Many-Valued Protothetic

Bolesław Sobociński

The Fifth International Symposium on Multiple-Valued Logic was held at Indiana University on May 13–16, 1975. On the evening of May 14, Sobociński presented a one-hour lecture and these notes are in a file dealing with that meeting.

The first six pages are the sort of notes that one would make in preparation for a talk. They lack detail, yet are quite informative. The next nine pages are very sketchy, clearly work in progress.

Sobociński returned to this topic in the Spring of 1965 in a course entitled "Advanced Calculus of Propositions" (Phil 278). See Rickey's notes for that course, pp. 216–296. This is active research.

Some documents about the meeting are included.

Sobociński never published on this topic.

Summary prepared by V. Frederick Rickey, July 3, 2018

Many-valued protothetics.

1. Protothetic of Leśniewski. $\Pi\rho\omega\tau o\varsigma$ – first theory.

Protothetic – bi-valued propositional calculus generalized by introducing ~~tt~~ ~~t~~ interior quantifiers and the variable functors into its field.

Remark: arguments and functors: $\sim p$ $p \supset q$ a.s.o.

~~We have~~ semantical categories in protothetic? $p \supset q$

~~It was known to Russell tha~~

~~We have, e.g.~~ It is known to Russell that:

~~[p]: ~ p ≡~~

$0 \equiv . [q] . q$, and, therefore, $[p] : \sim p . \equiv : p \supset . [q] . q$

Hence, full implicational calculus with interior quantifiers contains bi-valued propositional calculus.

We have 4 and only 4 constant functors for one propositiona argument, viz $\sim$, Fl., Vr ~~can~~ and assertum. + In protothetic we ~~can~~ have a variable, say f, which belong to the same semantical category as $\sim$ a. similarly, we can introduce a variable φ belonging to the same semantical category as $\supset$, $\vee$, $\wedge$ (16) a.s.o.

The addition of variable functors to pr. calculus allows us

a) to define (Tarski) "and" by "$\equiv$", viz.

$[pq] :. p . q . \equiv : [f] : f(p) \equiv f(q) . \equiv q$ (different forms)

also ~~to prove~~

b) to prove that of the formulas: (Tarski):

I $[pq] :. p \equiv q . \equiv : [f] : f(p) \equiv f(q)$ (law of extensionality)

and

II $[fp] : f(0) \supset . f(1) \supset f(p)$ (the principle of bivalency)

are inferentially equivalent in the field of classical logic.

Moreover, we have:

I a) $[\varphi f]: \varphi\{\sim\}.\varphi\{\vee z\}.\varphi\{\exists e\}.\varphi\{A 1\} \supset .\varphi\{f\}.$

b) $[fg]:.[p]: f(p).\equiv.g(p): \equiv :[\varphi]:\varphi\langle f\rangle.\equiv.\varphi\langle g\rangle$

and the same holds for arbitrary expressions of any higher semantical category. Thus, for $f(pq)$ we have:

Protothetic can ~~be axiomatized~~ be based on different primitive notions, e.g. on implication only. In such a case, it is sufficient to assume axiomatically a small fragment of implicational calculus and formula $[fp]: f([u].u) \supset : f([u].u. \equiv .[u].u) \supset . f(p)$

But, for several reasons Leśniewski accepted as a primitive notion $\equiv$.

The shorter single axiom (?, 1945) is:

A $[pq]:: p \equiv q. \equiv :: [f]:. f(p\, f(p\, [u].u)). \equiv : [r]: f(qr). \equiv . q \equiv p$

And five rules of procedure:

1. Definitions: $[p_1 \ldots p_n]: A(p_1 \ldots p_n). \equiv . \phi(p_1 \ldots p_n)$

   A definition is not an abbreviation, but a proven formula.

2. Rule of distribution of quantifiers:

   There are no vacuous quantifiers a.s.o. No particular quantifier.

3. Rule of detachment: $\vdash A \equiv B$ and $\vdash A$, then $\vdash B$ There is no detachment under quantifiers.

4. Rule of substitution: No use of incomplete symbols.

5 Rule of extensionality: (except extensionality of propositions).

Remark: No theorem of protothetic has the free variables.

~~It is~~

It is proved that

a) Protothetic is a consistent system. b) It is functionally complete and c) Protothetic is strongly complete system in the sense that if A is well-formed formula ~~A without free variables~~ in the field of protothetic and A does not possess the free variables, then either A or $\sim A$ is a consequence of axiom A.

Moreover, although we have only the rule concerning the distribution of quantifiers, any formula concerning the use of quantifiers is provable as a thesis of a system. E.g. we have:

$$[pf] :. [q] : p . \supset . f(q) : \equiv : p \supset [q] . f(q)$$

It can be easily proved that if $\varphi(x)$ is a propositional function belonging to a theory based on protothetic, then no matter to what theory of types ~~$\varphi(x)$ belong~~ φ and x belong, we have all quantification theory for ~~x~~ a.s.o.

