Freeform surface of progressive addition lens represented by Zernike polynomials

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ABSTRACT

We used the explicit expression of Zernike polynomials in Cartesian coordinates to fit and describe the freeform surface of progressive addition lens (PAL). The derivatives of Zernike polynomials can easily be calculated from the explicit expression and used to calculate the principal curvatures of freeform surface based on differential geometry. The surface spherical power and surface astigmatism of the freeform surface were successfully derived from the principal curvatures. By comparing with the traditional analytical method, Zernike polynomials with order of 20 is sufficient to represent the freeform surface with nanometer accuracy if dense sampling of the original surface is achieved. Therefore, the data files which contain the massive sampling points of the freeform surface for the generation of the trajectory of diamond tool tip required by diamond machine for PAL manufacture can be simplified by using a few Zernike coefficients.

Keywords: freeform, progressive addition lens, Zernike polynomial

1. INTRODUCTION

For presbyopic wearers, the value of the power correction is different for far vision and near vision, due to the difficulties of accommodation in near vision. Lenses suitable for presbyopic wearers are progressive addition lenses (PAL) which can include a complex freeform multifocal surface, for example the surface facing the person wearing the glasses, and a spherical surface on the other side. There are also freeform PAL designs based on the freeform surfaces for both sides of the lens. In order to guarantee the optical performance of PAL which relies on the freeform surface quality, the high accuracy of freeform surface representation is required to generate the diamond tool path trajectory for surface finishing [1]. The direct method to describe the freeform surface is surface sampling with high density, which usually leads to the increasing data redundance and unfavourable data access efficiency in manufacture process. We propose the indirect method which is fitting the freeform surface by Zernike polynomials and saving the coefficients of Zernike polynomials in data file that can be accessed by the control system of diamond turning machine to reconstruct the freeform surface for manufacture. Zernike polynomials have been used to describe the physical dimensions of the freeform PAL surface measured by a coordinate measuring machine (CMM) [2, 3]. However, the accuracy of the surface reconstruction is subject to the limited spatial sampling and accuracy of CMM, which results in the deviation of the optical properties of freeform PAL between the theoretical design and those derived from measured surface.

We discussed the possibility of the freeform surface fitting of PAL by Zernike polynomials to assist the manufacture process. The data sampling was performed on the original freeform surface generated by the software developed for PAL design and fitted with a set of Zernike polynomials using least-square method. The accuracy of freeform fit was analyzed with respect to the number of Zernike basis functions and the size of sampling grid. Surface power map of PAL were

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also derived with the coefficients of Zernike polynomials and compared with the ideal surface power distribution to verify the effectiveness of surface fitting.

2. ZERNIKE POLYNOMIALS IN CARTESIAN COORDINATE

A circular aperture surface can be represented in terms of Zernike polynomials, which is a complete and orthogonal basis over a normalized unit circle, as

$$W(x,y)=\sum_{n,m} C_n^m Z_n^m(x,y) \tag{1}$$

where C_n^m is the Zernike coefficient and $Z_n^m(x,y)$ is Zernike basis function defined in Cartesian coordinates as

For m>0

$$Z_n^m(x,y)=\sqrt{2(n+1)}\sum_{s=0}^{\frac{n-|m|}{2}}\sum_{j=0}^{\frac{n-|m|}{2}-s}\sum_{k=0}^{\frac{|m|}{2}}\frac{(-1)^{s+k}(n-s)!}{s!\left(\frac{n+|m|}{2}-s\right)!\left(\frac{n-|m|}{2}-s\right)!}\binom{\frac{n-|m|}{2}-s}{j}\binom{|m|}{2k}x^{n-2(s+j+k)}y^{2(j+k)} \tag{2}$$

For m<0

$$Z_n^m(x,y)=\sqrt{2(n+1)}\sum_{s=0}^{\frac{n-|m|}{2}}\sum_{j=0}^{\frac{n-|m|}{2}-s}\sum_{k=0}^{\frac{|m|-1}{2}}\frac{(-1)^{s+k}(n-s)!}{s!\left(\frac{n+|m|}{2}-s\right)!\left(\frac{n-|m|}{2}-s\right)!}\binom{\frac{n-|m|}{2}-s}{j}\binom{|m|}{2k+1}x^{n-2(s+j+k)-1}y^{2(j+k)+1} \tag{3}$$

For m=0

$$Z_n^m(x,y)=\sqrt{n+1}\sum_{s=0}^{\frac{n}{2}}\sum_{j=0}^{\frac{n}{2}-s}\frac{(-1)^{s}(n-s)!}{s!\left(\frac{n}{2}-s\right)!\left(\frac{n}{2}-s\right)!}\binom{\frac{n}{2}-s}{j}x^{n-2(s+j)}y^{2j} \tag{4}$$

where m and n are integers with $n \ge m$ and $x^2+y^2 \le 1$. Therefore, the derivatives of Zernike polynomials with x or y can be easily derived as Zernike basis functions are expressed in terms of powers of x and y.

