducing oux defs

eg Defn $\underbrace{\sim p}_{\text{prop form funct for 1 prop arg.}} . \equiv . p \equiv [a]. a$

So we have this sem cat in the system.

7th Class

Two more rules – Extentionality.

Two are necessary since we have Protothetic

& Ontological Forms.

$p \equiv q$ proto equivalence

$a = b$ ont equality

$\varphi(p)$ in proto prop form fnct 1 prop arg

How do we define equivalence for such things?

Easy $\varphi \doteqdot \psi$ $\doteqdot$ here is analogous to $\equiv$ in $p \equiv q$, but a different symbol.

$[p]: \varphi(p) \equiv \psi(p)$

says functor φ is equiv to funct ψ, where our equiv is of the higher degree.

So $\varphi(p) \equiv \psi(p) \;.\underset{Df.}{\equiv}.\; \varphi \doteqdot \psi$

$$[p_1 \ldots p_n] : \varphi(p_1 \ldots p_n) \equiv \psi(p_1 \ldots p_n) :$$

$$\equiv . \ \varphi \overset{*}{\equiv} \psi$$

Equivalence of a higher degree.

The same can be done with names.

we use $a \circ b$ instead of the

customary $a = b$

$$a \circ b . \equiv . [A] : A \varepsilon a . \equiv . A \varepsilon b$$

Say we would like to define $\varphi \overset{*}{\circ} \psi$

when φ & ψ are prop form fnct for 1 name arg

We can define this as:

$$[a] : \varphi\{a\} \equiv \psi\{a\} : \equiv . \ \varphi \overset{*}{\circ} \psi$$

$\overset{*}{\circ}$ is in a higher semantical level.

Concerning rule of Ext in proto function in Ontology.

This rule says that under certain

later explained conditions, you can add to the system a thesis of the following form. (please remember that this is not an exact formulation & that we can have many link function).

VI

$[\varphi\psi] :. [a_1 \cdots a_n] : \varphi(a_1 \cdots a_n) . \equiv . \psi(a_1 \cdots a_n) :$

$\equiv : [\phi] : \phi[\varphi] . \equiv . \phi[\psi]$

args belong to arb sem cat already in the syst.

This can be a many link function
φ

This rule allows us to deal in an extensional way with anything in the syst.

→ Anything you can say abt φ you can say abt ψ & v.v.

← banal – since we can always prove this.

When can you add such a thesis to the syst.

Any variable belonging to this formula must belong to a sem cat already introduced in the syst. So any thing in the formula must have a sense. If such a thing is not in the syst we must add the appropriate def before we use this rule. eg the foll def works

$[\varphi] \therefore \varphi(a_1 - a_n) \equiv \varphi(a_1 - \cdot a_n) :\equiv . \, {}^{V}\!/_{2} \{\varphi\}$

Rule VII says the same thing for the sem cat of names

VII $[\varphi \psi] : [A \; a_1 - - a_n] . A \varepsilon \varphi \{ a_1 - a_n \} . \equiv .$

$A \varepsilon \psi \{ a_1 - a_n \} : \equiv : [\chi] : \chi \{ \varphi \} .$

$\equiv . \chi \{ \psi \} .$

$a_1 - a_n$ can be no args. In this case φ & ψ are reduced to names.

The same conditions as before must hold.
all variables in $\boxed{VII}$ must be senseful in our language, i.e belongs to an already existing sem cat.

Once an Ext therm is introduced in the syst it behaves as any other formula. It has no special rules governing its use.

One thing is unsolved: whether the theses added by Rule $\underline{VI}$ / $\underline{VII}$ are mutually independent. or Mixture.

We do know one very peculiar thing. In this system we have to distinguish identity from equality. w gen names a equals b (a ob). We can distinguish identity for individuals $A = B$ means A & B are the same individual.

For identity for individuals we can prove extentionality w/o accepting any rule of Ext.

A full Frege calc requires, besides extentionality, another rules which allows them to add an inf no of axs called pseudo-defs.

Sim to Ax of Comprehension in set theory.

Anything provable w p-def can be proved using ontological defs & from a ont def a p-def follows by quant theory.