This superficial description of protothetic shows that this system is so bivalued, as possible. A problem which I like to discusse here is a ~~construction of~~ possibility of a construction of many-valued protothetic. At once we can remark that there is no infinite-valued propositional system which ~~can be~~ could be considered as protothetical system, since infinite-valued systems cannot be, obviously, functionally complete. On the other hand, for any natural number n, we are able to construct n-valued protothetical system.

The main difference between protothetic and many-valued protothetical systems is that the ~~principle of bivalency~~ law of extensionality is false in the latter systems, and in the field of such systems we have no principle of bivalency, but the principles o n-valency. E.g. in 3-valued system we have:

$$[fp] :: f(1) . \supset : f(2) \supset . f(3) \supset f(p)$$

In order to understand a method which allows us to construct for the given n, n-valued protothetic we have to analyse another formulization of Leśniewski's protothetic, i.e., the, so called, computable protothetic. Viz. System B

Axiom: $C00$

Rules;

a) if α and β are already theses of the system, then $C\alpha\beta$ is also.
b) if α and $C\beta 0$ " " " " " system, then $CC\alpha\beta 0$ is also
c) if $C\alpha 0$ and β " " " " " system, then $C\alpha\beta$ is also
d) if $C\alpha 0$ and $C\beta 0$ " " " ", then $C\alpha\beta$ is also.

obviously rules a – d describe matrix

C	0	1
0	1	1
1	0	1

~~Ex~~ $C\beta 0 = 1 \rightarrow \beta = 0$
~~$C\alpha 0 = 0$~~

e) $CCC\alpha\beta CC\beta\alpha 00$, α definiens, β definiendum
f) If definiens ~~of the given~~ (or substitution instance) belongs to the system, then also definiendum (or its corresponding instance) can be added to the system.
g) If $C\alpha 0$, then also $C\beta 0$ (analogous to f)
h) If for the given semantical category σ we have n constances (e.g. for $\underline{f}(p)$ 4: α, Vr, Fl, As) and in the system we have already n proven theses: $A(n_1)$, $A(n_2)$ ... $A(n_n)$, then we can add a thesis $[f]\cdot A(f)$

Thus e.g. we have $C00$, Define $1 = C00$, prove $C11$, then $[p]\cdot Cpp$.

i). If α begins with univ. quantifier, β can be obtain from α by substitution of the basic ~~constances~~ constances for its variables, $\vdash C\beta 0$, then $\vdash C\alpha 1$

It can be proved that system A i B are inferentially equivalent. and, moreover, the proof A → B gives the completeness proof, and B → A consistency proof.

Similarly, we can construct analogous systems for ≡ and several other functors.

The computable systems of protothetic give a hind how we can construct n-valued protothetic. Namely, in order to obtain n-valued protothetic we have to choose n-valued matrix which is functionally complete. Then, we define the rules analogous to Leśniewski's rules discussed above, but based on the matrix and we accept an analogous axiom. say, CΛΛ, where Λ is a constant. ... But in order to formalize such systems we have to modify considerably Leśniewski's approach. Namely, ~~e.g. consider~~ for any natural n, there are n-1 functionally complete different propositional systems - e.g. consider a matrix:

M1.

C	1	2	3	H
1	3	2	3	2
2	1	3	3	3
3	1	2	3	1

This matrix is functionally complete, but using it we can obtain two systems, viz accepting that 3 and 2 are designated values, or only ~~one~~ 3 (≡ Łukasiewicz - Słupecki). In order to obtain this two systems we have to accept 9 rules analogous to rules a - d, ~~and several~~ rules 3 rules analogous to rules of definitions and several rules replacing the rules h and i, and in addition the rules concerning functor H according to whether we like to obtain stronger or weaker system. ~~It~~ About ~~this~~ the stronger systems we can prove later ~~that the~~ that it possesses the properties similar to ordinary protothetic. Finally, we can ~~axio~~ formalize such systems in the ordinary way.

Similarly, we can obtain for arbitrary n, n-valued n-1 systems of protothetics. The proof that it holds for any natural n is rather involved. Dr. J. Słupecki constructed the algebraic systems analogous to such protothetical calculi.

No pag 2 has been fo...