3. PRINCIPAL CURVATURE AND SURFACE POWER

The freeform surface $z=W(x,y)$ of PAL and its partial derivatives are continuous everywhere. At each point p of the freeform surface there must be a unit normal vector. The normal plane at point p will cut the surface in a plane curve. This curve will have different curvatures for different normal planes. The principal curvatures, denoted κ_1 and κ_2, are the maximum and minimum values of this curvature, which are the roots of the following fundamental quadratic equation

$$\left(EG-F^2\right)\kappa^2-\left(EN+GL-2FM\right)\kappa+\left(LN-M^2\right)=0 \tag{5}$$

Where $E=1+W_x^2$, $F=W_xW_y$, $G=1+W_y^2$, $L=\frac{W_{xx}}{\sqrt{1+W_x^2+W_y^2}}$, $M=\frac{W_{xy}}{\sqrt{1+W_x^2+W_y^2}}$ and $N=\frac{W_{yy}}{\sqrt{1+W_x^2+W_y^2}}$

Here the curvature is defined by the reciprocal of the radius of an osculating circle in the normal plane. The directions of the normal plane where the curvature takes its maximum and minimum values are always perpendicular.

The principle curvatures across the lens surface will give two different local surface spherical powers which we call the high power $P_1(x, y)$ and the low power $P_2(x, y)$, respectively

$$P_i(x,y)=1000(1-RI)\cdot\kappa_i(x,y),\quad i=1,2 \tag{6}$$

where RI is the refractive index of the lens material. The radius of principle curvature is measured in millimeters, and $P_i(x, y)$ is measured in diopters. The value of $P_1(x, y)$ is larger than $P_2(x, y)$ at each point on the surface. The difference between the two surface spherical powers is related to the surface cylindrical power or called surface astigmatism

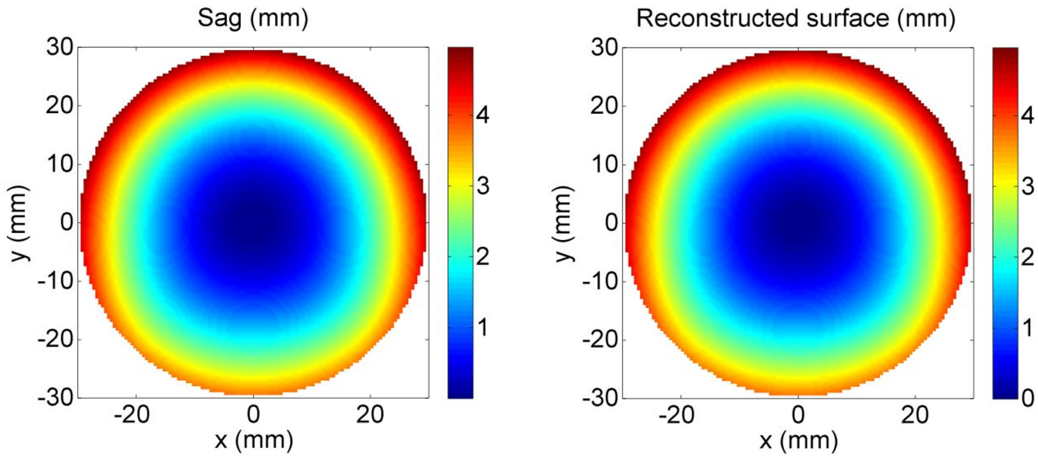
$$A(x,y)=P_2(x,y)-P_1(x,y) \tag{7}$$

where $A(x,y)$ is a negative value in order to be consistent with the conventional sphero-cylindrical notation (in "negative-cylinder") used in clinic.

