Emperically Derived Fermion Higgs Yukawa Couplings and Pole Masses

Kevin Loch*

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Empirically derived formulas are proposed for calculating the Higgs field Yukawa couplings and pole masses of the twelve known fundamental fermions with experimental inputs m_e, m_μ and the Fermi constant $G_{\rm F}^0$.

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I. INTRODUCTION

An explanation of the seemingly unrelated masses appearing as free inputs in the standard model as been eluding physicists for many decades. In a previous paper [1] the author introduced a system of empirically derived formulas that appeared to relate the fundamental fermion masses to each other at least to a close approximation. They were thus related approximately to the Higgs vacuum expectation value $v = \left(\sqrt{2}G_{\rm F}^0\right)^{1/2} \simeq 246.22$ GeV$/c^2$ through the top quark Higgs Yukawa coupling y_t. We now seek a formula for the top quark Higgs Yukawa coupling so that masses can be calculated directly from the Higgs field vacuum expectation value. This will hopefully provide insight into the underlying physics and help efforts to relate these formulas to the standard model.

II. REVIEW OF EXISTING FORMULAS

The basic structure for deriving the mass formulas in [1]

$$m_{\nu_1} \simeq \left(2\pi\alpha_f^7\right)\frac{y_t v}{\sqrt{2}}, \qquad m_e \simeq \left(2\pi\alpha_f^3\right)\frac{y_t v}{\sqrt{2}}, \qquad m_d \simeq \left(2\pi^3\alpha_f^3\right)\frac{y_t v}{\sqrt{2}}, \qquad m_u \simeq \left(\pi^3\alpha_f^3\right)\frac{y_t v}{\sqrt{2}},$$

$$m_{\nu_2} \simeq 9\left(\frac{2\pi\alpha_f^7}{k_\nu^{\frac{1}{3}}}\right)\frac{y_t v}{\sqrt{2}}, \qquad m_\mu \simeq 9\left(\frac{2\pi\alpha_f^{\frac{7}{3}}}{k_e^{\frac{1}{3}}}\right)\frac{y_t v}{\sqrt{2}}, \qquad m_s \simeq \left(\frac{2\pi^{\frac{4}{3}}\alpha_f^{\frac{5}{3}}}{k_d^{\frac{1}{3}}}\right)\frac{y_t v}{\sqrt{2}}, \qquad m_c \simeq \left(\frac{\pi\alpha_f}{k_u^{\frac{1}{3}}}\right)\frac{y_t v}{\sqrt{2}},$$

$$m_{\nu_3} \simeq 27\left(2\pi\alpha_f^7\right)\frac{y_t v}{\sqrt{2}}, \qquad m_\tau \simeq 27\left(2\pi\alpha_f^2\right)\frac{y_t v}{\sqrt{2}}, \qquad m_b \simeq \left(2\pi^{\frac{1}{2}}\alpha_f\right)\frac{y_t v}{\sqrt{2}}, \qquad m_{\rm top} = \frac{y_t v}{\sqrt{2}}. \tag{1}$$

is a generic sector residual formula based on a charged lepton relationship proposed by Terazawa [2]

$$k_S = \frac{m_1 m_3^2}{m_2^3}, \tag{2}$$

where k_S is the dimensionless sector residual and m_1, m_2, m_3 are the first, second and third generation fermion masses for sector S. As with the Koide formula [3], most of the factor(s) determining the widely different input masses in the charged lepton sector cancel out leaving a small residual value. In an attempt to discover factor(s) that do cancel out under the structure formula we introduced the parameter

$$\alpha_f = 27\frac{m_e}{m_\tau}. \tag{3}$$

This factor was applied to each fermion with an exponent corresponding to the relative mass scale of that fermion. While α_f has a value similar to $\alpha_{\rm QED}(M_Z^2)$ we assume it is a separate parameter related only to mass generation and not QED. Along with α_f, only 'simple' integer coefficients (1, 2, 9, 27) and factors of π were then required to relate