1. HHCqq $(=\varphi(q))$
2. q [1, L1]
3. $\varphi(C22)$ [1; C5]
D1. $3 = C22$ [DI]
4 $\varphi(3)$ [D1; DIV]
5 $\varphi(C33)$ [4; C9]
D2 $1 = HH2$ [DI]
6 $\varphi(H2)$ [1; H2]
7 H2 [6; L2]
8 $\chi(HH2)$ [6; H3]
9 $\chi(1)$ [8, D2, DII]
10 $\varphi(C11)$ [9; C1]
11. $\varphi(\Pi p C pp)$ [S1; 10; 3; 5]
12 $\Pi p C pp$ [11; L2]
13 $\varphi(C32)$ [4; 1; C8]
14 $\varphi(C12)$ [9; 1; C2]
15 $\varphi(\Pi p C p2)$ [SII; 14; 13; 5]
16 $\Pi p C p2$ [15; L1]
17. $\varphi(H\Pi p C p2)$ [15, H2]
18 $H\Pi p C p2$ [17, L2]
19 $\varphi(HHC22)$ [3; H3; H1]
20 $\varphi(HHC11)$ [10; H3, H1]
21 $\varphi(HHC33)$ [5; H3, H1]
22 $\varphi(\Pi p HHC pp)$ [SI; 19, 20; 21]
23 $\Pi p HHC pp$ [22; L1]

$SI \neq SII$

29 $\varphi(C\Pi f C f(2)\, C\, 3\, f(3)\, H3)$ [26, 27, C1]

$\mathfrak{D}4$ $N p = C p 1$ [D I]

30 $\chi(C21)$ [1, 9; C4)

31 $\chi(N2)$ [D4, 30; D II]

32 $\chi(C31)$ [4; 9; C7]

33 $\chi(N3)$ [

i t.d.

$$\left.\begin{array}{l} 30\quad C H p \Pi f C f(2)\, C p f(p) \\ 31\quad C \Pi f C f 2\, C p f(p) H p \end{array}\right\} \mathfrak{D} f\ H p = [f] : f(2) . p . f(p)$$

64 $0 . p \supset 0$ [10, 52]

65 $p \cdot q \supset p$ [64, 9, 3V]

66. $p \cdot 0 \supset 0$ [38, 10]

67 $p \cdot q \supset q$ [66, 9, 3V]

68 $\cancel{p \supset} 1 \supset . q \supset 1 \cdot q$ [C; E; 8, 12, 18]

69 $p \supset . q \supset p \cdot q$ [68, 11, 1V]

70 $p \supset . \sim(p \cdot q) \supset \sim q$ [69; {C, N}]

71 $\sim(p \cdot q) \supset . p \supset \sim q$ [70, {C, N}]

72 $\sim(p \supset \sim q) . \supset . p \cdot q$ [11, {C, N}]

73 $p \supset q \supset \cancel{N C p N q}\ \sim(p \supset \sim q)$ [35, {C, N}]

74 $p \cdot q \supset . \sim(p \supset \sim q)$ [65, 67, {C, N}, K].

75 $p \cdot q . \equiv . \sim(p \supset \sim q)$ [62, 72, 74]

76 $p \supset q \cdot q \supset p . \supset . p \equiv q$ [65, 67, 62].

77 $p \equiv q . \supset . p \supset q \cdot q \supset p$ [64, 52, 54].

78 $p \equiv q . \equiv . p \supset q \cdot q \supset p$ [62, 76, 77]

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79. $p \equiv . 1 \supset p$ [P.C, $\cancel{3V}$]

80. $[f] \therefore [x] : 1 \supset f(x) : \equiv : 1 \supset [x] . f(x)$ [79; E]

81 $[f] \therefore [x] : 0 \supset f(x) : \equiv : 0 \supset [x] . f(x)$ [11, E]

82 $[fp] \therefore [x] : p \supset f(x) : \equiv : p \supset . [x] . f(x)$ [80, 81, 1V]

83 $[fx] : [x] . f(x) . \supset . f(x)$

D3. $[pq] : f(p) . \equiv f(q) . \equiv . p \alpha \langle pq \rangle fff$

84 $[f] : f(0) \equiv f(1) : \supset . 0 \equiv 1$ [D3; 83, 8.C]

85 $[f] : f(1) \equiv f(0) : \supset . 1 \equiv 0$ [D3, 88, 8.C]

86 $[pq] : f(p) \equiv f(q) : \supset . p \equiv q$ [84, 85, 23, 8C]

$\cancel{q \supset \sim(p \supset \sim q)}$

$\cancel{p \cdot 0} \supset . \sim(0 \supset \sim q)$

$0 \cdot p . \supset$

III.