4. FREEFORM SURFACE DESIGN AND RECONSTRUCTION

PAL was designed on a circular region with the diameter of 60 mm and has the freeform, progressive power surface on the concave face. The lens specification is −2.00 diopters for distance power with a +2.00 diopters add and base curves of +4.00 diopters. The method proposed by Winthrop [4,5] is utilized to design the original analytical freeform surface of PAL. This method first assigns the refractive optical power along the meridian line on the lens, then generates the surface on the lens by prescribing curves which are transverse to the meridian line. The shape of the curves are chosen to have the desired surface curvature which is pre-determined by the required local refractive optical power and described in the conic form

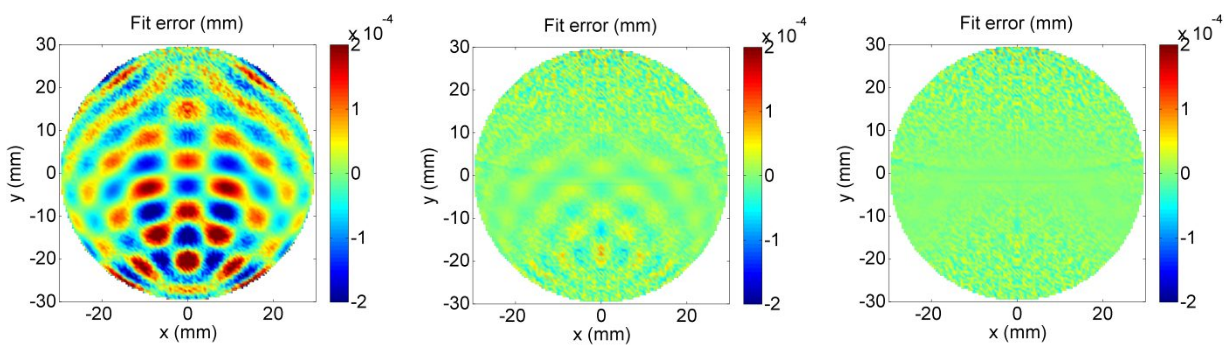
Fig.1(a) shows the sag of the freeform surface designed with this method. Then, the surface was discretized and sampled with equally spaced grid points, which produces data file that consist of x, y, z position. These data were imported into Matlab for freeform surface fitting with Zernike polynomials to generate Zernike coefficients. We have used least-square method in fitting a set of Zernike polynomials to the continuous surface. Then, the Zernike coefficients were used to reconstruct the surface as shown in Fig. 1(b) which was compared with the original surface to get the fit error. Here, the subgrids used in surface sampling were 0.5 mm × 0.5 mm and the Zernike basis functions up through order of N=20 were employed in surface fitting.



(a) (b)

Fig. 1. Freeform surface of the PAL. (a) original analytical surface designed; (b) reconstructed freeform surface.

We carried out the least square fit with increasing numbers of Zernike basis functions and demonstrated the fit error in Fig. 2. When 153 polynomials (N=16) are used in fitting and reconstruction, the local fit error (i.e. difference between the original and the reconstructed surfaces) increases along the corridor line from distance vision region to near vision region. In contrast, the fit error for 325 polynomials (N=24) is minimized and is almost symmetrically distributed about the horizontal axis through the lens center. The high order of Zernike basis functions can help to lower the level of fitting error. Table 1. has summarized the results of RMS fit error. We can find that the effectiveness of surface fitting is improved remarkably when the order of Zernike polynomials increases from 8 to 20. However, very limited improvement can be observed when the order is increased further.



(a) (b) (c)

Fig. 2. Fit error of freeform surface with sampling grid point of 0.5 mm apart. (a) N=16; (b) N=20; (c) N=24.

Table 1. Fit error of freeform surface (0.5×0.5 mm^2 sample subgrid)

N	*Original surface RMS* (mm)	*Reconstruction surface RMS* (mm)	*RMS fit error* (mm)	*Normalized fit error* (%)
8	1.3139	1.3165	2.6000×10^{-3}	2.0000×10^{-1}
12	1.3139	1.3165	4.6195×10^{-4}	3.5159×10^{-2}
16	1.3139	1.3165	8.1975×10^{-5}	6.2390×10^{-3}
20	1.3139	1.3165	2.3628×10^{-5}	1.7983×10^{-3}
24	1.3139	1.3165	2.0849×10^{-5}	1.5868×10^{-3}

The performance of surface fitting with respect to the number of sampling grid points was also evaluated with fixed number of Zernike polynomials as shown in Fig. 3. In comparison to Fig. 2(b), it is obvious that the dense sampling can help to improve the surface fitting accuracy. If we lower the density of sampling, the greatest fitting error will most likely to happen at the peripheral region of the freeform surface, which implies that the nonuniform edge clustered sampling grid pattern [6] may be a better choice for surface fitting than the uniform sampling method. Table 2. has summarized the results of RMS fitting error for different size of sampling subgrid. The effectiveness of surface fitting is improved obviously when the size of sampling subgrid is decreased. However, if the subgrid size is smaller than 1.0×1.0 mm^2, slight improvement can be found.