* kevinmloch@gmail.com

all 12 known fundamental fermions to each other within experimental constraints. The charged lepton residual k_e was derived from the experimental electron, muon and tau lepton masses. The neutrino residual k_ν was set to

$$k_\nu = 8 - k_e \tag{4}$$

which provided an excellent match for experimental mass squared differences derived from neutrino oscillation experiments. This can also be expressed as an angle $\theta_{k_{e\nu}} \simeq 24^{\text{o}}$ which differs from the weak mixing angle θ_W by approximately $\pi/48$ radians such that

$$k_\nu = 8\cos^2\theta_{k_{e\nu}}, \tag{5}$$

$$k_e = 8\sin^2\theta_{k_{e\nu}}. \tag{6}$$

The down and up type quark residuals were set to

$$k_d = 128, \tag{7}$$

$$k_u = 16. \tag{8}$$

to match heavy quark experimental pole masses and the rough order of magnitude for experimental/theoretical light quark masses. The relationship between the sector residuals is then

$$k_\nu + k_e = \frac{k_d}{k_u} = 8. \tag{9}$$

III. PROPOSED HIGGS FIELD YUKAWA COUPLINGS

In searching for an empirical formulas for the top quark Higgs Yukawa coupling we find

$$y_t = 1 - \frac{\alpha_f}{k_u^{\frac{1}{3}}} \tag{10}$$

provides an excellent match to the experimental value. Two puzzling results from the proposed mass formulas (1) were the sector residuals appearing in only the second generation masses, and the top quark Yukawa coupling appearing as a factor in all masses. This proposed formula for y_t could resolve both puzzles if we assume fermions in each sector use their sector residual in their 'version' of y_t and if we assume the residual appears in the left hand side in the second generation y_t-like terms and right hand side in the first and third generations. We find that formulas with factors of π then match experimental values only when the π factor is moved from outside to the left side of the y_t-like term. This results in new Higgs Yukawa coupling formulas for each of the known fermions

$$y_{\nu_1} = 2\alpha_f^7\left(\pi - \frac{\alpha_f}{k_\nu^{\frac{1}{3}}}\right), \quad y_e = 2\alpha_f^3\left(\pi - \frac{\alpha_f}{k_e^{\frac{1}{3}}}\right), \quad y_d = 2\alpha_f^3\left(\pi^3 - \frac{\alpha_f}{k_d^{\frac{1}{3}}}\right), \quad y_u = \alpha_f^3\left(\pi^3 - \frac{\alpha_f}{k_u^{\frac{1}{3}}}\right),$$

$$y_{\nu_2} = 18\alpha_f^7\left(\frac{\pi}{k_\nu^{\frac{1}{3}}} - \alpha_f\right), \quad y_\mu = 18\alpha_f^{\frac{7}{3}}\left(\frac{\pi}{k_e^{\frac{1}{3}}} - \alpha_f\right), \quad y_s = 2\alpha_f^{\frac{5}{3}}\left(\frac{\pi^{\frac{4}{3}}}{k_d^{\frac{1}{3}}} - \alpha_f\right), \quad y_c = \alpha_f\left(\frac{\pi}{k_u^{\frac{1}{3}}} - \alpha_f\right),$$

$$y_{\nu_3} = 54\alpha_f^7\left(\pi - \frac{\alpha_f}{k_\nu^{\frac{1}{3}}}\right), \quad y_\tau = 54\alpha_f^2\left(\pi - \frac{\alpha_f}{k_e^{\frac{1}{3}}}\right), \quad y_b = 2\alpha_f\left(\pi^{\frac{1}{2}} - \frac{\alpha_f}{k_d^{\frac{1}{3}}}\right), \quad y_t = 1 - \frac{\alpha_f}{k_u^{\frac{1}{3}}},$$

$$k_\nu + k_e = 8, \quad k_d = 128, \quad k_u = 16. \tag{11}$$

Predicted masses are then obtained by multiplying by the Higgs vacuum field

$$m_f = \frac{y_f v}{\sqrt{2}}. \tag{12}$$

One consequence of these changes is the sector residual formula (2) is no longer exact but only a close approximation

$$k_S \simeq \frac{m_1 m_3^2}{m_2^3}. \tag{13}$$

This also means we will need a new method to derive precision values of k_e and α_f. We continue to use the assumptions $k_\nu = 8 - k_e$, $k_d = 128$ and $k_u = 16$ as they provide excellent matches to experimental values.