11. $\Pi p\, Cp\frac{1}{2}$ [10; $\mathcal{L}$ I]

12. $\varphi(H\Pi p\, Cp\frac{1}{2})$ [10; H II]

13. $H\Pi p\, Cp\frac{1}{2}$ [12; $\mathcal{L}$ II]

14. $\chi(HC\frac{1}{2}\frac{1}{2})$ [3; H III]

15. $\varphi(HHC\frac{1}{2}\frac{1}{2})$ [14; H I]

16. $\varphi(\Pi p\, HHCpp)$ [2 15 i analogicznych formuł]

Dowodzimy kolejno następujące:

I. CCCpqrCCrpCsp

II. CCHqCHHqHpCHHpCpq

III. CCHHpHHqHHCpq

IV. CHHCpqCHHpHHq

V. $Cf(0)Cf(\frac{1}{2})Cf(1)f(q)$

Eliminacja kwantyfikatorów:

1) $0 = [p].p$

2) $1 = C00 = C\Pi pp\Pi pp$

3) $\frac{1}{2} = [p]:[p].p.\supset p = [p]:0\supset p$

4) $Hp = [f]: f(\frac{1}{2})\supset.p\supset f(p)$

III v

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$C\Pi p C\varphi p N p\ C\varphi(\Pi p C\varphi(p) N p\ N\Pi p C\varphi(p) N p$

$C\Pi p C p N p C\Pi p C p N p\ N\Pi p C p N p$

$C\Pi p C p N p\ N\Pi p C p N p$

$N\Pi p C p N p$

$\cancel{CN}$

$C\Pi p C p N p\ 0$

$\cancel{C\Pi}$

$C\varphi\Pi p C\varphi(p) N p\ K\cancel{K}\Sigma p K\varphi(p) p\ \Sigma p K\varphi(p) N p$

$\underline{\Pi p C p N p}$

$C\Pi p N E\varphi p p C\Pi p q C K\varphi p \varphi q E p q\ \Pi p C\varphi\varphi p N\varphi p$

$CCCpCp\tfrac{1}{2}pp$

max(1, q − p + 1)

	0	1/3	2/3	1	N
0	1	1	1	1	1
1/3	2/3	1	1	1	2/3
2/3	1/3	2/3	1	1	1/3
1	0	1/3	2/3	1	0

1) $C\tfrac{1}{3}C\tfrac{1}{3}0$

$CC\tfrac{2}{3}0\tfrac{1}{3}$

$C\tfrac{2}{3}C\tfrac{1}{3}0$

$CC\tfrac{1}{3}0\tfrac{2}{3}$

2) $CCC\tfrac{1}{3}00\tfrac{1}{3}$

$CC\tfrac{1}{3}0\,2\,C\tfrac{1}{3}2$

$Cp0$: $\cancel{1}$, 2/3, 1/3, 0

2 − 1 + 1 = 2

3 − 4 + 1

$\underline{4 - 3 + 1}$

2

4 − 2 + 1

3 − 2 + 1

$\cancel{3} - \cancel{3} + 1$

2 − 1 + $\cancel{1}$

$CCCpCpqpp$

$CCCpCpCpqpp$

$CCNpNqCqp$ $\tfrac{2}{3}\ \tfrac{1}{3}$

$CC\tfrac{2}{3}0\tfrac{2}{3}$

$\cancel{C\tfrac{2}{3}C\tfrac{2}{3}0}$

$\cancel{\tfrac{1}{3}0}$

$\tfrac{2}{3}\cancel{0}$

$CCCpqpp$

$\tfrac{1}{3}0 = \tfrac{2}{3}$

$\overline{\text{VI}}$.

$z_1 p = CHH\tfrac{1}{2}HC\tfrac{1}{2}p$ | $\tfrac{1}{2}$, 1, 1

$z_2 p = HHCHH\tfrac{1}{2}HHp$ | $\tfrac{1}{2}$, $\tfrac{1}{2}$, 0

$C_1 pq = HHCz_2 pHCz_1 pC_1 pq$

$Np = CC\tfrac{1}{2}pHHCp\tfrac{1}{2}$

$CHHCpqCHHpHHq$

$CHHHpp$

—

$CCHHpHHqHHCpq$

F_1	F_2	F_3	F_4	F_5	F_6	F_7	F_8	F_9	F_{10}	F_{11}	F_{12}	F_{13}	F_{14}	F_{15}	F_{16}	F_{17}	F_{18}	F_{19}	F_{20}	F_{21}	F_{22}	F_{23}	F_{24}	F_{25}	F_{26}	F_{27}
1	1	1	1	1	1	1	1	1	2	2	2	2	2	2	2	2	2	3	3	3	3	3	3	3	3	3
1	2	1	3	1	2	3	2	3	2	1	2	3	1	2	3	1	3	3	1	2	1	3	2	3	1	2
1	1	2	1	3	3	2	2	3	2	2	1	1	3	3	2	1	3	3	2	1	3	1	3	2	1	2