There is an essential diff between Ont & FC (even FC ω). Certain variables (all) are tacitly or explicitly prescribed to really existing individuals. We have no such thing in Ont.

We have semantical notion of a name every lang has this & we distinguish individual names.

$A \varepsilon a$ implies A is an individual.

If we can prove then A is a really existing object.

But this is not the business of logic. Let Metaphysics do it.

If an individual says he is the only existing object our syst is still ok for him.

In FC we prove $[\exists x] . \varphi\{x\}$

It means in FC not only that $\varphi\{x\}$ is true for a certain given x but it means also that x really exists in the world. From the truth of this formula you can prove $\exists$ at least one indiv in the World. in PM ∂ 22 they prove this.

Philosophers interested in logic have difficulty here. They say (Quine) they can prove existence of Pegasus.

In our syst Ont. $\exists$ does not give existence. All it says in Ont is that there is a name s' $\varphi(x)$ is a true formula. x can even be the empty name. In our syst we have to prove $x \varepsilon a$.

In Ont you cannot prove

$$[\exists A]. A \varepsilon A.$$

This can however be added as an axiom.

Therefore $[\exists x]. \varphi\{x\}$ is in FC or set-theory. not in Ont

The difficulty is in understanding Quants. They have an existential notion on the quant ∃. Our particular quant has no existential import. We use it in the unrestricted or linguistic sense.

You can prove easily the consistency of Ontology even if you add axioms saying that there are an arbitrary finite number of objects really existing. For an inf no of objects we have no proof of consistency — but neither does any other system with an axiom of infinity.

The proof of the consistency of ontology is done by interpretation in protothetic.

Let names be considered as props & let ε be considered as conjunction. Then all our axioms & rules become true formulas in proto.

AC does not follow from our syst. But it can be added as a rule sim to extensionality ie we need it for every sem set.

But if we have AC for the sem set $n+1$ we can prove it for $n(-1)$.

Next week we begin formal deductions.

8 Th Class

Preliminary Deductions.

B. $[A a] :: A \varepsilon a . \equiv : [\exists B] . B \varepsilon A : [A B] :.$

1st Axiom of Ontology $B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C :$

Nothing else than a translation of the condition for ε expressed using ε $[B] : B \varepsilon A . \supset . B \varepsilon a$

In the Original symbolism, which is unpractical for some reasons, we write $\varepsilon\{A\ a\}$.

Here we have addopted not only the axiom, but also two prim sem cats.

From rules concerning defs the parenthesis { } is assigned forever in this system to functors whose args are names (one or more variables).

Prescribe certain letters to this or that sem cat. eg $x, y, z \ldots$ used as variables for names.

This is rather restrictive in use

f, g ... variables whose args are names

φ ... R

Specific symbols for variables. This is convenient if we have a limited number of types eg in PC Cpq all's are variables. In our system we have an unlimited number of types so this is difficult to formulate the rules of procedure by the use of specific letters.

The special kind of parenthesis is much more convenient.

This is a single axiom of Ontology. Analyze it from the former point of view. What is the structure of this formula. it is

$p \equiv q . r . s$

So it is conj of $p \supset q$, $p \supset r$, $p \supset s$, $qrs \supset p$

B1. $[A\,a] : A \varepsilon a . \supset . [\exists B] . B \varepsilon a$

There is no difference between cap & small letters

But it is a practical custom to use caps for indiv.

B2. $[A\,B\,C\,a] : A \varepsilon a . B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C$

B3. $[A\,B\,a] : A \varepsilon a . B \varepsilon A . \supset . B \varepsilon a$

B4. $[A\,B\,a] :. B \varepsilon A : [B\,C] : B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C :$

$[B] : B \varepsilon A . \supset . B \varepsilon a : \supset . A \varepsilon a$

B2 gives uniqueness of individuals.

B3 is just $[A\,B\,a] : B \varepsilon A . A \varepsilon a . \supset . B \varepsilon a$

from the suggestion given by Peano & adopted by Russell logicians are accustomed to having membership non transitive.

$x \in \alpha . \alpha \in \beta . \supset . x \in \beta$

by theory of types this has no sense. It is not a well constructed formula.