IV. MONTE CARLO SOLUTION FOR DERIVING PRECISION α_f AND k_e

An adaptive Monte Carlo technique is used to simultaneously solve the electron and muon equations

$$\text{e\_test} = \frac{\alpha_f^4}{k_e^{1/3}} - \pi\alpha_f^3 + \frac{m_e}{\sqrt{2}v} = 0, \tag{14}$$

$$\text{u\_test} = \alpha_f^{10/3} - \frac{\pi\alpha_f^{7/3}}{k_e^{1/3}} + \frac{m_\mu}{9\sqrt{2}v} = 0, \tag{15}$$

for α_f and k where $v \simeq 246.22$ GeV is the Higgs field vacuum expectation value. The following constraints are used in the Monte Carlo solution to improve and speed up convergence:

- α_f (initial) $= 7 \times 10^{-3} \pm 0.5$; k_e (initial) $= 1.0 \pm 1$
- $k_e > 1$
- e_test > 0; u_test > 0
- score $=$ (e_test $+$ u_test) > 0
- score $<$ score_last
- u_test / e_test < 20; e_test / u_test < 20.

The range of random guesses for α_f and k_e is adjusted after each successful guess matching the above criteria to greatly speed up convergence. After a fixed limit of 5×10^6 guesses the answer is evaluated and accepted if score $< 1 \times 10^{-19}$. This process is repeated while varying m_e, m_μ, v input parameters over their uncertainty ranges to determine the uncertainty of α_f, k and the predicted fermion masses and derived parameters. As the known inputs are all linear in the test equations this can be simplified by only testing combinations of the extremes of their uncertainty ranges. Source code for the Monte Carlo solution is provided in [4]. By using only m_e, m_μ, and v as inputs, it is possible to determine α_f and k_e to higher precision than in our previous paper as well as provide a prediction of the tau lepton mass.

The results are presented in table I. For the leptons and heavy quarks all predicted values are closer than the previous formulas (1) to the central values of the experimental comparisons. For light quarks only reference masses in a renormalization scheme (like $\overline{\text{MS}}$) are available and it is not appropriate to compare them directly. These references are shown in Table I only to show matches to the rough order of magnitude for light quarks. The possibility that $\theta_W = \theta_{k_{e\nu}} + \frac{\pi}{48}$ discussed in [1] is less convincing now that the resulting m_W is near the high end of the reference experimental uncertainty.

V. CONCLUSION

The fermion Higgs field Yukawa coupling formulas presented here provide an excellent match to experimental results for the known neutrinos, charged leptons and heavy quarks. These matches are an improvement on the prior work [1] in form as well as closeness of match. We are now able to calculate predicted masses from just the muon mass, electron mass and Fermi coupling constant, all of which are known experimentally to high precision. The resulting high precision predicted masses can be tested against improved experimental results as they become available. As with any empirically derived formulas, great caution is advised until both experimental confirmation and theoretical understanding is achieved. Some of the experimental values are not yet known to high precision which