$\cancel{F_7}$

$\downarrow$

Hp

$F_1 = HCpp = CCpp1$

$F_2 = HHCp2$

$F_3 = HC2Hp$

$F_4 = CHHp1 = HCCp22$

$F_5 = CHp1$

$F_6 = p = A_3(p)$

$F_7 = \cancel{CCp11}$

$F_8 = HCp1$

$F_9 = CCp11 = C2p$

$F_{10} = HHCpp = CCpp2$

$F_{11} = HCHHp1 = HHCCp22$

$F_{12} =$

$F_{13} = Hp$

$F_{14} =$

$F_{15} = \cancel{Hp}\ HHC2Hp$

$F_{16} = Cp2$

$F_{17} = HCCp11$

$F_{18} = HHCp1$

$F_{19} = Cpp$

$F_{20} = HHp$

F_{21}

$F_{22} = HCp2$

$F_{23} = C2Hp$

$F_{24}\ CCp22$

F_{25}

$F_{26} = Cp1$

$F_{27} = HHCCp11 = HHC2p$

1) $p \supset p$

2) $1 \supset 1$ (1)

2) $p \supset . q \supset p$

3) $1 \supset 3$ (2)

4) $1 \supset : [p] : 1 \supset p$ (2)

5) $1 \supset 2$ [4, Df].

6) $[p] : 1 \supset p$ [S, 2, 3, 5]

7) 2 (Df, 6).

8) 3 (Df 1)

9) $[f] : 1 \supset [x] . f(x) . \supset . [x] : 1 \supset f(x)$ [6, A].

10) $2 \supset q . \supset q$ [7, A]

11) $3 \supset q . \supset q$ [8, A]

12) $2 \supset [x] . f(x) : \supset : [x] : 2 \supset f(x)$ (A, 10, 2)

13) $3 \supset [x] . f(x) : \supset . [x] : 3 \supset f(x)$ (A, 11, 2)

14) $[fp] :: p \supset [x] . f(x) : \supset : [x] : p \supset f(x)$ [S, S, 2, 13]

15) $p \supset q . \supset : p \supset . r \supset q$

16) $[x] : 2 \supset f(x) . \supset . 2 \supset [x] . f(x)$ [10, 15]

17) $[x] : 3 \supset f(x) . \supset . 3 \supset [x] . f(x)$ [11, 15]

18) $[x] : 1 \supset f(x) . \supset . 1 \supset [x] . f(x)$ [A, 6]

19) $[pf] : \cancel{p \supset f(x)} \supset [x] : p \supset f(x) : \supset : p \supset . [x] . f(x)$ [S, 18, 16, 17]

20) $p \supset 2$ [2, 7]

21) $p \supset 3$ [3, 7]

$[fp] : p \supset [x] . f(x) . \supset . [x] . p \supset f(x)$.

$[p] : [x] : p \supset f(x) . \supset . p \supset [x] . f(x)$

$[x] : 1 \supset f(x) . \supset . \underline{1 \supset [x] . f(x)}$

2 2

3 3

$[x] \boxed{2 \supset f(x)} . \supset . 2 \supset [x] . f(x)$

$\underline{CCpqCpCrq}$

Df1 $A_1 pq = CCpqq$

A_1	1	2	3
1	1	3	3
2	2	2	3
3	3	3	3

Df6 $F_3 p = HCHp1$

p	F_3
1	2
2	2
3	1

Df2. $F_1 p = Cp1$

p	$F_1 p$
1	3
2	1
3	1

D7 $V_3 pq = V_1 F_3 p F_3 q$

V_3	1	2	3
1	1	1	1
2	1	1	1
3	1	1	3

Df3. $F_1 A_1 pq = V_1 pq$

V_1	1	2	3
1	3	1	1
2	1	1	1
3	1	1	1

D8 $V_4 pq = A_1 A_1 V_1 pq V_2 pq V_3 pq$

V_4	1	2	3
1	3	1	1
2	1	3	1
3	1	1	3

Df4 $F_2 p = HCp2$

p	F_2
1	3
2	1
3	3

Df $\underline{I}$ $V_4 \alpha\beta$

D5 $V_2 pq = V_1 F_2 p F_2 q$

V_2	1	2	3
1	1	1	1
2	1	3	1
3	1	1	1

FIFTH INTERNATIONAL SYMPOSIUM ON MULTIPLE-VALUED LOGIC

IEEE COMPUTER SOCIETY

ONR

The *1975 INTERNATIONAL SYMPOSIUM ON MULTIPLE-VALUED LOGIC* continues to bring together those having interests related to the theory and applications of multiple-valued logic.