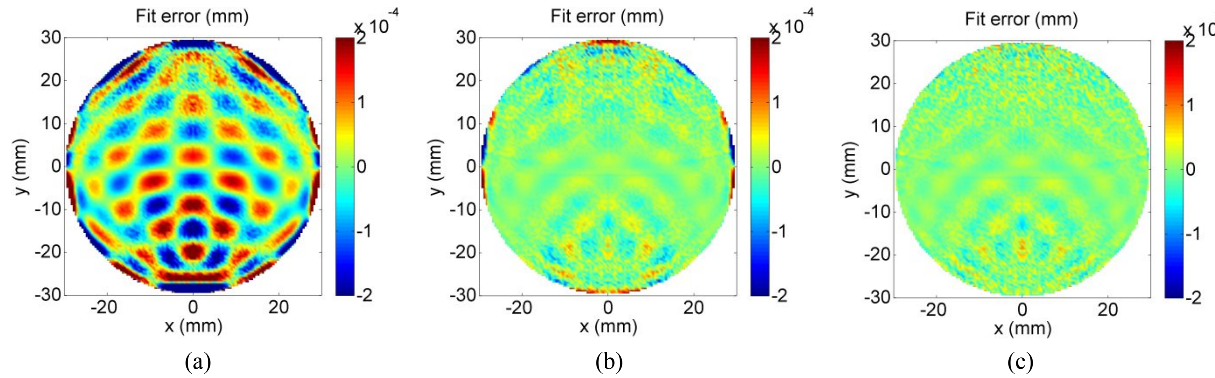


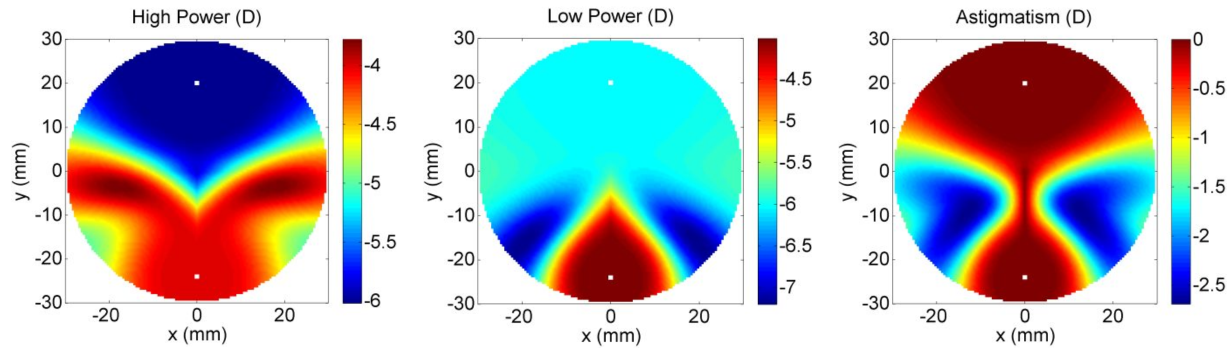
Fig. 3. Fit error of freeform surface with N=20 and sampling grid point of (a) 3.0 mm; (b) 2.0 mm; (c) 1.0 mm apart.

Table 2. Fit error of freeform surface (Zernike polynomials up through 20th order)

Sample subgrid size (mm^2)	*Original surface RMS* (mm)	*Reconstruction surface RMS* (mm)	*RMS fit error* (mm)	*Normalized fit error* (%)
3.0×3.0	1.3303	1.3168	1.9134×10^{-4}	1.4384×10^{-2}
2.0×2.0	1.3201	1.3165	7.0367×10^{-5}	5.3303×10^{-3}
1.0×1.0	1.3130	1.3165	2.4407×10^{-5}	1.8589×10^{-3}
0.5×0.5	1.3139	1.3165	2.3628×10^{-5}	1.7983×10^{-3}