	Parameter	THIS WORK	Reference Value	Rel. Diff.	Ref.
Model Inputs	m_e		0.5109989461(31) MeV		CODATA 2014 [5]
	m_μ		105.6583745(24) MeV		CODATA 2014 [5]
	G_F^0		$1.1663787(6) \times 10^{-5}$ GeV^{-2}		CODATA 2014 [5]
	$v = (\sqrt{2}G_F^0)^{-1/2}$		246.219651(63) GeV		
Assumptions	$k_\ell = k_e + k_v$	8 (exact)			
	k_u	16 (exact)			
	k_d	128 (exact)			
Model Parameters	α_f	$7.76486074(68) \times 10^{-3}$			
	α_f^{-1}	128.785310(11)			
	k_e	1.36564100(35)			
	$k_\nu = 8 - k_e$	6.63435900(35)			
	$\sin^2\theta_{k_{e\nu}}$	0.170705125(43)			
Tau Lepton	m_τ	1776.84726(16) MeV	1776.86 ± 0.12 MeV	7.2×10^{-6}	PDG 2017 [6]
Neutrino	m_1	$1.85930930(67) \times 10^{-3}$ eV			
	m_2	$8.8758739(31) \times 10^{-3}$ eV			
	m_3	$5.0201351(18) \times 10^{-2}$ eV			
	Δm_{21}^2	$7.5324106(53) \times 10^{-5}$ eV^2	$7.53 \pm 0.18 \times 10^{-5}$ eV^2	3.2×10^{-4}	PDG 2017 [6]
	Δm_{31}^2	$2.5167186(18) \times 10^{-3}$ eV^2	$2.524^{+0.038}_{-0.041} \times 10^{-3}$ eV^2	2.9×10^{-3}	Esteban 2016 [7]
	Δm_{32}^2	$2.4413945(18) \times 10^{-3}$ eV^2	$2.45 \pm 0.05 \times 10^{-3}$ eV^2	3.5×10^{-3}	PDG 2017 [6]
	$\sum m_\nu$	$6.0936534(22) \times 10^{-2}$ eV	$< 9.68 \times 10^{-2}$ eV		Giusarma 2016 [8]
Down-type	m_d	5.054366836(32) MeV	$4.7^{+0.5}_{-0.4}$ MeV ($\overline{\text{MS}}$)[1]		PDG 2017 [6]
	m_s	95.972667(11) MeV	96^{+8}_{-4} MeV ($\overline{\text{MS}}$)[1]		PDG 2017 [6]
	m_b	4.78815976(83) GeV	4.78 ± 0.06 GeV	1.7×10^{-3}	PDG 2017 [6]
Up-type	m_u	2.527057833(16) MeV	$2.2^{+0.6}_{-0.4}$ MeV ($\overline{\text{MS}}$)[1]		PDG 2017 [6]
	m_c	1.67496074(29) GeV	1.67 ± 0.07 GeV	3.0×10^{-3}	PDG 2017 [6]
	m_{top}	173.567087(45) GeV	173.5 ± 1.1 GeV	3.9×10^{-4}	PDG 2017 [6]
	y_t	0.99691851297(27)			
Electroweak if $\theta_W = \theta_{k_{e\nu}} + \frac{\pi}{48}$	$\sin^2\theta_W$	0.222632911(48)	0.22290(33)	1.2×10^{-3}	PDG 2017 [6]
	m_W/m_Z	0.881684234(27)	0.88153 ± 0.00017	1.7×10^{-4}	PDG 2017 [6]
	m_W	80.3987(19) GeV	80.385 ± 0.015 GeV	1.7×10^{-4}	PDG 2017 [6]

Table I. Predicted pole masses and derived values. [1]It is not appropriate to compare predicted pole masses and $\overline{\text{MS}}$ reference masses directly. These values are shown to reference rough order of magnitude only.

leaves considerable room for various formulas to match those results. There is no current theory which can derive reliable pole masses for the light quarks which casts some doubt on the reliability of our light quark formulas and in particular our assumption of $k_d = 128$.

[1] Kevin Loch, http://kl.net/papers/fermion-mass-hierarchy.pdf
[2] H.Terazawa and M.Yasu'e, Phys.Lett. B307, 383(1993), doi:10.1016/0370-2693(93)90237-C
[3] Y. Koide, Nuovo Cim. Lett. 34, 201 (1982).
[4] Kevin Loch, http://kl.net/papers/src/fmmc-src-1.0.tar.gz
[5] Peter J. Mohr, David B. Newell, Barry N. Taylor, CODATA Recommended Values of the Fundamental Physical Constants: 2014, doi:10.1103/RevModPhys.88.035009, arXiv:1507.07956 [physics.atom-ph].
[6] C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016).
[7] Ivan Esteban, M. C. Gonzalez-Garcia, Michele Maltoni, Ivan Martinez-Soler, Thomas Schwetz, Updated fit to three neutrino mixing: exploring the accelerator-reactor complementarity, IFT-UAM/CSIC-16-114, YITP-SB-16-45, arXiv:1611.01514 [hep-ph].
[8] Elena Giusarma, Martina Gerbino, Olga Mena, Sunny Vagnozzi, Shirley Ho, Katherine Freese, On the improvement of cosmological neutrino mass bounds, Phys. Rev. D 94, 083522 (2016), doi:10.1103/PhysRevD.94.083522, arXiv:1605.04320 [astro-ph.CO].