This *ADVANCE PROGRAM* provides an early schedule indicating the variety of interest areas for papers and some of the distinguished participants from different countries. This FIFTH ANNUAL SYMPOSIUM is supported by the following organizations: ACM Indiana University
IEEE Computer Society
Society for Exact Philosophy
U.S. Office of Naval Research

Tuesday, May 13, evening
- *Registration*

Wednesday, May 14, morning
- *Welcome to Indiana University*
- *Welcome Address by the President of the International Society and Organizing Group for Multiple-Valued Logic: G. Epstein* (IU, USA)

CHAIRMAN: *L. Zadeh* (UCB, USA)
- *Invited Address: R. Bellman* (USC, USA), Local Logics

CHAIRMAN: *E. Santos* (Youngstown SU, USA)
1. R. Giles (Queen's U, CAN), Lukasiewicz logic and fuzzy set theory
2. D. Rine (WVU, USA), Associative and multi-valued logic techniques to improve some X-ray image processing
3. H. Wechsler (UCI, USA), Applications of fuzzy logic to medical diagnosis.
4. Y. Pao and F. Merat (Case WRU, USA), Distributed associative memory for patterns
5. P. Schotch (Dalhousie U, CAN), Fuzzy modal logic

Wednesday, May 14, afternoon
CHAIRMAN: *P. Halmos* (IU, USA)
- *Invited Address: P. Dwinger* (U Ill. Chicago, USA), Recent developments in the theory of Post algebras

CHAIRMAN: *M. Allen* (UNCC, USA)
1. E. DuCasse (CUNY, USA), Reducibility of Post functions
2. R. Cignoli (U Ill. Chicago, USA), Free non-symmetric n-valued Lukasiewicz algebras
3. J. Loader (Brighton Polytechnic, ENG), Second order and higher order universal decision elements in m-valued logic
4. G. Malinowski (Lodz U, POL), Matrix representation for the dual counterparts of Lukasiewicz m-valued sentential calculi and the problem of their degrees of maximality
5. S. Hu (Cleveland SU, USA), A ternary algebra for probability computation of digital circuits
6. R. Michalski (U Ill. Urbana, USA), Synthesis of optimal and quasi-optimal variable-valued logic formulas
7. T. Higuchi and M. Kameyama (Tohoku U, JAPAN), Synthesis of multiple-valued logic networks based on tree type universal logic modules

Wednesday, May 14, evening
- *Additional Papers and Informal Presentations*

Thursday, May 15, morning
CHAIRMAN: *A. Svoboda* (UCLA, USA)
- *Invited Address: M. Yoeli* (Technion, ISRAEL), Binary and multiple-valued models of binary gate networks

CHAIRMAN: *D. Givone* (SUNY Buffalo, USA)
1. H. Mouftah & I. Jordan (U Laval, CAN), A design technique for an integrable ternary arithmetic unit
2. Z. Vranesic & V. Hamacher (U Toronto, CAN), Threshold logic in fast ternary multipliers
3. K. Hickin & J. Plotkin (MSU, USA), Compactness and ρ-valued logics
4. D. Etiemble (Paris VI U, FRANCE) & M. Israel (CNAM, FRANCE), Implementation of a complete ternary algebra with elementary operators—application to ternary flip-flop
5. T. Kitahashi (Osaka U, JAPAN), A survey of studies on multiple-valued logic in Japan
6. S. Lee & Y. Keren-Zvi (U Houston, USA), A generalized Boolean algebra and its application to logical design

Thursday, May 15, afternoon
CHAIRMAN: *G. Abraham* (Naval Res. Lab., USA)
- *Invited address: G. Metze* (U Ill. Urbana, USA), Some multi-valued approaches to two-valued switching problems

CHAIRMAN: *S. Su* (CUNY, USA)
1. C. Moraga (U Dortmund, W. GERMANY & U Santa Maria, CHILE), Hybrid logic—a fast ternary adder
2. J. Deschamps & A. Thayse (MBLE Res. Lab., BELGIUM), Representations of discrete functions
3. P. Cheung & D. Purvis (Packard Ins., USA), A computer oriented, heuristic minimization algorithm for multiple-output, multiple-valued switching functions
4. T. Higuchi & M. Kameyama (Tohoku U., JAPAN), Ternary logic system based on T-gate
5. V. Pinkava (Severalls Hospital, ENG), Some further properties of the Pi-logics
6. T. Wesselkamper (Virginia Polytech. & SU, USA), The logical foundations of microlanguages

Thursday, May 15, evening
- *Banquet*

Friday, May 16, morning
CHAIRMAN: *A. Tarski* (UCB, USA)
- *Invited Address: H. Rasiowa* (U Warsaw, POL), Multiple-valued algorithmic logics as a tool to investigate programs