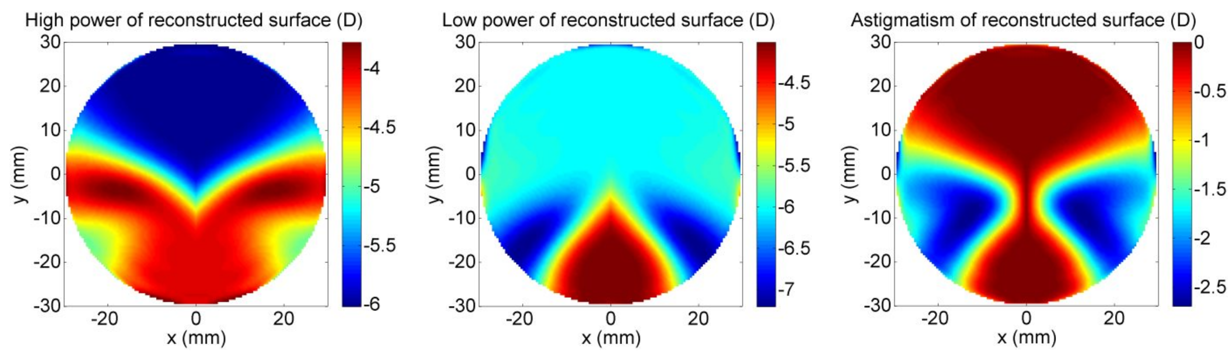
5. ANALYSIS OF SURFACE POWER

In clinic, the characteristics of PAL are represented as spherical power and astigmatism (or local cylindrical power). So, we expanded the study to quantify the power distribution of the reconstructed freeform surface. Here, the value of spherical power and astigmatism were derived from the surface properties, and no adjustment was made for variation in vertex distance and oblique incidence associated with a wearer's eye rotating behind the lens. Fig. 4 shows the surface power including high power, low power and the associated astigmatism derived from the principle curvatures of the analytical freeform surface. The average value of high power and low power results in the equivalent spherical power which is most useful in clinic. The unwanted astigmatism generally increases laterally, away from the progressive corridor where there is less astigmatism. Fig. 5 and Fig. 6 illustrate the effect of sampling on the fidelity of the surface power of the reconstructed freeform surface. We have made use of approximately 709 samples (2.0×2.0 mm^2 sample subgrid) and 2821 samples (1.0×1.0 mm^2 sample subgrid) in Fig. 5 and Fig. 6, respectively and 231 polynomials (N=20) for both cases. The dense sampling in Fig. 6 results in a remarkable improvement on the overall surface power profile. The relation between the number of samples and the order of Zernike polynomials was established as 7×N^2.



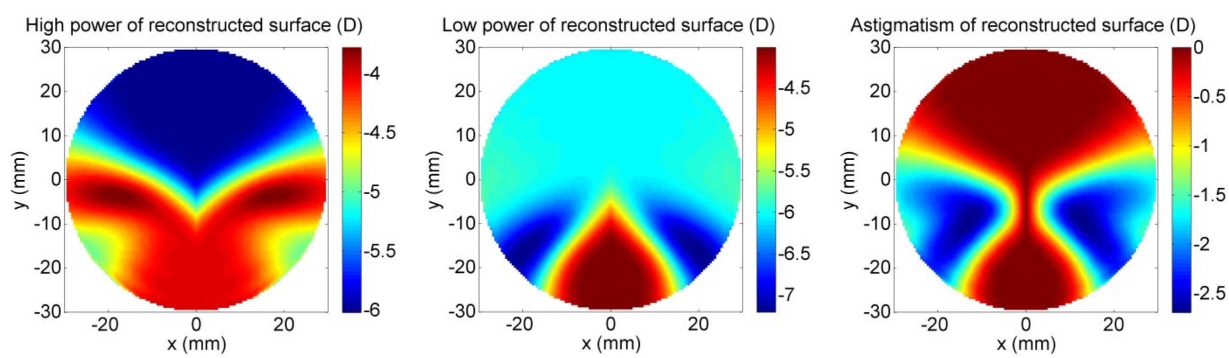
(a) (b) (c)

Fig. 4 Surface power of the original analytical surface designed. (a) High power; (b) Low power; (c)Astigmatism.



(a) (b) (c)

Fig. 5. Surface power of the reconstructed freeform surface with sampling grid point of 2.0 mm apart and N=20. (a) High power; (b) Low power; (c)Astigmatism.



(a) (b) (c)

Fig. 6. Surface power of the reconstructed freeform surface with sampling grid point of 1.0 mm apart and N=20. (a) High power; (b) Low power; (c)Astigmatism.

6. CONCLUSION

In order to achieve an acceptable Zernike polynomial fit with nanometer accuracy to an asymmetric freeform surface of PAL, many terms of Zernike basis function, on the order of hundreds, must be required. We have also observed that sampling density plays an important role in the freeform surface fitting and reconstruction. It is recommended that the freeform surface of PAL should be sampled with subgrid size smaller than $1.0\times1.0\ mm^2$ and fitted with Zernike polynomials up through at least 20th order. We also illustrated the relationship between the surface shape geometry and the surface power gradients across the freeform surface. The representation of freeform surface with Zernike polynomials makes it easy to derive the principle curvatures and the associated surface spherical power and surface astigmatism which is another approach to verify the effectiveness of surface fitting and is believed to be potentially useful for characterizing and designing PALs.

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