In their syst you have no transitivity for membership. They say instead:

$$\underbrace{x \in \iota y}_{x=y} . y \in \beta . \supset x \in \beta$$

Peano gave the example.

St Peter is an Apostle.

Apos belongs to class of 12

∴ St P is 12.

This cannot be applied in our syst since Apost is a common name & we cannot write the second hyp.

We have given two names to the same indiv

Nap Bonapart is Emperor

Victor of M is N B

∴ Victor of Marengo is Emperor.

The future analysis when this Ax. was established has shown that B1 – B4 are not mutually indep. Namely in 1923 Dr TARSKI proved that B2 follows from B3. So Lesniewski constructed a new Axiom

$$\text{BII} \quad [Aa] :. A \varepsilon a . \equiv : [\exists B] . B \varepsilon A . B \varepsilon a :$$
$$[BC] . B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C$$

$$\{BI\} \rightleftarrows \{B\,II\}$$

These proofs can be given in elementary ontology – ie ontology in which no extensionality is used. This proof is rather easy.

In 1929 Sobociński proved that from B3 B4 follows. But S's proof is rather complicated & requires the use of extensional reasoning.

These deductions from the short axiom are also in Ontology, Notebook 3, pp 3 – 6. The format is slightly — but insignificantly different (added Aug 2, 2018).

When Dr presented this result to Lesniewski he remarked at once that B1 & B3 can be connected in a single formula

A1. $[A a] : A \varepsilon a . \equiv . [\exists B] . A \varepsilon B . B \varepsilon a .$

Extentionality is necessary to prove A1 ⇌ B1

These results were proved in 1931 in Polish.

They have been translated into English

Polish Logic by McCall publ 1965?

Two of 5 papers have been translated here.

We adopt A1 as a single axiom of ontology. We can define term isomorphic to any mathematical term. AC + Ax of Inf can be added.

A2. $[Aa] : A \varepsilon a . \supset . [\exists B] . A \varepsilon B . B \varepsilon a$ [A1]

A3. $[ABa] : A \varepsilon B . B \varepsilon a . \supset . A \varepsilon a$ [A1]

This is just B3.

D1. $[ABa] : A \varepsilon a . B \varepsilon A . \equiv . A \varepsilon * \langle Ba \rangle$

This is an ontological Def.

It is a correct Def. Free variable on right occur once & are free on left. Definiens is a conj one member of which is indiv prop ? subj in Definiendum is a subject in definiens.

This defines a certain name function

'$*$' is a const which is of sem cat of name form fn for two name args.

We introduce a new kind of parenthesis.

$\langle \ \rangle$ must now always be used for name forming function with name args.

9th Class.

D1 $[ABa]: A \varepsilon a . B \varepsilon A . \equiv . A \varepsilon * \{Ba\}$

$\varepsilon \{A * \{Ba\}\}$

The meaning of $*$ is not interesting to us as it is an aux def.

A4 $[ABCa]: A \varepsilon a . B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C$

i.e. B2.

Dem

$[ABCa]:$

1) $A \varepsilon a .$

2) $B \varepsilon A .$

3) $C \varepsilon A . \supset .$

4) $A \varepsilon * \{Ca\} .$ $[\overrightarrow{D1}, B/C ; 1 ; 3]$

5) $B \varepsilon * \{Ca\} .$ $[A3\ a/* \{Ca\} ; 2 ; 4]$

$B \varepsilon C .$ $[\overleftarrow{D1}, A/B, B/C ; 5]$

Tarski's proof that from A3 follows A4

This is an example of a creative def

w/o this def you cannot prove this. A4.

This proof is elementary as extensionality is not used.

A5. $[Aa]: A \varepsilon a . \supset . A \varepsilon A$

An individual is itself.

$[A]: A \varepsilon A$ This is not true in our syst

. for to have this we must know A is an individual.

<u>Dem</u> $[A a]$:

1 $A \varepsilon a . \supset .$

$[\exists B]$.

$$\left.\begin{array}{ll} 2. & A \varepsilon B. \\ 3. & B \varepsilon a. \end{array}\right\} \quad [A2; 1]$$

$A \varepsilon A.$ $[A4, A/B, B/A, C/A; 3, 2]$

The law of identity for individuals cannot be formulated

w/o the hyp $A \varepsilon a$.