CHAIRMAN: *T. Traczyk* (Warsaw Tech. U, POL)
1. R. Grigolia (Tbilisi U, USSR), On the algebras corresponding to the n-valued Lukasiewicz-Tarski logical systems
2. I. Rosenberg (U Montreal, CAN), Functional completeness in heterogeneous multiple-valued logics
3. R. Wojcicki (Wroclaw U, POL), A theorem on the finiteness of the degree of maximality of the n-valued Lukasiewicz logic
4. J. Nazarala & C. Moraga (U Chile Santiago, CHILE & U Santa Maria, CHILE), Bilineal separation of ternary functions
5. J. Kabzinski & A. Wronski (Jagiellonian U., Cracow, POL), On equivalential algebras
6. B. Matilal (U Toronto, CAN), On the Navya-Nyaya logic of property and location
7. D. Ulrich (Purdue U, USA) Many-valued logics with non-many-valued extensions—two examples

Friday, May 16, afternoon
CHAIRMAN: *B. Sobocinski* (Notre Dame U, USA)
- *Invited Address and Welcome by the President of the Society for Exact Philosophy to its Annual Meeting: N. Belnap, Jr.* (U Pittsburgh, USA), A useful four-valued logic

CHAIRMAN: *S. Surma* (Jagiellonian U, Cracow, POL)
1. J. McCawley (U Chicago, USA), Truth functionality and natural deduction
2. J. Muzio (U Manitoba, CAN), Ternary two-place functions that are complete with constants
3. H. Leblanc (Temple U, USA), A Completeness theorem for 3-valued logic with quantifiers
4. H. Ellozy (IBM T. J. Watson Res. Center, USA) and Y. Patt (NCSU, USA), The linearity property and functional completeness in M-valued logic
5. C. Morgan (U Alberta, CAN), Similarity as a theory of graded equality for a class of many-valued predicate calculi
6. H. Herzberger (U Toronto, CAN), Supervaluations in two dimensions

Friday, May 16, evening
- *The annual meeting of the Society for Exact Philosophy continues into the weekend*

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1975 International Symposium on Multiple-Valued Logic

May 13-16, 1975 at Poplars Center, Indiana University, Bloomington, Indiana 47401 USA

1975 Symposium Committee:
Dr. G. Epstein
Symposium Chairman

Dr. M. Dunn
Program Chairman

ONR

The *1975 INTERNATIONAL SYMPOSIUM ON MULTIPLE-VALUED LOGIC* continues to bring together those having interests related to the theory and applications of multiple-valued logic.

This *ADVANCE PROGRAM* provides an early schedule indicating the variety of interest areas for papers and some of the distinguished participants from different countries. This FIFTH ANNUAL SYMPOSIUM is supported by the following organizations: ACM Indiana University
IEEE Computer Society
Society for Exact Philosophy
U.S. Office of Naval Research

Tuesday, May 13, evening
- *Registration*

Wednesday, May 14, morning
- *Welcome to Indiana University*
- *Welcome Address by the President of the International Society and Organizing Group for Multiple-Valued Logic: G. Epstein* (IU, USA)

CHAIRMAN: *L. Zadeh* (UCB, USA)
- *Invited Address: R. Bellman* (USC, USA), Local Logics

CHAIRMAN: *E. Santos* (Youngstown SU, USA)
1. R. Giles (Queen's U, CAN), Lukasiewicz logic and fuzzy set theory
2. D. Rine (WVU, USA), Associative and multi-valued logic techniques to improve some X-ray image processing
3. H. Wechsler (UCI, USA), Applications of fuzzy logic to medical diagnosis.
4. Y. Pao and F. Merat (Case WRU, USA), Distributed associative memory for patterns
5. P. Schotch (Dalhousie U, CAN), Fuzzy modal logic

Wednesday, May 14, afternoon
CHAIRMAN: *P. Halmos* (IU, USA)
- *Invited Address: P. Dwinger* (U Ill. Chicago, USA), Recent developments in the theory of Post algebras

CHAIRMAN: *M. Allen* (UNCC, USA)
1. E. DuCasse (CUNY, USA), Reducibility of Post functions
2. R. Cignoli (U Ill. Chicago, USA), Free non-symmetric n-valued Lukasiewicz algebras
3. J. Loader (Brighton Polytechnic, ENG), Second order and higher order universal decision elements in m-valued logic
4. G. Malinowski (Lodz U, POL), Matrix representation for the dual counterparts of Lukasiewicz m-valued sentential calculi and the problem of their degrees of maximality
5. S. Hu (Cleveland SU, USA), A ternary algebra for probability computation of digital circuits
6. R. Michalski (U Ill. Urbana, USA), Synthesis of optimal and quasi-optimal variable-valued logic formulas
7. T. Higuchi and M. Kameyama (Tohoku U, JAPAN), Synthesis of multiple-valued logic networks based on tree type universal logic modules

Wednesday, May 14, evening
- *Additional Papers and Informal Presentations*

Thursday, May 15, morning
CHAIRMAN: *A. Svoboda* (UCLA, USA)
- *Invited Address: M. Yoeli* (Technion, ISRAEL), Binary and multiple-valued models of binary gate networks