A6. [ABa]: $A \varepsilon a . B \varepsilon A . \supset . A \varepsilon B$

elem, but very interesting thesis. You will not find it in any other system.

Characteristic Formula of Ontology

When indiv syll was discovered & officially used there is no trace that anyone formulated such a thesis before this system.

St Thomas in Summa Theologica proves the uniqueness (one) of God. His reasoning is intuitively just the above thm.

Dem: [ABa]:

1. $A \varepsilon a.$

2. $B \varepsilon A . \supset .$

3. $A \varepsilon A.$ [A5; 1]

$A \varepsilon B.$ [A4; B/A, c/B; 1; 3, 2]

A7. $[A] : A \varepsilon A . \equiv . [\exists a] . A \varepsilon a .$ [A5]

Imp Later. Use Quant.

Now we can prove several other elem thms.

Now we shall prove B4.

D2. $[a] : [\exists A] . A \varepsilon a . \equiv . ! \{a\}$

This def is neces here only to get a sem cat. This isn't necessary 1/7/73

a is not empty — this def is however important

in its own right.

D3. $[Aa] : A \varepsilon a . \equiv . \varepsilon \{a\} \{A\}$

many-link def in which $\varepsilon \{ a \}$ belongs

to some sem cat as ! But ε here is not

the ε of $A \varepsilon a$. This ε belongs to sem cat

of functor form functor w 1 name arg

which forms a prop form funct for 1 name arg.

No misunderstanding as the parenthesis

Distinguish $\varepsilon\{Aa\}$ & $\varepsilon\{a\}\{A\}$

We can obviously define

[D3' $[Aa]: A \varepsilon a . \equiv . \varepsilon_1\{A\}\{a\}$]

ε cannot be put here as it is already used here in this sem cat.

E1 $[ab]:.[A]: A \varepsilon a . \equiv . A \varepsilon b : \equiv : [\varphi]:$

$\varphi\{a\} . \equiv . \varphi\{b\}.$

This follows directly from the rule just as a def does. But you must check if it is possible at this term to add this thesis.

We here are 2nd rule of Ext. The form is OK

φ is variable function for one name arg. Same sem cat as ! So E1 is OK.

This is an extremely strong formula.

A8 $[ABa] :. B \varepsilon A : [BC] : B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C :$

$[B] : B \varepsilon A . \supset . B \varepsilon a : \supset . A \varepsilon a$

Just B4

<u>Dem</u> $[ABa] :.$

1. $B \varepsilon A :$

2. $[BC] : B \varepsilon A . C \varepsilon A . \supset . B \varepsilon C :$

3. $[B] : B \varepsilon A . \supset . B \varepsilon a : \supset :$

4. $[C] : C \varepsilon A . \supset . C \varepsilon B .$ $[2, B \| C, \cancel{C/B}; 1]$

B‖C means that we interchange B & C

5. $[C] : C \varepsilon B . \supset . C \varepsilon A :$ $[A3, A/C, a/A; 1]$

6. $[C] : C \varepsilon B . \equiv . C \varepsilon A :$ $[PC, 4, 5]$

7. $[\varphi] : \varphi\{B\} . \equiv . \varphi\{A\} :$ $[E1, a/B, b/A, A \| C]$

? change A into C in interior quant.

8. $B \varepsilon a .$ $[3, C/B; 1]$

9. $\varepsilon \{a\} \{B\}$ $[D3, A/B; 8]$

10. $\vdash \varepsilon\{a\}\{A\}$. [7, $\frac{\varphi}{\varepsilon\{a\}}$; 9]

$A \varepsilon a$. [D3; 10]

Sobociński's proof is done

This is impossible to prove from A3 & A7 w/o Extentionality.

A9 $[Aa]:. A \varepsilon a. \equiv: [\exists B]. B \varepsilon A: [B C]: B \varepsilon A. C \varepsilon A. \supset B \varepsilon C:. [B]: B \varepsilon A. \supset. B \varepsilon a$. [A5, A4, A3, A8]

This proves the 1st Axiom of ontology.

This proof was given when S was a student of the second year & immediately published.