CHAIRMAN: *D. Givone* (SUNY Buffalo, USA)
1. H. Mouftah & I. Jordan (U Laval, CAN), A design technique for an integrable ternary arithmetic unit
2. Z. Vranesic & V. Hamacher (U Toronto, CAN), Threshold logic in fast ternary multipliers
3. K. Hickin & J. Plotkin (MSU, USA), Compactness and p-valued logics
4. D. Etiemble (Paris VI U, FRANCE) & M. Israel (CNAM, FRANCE), Implementation of a complete ternary algebra with elementary operators—application to ternary flip-flop
5. T. Kitahashi (Osaka U, JAPAN), A survey of studies on multiple-valued logic in Japan
6. S. Lee & Y. Keren-Zvi (U Houston, USA), A generalized Boolean algebra and its application to logical design

Thursday, May 15, afternoon
CHAIRMAN: *G. Abraham* (Naval Res. Lab., USA)
- *Invited address: G. Metze* (U Ill. Urbana, USA), Some multi-valued approaches to two-valued switching problems

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dvance Registration Closes May 1, 1975

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Indiana University

Dr. N. Cocchiarella
Program Vice-Chairman
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Dr. M. Dunn
Program Chairman
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Dr. S. Shapiro
Program Vice-Chairman
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Symposium Registrants please make hotel reservations directly with:
Poplars Research & Conference Center
400 E. 7th Street
Bloomington, IN 47401 USA

INDIANA UNIVERSITY
Computer Science Department
101 LINDLEY HALL
BLOOMINGTON, INDIANA 47401

TEL. NO. 812—337-6486

April 7, 1975

Dear Colleague:

This is to bring to your attention the existence of a Wednesday evening session, May 14, at the 1975 International Symposium on Multiple-Valued Logic, for informal presentations and additional papers which were received too late for refereeing or inclusion in the Proceedings.

We would be pleased for you to discuss or present your lately received work at this session. It is planned for Professor Sobocinski to begin the session with a one-hour talk, with subsequent presentations each of 15 minutes in length, as in the daytime sessions.

Since these 15 minute time slots may fill up rapidly once the Sumposium begins, it would be best for me to hear from you about this as soon before May 13 as possible.

Sincerely,

George Epstein

George Epstein
Symposium Chairman

GE/bh
Information Copy to Professor Boleslaw Sobocinski

PARTICIPANTS

1975 International Symposium on Multiple-Valued Logic

Allen, Michael C.
Altman, Jeffrey
Altman, Robert
Anderson, Susan
Aron, Ellen
Broere, John
Brubecker
Burkowski, Forbes J.
Butler, Jean
Byrne, Rodney P.
Cignoli, Roberto L.
Clifford, John E.
Collier, Ken
Deschamps, Jean-Pierre
Ducasse, Edgar
Dwinger
Eberhart
Ellozy, Hamed A.
Etiemble, Daniel
Gale, Stephen
Giles, Robin
Hamacher, V.C.
Hansen, John C.
Hardegree
Herzberger, Hans G.
Higuchi, Tatsuo
Hu, Sung
Huang, H.K.
Hunter, David
Israel, Michael
Jaworowski, Jan
Kameyama, Michitaka
Kittel, Phyllis M.
Larson, James
Levy, Gerard
Liddell, Gerrad
Loader, John
McCawley, James D.
Macvicar-Whelen, P.J.
Maraga
Metze, Gernot
Michalski, R.S.
Micheel, L.
Miller, David Michael
Moore, John
Morgan, Charles G.
Mouftah, Hussein T.
Muzio, Jon
Pao, Yoh-Han
Pinkava, Vaclav
Rapaport, William J.
Rasiowa, Helena
Rine, David
Rosenberg, Ivo G.
Santos, Eugene
Schotch, Peter
Smith, K.C.
Sobocinski
Su, Stephen Y.H.
Surma, S.
Swartwout, Robert E.
Tadahiro, Kitahashi
Thulin, Fred
Tront, Joseph G.
Ulrich, Dolph
Vranesic, Z.G.
Wasserman, Howard C.
Weschler, Harry M.
Wesselkamper, T.
Wherritt, Robert
Wolf, Robert
Yoeli, Michael
Zadeh, Lotfi

Pre-registered, did not attend as of Wed. 5 pm

Kirin, Vladamir
LaGrand, Paul
Leblanc, Hugues
Ledley, Robert S.
Plotkin, Jacob
Purvis, David M.
Stewart, William J.
Thomas, Robert J.
Wojcik, Anthony
Wolf, Robert G.

Prototetyka wielowartościowa

II Semestr 1964/65

[illegible] Indiana Univ., 1985

Spring 1965
Phil 298
[illegible] Calculus of Propositions